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INELASTIC CONTAMINATION OF ELASTIC NUCLEON-NUCLEUS POLARIZATION MEASUREMENTS

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NUCLEON-NUCLEUS POLARIZATION MEASUREMENTS

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ABSTRACT

A prescription for estimating the inelastic contamination present in elastic nucleon-nucleus polarization measurements is given with the use of methods discussed by T. K. Fowler for the corresponding problem in differential scattering cross-section measurements. It is shown that in many cases only a knowledge of readily accessible data is necessary to make such estimates. An example calculation is discussed for proton-carbon scattering at 310 MeV to demonstrate the ease with which one may determine the extent of inelastic contamination.
1. INTRODUCTION

Experimental studies of the elastic scattering from complex nuclei are complicated by the presence of contamination from almost-elastic scattering. Hence comparison with the predictions of theory is somewhat uncertain without some measure of the extent of contamination. A number of research groups have reported experimental work with sufficient energy resolution to make precise separation of inelastic from elastic scattering, but this work is limited to targets with well-spaced low-lying levels that greatly facilitate the separation. Thus, in general, the difficulty still remains.

A rather complete theoretical analysis of "quasi-elastic" scattering was presented by T. K. Fowler in hope of gaining some insight into the degree of contamination that may be expected with experimental techniques currently available. Fowler limited his discussion to the scattering cross section, but it is clearly possible to apply the general formalism to estimate the contamination expected in elastic polarization experiments arising from the excitation of low-lying levels of the target nucleus. It is the purpose of this paper to apply this formalism to such an analysis. An expression is given for the average polarization expected in an experiment that includes inelastics up to some small excitation, \( \Delta E \). The resulting nucleon-nucleus scattering matrix is related to the corresponding nucleon-nucleon scattering operators. Finally, contamination is defined and a simplified expression is presented.

Because we are asking questions about excited states of the target nucleus, one might expect that the target wave functions should play a significant role in the discussion; however, it will soon become apparent that a crude knowledge of only the ground-state wave function is important.
Thus, rather simple models of the nucleus suffice as adequate characterization of the properties of the ground state. A brief discussion of some useful models is presented in a later section. That only the ground state need be considered follows from an improved closure approximation suggested by Fowler.
II. DEFINITIONS AND APPROXIMATIONS

A. AVERAGE POLARIZATION

For scattering with a pure incident beam and for excitation of a particular final state, the polarization becomes

\[ \langle P \rangle = \frac{\langle X_i | T^+ a_0 T | X_i \rangle}{\langle X_i | T^+ T | X_i \rangle} \quad (2.1) \]

where \( T = (\psi(v_t') | T_N | \psi_0 (v_t)) \); \( T_N \) is the full scattering matrix, and \( X_i \) is the spin state for the incident beam. Here \( \psi, \psi_0 \) are final and initial momentum states for the incident particle; \( \gamma(v_t'), \psi_0 (v_t) \) correspond to initial and final target states, respectively. However, we are interested in an average polarization, which is determined by a special scattered beam. This beam may be as precise or crude as we wish, depending on our desire for detailed observation or our resolution. Thus, a final beam may be all those particles which have excited a particular target state and have a particular momentum and spin projection, or it may be that collection for which any one of these final-state designations is relaxed. Corresponding to this special beam there will be special beam averages. Here average polarization is defined by that scattered beam which

(a) is initiated by a particular momentum and two spin directions with relative weights \( f_1 \) and \( f_2 \); 

(b) results in (i) all energy eigenstates \( 1, 2, \ldots, p \) with all spin projections possible to these states and (ii) a final scattered-particle momentum direction with either spin direction.

For average polarization we thus have
\[
(p_n)_{av} = \frac{\sum_{\lambda} \sum_{i} \sigma_{\lambda}^{\nu_i}(\theta) \langle p_{n\lambda} \rangle \nu_i}{\sum_{\lambda} \sum_{i} \sigma_{\lambda}^{\nu_i}}.
\]

where

\[
\sigma_{\lambda}^{\nu_i}(\theta) = \sum_{\nu_j} \int \frac{(2\pi)^4}{\nu_1^2} k^2 dk \delta(E_k - E_{k_0} + \chi_{\gamma}) \chi |\gamma(v_t^{'}) k v_j^{'}) |T_N| 0(v_t) k_0 v_i)|^2
\]

and \(k_0, k\) = initial and final momentum of nucleon; \(E_{k_0}, E_k\) = initial and final kinetic energy of the incident particle; \(\theta\) = scattering angle; \(\chi_{\gamma}\) = energy of excitation of the target, and \(\nu_1\) = initial velocity of the particle. To be completely precise, it is necessary to introduce a weighting function \(W_{\gamma}\) to account for the a priori weighting dictated by the experimental setup. We refer particularly to energy resolution. Furthermore, \(W_{\gamma}\) will have the property of limiting a sum over all states to those in which the nucleus is excited by no more than some specified small amount, \(\Delta E\). If we introduce this weighting function, we may write

\[ \sum_{\lambda} = \sum_{\gamma(v_t^{'})} W_{\gamma}. \]

A typical form suggested by Fowler is a step function that is unity if \(E_{k_0} - E_k < \Delta E\) and zero otherwise. Finally, if \(W_{\gamma}\) is regarded as a function of \(E_k\) and introduced under the integral, we may perform closure over the target states with the result
\langle p_n \rangle_{\text{av}}
\begin{align*}
\sum_i \frac{(2\pi)^4}{V_i} \int_0^\infty \frac{d^4 k}{2k^2} \frac{W_{E_k}}{E_k} \langle 0|T^\dagger_i \sigma_n \delta(E_k - E_{k_0} + H_N - W_0)T|0\rangle_i \\
\sum_i \frac{(2\pi)^4}{V_i} \int_0^\infty \frac{d^4 k}{2k^2} \frac{W_{E_k}}{E_k} \langle 0|T^\dagger_i \delta(E_k - E_{k_0} + H_N - W_0)T|0\rangle_i \\
\equiv \frac{N}{D}\gamma
\end{align*}
(2.2)

where $|0\rangle_i \equiv |0(v_i)v_i\rangle$, and $T$ is to be regarded as an operator in the spin of the scattered particle and the coordinates of the target but a matrix element in the momentum states of the scattered particle. This expression is exact, but useless without knowledge of the action of $T$ on $|0\rangle_i$. 
B. SCATTERING OPERATOR

The multiple-scattering features of the collision can be made explicit by considering the multiple-scattering solution to the Schrödinger equation as proposed by Watson in his formal theory of multiple scattering. For the transition operator we have

\[ T_N = V \Omega, \]

(2.3)

where \( \Omega \) is the Moller wave operator and \( V \) is the interaction of the target with the incident particle. Thus,

\[ V = \sum \chi_\alpha, \]

where \( V \) is the two-particle interaction. Watson writes

\[ \Omega = \Omega_I \Omega_c \]

(2.4)

where \( \Omega_c \chi_\alpha = \psi_c(a)^\dagger \), the exact solution to the elastic-scattering problem, and \( \Omega_I \psi_c(a)^\dagger = \psi_a^\dagger \), affecting the "incoherent" portion of the solution. Now \( \Omega_c \) is given by the integral equation
$$\Omega_c = 1 + \frac{1}{d} \mathcal{O} \Omega_c,$$  \hspace{1cm} (2.5)

where $\mathcal{O}$ is the optical potential. A "multiple-scattering solution" may be produced for $\Omega_I$:

$$\Omega_I = 1 + \frac{P_0}{d} \sum_{\alpha=1}^{\infty} t_\alpha \Omega_I(\alpha)$$

and

$$\Omega_I(\alpha) = 1 + \frac{P_0}{d} \sum_{\beta \neq \alpha} t_\beta \Omega_I(\beta).$$  \hspace{1cm} (2.6)

The obvious interpretation of the $\Omega_I(\alpha) \Omega_c$ operator is the "effective field" incident on particle $\alpha$. The propagator $d$ is augmented by the presence of the optical potential, $d = a - \mathcal{O}$, which corresponds to propagation between collisions through a dispersive medium characterized by the optical potential. Here

$$a = E + i\eta - h - H_N,$$

where $h$ is the nucleon kinetic-energy operator, and $H_N$ is the Hamiltonian for the nucleus. Finally, the action of $P_0$ is to ensure complete inelas-
ticity for virtual excitations by disallowing repetition of a virtual excitation between two scattering operators, $t_{\alpha}$.

From the multiple-scattering equations it is possible to write

$$V_F = \sum_{\alpha} t_{\alpha} \Omega I(\alpha)$$

or

$$T_N = \sum_{\alpha} t_{\alpha} \Omega I(\alpha) \Omega_c$$

(2.7)

We make the following simplifying assumptions: The coherent field produced by $\Omega_c$ will be taken to be the incident wave. This is generally not a good approximation for the differential-scattering cross section, but because we deal here with a ratio, $N/D$, it is expected that this rather crude approximation is not too drastic. Furthermore, we may treat the elastic portion of the scattering only approximately, as we are focusing attention on contamination. Only a single inelastic scattering will be allowed; thus $\Omega I(\alpha) \approx 1$. It is to be expected that multiple inelastic scattering will contribute only negligibly to quasi-elastic scattering because, here, we are limiting our interest to low-lying excitations of the target. In order that multiple inelastic scattering make a significant contribution, it is necessary that the distance between two consecutive scatterings be small compared with the correlation distance, $R_c$. This follows since the subsequent scatterings must essentially "undo" one another. But if $\lambda_{mfp} >> R_c$, where $\lambda_{mfp} = \text{mean free path for collision}$, such a sequence of events is not very probably. Finally,
we make the high-energy approximation in which we approximate the bound $t_\alpha$ by the free two-nucleon scattering operator, $t_\alpha$. The error incurred by this approximation has been estimated to be small if 

$$\left( \frac{f}{\hbar} \right) \left( \frac{B_{av}}{E_{k_0}} \right)^2 \ll 1,$$

where $B_{av}$ is the average binding potential of a nucleon in the nucleus, $f$ is the scattering amplitude, and $\hbar$ is the de Broglie wavelength of the incident particle.

Taking these approximations into account, we may write

$$T_N = \sum_{\alpha=1}^{A} t_\alpha$$

or

$$T = \sum_{\alpha=1}^{A} \langle \hbar k | t_\alpha | \hbar k_0 \rangle.$$  \hspace{1cm} (2.8)
III. NUCLEON-NUCLEUS SCATTERING

If we make a partial Fourier analysis of $|0\rangle_1$ in the momentum of the nucleon, the two-nucleon scattering aspect of the problem is made clear. Thus

$$\xi_\alpha |0\rangle = \int \frac{d^3 P_\alpha}{(2\pi)^3/2} e^{iP_\alpha \cdot Z\alpha} \langle P_\alpha |0\rangle_1.$$  

Noting that the two-nucleon scattering is on the momentum shell, we find

$$\langle k | t_\alpha | k_0 \rangle = e^{-iq \cdot Z\alpha} t_\alpha^0 |0\rangle_1,$$

where $t_\alpha^0 \equiv \langle (k - P_\alpha + q)|t_\alpha^0(g_\alpha)| (k_0 - P_\alpha)\rangle$, $q = (k - k_0)$, the momentum transfer. Since the $P_\alpha$ is in a relative momentum combination with the scattered particle, we may neglect its presence in first approximation. The significance of $P_\alpha$ has been carefully discussed by Fowler. It is not expected to be important for large incident momentum.

The average polarization may now be written

$$\langle P_n \rangle_{av} = (N^d + N^{nd})/(D^d + D^{nd}),$$

where
Before these expressions can be related to nucleon-nucleon scattering, it is necessary to determine the action of the energy-conserving $\delta$ function. Formally, it may be expanded about some point of mean excitation of the nucleus. However, the best point of expansion is not necessarily the same for the "diagonal" and "nondiagonal" terms. That the two types of terms are fundamentally different was noted by Wick. He interpreted the operator $e^{-igZ\alpha}t^0_\alpha$ as representing excitation through collision, $\delta(\sum E)$ as free propagation of the whole system, and finally $t^0_\alpha e^{igZ\alpha}\sigma_n$ as de-excitation through collision. Thus the difference for the two types of terms rests in the fact that de-excitation for the nondiagonal term is through collision with a different particle of the target than that one which was responsible for the initial excitation. With this interpretation of the matrix element, the proper point of expansion becomes clear. In the "diagonal" case we deal with a simple two-particle collision characterized
by a mean excitation energy, $q^2/2m$. On the other hand, the "nondiagonal" case represents scattering off the "correlation structure" of the target, which makes anything but low-energy excitations very unlikely. To first order we might expect $W_{\text{ex}}^{\text{nd}} \approx 0$.

The final expressions for $N^d$ and $N^{\text{nd}}$ become

\[
N^d = \sum_\alpha \sum_i f_i \frac{(2\pi)^4}{V_I} \int_0^{\infty} W_k k^2 dk \delta(E_k - E_{k_0} + q^2/2m) \chi \langle 0 | t_\alpha^0 \sigma_i t_\alpha^0 | 0 \rangle_i
\]

and

\[
N^{\text{nd}} = \sum_{\alpha \neq \beta} \sum_i f_i \frac{(2\pi)^4}{V_I} \int_0^{\infty} W_k k^2 dk \delta(E_k - E_{k_0}) \chi \langle 0 | t_\alpha^0 \sigma_i t_\beta^0 | 0 \rangle_i
\]

\[
+i q \cdot (Z_\alpha - Z_\beta) \chi \cdot e^{-i q \cdot (Z_\alpha - Z_\beta)} \sigma_i t_\beta^0 | 0 \rangle_i.
\] (3.2)

To evaluate the matrix elements that appear above would require a detailed knowledge of the ground-state wave function. For each state of isospin, spin, and parity of a nucleon pair $(\alpha, \beta)$, we can expect a distinct pair-correlation function. Such a detailed analysis is probably not necessary, however. Thus only two correlation functions, space-symmetric and space-antisymmetric, will be regarded as significant in the evaluation of these matrix elements. Put another way, to evaluate $\langle 0 | e^{-i q \cdot (Z_\alpha - Z_\beta) \cdot Q_{\alpha \beta}} | 0 \rangle$, where $Q_{\alpha \beta}$ is a function of all the coordinates of $(\alpha, \beta)$ except $Z_\alpha$, $Z_\beta$. 
it would be necessary to expand \( |0\rangle \) in a complete set of two-particle product functions, with each term corresponding to coordinate functions, spin function, etc. We will assume, however, that separation of coordinate dependence from the other dynamical variables is immediate when \( |0\rangle \) is decomposed into space-symmetric and space-antisymmetric functions. This procedure is equivalent to the assumption of reference 5.

The operator that exchanges \( Z_\alpha, Z_\beta \) may be written

\[
\hat{P}_{\alpha\beta} = -\frac{1}{4} \left[ 1 + \vec{\sigma}_\alpha \cdot \vec{\sigma}_\beta \right] \left[ 1 + \vec{\tau}_\alpha \cdot \vec{\tau}_\beta \right].
\]

Thus

\[
|s\rangle \equiv \frac{1}{2}(1 + \hat{P}_{\alpha\beta})|0\rangle = P_{\alpha\beta}^s|0\rangle,
\]

\[
|a\rangle \equiv \frac{1}{2}(1 - \hat{P}_{\alpha\beta})|0\rangle = P_{\alpha\beta}^a|0\rangle
\]  

(3.3)

are the respective projections of \( |0\rangle \) that are symmetric and antisymmetric in the coordinates \( Z_\alpha, Z_\beta \). The assumption discussed above may now be written

\[
\langle s | e^{i q \cdot Z_\alpha - Z_\beta} Q_{\alpha\beta} | s \rangle = F_s(q) \langle s | Q_{\alpha\beta} | s \rangle , \quad \langle a | e^{i q \cdot Z_\alpha - Z_\beta} Q_{\alpha\beta} | a \rangle = F_a(q) \langle a | Q_{\alpha\beta} | a \rangle.
\]
We now have

\[ \sum_{\alpha \neq \beta} \langle 0 | e^{i q \cdot (z_\alpha - z_\beta)} q_{\alpha \beta} | 0 \rangle = F_s(q) \sum_{\alpha \neq \beta} \langle 0 | q_{\alpha \beta} P_{\alpha \beta}^a | 0 \rangle + F_a(q) \]

\[ \times \sum_{\alpha \neq \beta} \langle 0 | q_{\alpha \beta} P_{\alpha \beta}^a | 0 \rangle . \]

It is useful to introduce the symmetric and antisymmetric two-particle probability densities defined by

\[ F_s(a)(q) \equiv \int P_s(a)(x, x') e^{i q \cdot (x-x')} d^3 x d^3 x' , \]

which requires

\[ P_s(a)(x, x') = \sum_{\alpha \neq \beta} \frac{\langle 0 | s(x-x_{\alpha}) s(x'-x_{\beta}) P_{\alpha \beta}^s(a) | 0 \rangle}{\sum_{\alpha \neq \beta} \langle 0 | P_{\alpha \beta}^s(a) | 0 \rangle} . \quad (3.5a) \]

These functions may be written in a convenient form that explicitly demonstrates the two-particle correlations.

Let

\[ P_{2s}(a)(x, x') = F(x) P(x') \left[ 1 + G_s(a)(x-x') \right] , \quad (3.5b) \]

where \( F(x) \) corresponds to the density of particles, and we assume that
the correlation term is a function of the difference of the coordinates. We postpone the calculation of $G$ until after the spin, isospin averages indicated in Eq. (3.4) are made.
IV. SPIN, ISOSPIN AVERAGES AND NUCLEON-NUCLEON SCATTERING

For proton-neutron scattering in the c.m. frame, the scattering operator takes the form

\[ t^{\text{PN}}(\alpha) = A^{\text{PN}} + B^{\text{PN}} \sigma_n \sigma_n(\alpha) + C^{\text{PN}} \sigma_{n+} \sigma_{n+}(\alpha) + D^{\text{PN}} \sigma_{n-} \sigma_{n-}(\alpha) \]
\[ + E^{\text{PN}} (\sigma_n + \sigma_n(\alpha)), \]

with the requirement of invariance under space rotation, time inversion, and charge symmetry; for proton-proton scattering in the c.m. frame, there is a similar expression. Now \( \hat{n}_+ \), \( \hat{n}_- \), \( \hat{n}_x \) are unit vectors defined by

\[ \hat{n}_+ = \frac{P_{\text{in}} + P_{\text{out}}}{|P_{\text{in}} + P_{\text{out}}|}, \quad \hat{n}_- = \frac{P_{\text{out}} - P_{\text{in}}}{|P_{\text{out}} - P_{\text{in}}|}, \]

and

\[ \hat{n}_x = \frac{P_{\text{in}} \times P_{\text{out}}}{|P_{\text{in}} \times P_{\text{out}}|} \]

For proton scattering from either protons or neutrons we may write

\[ t(\alpha) = t_1(\alpha) + t_2(\alpha) \tau^Z_{\alpha}, \]
where $\tau_{Z\alpha} |n\rangle = |n\rangle$ and $\tau_{Z\alpha} |P\rangle = -|P\rangle$ for neutron or proton states, respectively. Here

$$t_1 = \frac{1}{2}(t_{PN} + t_{FP})$$

and

$$t_2 = \frac{1}{2}(t_{PN} - t_{FP}).$$

To evaluate the nuclear matrix elements described in the preceding section, we must determine the average over the ground state of the target of the operators,

(a) $I_1^{\alpha\beta}(N) \equiv t_1^\dagger \sigma_n t_1 \alpha \beta,$  
(b) $I_2^{\alpha\beta}(N) \equiv t_2^\dagger \sigma_n t_2 \alpha \beta,$

(c) $I_1^{\alpha\beta}(P) \equiv t_1^\dagger t_1 \alpha \beta,$
(d) $I_2^{\alpha\beta}(P) \equiv t_2^\dagger t_2 \alpha \beta.$

To illustrate, consider (a). Expanding to explicitly demonstrate the isospin dependence, we have

$$I^{\alpha\beta}(N) = z_{11} + z_{21} \tau_{Z\beta} + z_{12} \tau_{Z\alpha} + z_{22} \tau_{Z\alpha} \tau_{Z\beta}, \quad (4.2)$$
where, for example,

\[ z_{12} = t_1^+ (\beta) \sigma_n t_2 (\alpha). \]

If we write

\[ t_1 = t_0^1 + \sum_{\mu} t_{\mu}^1 \sigma_\mu \]

and

\[ t_2 = t_0^2 + \sum_{\mu} t_{\mu}^2 \sigma_\mu, \]

Eq. (4.2) may be expanded to show spin dependence. Thus

\[ z_{12} = t_0^1 \sigma_n t_0^2 + \sum_{\mu} t_{\mu}^1 \sigma_\mu t_0^2 \sigma_\mu (\beta) + \sum_{\mu'} t_0^1 \sigma_n t_{\mu'}^2 \sigma_\mu (\beta) \sigma_\mu (\alpha) \]

or

\[ z_{12} = p_{12} + q_{12} \cdot \sigma (\beta) + q_{21}^* \cdot \sigma (\alpha) + p_{12} : \sigma (\alpha) \sigma (\beta) , \quad (4.3) \]

where these coefficients are now only functions of the spin of the incident
particle. Similar expressions may be written for (b), (c), and (d).

The spin and isospin averages may now be performed with the aid of the tabulation of the relevant matrix elements, as given in reference 6. However, if we consider the important special case of dynamical spin, \( S = 0 \), and isospin, \( T = 0 \), the result is quite simple. (The corrections for nonvanishing spin and isospin are of relative order \( s/A \) or \( 1/A \).)

With the aid of these results the matrix elements of interest become (in two-body c.m.):

\[
\begin{align*}
(a) \sum_{\alpha \neq \beta} \langle 0 | t^0_\beta t^0_\alpha t^0_\alpha | 0 \rangle &= -A [(P_{11} + \text{tr} \bar{P}_{11}) + (P_{22} + \text{tr} \bar{P}_{22})] \\
&+ A^2 P_{11}, \quad (4.4a) \\
(b) \sum_{\alpha \neq \beta} \langle 0 | t^0_\beta t^0_\alpha t^0_\alpha P_{0\beta} | 0 \rangle &= -\frac{1}{4} A^2 [(P_{11} + \text{tr} \bar{P}_{11}) \\
&+ (P_{22} + \text{tr} \bar{P}_{22})] + 4A P_{11}, \quad (4.4b) \\
(c) \sum_{\alpha \neq \beta} \langle 0 | t^0_\beta t^0_\alpha t^0_\alpha | 0 \rangle &= -A [(P^t_{11} + \text{tr} \bar{P}^t_{11}) \\
&+ (P^t_{22} + \text{tr} \bar{P}^t_{22})] + A^2 P^t_{11}, \quad (4.4c)
\end{align*}
\]

and

\[
\begin{align*}
(d) \sum_{\alpha \neq \beta} \langle 0 | t^0_\beta t^0_\alpha t^0_\alpha P_{0\beta} | 0 \rangle &= -\frac{1}{4} [(P^t_{11} + \text{tr} \bar{P}^t_{11}) \\
&+ (P^t_{22} + \text{tr} \bar{P}^t_{22})] + 4A P^t_{11}. \quad (4.4d)
\end{align*}
\]
The coefficients $P_{11}$, $\text{tr} \Pi_{11}$, etc., are listed in the Appendix in terms of the corresponding components of the two-nucleon scattering matrix. However, the use of these explicit forms is not necessary in most cases because the combinations in which they appear above are directly related to two-nucleon and elastic nucleon-nucleus data.

For example, consider the expression for the barycentric two-body scattering cross section averaged over initial spin state of target nucleon and summed over final spin states of both nucleons for which an average over an equal number of neutrons and protons has been made. We find

$$\bar{\sigma}_{\text{c.m.}} v_k = \frac{(2\pi)^{1/2}}{4} \left( E_2 \right)^{2\text{c.m.}} \langle v_k \mid (P_{11} + \text{tr} \Pi_{11}) + (P_{22} + \text{tr} \Pi_{22}) \rangle$$

Before comparison can be made with nucleon-nucleus scattering, it is necessary to take into account the transformation between the two-body c.m. and the lab frame, because our original expressions refer to the lab frame. Nonrelativistically, the transition matrix is an invariant. Relativistically, we have

$$\left[ (E_1', E_2')^{1/2} \langle T \rangle (E_1, E_2)^{1/2} \right]_{\text{lab}} = \left[ (E_1', E_2')^{1/2} \langle T \rangle (E_1, E_2)^{1/2} \right]_{\text{c.m.}}$$
Further, we must realize that the direct comparison of the nucleon-nucleon amplitudes with the corresponding nucleon-nucleus amplitude is hampered by the fact that the energy-momentum conditions relating $k$ to $k_0$ are different. In other words, the relation between $k$ and $k_0$ is fixed by the kinematics of the nucleon-nucleus system, so we are off the two-body energy shell. This difference is expected to be unimportant at small forward angles, however.

If we define

$$\sigma_f \equiv \frac{\nu_k}{\sigma_{\text{c.m.}}} \left( \frac{E}{m} \right)^2 \frac{J(0)}{J(\theta)} \sigma_{\text{lab}}, \quad E = 2E_1$$

and

$$\sigma_c \equiv \langle \nu_k | P_{11} | \nu_k \rangle (2\pi)^4 \left[ \frac{E_1}{m} \right]^2$$

($J(\theta)$ is just the transformation function between the lab and the two-nucleon c.m. frame, $\sigma_{\text{lab}} = J(\theta) \sigma_{\text{c.m.}}$), we obtain, for $D^{nd}$ and $D^d$,

$$D^{nd} = w(k_0) \left\{ \frac{1}{2} [F_s(q) + F_a(q)] [-A \text{ tr} \Lambda \sigma_f + A^2 \text{ tr} \Lambda \sigma_c] 
+ 1/2 \left[ F_s(q) - F_a(q) \right] [-1/4 A^2 P_n^{1/2} \text{ tr} \Lambda \sigma_f + 4A P_n^{\text{el}} \text{ tr} \Lambda \sigma_c] \right\}$$

(4.5)
and

$$D^d = W(k_f) \Delta \text{tr} \Lambda \sigma_{\text{lab}},$$  \hspace{1cm} (4.6)$$

as in reference 6. Here $\Lambda$ is the density matrix for the spin of the incident particles.

Although $\sigma_k^\nu$ requires a detailed knowledge of the scattering matrix, it is directly related to the single-scattering-impulse approximation for the elastic-scattering cross section,

$$\sigma_{\text{el}}^\nu = [\Lambda C(\theta)]^2 \sigma_k^\nu,$$  \hspace{1cm} (4.7)$$

where

$$C(\theta) = \int e^{-i\mathbf{q} \cdot \mathbf{r}} P(\mathbf{r}) d^3 \mathbf{r}.$$ 

In a similar fashion we find, for the polarization averages,

$$N^\text{nd} = W(k_0) \left\{ \frac{1}{2} [F_s(q) + F_a(q)] [-A P_n^\dagger \text{tr} \Lambda \sigma_f + A^2 P_n^\text{el} \text{tr} \Lambda \sigma_C] \right. 
\left. + \frac{1}{2} [F_s(q) - F_a(q)] [1/4 A^2 P_n^\dagger \text{tr} \Lambda \sigma_f + 4A P_n^\text{el} \text{tr} \Lambda \sigma_C] \right\},$$

where
\[ p_n^{\text{el}} = \frac{\sum_k f_k (\nu_k | P_{11} | \nu_k)}{\sum_k f_k (\nu_k | P_{11} \nu_k)} \]

is the elastic polarization in the approximation that \( \sigma_{el} \) is the elastic-scattering cross section, and \( P_n^f \) is the average polarization in nucleon-nucleon scattering.
V. POLARIZATION CONTAMINATION

The expressions listed in the preceding section take on a slightly simpler form if we separate out the correlation portion of $F_s(a)$. Thus,

$$F_s(a) = c^2(\phi) + \lambda^{-1} c_s(a(q),$$

where $\lambda$ is the effective volume of the nucleus. Define

$$\Gamma(A/4) = \frac{1}{2} \left\{ \left( \frac{A}{4} + 1 \right) c_s + \left[ \left( \frac{A}{4} - 1 \right) c_w \right] \right\}$$

(5.1a)

and

$$\Lambda(A/4) = \frac{1}{2} \left\{ \left( \frac{A}{4} + 1 \right) c_s + \left( \frac{A}{4} - 1 \right) c_w \right\}.$$  

(5.1b)

The closure approximation to the quasi-elastic polarization becomes

$$\langle P_n \rangle_{\text{total}} = \frac{N}{D} = \frac{W(k_o) \rho [-\partial\Gamma(A/4) P_r^f + A R \Gamma(4/A) P_n^f] + W(k_f) A P_n^f}{W(k_o) \rho [-\partial\Gamma(A/4) + A R \Gamma(4/A)] + W(k_f) A}.$$  

where

$$R = \frac{\text{tr} A \sigma}{\text{tr} A \sigma_{ab}} \quad \text{and} \quad \Omega = \frac{J(0)}{J(\phi)}.$$
where
\[
n = D \text{ for } \Delta E = 0
\]
and
\[
n = 1/2 D \text{ for } \Delta E > 0.
\]

Here \( \Delta E \) is defined as in Section II, and \( D \) is the energy resolution. From this example it is clear that \( W(k_0) \) and \( W(k_f) \) do not differ greatly for forward scattering, and thus their effect essentially cancels out of \( \Delta P \), as anticipated above.

As our final expression for contamination, we take
\[
\Delta P = \frac{(p_f^e - p_e^f)}{n \frac{\Delta}{} (4/\Delta \sqrt{A}) \frac{1}{[A - \rho \Delta (A/4)]}}
\]
which is strictly valid for forward scattering.

To evaluate this expression, we must determine one- and two-particle distributions. This is described in the next section.
For contamination we write

$$\Delta P = P_n^{\text{total}} - P_n^{\text{el}},$$

where $P_n^{\text{el}}$ is defined in Eq. (4.9).

Thus

$$\Delta P = \frac{(P_n^f - P_n^{\text{el}})}{W(k_0) \rho A K \Gamma(4/\lambda)} \cdot \frac{1}{1 + \left[A W(k_f) - \rho A K \Gamma(4/\lambda) W(k_0)\right]}.$$ 

In the limit of small-angle scattering the contamination should be vanishingly small. For this special case we must take $W(k_f) = W(k_0)$, because there is no difference between "diagonal" and "nondiagonal" scattering at forward angles. And since $C_s(0) \approx C_a(0) \approx 0$ with $C(\theta) \to 1$, we may conclude $\Delta P \to 0$ for $\theta \to 0$.

Since we are essentially interested in small angles of scattering, we must expect the "diagonal" terms to make their full contribution. Hence $W(k_0) \approx W(k_f)$, and we see that in this approximation the contamination is independent of the detailed shape of the $W$'s.

Fowler suggests the following form for $W(k)$ of the shape discussed in Section II:

$$W(k) = \left[\exp\left(-n^{-1}\Delta E\right) + 1\right] / \left[\exp\left[-n^{-1}(\Delta E - E + E_k)\right] + 1\right],$$
VI. CORRELATION FUNCTIONS

Detailed knowledge of realistic correlation functions for nuclei is unavailable at present. At best one may propose certain models as close facsimiles of the nucleus and calculate these quantities within the framework of these special models. This procedure is usually quite successful, however, because the essential properties of the nucleus are generally dominated by a few important factors that may be readily incorporated into a simple model. For example, in an infinite nucleus the exclusion principle prevents a nucleon from being excited into a nearby state except near the Fermi surface. Thus the correlation between two distant particles is forbidden, as such correlation corresponds to very small energy excitation. The effective two-particle wave function rapidly approaches that of two uncorrelated particles. This rapid return to negligible correlation is what Gomez et al. refer to as the short "healing distance" of nuclear matter.

If we may assume that correlations in nuclear matter are relevant to those in finite nuclei, then it is clear that the Pauli exclusion plays a most significant role. Thus we will first consider a pure Fermi gas model and then modify it by the introduction of some dynamical correlation.

PURE FERMI GAS

We wish to know

\[ P_s(a) = \sum_{\alpha \neq \beta} \frac{\langle 0| s(x - z_{\alpha}) s(x' - z_{\beta}) P_{\alpha \beta} s(a)|0\rangle}{\sum_{\alpha \neq \beta} \langle 0| P_{\alpha \beta} s(a)|0\rangle}. \]
For the present model the nuclear wave function is

\[ \sqrt{N!} \phi (1, \ldots, N) = \sum_{p} (-1)^{p} \phi_{1}(1) \cdots \phi_{N}(N), \]

where

\[ \phi_{1}(1) = \frac{1}{\sqrt{1/2}} e^{i k_{1} \cdot z_{1}} i_{1}(1) s_{1}(1). \]

Here \( i(1) \) and \( s(1) \) are isospin, spin eigenfunctions. Thus

\[ \sum_{\alpha \neq \beta} \langle 0 | \delta(x - z_{\alpha}) \delta(x' - z_{\beta}) p_{\alpha \beta} s^{(a)} | 0 \rangle = \sum_{\alpha \neq \beta} \frac{1}{\sqrt{2}} \]

\[ \times \left[ e^{-ik_{\alpha} \cdot x} e^{-ik_{\beta} \cdot x'} \frac{1}{e} (l_{\alpha} p_{xx'}) \right] \left[ e^{ik_{\alpha} \cdot x} e^{ik_{\beta} \cdot x'} \right] \]

\[ - \delta_{\alpha \beta} \delta_{\tau \alpha} \delta_{\tau \beta} e^{ik_{\alpha} \cdot x} e^{ik_{\beta} \cdot x}. \]

Let

\[ x'_{F} = \left\{ \frac{A}{4} \right\} \left( \frac{A}{4} - 1 \right)^{-1} \sum_{k \neq k_{F}} e^{i(k_{\alpha} - k_{\beta}) \cdot (x - x')} \]
\[ P_s(a) (x, x') = \frac{1}{y^2} \left[ 1 + G_s(a) \right], \]

where

\[ G_s = \left[ \frac{1 - \left( \frac{1}{4A} \right)}{1 + \left( \frac{1}{4A} \right)} \right] J_F, \quad G_a = - J_F, \]

and

\[ J_F \approx \frac{9}{2} \pi \frac{J_{3/2}^2 (k_F |x - x'|)}{(k_F |x - x'|)^3}, \]

with \( k_F \) the Fermi momentum for the nucleus. If we designate

\[ \mathcal{F} [f] = \int f(x, x') e^{i q \cdot (x - x')} d^3x d^3x', \]

then

\[ \rho \Gamma(A) = A \Phi^2(\theta), \quad (6.1a) \]
and

\[ \rho \Gamma(A/4) = [(A/4) - 1] \rho \left( \frac{nF}{V} \right) + A C^2(\theta). \]  \hspace{1cm} (6.1b)

In this model we have taken

\[ F(z) = 1/y \quad \text{if} \quad |z| < R, \quad \text{or} \quad 0 \quad \text{if} \quad |z| > R. \]

Hence

\[ \rho \Gamma(A/4) = \frac{9}{2} \pi A j_{3/2}^2(qR) \]  \hspace{1cm} (6.2)

where \( q = 2P_0 \sin \frac{\theta}{2} \), which corresponds to the momentum-energy condition of elastic scattering. \( \Gamma(A/4) \) may be treated in a similar fashion.

If the upper limit of the above integral is extended to infinity (this results in negligible error for small momentum transfer), we find

\[ \rho \Gamma(A/4) = 2(A/4 - 1)(z - 1)^2 (z + 2) + A C^2(\theta), \]  \hspace{1cm} (6.3)
where

\[ z = \frac{q}{2k_F} \]  

Note that in the limit \( q \to 0 \) and \( \rho \Gamma(A/4) \to A \). 

Thus the expected result of zero contamination for vanishingly small momentum transfer is explicitly demonstrated for this particular model. In fact (6.4) is a rather general result, quite independent of any special model. This can be seen by observing that both \( P_2 \) and \( P \) are normalized in (3.5b). Hence

\[ \int g_s(a) \left| \frac{1}{r} \right| d^3r = 0 \text{ or } \lim_{q \to 0} \left[ g_s(a) \right] = 0, \]

which leads to (6.4) directly.
FERMI GAS WITH CORE CORRELATION

The model described above can be improved somewhat by introducing additional correlation of the "core" type. Many sophisticated methods are available for the introduction of dynamical correlation in the nucleus. However, these techniques are rather cumbersome and, in fact, unnecessary because we are here interested only in rough estimates of polarization contamination. Thus we limit ourselves to a simple calculation that should be adequate.

Jastrow proposed a nuclear wave function in which the core correlation was introduced as a variational parameter. We have

$$\tilde{\psi}(1, \cdots, N) = \prod_{i>j} [1 + \lambda(i,j)] \tilde{\psi}_F,$$

in which $\tilde{\psi}_F$ is the Fermi gas-model wave function described above. A number of choices are possible for $\lambda(i,j)$, but they all have the common feature that the probability for two particles to be closer than some small distance must be negligible, and the effect of $\lambda(i,j)$ must disappear for large separations. To incorporate these two features in a simple manner, we take $\lambda(1,2) = e^{-\left(\frac{r}{r_c}\right)^2}$, where $r_c$ is related to a mean correlation distance. Again we must calculate

$$P_{s(a)} \propto \sum_{\alpha \neq \beta} \langle 0| \delta(x - z_\alpha) \delta(x' - z_\beta) P_{\alpha \beta} s(a), \langle 0 |$$

(Eq. cont.)
\[ \psi^2 = |1 + \lambda(x, x')|^2. \] Hence

\[ P_{S(a)}^J(x, x') = |1 + \lambda(x, x')|^2 P_{S(a)}^F(x, x'). \] (6.6)

This same result may be obtained in a more rigorous fashion by noting that the effect of the core correlation can be introduced by developing the wave function in an Ursell-Mayer-type cluster expansion and keeping only the first term, which corresponds to neglect of all but two-particle clusters. That is, we write
\[ \Pi_{i>j} |1 + \lambda(i,j)|^2 = 1 + \sum_{i>j} \left\{ |1 + \lambda(i,j)|^2 - 1 \right\} \]

and introduce this into (6.5). Note that we still deal with a uniform density for the nucleus.

With this additional correlation, instead of (6.4a) and (6.4b) we have

\[ \rho \Gamma(\frac{A}{A}) = A \mathcal{F} \left[ \frac{V^2}{\psi^2} \right] \]  \hspace{1cm} (6.7a)

and

\[ \rho \Gamma(\frac{A}{4}) = \left[ (\frac{A}{4}) - 1 \right] \rho \mathcal{F} \left[ \frac{V^2}{\psi^2} \right] + A \mathcal{F} \left[ \frac{V^2}{\psi^2} \right]. \]  \hspace{1cm} (6.7b)

Hence the net result of introducing the core correlation is the modulating factor \( \psi^2 \). For (6.7a) we have

\[ \rho \Gamma(\frac{A}{4}) = \rho \Gamma_0(\frac{A}{4}) + \Delta(\frac{A}{4}), \]  \hspace{1cm} (6.8)

where the small correction
Here \( N_{R_c} \) denotes the number of particles in the correlation sphere described by \( R_c \). Thus we may expect only a small correction to the result for pure Pauli correlation. Now (6.7b) may be treated in a similar fashion; however, the first term is slightly complicated by the presence of \( \psi^2 \) as a factor in the transform \( \mathcal{F} \). Dealing essentially with a Fourier transform, we can take advantage of the convolution theorem for the Fourier transform of the product of two functions whose transforms are already known. Again the result may be expressed as a correction to the pure Pauli result.

Thus
\[
\rho \Gamma(A/4) = \rho \Gamma_0(A/4) + \Delta(A/4),
\]

where
\[
\Delta(A/4) = \left[ (A/4) - 1 \right] \left\{ \lambda(z, \gamma/\sqrt{2}) + \lambda(-z, \gamma \sqrt{2}) \right\} - 2\left[ \lambda(z, \gamma) + \lambda(-z, \gamma) \right].
\]

(6.9)

Here
\[ \lambda(z, \gamma) = 2/\sqrt{\pi} \frac{\gamma}{z} \int_0^1 x(x+2)(x-1)^2 e^{-(x-z)^2\gamma^2}, \]

where

\[ \gamma = \frac{R_c}{R_F} \]

and

\[ z = \frac{q}{2R_F}, \]

as before.

Let us discuss the results of these two models. We notice that \( \rho \Gamma(A/4) \) is dominated by the mean radius of the target. Corrections from core correlation introduce an effect that is of the order \( \left( \frac{R_c}{R} \right)^3 \Delta \); thus for small correlation length, we can expect the simple pure Fermi model to be adequate. The form of \( \rho \Gamma(A/4) \) is prescribed by the two correlation lengths, their relative importance being determined by the ratio \( \gamma = \left( \frac{R_c}{R_F} \right) \). It is not obvious that \( \Delta(A/4) \) in (6.9) constitutes a small correction without closer inspection of its form. The important fact is that for the term \( \lambda(z, \gamma) \) there is a corresponding term \( \lambda(-z, \gamma) \). Thus an appreciable mutual cancellation can be expected the net effect of which is a reduction of the importance of \( \Delta(A/4) \). This argument is not complete owing to the presence of the \( 1/z \) factor in \( \lambda \); however, an estimate of the integral for small \( z \) indicates that \( \lambda(+) - \lambda(-) \)
is in fact independent of momentum transfer and small compared with the corresponding term in $\rho \Gamma_0(A/4)$. We conclude that the Fermi gas model should be adequate.

For small nuclei we may account for the finite size of the nucleus with a gaussian shape for the density, $e^{-x^2/R^2}$. In (6.1a) we now have

$$C^2(\theta) = e^{-1/2 \frac{q^2}{R^2}}$$

where $R$ is the effective radius.
VII. DISCUSSION

Equation (5.2) and the expressions for $\rho \Gamma (A/4)$ and $\rho \Gamma (4/A)$ may be used to estimate the inelastic contamination expected in elastic polarization experiments. Its validity is limited to small-angle scattering, but this is no serious drawback, as we are interested in how much contamination may be expected even at small angles. That is, the amount of contamination will certainly increase with angle of scattering [as Eq. (5.2) shows], and it is important to get some idea how bad things can get even in a region where the problem might not be expected to be very serious. Finally, because we have used the closure approximation, $\Delta P$ will always overestimate the contamination.

To evaluate $\Delta P$ we must know:

(a) $P_n^f$ = free nucleon-nucleon polarization averaged over target identity and initial spin orientation of target,

(b) $P_n^{el}$ = elastic nuclear polarization for $T = 0$, $S = 0$ target in single-scattering impulse approximation,

(c) $\bar{\sigma}_{lab} = \text{lab nucleon-nucleon cross section averaged as in (a)},$

(d) $\bar{\sigma}_{el} = \text{elastic nuclear cross section for same conditions as in (b)}.$

Now $P_n^f$ and $\bar{\sigma}_{lab}$ may be determined directly from experimental data for nucleon-nucleon scattering,\textsuperscript{12}

\[
\bar{\sigma}_{lab} = \frac{1}{2} \left( \bar{\sigma}_{lab}^{PP} + \bar{\sigma}_{lab}^{PN} \right) \quad P_n^f = \frac{1}{2} \left( P_n^{PP} + \bar{\sigma}_{lab}^{PN} P_n^{PN} \right)
\]
Unfortunately $\bar{\sigma}_{el}$ and $P_{el}^{n}$ are not immediately related to experimental data. Recall that these are not the actual elastic-scattering parameters, but rather the Born approximations. In some cases, if the Born-approximation results are expected to be fair estimates to the actual scattering data, it is possible to use the experimental data directly. This procedure is actually quite useful in determining $P_{el}^{n}$, since it is known that Born-approximation calculations of the elastic polarization are actually quite good for angles not too near the first diffraction minimum. Furthermore, in many cases it is possible to take advantage of the numerous Born-approximation calculations for nucleon-nucleus scattering. In general, however, detailed knowledge of combinations of nucleon-nucleon scattering amplitudes is necessary to determine these quantities, in particular, $P_{ll}$, $P_{ll}$.

To get some idea how one can make a rough estimate of contamination in a particular case, consider proton-carbon scattering at 310 MeV. Considerable experimental data are available for this example. Here, rather than use actual scattering amplitudes, we may take advantage of the numerous Born-approximation calculations.

We write, for contamination,

$$\Delta P = \frac{(P_{el}^{n} - P_{el}^{f})}{1 + A \frac{\bar{\sigma}_{el} / \sigma_{lab}}{[1 - (\cos \theta_{L})^{-1} (1/A) \Gamma(A/4)]}}$$

$$= \frac{(P_{el}^{n} - P_{el}^{f})}{1 + \frac{A}{\bar{\sigma}_{el} / \sigma_{lab}} Q(A, \theta)}$$
Here \( \sigma_{el} \) is the Born-approximation differential-scattering cross section and \( (\cos \theta_L)^{-1} \) is the c.m.-lab conversion factor in the nonrelativistic approximation. In \( \Gamma(A/4) \) we take \( C^2(\theta) = \exp \left( -\frac{\theta}{2.69} \right) \), \( \theta \) in degrees, resulting from \( p(r) = \pi^{-3/2} a^{-3} \exp \left( \frac{-r^2}{a^2} \right) \); where \( a = 1.96 \) as determined to give the correct root-mean-square radius. Here \( \sigma_{el} \), \( P_{el} \) are taken from Born-approximation calculations of Saperstein and Feldman. Nucleon scattering data are taken from a review article. The result is illustrated graphically for the angular range 0 to 20 deg. in terms of percent correction of the Born-approximation calculation by Saperstein and Feldman.

As expected, we see that the correction is truly negligible for angles less than 10 deg., say. This result stems from the fact that \( \sigma_{el} \) is so much larger than \( \sigma_{lab} \). At angles \( \theta > 17^\circ \), the correction seems anomalously large; however, recall that we are in a position only to give upper limits, as we have employed closure.

Some useful observations can be made from this approximate calculation that can be of some use in general. For example, from the summary of data presented by Hess we see that \( \sigma_{lab} \) rapidly stabilizes to a cross section of approx. 4 nib and is roughly independent of energy in the range 200 to 400 MeV. The \( Q(A, \theta) \) is stable against variation after approx. 6 deg. in this particular calculation, and it seems that this stability against variation, at least for angles having \( \theta < 25^\circ \), is a rather general result. Furthermore, as an order-of-magnitude estimate we may take \( Q(A, \theta) \approx \frac{1}{A} \). Hence, purely as a "rule of thumb" procedure, we may take
\[ \Delta P \approx \frac{P_n^f - P_n^e}{1 + \frac{1}{A} \frac{\sigma_e}{\sigma_{\text{lab}}}} \quad \text{for } \theta > \theta_{\text{min}}, \]

where \( \sigma_{\text{lab}} \) may usually be taken as some fixed value appropriate for the energy range considered: \( \theta_{\text{min}} \approx 6^\circ \) in the example considered above.

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The various combinations of nucleon-nucleon amplitudes that arise in Eq. (4.12) are

\[ P_{11} = 2 \Re A_1^* E_1 + (|A_1|^2 + |E_1|^2) \sigma_n, \]

\[ P'_{11} = 2 \Re A_1^* E_1 \sigma_n + (|A_1|^2 + |E_1|^2), \]

\[ \text{tr}_{\mathbb{H}11} = 2 \Re E_1^* B_1 + (|B_1|^2 - |C_1|^2 - |D_1|^2 + |E_1|^2) \sigma_n, \]

and

\[ \text{tr}_{\mathbb{H}11}' = 2 \Re E_1^* B_1 \sigma_n + (|B_1|^2 + |C_1|^2 + |D_1|^2 + |E_1|^2). \]

The following matrix elements were used in Section IV:

(a) \[ \sum_{\alpha \neq \beta} \langle 0 | \sigma_\alpha \cdot \sigma_\beta | 0 \rangle = \frac{\Lambda}{2} (A + 1), \]

(b) \[ \sum_{\alpha \neq \beta} \langle 0 | \sigma_\alpha \cdot \sigma_\beta | 0 \rangle = -3A, \]

(c) \[ \sum_{\alpha \neq \beta} \langle 0 | \tau_\alpha \cdot \tau_\beta | 0 \rangle = -3A, \]

(d) \[ \sum_{\alpha \neq \beta} \langle 0 | \tau_\alpha \cdot \tau_\beta \sigma_\alpha \cdot \sigma_\beta | 0 \rangle = -9A, \text{ and (h)} \sum_{\alpha \neq \beta} \langle 0 | \sigma_\alpha \cdot \sigma_\beta \tau_\alpha \cdot \tau_\beta | 0 \rangle = 3A, \]
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Fig. 1. Polarization contamination in proton-carbon scattering at 310 MeV.
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