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Author
Glendenning, N.K.

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Norman K. Glendenning

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NUCLEAR COLLISIONS AT VERY HIGH ENERGY

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1. **Introduction, Equation of State, Energy Thresholds**

What do we know about nuclei? The literature of the last 20 or 30 years contains a wealth of fascinating detail about their structure, their energy levels and single particle aspects, their collective motion, and the way they interact with each other in collisions. Both the quantity and detail of the experimental data, and the sophistication of some of the theory is impressive. Yet what we know about nuclei concerns their properties at only one point on the graph of the equation of state of nuclear matter which is illustrated in Fig. 1. Aside from the trivial point at the origin, and the energy per nucleon at normal density, the curve drawn is a guess. The point where it crosses the axis at $\rho/\rho_0 \sim 2$ is based on nuclear matter calculations. We do not even know the curvature (compressibility) at normal density. Virtually everything we know about nuclei concerns their normal state!

Some interesting possibilities for the state of nuclear matter at high density are illustrated in Fig. 1. The Lee-Wick super dense state is illustrated, as is the effect of a phase transition, corresponding to a situation where a state of special correlation having the quantum numbers of the pion (pion condensate) becomes degenerate with ground state.

Perhaps the ultimate goal of research with relativistic energy nuclei is to study nuclear matter under abnormal conditions of high particle and energy density. This is a break
from the past. Nuclear physicists have concentrated on studying nuclei under normal conditions of low energy and temperature. High energy physicists have concentrated on putting higher and higher energy into a small volume. We do not know what surprises await us, but several possible rewards will be mentioned later.

To make it plausible why we expect to encounter new and interesting phenomena it is useful to examine Fig. 2, prepared by Swiateckii. There the projectile mass for a symmetric collision is plotted on one axis, and a bombarding energy per nucleon on the other. The shaded areas indicate thresholds where qualitatively new physical features take over. The low energy region is the domain of conventional nuclear physics, and is being intensively studied at many laboratories. The region immediately adjacent to the x-axis extending to very high energies is the domain of particle physics, studied at the very large accelerators. Most of the plane is completely unknown territory. We discuss briefly the thresholds following the subsonic region of conventional nuclear physics.

**Supersonic Threshold.** The energy of 20 MeV per nucleon corresponds to

1) the average kinetic energy of nucleons in nuclei $\sim \frac{3}{5} E_F$

2) minimum energy needed to compress nuclear matter to something like twice normal density (see Fig. 1)

3) estimated speed of sound in normal nuclear matter (involves the curvature at the minimum in Fig. 1).
We anticipate qualitatively different behavior as this threshold is crossed into the region of what can be labelled supersonic.

**Meson Threshold.** This is the threshold for particle production, starting with the pion, followed by the nucleon isobars and heavier mesons. Hot nuclear matter can be cooled by the production of these particles.

**Relativistic Threshold.** This corresponds to the mass of the nucleon and brings us to the full relativistic regime where not even the wave equation for \( s > 1/2 \) is known. Far into this region I have the hope that it may be possible to discover the general form of the hadronic mass spectrum, one of the most fundamental properties of matter.\(^1\)

2. **Two Classes of High Energy Nucleus-Nucleus Collisions**

The experimentally observed events reveal two extreme limiting types of collisions at energies in the few hundred MeV to several GeV per nucleon range.

1) **Peripheral Collisions.**\(^2\) This is the most frequently observed class of collisions and is characterized by the fact that a few particles are observed in the extreme forward cone and they have almost the same speed as the projectile. They are presumably fragments of the projectile and they range in mass from one to a number of mass units, but less than the projectile. Presumably these collisions are geometrically peripheral so that a few nucleons in the overlap region are knocked out. Both the projectile and target residues are
presumably excited by the sudden removal of a part of their mass, and may radiate particles after the collision (Fig. 3).

2) Central Collisions.\(^3\) In about 10% of the collisions no fast particles having the projectile speed are observed. Instead, many particles, up to 130 charged particles are emitted even in collisions with a total initial charge of 100. Many mesons are evidently produced. No remnant of the projectile in the forward cone is observed. The projectile is presumably stopped in the target and the energy shared by many particles. Fairly fast ones come out in the forward hemisphere while high Z particles (bright tracks) come out in all directions (Fig. 4,5).

3. What Happens in a Central Collision?

Are nuclei opaque or transparent to an incident high energy (2 GeV/nucleon) nucleus? The de Broglie wave length is short so that we argue at first on the basis of a sequence of individual nucleon-nucleon collisions. The mean free path between collisions based on the nuclear density and N-N cross section is

\[ \ell \sim \frac{1}{\rho \sigma} = \left[ (0.17/F^3) \times 40 \text{ mb} \right]^{-1} \sim 1.5 F \quad . \]

The energy loss per collision is \(\sim\) 100 MeV so that a 2 GeV nucleon might lose a GeV energy in traversing a lead nucleus. In other words there might be a high degree of transparency.
On the other hand, in roughly 10% of collisions there is an absence of high energy particles: the projectile appears to be stopped and the composite system decays or explodes. Since we deal with systems of at most several hundred particles, deviations from the mean can be large! Clearly the opaque collisions are very interesting.

So far there is no convincing evidence for the propagation of shock waves in these collisions but the formation of regions of high density is expected in any case. This is very interesting from several points of view. One is the Lee-Wick density isomeric state of nuclear matter (see Fig. 1). The other is the phase transition sometimes referred to as pion condensation. At some critical density not so much greater than normal, it is believed that a collective state of special correlation having the quantum numbers of the pion will become degenerate with the ground state.

Aside from high density phenomena there are other very interesting features of central collisions. If the projectile is stopped in the target or a part thereof the resulting system is very hot. After taking account of the escape of a certain fraction of the prompt pion production, energy and momentum conservation can be used to calculate the internal excitation or temperature (Fig. 6). The temperature can be lower than this however because 1) the prompt π's that do not immediately escape can interact with nucleons to form the nucleon isobars. This lowers the number of
fermions of given type which allows cooling. 2) Collisions of hot nucleons can produce secondary π's reducing thus the kinetic energy.

The high velocity (near c) of the pions and their strong interaction with nucleons provides a fast mechanism for thermalization of the composite system in addition of course to the nucleon nucleon collisions. Indeed computer studies suggest that thermalization can occur already after only 3 or 4 collisions. Chemical equilibrium between the various species, π, N, N*, D, t takes longer but may still be fast compared to the disassembly time of the composite. Therefore a thermodynamical description may be reasonable and indeed a free ideal gas treatment of the composite, called a nuclear fireball does qualitatively account for some of the proton and composite particle spectra observed\textsuperscript{6,7,8} (Figs. 7, 8). If a thermodynamic description does apply, it opens up a very exciting possibility which I mention later after some preparatory comments.

First we recognize that at energies where π,ρ... nucleon isobars and more massive particles are produced, the thermodynamics has to be geared to strongly interacting particles. This leads now to a diversion.

4. **Elementary Particles**

An age old question has intrigued mankind, at the latest, since the early Greek philosophers. Is matter divisible
only down to fundamental particles which are not further
divisible or is matter infinitely divisible into smaller pieces? The modern experience does not come down on one side of this
either-or question. Instead we find that matter can be divided
again and again but not into smaller and smaller particles.
Instead always some of the same particles we started with
reappear together with other particles. A nucleon cannot be
broken up only into smaller particles! This was not known
to the Greeks, and it runs counter to our experience of all
matter from the macroscopic down to and including nuclei.

Einstein's law of equivalence of mass and energy tells us that in a high energy collision between nucleons mass can
be created. We see mesons and baryon anti-baryon pairs pro-
duced and the nucleons may reappear as nucleon isobars. Is
this an entirely trivial consequence of Einstein's law? Does
a law or physical principle underlie the indeterminancy of the
outcome? Presumably so. In other areas of physics we are
accustomed to considering that the state of a system comprises,
virtually, all possible configurations of the same symmetry.
What these "configurations" are is the goal of high energy
physics. Whatever the mathematical description of the modern
answer to the age old question, whether it be bootstraps or
quarks or whatever, it seems quite certain that we know only
a few of the particles and resonances that can be produced
in high energy collisions; that their number and variety is
staggering.
5. Can the Hadronic Mass Spectrum be Determined by High Energy Nuclear Collisions

One of the fundamental properties of matter is the mass spectrum of the hadrons. Besides being an object of interest in particle physics, it has profound cosmological significance. The spectrum starts with the pion and rapidly becomes dense up to masses of about 2 GeV. Thereafter, according to our present knowledge, but most likely due to the difficulty of measurement, there are only a few additional known particles and resonances. The experimental spectrum is shown in Fig. 9 with the exception of isolated recent discoveries. According to the bootstrap hypothesis the spectrum continues beyond the known region and in effect, exponentially. The hypothesis can be stated simply as follows: from among the known particles or resonances select two (or more) and combine their quantum numbers. The multiplet so obtained are also particles or resonances (at something like the sum of the masses). Add these to the pool of particles and continue. The spectrum so obtained by Hamer and Frautschi is also shown in Fig. 9. The implication is astonishing. The number of particles and resonances grows so fast that at masses of only 2.5 GeV the number in a mass interval of the pion mass, expected from the bootstrap hypothesis, is $\sqrt[4]{10}$. The number of known hadrons is $\sqrt[2]{10}$. If new particles were discovered at the rate of one a day it would require a hundred years to confirm the bootstrap prediction by a direct count, and that in only such a small mass interval and at such a low mass!
We suggest as an alternative that it may be possible to determine the general behavior of the spectrum without the necessity of discovering the individual particles and resonances. The decay of hadronic matter produced in ultra high energy collisions between nuclei will depend upon the type and masses of particles that can energetically be produced, that is, on the mass spectrum of the hadrons. After that obvious statement, there remain two important questions. At what energy does the outcome of the collision depend sensitively on the (unknown) spectrum and what is the dynamical description of the reaction? We shall here assume that the colliding nucleons, assisted by particle production and the resulting subsequent collisions, attain a temporary state of equilibrium among the large number of hadrons, and shall answer the first question in the context of that dynamics. We conjecture that the energy at which sensitivity is achieved will not depend crucially on the dynamics assumed, although the observable signals may. To the nuclear physicist, the possibility that an equilibrium state would be temporarily reached in a nuclear collision at high energy may seem remote. Nonetheless thermodynamic models in particle physics are not new, and they enjoy some success. A collision involving initially many nucleons is even more likely to reach an equilibrium state than one involving initially only two.

It is the aim of this research to determine whether the thermodynamic properties of hot hadronic matter occupying a volume of the order of nuclear dimensions is sufficiently sensitive to the unknown hadronic mass spectrum as to encourage an attempt to discover it experimentally in this way. We shall refer hereafter
to such hadronic matter as a nuclear fireball, it being suggested by the thermodynamic bootstrap theory of hadrons due to Hagedorn\textsuperscript{11} and already used in the context of nuclear collisions.\textsuperscript{12,13}

The partition function and momentum distribution function for an ideal relativistic gas of Fermions or Bosons of mass $m$ and statistical weight $g = (2J+1)(2\ell+1)$ occupying a volume $V$ at temperature $T$ are (Hagedorn\textsuperscript{11})

$$Z(V,T) = \frac{gV}{2\pi^2} m^2 T \sum_1^\infty \frac{\Gamma(\ell+1)/2}{n^2} \frac{K_2\left(\frac{\pi nm}{T}\right)}{n^2}$$

$$f(p,T)d^3p = \frac{gV}{2\pi^2} \frac{p^2 dp}{\exp\left(\frac{p}{T} + \sqrt{p^2 + m^2}\right) + 1}$$

from which the various thermodynamic quantities can be calculated. We want to describe a gas of Baryons and Mesons distributed in mass according to some unknown functions $\rho_B(m)$ and $\rho_M(m)$ for which Baryon number is conserved. This conservation can be achieved as usual by introducing a chemical potential $\mu$ for the Baryons.\textsuperscript{14}

The average number and energy for Baryons and Mesons are

$$B = \frac{VT}{2\pi^2} \int_{m_N}^\infty dm \rho_B(m) m^2 \sum_1^\infty \frac{(-)^{n+1}}{n} K_2\left(\frac{\pi nm}{T}\right) \exp\left(\frac{\mu}{T}\right)$$

$$E_B = \frac{VT}{2\pi^2} \int_{m_N}^\infty dm \rho_B(m) m^3 \sum_1^\infty \frac{(-)^{n+1}}{n} \left[ K_1\left(\frac{\pi nm}{T}\right) + \frac{3T}{nm} K_2\left(\frac{\pi nm}{T}\right) \right] \exp\left(\frac{\mu}{T}\right)$$
\[ M = \frac{\sqrt{T}}{2\pi^2} \int_{m_{\pi}}^{\infty} \, dm \, \rho_M(m) m^3 \sum_{n=1}^{\infty} \frac{1}{n} K_2 \left( \frac{nm}{T} \right) \]  \hspace{1cm} (5)

\[ E_M = \frac{\sqrt{T}}{2\pi^2} \int_{m_{\pi}}^{\infty} \, dm \, \rho_M(m) m^3 \sum_{n=1}^{\infty} \frac{1}{n} \left[ K_1 \left( \frac{nm}{T} \right) + \frac{3T}{nm} K_2 \left( \frac{nm}{T} \right) \right] \]  \hspace{1cm} (6)

We shall consider collisions between identical nuclei with isospin \( I=0 \). In that case charge conservation is implied already. For a grid of values of \( \mu \), we solve the first equation for the value of \( T \) which yields the baryon density desired. The remaining equations then tell us the corresponding energy and meson number.

It may seem strange that a strongly interacting hadronic system is discussed in terms of an ideal gas. However Hagedorn has argued convincingly, on the basis of statistical mechanical techniques introduced by Beth Uhlenbech and Belenki,\(^{15}\) that the hadronic spectrum is the manifestation of the hadronic interactions; that by introducing the complete hadronic mass spectrum one has accounted for their interactions completely.

We consider two extreme possibilities for the non-strange hadrons (Fig. 10). One is a representation of the current experimental situation. We use a discrete spectrum of the known hadrons up to mass \(~1680\) MeV with their widths (14 mesons and 6 baryons). We continue this with a continuum of constant density equal to the average in the region \( 1820-2520 \) MeV, which is 27.5 per pion mass. The other is a Hagedorn mass spectrum normalized to agree with the above experimental situation at \( 1400 \) MeV and with a slope in agreement with the bootstrap iteration of Fig. 9. This continuous spectrum we use for \( m > 1680 \), and we use the discrete experimental spectrum of 20 hadrons mentioned above for the low mass end. The
number of baryons and mesons in the continuous region are assumed to be equal. In summary the extreme assumptions are

\[ \rho_{\text{exp}}(m) = \begin{cases} 
20 \text{ discrete hadrons}, & m < 12 \, m_\pi \\
27.5/\text{per pion mass}, & m > 12 \, m_\pi 
\end{cases} \]  

(7)

\[ \rho_{\text{Hagedorn}}(m) = \begin{cases} 
20 \text{ discrete hadrons}, & m < 12 \, m_\pi \\
\frac{1.12 \, e^{m/T_0}}{(m/T_0)^3}, & m > 12 \, m_\pi 
\end{cases} \]  

(8)

\[ T_0 = 0.958 \, m_\pi, \quad m_\pi = 140 \, \text{MeV} \]

The temperature is plotted in Fig. 11 as a function of the center of mass kinetic energy per nucleon for the symmetric collision. For an exponentially increasing mass spectrum the temperature is limited to \( T_0 \), but the temperature can increase without bound for the less rapidly increasing spectrum. What we see is a large difference in the temperature of the two assumed spectra at energies in range of the CERN storage rings. This encourages us to believe that even at presently attainable energies, one might be able to distinguish between various hadronic spectra.

The next steps in our research will involve invoking several different models for the expansion of the fireball to a freezeout density. We hope that the large difference between the two assumed spectra will persist to the freezeout density where thereafter it will be preserved in the spectra of observed particles.

We interpret our suggestion and result as follows. The true dynamics is without doubt much more complicated than equilibrium thermodynamics. Extensive theoretical developments in understanding
the dynamics of high energy nuclear collisions, assisted by much experimentation are needed before we can even contemplate extracting the hadronic mass spectrum from data on nuclear collisions. What this paper shows is that the sensitivity to the mass spectrum is most probably present in ultra high energy nuclear collisions and it therefore provides the motive for pursuing what is surely a long and difficult task.

References


2) Heckman, Crawford, Greiner, Lindstrom, and Wilson, Lawrence Berkeley Laboratory report, LBL-6562.


7) A. Mekjian, Lawrence Berkeley Laboratory report, LBL-5819.

8) W. D. Myers, Lawrence Berkeley Laboratory report, LBL-6569.


10) There is a long literature but perhaps the paper of most direct interest here is Ref. 11.
11) R. Hagedorn in Cargèse Lectures in Physics, Vol. 6, ed. by

12) G. F. Chapline, M. H. Johnson, E. Teller, and M. S. Weiss,

13) Westfall, Gosset, Johansen, Poskanzer, Meyer, Gutbrod,

14) J. I. Kapusta, (LBL-6504) has considered pion production in
nuclear collisions also assuming a relativistic ideal gas
of \( \pi, N, \) and \( \Lambda \). Our equations are a natural extension of
his, and both follow from the literature on ideal rela-
tivistic gases.


Figure Captions

Fig. 1. Schematic of energy per nucleon versus density of nuclear
matter (equation of state) showing several possible high
density behaviors.

Fig. 2. Illustration by W. Swiatecki of various thresholds in the
mass of projectile versus collision lab energy for symmetric
collisions.

Fig. 3. A typical peripheral collision showing forward cone of
(charged) projectile fragments having virtually the projec-
tile energy.

Fig. 4. A central collision showing high multiplicity of charged
particles.
Fig. 5. Charged particle multiplicity distributions from Fung, Gorn, Kiernan, Liu, Lu, Oh, Ozawa, Poe, Van Dalen, Schroeder, and Steiner (unpublished, 1977).

Fig. 6. Schematic showing geometrical assumptions of fireball model. The fireball is the portion swept out by the projectile having an intermediate velocity $\beta$ and a temperature corresponding to an excitation energy given by application of energy and momentum conservation.

Fig. 7. Comparison of the fireball calculation $^6$ (dashed) and firestreak $^8$ (solid) with data of proton spectra.$^3$

Fig. 8. Thermodynamic calculations $^7$ of composite particle spectra compared with data.$^3$

Fig. 9. The density of different hadrons, $\rho$, in unit interval is plotted as a function of the mass in units of the pion mass. The experimentally known particles and resonances with their multiplicities are shown in the shaded areas. The dotted histogram is a bootstrap iteration $^9$ on the known spectrum and the solid curve is a Hagedorn type spectrum, fitted to the bootstrap.

Fig. 10. Similar to Fig. 9 but the non-strange hadrons only are shown and no anti-baryons. Additional more recent data is also plotted. The curve is a Hagedorn spectrum with slope as determined in Fig. 9 and normalized to the average in 5 pion mass intervals centered at $m = 10 \, m_\pi$.

Fig. 11. The temperature of hot hadronic matter assumed to be produced in a symmetric nuclear collision is plotted as a
function of the C.M. kinetic energy per nucleon of the colliding nuclei for a volume corresponding to about the initial density of nucleons. The curve labelled "Experiment" corresponds to a mass spectrum that approximates the known spectrum Eq. (7), while that labelled Bootstrap corresponds to a Hagedorn spectrum, Eq. (8). (B/V = 1/4)

Fig. 12. Corresponding to the two situations of Fig. 11, the baryon and meson populations in hot hadronic matter. Note that the number of low mass baryons decreases as more heavy baryon states (not shown) are populated.
Event 10. Hybrid event. The 6 forward going particles are all $\alpha$'s. Careful observation discloses that there are 6 additional small angle protons (or pions). If they are protons then all 18 charges of the incident argon projectile are accounted for.

Fig. 3
PRELIMINARY MULTIPLICITY DISTRIBUTIONS
FOR 1.8 GeV/n Ar

LiH tgt.
736 Events

Pb$_3$O$_4$ tgt.
425 Events

Fig. 5
Fig. 7

Proton laboratory energy spectrum $d^2\sigma/dE$ (mb/MeV sr)

- $^{20}\text{Ne} + ^{238}\text{U}$
- 250 MeV/n
- $\theta_{\text{lab}} = 30^\circ$
- $60^\circ$ ($\times 10^{-1}$)
- $90^\circ$ ($\times 10^{-2}$)
- $120^\circ$ ($\times 10^{-3}$)

- $400$ MeV/n
- $\theta_{\text{lab}} = 30^\circ$
- $60^\circ$ ($\times 10^{-1}$)
- $90^\circ$ ($\times 10^{-2}$)
- $120^\circ$ ($\times 10^{-3}$)
- $150^\circ$ ($\times 10^{-4}$)

Proton laboratory kinetic energy (MeV)
$^{20}$Ne + U 400 MeV/nucl.

$kT = 40$ MeV, $v_{c.m.} = 0.15 C$

Fig. 8
Fig. 9
Fig. 10

NON-STRANGE HADRONS
(ANTIBARYONS NOT COUNTED)
FOR HAGEDORN SPECTRUM
AND EXPERIMENTAL SPECTRUM

EXPERIMENTAL

HAGEDORN

T (MEV)

K (CM) IN MEV PER NUCLEON

Fig. 11
FOR HAGEDORN SPECTRUM
BARYON AND MESON POPULATIONS

FOR EXPERIMENTAL SPECTRUM + 27.5
BARYON AND MESON POPULATIONS

Fig. 12
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