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Author
Goldhaber, Gerson.

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MULTIPION AND BARYON RESONANCES IN THE $\pi^+ + p$ REACTION

Gerson Goldhaber

July 1964
MULTIPION AND BARYON RESONANCES IN THE $\pi^+ + p$ REACTION

Gerson Goldhaber
Lawrence Radiation Laboratory and
Department of Physics
University of California, Berkeley, California

I. INTRODUCTION

Various stages of the work I want to discuss today have been carried out by John Brown, Sulamith Goldhaber, John Kadyk, Thomas O'Halloran, Benjamin Shen, George Trilling, and myself. What I want to discuss today concerns meson-nucleon interactions which lead to four particles in the final state. Most of my talk will be devoted to the reaction

$$\pi^+ + p \rightarrow \pi^+ + \pi^- + \pi^+ + p$$

at 3.65 BeV/c. \(1\)

The data comes from an experiment which we have carried out in the last year by use of the 20-inch Brookhaven bubble chamber exposed in the Brookhaven-Yale separated beam at the A.G.S. All the analysis was carried out in Berkeley using the PANG and KIC/K programs, as adapted to this chamber.

II. AN EXAMPLE OF THE FOUR-PARTICLE PHASE SPACE TRIANGLE

Before I start with the subject proper, I would like to discuss an earlier experiment which has already been reported in the literature,\(^1,2,3\) which, however, illustrates why we got interested in the four-particle final state. This experiment was on the reaction

$$K^+ + p \rightarrow K^+ + \pi^- + p + \pi^+$$

at 1.96 BeV/c \(2\)

The experiment was carried out in the same chamber and beam. In this experiment and for these types of events we have observed an effect which we have called "double resonance formation" where we effectively observe a dominance of what
might be called a two "particle" interaction. Here these two particles (or composites) are resonances which then decay into two particles each. The reaction is thus

$$K^+ + p \rightarrow K^* + N^{*+}$$

$$K^+ + \pi^- \rightarrow p + \pi^+$$

(3)

In this experiment we found that 64% of all the interactions leading to four particles in the final state proceeded through this double resonance channel. In connection with this work, we have noted that the kinematical boundaries for the reaction are particularly simple if we plot the mass of one of these two particle composites against the mass of the other of these composites. This is, of course, true irrespective of whether any resonances are present between either set of two particles. The resulting kinematical boundaries are given simply by an isosceles triangle which we have called the "four-particle phase space triangle". This is illustrated in Fig. 1. The phase space triangle has the nice feature that the x and y axes remain fixed. A change in the total available energy or in Q simply moves the hypoteneuse away from the origin. Any resonance band which may be present between the two participating particles in one or both of the composites will thus remain fixed. While the kinematical boundaries are extremely simple in this instance the phase space is not as nice as in the case of the Dalitz plot, namely the phase space is given by the product of three momenta as indicated in the figure. In Fig. 2, we have illustrated this phase space distribution in a three dimensional plot for the particular example we are discussing. Reaction 2 turned out to be a rather fortunate choice as our first example of double resonance formation for the following reasons: the $K^+$ and $\pi^-$ form a $K^*(890)$, while the $p$ and $\pi^+$ form an $N^{*+}(1238)$ and there are no other strong competing resonance effects that we know of, that is to say, the $K^+$ and $p$ do not give any known resonance effect, the $\pi^+$ and $\pi^-$ are below the
ρ threshold and thus do not appear to give any known resonance effect. The only other known resonance which does occur is the \( N^{0}(1238) \) which, as is well known, amounts to about 10% of the \( N^{++}(1238) \) resonance. Thus, basically, except for about 10% contamination effects, the double resonance formation is the most important process that occurs here. This is illustrated in Fig. 3 where we show the triangle plot as well as the projection on the two mass axes. For reaction 2 we were able to show that the double resonance formation proceeds through the exchange of a spin 0 object, possibly a pion, as far as angular distributions are concerned. In fact, this reaction allowed us to get a conclusive measurement on the spin of the \( K^{*} \), because of the alignment caused by the exchange of a spin 0 object.\(^1\) As far as absolute cross sections are concerned, however, the one pion exchange model is inadequate to explain the experimental data and rather severe form factors had to be introduced to obtain a quantitative fit.\(^3\)

III. APPLICATION TO THE \( π^{+} + p \) REACTION

Now, I wish to come to the main topic of my talk today, namely, Reaction (1). The principal channels for this reaction are given in Table I. Here we again form four particles in the final state, however the situation is somewhat more complicated. First of all we have more energy in the center of mass, namely, \( E^{*} = 2.78 \) BeV. This allows for considerable portions of the resonance bands in the triangle plot to lie outside the double resonance region in Fig. 4. Furthermore, we have the complication that we are dealing with two \( π^{+} \) mesons and hence each event can contribute two points to the triangle plot.

Since one of the dominant processes in Reaction (1) is the formation of the \( N^{++} \) resonance, we found it expedient to classify our events into types according to whether or not a \( π^{+} p \) mass value falls inside the \( N^{++} \) band, defined
by $1.12 \leq M(p\pi^+) \leq 1.32$ BeV. We call that group of events for which one $\pi^+$ mass falls inside the $N^{*++}$ band "type 1" (1024 events). Those events for which both $\pi^+$ mesons form a $\pi^+p$ mass which lies inside the $N^{*++}$ band we call "type 2" (118 events). Finally, "type 3" (642 events) refers to events for which both $\pi^+p$ mass combinations lie outside the $N^{*++}$ band. This breakdown into types is illustrated in Fig. 4. In Fig. 4 we present a scatter plot of the $\pi^+p$ mass against the $\pi^+\pi^-$ mass. Since each event has two $\pi^+$ mesons it is represented by two points in this triangle plot. For clarity we have left out from Fig. 4a those points of type 1 events which correspond to the reflection of the $N^{*++}$ band. These points are shown separately in Fig. 4e, with their projection (Figs. 4f and 4g). The most prominent feature of the $M(\pi^+\pi^-)$ distribution is $\rho^0$ production, defined here by $0.65 \leq M(\pi^+\pi^-) \leq 0.85$ BeV (Figs. 4c and 4d) proceeds with comparable cross sections through channels (1a) and (1b). When we examine the projection of points due to type 1 events in which we form the $\pi^+\pi^-$ mass by combining the $\pi^+$ of the $N^{*++}$ isobar with the $\pi^-$ meson we find no appreciable $\rho^0$ production above background.

A. The Enhancement in the $\pi^-p$ Mass Distribution

By considering the events outside the $N^*$ band (type 3) but inside the $\rho^0$ band we have observed a strong mass enhancement in the $\pi^+\pi^-\pi^+$ mass distribution. We first noted a marked difference in the four momentum transfer, $\Delta^2$, to the $\rho$ meson for events of type 1, i.e., with $N^{*++}$ production, and those of type 3 (see Fig. 5). In the latter case the $\Delta^2$ distribution is much wider with a half width of about $50 m_{\pi}^2$. This gave us the clue that the $\rho$'s in the type 3 events are produced by a different mechanism. In Fig. 7 we show the Dalitz plot in which the mass squared of the $p\pi^+$ is plotted against the mass squared of the $\rho^0\pi^+$ system. For events in which either $\pi^+\pi^-$ combination fall inside the $\rho^0$ band, the so-called "double $\rho$" events, we have plotted only one point.
chosen arbitrarily. We have observed a clear mass enhancement in the region of 1 to 1.4 BeV. We refer to this as the $A^+$ band where the $A^+$ is produced in the reaction (see Fig. 6c)

$$\pi^+ + p \rightarrow A^+ + p$$

(4)

and

$$A^+ \rightarrow \rho^0 + \pi^+$$

(5)

In Fig. 7c, which shows the projection of the Dalitz plot excluding the events inside the $N^{++}$ band, this mass enhancement can be clearly observed. We have also noted that there appears to be some structure in the mass enhancement. In Fig. 7d, we show a plot in which the $N^+$ band is included as well. Here, too, a clear-cut enhancement above phase space can still be discerned. Furthermore, we found that the mass enhancement was also associated with low four momentum transfer to the $\rho^0 \pi^+$ system as illustrated in Fig. 8.

Shortly after the publication of our results this mass enhancement was confirmed by Chung et al.\textsuperscript{5} in the Alvarez Group at Berkeley working with the $\pi^- p$ reaction at 3.2 BeV/c and by the Anglo-German collaboration group\textsuperscript{6} working with the $\pi^+ p$ reaction at 4 BeV/c. The suggestion of structure which we commented on in our paper shows up extremely clearly in these new data and we are apparently dealing with two distinct peaks: the $A_1$ at about 1090 MeV and the $A_2$ at around 1310 MeV. (See Figs. 9 and 1A) What is more, in the work of Chung et al., a peak corresponding to $A_2$ has been observed in the $K^0 L_1$ and $K^- K^0$ mass distributions. While in our old work we could only state that the $A$ enhancement has isotopic spin $T = 1$ or 2 and odd $G$ parity, with this new result, the isotopic spin must clearly $T = 1$, for the $A_2$ meson.

In Fig. 10 we reproduce the results of the Anglo-German collaboration experiment in which they present evidence for the decay mode
In Fig. 11 is our own data relating to this decay mode. The \( \eta^0 \) mesons from our data are the ones decaying into \( \pi^+ - \pi^- \). The events come from the reaction,
\[
\pi^+ + p \rightarrow \pi^+ \pi^- \pi^0 + p
\]
(7)
The corresponding \( \pi^+ - \pi^- \) mass distribution is given in Fig. 12. As can be noted there is some enhancement in the \( A_2 \) region and hardly any in the \( A_1 \) region. The spin analysis is given schematically below.

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**SPIN ANALYSIS FOR THE \( A_2 \) MESON**

Decay Mode: \( A_2^\pm \rightarrow \rho^0 + \pi^\pm \) (1) \( T = 1 \) or \( 2 - G \) is odd

<table>
<thead>
<tr>
<th>( J^P(\pi) )</th>
<th>( J^P(\rho) )</th>
<th>( J^P(A) )</th>
<th>Ruled out by:</th>
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<tr>
<td>0^-</td>
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<tr>
<td>0^-</td>
<td>2^+</td>
<td>2^-</td>
<td>3^-</td>
</tr>
</tbody>
</table>

Decay mode: \( A_2^\pm \rightarrow \eta^0 + \pi^\pm \) (2) \( T = 1 - G \) is odd

<table>
<thead>
<tr>
<th>( J^P(\pi) )</th>
<th>( J^P(\eta^0) )</th>
<th>( J^P(A) )</th>
<th>Ruled out by:</th>
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<td>0^-</td>
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<td>( \pi^+ - \pi^- ) Dalitz plot</td>
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Decay mode: \( A_2 \rightarrow K^+ K^- \) (3) \( T = 0 \) or \( 1 \), since \( G \) is odd, and together with (1) \( T = 1 \) must apply. But \( G = (-1)^{I+L} \) hence \( 0^+, 2^+, 4^+ \), ... are the only allowed \( J^P(A) \) values. Thus \( J^{PG}(A_2) = 2^+ \) follows from decay modes (1) and (3). This value is also consistent with (2).
In Fig. 13 we show our latest data on the $A$ mesons. Here we have combined our old data on reaction (1) with new data from the equivalent $\pi^-p$ reaction. In both instances the $N^{++}(1238)$ band has been excluded. In this plot the $A_2$ resonance stands out clearly at $E(A_2) = 1335 \pm 10$ MeV and $\Gamma(A_2) = 90 \pm 10$ MeV. It must be pointed out, however, that while we observe a clear enhancement in the $A_1$ region above background, and distinct from the $A_2$ peak, we do not observe a clear-cut peak there. As we have pointed out before, the $A_1$ enhancement may be due to the Peierl's mechanism as discussed by Nauenberg and Pais and more recently by Oakes and also by Franklin. Here we must also note that the $A_1$ enhancement coincides in mass with the $\pi^+\pi^-\pi^+$ mass enhancement observed in the $\pi^-p$ reaction from 6 to 16 BeV/c in the Ecole Polytechnique heavy liquid bubble chamber exposed at CERN.

Finally, if the $K\bar{K}$ and $\eta\pi$ decay modes are indeed the decay products of the $A_2$ meson, we must be observing an appreciable violation of the $A$ quantum number introduced by Bronzon and Low.

B. Mass Shift and Resonance Narrowing Effects in the $\rho$ Mesons from a Decay

We have looked at the $\pi^+\pi^-$ mass distributions for events with $N^{++}$ removed. There we have further considered events from the $A_1$ band ($1000 \leq M_{\pi^+\pi^-} \leq 1200$ MeV) and $A_2$ band $1250 \leq M_{\pi^+\pi^-} \leq 1450$ MeV, separately. The idea behind this approach was that while in previous studies of $\rho$ mesons these were always produced at an interaction vertex. In the present data they come from the decay of a resonance (or resonances). Thus, if two $\rho$ mesons with different quantum numbers exist, such differences may show up in the present case.

As may be noted from Fig. 14 we indeed find a mass shift between the $\rho$ mesons associated with the $A_1$ and $A_2$ bands. We find $E_\rho(A_1) = 750$ MeV and $E_\rho(A_2) = 785$ MeV respectively and $\Gamma = 80$ MeV in both cases. Thus there appears to be
a mass shift and resonance narrowing effect occurring here. At first sight one might think that such a mass shift could be due to the selection of different three particle masses in the two cases. To investigate this point we have made careful comparisons with similar "events" corresponding to $\pi + p \rightarrow \rho + \pi + p$ generated artificially by a Monte Carlo method. For these artificial events we introduced a "$\rho$ resonance" with $E_\rho = 770$ MeV and $\Gamma_\rho = 120$ MeV to reproduce the observed distribution with no three pion mass band selection. We also removed events with $p\pi$ masses lying in the $N^{*+}$ band in such a manner as to simulate reactions (4) and (5).

We conclude from these tests that no such mass shift is introduced in the "artificial events" purely by the three pion mass selections we have made (see Fig. 14). Thus it would appear that the observed mass shift is a physical effect related to the two mass peaks $A_1$ and $A_2$. Here we must also note that centrifugal barrier effects expected in the $A_2$ would give a mass shift in the opposite direction, i.e., a preference for higher velocity and thus lower mass $\Pi$ mesons. The peculiar structure of $\rho$ peaks has often been attributed to the presence of the two-pion decay of the $\omega$. To the extent that the $\rho$ mesons shown here indeed come from $A$ decay this effect cannot be of importance here. To the extent that "double $\rho$" formation contributes to $A_1$ formation, a mass shift may be introduced here in the observed direction due to the location of the boundary of the Dalitz plot which cuts the "double $\rho$" region.

IV. ANGULAR DISTRIBUTIONS FOR EVENTS WITH $N^{*+}$ PRODUCTION

We will now focus our attention on those events giving $N^{*+}(1238)$ production. In particular, I want to discuss the angular distribution in the $\pi^+\pi^-$ system in the channel:

$$\pi^+ + p \rightarrow N^{*+} + \pi^+ + \pi^-$$ (8)
Let us define the decay angle of the $\rho^0$ meson to be between the decay $\pi^+$ and incident $\pi^+$ in the $\rho^0$ center of mass and that of the $N^{*+}$ between its decay pion and the target proton in the $N^{*+}$ center of mass. We will call these angles $\alpha_{\rho^0}$ and $\alpha_{N^*}$ respectively. We must note here that we actually use $\alpha_{\rho^0}$ to designate the decay angle even when the $\pi^+\pi^-$ mass value lies outside the $\rho$ band.

For one pion exchange one considers the $\rho^0$ and $N^*$ vertices to be $\pi-\pi$ and $p-\pi$ scattering vertices respectively. Therefore for pure $\rho^0$ and $N^*$ formation one expects distributions of the form $I(\alpha_{\rho^0}) = \cos^2 \alpha_{\rho^0}$ and $I(\alpha_{N^*}) = 1 + 3 \cos^2 \alpha_{N^*}$ respectively. We note, however, that both experimental distributions exhibit interference terms in $\cos \alpha$ leading to strong forward peaking in $\alpha_{\rho^0}$ as well as appreciable forward peaking in $\alpha_{N^*}$. Concentrating on the events for which $M(p_{1\pi^+})$ lies in the $N^{3/2}$ band our data exhibit the feature that the forward (F), backward (B) asymmetry of $\alpha_{\rho^0}$ as expressed by the ratio $(F-B)/(F+B)$ remains positive as the $\pi^+\pi^-$ mass traverses the $\rho^0$ mass region. (See Fig. 15b and 15c). We illustrate this asymmetry in Fig. 15 by a scatter plot of $\cos \alpha_{\rho^0}$ versus $M(\pi_{2\pi^-})$. A quantitative description of the asymmetry phenomenon is obtained by expanding the $\rho$ and $N^*$ decay angular distributions in the form of a polynomial $I(\alpha) = A + B \cos \alpha + C \cos^2 \alpha$. Tables II and III give the coefficients for $I(\alpha_{\rho^0})$ and $I(\alpha_{N^*})$ as a function of $M(\pi_{2\pi^-})$ and $M(p_{1\pi^+})$ respectively. Similar asymmetry phenomena have been observed in the reaction

$$\pi^- + p \rightarrow \pi^- + \pi^+ + n$$

(9)

The cause of the asymmetry in the decay of the $\rho^0$ is a mystery of long standing. In this respect the $\rho^0$ behaves quite differently from the charged $\rho$. In the case of the charged $\rho$ in the reaction

$$\pi^- + p \rightarrow \pi^- + \pi^0 + p$$

(10)
for example, some asymmetry is present. The asymmetry however, changes sign as the ρ⁻ mass is traversed and can hence be attributed to a small amount of essentially constant I = 2, s-wave contribution. On the other hand, for the π⁺π⁻ angular distribution in channel 8, ρ remains forward peaked and persists throughout the region of the ρ up to about 1.1 BeV at which point the asymmetry goes to 0 and changes sign in the region of the f° meson. Of course, in this latter region, the statistics become rather poor. The plot shown in Fig. 15, is without Δ² cutoff, however, the behavior in the ρ° region is the same even for low Δ² values.

V. ANGULAR CORRELATION EFFECTS

One of the basic consequences of the OPE model is that a pseudo scalar meson cannot transmit angular information. Thus, on the OPE model we expect the decay angular distribution of the ρ° meson to be uncorrelated with that of the N*++. Experimentally, we observe very considerable correlations between ρ and N*. The presence of such marked correlations between decay angles ρ and N* indicate that processes other than one pion exchange must contribute here. We see this correlation clearly in a scatter plot of cos ρ versus cos N*. We show such a plot for events contributing to the channel

$$\pi^+ + p \rightarrow N^{*++}(1238) + \rho^0$$

and Δ² ≤ 20 m₂ (see Fig.16). Without correlations the angular distributions in ρ, for example, should be independent of the value of N*. Instead we find on choosing the two polar and the equatorial intervals in N* that the corresponding distribution in cos ρ differs appreciably. (See Figs.16b,16c, and 16d). Expansion coefficients of I(ρ) and I(N*) for the distributions in Fig.16 are listed in Table IV. Examining such distributions for events with
\( \Delta^2 \) as low as \( \Delta^2 \approx 5m^2 \) we find that the same characteristics still persist. Thus in the present work we are unable to isolate the pure one-pion exchange process.

We have similarly searched for correlations between the Treiman–Yang angles \( \phi^p \) and \( \phi^N \) at the two vertices and found none. Although considerable non–OPE contributions are present here, the two Treiman–Yang angle distributions show only slight deviations from isotropy, Fig. 17. Thus, the correlation between the cosines of the two scattering angles \( \cos \alpha^p \) and \( \cos \alpha^N \) would appear to represent in this instance a more sensitive test for one-pion-exchange. The presence of these angular correlations indicate that other diagrams aside from OPE may be involved. In a general sense this was already clear from the fact that in order to fit cross sections on the OPE model one needs the introduction of very considerable form factors. A complete treatment of the observed effects would thus have to take into account a superposition of all such diagrams with suitable Bose symmetrization. An alternative approach explored by A.S. Goldhaber is to consider the exchange of more than one spin zero object in which case the factoring at each vertex, which leads to the lack of correlation on the OPE model, is no longer possible.

VI. KINEMATICAL PEAKS ASSOCIATED WITH POSSIBLE NUCLEON ISOBAR FORMATION

Let us now consider the observed phenomena from the point of view of nucleon isobar formation. If we study the three-particle mass distribution \( p^+ n^- \), and particularly that combination with small four-momentum transfer to the \( p^+ n^- \) system, viz. \( \Delta^2(p^+ n^-) \leq 15m^2 \). We observe a distinct mass peak at \( E(p^+ n^-) = 1480 \pm 10 \text{ MeV} \). If this is indeed a definite state, the simplest interpretation is to consider this peak as the inelastic decay of the \( N_{1/2}^* (1, 10) \) isobar, Fig. 18. The reason for the mass shift is not clear at present, however, we can point out that mass shifts of this order do occur.
As each event gives two $p\pi^+\pi^-$ mass triplets we have separated these by ordering them according to the production angle of the $\pi^+$ in the overall center of mass system. This is essentially the same as ordering them according to the $\Delta^2(p\pi^+\pi^-)$ value. The pion produced with the smaller angle we call $\pi_a^+$, the other $\pi_b^+$. The distribution of the $\pi_a^+$ production angle shows a very pronounced peaking at small angles indicative of small four momentum transfer squared, $\Delta^2$, to the $p\pi_b^+\pi^-$ system. The mass squared distribution of the $p\pi_b^+\pi^-$ system and the corresponding Chew-Low plot show the following distinctive features (see Fig. 19): there is an excess of events with low $p\pi^+\pi^-$ mass values above the number expected from phase space as well as from phase space modified by the presence of the $\rho^0$ and $N_{3/2}^{*+}(1238)$ resonance. These curves which were calculated by a Monte Carlo method do not include the effects of angular distribution in $\rho^0$ and $N^*$ center of mass systems. Furthermore, we find that the clustering of points in a strip at low mass values is associated with an enhancement of small $\Delta^2$ values, to the $p\pi_b^+\pi^-$ system. If we accept the conjecture that this $p\pi^+\pi^-$ mass band is associated with the lower vertex in Fig. 20, then this mass strip sweeps through the entire set of higher $I = 1/2$ and $I = 3/2$ nucleon isobars. In Fig. 19a, and 19b we present distributions for all mass triplets, (i.e., two per event), while in Fig. 19c and 19d we selected only one mass triplet per event, namely $p\pi_b^+\pi^-$, in order to isolate the above mentioned effect.

We wish to emphasize here that the points' clustering around high $\Delta^2$ and large mass squared values are almost entirely due to the $p\pi_a^+\pi^-$ triplets. This feature is a clear kinematic reflection produced by forming a $p\pi\pi$ mass triplet with the strongly forward peaked pion, $\pi_a^+$. 
Let us now focus our attention on those events with small $\Delta^2(p_n^+\pi^-)$ values. We chose a cutoff in $\Delta^2$ (i.e., $\Delta^2(p_n^+\pi^-) \leq 15 m^2$) such as to allow all possible $p_n^+\pi^-$ mass values. By these means we select 668 events out of a total of 1787 events. It is for these events that we note a peaking centered at $1.48 \pm 0.01$ BeV which may be associated with the three particle decay of the $N^{*+}_{1/2}(1510)$ resonance. When we now examine the three particles participating in this peak, defined here by $1.4 \leq M(p_n^+\pi^-) \leq 1.6$ BeV, they appear to interact in pairs with each other. Namely, the proton and $n^+_b$ give $N^{*+}_{3/2}(1238)$ formation exclusively (see Fig. 20d), the proton and the $\pi^-$ mass combination lies principally in the $N^{*0}_{3/2}(1238)$ band, (see Fig. 20e). In this kinematically highly constrained system, the $p_n^+\pi^-$ mass distribution peaks strongly at $\sim 390$ MeV. We find $E_{\pi\pi} = 390 \pm 10$ MeV and a width of $\Gamma_{\pi\pi} = 100$ MeV. This peak may be the same effect as the one observed by Samios et al.\textsuperscript{19} Here we must note that the masses of the four composites involved ($p_n^+\pi^-, p_n^+\pi^+, p^-\pi^+, n^+_b\pi^-$) are no longer independent. For a three particle composite of mass $M_{123}$ the following kinematical relation results from energy conservation:

$$M_{13}^2 = M_{123}^2 - m_1^2 - m_2^2 + m_3^2$$

(12)

while momentum conservation further restricts the physically allowed region by the triangle inequality:

$$p_1 + p_2 - p_3 \equiv 0, \ p_1 - p_2 + p_3 \equiv 0, \ \text{and} \ -p_1 + p_2 + p_3 \equiv 0$$

(13)

where

$$p_i^2 = [(M_{123}^2 - (m_i^2 + m_j^2))^2 - 4 m_i^2 m_j^2] / 4M_{123}^2$$

(14)

and $m_j$ and $m_i$ are the masses of the two particle composites and of the individual particles respectively. These relations determine the physically allowed kinematical boundary, i.e., the Dalitz plot. Thus by using the Breit Wigner distribution in $M_{123}$ the observed peak at $1480$ MeV and in $M_{12}$ and $M_{23}$ the
We have also selected events of types 1 and 2 with the four-momentum transfer squared to the $p\pi^+\pi^-$ system smaller than 15 m$_\pi^2$, $\Delta^2(p\pi^+\pi^-) \leq 15$ m$_\pi^2$. In Fig. 21, we show the correlation plots of the mass squared of various two particle systems against that of $p\pi^+\pi^-$ as well as the distributions in mass squared of the corresponding systems for these events.

We will now illustrate this kinematical relation which exists between the $N^*_{1/2}(1510)$ and the $N^*_{3/2}(1238)$ resonances by the Dalitz plot. Namely, we note that the intersection of the two $N^*_{3/2}$ bands, $N^o_{3/2}(1238)$ and $N^o_{3/2}(1238)$, in the $M^2(p\pi^+), M^2(p\pi^-)$ plane occurs within the Dalitz plot corresponding to a total energy of $\sim$1500 MeV for the $p\pi^+\pi^-$ system (see Fig. 22.). It is very tempting to speculate that the location of the 1510 isobar relative to the 1238 isobar is not purely an accident. We will refer to the above phenomenon as "compound resonance" formation. There is no doubt in our experiment that $N^*_{1/2}(1510)$ formation occurs strongly. If we accept the current ideas, in which the $N^*_{1/2}(1510)$ is considered as a bonafide resonance$^{20}$ then we must conclude that the $\pi\pi(390)$ peak occurs as a kinematical consequence. Alternately, one can consider the possibility that the $N^*_{1/2}(1510)$ is an enhancement which occurs due to the superposition or perhaps constructive interference between the two $N^*_{1/2}(1238)$ bands. Should it turn out finally that the $\pi\pi(390)$ peak has some dynamical origin, it could conceivably also contribute to the $N^*_{1/2}(1510)$ formation. This alternative appears less likely to us in view of the fact that $\pi\pi(390)$ formation does not appear to occur to any appreciable extent in our data other than in the events
associated with the 1480 peak. A similar compound resonance has recently been observed in the \( K\bar{K} \) system\(^\text{21}\) at a mass value of 1410 MeV. In that system the pion appears to resonate both with the \( K^0 \) and \( K^- \) (or \( \bar{K}^0 \) and \( K^+ \)) simultaneously giving \( K^* \) and \( \bar{K}^* \) formation. Furthermore, on examining the mass distribution of this charged \( K\bar{K} \) composite, which is again a highly constrained system, a peaking at a mass value of \( \approx 1020 \) MeV is observed. Here, too, a kinematical computation based on (2) reproduces the experimental \( M(K\bar{K}) \) distribution rather well. Thus the observed \( K\bar{K} \) enhancement (in the \( K^0 K^- \) and \( \bar{K}^0 K^+ \) states) may also be kinematical in nature. Alternately, if the above \( K\bar{K} \) state has a dynamical origin, the peak at 1410 MeV in the \( K\bar{K} \) system may be a consequence of three particles "tied together" into a compound resonance by the action of resonances between each pair of them.

If we now examine the \( \pi^+ \pi^- \) mass distribution (see Fig. 20 g), we note that about half of the negative pions which we have attributed to the decay of the 1480 MeV peak appear to participate in \( \pi^0 \) formation as well. In fact, the \( \pi^0 \) mesons so produced contribute about 20% to the events classified in the sample: \( \pi^0 + N^*(1238) \). Furthermore, if we examine the distribution in \( \cos\theta \) for this sample of \( \pi^0 \) mesons, we find that all the \( \cos\theta \) values are very strongly forward peaked. Thus, as pointed out above, the observed asymmetry in the decay angular distribution of the \( \pi^0 \) meson is intimately connected with the 1480 mass peak.

VII. OBSERVATIONS ON THE \( \pi^+ \pi^+ \) MASS DISTRIBUTION

Due to isotopic spin conservation an \( I = 5/2 \) isobar cannot be produced elastically in a \( np \) collision. No such restriction exists, however, for inelastic production processes. We have thus looked for such an isobar in Reaction (1). Aside from other effects which occur in this reaction, we have noted a marked
forward peaking of the \( \pi^- \) meson in the overall center of mass system which, of course, corresponds to an enhancement in the small four-momentum transfer \( \Delta^2(p\pi^+\pi^-) \) to the \( p\pi^+\pi^- \) system.

Furthermore, we note from a Chew-Low plot that the enhancement at small \( \Delta^2(p\pi^+\pi^-) \) values is associated with an enhancement in \( p\pi^+\pi^- \) mass distribution where this particle combination corresponds uniquely to the \( I = 5/2 \) state, Fig. 23. If we consider this enhancement as a possible \( I = 5/2 \) isobar \( N_{5/2}^* \) the corresponding total energy and full width at half maximum are \( E_{5/2} = 1.56 \pm 0.02 \text{ BeV}, \Gamma_{5/2} = 0.22 \pm 0.02 \text{ BeV} \).

In Fig. 24, we present the distribution of \( \cos\theta^*_\pi \) where \( \theta^*_\pi \) is the angle between the \( \pi^- \) meson direction and the incident \( \pi^+ \) meson direction in the overall center of mass system. It is evident that the \( \pi^- \) meson is forward peaked in events associated with \( N_{*++} \) production (type 1 events) and becomes extremely forward peaked for events with "double \( N_{*++} \)" production (type 2 events). No such marked peaking occurs for events without \( N_{*++} \) production (type 3 events). These are the events which are connected with \( A_1^- \) and \( A_2^- \) formation as was shown above. The corresponding distributions in \( \Delta^2(p\pi^+) \) and \( \Delta^2(p\pi^+\pi^-) \) are also shown in Fig. 24.

Since the phenomenon is associated with \( N_{*++} \) formation we will now consider the 1142 events of types 1 and 2. This very striking phenomenon can be expressed in the language of four-momentum transfer to the \( p\pi^+\pi^- \) system, \( \Delta^2(p\pi^+\pi^-) \), and can be correlated with mass distributions of the \( p\pi^+\pi^- \) system. The Chew-Low plot in Fig. 23 illustrate this point. We note two regions of clustering of events. The one at low \( \Delta^2(p\pi^+\pi^-) \) values, (i.e., forward peaking of the \( \pi^- \)) corresponds to a \( M(p\pi^+\pi^-) \) mass peaking around 1.56 BeV, which may represent evidence for an \( N_{5/2}^*(1560) \) isobar. The clustering at higher \( \Delta^2 \) values corres-
ponds to the $M(p^+\pi^+)$ mass peaking near 2.56 BeV. This is probably a reflection of mass peaking effects in the $M(p^+\pi^-)$ system.

If we plot the mass squared of $p^+_1$ against that of $p^+_2$ and vice versa we obtain a scatter plot which corresponds to the superposition of Dalitz plots for various $p^+\pi^+$ masses. In this plot every point occurs twice symmetrically with respect to the 45° axis (see Fig. 25). We note the $N^{*++}$ bands and particularly the enhancement at the overlap of the bands. The overlap region is particularly enhanced when we consider events with $\Delta^2(p^+\pi^+) \leq 15 m^2_\pi$ (see Fig. 25b). The density distribution along one of the $N^*$ bands is given in the inserts. By our plotting procedure we would expect a peaking by a factor of 2 at the overlap region. Actually, we observe a peaking of about 4 times the adjacent region. This effect is particularly accentuated if we limit ourselves to events with $\Delta^2(p^+\pi^+) \leq 15 m^2_\pi$ (see Fig. 25b). In contrast we show the same distribution for the $\pi^-p$ reaction (see Fig. 26). In this case no particular enhancement is obtained at the crossing of these two $N^{*0}$ bands. It is noteworthy that for an $I = 5/2$ isobar the expected intensity of $p^+\pi^-\pi^+$ production is $1/50$ of that for $p^+\pi^+$ production.

Here again if we are indeed dealing with an $N^{*}_{5/2}$ state we note that the decay of this state proceeds in such a manner that by and large the proton forms mass values with both $\pi^+$ mesons which lie in the $N^*(1238)$ bands. This in turn determines the $\pi^+\pi^+$ mass distribution for energy and momentum conservation as in Equation (12). The computed curve is shown in Fig. 27 together with the experimental distribution for $1420 \leq M(p^+\pi^+) \leq 1760$ MeV. The $\pi^+\pi^+$ mass distribution does not give as pronounced a peak as the $\pi^+\pi^-$ distribution near 390 MeV, however, a general enhancement near 500 MeV is observed.
In Fig. 28, we show the correlation plots of the mass squared of various two particle systems against that of $p\pi^+\pi^+$ as well as the distributions in mass squared of the corresponding systems for the events of types 1 and 2 and with the four-momentum transfer squared to the $p\pi^+\pi^+$ system smaller than $15 m_{\pi}^2$.

VIII. CONCLUSION

It is clear from the data presented here that angular distributions in the $\rho$ decay (and the $\pi^+\pi^-$ system in general) are related to the mass peaks in the $p\pi^-\pi^-$ and $p\pi^+\pi^+$ systems. This is summarized in Fig. 29 where we show the Chew-Low plots for the two systems, the Dalitz plots for the "particles" $N^*$, $\pi^+$, $\pi^-$ and correlation plots between the three-particle masses and the angle $\alpha_p$. The problem is to ascertain which of the observed phenomena is the "cause" and which the "effect". To study the problem as to whether the mass peaks do or do not have a dynamical origin we have made the following considerations:

(a) A Monte Carlo calculation which takes into account the angular distributions in $\rho^0$ and $N^*$ formation in order to see whether these cause mass peaks.\(^{22}\) Here we have carried out a phase space calculation by using a modified version of the program FAKE,\(^{13}\) a Monte Carlo simulation of the events.

To investigate the effect of the reaction $\pi^+ p \rightarrow N^* + \rho^0$ on the three particle mass distribution we have taken the following approach:

1. Generate 11,000 artificial events with the mass distributions in the $p\pi$ and $\pi\pi$ pairs simulated according to the Breit Wigner equation for the $N^*(1238)$ and $\rho$ resonances respectively.
2. These events were oriented with respect to the incident direction to simulate experimental $\Delta^2(\pi\pi)$ distribution.
3. The events were then weighted according to angular distribution in $\alpha_p$ and $\alpha_N^*$.\(^*\)
(4) The events were selected according to mass bands to simulate type 1 and type 2 (i.e., \( N^*(1238) \) band) as was done for the experimental data.

With these criteria we used three different angular distributions in \( \alpha_{p} \) and \( \alpha_{N^*} \).

1. The ideal distribution (complete alignment and no interference)
   \[
   I(\alpha_p) = \cos^2 \alpha_p \quad \text{and} \quad I(\alpha_{N^*}) = 1 + 3 \cos^2 \alpha_{N^*}.
   \]

2. The distribution observed experimentally. Here we used the values for \( A, B, \) and \( C \) the expansion coefficients in \( \cos \alpha \) given in Tables II and III.

3. The distribution corresponding to \( \pi \pi \) scattering in the reaction \( \pi^- + p \rightarrow \pi^+ + \pi^- + n \). The special feature in this reaction is that the \( \pi \pi \) angle is forward peaked with hardly any peaking in the backward direction. This we represented by the form
   \[
   I(\alpha_p) = \frac{1}{4} + \cos \alpha + \cos^2 \alpha
   \]
   while for \( \alpha_{N^*} \) we used \( I(\alpha_{N^*}) = 1 + 3 \cos^2 \alpha_{N^*} \) again.

The conclusion from these calculations is that the angular distributions cannot reproduce the observed mass peaks.

(b) Evaluation of the amplitudes corresponding to the Feynman diagram for \( \rho N^*** \) formation, including Bose symmetrization for the two \( \pi^+ \) mesons. Here we have computed the \( p\pi^+\pi^+ \) mass and \( \Delta^2(p\pi^+\pi^+) \) distributions based on the Feynman diagram for \( \rho^0 \) and \( N^* \) formation. This model was simplified by neglecting spin dependent factors. These calculations are still in progress but preliminary results indicate that some of the features of the experimental distributions related to the \( p\pi^+\pi^+ \) system can be reproduced qualitatively.
REFERENCES


10. J. Franklin, Preprint


13. The phase space is calculated by a Monte–Carlo simulation of the events using a modified version of the program FAKE (See G. R. Lynch, Program FAKE: Monte–Carlo Simulation of Bubble Chamber Events, Lawrence Radiation Laboratory Report UCRL–10335, 1962, unpublished).

14. Dr. Ian Butterworth, private communication.


The forward peaking in \( \rho \) in the reaction \( \pi^- + p \rightarrow \pi^+ + \pi^- + n \) has recently been interpreted in terms of \( N^*_{1/2} (1238) \) formation effects by P. Eberhard and M. Pripstein (preprint).


18. Detailed discussions on angular correlation effects at two scattering vertices have recently appeared. S. M. Berman and R. J. Oakes, Preprint.


20. The dynamical approach suggested by Peierls and extended by R. C. Hwa (Phys. Rev. 130, 2580 (1963)), indicates that while the \( N^* (1510) \) isobar is related to the \( N^* (1238) \) isobar its occurrence can still be ascribed to a pole. Considerations were further extended by I. P. Gyuk and S. F. Tuan, Nuovo Cimento 32, 227 (1964).

22. This work is due to Dr. Thomas A. O'Halloran.

23. We wish to acknowledge the help given us by Dr. F. Low in connection with this approach.
TABLE I

Partial Cross Sections for Channels Leading to $\rho^0$, $f^0$ and $N^*(1238)$ Formation in the Reaction $\pi^+ p \rightarrow \pi^+ \pi^0 p$ at 3.65 BeV/c (a)

<table>
<thead>
<tr>
<th>Final State</th>
<th>Per Cent</th>
<th>(mb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>la $\rho^0 N^{*++}$</td>
<td>30.5</td>
<td>1.17 ± 0.12</td>
</tr>
<tr>
<td>lb $\rho^0 \pi^+ p$</td>
<td>23.0</td>
<td>0.86 ± 0.09</td>
</tr>
<tr>
<td>lc $\pi^+ - N^{*++}$</td>
<td>30.1</td>
<td>1.16 ± 0.12</td>
</tr>
<tr>
<td>ld $f^0 N^{*++}$</td>
<td>3.4</td>
<td>0.13 ± 0.04</td>
</tr>
<tr>
<td>le $\pi^+ - p$ (&quot;non resonant&quot;)</td>
<td>13.0</td>
<td>0.53 ± 0.1</td>
</tr>
<tr>
<td>TOTAL</td>
<td>100.0</td>
<td>3.85 ± 0.30 (b)</td>
</tr>
</tbody>
</table>

(a) This table refers only to the most prominent features. Finer features such as the $A^+$ effect, $N^* (1238)$ formation etc. are not explicitly incorporated.

(b) The cross sections were calculated by calibration of the total number of $\pi^+ p$ interactions against the cross section measurements of Longo and Moyer, Phys. Rev. 125, 701, (1962).
Table II  Expansion Coefficients of \( I(\alpha_p) = A + B \cos \alpha + C \cos \alpha \) as a function of \( M(\pi^+_p) \) normalized to \( A = 1.0 \).

Events are selected so that \( M(\pi^+_p) \) lies inside the \( N^{*++}_{3/2} \) band. For "double-\( N^* \)" events (118 out of a total of 1142 events) \( \pi^+_p \) represents an arbitrary choice. Also shown is the \((F-B)/(F+B)\) ratio. These data correspond to Fig. 1.

The last two lines give the coefficients for all events in the \( N^{*++}_{3/2} \) band and in the double \( N^{*++}_{3/2} \) resonance region respectively in millibarns per steradian.

<table>
<thead>
<tr>
<th>( M(\pi^+_p) ) BeV</th>
<th>( A )</th>
<th>( B )</th>
<th>( C )</th>
<th>( (F-B)/(F+B) )</th>
<th>Number of Events</th>
<th>( \Delta^2 \leq 50 ) Number of Events</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0.40 \pm 0.16 )</td>
<td>( 1.0 \pm 0.2 )</td>
<td>( 0.4 \pm 0.2 )</td>
<td>( 0.4 \pm 0.4 )</td>
<td>( 0.24 \pm 0.1 )</td>
<td>( 90 )</td>
<td>( 0.27 \pm 0.11 )</td>
</tr>
<tr>
<td>( 0.65 \pm 0.05 )</td>
<td>( 1.0 \pm 0.2 )</td>
<td>( 0.7 \pm 0.2 )</td>
<td>( 1.0 \pm 0.4 )</td>
<td>( 0.27 \pm 0.08 )</td>
<td>( 129 )</td>
<td>( 0.35 \pm 0.09 )</td>
</tr>
<tr>
<td>( 0.725 \pm 0.25 )</td>
<td>( 1.0 \pm 0.3 )</td>
<td>( 1.1 \pm 0.4 )</td>
<td>( 5.0 \pm 0.8 )</td>
<td>( 0.27 \pm 0.08 )</td>
<td>( 150 )</td>
<td>( 0.27 \pm 0.08 )</td>
</tr>
<tr>
<td>( 0.775 \pm 0.025 )</td>
<td>( 1.0 \pm 0.2 )</td>
<td>( 1.3 \pm 0.3 )</td>
<td>( 1.6 \pm 0.4 )</td>
<td>( 0.36 \pm 0.07 )</td>
<td>( 163 )</td>
<td>( 0.42 \pm 0.08 )</td>
</tr>
<tr>
<td>( 0.825 \pm 0.05 )</td>
<td>( 1.0 \pm 0.2 )</td>
<td>( 1.1 \pm 0.3 )</td>
<td>( 3.1 \pm 0.5 )</td>
<td>( 0.25 \pm 0.07 )</td>
<td>( 207 )</td>
<td>( 0.35 \pm 0.07 )</td>
</tr>
<tr>
<td>( 0.950 \pm 0.05 )</td>
<td>( 1.0 \pm 0.3 )</td>
<td>( 0.3 \pm 0.5 )</td>
<td>( 3.8 \pm 1.0 )</td>
<td>( 0.11 \pm 0.11 )</td>
<td>( 81 )</td>
<td>( 0.21 \pm 0.12 )</td>
</tr>
<tr>
<td>( 1.050 \pm 0.05 )</td>
<td>( 1.0 \pm 0.5 )</td>
<td>( 0.9 \pm 0.8 )</td>
<td>( 5.8 \pm 0.1 )</td>
<td>( 0.10 \pm 0.13 )</td>
<td>( 62 )</td>
<td>( 0.20 \pm 0.15 )</td>
</tr>
<tr>
<td>( 1.150 \pm 0.05 )</td>
<td>( 1.0 \pm 0.5 )</td>
<td>( -1.2 \pm 0.8 )</td>
<td>( 6.1 \pm 1.3 )</td>
<td>( -0.14 \pm 0.12 )</td>
<td>( 74 )</td>
<td>( -0.18 \pm 0.15 )</td>
</tr>
<tr>
<td>( 1.250 \pm 0.05 )</td>
<td>( 1.0 \pm 0.4 )</td>
<td>( -1.3 \pm 0.6 )</td>
<td>( 5.8 \pm 1.3 )</td>
<td>( -0.15 \pm 0.10 )</td>
<td>( 92 )</td>
<td>( 0.14 \pm 0.15 )</td>
</tr>
<tr>
<td>( 1.465 \pm 0.165 )</td>
<td>( 1.0 \pm 0.3 )</td>
<td>( -0.2 \pm 0.4 )</td>
<td>( 2.6 \pm 0.1 )</td>
<td>( 0.15 \pm 0.10 )</td>
<td>( 94 )</td>
<td>( -0.05 \pm 0.16 )</td>
</tr>
</tbody>
</table>

| \( 0.280 \) to \( 1.450 \) | \( 1.0 \pm 0.07 \) | \( 0.67 \pm 0.10 \) | \( 2.48 \pm 0.20 \) | \( 0.19 \pm 0.03 \) | \( 1142 \) | \( 0.38 \pm 0.04 \) |

<table>
<thead>
<tr>
<th>A (mb/sr)</th>
<th>B (mb/sr)</th>
<th>C (mb/sr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.280 - 1.450</td>
<td>( 0.11 \pm 0.007 )</td>
<td>( 0.72 \pm 0.01 )</td>
</tr>
<tr>
<td>0.650 - 0.850</td>
<td>( 0.048 \pm 0.005 )</td>
<td>( 0.055 \pm 0.007 )</td>
</tr>
</tbody>
</table>
Table III: Expansion coefficients of \( I(\alpha_N^*) = A + B \cos \alpha_N^* + C \cos^2 \alpha_N^* \) as a function of \( M(\pi^+ p) \) normalized to \( A = 1.0 \). The 522 events analyzed here correspond to channel (1). Also shown is the ratio \( (F-B)/(F+B) \). The last line gives the coefficients in the "double resonance" region in millibarns per steradian.

<table>
<thead>
<tr>
<th>( M(\pi^+ p) ) BeV</th>
<th>A (mb/sr)</th>
<th>B (mb/sr)</th>
<th>C (mb/sr)</th>
<th>( (F-B)/(F+B) )</th>
<th>Number of Events</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.16 ± .04</td>
<td>1.0 ± 0.1</td>
<td>-0.1 ± 0.2</td>
<td>0.4 ± 0.4</td>
<td>0.01 ± 0.08</td>
<td>138</td>
</tr>
<tr>
<td>1.225 ± 0.025</td>
<td>1.0 ± 0.1</td>
<td>0.2 ± 0.2</td>
<td>1.0 ± 0.4</td>
<td>0.05 ± 0.07</td>
<td>192</td>
</tr>
<tr>
<td>1.28 ± 0.035</td>
<td>1.0 ± 0.1</td>
<td>0.8 ± 0.2</td>
<td>2.2 ± 0.5</td>
<td>0.23 ± 0.07</td>
<td>192</td>
</tr>
<tr>
<td>1.12 - 1.32</td>
<td>1.0 ± 0.1</td>
<td>0.27 ± 0.1</td>
<td>1.1 ± 0.2</td>
<td>0.11 ± 0.4</td>
<td>522</td>
</tr>
</tbody>
</table>

Number of Events:

- \( A(\text{mb/sr}) \):
  - 1.12 - 1.32: 0.066±0.007
  - 1.12 - 1.32: 0.028±0.007
  - 1.12 - 1.32: 0.072±0.013

- \( B(\text{mb/sr}) \):
  - 1.12 - 1.32: 0.028±0.007

- \( C(\text{mb/sr}) \):
  - 1.12 - 1.32: 0.072±0.013
Table IV. Expansion coefficients of $I(\alpha_p)$ and $I(\alpha_N^*)$ as functions of the polar and equatorial intervals in $\cos \alpha_N^*$ and $\cos \alpha_p$ respectively. The variation of the expansion coefficient for $I(\alpha_p)$ with $\cos \alpha_N^*$ and of $I(\alpha_N^*)$ with $\cos \alpha_p$ indicates the correlation effect discussed in the text. The 289 events analyzed here correspond to channel (1) for $\Delta^2 - 2m^2$. They are illustrated in Fig. 2. Also shown in the $(F-B)/(F+B)$ ratio.

Coefficients for $\rho$ decay

<table>
<thead>
<tr>
<th>Curve in Fig. 2</th>
<th>$\cos \alpha_N^*$</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>$(F-B)/(F+B)$</th>
<th>Number of Events</th>
</tr>
</thead>
<tbody>
<tr>
<td>e</td>
<td>$-1.0$ to $-0.4$</td>
<td>$1.0 \pm 0.2$</td>
<td>$5.6 \pm 0.9$</td>
<td>$12.4 \pm 1.6$</td>
<td>$0.53 \pm 0.07$</td>
<td>120</td>
</tr>
<tr>
<td>f</td>
<td>$-0.4$ to $0.4$</td>
<td>$1.0 \pm 0.2$</td>
<td>$0.6 \pm 0.3$</td>
<td>$1.3 \pm 0.5$</td>
<td>$0.17 \pm 0.09$</td>
<td>116</td>
</tr>
<tr>
<td>g</td>
<td>$0.4$ to $1.0$</td>
<td>$1.0 \pm 0.3$</td>
<td>$4.0 \pm 0.7$</td>
<td>$9.5 \pm 1.2$</td>
<td>$0.48 \pm 0.07$</td>
<td>153</td>
</tr>
</tbody>
</table>

Coefficients for $N^*$ decay

<table>
<thead>
<tr>
<th>Curve in Fig. 2</th>
<th>$\cos \alpha_p$</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>$(F-B)/(F+B)$</th>
<th>Number of Events</th>
</tr>
</thead>
<tbody>
<tr>
<td>e</td>
<td>$-1.0$ to $-0.4$</td>
<td>$1.0 \pm 0.2$</td>
<td>$0.2 \pm 0.3$</td>
<td>$0.9 \pm 0.5$</td>
<td>$0.08 \pm 0.10$</td>
<td>91</td>
</tr>
<tr>
<td>f</td>
<td>$-0.4$ to $0.4$</td>
<td>$1.0 \pm 0.2$</td>
<td>$0.3 \pm 0.3$</td>
<td>$-0.3 \pm 0.4$</td>
<td>$0.83 \pm 0.12$</td>
<td>72</td>
</tr>
<tr>
<td>g</td>
<td>$0.4$ to $1.0$</td>
<td>$1.0 \pm 0.2$</td>
<td>$0.6 \pm 0.3$</td>
<td>$3.4 \pm 0.5$</td>
<td>$0.14 \pm 0.07$</td>
<td>226</td>
</tr>
</tbody>
</table>
Table V. Location of resonances and mass peaks in the reaction \( \pi^+ + p \rightarrow \pi^+ + \pi^- + \pi^+ + p \) at 3.65 BeV/c. Also shown are the "resonance shift" and "resonance broadening" effects.

<table>
<thead>
<tr>
<th></th>
<th>( E(\text{MeV}) )</th>
<th>( \Gamma(\text{MeV}) )</th>
<th>Other Restrictions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N^{*++} )</td>
<td>1225 ± 10</td>
<td>110 ± 10</td>
<td>None</td>
</tr>
<tr>
<td>( \rho^0 )</td>
<td>770 ± 10</td>
<td>130</td>
<td>None</td>
</tr>
<tr>
<td>( p\pi^+\pi^- )</td>
<td>1480 ± 10</td>
<td>~120</td>
<td>( \Delta^2 (p\pi^+\pi^-) \leq 15 , m^2 )</td>
</tr>
<tr>
<td>( p\pi^+\pi^+ )</td>
<td>1560 ± 20</td>
<td>220 ± 20</td>
<td>( \Delta^2 (p\pi^+\pi^+) \leq 15 , m^2 )</td>
</tr>
<tr>
<td>( \rho^0 )</td>
<td>~770</td>
<td>~180</td>
<td>In &quot;1480&quot; peak</td>
</tr>
<tr>
<td>( \rho^0 )</td>
<td>750</td>
<td>~80</td>
<td>In ( A_\perp ) band</td>
</tr>
<tr>
<td>( \rho^0 )</td>
<td>785</td>
<td>~80</td>
<td>In ( A_2 ) band</td>
</tr>
<tr>
<td>( N^{*0} )</td>
<td>1190 ± 10</td>
<td>~160</td>
<td>In &quot;1480&quot; peak</td>
</tr>
<tr>
<td>( \pi^+\pi^- (390) )</td>
<td>390 ± 10</td>
<td>~110</td>
<td>In &quot;1480&quot; peak</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>1335 ± 10</td>
<td>90 ± 10</td>
<td>In ( \rho^0 ) band ( N^{*++} ) out</td>
</tr>
</tbody>
</table>
Four-particle phase space triangle

\[ a + b \rightarrow x + y \]
\[ L_{1+2} \rightarrow L_{3+4} \]

\[ Q = W - \sum_{i=1}^{4} m_i \]

\[ \phi \sim \frac{1}{W} \int k_x k_y p_{xy} \, dm_x \, dm_y \]

Fig 1
$K^+ + p \rightarrow K^+ \pi^- + p + \pi^-$

Fig 2
Fig. 3
Fig 5

Type 1: $\pi^+\pi^- \text{ only.}$

(a)

Type 3

1.0 $\leq M^2(\rho^0\pi^+) \leq 2.0 \text{ BeV}^2$

(b)

Number of events per 20 $M^2_\pi$ units

$\Delta^2_{2\pi}$ in $M^2_\pi$ units

MUB-2297
(a) \[
\begin{array}{c}
\pi^+ \rightarrow \rho^0 \rightarrow \pi^+ \\
p \rightarrow N^{***} \rightarrow \pi^-
\end{array}
\]

(b) \[
\begin{array}{c}
\pi^+ \rightarrow \pi^+ \\
p \rightarrow N^{***} \rightarrow \pi^-
\end{array}
\]

(c) \[
\begin{array}{c}
\pi^+ \rightarrow A^+ \rightarrow \rho^0 \\
p \rightarrow p
\end{array}
\]

(d) \[
\begin{array}{c}
\pi^+ \rightarrow \pi^+ \\
p \rightarrow N_{5/2}^{*} \rightarrow \rho^0
\end{array}
\]

Fig. 6
\[ \pi^+ + p \rightarrow \rho^0 + \pi^+ + p \]

983 events

"Double \( \rho \)" events counted once only

0.65 BeV \( \times \) \( M_{\rho} \)

0.85 BeV \( \times \) \( M_{\rho} \)

(c) Type 3 (\( N^* \) band excluded)

428 events

Phase space normalized to all events

Phase space normalized to events with \( M^2 > 2 \text{BeV}^2 \)

Double \( \rho \) events

(d) 983 events

\( N^* \) band included

Fig. 7
Figure 8

$\pi^+ p \rightarrow \pi^0 \pi^+ p$

428 events
Type 3

$M^2 (\rho^0 \pi^+) \ (\text{BeV}^2)$

Number of events

$\Delta^2 = 50 m^2_{\pi}$

$0 \leq \Delta^2 \leq 50 M^2_{\pi}$

$50 M^2_{\pi} \leq \Delta^2$

Number of events per 0.2 BeV$^2$
FIG. 9. Effective-mass distributions for (a) $K^-K^0$ and $K^0K^0$ pairs; (b) $\pi^+\pi^-$ combinations for events with $M(\pi\rho)$ outside the $N^{*++}$ interval. The smooth curves represent $3\pi$ phase space normalized to events outside the peaks. Bars indicate mass intervals used in the Dalitz plots.

FIG. 10. Dalitz plot for events with $M(\pi^-\pi^+\pi^-)$ between 1.00 and 1.18 BeV, and projection onto the $M^2(\pi^+\pi^-)$ axis; (c, d) Dalitz plot for events with $M(\pi^-\pi^+\pi^-)$ between 1.22 and 1.40 BeV, with projection onto $M^2(\pi^+\pi^-)$ axis. The shaded areas are projections for $p^0$ bands only, $M(\pi^-\pi^-)$ between 0.64 and 0.66 BeV. Events with $\Delta^2(p) > 0.65 (\text{BeV}/c)^2$ and $M(\pi^+\rho)$ in the $N^{*++}$ interval are not shown. The solid curves represent the phase-space limits for 1.18 BeV in (a) and 1.40 BeV in (c).

Fig. 9
Fig. 10: $n^+n^-n^-$ effective mass distribution for reaction $n^p - pn^+n^-$, both $pn^+$ masses outside $N^*$ region. Solid histogram: at least one $n^+n^-$ combination in the $p$ region; dashed histogram: both $n^+n^-$ combinations in the $p$ region. The curves show a superposition of a smooth background and two Breit-Wigner distributions normalized to the solid histogram.

Fig. 11: Chew-Low plot of $M_{e^+e^-}$ versus $t$ (four momentum transfer to the proton); both $pn^+$ masses outside $N^*$ region and at least one $n^+n^-$ combination in the $p$ region.

Fig. 12: (a) Missing mass distribution for reaction $n^p - pn^+ +$ neutrals. (b) Effective mass distribution of $n^+$ combined with $\pi^0$ candidate. Solid histogram: for both $n^p - pn^+ +$ neutrals and $n^p - pn^+n^-$; shaded histogram: only from $n^p - pn^+n^-$, $N^*$ region removed and $t < 0.6$ GeV$^2$. (c) Effective mass distribution of $n^+$ combined with missing neutrals for control regions on either side of the $\pi^0$ region of the reaction $n^p - pn^+ +$ neutrals; $N^*$ region removed and $t < 0.6$ GeV$^2$. 
Background

$600 \leq M(\pi^+\pi^-\pi^0) \leq 650$ MeV

57 events

$530 < M(\pi^+\pi^-\pi^0) < 570$ MeV

Number of events per 40 MeV

$M(\eta^0 \pi^+)$ (MeV)
\[
\pi^+ p \rightarrow \pi^+ \pi^- \pi^0 + p
\]
Number of events per 20 MeV

Number of events per 40 MeV

(c) \(\pi^+\pi^-\pi^0\) and \(\pi^+\pi^-\pi^0\)
2 x 645 triplets
Type 3

225 above phase space

17 above phase space

(d) \(\pi^+\pi^-\pi^0\) 1352 triplets
Types 1 and 2

18 above phase space

(e) \(\pi^+\pi^-\pi^0\) 1144 triplets
Type 1

310 above phase space

(f) \(\pi^+\pi^-\pi^0\) and \(\pi^+\pi^-\pi^0\)
2 x 1997 triplets
All types

43 above phase space

(g) \(\pi^+\pi^-\pi^0\)
"N reflection"

320

(h) \(\pi^+\pi^-\pi^0\) and \(\pi^+\pi^-\pi^0\)
630 triplets above phase space
Fig. 13
Fig. 15
Fig. 17
$\sigma(\pi^+p)$ and $\sigma(\pi^-p)$ vs $E_{c.m.}$ in BeV

\[ \pi^+p \rightarrow N^{***}(1238) + \pi^- + \pi^+ \]

$\Delta^2(p\pi^+\pi^-) \leq 15 \text{ m}^2$

Fig. 18
(a) $2 \times 1787$ mass triplets

(c) 1787 mass triplets

$\theta_{\pi_d} < \theta_{\pi_b}$

(b)

(d)
\[ \theta_{\pi^0} < \theta_{\pi^+} \]
\[ \Delta^2 < 15 \text{m}^2_{\pi} \]
\[ 1.4 \leq M(p\pi^+_\pi^-) \leq 1.6 \text{BeV} \]
\[
\nu + p \rightarrow N^{**} + \pi^{+} + \pi^{-}
\]
\[
\Delta^2 (p \pi^+ \pi^-) < 15 m_p^2
\]
525 events

Fig. 21
Fig. 22
\[ \pi^+ p \rightarrow N^{*++}(1238) + \pi^+ \pi^- \]
\[ \pi^+ p \rightarrow \pi^+ p + \pi^+ \pi^- \]

(a) \(N^{++}\) events (type 1)
1024 events

(b) "Double" \(N^{++}\) events (type 2)
118 events

(c) No \(N^{++}\) events (type 3)
645 events

(d) 
Number of events per 5 \((m_{\pi})^2\)

(e) 
\(\Delta^2 (p_{\pi} + m_{\pi})\) vs \((m_{\pi})^2\)

(f) 

(g) 
2 \times 1028 doublets

(h) 
2 \times 118 doublets

(i) 
2 \times 645 doublets

Fig. 24
\[ \pi^+ + p \rightarrow \pi^+ + \pi^- + \pi^+ + p \]

All \[ \Delta^2(p\pi^+\pi^+) \]

\[ \Delta^2(p\pi^+\pi^+) \leq 15 m_{\pi}^2 \]

N* (1238) band

(a)

N*++ (1238) band

(b)
\[ \pi^- + p \rightarrow \pi^- + \pi^+ + \pi^- + p \]

All \( \Delta^2 (p \pi^\pi^-) \)

\( \Delta^2 (p \pi^\pi^-) \leq 15 m_\pi^2 \)

\( N^{*0}(1238) \) band

\( N^{*0}(1238) \) band

Fig 26
\[ \Delta^2 (p \pi^+ \pi^+) \leq 15 m^2 \]

\[ E (p \pi^+ \pi^+) = 1560 \text{ MeV} \quad \Gamma = 220 \text{ MeV} \]

\[ \Delta^2 (p \pi^+ \pi^+) \leq 15 m^2 \]

\[ 1.42 \leq M (p \pi^+ \pi^+) \leq 1.76 \text{ BeV} \]

Fig. 27
Fig. 28
Fig. 29
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