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THE ROLE OF GOVERNMENTAL POLICY IN AGRICULTURAL
LAND APPRECIATION AND WEALTH ACCUMULATION

by

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THE ROLE OF GOVERNMENTAL POLICY IN AGRICULTURAL LAND APPRECIATION
AND WEALTH ACCUMULATION

Introduction

In the evaluation of governmental intervention, land and rental markets along with tenant arrangements must be given special attention. Over the last decade, there has been a rapid escalation in farmland prices. For the 1970s, as a whole, the Agriculture Department's index of the price of land rose at an annual rate of 13 percent, nearly double the 7.4 percent annual rise in the general Consumer Price Index. This rapid appreciation has been associated with another emerging phenomenon, namely, the disruption of the traditional unity between ownership and operation of farm units. The average size of production in units increased from 216 acres in 1950 to 390 acres in 1976.

In analyzing agricultural land appreciation and its associated implications for wealth accumulation, the important features of the U.S. agricultural sector must be recognized. Following Shultz and Rausser, Zilberman, and Just, these features include (1) inelastic domestic demand, (2) elastic export demand, (3) competitiveness, (4) asset fixity, (5) rapid technological change, (6) variable asset qualities including human and managerial capital, (7) institutional limits to credit availability, and (8) partial separation of asset ownership and utilization. Much of the variation observed within the agricultural production sector emanates from differences in production techniques, land quality, human capital, and wealth controlled by individual producers. The limitations of credit availability for producers in different size classes have been noted by recent empirical evidence. This evidence suggests that larger farmers borrow more; they borrow more to invest in capital; and their
ability to borrow more stems, in part, from their high repayment capacity (Baker; Quinn; Riboud).

As Harold Carter and Warren Johnston have observed for U. S. agriculture, world credit markets have become an important determinant of redistribution within the U. S. agricultural sector. They have cautioned that the intense pressure toward a heavy reliance of capital markets in order to purchase land and equipment may pose a real threat to the existence of the family farm. The basis for this observation is that:

"the proportion of [farmland] transfers on which debt was incurred rose from 58 percent in 1950 to 88 percent in 1977 and the ratio of debt to purchase price of credit finance transfers rose from 57 percent in 1950 to 77 percent in 1977... [page 74]."

A recent theoretical framework has been advanced by Feldstein to explain the link between general price inflation and the relative price of land. In essence, his general equilibrium framework places relative farmland price appreciation at the door of one form of governmental intervention, namely, U. S. tax laws. Inflation and the tax laws interact to raise the return on land and lower the return on reproducible capital. The culprit is the difference between ordinary income and capital gain tax rates in an inflationary environment.

In addition to U. S. tax laws, a number of other governmental policies play a significant role in land and other agricultural asset markets and, thus, wealth accumulation. These policies include specific sectoral interventions which assumed a number of alternative forms over the post-World War II period including price supports, accumulation of public stocks, acreage set-asides and diversions, deficiency payments, diversion payments, stock-holding
subsidies, and target prices under both mandatory and voluntary participation. Governmental programs have focused on wheat, feedgrains, cotton, and rice as well as a number of other commodities.

Along with sectoral policies, U.S. monetary policy has begun to assume an increasingly important role in the evaluation of assets employed in the agricultural sector. The recent volatility of both short- and long-term interest rates resulting from the change in federal reserve policy in October, 1979, have only begun to be seriously felt in the evaluation of the agricultural asset base. This change, along with other banking deregulations, have eliminated the wedge that has existed over much of the post-World War II period between rural and general economic credit markets (Baker). U.S. farm credit policy implemented by the Federal Land Bank (long-term credit market) and the Production Credit Association (short-term credit market) no longer is able to isolate rural credit from other U.S. credit markets. It is argued here that not only income tax policies but, in addition, monetary policy, agricultural sector policies, and rural credit policies play an important, and often conflicting, role in the formation of land price expectations, land appreciation, and related wealth accumulation in the agricultural sector.

This paper advances a theoretical model for capturing the effects of each of these different forms of governmental intervention. The model assumes that each firm maximizes its expected net wealth, period by period, where changes in wealth are affected by farming operations, capital gains on land assets, capital gains on alternative investments, debt payments on both operating capital and land capital, and the rate of taxation on these various forms of gains and losses. The resources of each firm consist of cash on hand or an alternative liquid assets, owned land, and credit availability which depends
on the farm's asset position. Each firm makes decisions regarding how much land to buy or sell, how much land to rent, how much debt to carry on both utilization and ownership, how much to invest outside of agriculture, and whether to voluntarily participate in government agricultural programs.

The sectoral policies included in the model are much like those instituted during the 1970s and consist of a subsidy or deficiency payment tied to either individual or regional production norms, along with the associated set-aside requirements. With some alternative interpretations, the framework also lends itself to analysis of price-support and diversion policies. Each firm is assumed to face uncertainty with risk neutrality and diversified land price expectations.
II. The Model of the Individual Decision-Maker

Assume that I individuals are either active or potential holders of agricultural land and are denoted by \( i = 1, \ldots, I \). Suppose that each holds the objective of maximizing its expected annual gains in wealth from ownership and/or operation,

\[
G_i = T_i + C_i - F_i(T_i + \tau C_i),
\]

(1)

where

\( T_i \) = expected net taxable income,

\( C_i \) = expected capital gains,

\( \tau \) = proportion of capital gains which are taxable, and

\( F_i \) = a linear tax function associated with the marginal tax bracket for individual \( i \), \( F_i(T_i + \tau C_i) = f^0_i + f'_i(T_i + \tau C_i) \).

Capital gains consist of expected appreciation in the value of owned land,

\[
C_i = (W_i - W) L_i,
\]

(2)

where

\( W_i \) = land prices at the end of the period expected by agent \( i \),

\( W \) = land prices at the beginning of the period, and

\( L_i \) = land owned by individual \( i \) after current land transactions.

Consider the general case with \( J \) types or qualities of land where

\[
W_i^* = (W_{i1}^*, \ldots, W_{iJ}^*),
\]

\[
W = (W_1, \ldots, W_J),
\]

and

\[
L_i = (L_{i1}, \ldots, L_{iJ})'.
\]
Suppose the land qualities are ordered according to profitability so that quality 1 is the poorest land and quality \( J \) is the highest quality.

Suppose that land prices expected at the end of the period possibly depend on current land prices,

\[
W_{i}^{+} = W_{i}^{0} + \psi_{i} W_{i},
\]

(2')

where \( \psi_{i} \) is a scalar parameter reflecting the rate by which individual \( i \) revises his expectations in response to current land price adjustments. For example, if \( W_{i}^{0} = 0 \) and \( \psi_{i} = 1 \), then individual \( i \) myopically assumes that land prices will not change; if \( \psi_{i} = 0 \), then individual \( i \) does not adjust his land price expectations from \( W_{i}^{0} \) as current land prices adjust.

Expected net taxable income consists of expected income from farming plus rental income plus net interest income (expense). Specifically,

\[
T_{i} = (\pi_{i}+ \lambda_{i} \gamma) A_{i} - RZ_{i} - \theta_{i} \check{D}_{i} - \bar{\theta}_{i} \check{D}_{i} + \bar{\theta}_{i} H_{i},
\]

(3)

where

\[ \pi_{i} = (\pi_{i1} \ldots , \pi_{ij}) \] is a vector of expected quasi-rents per acre for individual \( i \) associated with various land qualities which account for differences in human capital among individuals \( (\pi_{i1} < \ldots < \pi_{ij}) \),

\[ \lambda_{i} \] is a scalar variable indicating participation in a government price support and/or diversion program \( (\lambda_{i} = 1 \text{ for participation and } \lambda_{i} = 0 \text{ for nonparticipation}) \),

\[ \gamma = (\gamma_{i1} \ldots , \gamma_{ij}) \] is a vector of expected government payments per acre associated with various land qualities for individual \( i \),

\[ A_{i} = (A_{i1} \ldots , A_{ij})' \] is a vector of acreages of various qualities utilized by individual \( i \) for production,
\( R = (R_1, \ldots, R_J) \), a vector of rental rates on lands of various qualities paid at the beginning of the production period,

\( Z_i = (Z_{i1}, \ldots, Z_{ij}) \), a vector of net rentals of various land qualities by individual \( i \) \((Z_{ij} > 0\) implies obtaining the use of land through leasing from someone else, while \( Z_{ij} < 0\) implies renting the use of land to someone else),

\( \theta_i \), a scalar parameter representing the long-term interest rate on land debt for individual \( i \),

\( D_i \), a scalar variable representing the (land) debt accumulated by individual \( i \) after current land transactions,

\( \hat{\theta}_i \), a scalar parameter representing the short-term interest rate on operating debt for individual \( i \),

\( \hat{D}_i \), a scalar variable representing short-term operating debt carried through the growing season by individual \( i \),

\( \hat{\theta}_i \), an opportunity return on funds (e.g., the rate of interest on savings or alternative liquid investments) for individual \( i \),

and

\( H_i \), liquid reserves carried by individual \( i \) after current land transactions and expenses at the beginning of the current growing season.

Each farmer faces several major constraints. The utilization constraint implies that a farmer cannot utilize more land than he controls through ownership and rentals,

\[ A_i + V_i = L_i + Z_i, \quad (4) \]
where

\[ V_i = (V_{i1}, \ldots, V_{ij}) = \text{a vector of acreages of various qualities diverted or idled by individual } i. \]

The rental constraint implies that a farmer cannot rent more land than he owns,

\[ -z_i \leq L_i. \tag{5} \]

The sale constraint implies that a farmer cannot sell more land than he owns,

\[ L_i \geq 0. \tag{6} \]

The inequalities in (4) through (6) characterize the physical constraints on land.

- The long-term credit constraint implies that a farmer can borrow against his land but only up to a fixed ratio, \( \rho \),

\[ D_i \leq \rho W L_i. \tag{7} \]

The short-term credit constraint implies that a farmer can borrow up to some fixed proportion, \( \delta \), of the cost of planting and growing a crop,

\[ \delta_i \leq \delta u_i A_i. \tag{8} \]

where

\[ u_i = (u_{i1}, \ldots, u_{ij}) = \text{a vector of operating capital requirements (the cost of planting and growing a crop to maturity) per acre associated with various land qualities for individual } i. \]
Land transactions and operating capital can be financed by either cash or debt as implied by the transaction identity,

\[ W(L_i - L_i^0) + \mu_i A_i + RZ_i = (D_i - D_i^0) + \hat{D}_i - (H_i - H_i^0), \quad (9) \]

where \( L_i^0, D_i^0, \) and \( H_i^0 \) represent land holdings, land debt, and cash carried over from the previous decision period, respectively. In addition, each farmer faces physical financial constraints that debt and cash on hand must be nonnegative,

\[ D_i \geq 0, \quad \hat{D}_i \geq 0, \quad H_i \geq 0. \quad (10) \]

Finally, in the event of government program participation \( (\lambda_i = 1) \), a farmer must consider the associated diversion constraint,

\[ eV_i \geq \omega A_i, \quad (11) \]

where \( e = (1, \ldots, 1) \) and \( \omega \) is the amount of land which must be diverted under program participation for each acre utilized, \( 0 < \omega < 1. \)

The individual's problem is, thus, to maximize the objective function in (1) subject to the constraints in (4) through (11) using decision variables \( A_i, Z_i, L_i, V_i, D_i, \hat{D}_i, H_i, \) and \( \lambda_i. \)

III. Individual Behavior

Before proceeding to delve into issues of wealth accumulation and land prices, a few results are needed regarding how individuals use accumulated land and wealth. First, note that, without participation in the government
program, the optimal choice of \( V_i \) will be \( V_i = 0 \) if \( R > 0 \) (all land will either be utilized or rented out) and, hence, both \( V_i \) and \( Z_i \) may be eliminated as decision variables \([Z_i \text{ is determined by } A_i \text{ and } L_i \text{ in (4)}]\). Alternatively, with government participation, \( Z_i \) may be eliminated as a decision variable since it is determined in (4) by \( A_i, L_i, \text{ and } V_i \). Furthermore, a farmer will always select the land quality with the lowest rental rate, say, \( j = 1 \), to satisfy diversion requirements and, hence,

\[
V_{i2} = \ldots = V_{ij} = 0.
\]  

(12)

To see this, note that a farmer is obtaining diversion land by means of leasing which minimizes his cost through selecting the lowest rental rate; alternatively, a farmer who owns only land with a higher rental rate would always be better off to rent out some of his land and lease diversion land with the lowest rental rate from someone else. Next, note that no more land will be diverted than required under the government program if \( R > 0 \) so that (13) implies

\[
V_{i1} = \omega \sum_{j=1}^{J} A_{ij}.
\]  

(13)

Having eliminated \( Z_i \) and \( V_i \) as decisions using (4), (12), and (13), the individual decision-maker's problem can be further simplified by considering an equivalent problem with decision variables \( \bar{A}_i, \bar{L}_i, \hat{A}_i, \hat{L}_i, \text{ and } H \), where

\[
\bar{A}_i = (\bar{A}_{i1}, \ldots, \bar{A}_{ij}) = \text{a vector of acreages of various qualities utilized by individual } i \text{ which are not used as collateral in raising short-term operating capital,}
\]
\( \mathbf{\tilde{L}}_i = (\mathbf{\tilde{L}}_{i1}, \ldots, \mathbf{\tilde{L}}_{iJ}) \) = a vector of acreages of various qualities owned by individual \( i \) which are not used as collateral in raising long-term land-investment capital,

\( \mathbf{\hat{A}}_i = (\mathbf{\hat{A}}_{i1}, \ldots, \mathbf{\hat{A}}_{iJ}) \) = a vector of acreages of various qualities utilized by individual \( i \) which are used as collateral in raising short-term operating capital,

and

\( \mathbf{\mathcal{L}}_i = (\mathbf{\mathcal{L}}_{i1}, \ldots, \mathbf{\mathcal{L}}_{iJ}) \) = a vector of acreages of various qualities owned by individual \( i \) which are used as collateral in raising long-term land-investment capital.

Thus,

\[
\mathbf{A}_i = \mathbf{\tilde{A}}_i + \mathbf{\hat{A}}_i,
\]

\[
\mathbf{L}_i = \mathbf{\mathcal{L}}_i + \mathbf{\hat{L}}_i,
\]

\[
\mathbf{D}_i = \kappa \mathbf{\mathcal{L}}_i,
\]

and

\[
\mathbf{\hat{D}}_i = \kappa \mathbf{\mathcal{L}}_i.
\]

Using these relationships together with (4), (12), and (13), the problem in (1) through (11) can be rewritten as

\[
\max_{\mathbf{\tilde{A}}_i, \mathbf{\mathcal{L}}_i, \mathbf{\hat{A}}_i, \mathbf{\mathcal{L}}_i, \mathbf{H}_i, \lambda_i} \quad G_i = (1 - f_i) \left( \pi_i + \lambda_i \gamma_i - R - \lambda_i \kappa R_{\text{we}} \right) \mathbf{\tilde{A}}_i + (1 - f_i) \left( \pi_i + \lambda_i \gamma_i - R - \lambda_i \kappa R_{\text{we}} - \theta_i \lambda_i \right) \mathbf{\hat{A}}_i + \left[ (1 - f_i) R + (1 - f_i \tau) (W_i^T - W) \right] \mathbf{\mathcal{L}}_i \quad \text{ (15)}
\]

\[
+ \left[ (1 - f_i) (R - \theta_i \kappa W) + (1 - f_i \tau) (W_i^T - W) \right] \mathbf{\hat{L}}_i + (1 - f_i) \theta_i H_i - f_i^0
\]
subject to

\[ (\mu_i + R + \lambda_i \overline{R}_w) \hat{A}_i + (\mu_i + R + \lambda_i \overline{R}_w - \delta \mu_i) \hat{A}_i \]
\[ + (W - R) \bar{L}_i + (W - R - \omega W) \hat{L}_i + H_i E_i \]

\[ \Lambda = \hat{\Lambda}, \bar{A}_i, \hat{L}_i, \bar{L}_i, H_i \geq 0, \quad \lambda = 0, 1 \]

(17)

where \( E_i \) is initial wealth

\[ E_i = W_i^0 + H_i^0 - D_i^0. \]

The constraint in (15) follows from (9) upon substituting (4), (12), (13), and (14) while all other constraints in (5) through (11) are satisfied by (14) and (17).

For given \( \lambda_i \), the problem in (15) through (17) is a portfolio choice problem under risk neutrality for which the rates of return per dollar of investment on \( \hat{A}_i, \bar{A}_i, \bar{L}_i, \hat{L}_i, \) and \( H_i \) are

\[ r_{1ij}^\lambda = \frac{(1 - f_i) \left( \pi_{ij} + \lambda_i \gamma_{ij} - R_j - \lambda_i \overline{R}_w \right)}{\mu_{ij} + R_j + \lambda_i \overline{R}_w} \quad j = 1, \ldots, J, \]  

(18)

\[ r_{2ij}^\lambda = \frac{(1 - f_i) \left( \pi_{ij} + \lambda_i \gamma_{ij} - R_j - \lambda_i \overline{R}_w - \delta \lambda \delta \mu_{ij} \right)}{\mu_{ij} + R_j + \lambda_i \overline{R}_w - \delta \mu_{ij}} \quad j = 1, \ldots, J, \]  

(19)

\[ r_{3ij}^\lambda = \frac{(1 - f_i) R_j + (1 - f_i \tau) \left( W_{ij}^* - W_j \right)}{W_j - R_j} \quad j = 1, \ldots, J, \]  

(20)
\[ r_{4ij} = \frac{(1 - f_i)(R_j - \omega_i\theta_j) + (1 - f_i\tau)(W_j - W_j)}{W_j - R_j - \omega_j} \quad j = 1, \ldots, J, \quad (21) \]

and

\[ r_{5i} = (1 - f_i) \bar{o}_i. \quad (22) \]

These rates of return are derived as the coefficients in the objective function divided by the corresponding coefficients in the constraint.

Note that \( r_{1ij}^\lambda \) reflects the expected after-tax rate of return (including any government-program payments) on operating capital used in farming land type \( j \) for individual \( i \); \( r_{3ij} \) reflects the expected after-tax rate of return on investment in land type \( j \) after correcting for capital gains tax advantages for individual \( i \); \( r_{2ij}^\lambda \) reflects the expected rate of return (including government-program payments) on operating capital used in farming land type \( j \) after further considering the collateral effects of additional acreage for individual \( i \); and, finally, \( r_{4ij} \) reflects the expected rate of return on type \( j \) land investment after further considering the land purchase possibilities associated with increased collateral for individual \( i \). To further understand this interpretation, suppose one invests \$1.00 in farming operations for land type \( j \) thus earning a direct return of \( r_{1ij}^\lambda \). One can use only \( u_{ij}/(u_{ij} + R_j + \lambda_i R_i) \) of this investment as collateral (recall that rental payments cannot serve as collateral under the assumptions of this paper). If the debt constraint is binding, this increases the debt ceiling by \( \rho u_{ij}/(u_{ij} + R_j + \lambda_i R_i) \) which, if used for farming land type \( j \), returns \([r_{1ij} - \bar{o}_i(1 - f_i)] \rho u_{ij}/(u_{ij} + R_j + \lambda_i R_i)\) after tax and interest expenses. But, this additional investment further increases the debt ceiling by \( \rho^2 u_{ij}^2/(u_{ij} + R_j + \lambda_i R_i)^2 \), and so on. Thus, the rate of return on the original dollar after considering the collateral effects is
\[
\sum_{k=1}^{\infty} \left( \frac{e^{u_{ij}}}{v_{ij} + R_j + \lambda_i R_W} \right)^k \left[ r_1^{ij} - \hat{e}_i (1 - f_i) \right]
\]  

A similar explanation implies

\[
\sum_{k=1}^{\infty} \left( \frac{e^{W_j}}{W_j - R_j} \right)^k \left[ r_3^{ij} - \hat{e}_i (1 - f_i) \right]
\]

If any one of the rates of return in (18) through (22) dominates all others, then the individual will allocate all his initial wealth to that activity. If any subset of these rates of return are equal and dominate all others, then expected gains in wealth are maximized by choosing any combination of that subset of activities which just exhausts initial wealth.

Next, consider the participation choice. Since the optimal gains without participation are max \( \{ r_1^{ij}, r_2^{ij}, r_3^{ij}, r_4^{ij}, r_5^{ij}, j = 1, \ldots, J \} \cdot E_i \) and the optimal gains with participation are max \( \{ r_1^{ij}, r_2^{ij}, r_3^{ij}, r_4^{ij}, r_5^{ij}, j = 1, \ldots, J \} \cdot E_i \), the optimal gains are max \( \{ r_1^{ij}, r_2^{ij}, r_3^{ij}, r_4^{ij}, r_5^{ij}, j = 1, \ldots, J \} \cdot E_i \) and \( \lambda_i = 0 \) if \( r_1^{ij} \) or \( r_2^{ij} \) provides the maximum and \( \lambda_i = 1 \) if \( r_1^{ij} \) or \( r_2^{ij} \) provides the maximum.

These results obtain Propositions 1 through 7. For the purpose of stating these results, let \( \mathcal{A}_i \) be the set of rates of return of interest to the decision-maker, \( \mathcal{A}_i = \{ r_1^{ij}, r_2^{ij}, r_3^{ij}, r_4^{ij}, r_5^{ij}, j = 1, \ldots, J \} \).
PROPOSITION 1: Suppose $r_{1ij^*}^0$ provides a unique maximum of $J_i$. Then, the individual will sell all (hold no) land, retire all land debt, channel all liquid assets into farming operation, take no operating debt, and rent and farm without government program participation just as much land of quality $j^*$ as for which his assets are sufficient to cover operating capital needs,

$$A_{ij^*} = Z_{ij^*} = \frac{E_i}{\mu_{ij^*} + R_{j^*}}$$

$$A_{ij} = Z_{ij} = 0, \quad j \neq j^*$$

and

$$L_{ij} = D_i = \hat{D}_i = H_i = 0, \quad j = 1, ..., J.$$

PROPOSITION 2: Suppose $r_{1ij^*}^1$ provides a unique maximum of $J_i$. Then, the individual will sell all (hold no) land, retire all land debt, channel all liquid assets into farming operation, take no operating debt, and rent and farm under government program participation just as much land of quality $j^*$ as for which his assets are sufficient to cover operating capital needs,

$$A_{ij^*} = Z_{ij^*} = \frac{E_i}{\mu_{ij^*} + R_{j^*} + R_o}$$

$$A_{ij} = Z_{ij} = 0, \quad j \neq j^*$$

and

$$L_{ij} = D_i = \hat{D}_i = H_i = 0, \quad j = 1, ..., J.$$
PROPOSITION 3: Suppose \( r_{2ij^*} \) provides a unique maximum of \( \lambda_i \).

Then, the individual will sell all (hold no) land, retire all land debt, channel all liquid assets into farming operation, borrow as much money for farming operation as possible, and rent and farm without government program participation just as much land of quality \( j^* \) as for which his assets are sufficient to cover operating capital needs,

\[
A_{ij^*} = Z_{ij} = \frac{E_i}{\nu_{ij^*} + R_{j^*} - ãu_{ij^*}}
\]

\[
\hat{D}_i = \frac{\hat{u}_{ij^*}E_i}{\nu_{ij^*} + R_{j^*} - ãu_{ij^*}}
\]

\[
A_{ij} = Z_{ij} = 0, \quad j \neq j^*
\]

and

\[
L_{ij} = D_i = H_i = 0, \quad j = 1, \ldots, J.
\]

PROPOSITION 4: Suppose \( r_{2ij^*}^l \) provides a unique maximum of \( \lambda_i \).

Then, the individual will sell all (hold no) land, retire all land debt, channel all liquid assets into farming operation, borrow as much money for farming operation as possible, and rent and farm under government program participation just as much land of quality \( j^* \) as for which his assets are sufficient to cover operating capital needs,
\[ A_{ij}^* = Z_{ij}^* = \frac{E_i}{\mu_{ij}^* + R_{ij}^* + K_{ij}^*} \]

\[ \hat{D}_i = \frac{\hat{\sigma}_{ij}^* E_i}{\mu_{ij}^* + R_{ij}^* + K_{ij}^*} \]

\[ A_{ij} = Z_{ij} = 0, \quad j \neq j^* \]

and

\[ L_{ij} = D_i = H_i = 0, \quad j = 1, \ldots, J. \]

PROPOSITION 5: Suppose \( r_{3ij}^* \) provides a unique maximum of \( \Delta_j \).

Then, the individual will discontinue any farming operations, hold no operating debt, channel all his liquid assets into land investment, retire all (hold no) land debt, and purchase and rent out as much land of quality \( j^* \) as for which his liquid assets are sufficient after retiring all debt,

\[ L_{ij}^* = -Z_{ij}^* = \frac{E_i}{W_{ij}^* - R_{ij}^*} \]

\[ L_{ij} = Z_{ij} = 0, \quad j \neq j^* \]

and

\[ A_{ij} = D_i = H_i = 0, \quad j = 1, \ldots, J. \]

PROPOSITION 6: Suppose \( r_{4ij}^* \) provides a unique maximum of \( \Delta_j \).

Then, the individual will discontinue any farming operations, retire any operating debt, channel all his liquid assets into land investment, borrow as
much money for land purchases as possible, and both purchase and rent out as much land of quality $j^*$ as for which his liquid assets plus mortgage debt ceiling are sufficient to cover purchases,

$$L_{ij^*} = -Z_{ij^*} = \frac{E_i}{W_{j^*} - R_{j^*} - \rho W_{j^*}}$$

$$D_i = \frac{\rho W_{j^*} E_i}{W_{j^*} - R_{j^*} - \rho W_{j^*}}$$

$$L_{ij} = Z_{ij} = 0, \quad j \neq j^*$$

and

$$A_{ij} = \hat{D}_i = H_i = 0, \quad j = 1, \ldots, J.$$

**PROPOSITION 7:** Suppose $r_{5i}$ provides a unique maximum of $\mathcal{D}_i$.

Then, an individual will sell all (hold no) land, retire any debt, and cease any farming operations; in other words, the individual will channel all net worth into nonagricultural uses,

$$H_i = E_i$$

$$L_{ij} = A_{ij} = Z_{ij} = D_i = \hat{D}_i = 0, \quad j = 1, \ldots, J.$$

**PROPOSITION 8:** A farmer who farms land quality $j$ will (not) participate in a voluntary government program if the expected rate of return from participation is greater (less) than the expected rate of return from farming operation, i.e., if
\[
\frac{\gamma_{ij} - \bar{r}_w}{\bar{r}_w} (1 - f_i) > (<) \max \left\{ r_{1ij}^0, r_{2ij}^0 \right\}.
\]

PROOF: Immediate from equations (18) and (19) and Propositions 1 through 4.

Propositions 1 through 8 jointly give the plausible result that an individual will channel his net worth into the alternative yielding the highest rate of return. These results are summarized in Table 1. Of course, these solutions are only the corner solutions. When several activities yield (the same) maximum rate of return, then (the same) optimum annual gains in Table 1 not involving both participation and nonparticipation.

Based on Propositions 1 through 8 and the underlying definitions in (18) through (22), the results for individuals which are immediately apparent can be summarized as follows:

1. Farmers will tend to divert the lowest quality land available for purposes of satisfying the diversion or set-aside requirements of government programs.

2. Farmers participating in government programs will tend to utilize higher quality land in operation than used by those farmers not participating in government programs.

3. Good farmers who can expect relatively higher returns from farming operations will tend to increase their involvement in farming operations, reduce their involvement in land investment, and reduce their investment outside agriculture while renting land from others to expand their farming operation.

4. Individuals who expect a higher appreciation of agricultural land values will tend to increase their investment in agricultural land and to
reduce their involvement in farming operations and in investments other than agriculture and, thus, to rent the land to others for farming operation.

5. Individuals who expect a higher rate of return on investment outside agriculture will reduce their involvement in farming operations and agricultural landownership to increase investment outside agriculture.

6. Better farmers who expect relatively greater returns from farming operation will tend to finance more of their operations through the use of operating capital.

7. Individuals who expect higher appreciation of land prices will tend to finance more of their land purchases through credit.

8. Individuals in higher tax brackets will tend to invest more in agricultural land to take advantage of capital gains tax breaks.

These results basically determine individual demand curves for land and credit. For the purposes of discussing the graphical properties of these relationships, define the opportunity cost of farming land type j for individual i,

$$\eta_{ij} = \max \left\{ \max_{j \neq j'} r_{1ij}'', \max_{j \neq j'} r_{2ij}'', \max_{j \neq j'} r_{3ij}', \max_{j \neq j'} r_{4ij}'', r_{5ij}' \right\}$$

and the opportunity cost of owning land of type j for individual i,

$$\nu_{ij} = \max \left\{ \max_{j \neq j'} r_{1ij}', \max_{j \neq j'} r_{2ij}', \max_{j \neq j'} r_{3ij}', \max_{j \neq j'} r_{4ij}', r_{5ij}' \right\}$$

Next, define the reservation rent for type j land by individual i as the highest rental rate at which individual i will lease (and farm) land type j.
| Unique maximizer | $A_{ij}$ | $A_{ij}(j 
eq j^*; j^* > R)$ | $Z_{ij}(j; R_{j} > R)$ | $Z_{ij}(j; R_{j} = R)$ | $L_{ij}$ | $L_{ij}(j) = j^*$ |
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Note: Blanks indicate no data available.
This is the rental rate, \( R_j \) which just equates \( \max(\max(\lambda_{r_{1ij}}, \lambda_{r_{2ij}})) \) with \( n_{ij} \). Since \( \lambda_{r_{2ij}} \) is a weighted average of \( \lambda_{r_{1ij}} \) and \( (1 - f_i) \), this yields \( \lambda_{r_{1ij}} \) as \( \lambda_{r_{2ij}} \leq (1 - f_i) \).

Thus, the reservation rent is

\[
\hat{R}_{ij} = \begin{cases} 
\max_{\lambda} \frac{(1 - f_i)(\pi_{ij} + \lambda\eta_{ij} - \lambda\bar{R}_o) - n_{ij}(u_{ij} + \lambda\bar{R}_w)}{(1 - f_i) + n_{ij}} & \text{if } n_{ij} \leq (1 - f_i) \theta \\
\max_{\lambda} \frac{(1 - f_i)(\pi_{ij} + \lambda\eta_{ij} - \lambda\bar{R}_w - \theta\phi_{ij}) - n_{ij}(u_{ij} + \lambda\bar{R}_w - \theta\phi_{ij})}{(1 - f_i) + n_{ij}} & \text{if } n_{ij} \geq (1 - f_i) \theta 
\end{cases}
\]

(25)

Similarly, the reservation price for type \( j \) land by individual \( i \) is the land price which just equates \( \max(\lambda_{r_{3ij}}, \lambda_{r_{4ij}}) \) with \( v_{ij} \) and is defined by

\[
W_{ij} = \frac{R_j(v_{ij} + 1 - f_i) + W_{ij}^0(1 - f_i \tau_i)}{v_{ij} + (1 - f_i \tau_i) (1 - \psi_i)} \quad \text{if } v_{ij} \leq (1 - f_i) \theta \\
\frac{R_j(v_{ij} + 1 - f_i) + W_{ij}^0(1 - f_i \tau_i)}{v_{ij}(1 - \rho) + (1 - f_i) \theta \phi + (1 - f_i \tau_i) (1 - \psi_i)} \quad \text{if } v_{ij} \geq (1 - f_i) \theta 
\]

(26)

assuming that the relevant denominator in (26) is positive (recall from (2')) that \( W^*_{ij} = W_{ij}^0 + \psi_i W_{ij} \) and note using (24) that \( \lambda_{r_{3ij}} \leq \lambda_{r_{4ij}} \) as \( \lambda_{r_{3ij}} \leq (1 - f_i) \).

From (25), the individual's demand for rental land of type \( j \) for utilization is a kinky function such as depicted in Figure 1. Above the individual's reservation rent, \( \hat{R}_{ij} \), he leases no land. Below the reservation rent, he leases a quantity of land type \( j \) given by Proposition 1.
Figure 1: Individual Demand for Rental Land
if \((1 - f_i)\hat{\Theta} > r_{1ij}^0 > r_{1ij}^1\), by Proposition 2 if \((1 - f_i)\hat{\Theta} > r_{1ij}^1 > r_{1ij}^0\), by Proposition 3 if \(r_{2ij}^0 > r_{2ij}^1 (1 - f_i)\hat{\Theta}\), and by Proposition 4 if \(r_{2ij}^1 > r_{2ij}^0 (1 - f_i)\hat{\Theta}\). Thus, the demand for rental land for utilization is initially flat at \(\hat{R}_{ij}\) and then is downward sloping as in the case of \(Z_{1ij}\) in Figure 1 (note that each nonzero \(A_{ij}\) in Table 1 is a rectangular hyperbola). Note, however, that as \(R_j\) declines, the quantity of rental land demanded may change abruptly as the conditions of an alternative proposition become applicable thus giving rise to demand curves such as \(Z_{2j}\), \(Z_{3j}\), \(Z_{4j}\), and \(Z_{5j}\) in Figure 1. The changes that can possibly occur as \(R_j\) declines are from nonborrowing to borrowing (from Propositions 1 through 3 or 2 through 4) and from participation to non-participation (from Propositions 2 to 1 or 4 to 3). To see that a change from borrowing to nonborrowing is not possible, note that \(r_{2ij}^\lambda\) increases as the rental rate declines and; hence, if \(r_{2ij}^\lambda > (1 - f_i)\hat{\Theta}\), it cannot fall below \(\hat{\Theta}\). To see that a change from nonparticipation to participation is not possible, note that \(r_{1ij}^0\) and \(r_{2ij}^0\) increases \(R_j\) declines and; hence, once they become greater than \((1 - f) (y_{ij} - \bar{R}_w)/(\bar{R}_w)\) in Proposition 8, they can never fall below. These results, together with Table 1, imply that the individual rental quantity demanded for utilization is strictly decreasing in the rental rate. The switch points, \(R_{ij}^*\) and \(R_{ij}^0\), in Figure 1 can be found by equating the conditions of the Propositions between which the switches are taking place. This yields the switch point from nonborrowing to borrowing,

\[
R_{ij}^* = \max_{\lambda} \frac{\left(\gamma_{ij} + \lambda y_{ij} - \lambda \bar{R}_w\right) - \hat{\Theta}(\alpha_{ij} + \lambda \bar{R}_w)}{1 + \hat{\Theta}}
\]

(27)
and the switch point from participation to nonparticipation,

\[
\mathcal{R}^0_{ij} = \begin{cases} 
\pi_{ij} \mathcal{R}_w - (\gamma_{ij} - \mathcal{R}_w) v_{ij} & \text{if } r^0_{1ij} \leq (1 - f_i)\theta \\
\pi_{ij} \mathcal{R}_w - (\gamma_{ij} - \mathcal{R}_w) v_{ij}(1 - \rho) & \text{if } r^0_{1ij} > (1 - f_i)\theta
\end{cases}
\]  

(28)

Similarly, from (26), the individual's demand for ownership of land type \( j \) is also a kinky function. Above his reservation price, \( \hat{W}_{ij} \), an individual will demand no land. Below his reservation price, he demands a quantity of land given by Proposition 3 if \( (1 - f_i)\theta > r_{3ij} (> r_{4ij}) \) and by Proposition 4 if \( (1 - f_i)\theta < r_{3ij} (< r_{4ij}) \). Thus, the demand for land is initially flat by \( \hat{W}_{ij} \) and then is downward sloping just as in the case of rental demand. As \( W_j \) declines, however, nonborrowing behavior may switch to borrowing behavior (thus leading to a second flat segment) at the point

\[
\hat{W}^*_{ij} = \frac{R_j (v_{ij} + 1 - f_i) + W^0_{ij} (1 - f_i \theta)}{(1 - f_i)\theta + (1 - f_i \theta) (1 - \psi_i)}
\]  

(29)

(found by equating \( r_{3ij} \) and \( r_{4ij} \)). Since demand with borrowing is greater than without, the quantity of land type \( j \) demanded is strictly decreasing in its price.

In a similar manner, one can derive the demand for diversion rental land for which a reservation rental rate can be determined, as in deriving the demand for rental land for utilization; and a second flat segment in the
demand curve can occur in switching from participation without borrowing to participation with borrowing. Between and below these two flat segments, the demand for diversion rental land is declining according to the hyperbolas in \( R \) given in the fourth column of Table 1. Also, one can determine the individual demand for short-term operating credit and for long-term land credit. In this case the demands will be pure step functions since the quantities of credit in Table 1 do not depend on the respective interest rates. Steps will occur in these functions in switching from nonborrowing to borrowing and then possibly in switching between land types or between participation and nonparticipation. With some tedious work, these demands can be shown to be nondecreasing in interest rates under the assumptions above.

IV. Agricultural and Land Market Equilibrium

Given behavior by a large number of individuals as described in Section III, market relationships for supply and demand of landownership and land rental services and for supply of agricultural products can be determined by aggregation. The aggregate demand for landownership is \( L^D = \sum_{j=1}^{I} L_j \), which in equilibrium must satisfy

\[
L^D = \bar{L} \tag{30}
\]

where \( \bar{L} \) is a vector giving the total quantity of land of each quality.

Similarly, the aggregate excess demand for land rental is given by \( \sum_{i=1}^{I} Z_i \), which in equilibrium must satisfy

\[
\sum_{i=1}^{I} Z_i = 0. \tag{31}
\]
Finally, the physical constraints on utilization imply

$$\sum_{i=1}^{I} (A_i + V_i) = C. \tag{32}$$

Note, however, that the condition in either (31) or (32) is redundant since all individuals satisfy (4); hence, the condition in (31) will not be examined further. Equations (30) and (31) are sufficient in principle to determine land prices $W$ and rental rates $R$ given land price expectations $W_i^*$, quasi-rent expectations $\pi_i$, and government payment expectations, $v_i$ for $i = 1, \ldots, I$.

The aggregate demand for short-term operating capital is

$$\hat{K}^D(\hat{e}) = \sum_{i=1}^{I} \hat{D}_i$$

and the total demand for agricultural real estate capital is

$$K^D(\hat{e}) = \sum_{i=1}^{I} D_i$$

assuming that agricultural credit markets operate so that all individuals face the same interest rate, $\hat{\theta}_i = \hat{\theta}$ and $\theta_i = \theta$, $i = 1, \ldots, I$. Suppose also that the supply of operating capital for agricultural purposes is given by $\hat{K}^S(\hat{e})$, and the supply of capital for agricultural land investment is given by $K^S(e)$ so that agricultural credit market equilibrium is characterized by

$$\hat{K}^S(\hat{e}) = \hat{K}^D \tag{33}$$
Finally, suppose transfer of ownership and rental agreements take place at the beginning of the growing season. Then, at the end of the growing season, the state of nature is revealed, yields are realized, and agricultural prices are determined. Finally, land price and agricultural quasi-rent expectations are revised and the succeeding production period commences. In this context, agricultural supply (at the end of the production period) is

\[ x^S = \sum_{i=1}^{I} \sum_{j=1}^{J} y_{ij} A_{ij} \]

where \( y_{ij} \) is the realized yield on land of quality \( j \) for individual \( i \).

Suppose, also, that the demand for agricultural output is given by \( X^D(P) \) so that equilibrium in the output market must satisfy

\[ x^S = X^D(P). \]

Before turning to the analysis of these equilibrium conditions, a classification of markets according to the following proposition is useful.

**PROPOSITION 9:** Suppose expected net government benefits per acre and expected profit per acre from operation are ordered consistently according to land quality (the ranking of land qualities does not depend on human capital or government intervention) and, moreover, that expected government payments to one individual on a poorer land quality can never exceed expected government payments to another individual on a better land quality. (a) If a voluntary government program results in partial participation, then two
critical land qualities will result such that all land poorer than the lower critical quality, $j_a$, will be diverted, all land better than the higher critical quality, $j_b$, will be utilized under compliance, and all land with qualities between the two critical levels will be utilized under non-compliance. (b) If a voluntary government program results in full participation or if participation is required, then one critical land quality, $j_a$, will result such that all lower quality land is diverted and all higher quality land is utilized under participation.

PROOF: To prove part (a) by negation, suppose individual 1 participates utilizing land quality $j$, and individual 2 does not participate while utilizing land quality $j' > j$. Then $\gamma_{1j} > \gamma_{2j}$, which contradicts the assumptions of the position. This proves the existence of $j_b$. The existence of $j_a$ follows from competition which equates $\bar{R}_i$ over $i$ and the fact that every individual uses the lowest rental rate available for diversion land. Part (b) is proved similarly.

The proof of some of the propositions of the following section can be easily facilitated by graphical means. Thus, consider the aggregate demands which result from individual demands of the forms in the previous section. Aggregate demands are obtained by horizontal summation of individual demands and thus must follow the shapes indicated in Figure 2. The demand for ownership of each land quality and the demand for utilization of each land quality will be kinky as in Figure 2a where the flat segments correspond to reservation prices or switching prices of various individuals. Since the supply
of various land qualities is fixed, the equilibrium can occur either in a flat
segment of demand such as where supply is \( L_j \) or in a declining segment such
as where supply is \( L_j' \) in Figure 2a. In any case all individuals who utilize
land type \( j \) must satisfy either (18) or (19) for either \( \lambda = 0 \) or \( \lambda = 1 \) and, in
particular,

\[
R_j = \begin{cases} 
(1 - f_i) (\pi_{ij} + \gamma_{ij} - R_w) - r_i^*(\nu_{ij} + R_w) & \text{for } r_i^* \leq \hat{\theta}; A_{ij} > 0; \lambda_i = 1; \\
\frac{1 - f_i + r}{1 - f_i + r} & \text{for } j = j_b, \ldots, J; i = 1, \ldots, I; \\
(1 - f_i) (\pi_{ij} + \gamma_{ij} - \hat{\theta}\nu_{ij} - \hat{\theta}u_{ij}) - r_i^*(\nu_{ij} + R_w - \hat{\theta}u_{ij}) & \text{for } r_i^* > \hat{\theta}; A_{ij} > 0; \lambda_i = 1; j = j_b, \ldots, J; \\
\frac{1 - f_i - r_i^*}{1 - f_i - r_i} & \text{for } j = j_a, \ldots, j_b - 1; \\
(1 - f_i) (\pi_{ij} - \hat{\theta}\nu_{ij}) - r_i^*(\nu_{ij} - \hat{\theta}u_{ij}) & \text{for } r_i^* \geq \hat{\theta}; A_{ij} > 0; \lambda_i = 0; \\
\frac{1 - f_i - r_i^*}{1 - f_i - r_i} & \text{for } j = j_a, \ldots, j_b - 1.
\end{cases}
\]

where \( r_i^* \) is the maximum rate of return
individual \( i \) can earn.

Similarly, all individuals who choose to own land must satisfy either (20)
or (21) and, hence,
In the case of land rental, however, one must consider the demand for diversion in addition to the demand for utilization. The quantity of land demanded for diversion is proportional to the quantity of land demanded for utilization under participation according to the government program diversion requirement. Since an increase in $R$ of 1 is equivalent to an increase in $R_j$ of $\omega$ in either (18) or (19), the demand for diversion land associated with participation in utilization of type $j$ land is proportional to the demand for utilization of type $j$ land. Thus, summing these demands for diversion over land types obtains an aggregate demand for diversion land of the same general form as depicted by $D^D$ in Figure 2b. To this demand, one must add the demand for utilization of diversion quality land. The aggregate demand for diversion quality land is then the horizontal summation of the demand for diversion, $D^D$, and the demand for utilization of diversion quality land, $\sum_{j=1}^{J_a} \sum_{i=1}^{I} A_{ij}$. However, Proposition 9 implies that no lower demand quality than $j_a$ will be utilized. Hence, demand for utilization of diversion quality land must consist solely of demand for utilization of land type $j_a$ as in Figure 2b. This will hold only if the equilibrium diversion quality rental rate $\bar{R}$ determined by the intersection of $L_{ja} = \sum_{j=1}^{J_a} L_j$ and the total diversion demand, $D^T$, is such that $\bar{R}_{ja} \geq \bar{R} \geq \bar{R}_{ja-1}$ where $R_j$ is the highest rental rate at which land type $j$ is utilized.
Using (18) and (19), all individuals who participate in the government program must satisfy

\[
\hat{R}_i = \max_i \hat{R}_{ij}.
\]

\[
\begin{align*}
\hat{\theta} = \left\{ \begin{array}{l}
(1 - f_i) \left( \pi_{ij} + \gamma_{ij} - R_j - \hat{\omega}u_{ij} \right) - r^*_i (u_{ij} + R_j + \hat{\omega} - \hat{\omega}u_{ij}) \\
(1 - f_i) \hat{\omega} \hat{u}_{ij} & \text{for } D_i > 0; \lambda_i = 1; \\
(1 - f_i) \left( \pi_{ij} - R_j - r^*_i (u_{ij} + R_j - \hat{\omega}u_{ij}) \right) & \text{for } D_i > 0; \lambda_i = 0.
\end{array} \right.
\end{align*}
\]

The demands for credit will be step functions since they are horizontal summations of step functions (see Figure 2c). The flat segments here also correspond to reservation prices or switching points. Equilibrium occurs either on a vertical segment such as at \( \hat{\theta}_0 (e_0) \) or on a horizontal segment such as at \( \hat{\theta}_1 (e_1) \). Using (18) and (19), every individual who uses short-term credit must satisfy

\[
\begin{align*}
\hat{R}_i = \left\{ \begin{array}{l}
(1 - f_i) \left( \pi_{ij} + \gamma_{ij} - R_j \right) + r^*_i (u_{ij} + R_j) \\
(1 - f_i) \omega + r^*_i \omega & \text{for } r^*_i \leq \hat{\theta}; \lambda_i = 1; eA_i > 0; \\
(1 - f_i) \left( \pi_{ij} + \gamma_{ij} - R_j - \hat{\omega}u_{ij} \right) + r^*_i (u_{ij} + R_j - \hat{\omega}u_{ij}) \\
(1 - f_i) \omega + r^*_i \omega & \text{for } r^*_i > \hat{\theta}; \lambda_i = 1; eA_i > 0.
\end{array} \right.
\end{align*}
\]
and using (20) and (21), every individual who uses long-term credit must satisfy

\[
e = \frac{(1 - f_1)R_j + (1 - f_1)R_j^* W_{ij} - W_j^* - r_i^*(W_j - R_j - \rho W_j)}{(1 - f_1)\rho W_j} \text{ for } D_i > 0. \tag{40}
\]

Equations (36) through (40) give price equations which, together with the quantity equations in (30) through (34), determine the general equilibrium at the beginning of the production period if individuals have fixed subjective distributions of output price. If individuals have rational expectations for output price, then (35) must also be considered in determining the general equilibrium at the beginning of the production period. Obviously, from the above results, a unique equilibrium price and quantity exists in each market given all other prices assuming credit supply and output demand are not perfectly inelastic (except under peculiar circumstances as pointed out below). With some further tedious derivation which will be presented in a later paper, one can also show that the general equilibrium for this framework exists and is unique (except under the peculiar circumstances). Thus, the general model provides an adequate basis for developing comparative static results.

V. The Comparative-Static Results

To examine the comparative static properties of the model, a temporary simplification of assumptions is convenient.

PROPOSITION 10: Suppose that all individuals possess equal human capital \((\pi_{ij} = \pi_j, \gamma_{ij} = \gamma_j, \mathcal{W}_{ij} = \mathcal{W}_j, i = 1, \ldots, I)\) and alternative investment possibilities \((\mathcal{G}_1 = \mathcal{G})\) and that the supply of short term operating credit is neither so large that everyone can finance farming
operations to the collateral limit nor so small that no one can finance farming operations if the short-term rate of interest is equal to \( \overline{\sigma} \). (a) If a voluntary government program results in partial participation, then the rental rate structure is given by

\[
R_j = \begin{cases} 
(1 + \hat{\sigma})^{-1} \left[ \pi_j + \gamma_j - \overline{R} - \hat{\sigma}(u_j + \overline{R}) \right] & \text{if } j \geq j_b \\
(1 + \hat{\sigma})^{-1} \left( \pi_j - \hat{\sigma} u_j \right) & \text{if } j_a < j < j_b \\
\overline{R} & \text{if } j < j_a 
\end{cases}
\]

\[ (41) \]

where

\[
\overline{R} = (1 + \hat{\sigma})^{-1} \min \gamma_{jb} / \omega, \gamma_j - \hat{\sigma} u_j 
\]

(b) If a voluntary government program results in full participation or if participation is required, then the rental rate structure is given by

\[
R_j = \begin{cases} 
(1 + \hat{\sigma})^{-1} \left[ \pi_j + \gamma_j - \overline{R} - \hat{\sigma}(u_j + \overline{R}) \right] & \text{if } j \geq j_a \\
\overline{R} = (1 + \hat{\sigma})^{-1} (1 + \omega)^{-1} \left( \pi_j + \gamma_j - \hat{\sigma} u_j \right) & \text{if } j > j_a. 
\end{cases}
\]

\[ (42) \]

PROOF: To prove part (a), note that the rate of return must equate across land qualities since every individual has identical rates of return on individual land qualities. Thus, any inequality would lead to arbitrage, i.e., bidding the rental rate up where a high return prevails and undercutting of rental rates where a low return prevails. Similarly, arbitrage will lead to equating \( \overline{\sigma} \) and \( \hat{\sigma} \) since otherwise either all individuals would prefer
farming to alternative investments or vice versa. Any inequality would be dissipated through competitively bidding rental rates or alternative investment prices up or down. The rental rates for utilized land in (41) are obtained by equating (18), (19), and (22) where \( \hat{\Theta} = \Theta \). To find \( \bar{R} \), note that 

\[
\bar{R} = \left( \pi_{j_a} - \hat{\Theta}_{j_a} \right) / \left( 1 + \Theta \right)
\]

from Proposition 9 since otherwise individuals will prefer to use land quality \( j_a \) for diversion where the rental rate is \( \pi_{j_a} - \hat{\Theta}_{j_a} \). If some of land type \( j_a \) is diverted and some is utilized, then \( \bar{R} = R_{j_a} \) by competition. Alternatively, if none of land type \( j_a \) is diverted, then \( \bar{R} = \gamma_{j_b}/\omega > R_{j_a} \) since \( j_b \) is the marginal quality land utilized under participation and \( \gamma_{j_b} - \omega \bar{R} = 0 \) is the marginal condition for optimization. Part (b) is proved similarly.

PROPOSITION 11: Suppose that all individuals possess equal human capital and alternative investment possibilities and that the supply of long-term credit is perfectly elastic at the rate of return on alternative investments (\( \Theta = \bar{\Theta} \)). Suppose further that everyone is in the same marginal tax bracket (\( f_i = f \)) and that land price expectations are proportional to current land prices (\( W_i^0 = 0, \psi_i > 0; i = 1, ..., I \)). Then land transactions will take place only if the minimum expected rate of land appreciation, \( \psi_i \), among those individuals carrying land into the current period is less than the rate of return on alternative investments after correcting for capital gains tax advantages,

\[
\min_{i} \left\{ \psi_i; L_{i,j}^0 > 0 \text{ for some } j \right\} < 1 + \hat{\Theta}(1 - f) \frac{1}{1 - i} = \bar{\psi}
\]

If this condition does not hold, there will be no market for land since no land will be sold. If this condition holds, then the land market equilibrium will follow one of two alternatives. (i) If the set of individuals who have
infinite reservation prices \((\bar{\psi}_i \geq \bar{\psi})\) has a combined wealth sufficient to buy all land at prices higher than the highest reservation price among those individuals for whom reservation prices exist \((\bar{\psi}_i \leq \bar{\psi})\), i.e.,

\[
\sum_{i=1}^{I} E_i > \max \sum_{j=1}^{J} (1 - \rho) (\hat{W}_{ij} - R_j) \delta_j
\]

\(\bar{\psi} \leq \bar{\psi}\)  

then the land market will reach equilibrium where all individuals with \(\bar{\psi}_i \geq \bar{\psi}\) own land and bid up prices to just exhaust their wealth

\[
\sum_{i=1}^{I} E_i = (1 - \rho) (W - R) \delta
\]

\(\bar{\psi}_i > \bar{\psi}\)

such that land prices are proportional to rental rates

\[
W = \frac{\bar{\psi}_i > \bar{\psi}}{(1 - \rho)R \delta}
\]

all individuals with \(\bar{\psi}_i < \bar{\psi}\) will not own land. (ii) If the combined wealth of individuals with infinite reservation prices is not sufficient to satisfy (44), then there will be some critical expected rate of land price appreciation, \(\bar{\psi}^*\), above which everyone will own land and below which no one will own land; equilibrium land prices will be
\[ W = \frac{(1 - f)(1 + \bar{\psi})}{(1 - f)e + (1 - \bar{\psi})(1 - \psi^*)} R. \]  

(47)

**PROOF:** The inequality in (43) follows from (26). Equations (44) and (45) follow upon noting that individuals with \( \psi_j \geq \bar{\psi} \) will use all their wealth to buy land using as much credit as possible at any price and anyone with \( \psi_j < \bar{\psi} \) will not buy land at prices above their respective reservation prices. Equation (46) from arbitrage and (45) in equating rates of return among alternative land holdings. Equation (47) follows from (42) and arbitrage for \( e = \bar{e} \).

From equation (47) one can see that the results of this paper contain both the Tweeten and Feldstein results as special cases. That is, if capital gains are taxed at the same rate as other income, then \( W_j = (1 + \bar{\psi})/(1 + \bar{\psi} - \psi^*) \) for \( R_j \) which is the same as the Tweeten result that land prices equate to the gains from operation divided by the real interest rate. More generally, one finds the Feldstein result that

\[ W_j = \frac{(1 + \bar{\psi})R_j}{\bar{\psi} + \frac{1 - f\gamma}{1 - f} \psi^*}. \]

from which he argues that land prices grow more rapidly than inflation if capital gains are taxed at a lower rate than other income. However, in the more general model of this paper, the possibility of land prices growing more rapidly than inflation is further underscored by the result in (46).
Note that somewhat more general results can be obtained when an absolute component is added to land price expectations following equation (2'); however, this paper is too limited in space and time to fully present all the related proofs.

Furthermore, the results can be easily generalized for the purpose of performing general equilibrium comparative statics by introducing differences among individuals in marginal tax brackets, human capital, and alternative investment possibilities. Consider, for example, generalizing Proposition 11 in this context. Here one can simply define a set of individuals, $\mathcal{G}$, which has infinite reservation prices for land, i.e.,

$$\mathcal{G} = \{i; v_{ij} + (1 - f_i)(1 - \psi_i) (1 - \psi_i) \leq 0 \text{ for some } j\}.$$

Then equations (44) through (47) follow immediately where the summation is over individuals in $\mathcal{G}$ rather than over individuals with $\psi_i \geq \overline{\psi}$ except that equation (47) must be stated in terms of the $f_i, \psi_i$, and the marginal rate of return of the marginal individual, and thus (47) should be replaced with (37). Proposition 10 can be generalized similarly.

For the remainder of this paper, a number of results which are implied rather directly from the above line of argument are simply summarized as follows:

9. Equilibrium land prices will equate to a linear combination of the rental rate vector and a vector of the absolute component of land price expectations.

10. A change in monetary policy that increases the cost of short-term capital tends to reduce rental rates and land prices on all qualities of
land. The effect of an increase in the cost of long-term capital associated with agricultural land investment is to reduce the ratio of agricultural land prices to rental rates. Thus, the effect of tighter monetary policy is to reduce the effect of land prices through an effect on the ratio of land price to rental rate as well as through the effect on rental rates of more costly short-term capital. As a result, the tendency to participate in government programs will increase and product prices will increase accordingly.

11. An increase in land price expectations normally causes the ratio of rental rates to land prices to decrease.

12. If the expected rate of land price appreciation corrected for tax considerations among those individuals who hold land exceeds the cost of long-term capital for land investment and the rate of return on alternative investments, then a disequilibrium will result in the land market such that the only sales of land are involuntary (connected with death of the owner) where the resulting upward spiraling prices fuel higher land price expectations and less interest on the part of owners in selling land. (The same phenomena could occur in a downward price spiral).

13. If some individuals who own land have reservation prices, these reservation prices will tend to bound the price spirals. A key determining factor in this regard is whether individuals formulate land price expectations in absolute or in relative terms.

14. The effect of a higher rate of exemption on capital gains for tax purposes and the effect of escalation in the tax structure in general is to increase the ratio of land prices to rental rates and to encourage inflationary land price spirals.
15. The effect of an increase in the down-payment requirement on long-term debt is to reduce the cost of long-term capital; the effect on the ratio of land prices to rental rates may be in either direction depending on the distribution of land price expectations of those individuals who are marginally affected.

16. The effect of an increase in the down-payment requirement on short-term operating capital in agriculture is to reduce the interest rate on short-term debt and, as a result, to increase the rental rate on all qualities of land and reduce participation in voluntary government programs. Land prices will increase accordingly.

17. The effect of an increase in the return from alternative investments outside agriculture is to reduce land prices and rental rates; to increase participation; and, thus, to increase product prices. The entrance of foreign investment into U. S. agriculture (which suggests a reduction in the rate of return on alternative investments) would have the opposite effect. If existing land holders hold absolute rather than relative price expectations, the entrance of foreign investors can occur more rapidly although with less impact on land prices.

18. Land prices and rental rates of higher quality lands utilized under government program participation will be higher than in the absence of government programs reflecting the expected net benefits of government-program participation, the returns from operation, and the cost of financing the operation.

19. The land prices and rental rates of land utilized without participation in government programs will reflect expected returns from operation less the cost of financing the operation.
20. Land prices and rental rates of land utilized for diversion and set-aside will be determined by the gains from operation on the marginal quality land utilized without participation and the expected government program payments and diversion requirement on the marginal quality land utilized under participation.

21. An increase in demand for agricultural products will cause rental rates and land prices to increase on all qualities of land.

22. The effect of technological change, also, is to increase land prices and rental rates on all qualities of land; however, increases in the cost of operating capital tend to reduce the effects of cost-reducing technology.

23. An increase in government-program price supports or diversion payments will tend to increase participation in government programs and to increase land values and rental rates on all qualities of land; however, the main effect on land prices and rental rates for land qualities not utilized under participation is through the product price whereas the main effect on land prices and rental rates of diversion quality lands is through an increase in the rental rates and land values on marginal quality land.

24. The effect of increasing a diversion requirement of government programs is to reduce program participation; land prices and rental rates tend to decrease on all qualities of land (assuming partial participation), and product prices will decrease.
FOOTNOTES

1 Farmland debt has increased from approximately $30 billion in 1971 to about $78 billion in 1979. Recent U. S. Department of Agriculture forecasts suggest that total foreign debt is likely to increase to approximately $500 billion by 1990 with a healthy general economy, while it may rise to almost $1 trillion under poor economic conditions.

2 The term "individuals" is used in a broad sense to mean individuals, partnerships, or corporations whether involved in a farming venture or simply in investment.

3 One can also note that the declining segments are of smaller slope as quantity increases.

4 Here we assume that the quantities of different land types available do not by chance satisfy $\omega \sum_{j=1}^{T} L_j = \sum_{j=1}^{T+1} L_j$, in which case the marginal condition does not determine a unique diversion quality rental rate but rather a small range of possibilities $[y_{j_b}/\omega, (\pi_{j_a} - \bar{\pi}_{j_a})/(1 + \nu)]$. If the difference in productivities between land types $j_a$ and $j_a - 1$ is small, then this nonuniqueness is substantively inconsequential.
REFERENCES


L. Tweeten, "Farmland Pricing in an Inflationary Economy," Department of Agricultural Economics, Oklahoma State University, 1980.