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Ever since Bethe\(^1\) first showed the existence of a bound state of \(H^-\) by the Rayleigh-Ritz method for upper bounds to eigenvalues, it has been uncertain whether any other bound states existed, though physical arguments suggested the negative. In addition to its importance in astro-physics this question has also come up recently in discussions of the scattering of electrons from hydrogen.\(^2\)

To settle this problem we need to calculate lower bounds for the eigenvalues of the Schrödinger equation, but the best known method for lower bounds, involving the difference of energy levels, is useless here.

In a recent paper Bazley\(^3\) has given a method for finding lower bounds for the eigenvalues \(E_i\) of the Hamiltonian

\[
H = H_0 + V
\]

where \(H_0\) can be exactly solved and \(V\) is positive definite. If \(\phi_i\) is the normalized eigenfunction of \(H_0\) with eigenvalue \(\epsilon_i\), then the first order results of Bazley is

\[
E_i \geq \epsilon_i + (\phi_i^T V^{-1} \phi_i)^{-1}
\]

We have carried out this simple calculation for the problem of the negative hydrogen ion, \(V = e^2/r_1^2\), with the following results. All states, both singlet and triplet, arising from the hydrogen states \(1s, n\ell (n = \ell + 1 = 2, 3, \ldots)\), as well as the \(1s, 2s \, 3S\) state, have an energy greater than -1\(\text{ry}\) and are thus subject to auto-deionization leaving one electron in \(1s\) and the other free. The \(1S\) state arising from \(1s^2\) has an energy greater than -1.086\(\text{ry}\), but this state is known to be bound at -1.0555\(\text{ry}\);\(^4\) however the next member of this series \(1s, 2s \, 1S\) is unbound.

There is a second series of levels (not involved in the scattering problem) which for angular momentum \(\ell\) have parity \((-1)^{\ell+1}\) and thus cannot decay into \(1s, k\ell\).\(^*\) However, we have shown that all such states lie higher than \(2p, k\ell\) into which they can auto-deionize.

Our calculations were for the lowest state (in \(H_0\)) of each kind, but the order of levels with a given set of good quantum numbers must remain unchanged in
going from $H_0$ to $H$.\textsuperscript{5} Thus it is proved that there are no bound states of $H^-$ other than the one well known\textsuperscript{1} $S$.

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\textsuperscript{5}$k$ denotes a state in the continuum.

References

1. H. A. Bethe, Z. Physik 57, 815 (1929)
5. J. V. Neumann and E. Wigner, Physik Z. 30, 467 (1929)