Title
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MODELING THE DYNAMIC BEHAVIOR OF A SINGLE PILE IN DRY SAND USING A NEW P-Y MATERIAL MODEL

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ABSTRACT The lateral response of pile foundations in sand is commonly analyzed using p-y elements attached to the pile, typically using the API sand p-y relationship. The API relationship was developed for static loading conditions, with cyclic correction factors intended to represent degradation due to many slow loading cycles. However, the API model is often applied for dynamic loading conditions (e.g., earthquake shaking) because suitable alternatives have not been formulated. This study demonstrates that the API sand functional form is inappropriate for dynamic analysis of piles, and presents a new functional form that better captures the nonlinear p-y behavior of piles in sand during earthquake loading. The new functional form is in the process of being implemented in OpenSees, an open source finite element modeling platform that is freely available to users.

Introduction

Predicting dynamic behavior of piles during an earthquake is an important challenge for geotechnical engineers. The analysis of the lateral behavior of pile foundations has been investigated using 2D and 3D modeling of pile-soil system using finite element or finite difference methods [Kim et al. (2012)]. The beam on Nonlinear Winkler Foundation (BNWF) approach is also used extensively because it is considerably less complex than continuum methods [Boulanger et al. (1999)]. For BNWF analysis, p-y material model behavior is very important because the surrounding soil is represented entirely by nonlinear springs.

The API p-y curve (API, 1987), which was developed for static loading conditions, is often utilized for dynamic loading conditions because suitable alternatives have not been formulated. Researchers have investigated p-y behavior for dynamic loading conditions. Dobry et al. (2003) reported that the API p-y curve underestimates the ultimate resistance of soil under dynamic loading. Yang et al. (2011) and Yoo (2013) investigated the behavior of p-y curves under dynamic loading conditions from a series of shaking table tests. They reported that the API p-y curve not only significantly underestimates the ultimate resistance of soil for shallow depths but also overestimates the subgrade reaction modulus of soil when the displacement of a pile is less than 1% of the pile diameter.

In this study, a new p-y functional form that better captures the nonlinear p-y behavior in sand is developed using bounding surface plasticity theory. Centrifuge model data analyzed using the proposed model and the API sand model demonstrates the improved model.

Centrifuge tests

Dynamic centrifuge model tests for a single pile were conducted at the Korea Construction Engineering Development Collaboratory Program (KOCED), Geo-Centrifuge Center, on a centrifuge which has a radius of 5 m, 2.5 ton payload and up to 100 g centrifugal acceleration. All tests in this study were carried out at a centrifugal acceleration of 40g.

The model container used for the centrifuge tests was an Equivalent Shear Beam (ESB) box. The ESB model container was formed by stacking 10 light-weight aluminum alloy rectangular frames on a base plate to create internal dimensions of 49 cm × 49 cm × 63 cm and external dimensions of 65 cm × 65 cm × 65 cm in length, width, and height, respectively. Each frame is 6 cm in height and is separated by inside ball bearings and rubber spacing layers. The design concept is to match the deflection of each frame with that of the soil column at the middle of the frame and
the performance of the ESB box was verified by Kim et al. (2010).

Fig. 1 shows the schematic layout of a single pile and instrumentation for the centrifuge tests. The model pile was fabricated with a closed-ended aluminum pipe with 2.5 cm external diameter and a 0.1 cm wall thickness, and the embedment depth of a pile was 57 cm and it was fixed against translation at the base. The concentrated mass of 1.0 kg was attached to the pile at the height of 13 cm from the surface to simulate a superstructure. The properties of the soil-pile system are summarized in Table 1.

Table 1. Properties of model pile

<table>
<thead>
<tr>
<th></th>
<th>Prototype</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter of pile (cm)</td>
<td>76</td>
<td>2.5</td>
</tr>
<tr>
<td>Thickness of pile (cm)</td>
<td>1.7</td>
<td>0.1</td>
</tr>
<tr>
<td>Material</td>
<td>Steel</td>
<td>Aluminum</td>
</tr>
<tr>
<td>Flexural rigidity (N*cm²)</td>
<td>5.75E+12</td>
<td>2199280</td>
</tr>
<tr>
<td>Concentrated mass on the pile head (kg)</td>
<td>48,000</td>
<td>1.0</td>
</tr>
<tr>
<td>Embedment depth (cm)</td>
<td>2,200</td>
<td>55</td>
</tr>
</tbody>
</table>

Strain gauges were attached on both sides of the pile to measure curvature in the pile during vibrations. Bending moments were computed from measured curvatures based on the observation that the pile behavior remained elastic during shaking. Strain gauges were protected by tape applied after attaching them to the pile. Accelerometers were used to measure the acceleration responses of the soil and the superstructure. Eight accelerometers were installed in the soil with varying depths and one accelerometer was attached to the upper mass.

The model soil was Jumoonjin sand, characterized as fine-grained uniform sand (Table 2). The dry sand deposit was prepared to have a relative density of 80% to minimize changes in relative density during the shaking sequence.

A sequence of sinusoidal wave excitations were imposed on the base of the model. The amplitude of the input sinusoidal wave varied from 0.05g to 0.3g and the frequency was 1Hz in the prototype scale. Table 3 shows the test programs.

Table 2. Properties of Jumoonjin Sand

<table>
<thead>
<tr>
<th>D₁₀ (mm)</th>
<th>φ (deg)</th>
<th>Cᵤ</th>
<th>Gₛ</th>
<th>γ_d, max (kN/m³)</th>
<th>γ_d, min (kN/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.38</td>
<td>40</td>
<td>1.58</td>
<td>2.65</td>
<td>15.99</td>
<td>13.05</td>
</tr>
</tbody>
</table>

Table 3. Test programs (in prototype)

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Amplitude of acceleration (g)</th>
<th>Input frequency (Hz)</th>
<th>Relative density</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.05</td>
<td>1.5</td>
<td>80%</td>
</tr>
<tr>
<td>2</td>
<td>0.1</td>
<td>1.5</td>
<td>80%</td>
</tr>
<tr>
<td>3</td>
<td>0.15</td>
<td>1.5</td>
<td>80%</td>
</tr>
<tr>
<td>4</td>
<td>0.2</td>
<td>1.5</td>
<td>80%</td>
</tr>
<tr>
<td>5</td>
<td>0.3</td>
<td>1.5</td>
<td>80%</td>
</tr>
</tbody>
</table>

**Experimental p-y backbone curve**

From a series of centrifuge test results, dynamic p-y curves were determined for each input motion. Lateral resistance p and pile displacement y_pile were calculated based on the simple beam theory, as shown in equation [1]

\[
p = \frac{d^2}{dz^2} M(z) \quad y_{pile} = \int \frac{M(z)}{EI} dz
\]

where M(z) is bending moment along a pile and EI is flexural stiffness of a pile.

The weighted residual method, demonstrated by Brandenberg et al. (2011) to provide reasonable results for differentiation of experimental bending
moment data, was used to determine the lateral resistance \( p \). Pile displacement was determined by double integration of the bending moment along a pile using the trapezoidal rule.

The displacement \( y \) in the dynamic p-y curve is the relative displacement between the soil and pile because the displacement of soil occurs during an earthquake. Therefore, to obtain the relative displacement \( y \) for the dynamic p-y curve, the soil movement \( y_{soil} \), obtained from the free field accelerometers, was subtracted from the pile displacement \( y_{pile} \) at each time step.

Fig. 2 shows the experimental dynamic p-y curve at depth of 0.5m for various input acceleration amplitude in prototype scale.

The dynamic p-y backbone curve was determined by taking the peak points of dynamic p-y curves, which corresponded to the maximum subgrade reaction. Two more test cases for input ground acceleration of 0.1g and 0.2g were added to construct the p-y backbone curve shown in Fig. 3.

**Fig. 2.** Experimental dynamic p-y curves (0.5m depth)

![Experimental dynamic p-y curves (0.5m depth)](image)

**Description of BNWF analysis**

In this study, Beam on Nonlinear Winkler Foundation (BNWF) analyses were carried out to analyze the lateral behavior of a pile by implementing p-y curves in OpenSees, an open source finite element modeling platform that is freely available to users. Fig. 4 shows the description of BNWF analysis. The soil-pile system is modeled utilizing elastic beam column elements for the pile and nonlinear spring elements which represents the lateral response of the surrounding soil. The vertical displacement is fixed at the bottom of the pile. Input motions from recorded acceleration along the soil depth were imposed at the end of each spring node.

**Fig. 4.** Schematic diagram of BNWF analysis

![Schematic diagram of BNWF analysis](image)

**BNWF analysis using API sand**

The PySimple1 uniaxial material in OpenSees using soil type 2 approximates the functional form of the API sand equations, and this relation was used herein. The API p-y curve can be defined using equation [2].

\[
\text{tanh} \left( \frac{u}{k} \right) = \frac{p}{A_p} \tan \left( \frac{ky}{A_p} \right)
\]

where \( A \) is factor to account cyclic or static loading, \( p_u \) is ultimate soil resistance, and \( k \) is initial modulus of subgrade reaction.

The ultimate soil resistance was determined using the equations suggested by API (1987). API also recommended the subgrade reaction coefficient as 61 MN/m^3 for dense sand above the water table. The conventional API p-y curve is significantly different.
from the experimental p-y backbone curve as shown in Fig.3. The API p-y curve not only underestimates the ultimate resistance of soil but also overestimates the subgrade reaction modulus at the small displacement range.

Therefore, in this study, the initial modulus of subgrade reaction for each amplitude of input motion was modified to be equal to the secant modulus of experimental p-y backbone curve as shown in Fig 3. The initial modulus of subgrade reactions were set to be 14,610 kPa, 10,680 kPa, and 6,481 kPa for amplitude of input ground motion of 0.05g, 0.15g, and 0.3g, respectively.

Fig. 5 shows the comparison of the acceleration response at the upper mass between analysis results and test results in prototype scale. Three graphs at the first column show the results when the initial modulus of subgrade reaction was set to be 14,610 kPa, which is the secant modulus of the experimental backbone curve for the amplitude of input ground motion of 0.05g. It was found that the analysis result predict the centrifuge result reasonably well for the case of the amplitude of input ground motion is 0.05g. However, the analysis results overestimate the centrifuge test results for the case of the amplitude of input ground motion is 0.15g and 0.3g. For each input motion, a particular subgrade reaction modulus could be selected to provide reasonable agreement with the test data, but the appropriate subgrade reaction modulus was found to depend on shaking amplitude. For example, subgrade reaction moduli of 14,610, 10,680, and 6,481 kPa were required to provide a good match with the experimental data for input shaking intensities of 0.05, 0.15, and 0.3g, respectively. This is an undesired feature that indicates the functional form of the API sand p-y curve is incorrect.

Therefore, we now turn our attention to formulating a more appropriate p-y model for sand.

**Proposed model**

A new functional form that better captures the nonlinear behavior of p-y curves under dynamic loading conditions is proposed using the bounding surface plasticity theory.

The nonlinear p-y behavior can be conceptualized as consisting of an elastic component and a plastic component. The elastic component can be defined by the maximum subgrade reaction modulus of soil and the plastic component can be defined as shown in Equation [3].

\[
k_p = c_0 \left( \frac{\sigma_0}{\rho_a} \right)^{0.5} k_{\text{max}} \left( \frac{p_u - p \cdot \text{sign}(y)}{|p - p_{IN}|} \right)
\]

Equation [3]

where \(k_p\) is plastic stiffness, \(k_e\) is elastic stiffness, \(k_{\text{max}}\) is maximum subgrade reaction modulus, \(c_0\) is a material constant, \(p_a\) is atmosphere pressure, \(\sigma_0\) is mean effective stress, and \(p_{IN}\) is value of \(p\) at the start of the current plastic loading cycle.

The suggested p-y spring has an initial range of elastic behavior between \(-k_{\text{max}} \times \gamma_{\text{yield}} < p < k_{\text{max}} \times \gamma_{\text{yield}}\) where \(\gamma_{\text{yield}}\) is the yielding displacement. Beyond the elastic range, the elasto-plastic stiffness of the p-y spring is described by Equation [4].
Fig. 6 shows the example behavior of the proposed model for given mechanical properties of soil in Table 4. Note that some cyclic degradation is inherent to the model due to the evolution of ρ₂.⁰.

Table 4. Example mechanical properties of soil

<table>
<thead>
<tr>
<th>p₀</th>
<th>k_max</th>
<th>γ_y</th>
<th>c₀(σᵥ‴/p₀)⁰.⁵</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>0.001</td>
<td>10</td>
</tr>
</tbody>
</table>

Fig. 6. Example behavior of proposed model

**BNWF analysis using proposed model**

The material properties of Jumoonjin sand which characterize the behavior of p-y curve in the proposed model were determined by the following procedure.

**Maximum subgrade reaction modulus**

Gazetas and Dobry (1984) suggested relationship between the maximum subgrade reaction modulus and the elastic stiffness of soil as shown in equation [5].

[5] \[ k_{\text{max}} = 1.69 \left( \frac{E_p}{E_s} \right)^{-0.137} E_s \]

where \( E_p \) is elastic stiffness of a pile and \( E_s \) is elastic stiffness of soil.

The elastic stiffness of soil can be determined from the shear wave velocity of soil. Equation [6] shows the shear wave velocity of Jumoonjin sand at the relative density of 80% (Kim, 2010).

[6] \[ V_s (m/\text{sec}) = 214 \left( \frac{\sigma_v'}{p_0} \right)^{0.282} \]

where \( V_s \) is shear wave velocity, \( \sigma_v' \) is vertical effective stress, and \( p_0 \) is atmosphere pressure. The value of \( E_p \) is intended to be applied to a solid pile, but in our case a tubular pile was used. Therefore, the value of \( E_p \) was selected as the value for which a pile with a solid cross-section would have the same flexural stiffness as the tubular pile.

**Yielding displacement**

The yielding displacement of soil can be determined from the relationship between yielding strain and yielding displacement of a round shape pile as shown in equation [7].

[7] \[ \gamma_y = \frac{\gamma_v \times D}{0.4} \]

where \( \gamma_v \) is yielding strain and \( D \) is pile diameter. The constant 0.4 was determined using a two-dimensional plane strain finite element simulation (results not shown for brevity).

The yielding strain of Jumoonjin sand was defined from the equation suggested by Darendeli (2001) as shown in equation [8] assuming that \( G/G_{\text{max}} \) equals to 0.99 at the yielding strain.

[8] \[ \gamma_v = 2.79 \times 10^{-4} \times \sigma_0^{0.234} \]

where \( \sigma_0' \) is mean effective stress.

**Ultimate soil resistance**

The ultimate soil resistance was determined from the suggested equation by Yoo (2013) as shown in equation [9].

[9] \[ p_u = 13.3DK_p \gamma_v'z^{1.02} \]

where \( D \) is pile diameter, \( K_p \) is passive pressure coefficient, \( \gamma_v' \) is effective unit weight, and \( z \) is depth.

**Material constant, \( c_0 \)**

The material constant \( c_0 \) was determined by the trial and error method in order to reproduce the experimental p-y backbone curve by the proposed p-y model. Fig. 7 shows the behavior of the proposed p-y model for the material constant of 0.033. The
Simulated p-y curve is reasonably reproducing the experimental p-y backbone curve.

**Fig. 7.** Behavior of proposed p-y model (0.5m depth)

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**Analysis results by proposed model**

Fig. 8 shows the comparison between the analysis results using the proposed p-y model and the centrifuge test results in terms of the acceleration at the upper mass.

The analysis results predict the centrifuge results within error of 2% and 4% for the amplitude of input ground motion of 0.05g and 0.15g, respectively. In case of 0.3g input motion, the prediction is 20% larger than the measurements. We are investigating this error as we continue to develop the p-y material model. Nevertheless, these results show a significant improvement compared to the analysis results using API p-y curve because a single set of input parameters is able to capture the response over a range of shaking intensities.

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**Conclusions**

In this study, a new functional form that better captures the nonlinear p-y behavior of piles in sand during earthquake loading is proposed using the bounding surface plasticity theory.

Beam on a Nonlinear Winkler Foundation (BNWF) analysis which considering the surrounding soil as spring elements was performed to analyze the lateral behavior of a pile under earthquake loading condition. The spring elements were modeled using both the
API p-y curve and the proposed p-y curve. The analysis results were compared with the centrifuge test results.

The API p-y curve was significantly different from the experimental p-y backbone curve because the API p-y curve not only underestimates the ultimate resistance of soil but also overestimates the subgrade reaction modulus at the small displacement range. The BNWF analysis results implementing the API p-y curve as spring elements indicates that the API p-y curve is inappropriate to predict the behavior of piles under earthquake loading condition.

The procedure to determine the mechanical properties of the new functional form of p-y curve was suggested and the behavior of the proposed p-y curve was proved to capture the experimental p-y backbone curve reasonably well.

References


