Lawrence Berkeley National Laboratory
Recent Work

Title
PROPERTIES OF THE AND THE \( \pi^0 \) FROM K-p INTERACTIONS ABOVE 1.7 BeV/c

Permalink
https://escholarship.org/uc/item/1p6874s2

Authors
Merrill, Deane W.
Button-Shafer, Janice.

Publication Date
1967-07-10
PROPERTIES OF THE \( \Xi^- \) AND THE \( \Xi^+ \) (1817) FROM K\(^-\)p INTERACTIONS ABOVE 1.7 BeV/c

Deane W. Merrill and Janice Button-Shafer

July 10, 1967
DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.
UNIVERSITY OF CALIFORNIA
Lawrence Radiation Laboratory
Berkeley, California
AEC Contract No. W-7405-eng-48

PROPERTIES OF THE $\Xi^-$ AND THE $\Xi^*$ ($1817$) FROM
$K^-p$ INTERACTIONS ABOVE 1.7 BeV/c
Deane W. Merrill and Janice Button-Shafer

July 10, 1967
Properties of the $\Xi^-$ and the $\Xi^*(1817)$ from $K^-p$ Interactions above 1.7 BeV/c

Deane W. Merrill and Janice Button-Shafer

Lawrence Radiation Laboratory
University of California
Berkeley, California

July 10, 1967

ABSTRACT

A sample of 2529 $\Xi^-$ hyperons, produced in $\Xi^-K^+, \Xi^{0+}K^0\pi^+$, and $\Xi^-K^0\pi^-\pi^+$ final states by $K^-$ (in hydrogen) at incident momenta from 1.7 to 2.7 BeV/c, has been analyzed. The data are from an exposure of 26 events/µb ("K-63" run) in the Alvarez 72-inch bubble chamber; approximately 85% of the $\Xi^-$ events with visible $\Lambda$ decay have been analyzed. A maximum-likelihood fit (with $\alpha_{\Lambda} = 0.656 \pm 0.055$ and with the $\Xi^-$ spin = 1/2) yields the following values of $\Xi^-$ decay parameters: $\alpha_{\Xi^-} = -0.375 \pm 0.051$; $\Phi_{\Xi^-} = \tan^{-1} (\beta_{\Xi^-}/\gamma_{\Xi^-}) = 9.8^\circ \pm 11.6^\circ$. Our spin analysis of the 3400 $\Xi^-$ decays from the K-72 and K-63 experiments gives likelihood results which favor $J_{\Xi} = 1/2$ over $J_{\Xi} = 3/2$ by the equivalent of approximately 2.5 standard deviations. Analysis of the $\Xi^*(1817) \to \Xi^*(1530) + \pi$ decay mode indicates that the hypotheses $J^P = 1/2^+, 1/2^-, 3/2^-, 5/2^+, 7/2^-$, etc. are favored; but results are inconclusive because of high background as well as poor statistics. Analysis of the $\Xi^*(1817) \to \Lambda + K$ provides no spin or parity discrimination. The K-63 beam channel is briefly described.
Properties of the $\Xi^-$ and the $\Xi^*$ (1817) from $K^-p$ Interactions above 1.7 BeV/c

Deane W. Merrill and Janice Button-Shafer

Lawrence Radiation Laboratory
University of California
Berkeley, California

July 10, 1967

I. INTRODUCTION

Prior to this experiment approximately 2600 $\Xi^-$ had been analyzed to determine the $\Xi^-$ decay parameters $\alpha_{\Xi^-}$ and $\phi_{\Xi^-} = \tan^{-1}(\beta_{\Xi^-}/\gamma_{\Xi^-})$. The largest single sample previously analyzed consisted of 1004 events from the K-72 experiment (K$^-p$ at 1.2-1.7 GeV/c), for which the values $\alpha_{\Xi^-} = -0.368 \pm 0.057$ and $\phi_{\Xi^-} = 0.5^0 \pm 10.7^0$ (with $\alpha_{\Lambda} = 0.641 \pm 0.056$ and with the $\Xi$ spin assumed to be 1/2) were reported. In this paper we describe the analysis of 2529 $\Xi^-$ events in the K-63 experiment (K$^-p$ at 1.7-2.7 GeV/c), including 224 $\Xi^-K^+$ events previously analyzed. From K-63 data we obtain values $\alpha_{\Xi^-} = -0.375 \pm 0.051$ and $\phi_{\Xi^-} = 9.8^0 \pm 11.6^0$ (with $J_{\Xi^-} = 1/2$ and $\alpha_{\Lambda} = 0.656 \pm 0.055$), in good agreement with previously reported values. Combining our data with 902 K-72 events, we obtain (with $\alpha_{\Lambda} = 0.657 \pm 0.047$) $\alpha_{\Xi^-} = -0.394 \pm 0.041$ and $\phi_{\Xi^-} = 9.9 \pm 9.0^0$.

The $\Xi$ spin is also analyzed in a combined sample which includes 900 $\Xi^-$ and 150 $\Xi^0$ events from the K-72 experiment.
The decay properties of the \( \Xi^+(1530) \) and the \( \Xi^+(1817) \) have been analyzed in earlier published work. In these areas we have treated a somewhat larger data sample, but no essential modification of earlier results is indicated.

A later paper will treat other topics, such as \( \Xi^0 \) decay parameters and production systematics.
II. DESCRIPTION OF THE K-63 BEAM CHANNEL

The data analyzed in this report were obtained from photographs of K^-p interactions in the Laboratory's 72-inch bubble chamber. Most of the data are from a 1.7 to 2.7 BeV/c separated K^- beam (K-63) designed by Joseph J. Murray with the assistance of J. Button-Shafer; however, our analysis of E^- decay properties includes data from an earlier beam, K-72 (designed by Harold K. Ticho and others). 

Figures II-1 and II-2 illustrate the general features of the K-63 beam, which has been described in detail elsewhere. The Bevatron internal proton beam (operating at 6.1 BeV, 1.2x10^12 protons/pulse) strikes a copper target 4 in. long by 1/8 in. wide by 1/16 in. high. The secondary beam channel accepts ~ 0.10 msr at an angle of 0°. Momentum selection (~ ± 1.5% about the nominal momentum) is performed by collimators at mass slit 1, after horizontal dispersion in M3. Electrostatic separators S1 and S2 separate K^- from background (mostly π^-) in two stages, in the vertical plane. These separators, of the glass-cathode type described in Ref. 15, maintain a potential of 500 kV across a 2-inch gap. At the two mass slits, K^- and π^- are focused into images 1/16 in. high separated by ~ 1/8 in. The K^- pass through the slits to the bubble chamber, whereas the π^-, passing through uranium in the slit jaws (see Fig. II-3), lose ~ 8% of their momentum and are swept aside by the bending magnet M4.
The rather broad momentum bite necessitated the use of special cocked mass slits, a design feature utilized for perhaps the first time in a bubble-chamber beam. One of these slits (#1) is described in Fig. II-3; slit #2 is similar in design but more nearly parallel to the beam direction. Particles are focused at various distances (y) along the beam axis, the higher-momentum particles being focused further downstream. The bending in the Bevatron field and in magnet M3 produces horizontal dispersion; and the mass slit is designed so that a particle of any momentum (in the 3% interval) is focused at some point along the mass slit. Images in the horizontal and vertical planes approximately coincide and "track" linearly with momentum to follow the mass slit. (Degrading and multiple scattering in the tapered jaws were studied by computer calculations; uranium was found more effective in eliminating pions than iron or copper.)

The critical design requirements necessitated the use of many quadrupoles, which were carefully corrected for aberrations. Optimum quadrupole positions and currents were calculated with a special analog computer designed by Murray. The beam was tuned for initial running in July, 1963.

Over an 18-month period, 2376 rolls (average ~ 630 frames/roll) of K⁻ film were photographed, including:
(a) 897 rolls at 2.45 to 2.7 BeV/c; (b) 235 rolls at
2.1 BeV/c; (c) 249 rolls at 1.7 BeV/c; (d) 26 rolls at 2.9 BeV/c, which had unacceptably low $K^-$ yield and high background; (e) 321 rolls at 2.0 BeV/c, for UCLA; (f) 425 rolls at 2.1 BeV/c, with lead plates in the chamber; and (g) 223 rolls in $D_2$ (no lead plates), at 2.1 and 2.63 BeV/c. Only the data from (a), (b), and (c), amounting to $\sim 26$ events/mbarn, are discussed in this report; in this exposure, we observe 6 to 10 beam tracks per frame, including 15 to 35% non-$K^-$ background.

The same beam setup was used for a $\pi^-$ exposure from 1.6 to 4.2 BeV/c.
III. SELECTION OF EVENTS

After being topologically scanned, the events of the K-63 experiment were measured either on one of the SMP's Franckenstein measuring projectors or on one of the scanning and measuring projectors. The measured events were processed on the IBM 7094 or 7044 with the standard data-analysis programs of the Alvarez group—PANAL, PACKAGE, WRING, AFREET, and DST-EXAM. Failing events (events failing to fit acceptably any kinematic hypothesis) were remeasured and, when necessary, re-examined at the scanning table. For ambiguous events, ionization information was used wherever possible to distinguish between competing hypotheses.

The actual fitting of the events, done by PACKAGE, begins with a three-dimensional reconstruction of each measured track; appropriate corrections are made for energy loss, optical distortions, and non-uniformity of the magnetic field. The measured momenta and angles of each track at a production or decay vertex are adjusted to give a best fit to each of several particle-assignment hypotheses, and a $\chi^2$ is calculated. In DST-EXAM the $\chi^2$ values from individual vertices are combined to form an overall confidence level for each of several production and decay hypotheses, on the basis of which the most likely hypothesis is selected. Events passing no hypothesis with a confidence level $\geq 0.005$ are not used.
In this paper we consider only $\Xi^-$ events in which both the $\Xi^-$ and $\Lambda$ decay visibly in the chamber; these occur in the following final states and topologies:

Event-type 72 (vee with two prongs and negative decay vertex):

$$K^- + p \rightarrow \Xi^- + K^+ + \pi^- + \pi^+ + \pi^- + K^0 \text{ unseen}$$

Event-type 74 (vee with four prongs and negative decay vertex):

$$K^- + p \rightarrow \Xi^- + K^+ + \pi^- + \pi^+ + \pi^-$$

Event-type 12 (two vees, two prongs, and negative decay vertex):

$$K^- + p \rightarrow \Xi^- + K^0 + \pi^+; \quad K^0 \rightarrow \pi^+ + \pi^- \text{ (K$^0$ seen)}$$

$$\Xi^- + K^0 + \pi^+ + \pi^-; \quad K^0 \rightarrow \pi^+ + \pi^- \text{ (K$^0$ seen)}$$

In fitting each of the hypotheses, we made a 3C (three-constraint) fit at the $\Lambda$ decay vertex, and a 3C fit at the $K^0$ decay vertex where a $K^0$ was observed. The fitted $\Lambda$ momentum was used in a 4C fit (3C for events having short $\Xi^-$) at the $\Xi^-$ decay vertex. Finally the fitted $\Xi^-$ momentum was used in a 4C fit, a 1C fit, or a missing-mass calculation at the production vertex.

In Table III-1 we list, according to topology and final state, the number of events obtained at each
Table III-1. Final states and momenta of \( \Xi^- \) events analyzed.

<table>
<thead>
<tr>
<th>Expt.</th>
<th>( P ) (BeV/c)</th>
<th>( \Xi^-K^+ )</th>
<th>( \Xi^-K^+\pi^0 )</th>
<th>( \Xi^-K^0 \pi^+ )</th>
<th>( \Xi^-K^+) +neutrals</th>
<th>( \Xi^-\pi^+ ) +neutrals</th>
<th>( \Xi^-K^0 \pi^+ )</th>
<th>( \Xi^-K^0\pi^+\pi^0 )</th>
<th>( \Xi^-K^+\pi^+\pi^- )</th>
</tr>
</thead>
<tbody>
<tr>
<td>K-63</td>
<td>1.7</td>
<td>272</td>
<td>31</td>
<td>54</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>K-63</td>
<td>2.1</td>
<td>342</td>
<td>105</td>
<td>173</td>
<td>1</td>
<td>6</td>
<td>94</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>K-63</td>
<td>2.45</td>
<td>76</td>
<td>47</td>
<td>50</td>
<td>6</td>
<td>7</td>
<td>24</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>K-63</td>
<td>2.55</td>
<td>103</td>
<td>66</td>
<td>85</td>
<td>15</td>
<td>24</td>
<td>26</td>
<td>6</td>
<td>17</td>
</tr>
<tr>
<td>K-63</td>
<td>2.6</td>
<td>153</td>
<td>131</td>
<td>145</td>
<td>28</td>
<td>45</td>
<td>67</td>
<td>15</td>
<td>38</td>
</tr>
<tr>
<td>K-63</td>
<td>2.7</td>
<td>76</td>
<td>49</td>
<td>54</td>
<td>13</td>
<td>24</td>
<td>22</td>
<td>6</td>
<td>11</td>
</tr>
<tr>
<td>K-63 total</td>
<td></td>
<td>1022</td>
<td>429</td>
<td>561</td>
<td>63</td>
<td>106</td>
<td>233</td>
<td>37</td>
<td>78</td>
</tr>
</tbody>
</table>
momentum in the K-63 experiment. The events listed represent approximately 85% of the events that will eventually be available when the remeasurement process has been completed.

The identification of a \( \Xi^- \) with visible \( \Lambda \) decay, even without ionization information, is completely unambiguous. Of the 2181 type-72 events listed, not one was ambiguous with the other hypotheses tested, namely:

\[
\begin{align*}
K^- + p &\rightarrow \Xi^- + K^0 + \pi^+; \quad \Xi^- \rightarrow \Lambda + \pi^-; \quad K^0 \rightarrow \pi^0 + \pi^- \\
&\rightarrow \Xi^- + K^0 + \pi^+ + \pi^-; \quad \Xi^- \rightarrow \Lambda + \pi^-; \quad K^0 \rightarrow \pi^0 + \pi^-; \quad \Lambda \rightarrow n (4) \quad \text{or} \quad \Lambda \rightarrow 2 \pi^- n \quad \text{and} \quad \Xi^- + K^- + K^0; \quad \Xi^- \rightarrow \Lambda + \pi^-; \quad K^0 \rightarrow \pi^0 + \pi^-.
\end{align*}
\]

No competing hypotheses were tested for event type 74 or 12.

About 6% of the events listed are ambiguous between two or more hypotheses involving \( \Xi^- K^+ \pi^- \) and \( \Xi^- K^0 \pi^+ \) final states (in event type 72) are most numerous. Our analysis of \( \Xi^- \) decay parameters, however, is virtually independent of the final state in which the \( \Xi^- \) is produced, so that we retain in our sample events ambiguous between two or more final states involving \( \Xi^- \) with visible \( \Lambda \) decay.

Table II-2 lists the subsamples into which the data were divided for analysis. The older K-72 experiment is included.

For more detailed discussion of the selection of events, especially the separation of \( \pi^- p \) from \( K^- p \) interactions, see Ref. 21.
### Table II: Subsamples used in \( \hat{\epsilon} \cdot \hat{K} \) decay parameter analysis.

<table>
<thead>
<tr>
<th>Expt.</th>
<th>Subsample</th>
<th>Final state</th>
<th>( p ) (BeV/c)</th>
<th>Events</th>
<th>Subsamples</th>
<th>Events per subsample</th>
<th>( \hat{\epsilon} \cdot \hat{K} ) cutoff points</th>
</tr>
</thead>
<tbody>
<tr>
<td>K-63</td>
<td>( \Xi^- K^+ )</td>
<td>1.7</td>
<td>272</td>
<td>4</td>
<td>68</td>
<td>0.88, 0.70, -0.07</td>
<td></td>
</tr>
<tr>
<td>K-63</td>
<td>( \Xi^- K^+ )</td>
<td>2.1</td>
<td>342</td>
<td>5</td>
<td>68</td>
<td>0.90, 0.75, 0.45, -0.43</td>
<td></td>
</tr>
<tr>
<td>K-63</td>
<td>( \Xi^- K^+ )</td>
<td>2.45, 2.55</td>
<td>179</td>
<td>2</td>
<td>89</td>
<td>0.73</td>
<td></td>
</tr>
<tr>
<td>K-63</td>
<td>( \Xi^- K^+ )</td>
<td>2.6, 2.7</td>
<td>229</td>
<td>3</td>
<td>76</td>
<td>0.89, 0.50</td>
<td></td>
</tr>
<tr>
<td>K-63</td>
<td>( \Xi^- K^+ \pi^0 )</td>
<td>1.7, 2.1</td>
<td>136</td>
<td>2</td>
<td>68</td>
<td>0.43</td>
<td></td>
</tr>
<tr>
<td>K-63</td>
<td>( \Xi^- K^+ \pi^0 )</td>
<td>2.45, 2.55</td>
<td>113</td>
<td>2</td>
<td>56</td>
<td>a</td>
<td></td>
</tr>
<tr>
<td>K-63</td>
<td>( \Xi^- K^+ \pi^0 )</td>
<td>2.6, 2.7</td>
<td>180</td>
<td>2</td>
<td>90</td>
<td>0.43</td>
<td></td>
</tr>
<tr>
<td>K-63</td>
<td>( \Xi^- K^0 \pi^+ )</td>
<td>1.7, 2.1</td>
<td>321</td>
<td>4</td>
<td>80</td>
<td>0.58, 0.20, -0.36</td>
<td></td>
</tr>
<tr>
<td>K-63</td>
<td>( \Xi^- K^0 \pi^+ )</td>
<td>2.45, 2.55</td>
<td>185</td>
<td>3</td>
<td>62</td>
<td>0.70, 0.07</td>
<td></td>
</tr>
<tr>
<td>K-63</td>
<td>( \Xi^- K^0 \pi^+ )</td>
<td>2.6, 2.7</td>
<td>288</td>
<td>4</td>
<td>72</td>
<td>0.78, 0.43, -0.17</td>
<td></td>
</tr>
<tr>
<td>K-63</td>
<td>( \Xi^- K^\pi^\pi )</td>
<td>All</td>
<td>115</td>
<td>1</td>
<td>115</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>K-63</td>
<td>( \Xi^- K^\pi^\pi )</td>
<td>All</td>
<td>169</td>
<td>2</td>
<td>85</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>K-63</td>
<td>( \Xi^- K^+ + ) neutrals</td>
<td>All</td>
<td>2529</td>
<td>34</td>
<td>75</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>K-72</td>
<td>( \Xi^- K^+ )</td>
<td>1.2-1.4</td>
<td>194</td>
<td>3</td>
<td>65</td>
<td>0.69, 0.10</td>
<td></td>
</tr>
<tr>
<td>K-72</td>
<td>( \Xi^- K^+ )</td>
<td>1.5</td>
<td>470</td>
<td>7</td>
<td>67</td>
<td>0.90, 0.78, 0.59</td>
<td></td>
</tr>
<tr>
<td>K-72</td>
<td>( \Xi^- K^+ )</td>
<td>1.6, 1.7</td>
<td>166</td>
<td>2</td>
<td>83</td>
<td>0.29, -0.13, -0.53</td>
<td></td>
</tr>
<tr>
<td>K-72</td>
<td>( \Xi^- K^+ \pi^0 )</td>
<td>All</td>
<td>902</td>
<td>13</td>
<td>69</td>
<td>--</td>
<td></td>
</tr>
</tbody>
</table>

\( a \) Both subsamples contain events from \( \hat{\epsilon} \cdot \hat{K} = -1 \) to +1.
IV. THEORY

In this paper we shall discuss the decay properties of the $\Xi$ hyperon and the $\Xi^*(1817)$. The $\Xi$ decays weakly, principally via the non-leptonic modes

$$\Xi^- \rightarrow \Lambda + \pi^-$$
$$\Xi^0 \rightarrow \Lambda + \pi^0.$$  (5)

The $\Xi^*(1817)$ decays strongly, principally via

$$\Xi^*(1817) \rightarrow \Lambda + \bar{K}$$
$$\Xi^*(1817) \rightarrow \Xi^*(1530) + \pi.$$  (6)

We may schematically represent these processes by

$$F_J \rightarrow F_{J'} + B_0,$$  (7)

where $F_J$, $F_{J'}$, and $B_0$ are a fermion of spin $J$, a fermion of spin $J'$, and a spinless boson, respectively. The angular distributions in a decay process of this type have been investigated by a number of authors, including Capps, Gatto and Stapp, Byers and Fenster, Ademollo and Gatto, Button-Shafer, Zemach, and Berman and Jacob. We shall lean most heavily on the work of Byers and Fenster (cast into a maximum-likelihood framework); this uses the language of irreducible tensors $T_{LM}$ for the special case $J' = 1/2$. We shall also use Reference 30, an extension of the Byers-Fenster formalism, to treat the case $J' = 3/2$.

The Byers-Fenster/formalism has several appealing features: (1) the initial spin state of $F_J$ is described
by a complete set of independent parameters, without assumptions regarding the mechanisms that produced $F_J$; (ii) the mechanism describing the decay of $F_J$ may be described in terms of simple helicity amplitudes; (iii) the spin state of $F_J$, is expressed in terms of expectation values of a complete set of orthogonal spin operators. As a result, one can readily formulate tests to extract all possible information (about the spin and decay properties of $F_J$) from a given set of observed decays.

Coordinate Systems and Relativistic Transformations

Figure 7V-1 illustrates $\Xi$ production and decay via the sequence (A) $K^- + p + \Xi + K$; (B) $\Xi + \Lambda + \pi$; (C) $\Lambda + p + \pi$. In the c.m. frame (A), the axes $X,Y,Z$, and the $\Xi$ production angle $\Theta$ are defined in terms of the incoming $K^-$ direction $\hat{K}$ and the outgoing $\Xi$ direction $\hat{\Xi}$. In the $\Xi$ rest frame (B), the $\Lambda$ direction $\hat{\Lambda}$ is defined in terms of angles $\Theta$ and $\Phi$. In the rest frame (C), the $\Lambda$ polarization $\hat{P}_{\Lambda}$ and the proton direction $\hat{p}$ are described with reference to axes $x$ and $y$, illustrated in the expanded view (upper left) of system (B).

In a particle's own rest frame, its spin state, and the angular distribution of its decay products, are conveniently expressed in terms of tensors formed from the three components of a spin operator $\hat{S}$. When one wishes to describe multistep production and decay processes
similar to that of Fig. IV-1, the three-dimensional description of spin states may be used (even in the relativistic region), provided the observed momenta of the reaction are transformed successively through all intermediate rest frames, via successive "direct Lorentz transformations": \(^34, 35\)

Description of Initial Spin State

Using the language of Byers and Fenster, \(^36\) we expand the initial-state density matrix \(\rho_1\) in terms of irreducible tensor operators \(T_{LM}\):

\[
\left(\rho_1\right)_{jk} = \frac{1}{2J+1} \sum_{L=0}^{L} \sum_{M=-L}^{L} (2L+1) t^*_{LM} \left(T_{LM}\right)_{jk},
\]

where the quantities \(t_{LM} = \text{Tr}(\rho_1 T^\dagger_{LM})\) are expectation values of the \(T_{LM}\). The \(T_{LM}\) are normalized so that

\[
\text{Tr}(T^\dagger_{LM} T_{LM}) = \frac{2J+1}{2L+1} \delta_{LL'} \delta_{MM'},
\]

and are formed from spin operators \(S_x, S_y,\) and \(S_z\), as the spherical harmonics \(Y_{LM}\) are formed from coordinates \(x, y,\) and \(z\). [For example, \(T_{11}^\alpha(S_x + iS_y),\) in analogy with \(Y_{11}^\alpha(x + iy).\)] The tensor operators \(T_{LM}\) and the expectation values \(t_{LM}\) obey the relations

\[
T_{L,-M} = (-)^M T^\dagger_{LM},
\]

\[
t_{L,-M} = (-)^M t^*_{LM}.
\]
Hence $t_{L0}$ is real. The normalization condition $\text{Tr} \rho_i = 1$ implies that $t_{00} = 1$. The matrix representation of the $T_{LM}$ depends upon the dimensionality $(2J+1)$ of the spin space, and also upon the choice of basis vectors used to define the space. In a representation where $T_{L0}$ is diagonal, the matrix elements of the $T_{LM}$ are real, and equal to Clebsch-Gordan coefficients:

$$
\left( T_{LM} \right)_{M'' M}^{M'M} = C(JLJ; M'M) \delta_{M'' M', M'M} \quad (11)
= (-)^{J-M'} \left( \frac{2J+1}{2L+1} \right)^{1/2} C(JLJ; M'', -M') \delta_{M'' M; -M'.}
$$

(The notation for the Clebsch-Gordan coefficients corresponds to $C(J_1 J_2 J_3; m_1 m_2)$, where $J_1 + J_2 = J_3$ and $m_1 + m_2 = m_3$.)

Noting that $t_{10}$ is related to the expectation value of the spin operator $S_z$ by

$$
t_{10} = \left[ \frac{1}{J(J+1)} \right]^{1/2} \langle S_z \rangle,
$$

one obtains an upper limit for $|t_{10}|$, for any spin $J$:

$$
|t_{10}| \leq \left( \frac{J}{J+1} \right)^{1/2} \quad (13)
$$

Similar relations may be derived for other $T_{LM}$. An additional restriction on the permitted range of the $t_{L0}$ is imposed by the requirement that the diagonal elements of the density matrix be real and non-negative.

Substituting appropriate values of the matrix elements $\left( T_{LM} \right)_{jk}$ into Eq. (8), one obtains the following
inequalities:

For $J = 1/2$:

$$1 + \sqrt{3} t_{10} > 0 .$$  \hfill (14)

For $J = 3/2$:

$$1 + 3 \sqrt{\frac{3}{5}} t_{10} + \sqrt{\frac{5}{7}} t_{20} + \sqrt{\frac{7}{5}} t_{30} > 0$$

$$1 + \sqrt{\frac{3}{5}} t_{10} - \sqrt{\frac{5}{7}} t_{20} + 3\sqrt{\frac{7}{5}} t_{30} > 0 .$$  \hfill (15)

For $J = 5/2$:

$$1 + 3 \sqrt{\frac{5}{7}} t_{10} + 5 \sqrt{\frac{5}{14}} t_{20} + \sqrt{\frac{35}{6}} t_{30} + 3 \sqrt{\frac{3}{14}} t_{40} + \frac{11}{42} t_{50} > 0$$

$$1 + 9 \sqrt{\frac{1}{35}} t_{10} - \sqrt{\frac{5}{14}} t_{20} + 7 \sqrt{\frac{7}{30}} t_{30} - 9 \sqrt{\frac{3}{14}} t_{40} + 5 \sqrt{\frac{11}{42}} t_{50} > 0$$

$$1 + 3 \sqrt{\frac{1}{35}} t_{10} - 2 \sqrt{\frac{10}{7}} t_{20} + 2 \sqrt{\frac{14}{15}} t_{30} + 3 \sqrt{\frac{6}{7}} t_{40} + 5 \sqrt{\frac{22}{21}} t_{50} > 0 .$$  \hfill (16)

If all $t_{L0}$ having $L > 1$ are zero, these various inequalities reduce to the condition

$$|t_{10}| \leq \frac{1}{3} \sqrt{\frac{J+1}{J}} .$$  \hfill (17)

The inequalities (13) and (17) are equivalent to the two inequalities of Lee and Yang:

$$|\langle \cos \theta \rangle| \equiv |\langle \hat{F}_J \cdot \hat{n} \rangle| \leq \frac{1}{2J+2} ,$$  \hfill (18)

and

$$|\langle \cos \theta \rangle| \leq \frac{1}{6J} .$$  \hfill (19)

Equation (19) holds only if no powers higher than $(\cos \theta)$ appear in the $F_J$ decay distribution $I(\theta, \phi)$. 
Further inequalities restricting the values of the $t_{LM}$ have been pointed out by Byers and Fenster, and others, but in our analysis these inequalities provide no new information.

In certain cases some of the $t_{LM}$ may vanish due to symmetries in the production process. For example, for particles $J$ produced in a parity-conserving reaction of the type

$$A + B \rightarrow J + C + D + E + \ldots ,$$

the expectation values $t_{LM}$ describing the spin state of the particles $J$ in their rest frame vanish for odd $M$, provided:

(i) the axis of quantization of the $t_{LM}$ is the production normal

$$\hat{z} = \hat{A} \times \hat{J};$$

(ii) the beam and target particles ($A$ and $B$) are unpolarized, and averages are taken over the spin states of the final-state particles $C,D,E$, etc.;

(iii) averages are taken over all directions of $C,D,E$, etc. 

(This is a generalization of Capps' Checkerboard Theorem.)

Because of the symmetries in the production reaction, we choose to express the initial-state density matrix $\rho_1$ in the $(X,Y,Z)$ coordinate system, having as its $Z$-axis
the production normal \( \hat{n} \). (See Fig. IV-1.) Hence \( \rho_f \) must consist of two parts: a rotation matrix \( R(\phi, \theta, 0) \) transforming \( \rho_1 \) into the helicity representation, and the diagonalized transition matrix \( M' \) describing the actual decay. That is,

\[
\rho_f = M'R(\phi, \theta, 0) \rho_1 R^\dagger(\phi, \theta, 0) (M')^\dagger
\]

where \( M' \) and \( \rho_f \) are represented in the helicity system \((x, y, z)\) and \( \rho_1 \) in the production system \((X, Y, Z)\).

The element of the complete decay matrix \( M = M'R(\phi, \theta, 0) \) may be written

\[
[M]_{\lambda m} = A_\lambda [(2J+1)/4\pi]^{1/2} \sum_{m_\lambda} \phi^{J, \lambda}_{m_\lambda} (\phi, \theta, 0),
\]

where \( \lambda \) is the projection of spin \( J \) (and \( J' \)) on the helicity \( z \)-axis, and \( m \) is the projection of spin \( J \) on the production \( Z \)-axis. The "helicity amplitudes" \( A_\lambda \) are the elements of the diagonalized transition matrix \( M' \), each representing the probability amplitude for the process (with helicity \( \lambda \)).

\[
|J, \lambda\rangle \rightarrow |J', \lambda\rangle + B_0.
\]

The functions \( \phi_{J, m_1 m_2}^{\lambda}(\alpha, \beta, \gamma) \) are the usual matrix elements of the rotation operator \( R(\alpha, \beta, \gamma) \):

\[
\phi_{J, m_1 m_2}^{\lambda}(\alpha, \beta, \gamma) = \exp(-im_1 \alpha) \sum_{m_2} d_{m_1 m_2}^{J}(\beta) \exp(-im_2 \gamma).
\]

(See, for example, Jacob and Wick.)

In general, the decay process \( F_J \rightarrow F_{J'} + B_0 \) can proceed via several different partial waves \( \lambda \), where \( \lambda \) may assume values from \( |J - J'| \) to \( |J + J'| \). In strong decay,
only those partial waves consistent with parity conservation contribute to the decay amplitude. In terms of the usual complex partial-wave decay amplitudes $a_\lambda$, the helicity amplitudes of Eq. (22) have the form

$$A_\lambda = (-)^{\lambda - J'} \sum_\ell a_\ell C(JJ' \ell; \lambda, -\lambda) . \quad (24)$$

Byers and Fenster (and Button-Shafer) simplify the expression for $\rho_f$ by utilizing the orthonormality properties of the $\Theta^J_{M\lambda}$ functions and Clebsch-Gordan coefficients. The general form (in the helicity representation) is

$$\begin{align*}
\langle \rho_f \rangle_{J', \lambda} & \equiv \langle J', \lambda | \rho_f | J', \lambda' \rangle = (-)^{J - \lambda'} (A_\lambda A_\lambda^*/4\pi) (2J + 1)^{1/2} \\
\times \sum_{L=0}^{2J} \sum_{M=-L}^{L} \left[ (2L+1)^{1/2} C(JJL; \lambda, -\lambda') t_{LM}^* \Theta^L_{M, \lambda - \lambda'} (\phi, \theta, 0) \right],
\end{align*} \quad (25)$$

which is valid for any spin $J$ and $J'$ (integer as well as half-integer). We note that only the $\Theta^L_{M, \lambda - \lambda'} (\phi, \theta, 0)$ having integral indices $L, M$, and $(\lambda - \lambda')$ appear in Eq. (25) and that each $t_{LM}$ describing the initial $F_J$ multiplies a single $\Theta^L_{M, \lambda - \lambda'}$ function.

Having arrived at an expression for the final-state density matrix, one may calculate the angular distribution of the decay process

$$F_J \rightarrow F_{J'} + B_0 \quad \Rightarrow \quad I(\theta, \phi) = \text{Tr}(\rho_f) . \quad (26)$$
A complete description of the spin state of $F_{J'}$ as a function of $\theta$ and $\phi$ is obtained by calculating the quantities (with $T_{LM}$ operators now used in the $F_{J'}$ spin space, helicity representation)

$$I \langle T_{LM} \rangle (\theta, \phi) = \text{Tr} (\rho I T_{LM}) . \quad (27)$$

With slight deviation from the work of Byers and Fenster, we now evaluate (26) and (27) for the special case $J' = 1/2$. Then we discuss some of the results obtained by Button-Shafer for the special case of strong decay with $J' = 3/2$.

a. Spin $J$ = spin $1/2$ + spin 0.

For weak decay, the two partial waves $a \equiv a_{J-1/2}$ and $b \equiv a_{J+1/2}$ can contribute to the transition matrix $M$. One customarily defines

$$\alpha + i \beta \equiv (\text{Re} + i \text{Im})[a^* b/(|a|^2 + |b|^2)] \quad (28)$$

$$\gamma \equiv (|a|^2 - |b|^2)/(|a|^2 + |b|^2)$$

so that $\alpha^2 + \beta^2 + \gamma^2 = 1$. Alternatively, the parameters $\beta$ and $\gamma$ may be expressed in terms of independent parameters $a$ and $\phi$:

$$\beta + i \gamma \equiv [1 - a^2]^{1/2} e^{i\phi} . \quad (29)$$

Using Eqs. (26) and (27) with $T_{LM} \parallel \sigma_z$ and $\sigma_x \perp \sigma_y$ ($L,M = 1,0$ and 1,1) one arrives at the result
where

\[ I(e, \Theta) = a_{\text{IP}'} \int \ldots V \ldots \] 

\[ \sum_{L = 0}^{2J-1} \sum_{L \text{ even}} + \sum_{L = 1}^{2J} \sum_{L \text{ odd}} \sum_{M=-L}^{L} n_{L0}^{J} t_{LM} y_{LM}^{*}(\Theta, \Phi) \]  

\[ IP' \cdot z = \alpha \sum_{L = 0}^{2J-1} \sum_{L \text{ even}} + \sum_{L = 1}^{2J} \sum_{L \text{ odd}} \sum_{M=-L}^{L} n_{L0}^{J} t_{LM} y_{LM}^{*}(\Theta, \Phi) \]  

\[ IP' \cdot \hat{x} + 1 \cdot IP' \cdot \hat{y} = (-\gamma + i\beta)(2J+1) \sum_{L = 1}^{2J} \sum_{L \text{ odd}} \sum_{M=-L}^{L} n_{L0}^{J} t_{LM} E_{M, L}(\Theta, \Phi, 0) \]  

\[ X \left[ (2L + 1)/4\pi \right]^{1/2} \left[ L(L+1) \right]^{-1/2}, \]

where

\[ n_{L0}^{J} = (-)^{J - \frac{1}{2}} \left[ (2J + 1)/4\pi \right]^{1/2} c(J, J, L; \frac{1}{2}, \frac{1}{2}, \frac{1}{2}) \]  

For the two-step decay process

\[ \Xi + \Lambda + \pi; \Lambda + p + \pi, \]

the joint angular distribution of the \( \Lambda \) (in the \( \Xi \) rest frame) and of the decay proton (in the \( \Lambda \) rest frame) is given by

\[ I(\hat{\Lambda}, \hat{p}) = I(\hat{\Lambda}) [1 + \alpha_{\Lambda} \hat{p} \cdot \hat{A}] \]

\[ = I(\Theta, \Phi) + \alpha_{\Lambda} [\hat{\text{IP}'} \cdot \hat{A}(\hat{p}, \hat{\Lambda}) + \hat{\text{IP}'} \cdot \hat{x}(\hat{p}, \hat{x}) + \hat{\text{IP}'} \cdot \hat{y}(\hat{p}, \hat{y})], \]
where the Λ angular distribution \( I(\theta, \phi) \) and the Λ polarization components \( I_\Lambda \cdot x, I_\Lambda \cdot y, \) and \( I_\Lambda \cdot z = I_\Lambda \cdot \hat{\Lambda} \) are given by Eqs. (30).

The distribution in \( (\hat{p} \cdot \hat{\Lambda}) \) is obtained by integrating Eq. (33) over \( \theta, \phi, \) and \( \phi_p = \tan^{-1} [(\hat{p} \cdot y)/(\hat{p} \cdot x)] : \)

\[
I (\hat{p} \cdot \hat{\Lambda}) = 1 + \alpha_\Lambda \alpha_\Xi (\hat{p} \cdot \hat{\Lambda}); \tag{34}
\]

This relation holds for any spin \( J. \) Thus a spin-independent estimate of \( \alpha_\Xi \) is possible even if all \( t_{LM} \) with \( L > 0 \) are zero.

The presence of non-zero \( t_{LM} \) with \( L > 0 \) permits a more accurate (and spin-dependent) determination of \( \alpha_\Xi. \)

If the \( \Xi \) has spin 1/2 and polarization \( P_\Xi = \vec{P}_\Xi \cdot \hat{n} = \sqrt{3} \, t_{10}, \) the distribution function (33) reduces to the familiar form

\[
I(\hat{\Lambda}, \hat{p}) = 1 + \alpha_\Xi P_\Xi \cos \theta + \alpha_\Lambda (\hat{p} \cdot \hat{\Lambda}) [\alpha_\Xi + P_\Xi \cos \theta] \\
+ \alpha_\Lambda P_\Xi \sin \theta [\beta_\Xi (\hat{p} \cdot \hat{y}) - \gamma_\Xi (\hat{p} \cdot \hat{x})]. \tag{35}
\]

For either strong or weak decay, three features of the decay distribution enable one, in principle, to determine the spin \( J: \) (1) a lower limit for \( J \) is established by the maximum complexity of the observed distribution; i.e., \( J \geq L_{\text{max}}/2 \) where \( L_{\text{max}} \) is the \( L \)-value of the highest non-zero \( t_{LM}; \) (ii) if \( |t_{10}| \) or any other \( |t_{LM}| \) exceeds its \( J \)-dependent bounds, an upper limit for

The above equations hold also for a decay such as \( \Xi^*(1817) \rightarrow \Lambda + K; \) \( \Lambda \rightarrow p + \pi^- \) if \( \alpha_{\Xi^*} \) and \( \beta_{\Xi^*} \) are set equal to zero and \( \gamma_{\Xi^*} \) taken as ± 1.
J may be established by inequalities similar to the Lee-Yang inequalities (18) and (19); (iii) if any odd-L t_{LM} are non-zero, a best value of the factor (2\bar{J}+1) of Eq. (30\bar{c}) may be determined experimentally.

For further discussion of the above formalism and use of the likelihood method see Reference 33, 6, or 21.

b. Strong decay: Spin J \rightarrow spin \frac{3}{2} + spin \frac{1}{2}.

The strong decay process

\[ F_J + F_{3/2} + B_0 \]  

(where \( F_{3/2} \) is a spin-3/2 fermion) has been discussed by Button-Shafer, 30 Zemach, 31 and Berman and Jacob, 32 among others. We utilize the formalism of reference 39 which extends the Byers-Fenster theory to obtain a complete and general description of the decay process (36).

All equations are discussed more extensively in Ref. 30, except for the introduction here of the parameter \( \lambda_0 \) (Eq. 48) and of the momentum-barrier treatment of higher \( \lambda \)-waves.

A variety of tests may be performed to determine the spin and parity of \( F_J \); we shall describe here only those used in our analysis of the reaction

\[ \Xi^*(1817) + \Xi^*(1530) + \pi; \Xi^*(1530) + \Xi + \pi, \]  

which we denote symbolically by

\[ F_J \rightarrow F_{3/2} + B_0; F_{3/2} \rightarrow F_{1/2} + B_0 \]  

The spin state of \( F_{3/2} \) may be represented by a 4x4
density matrix $\rho_f$ whose elements (in the helicity representation) are given by Eq. (25). In particular, the diagonal elements are

$$
(\rho_f)_{\lambda\lambda} = \sum_{L=0}^{2J} \sum_{M=-L}^{L} t_{LM}^* |A_\lambda|^2 n_{L\lambda}^{(2\lambda)} Y_{LM}(\theta, \phi),
$$

(34)

where the $t_{LM}$ describe the spin state of $F_J$, and where $\theta$ and $\phi$ define the direction of $F_{3/2}$ in the $F_J$ rest frame. The helicity amplitudes $A_\lambda$ are given by (24), and the $J$-dependent constants $n_{L\lambda}^{(2\lambda)}$ by the relation

$$
n_{L\lambda}^{(2\lambda)} = (-)^{J-\lambda} [(2J+1)/4\pi]^{1/2} C(J; J; \lambda, \lambda).$$

(40)

The angular distribution of $F_{3/2}$ (in the $F_J$ rest frame) is

$$
I(\theta, \phi) = \text{Tr} (\rho_f)
= 2 \sum_{L=0}^{2J-1} \sum_{L\text{ even}}^{L} \sum_{M=-L}^{L} \left[ |A_{3/2}|^2 n_{L\lambda}^{(3)} + |A_{1/2}|^2 n_{L\lambda}^{(1)} \right] t_{LM}^* Y_{LM}(\theta, \phi)
$$

(41)

A lower limit on the spin $J$ is established by the maximum complexity of the observed $I(\theta, \phi)$ distribution; i.e., if the data require non-zero $t_{LM}$ through order $L$, then $J \geq L/2$. No spin information is obtained from Eq. (41) if all $t_{LM}$ having $L > 0$ are consistent with zero. One finds that $I(\theta, \phi)$ has a particularly simple form if $\phi$ is
ignored and only the lower \( \ell \)-wave included, \( \hat{4} \xi^2 + b_2 \hat{P}_2 + a_4 \hat{P}_4 \ldots \).

If \( F_{3/2} \) subsequently decays strongly via \( F_{3/2} \rightarrow F_{1/2} + B_0 \), the angular distribution of \( F_{1/2} \) (in the \( F_{3/2} \) rest frame) is given by the Byers-Fenster formalism:

\[
\mathcal{Q}(\psi) = \frac{1}{4\pi} I(\theta, \phi) \left[ 1 - \sqrt{5} \langle T_{20} \rangle \left( \theta, \phi \right) P_2(\cos \psi) \right],
\]

where \( \psi \) is the angle of \( F_{1/2} \) relative to the \( F_{3/2} \) direction of flight, and where we have ignored the azimuth of \( F_{1/2} \) is ignored, and where \( \langle T_{20} \rangle \) describes the \( F_{3/2} \) spin state. The quantity

\[
I \langle T_{20} \rangle(\theta, \phi) = \text{Tr} \left( \rho_F T_{20} \right).
\]

represents the \( \langle T_{20} \rangle \) component of \( F_{3/2} \) "polarization" referred to helicity axes, i.e., the \( F_{3/2} \) spin alignment along its direction of flight.

Combining Eqs. (41) through (43) and integrating over \( \theta \) and \( \phi \) (the angles describing the direction of \( F_{3/2} \)), we are left with only those terms containing \( t_{00} \), so that

\[
\mathcal{Q}(\psi) \propto 1 + b_2 P_2(\cos \psi) = 1 + b_2 \left( 3 \cos^2 \psi - 1 \right)/2,
\]

where

\[
b_2 = \left( |A_{1/2}|^2 - |A_{3/2}|^2 \right) \times \left( |A_{1/2}|^2 + |A_{3/2}|^2 \right)^{-1}.
\]

(We have used the relation \( n_{00}^{(1)} = n_{00}^{(3)} \).) After integration over \( \theta \) and \( \phi \), the azimuthal distribution of
Fl/2 about the F3/2 line of flight is isotropic.

If J = 1/2, then A3/2 = 0, so that b2 = 1.0 regardless of the parity of J. If J \geq 3/2, the helicity amplitudes A_\lambda have the following form for various \varepsilon^*(1817) spin and parity assumptions J^P [where P is the parity of \varepsilon^*(1817) relative to that of \varepsilon^*(1530)]:

\begin{align*}
J^P = 3/2^-, 5/2^+, 7/2^-, & \text{ etc.:} \\
A_{1/2} & = a(3J-3/2)^{1/2} - c(3J+3/2)^{1/2} \\
A_{3/2} & = a(3J+3/2)^{1/2} + c(3J-3/2)^{1/2} \\
J^P = 3/2^+, 5/2^-, 7/2^+, & \text{ etc.:} \\
A_{1/2} & = b(J - 1/2)^{1/2} - d(3J+9/2)^{1/2} \\
A_{3/2} & = b(3J+9/2)^{1/2} + d(J - 1/2)^{1/2},
\end{align*}

where a, b, c, and d are complex amplitudes for decay via partial waves \ell = J-3/2, J-1/2, J+1/2, and J+3/2 respectively. One may show that b_2 is of the form

\begin{equation}
b_2 = S \cos \lambda_0 - T \sin \lambda_0 \cos \Delta \phi,
\end{equation}

where S, T, \Delta \phi, and \cos \lambda_0 have the following values and

\begin{align*}
\begin{array}{cccc}
J^P & S & T & \Delta \phi \\
3/2^-, 5/2^+, 7/2^-, & \frac{|a|^2 - |c|^2}{2 |a| |c|} & \frac{\delta_c - \delta_a}{(2J-3)/4J} & \\
3/2^+, 5/2^-, 7/2^+, & \frac{|b|^2 - |d|^2}{2 |b| |d|} & \frac{\delta_d - \delta_b}{(-2J-5)/(4J+4)} & \\
\end{array}
\end{align*}
We note that \(-1 < b_2 < 1\), regardless of the magnitudes and relative phases of \(a, b, c, \) and \(d\). If only the lower partial wave (\(a\) or \(b\)) contributes (for those cases having \(J > 1/2\)), \(b_2\) has the following values:

<table>
<thead>
<tr>
<th>(J^P)</th>
<th>(b_2)</th>
<th>(J^P)</th>
<th>(b_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2(^+)</td>
<td>1.00</td>
<td>1/2(^-)</td>
<td>1.00</td>
</tr>
<tr>
<td>3/2(^-)</td>
<td>0.00</td>
<td>3/2(^+)</td>
<td>-0.80</td>
</tr>
<tr>
<td>5/2(^+)</td>
<td>0.20</td>
<td>5/2(^-)</td>
<td>-0.71</td>
</tr>
<tr>
<td>7/2(^-)</td>
<td>0.29</td>
<td>7/2(^+)</td>
<td>-0.67</td>
</tr>
<tr>
<td>limit (J \to \infty)</td>
<td>0.50</td>
<td>limit (J \to \infty)</td>
<td>-0.50</td>
</tr>
</tbody>
</table>

One expects the rate of decay via partial wave \(\ell\) to be suppressed (relative to 1.0 for \(\ell = 0\)) by a factor of the order of

\[
(qR)^2 \left[ 1 + (qR)^2 \right]^{-1} \quad \text{for } \ell = 1 \tag{49a}
\]

\[
(qR)^4 \left[ 9 + 3(qR)^2 + (qR)^4 \right]^{-1} \quad \text{for } \ell = 2 \tag{49b}
\]

\[
(qR)^6 \left[ 225 + 45(qR)^2 + 6(qR)^4 + (qR)^6 \right]^{-1} \quad \text{for } \ell = 3, \tag{49c}
\]

where \(q\) is the momentum of \(F_{3/2}\) in the \(F_J\) rest frame, and \(R\) is a characteristic radius of interaction, of the order of \((2m_\pi)^{-1}\). (For the decay process \(\Xi^*(1817) \to \Xi^*(1530) + \pi,\) \(q \approx 230\text{ MeV/c} \approx 1/R.\)) Taking \(qR \approx 1\), we estimate \(|P^2|/|S^2| \approx 0.08\) and \(|F^2|/|P^2| \approx 0.007\), where \(S, P, D, \) and \(F\) are decay amplitudes for \(\ell = 0, 1, 2, \) and 3, respectively.
Even with complete ignorance of the relative phase $\Delta \phi$, we may specify the permitted range of the coefficient $a_2$, allowing for the presence of higher partial waves:

<table>
<thead>
<tr>
<th>$J^P$</th>
<th>Partial waves $\ell$</th>
<th>$b_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1/2^+$</td>
<td>1</td>
<td>1.0</td>
</tr>
<tr>
<td>$1/2^-$</td>
<td>2</td>
<td>1.0</td>
</tr>
<tr>
<td>$3/2^-$</td>
<td>0,2</td>
<td>0.0±0.5</td>
</tr>
<tr>
<td>$3/2^+$</td>
<td>1,3</td>
<td>-0.8±0.1</td>
</tr>
<tr>
<td>$5/2^+$</td>
<td>1,3</td>
<td>0.2±0.2</td>
</tr>
</tbody>
</table>

Decay via the higher partial wave is negligible for higher spin hypotheses.
V. ANALYSIS OF DATA

A. Decay parameters of the $\Xi^-$

Because the overall average polarization of our $\Xi^-$ sample is small, the 2529 K-63 events were arbitrarily divided, according to final-state momentum, and values of ($\Xi \cdot k$), into 34 subsamples, defined in Table III-2. No attempt was made to optimize the binning criteria. Also listed, in 13 subsamples, are 902 events of the K-72 experiment, corresponding approximately to the sample analyzed earlier.$^6,1$ (The sample described in Ref. 1 contains additional events not in our K-72 sample, but polarization information from 3-body final states was not used.) In Table IV-1 we compare results obtained separately from K-63 and K-72 data, and we present results from a combined sample containing 2529 + 902 = 3431 $\Xi^-$ events.

Estimates of $\alpha_{\Lambda}, \alpha_{\Xi^-}$ were obtained from maximum-likelihood fits to a decay distribution of the form Eq. (34); these estimates are independent of the assumed $\Xi$ spin, and independent of the way in which subsamples are defined. Estimates of $\alpha_{\Lambda}, \alpha_{\Xi^-}, \phi_{\Xi^-}$ were obtained from fits to a decay distribution of the form (35); variable parameters in the fits were $\alpha_{\Lambda}, \alpha_{\Xi^-}, \phi_{\Xi^-}$, and the polarization of each subsample. Fits were performed (as indicated) both with $\alpha_{\Lambda}$ free, and with $\alpha_{\Lambda}$ weighted (by
Table V-1. Decay parameters for $\Xi^-$

<table>
<thead>
<tr>
<th>Sample</th>
<th>Events</th>
<th>$a_\Lambda a_{\Xi^-}$</th>
<th>Subsamples</th>
<th>$\ln L$</th>
<th>$a_\Lambda$</th>
<th>$a_{\Xi^-}$</th>
<th>$\phi_{\Xi^-}$ (deg)</th>
<th>Correlation matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>K-63</td>
<td>2529</td>
<td>$-0.262 \pm 0.033$</td>
<td>34</td>
<td>68.95</td>
<td>0.743 $\pm$ 0.122</td>
<td>$-0.344 \pm 0.063$</td>
<td>10.1 $\pm$ 11.4</td>
<td>$(a_\Lambda a_{\Xi^-})$ (a_\Lambda \phi_{\Xi^-}) (a_{\Xi^-} \phi_{\Xi^-})$</td>
</tr>
<tr>
<td>K-72</td>
<td>902</td>
<td>$-0.281 \pm 0.055$</td>
<td>13</td>
<td>41.16</td>
<td>0.685 $\pm$ 0.107</td>
<td>$-0.426 \pm 0.067$</td>
<td>9.7 $\pm$ 14.1</td>
<td>0.789 $-$ 0.027 $-$ 0.018</td>
</tr>
<tr>
<td>Combined</td>
<td>3431</td>
<td>$-0.267 \pm 0.028$</td>
<td>47</td>
<td>109.70</td>
<td>0.698 $\pm$ 0.069</td>
<td>$-0.381 \pm 0.045$</td>
<td>10.0 $\pm$ 8.9</td>
<td>0.295 $-$ 0.008 $-$ 0.020</td>
</tr>
<tr>
<td>K-72$^a$</td>
<td>1004$^b$</td>
<td>--</td>
<td>12</td>
<td>38.74</td>
<td>0.682 $\pm$ 0.104</td>
<td>$-0.362 \pm 0.058$</td>
<td>0.3 $\pm$ 10.6</td>
<td>0.653 $-$ 0.026 $-$ 0.025</td>
</tr>
</tbody>
</table>

$^a$ Previously published results, included for comparison with our K-72 sample (see Ref. 1).

$^b$ Includes 176 events providing information only on $a_\Lambda a_{\Xi^-}$.
a factor \( \exp \left[ -\frac{1}{2} \left( \alpha_\Lambda - 0.62 \right)^2 / 0.07^2 \right] \) in the likelihood.

Quoted errors on \( \alpha_\Lambda \), \( \alpha_{\Xi^-} \), and \( \Phi_{\Xi^-} \) were obtained from the error matrix \( G \), calculated as the negative of the inverse of the second-derivative matrix of \( w = \ln \mathcal{L} \) (where the likelihood \( \mathcal{L} = \prod_{i=1}^{N} \mathcal{L}(\hat{\Lambda}_i, \hat{p}_i) \)). If \( x_1, x_2, \ldots \) are variable parameters in the maximum-likelihood fit, the error \( \delta x_i \) of a parameter \( x_i \) is given by

\[
(\delta x_i)^2 = G_{ii}
\]

where \((G^{-1})_{jk} = \frac{\delta^2 w}{\delta x_i \delta x_j} \) \( \ldots \) \( (50a) \)

The correlation coefficients listed are off-diagonal elements of the normalized error matrix \( C_{jk} = G_{jk} (G_{jj} G_{kk})^{-1/2} \). A study of Monte Carlo events demonstrates that the calculated errors \( \delta x_i \) correspond to the rms deviation of independent measurements of \( x_i \), i.e.

\[
(\delta x_i)^2 = \langle (x_i - \langle x_i \rangle)^2 \rangle = \langle x_i^2 \rangle - \langle x_i \rangle^2 \ldots \) \( (51) \)

In Fig. \( \mathbf{V-1} \) we illustrate the correlation between \( \alpha_\Lambda \) and \( \alpha_{\Xi^-} \), for the combined K-72 and K-63 data. Correlations between \( \alpha_\Lambda \) and \( \Phi_{\Xi^-} \), and between \( \alpha_{\Xi^-} \) and \( \Phi_{\Xi^-} \), are negligible.

As a visual check on the results presented, we display certain angular correlations in the observed \( \Xi^- \) decay distributions.
In Fig. V-2 we display the distribution of $\hat{p} \cdot \hat{\lambda}$ for the 3431 $\Xi^-$ events listed in Table III-2. As expected, the observed distribution is proportional to $1 + \alpha_{\Lambda} \alpha_{\Xi^-} \hat{p} \cdot \hat{\lambda}$ (corresponding to the straight line plotted).

In Fig. V-3 we present distributions of the four quantities $N$ (no. of events), $\sum_{i=1}^{N} (\hat{p} \cdot \hat{\lambda})_i$, $\sum_{i=1}^{N} (\hat{p} \cdot \hat{y})_i$, and $\sum_{i=1}^{N} (\hat{p} \cdot \hat{x})_i$, as functions of $\hat{\lambda} \cdot \hat{n} = \cos \theta$. For purposes of plotting, events in subsamples/having $P_{\Xi^-} < 0$ (shaded) were rotated $180^\circ$ about the beam axis, effective/raising the overall average polarization from $0.04 \pm 0.04$ to $0.26 \pm 0.04$.

**B. Spin of the $\Xi$**

The existence, in the observed $\Xi$ decay distribution, of any non-zero $t_{LM}$ having $L > 1$ would immediately establish the $\Xi$ spin $J$ to be greater than $1/2$. In Table V-2 we present, for 15 subsamples of $\Xi^-$ and $\Xi^0$ events, values of $t_{LM}$ obtained from maximum-likelihood fits assuming $J = 3/2$, $L_{\text{max}} = 3$, $\alpha_{\Lambda} = 0.62$, $\alpha_{\Xi^-} = \alpha_{\Xi^0} = -0.40$, $\Phi_{\Xi^-} = \Phi_{\Xi^0} = 0$. The $\Xi^-$ data have been corrected for scanning biases, as explained in the Appendix (and in Appendix B of Reference 21). We compare values of $\ln \mathcal{L}$ from the $L_{\text{max}} = 3$ fits (seven parameters per sample) with values obtained assuming $L_{\text{max}} = 1$ (one parameter per sample). For the 15 samples, we observe an overall increase of 44.1 in $\ln \mathcal{L}$ as $L_{\text{max}}$ is increased from 1 to 3. (An increase of
<table>
<thead>
<tr>
<th>Subsample</th>
<th>Expt.</th>
<th>Final state</th>
<th>p(BeV/c)</th>
<th>w = fnL</th>
<th>tLM (×100)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Lmax=1</td>
<td>Lmax=3</td>
</tr>
<tr>
<td>1.</td>
<td>K-72</td>
<td>Ξ−K⁺</td>
<td>1.2-1.4</td>
<td>194</td>
<td>4.8</td>
</tr>
<tr>
<td>2.</td>
<td>K-72</td>
<td>Ξ−K⁺</td>
<td>1.5</td>
<td>470</td>
<td>3.8</td>
</tr>
<tr>
<td>3.</td>
<td>K-72</td>
<td>Ξ−K⁺</td>
<td>1.6, 1.7</td>
<td>166</td>
<td>6.5</td>
</tr>
<tr>
<td>4.</td>
<td>K-63</td>
<td>Ξ−K⁺</td>
<td>1.7</td>
<td>272</td>
<td>7.4</td>
</tr>
<tr>
<td>5.</td>
<td>K-63</td>
<td>Ξ−K⁺</td>
<td>2.1</td>
<td>342</td>
<td>4.9</td>
</tr>
<tr>
<td>6.</td>
<td>K-63</td>
<td>Ξ−K⁺</td>
<td>2.45, 2.55</td>
<td>179</td>
<td>6.9</td>
</tr>
<tr>
<td>7.</td>
<td>K-63</td>
<td>Ξ−K⁺</td>
<td>2.6, 2.7</td>
<td>229</td>
<td>7.7</td>
</tr>
<tr>
<td>8.</td>
<td>K-72, K-63</td>
<td>Ξ−K⁺π⁰</td>
<td>1.5-2.1</td>
<td>154</td>
<td>1.6</td>
</tr>
<tr>
<td>9.</td>
<td>K-63</td>
<td>Ξ−K⁺π⁰</td>
<td>2.45-2.7</td>
<td>301³</td>
<td>2.2</td>
</tr>
<tr>
<td>10.</td>
<td>K-72, K-63</td>
<td>Ξ−K⁰π⁺</td>
<td>1.5-2.1</td>
<td>154</td>
<td>1.6</td>
</tr>
<tr>
<td>11.</td>
<td>K-63</td>
<td>Ξ−K⁰π⁺</td>
<td>2.45-2.7</td>
<td>473</td>
<td>8.0</td>
</tr>
<tr>
<td>12.</td>
<td>K-63</td>
<td>Ξ−K⁺π</td>
<td>All</td>
<td>284</td>
<td>4.6</td>
</tr>
<tr>
<td>13.</td>
<td>K-72, K-63</td>
<td>Ξ⁰K⁰</td>
<td>All</td>
<td>194</td>
<td>1.2</td>
</tr>
<tr>
<td>14.</td>
<td>K-72, K-63</td>
<td>Ξ⁰K⁺π⁻</td>
<td>1.5-2.1</td>
<td>164</td>
<td>2.8</td>
</tr>
<tr>
<td>15.</td>
<td>K-63</td>
<td>Ξ⁰K⁺π⁻</td>
<td>2.45-2.7</td>
<td>271³</td>
<td>3.9</td>
</tr>
<tr>
<td></td>
<td>K-63</td>
<td>Ξ⁰K⁰π⁻</td>
<td>All</td>
<td>20</td>
<td>3.9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Subsample</th>
<th>Expt.</th>
<th>Final state</th>
<th>p(BeV/c)</th>
<th>w = fnL</th>
<th>tLM (×100)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-12</td>
<td>Ξ⁻ total</td>
<td>3431³</td>
<td>69.9</td>
<td>104.6</td>
<td>34.7</td>
</tr>
<tr>
<td>13-15</td>
<td>Ξ⁰ total</td>
<td>649</td>
<td>7.9</td>
<td>17.3</td>
<td>9.4</td>
</tr>
<tr>
<td>1-15</td>
<td>Ξ⁻ and Ξ⁰, total</td>
<td>4080</td>
<td>77.8</td>
<td>121.9</td>
<td>44.1</td>
</tr>
</tbody>
</table>

³Sample No. 9 contains eight K-72 Ξ−K⁰π⁺.
45.0 is expected; we conclude that the spin-1/2 hypothesis is permitted, although not required, by the data.

The spin-3/2 hypothesis could be discounted if one of the inequalities \[(13)\] or \[(15)\] were significantly violated. Although the spin-3/2 density matrix constraint \[(15)\] is violated by three subsamples (3, 6, and 7), the effect is less than one standard deviation in each case. Violation of the spin-5/2 density matrix constraint \[(16)\] is only slightly more significant.

Hence, only the presence of the \((2J+1)\) factor in the transverse \(\Lambda\) polarization distribution \[(30c)\] affords us a possibility of spin discrimination.

**\((2J+1)\) Spin Factor**

We have investigated the \((2J+1)\) spin factor using 3278 K-72 and K-63 \(\pi^-\) events. (Only 96% of the events appearing in Table II-1 were available at the time of this analysis.) The data were arbitrarily divided into 47 approximately equal subsamples according to final state, momentum, and cm. \(\pi^-\) production angle (see Table V-3). No attempt was made to optimize the binning criteria. Also listed in Table V-3 are seven \(\Xi^0\) subsamples.

Maximum-likelihood fits were performed to an assumed \(\Xi^-\) decay distribution of the form \[(33)\] with \[(30)\]; variable parameters in the fits were \(\alpha_\Lambda, \alpha_\Xi^-, \Phi_\Xi^-,\) and a value of \(t_{10}\).
Table V-3. Subsamples used in \( \Xi \) spin analysis (K-72 and K-63 data combined).

<table>
<thead>
<tr>
<th>Subsample</th>
<th>Final state</th>
<th>p(BeV/c)</th>
<th>Events</th>
<th>Subsamples</th>
<th>Events per subsample</th>
<th>(( \Xi \cdot K )) cutoff points</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Xi^- )K(^+)</td>
<td>1.2-1.4</td>
<td>194</td>
<td>3</td>
<td>65</td>
<td>0.69, 0.10</td>
<td></td>
</tr>
<tr>
<td>( \Xi^- )K(^+)</td>
<td>1.5</td>
<td>470</td>
<td>7</td>
<td>67</td>
<td>0.91, 0.78, 0.59, 0.29, -0.11, -0.51</td>
<td></td>
</tr>
<tr>
<td>( \Xi^- )K(^+)</td>
<td>1.6, 1.7</td>
<td>304</td>
<td>4</td>
<td>76</td>
<td>0.87, 0.65, -0.14</td>
<td></td>
</tr>
<tr>
<td>( \Xi^- )K(^+)</td>
<td>2.1</td>
<td>355</td>
<td>5</td>
<td>71</td>
<td>0.89, 0.76, 0.44, -0.44</td>
<td></td>
</tr>
<tr>
<td>( \Xi^- )K(^+)</td>
<td>2.45, 2.55</td>
<td>179</td>
<td>3</td>
<td>60</td>
<td>0.86, 0.26</td>
<td></td>
</tr>
<tr>
<td>( \Xi^- )K(^+)</td>
<td>2.6, 2.7</td>
<td>229</td>
<td>3</td>
<td>76</td>
<td>0.89, 0.50</td>
<td></td>
</tr>
<tr>
<td>( \Xi^- )K(^+) ( \pi^0 )</td>
<td>1.5-2.1</td>
<td>147</td>
<td>2</td>
<td>73</td>
<td>0.43</td>
<td></td>
</tr>
<tr>
<td>( \Xi^- )K(^+) ( \pi^0 )</td>
<td>2.45, 2.55</td>
<td>112</td>
<td>2</td>
<td>56</td>
<td>0.34</td>
<td></td>
</tr>
<tr>
<td>( \Xi^- )K(^+) ( \pi^0 )</td>
<td>2.6, 2.7</td>
<td>180</td>
<td>2</td>
<td>90</td>
<td>0.43</td>
<td></td>
</tr>
<tr>
<td>( \Xi^- )K(^0) ( \pi^+ )</td>
<td>1.5-2.1</td>
<td>350</td>
<td>5</td>
<td>70</td>
<td>0.69, 0.41, -0.02, -0.48</td>
<td></td>
</tr>
<tr>
<td>( \Xi^- )K(^0) ( \pi^+ )</td>
<td>2.45, 2.55</td>
<td>186</td>
<td>3</td>
<td>62</td>
<td>0.70, 0.07</td>
<td></td>
</tr>
<tr>
<td>( \Xi^- )K(^0) ( \pi^+ )</td>
<td>2.6, 2.7</td>
<td>288</td>
<td>4</td>
<td>72</td>
<td>0.78, 0.43, -0.17</td>
<td></td>
</tr>
<tr>
<td>( \Xi^- )K(^+) ( \pi^0 )</td>
<td>all</td>
<td>135</td>
<td>2</td>
<td>68</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>( \Xi^- )K(^+) ( \pi^0 )</td>
<td>all</td>
<td>149</td>
<td>2</td>
<td>74</td>
<td>-0.05</td>
<td></td>
</tr>
<tr>
<td>( \Xi^- ) sample, total</td>
<td>3278</td>
<td>47</td>
<td>70</td>
<td>--</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| \( \Xi^- \)K\(^0\) \( \pi^- \) | 1.2-1.7 | 106 | 1 | 106 | -- |
| \( \Xi^- \)K\(^0\) \( \pi^- \) | 1.7-2.7 | 88 | 1 | 88 | -- |
| \( \Xi^- \)K\(^+\) \( \pi^- \) | 1.5-1.7 | 70 | 1 | 70 | -- |
| \( \Xi^- \)K\(^+\) \( \pi^- \) | 2.1 | 94 | 1 | 94 | -- |
| \( \Xi^- \)K\(^+\) \( \pi^- \) | 2.45, 2.55 | 81 | 1 | 81 | -- |
| \( \Xi^- \)K\(^+\) \( \pi^- \) | 2.6 | 112 | 1 | 112 | -- |
| \( \Xi^- \)K\(^0\) \( \pi^- \) | 2.7 | 78 | 1 | 78 | -- |
| \( \Xi^- \)K\(^0\) \( \pi^- \) | all | 20 | 1 | 20 | -- |
| \( \Xi^- \) sample, total | 649 | 7 | 93 | -- |
for each of the 47 subsamples. (The method of analysis is similar to the likelihood treatment of References 6
and 1.) Having found $t_{20}$, $t_{22}$, $t_{30}$, and $t_{32}$ to be consistent with zero, we assumed $I_{\text{max}} = 1$ (i.e., $t_{20} = t_{22} = t_{30} = t_{32} = 0$); the assumption should not bias our determination of $(2J+1)$. Fits were performed in three ways: (i) with no information regarding $\alpha_A$ or $\alpha_{\Xi^-}$; (ii) with the use of $\alpha_A = 0.62\pm0.07$, the value of Cronin and Overseth; (iii) with weighting of $\alpha_A = 0.62\pm0.07$ and $\alpha_A \alpha_{\Xi^-} = 0.321\pm0.048$, a value roughly corresponding to the world average of spin-independent determinations of $\alpha_A \alpha_{\Xi^-}$, excluding Berkeley data. (These constraints were applied by including in the likelihood factors of the form $\exp\left(-\frac{1}{2}(\alpha_A - 0.62)^2/(0.07)^2\right)$ and $\exp\left[-\frac{1}{2}(\alpha_A \alpha_{\Xi^-} + 0.321)^2/(0.048)^2\right]$.)

In Fig. 5-4 we illustrate the behavior of $w = \ln \chi^2$ as a function of the assumed spin factor $(2J+1)$. From curve (ii) ($\alpha_A$ weighted by $0.62\pm0.07$, $\alpha_A \alpha_{\Xi^-}$ free) we estimate $(2J+1) = 2.0^{+0.7}_{-0.4}$, corresponding to a $\Xi^- \text{ spin } J = 1/2$. (At the likelihood maximum, $\alpha_A = 0.65\pm0.05$, $\alpha_{\Xi^-} = -0.41\pm0.04$, and $\phi_{\Xi^-} = 13^\circ\pm9^\circ$.) The $J = 1/2$ hypothesis is favored over $J = 3/2$ by $\gamma (2\times3.0)^{1/2} = 2.45$ standard deviations; higher spin hypotheses are excluded by $>3$ standard deviations. Violation of the spin-3/2 density matrix constraint in 16 subsamples causes a decrease of 7.03 in $w = \ln \chi^2$ when the constraint is applied; however, the violation is not statistically
significant.

Analysis of Monte Carlo Events

above

The conclusions of Sec. V.A.2 were checked by comparing experimental data with samples of computer-generated Monte Carlo events. For comparison with the 15 $\Xi^-$ and $\Xi^0$ subsamples of Table IV-2, we generated 75 Monte Carlo samples, having 272 events each, according to each of the following hypotheses:

(a) $J = 1/2$, $\alpha_\Lambda = 0.62$, $\alpha_\Xi = -0.40$, $\phi_\Xi = 0$, $t_{10} = 0$ (indistinguishable from $J = 3/2$ with $t_{10} = 0$);
(b) $J = 1/2$, $\alpha_\Lambda = 0.62$, $\alpha_\Xi = -0.40$, $\phi_\Xi = 0$, $t_{10} = t_{10}^{\max} = 0.57$;
(c) $J = 3/2$, $\alpha_\Lambda = 0.62$, $\alpha_\Xi = -0.40$, $\phi_\Xi = 0$, $t_{10} = t_{10}^{\max} = 0.43$.

For each sample, we performed two maximum-likelihood fits, assuming $\alpha_\Lambda = 0.62$, $\phi_\Xi = 0$, and $J = 1/2$ and 3/2, respectively; $\alpha_\Xi$ and $t_{10}$ were free parameters in the fits. In Figs. IV-5 and IV-6 we present distributions of $X \equiv \Delta \ln \chi \equiv \ln \chi (J = 1/2) - \ln \chi (J = 3/2)$ for the experimental data and for the Monte Carlo samples. See Ref. 21 for further discussion of Monte Carlo events and interpretation of likelihood results.

C. Analysis of $\Xi^*(1817) + \Xi^*(1530) + \pi$

In this section we investigate the spin and parity of $\Xi^*(1817)$, using a cleaner sample of $\Xi^0 K^0 \pi^+ \pi^-$ events than that studied previously, and utilizing two spin
tests proposed by Button-Shafer. (One of these has been previously applied.) The scarcity of data prevents the use of more elaborate tests.

In Fig. IX-7 we present a plot of $\pi$ mass squared vs. $\pi\pi$ mass squared for 164 $\pi K\pi\pi$ events from the K-63 experiment. Six events are at 2.1 BeV/c, and the remainder are from 2.45 to 2.7 BeV/c. Plotted are 135 unambiguous events ($78 \pi^+ K^+\pi^-\pi^-$, $20 \pi^- K^0\pi^+\pi^-$, and $37 \pi^- K^0\pi^+\pi^-$) with visible lambda decay, plus 29 $\pi^- K^0\pi^+\pi^0$ events without visible lambda decay. Only the 135 unambiguous events are further analyzed. Events designated $\pi^*(1817)$ have $\pi\pi\pi$ effective masses between 1775 and 1850 MeV, corresponding to an interval of $\approx 2 \times \Gamma_{\text{obs}}$. Events designated $\pi^*(1530)$ have at least one $\pi\pi$ pair with $I_z = \pm 1/2$ and $[1510 \text{ MeV} < m(\pi\pi) < 1550 \text{ MeV}]$; events designated $K^*$ have at least one $K\pi$ pair with $I_z = \pm 1/2$ and $[840 \text{ MeV} < m(K\pi) < 940 \text{ MeV}]$. Of the events designated as both $\pi^*(1817)$ and $\pi^*(1530)$, about half are due to non-resonant background.

Using the extended Byers-Fenster formalism for hyperon decay into a spin-$3/2$ fermion plus a spin-zero boson (see Sec. VIII and Ref. 30), and assuming the $\pi^*(1530)$ to have spin $3/2$, we examine the spin and parity [relative to $\pi^*(1530)$] of $\pi^*(1817)$ decaying via $\pi^*(1817) \rightarrow \pi^*(1530) + \pi$. The sample analyzed contains 41 unambiguous $\pi K\pi\pi$ events ($23 \pi^- K^+\pi^-\pi^-$, $13 \pi^- K^0\pi^+\pi^0$, and $5 \pi^0 K^0\pi^+\pi^-$) having both a $\pi^*(1817)$ and a $\pi^*(1530)$. Of the 13
$\Xi^- K^0 \pi^+ \pi^0$ events, 6 contain a $\Xi^*(1530)$ and 7 contain a $\Xi^*(1530)$; none contains both.

Let us designate as $\pi_2$ and $\pi_1$ the pions included and not included, respectively, in the $\Xi^*(1530)$; i.e.,

$$K^- + p \rightarrow \Xi^*(1817) + K \rightarrow \Xi^*(1530) + \pi_1 \rightarrow \Xi + \pi_2.$$  \hspace{1cm} (52)

In the $\Xi^- K^0 \pi^+ \pi^0$ final state, either $K\pi_1$ or $K\pi_2$ may form a $K^*$, whereas in the $\Xi^- K^+ \pi^+ \pi^-$ and $\Xi^0 K^0 \pi^+ \pi^-$ final states, only $K\pi_1$ may form a $K^*$.

From the 41-event sample, in order to avoid interference effects between $\Xi^*(1817)$ and $K^*(890)$, we removed 21 events having $m(K\pi_1) > 840$ MeV. The 20 events remaining include 15 $\Xi^- K^+ \pi^+ \pi^-$, 4 $\Xi^- K^0 \pi^+ \pi^0$, and 1 $\Xi^0 K^0 \pi^+ \pi^-$. None of the 4 $\Xi^- K^0 \pi^+ \pi^0$ events remaining has a $K\pi_2$ effective mass in the $K^*$ region.

In order that the angular distributions of interest be undistorted by the removal of $K^*$ events, we assigned double weight to certain non-$K^*$ events, selected as follows. For an event of the above type (Ed(52), the $K^*$ cutoff criterion $m(K\pi_1) > 840$ MeV may be re-expressed as a cutoff in $\cos \alpha = \hat{\Xi}^*(1530) \cdot \hat{\Xi}^*(1817)$, where the cutoff point in $\cos \alpha$ depends upon the c.m. energy of the $K^- p$ system and upon the effective masses $m(\Xi \pi_1 \pi_2)$ and $m(\Xi \pi_2)$ for the particular event. The curves plotted
in the left half of Fig. \( V - \theta \) represent the relation between c.m. energy and the value of \( \cos \alpha \) corresponding to \( m(K\pi_1) = 840 \text{ MeV} \) for events having both a \( \Xi^*(1817) \) and a \( \Xi^*(1530) \). In the right half of Fig. \( V - \theta \) the same curves appear, reflected about the line \( \cos \alpha = 0 \). For each event we imagine a single curve (and its reflection) corresponding to the particular values of \( m(\Xi\pi_1\pi_2) \) and \( m(\Xi\pi_2) \) for that event. Each \( K^* \) event falls to the left of the left-hand curve and is discarded; in order to correct for the events lost, each event falling to the right of the reflected curve (on the right-hand side) is assigned double weight in the analysis to follow. In effect, some events having \( \cos \alpha < 0 \) are replaced by other events having \( \cos \alpha > 0 \). The removal-and-replacement procedure does not systematically bias either of the two experimental distributions of interest in the following analysis.

In Fig. \( V - 9(a) \) we plot the distribution of \( |\cos \theta| = |\hat{\Xi}^*(1530) \cdot \hat{n}| \), where \( \hat{n} = \hat{KX}^*(1817) \) is the \( \Xi^*(1817) \) production normal. Assuming \( I(\theta) \) to be of the form \( 1 + a_2 P_2(\cos \theta) \), we calculate the coefficient \( a_2 \) as

\[
a_2 = 5 \langle P_2 \rangle = \left( \frac{5}{N} \right) \sum_{i=1}^{N} P_2 (\cos \theta_i) \tag{53}
\]

with an experimental error

\[
\delta a_2 = \left( \frac{5}{N} \right)^{1/2} \left[ 1 + \frac{2}{7} a_2 - \frac{1}{5} a_2^2 \right]^{1/2} \tag{54}
\]

(The \( a_2 \) is proportional to \( t_{20} \), a measure of \( \Xi^*(1817) \) alignment along \( \hat{n} \).)
Here \( N (= 20) \) is the actual number of events, and \( N' (= 28) \) is the number with doubly weighted events counted twice. For the non-\( K^* \) events of Fig. \( \gamma - 9 \) we obtain \( a_2 = 0.6 \pm 0.5 \), consistent with isotropy and with the \( J = 1/2 \) assignment. However, with the \( J = 1/2 \) assignment, we cannot rule out higher spin hypotheses. For \( \frac{3}{2} \) \( (\frac{3}{2}) \) \( a_2 \) may have any value between \(-1.0\) and \(1.0 (\pm 0.8\) and \(0.8\) Background events lying outside the \( \Xi^*(1817) \) region (1775 to 1850 MeV), but otherwise selected just as the \( \Xi^*(1817) \) events, also yield an \( I(\theta) \) distribution consistent with isotropy.

In Fig. \( \gamma - 9 (b) \) we plot the distribution of \( |\cos \psi| = |\hat{E} \cdot \hat{E}^*(1530)| \), i.e., the decay angle of \( \Xi^*(1530) \) relative to its line of flight. The expected distribution is of the form \( I(\psi) = 1 + b_2 P_2 (\cos \psi) \) for any value of the \( \Xi^*(1817) \) spin \( J \). For a pure sample of \( \Xi^*(1817) \) decaying via \( \Xi^*(1817) \rightarrow \Xi^*(1530) + \pi \), predicted values of the coefficient \( b_2 \) are as follows [where the parity of the \( \Xi^*(1817) \) is defined relative to that of the \( \Xi^*(1530) \)]:

<table>
<thead>
<tr>
<th>( J^+ )</th>
<th>( J^P )</th>
<th>Partial wave ( \ell )</th>
<th>( b_2 ) predicted</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2+</td>
<td>1</td>
<td>1</td>
<td>1.0</td>
</tr>
<tr>
<td>1/2−</td>
<td>2</td>
<td>2</td>
<td>1.0</td>
</tr>
<tr>
<td>3/2−</td>
<td>0,2</td>
<td>0,2</td>
<td>1.0</td>
</tr>
<tr>
<td>5/2+</td>
<td>1,3</td>
<td>1,3</td>
<td>1.0</td>
</tr>
<tr>
<td>7/2−</td>
<td>2,4</td>
<td>2,4</td>
<td>1.0</td>
</tr>
<tr>
<td>etc.</td>
<td>etc.</td>
<td>etc.</td>
<td>1.0</td>
</tr>
<tr>
<td>3/2+</td>
<td>1,3</td>
<td>1,3</td>
<td>1.0</td>
</tr>
<tr>
<td>5/2−</td>
<td>2,4</td>
<td>2,4</td>
<td>1.0</td>
</tr>
<tr>
<td>7/2+</td>
<td>3,5</td>
<td>3,5</td>
<td>1.0</td>
</tr>
<tr>
<td>etc.</td>
<td>etc.</td>
<td>etc.</td>
<td>1.0</td>
</tr>
</tbody>
</table>

\( \Xi^*(1817) \) region (1775 to 1850 MeV), but otherwise selected just as the \( \Xi^*(1817) \) events, also yield an \( I(\theta) \) distribution consistent with isotropy.
The observed value of \( b_2 \) is \( 1.4 \pm 0.5 \), which favors the hypotheses \( J^P = 1/2^\pm \) and \( J^P = 3/2^-, 5/2^+, 7/2^- \), etc. over \( J^P = 3/2^+, 5/2^-, 7/2^+ \), etc. Simply from angular-momentum barrier-penetration arguments, the lower \( \ell \)-wave hypotheses are more likely, so that the evidence points to \( J^P = 3/2^- \) (with \( \ell = 0 \)) as the most probable hypothesis.

Background events outside the \( \Xi^* (1817) \) region yield a value \( b_2 = 0.2 \pm 0.4 \), indicating that the observed anisotropy may indeed be associated with \( \Xi^* (1817) \). However, because the \( \Xi^* (1817) \) sample contains \( \gtrsim 50\% \) background, we cannot ignore the possibility that the anisotropy may be due to \( \Xi^* (1817) \) interference with nonresonant background. In conclusion, our analysis does not permit us to rule out conclusively any \( J^P \) hypothesis for \( \Xi^* (1817) \).

D. Results for \( \Xi^* (1530) \)

Events containing the \( \Xi^* (1530) \), of the type \( \Xi K\pi \) and \( \Xi K\pi\pi \), were also analyzed by likelihood techniques to determine the \( \Xi^* (1530) \) spin and parity. For the 251 \( \Xi^* \) from \( \Xi K\pi \) final states (not previously reported), the \( J = 1/2 \) hypothesis is roughly 3 or 4\% as likely as the \( J = 3/2 \) hypothesis. The hypothesis \( J^P = 3/2^- \) is favored over \( 3/2^- \) by \( \approx 2.1 \) standard deviations. For the \( \Xi^* (1530) \) events from \( \Xi K\pi \) and \( \Xi K\pi\pi \) samples combined, the spin result becomes 3.5\% (\( J = 1/2 \) compared with 3/2) and \( J^P = 3/2^- \) is favored over \( 3/2^- \) by \( \approx 2.8 \) standard deviations. Thus little improvement over the results from the \( \Xi K\pi\pi \) sample (Ref. 11) is obtained; spin discrimination is poorer than that of London et al. (1\% for \( J = 1/2 \)), but parity discrimination is better than that of Schlein et al. (0.035 confidence level for \( 3/2^- \)). \(^3, ^2\)
VI. DISCUSSION

A. Decay parameters of the $\Xi^-$

The values of $\Xi^-$ decay parameters reported in Table V-1 are in agreement with previously published results obtained from K-72 data and from K-63 $\Xi^-K^+$ data. The slight differences that exist are due to differences in the samples analyzed, in the binning criteria, and in the values assumed for $\alpha^\Lambda$. 

In Table VI-1 and Fig.VI-1 we compare values listed in Table V-1 with values previously reported in other experiments 1-5, and we list approximate world averages of $\alpha^\Xi$ and $\phi^\Xi$. Because different assumed values of $\alpha^\Lambda$ were used in various experiments, and because $\alpha^\Lambda$ and $\alpha^\Xi$ are highly correlated, values of $\alpha^\Xi$ were not averaged directly. For non-Berkeley data, we calculated $\alpha^\Lambda \alpha^\Xi = 0.325 \pm 0.047$; then assuming $\alpha^\Lambda = 0.657 \pm 0.047$ we obtained $\alpha^\Xi = 0.495 \pm 0.080$ which was averaged with the Berkeley value $\alpha^\Xi = 0.394 \pm 0.041$ to obtain the world average listed.

The positive value of $\gamma = (1 - \alpha^2)^{1/2} \cos \phi$ shows that the S-wave (parity-violating) amplitude dominates in $\Xi^-$ decay. The phase difference of S and P amplitudes, calculated from the Berkeley values of $\alpha$ and $\phi$, is $(\Delta_P - \Delta_S) = \tan^{-1} (\beta^\Xi/\alpha^\Xi) = 157^0 \pm 21^0$. In the absence of final-state interactions, T invariance in the decay
Table VI-1. Decay parameters: comparison of experimental results.

<table>
<thead>
<tr>
<th>Lab</th>
<th>Events</th>
<th>$\alpha_{\Lambda} \alpha_{\Xi^-}$</th>
<th>Fitted or assumed value of $\alpha_{\Lambda}$</th>
<th>$\alpha_{\Xi^-}$</th>
<th>$\Phi_{\Xi^-}$ (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LRL(K-72)$^a$</td>
<td>1004</td>
<td>--</td>
<td>$0.641 \pm 0.056$</td>
<td>$-0.368$</td>
<td>$0.5 \pm 10.7$</td>
</tr>
<tr>
<td>LRL(K-63)$^b$</td>
<td>2529</td>
<td>$-0.262 \pm 0.033$</td>
<td>$0.656 \pm 0.055$</td>
<td>$-0.375 \pm 0.051$</td>
<td>$9.8 \pm 11.6$</td>
</tr>
<tr>
<td>LRL combined$^b$</td>
<td>3431</td>
<td>$-0.267 \pm 0.028$</td>
<td>$0.657 \pm 0.047$</td>
<td>$-0.394 \pm 0.041$</td>
<td>$9.9 \pm 9.0$</td>
</tr>
<tr>
<td>BNL+S$^c$</td>
<td>700</td>
<td>$-0.34 \pm 0.09$</td>
<td>$0.62 \pm 0.07$</td>
<td>$-0.47 \pm 0.12$</td>
<td>$0 \pm 20$</td>
</tr>
<tr>
<td>EP$^d$</td>
<td>517</td>
<td>$-0.27 \pm 0.07$</td>
<td>$0.62 \pm 0.07$</td>
<td>$-0.44 \pm 0.11$</td>
<td>$-16 \pm 37$</td>
</tr>
<tr>
<td>UCLA$^e$</td>
<td>356</td>
<td>$-0.41 \pm 0.10$</td>
<td>$0.62 \pm 0.07$</td>
<td>$-0.62 \pm 0.12$</td>
<td>$54 \pm 25$</td>
</tr>
<tr>
<td>CERN$^f$</td>
<td>62</td>
<td>$-0.35 \pm 0.18$</td>
<td>$0.61 \pm 0.07$</td>
<td>$-0.73 \pm 0.21$</td>
<td>$45 \pm 30$</td>
</tr>
<tr>
<td>Average$^g$</td>
<td>5066</td>
<td>$-0.28 \pm 0.02$</td>
<td>$0.66 \pm 0.05$</td>
<td>$-0.42 \pm 0.04$</td>
<td>$113 \pm 7.4$</td>
</tr>
</tbody>
</table>

$^a$See Ref. 1

$^b$See Table V-VIII (bottom) of Ref. 21.

$^c$(Brookhaven National Laboratory and Syracuse University). See Ref. 3

$^d$(Ecole Polytechnique and others). See Ref. 4

$^e$See Ref. 2

$^f$See Ref. 5

$^g$Only entries below dashed line are included in average. See text regarding values of $\alpha_{\Xi^-}$.
transition requires \((\Delta_p - \Delta_s) = 0\) or \(\pi\), whereas C invariance requires \((\Delta_p - \Delta_s) = \pm \pi/2\). It appears that the hypothesis of \(T\) invariance is favored by the data.

Under the assumptions of \(SU(3)\) symmetry, octet dominance, and invariance under \(R\) (i.e., inversion through the origin \(I_z = 0, Y = 0\)) Lee has predicted a triangular relationship among the non-leptonic covariant decay amplitudes for the processes:

\[
\begin{align*}
\Xi^- : & \quad \Xi^- + \Lambda + \pi^- \\
\Lambda_0^- : & \quad \Lambda^0 + p + \pi^- \\
\text{and} & \\
\Xi_0^+ : & \quad \Xi^+ + p + \pi^0.
\end{align*}
\]

According to Lee, both the S-wave (parity-nonconserving) and P-wave (parity-conserving) amplitudes satisfy the relationship:

\[
2 \Xi^- - \Lambda_0^- = \sqrt{3} \Xi_0^+.
\]

The same relationship (for either parity-nonconserving or parity-conserving amplitudes, or both) has been derived by other authors under different assumptions.

Furthermore, the \(|\Delta I| = 1/2\) rule predicts a triangle relation:

\[
\Xi_0^+ = \frac{1}{\sqrt{2}} [\Xi^- - \Sigma^+]
\]
\[ \Sigma_0^+ : \Sigma^+ \rightarrow p + \pi^0 \]
\[ \Sigma_+^+ : \Sigma^+ \rightarrow n + \pi^+ \]
\[ \Sigma_-^- : \Sigma^- \rightarrow n + \pi^- , \]

so that the Lee triangle prediction and the $|\Delta I| = 1/2$ rule together require

\[ \frac{1}{\sqrt{3}} [2 \Sigma_- - \Lambda_0] = \frac{1}{\sqrt{2}} [\Sigma^- - \Sigma^+] . \]  

(59)

In Eqs. (56), (57), and (59), the covariant S- and P-wave amplitudes (denoted by A and B, respectively) are related to the partial decay rate $w$ by

\[ w = \frac{q}{8\pi M^2} \{ |A|^2 [ (M+m)^2 - \mu^2 ] + |B|^2 [ (M-m)^2 - \mu^2 ] \} , \]  

(60)

where $M$, $m$, and $\mu$ are the rest masses of the parent baryon, the decay baryon, and the decay pion, respectively, and $q$ is the pion momentum in the rest frame of the parent baryon. In terms of the phenomenological decay amplitudes $a$ and $b$ appearing in Eq. (28), $A$ and $B$ are given by

\[ \left( \frac{B}{A} \right)^2 = \left( \frac{b}{a} \right)^2 \left( \frac{(M+m)^2 - \mu^2}{(M-m)^2 - \mu^2} \right) . \]  

(61)

A recent determination of the $\Sigma$ decay parameters $\alpha(\Sigma_0^+)$, $\alpha(\Sigma_+^+)$, and $\alpha(\Sigma_-^-)$ has demonstrated that (57) is well satisfied, and also permits a more accurate test of Eqs. (56) and (59) than was previously possible. In Table VI-2 we present covariant amplitudes $A$ and $B$ for the processes of (55) and (58), calculated under the assumption that $A$ and $B$ are
Table VI-2. Non-leptonic hyperon decay amplitudes

<table>
<thead>
<tr>
<th>Decay</th>
<th>$\tau$ $\times 10^{10}$ sec$^{-1}$</th>
<th>Branching fraction</th>
<th>$\alpha$</th>
<th>$A^a$ $\times 10^{-5}$ (sec$^{-1/2}$)</th>
<th>$B^a$ $\times 10^{-5}$ (sec$^{-1/2}$)</th>
<th>Correlation coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma^- + A + \pi^-$</td>
<td>1.75±0.05</td>
<td>1.00</td>
<td>-0.394±0.041</td>
<td>2.022±0.029</td>
<td>-6.70±0.66</td>
<td>0.136</td>
</tr>
<tr>
<td>$\Lambda + p + \pi^-$</td>
<td>2.53±0.05</td>
<td>0.663±0.014</td>
<td>0.657±0.047</td>
<td>1.542±0.030</td>
<td>10.85±0.97</td>
<td>-0.532</td>
</tr>
<tr>
<td>$(\Sigma^+ + p + \pi^-)\gamma &gt; 0$</td>
<td>0.810±0.013</td>
<td>0.528±0.015</td>
<td>-0.960±0.067</td>
<td>1.558±0.142</td>
<td>-11.71±1.88</td>
<td>-0.959</td>
</tr>
<tr>
<td>$(\Sigma^+ + p + \pi^-)\gamma &lt; 0$</td>
<td>0.810±0.013</td>
<td>0.472±0.015</td>
<td>-0.006±0.043</td>
<td>-1.861±0.034</td>
<td>-0.05±0.41</td>
<td>-0.001</td>
</tr>
<tr>
<td>$(\Sigma^+ + n + \pi^-)\gamma &gt; 0$</td>
<td>0.821±0.013</td>
<td>0.472±0.015</td>
<td>-0.006±0.043</td>
<td>-1.861±0.034</td>
<td>-0.05±0.41</td>
<td>-0.001</td>
</tr>
<tr>
<td>$(\Sigma^+ + n + \pi^-)\gamma &lt; 0$</td>
<td>0.810±0.013</td>
<td>0.528±0.015</td>
<td>-0.960±0.067</td>
<td>1.558±0.142</td>
<td>-11.71±1.88</td>
<td>-0.959</td>
</tr>
<tr>
<td>$\frac{1}{2}(2\Sigma^- - \Lambda)$</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>1.455±0.040</td>
<td>-14.09±0.70</td>
<td>0.004</td>
</tr>
<tr>
<td>$\frac{1}{2}(\Sigma^- - \Sigma^+)$</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>1.327±0.037</td>
<td>-13.01±0.31</td>
<td>0.005</td>
</tr>
</tbody>
</table>

The $m_\pi$ can be disregarded if the q of Eq. 60 is expressed in units of $m_\pi c$. This solution is inconsistent with recent evidence that $\gamma(\Sigma^+_+ < 0$. See Ref. 55.
relatively real. Only the relative signs of A and B are experimentally observable; a further ambiguity exists in the case of the $\Sigma^+$ and $\Sigma^-$ decays [Eq. (55) and (58)], where the sign of $\delta$ has not been determined. A recent experiment has shown that $\delta < 0$ for $\Sigma^+$ decay. In Table VI-2, world-average values of lifetimes, branching fractions, and decay parameters are used, except that $\alpha_{\Xi^-}$ and $\alpha_{\Lambda}$ are our results from K-63 and K-72 data.

The consistency of the data with Eqs. (54), (57), and (59) is illustrated in Fig. VI-2. In plotting the error ellipse for $(2 \Xi^- - \Lambda^0)/\sqrt{3}$, we have taken into account the correlation between $\alpha_{\Xi^-}$ and $\alpha_{\Lambda}$. We conclude that Eqs. (54) and (57) are well satisfied by the experimental data; Eq. (59) is less well satisfied. [Fig. VI-2 actually represents $\Xi^-$ and $\Lambda^0$ amplitudes derived from $\alpha_{\Xi^-} = -0.381 \pm 0.037$ and $\alpha_{\Lambda} = 0.690 \pm 0.048$. Our $A_{\Xi}$ and $A_{\Lambda}$ (Table VI-2) differ from these by < 1%, whereas $B_{\Xi}$ is larger (3.7%) and $B_{\Lambda}$ smaller (5.5%); thus our Table VI-2 values yield a slightly better fit to the triangle hypothesis, in that point 1 is moved up by $\approx 1\%$ of its B coordinate.]

A veritable flood of predictions concerning non-leptonic hyperon decay has resulted from the advent of SU(6) and higher symmetry schemes. (As of August 1966, at least 70 papers containing specific predictions regarding decay amplitudes have appeared in the literature. Most of these deal with the Lee-Suzuki triangle prediction and the reasons for its apparent validity.) The theoretical situation is far too complex to discuss here, and the reader is referred to a recent review by Pais. Predictions of various theoretical models may be readily checked with the values in Table VI-1.)
B. Spin of the $E$

Our conclusion that the $E$ has spin $1/2$ is in agreement with the prediction of SU(3) and with the findings of previous investigations. A maximum-likelihood analysis identical with that of Sec. II: B. performed on 828 $K^- K^+$ events alone, yields a value $X = \ln(1/2) - \ln(3/2) = 2.60$, favoring the spin-$1/2$ hypothesis by 2.3 standard deviations. (Our analysis of 3278 events yields only slightly better spin discrimination (2.45 s.d.), partly because $K^- K^+$ events are not strongly polarized and partly because we have not optimized the binning criteria, as was done in Ref. 1. See Fig. VI-3.

In an alternative approach, one may calculate directly the factor $(2J+1)$ as a ratio of odd-$L$ moments of the transverse and longitudinal $A$ polarization distributions. If only moments proportional to $t^{10}$ are considered,

$$2J+1 = \left[ \frac{\langle P \hat{y} \sin \phi \rangle^2 + \langle P \hat{x} \sin \phi \rangle^2}{(1-\alpha_E^2)^{1/2} \langle P \hat{x} \cos \phi \rangle^2} \right]^{1/2}$$

(62)

For 356 $E^-$ events, Carmony et al. obtained a value $(2J+1) = 1.53$, assuming $\alpha_E = -0.48$. By calculating an expected distribution in $(2J+1)$ (presumably/the numerator and denominator of Eq. (62) to be normally distributed quantities) they claim an exclusion of the spin-$3/2$ hypothesis by 3.1 standard deviations.

For 749 $E^-$ events of the K-72 experiment, Button-Shafer et al. obtained values of $(2J+1) = 2.86$ and 2.18, assuming
\( \alpha_E^- = -0.48 \) and \(-0.34\), respectively. Here the 
2\( J+1 \) values and their expected distributions were cal-
culated as the ratio of two normally distributed quan-
tities. The resulting confidence levels, with \( \alpha_E^- \)
assumed to be \(-0.48\) \((-0.34)\), were \(0.22\) \((0.42)\) for \( J = 1/2 \), \(0.15\) \((0.015)\) for \( J = 3/2 \), and \(0.003\) \((0.0002)\) for 
\( J = 5/2 \).

C. \( E^*(1817) \) classification.

Our results from the analysis of \( E^*(1817) \rightarrow 
E^*(1530) + \pi \) are consistent with those obtained in
previous investigations. The hypotheses \( J^P = 1/2^+, 
3/2^-, 5/2^+, 7/2^-, \) etc. are favored over others, but
any assignment from our data is questionable because \( J^P = 3/2^- \) might be considered the most likely
of large background. \( \) \( \text{(See discussion near the end of Sec. V.C.)} \)

In an earlier analysis of essentially the same
data, the distribution appearing in Fig. \( \text{V-9(a)} \) was
found to be consistent with isotropy. In the same
analysis, the observed branching ratios of \( E^*(1817) \) into
\( \Xi^*(1530) + \pi, \Lambda + K, \) and \( \Xi + \pi \) were cited as evidence possibly favoring the \( J^P = 3/2^- \) and \( 5/2^+ \) hypotheses. (More recent information regarding branching ratios renders the same test somewhat less conclusive than it was considered earlier.)

In Ref. 10 Button-Shafer et al. performed a Byers-Fenster moment analysis of \( \Xi^*(1817) \) (in \( K-63 \Lambda K \bar{K} \) final states) decaying into \( \Lambda + K \). [We disregard a similar analysis of \( \Xi^*(1817) \rightarrow \Xi + \pi \), as it is now believed that the \( \Xi \pi \) enhancement near 1817 MeV maybe entirely due to \( \Xi^*(1933) \).] The analysis of \( \Xi^*(1817) \) in \( \Lambda K \bar{K} \) final states is complicated by the presence of interfering \( \phi \) (1020) and (in \( \Lambda K \bar{K}^0 \), final states) by the impossibility of distinguishing \( K^0 \) from \( \bar{K}^0 \). After removal of events in the \( \phi \) (1020) region, 71 (resonant plus background) events in the region [1775 MeV \( \leq m(\Lambda K) \leq 1850 \) MeV] were found to require 'spin' \( > 1/2 \), with the hypothesis \( J^P = 3/2^- \) slightly preferred over \( 3/2^+ \). However, no firm conclusions could be drawn; i.e. "background events outside the \( \Xi^*(1820) \) region also require a 'spin' greater than 1/2, but perhaps not so firmly as do the 'resonant' events," and "the evidence (for \( J^P = 3/2^- \) over \( 3/2^+ \)) is exceedingly weak because of large background in the \( \Xi^*(1820) \) decay channels." 10

We have attempted a maximum-likelihood analysis of a somewhat larger \( \Lambda K \bar{K} \) data sample than that analyzed
earlier. Here we removed events containing $\phi(1020)$ by requiring $(\Lambda \bar{\Sigma}^\ast(1817)) < 0$, whereas in the previous analysis (due to statistical limitations) only events in a narrow $\phi$ band were removed. For 59 events in the region $[1787 \text{ MeV} \leq m(\Lambda R) \leq 1847 \text{ MeV}]$ we obtain values of $w = \ln \Lambda = 0.00, 1.01, 3.36, \text{ and } 3.66$ for $J^P = 1/2^-, 1/2^+, 3/2^+, \text{ and } 3/2^-$, respectively. An increase of 3.0 is to be expected as $J$ is increased from 1/2 to 3/2, simply from the addition of six extra parameters, so that our analysis of $\Sigma^\ast(1817) \rightarrow \Lambda + \bar{K}$ provides no spin or parity discrimination whatever. [Comparable results are obtained for background events outside the $\Sigma^\ast(1817)$ region.]

No evidence has been found for a $\Sigma^\ast$ resonance near 1600 MeV, which had been suggested as the missing member of a $3/2^-$ unitary octet containing the $N_{1/2}^\ast(1525)$, the $Y_0^\ast(1520)$, and the $Y_1^\ast(1660)$. (The last is not firmly established as a $3/2^-$ state.)

Dalitz has suggested perhaps the most attractive scheme to accommodate the $\Sigma^\ast(1817)$, if it has spin and parity $3/2^-$. The $N_{1/2}^\ast(1525)$, $Y_0^\ast(1520)$, $Y_1^\ast(1660)$, and $\Sigma_{1/2}^\ast(1817)$ could form an octet, with the singlet $Y_0^\ast$ mixing with a new $3/2^-$ singlet state at perhaps 1670 MeV. However, as indicated above, other classifications are certainly possible for the $\Sigma^\ast(1817)$ [and perhaps for the $Y^\ast(1660)$].
VII. ACKNOWLEDGMENTS

Dr. Peter Berge has furnished useful programming assistance and much helpful advice. Thanks are due to Dr. Joseph Murray, who supervised the beam design and construction. The Bevatron and bubble-chamber crews, and also Alvarez Group Scanners and Measurers have provided indispensable services. We have appreciated the interest of Dr. Arthur Rosenfeld. Finally we thank Dr. Luis Alvarez for his encouragement.
APPENDIX: DISCUSSION OF SYSTEMATIC ERRORS

In Fig. A-1 we present, for the 3431 Ξ− events of Table III-2, distributions of \( \hat{\Lambda} \cdot \hat{\kappa} \) and \( \hat{\Lambda} \cdot \hat{\Xi} \). These distributions should be isotropic if \( J_{\Xi} = 1/2 \) and if scanning biases and other systematic effects are absent. (Even if \( J_{\Xi} > 1/2 \), these distributions must be even in \( \hat{\Lambda} \cdot \hat{\kappa} \) and \( \hat{\Lambda} \cdot \hat{\Xi} \).)

We attempt to determine the exact cause of the observed anisotropy, in order to see whether it can bias our measurement of Ξ− decay parameters. The following effects are considered:

(i) loss of events having π− (from Ξ− decay) nearly collinear with Ξ−;
(ii) loss of events having short Ξ− and/or Λ;
(iii) escape from chamber, prior to decay, of Ξ− or Λ;
(iv) precession of Ξ− and/or Λ polarization in magnetic field.

In order to facilitate our discussion we define two new coordinate systems, \((x,y,z)\) and \((x',y',z')\). The axes \( z \) and \( z' \) correspond to \( p_k \) (lab) and \( p_{\Xi} \) (lab) respectively, the lab directions of the incident K− and of the Ξ−. We define \( \hat{y} = \frac{\hat{p}_k}{p_k} \times \hat{z}_{ch} \) and \( \hat{y}' = \frac{\hat{p} _\Xi}{p_\Xi} \times \hat{z}_{ch} \), where \( \hat{z}_{ch} \) is the bubble chamber z-axis (essentially the optic axis and the direction of the magnetic field).

Directions of particles with respect to \((x,y,z)\) and
(x', y', z') are specified by angles (θ, φ) and (θ', φ') respectively. Incident beam tracks are nearly horizontal in the bubble chamber (i.e., \( \hat{x} \approx \hat{z}_{\text{ch}} \)), so we may regard \( \psi_{\text{(lab)}} \) and \( \psi_{\text{(lab)}}' \) (projected angles in the y-z plane) as the projected angles (relative to \( \hat{p}_k_{\text{(lab)}} \) and \( \hat{p}_E_{\text{(lab)}}' \), respectively) seen by the scanner.

1. Small-angle \( \Xi^- \) decay. In Figs. A-2 and A-3 we present, for the 2529 K-63 \( \Xi^- \) events listed in Table III-1, scatter plots and projections of \( \phi_{\Xi} \) (c.m.) vs. \( \cos \theta_{\Xi} \) (c.m.), angles describing \( \Xi \) production in the c.m. frame, relative to axes (x, y, z); and \( \phi'_{\Xi} = \phi_{\pi} + \pi \) vs. \( \cos \theta'_{\Xi} = -\cos \theta_{\pi} \), angles describing \( \Xi \) decay in the \( \Xi \) rest frame, relative to axes (x', y', z'). The quantity \( \cos \theta_{\Xi} \) (c.m.) is equivalent to \( \hat{\Xi} \cdot \hat{K} \) as defined by Fig. IV-1; however, \( \cos \theta'_{\Xi} \) is not equivalent to \( \hat{\Lambda} \cdot \hat{\Xi} \), because for Fig. A-3 we have transformed \( \hat{p}_\Lambda \) from the lab frame to the \( \Xi \) rest frame via a single Lorentz transformation along \( \hat{p}_{\Xi} \) (lab) (rather than through the intermediate c.m. frame).

In Fig. A-3, \( \phi'_{\Lambda}(\Xi) = \phi'_{\Lambda}(\text{lab}) \); i.e. \( \theta'_{\Lambda} = \tan^{-1}(\hat{p}_\Lambda \cdot \hat{y})/(\hat{p}_\Lambda \cdot \hat{x}) \) is the same in the \( \Xi \) rest frame as in the lab frame.

The distribution of \( \phi_{\Xi} \) (c.m.) in Fig. A-2 is consistent with isotropy. Hence, because the \( \Xi \) can be polarized only along \( \hat{n} = (\hat{K} \times \hat{\Xi}) \) and because \( \hat{n} \) is uncorrelated with the bubble chamber z-axis, the distribution in Fig. A-3 should be isotropic if the \( \Xi \) has spin 1/2 and if systematic biases are absent. [Even if \( J_{\Xi} > 1/2 \), the distribution must be even in \( \cos \theta'_{\Xi} \).]
We have sketched on the scatter plot (Fig. A-3), for $p_\Xi^{(lab)} = 1.0$ and 2.4 BeV/c, contours representing $\psi^{(lab)} = 5^\circ$, where $\psi^{(lab)}$ is the projected lab angle between the $\Xi^-$ and decay pion at the $\Xi^-$ decay vertex. The curve in the $\cos \theta_\Lambda^{(\Xi)}$ projection represents the expected distribution of events, calculated under the assumption that no events having $\psi^{(lab)} \leq 5^\circ$ are detected. The observed distribution is consistent with assumed cutoff values $[\psi^{(lab)}]_{\min} = 5^\circ \pm 2^\circ$, corresponding to a 10 to 20% loss of events.

The observed anisotropy in $\hat{\Lambda} \cdot \hat{K}$ and $\hat{\Lambda} \cdot \hat{\Xi}$ (Fig. A-1) is related to the anisotropy in $\cos \theta_\Lambda^{(\Xi)}$ (Fig. A-3), and may be entirely attributed to the loss of small-angle $\pi^-$ from $\Xi^-$ decay. As would be expected from such a bias, the anisotropy is most evident in $\cos \theta_\Lambda^{(\Xi)}$, less so in $\hat{\Lambda} \cdot \hat{K}$, and still less so in $\hat{\Lambda} \cdot \hat{\Xi}$.

Correcting for such a bias is somewhat difficult. For example, if one attempts to remove all events having $\psi^{(lab)}$ less than some minimum value, say $5^\circ$, applying a weight to the remaining events, one inevitably loses all events in a finite range of $\cos \theta_\Lambda^{(\Xi)}$ near $\cos \theta_\Lambda^{(\Xi)}$.

As a substitute measure, we have corrected the anisotropy in $\hat{\Lambda} \cdot \hat{K}$ (Fig. A-1) by weighting events with an empirical correction factor of the form

$$w(z) = \left[1 - C (z - z_0)^2\right]^{-1} \tag{63}$$

where $z = (\hat{\Lambda} \cdot \hat{K})$, $z_0 = -0.35$, and $C = 0.50$ (1.24) for
K-72 (K-63) events. (The curve plotted in Fig. A-1 corresponds to \([w(z)]^{-1}\).) After correction, the distributions in \(\hat{A} \cdot \hat{K}\) and \(\hat{A} \cdot \hat{\Xi}\) (Fig. A-1) are consistent with isotropy. The changes in measured decay parameters resulting from the correction are insignificant compared with statistical errors.

The loss of small-angle \(\Xi\) decays obviously distorts the observed distribution of \(\phi^{'}(\Xi)\) as well as \(\cos \theta^{'}(\Xi)\), but because we average over \(\phi^{'}(\Xi)\) the distortion does not bias our determination of \(\Xi^-\) decay parameters.

2. Short \(\Xi^-\) and/or \(\Lambda\).

Because the forward-produced \(\Xi^-\) have greater lab momenta and thus travel farther prior to the decay, on the average, than do backward-produced \(\Xi^-\), one expects a preferential loss of backward-produced \(\Xi^-\), which distorts the observed \(\Xi^-\) production distribution (Fig. A-2). The loss rate is about 6% in K-72 events and is most likely lower in K-63 events, where \(\Xi^-\) momenta are higher. However, loss of short \(\Xi^-\) cannot explain the anisotropy observed in the \(\Xi^-\) decay distribution (Figs. A-1 and A-3).

Due to the loss of short \(\Lambda\), one also expects a preferential depletion of events having small values of \(\cos \theta^{'}(\Xi)\). Noting that \(\tilde{P}^{(lab)}_{\Lambda}/m_{\Lambda} \approx \tilde{P}^{(lab)}_{\Xi}/m_{\Xi}\) (an approximation valid to about 10% in magnitude and 10° in angle for the events of this experiment) and that \(\gamma^{(\Xi)}_{\Lambda} = E^{(\Xi)}_{\Lambda}/m_{\Lambda} \approx 1\), we may approximately write
\[ p_\Lambda (\text{lab}) \sim p_\Lambda \sim p_\Xi (\text{lab}) \]

\[ e^{\eta_\Lambda (\text{lab})} \sim E_\Lambda (\Xi) + \gamma_\Xi (\text{lab}) p_\Lambda (\Xi) \cos \theta_\Lambda ' (\Xi) \]

\[ \sim M_\Lambda \eta_\Xi (\text{lab}) \left[ 1 + \frac{\beta_\Lambda (\text{lab})}{\beta_\Xi (\text{lab})} \cos \theta_\Lambda ' (\Xi) \right] \]

\[ \sim M_\Lambda \eta_\Lambda (\text{lab}) \left[ 1 + \frac{\beta_\Lambda (\text{lab})}{\beta_\Xi (\text{lab})} \cos \theta_\Lambda ' (\Xi) \right] \]

where \( (\hat{\eta}_\Xi (\text{lab}), \gamma_\Xi (\text{lab})) = \frac{1}{m_\Xi} (\hat{p}_\Xi (\text{lab}), E_\Xi (\text{lab})) \) and \( (\hat{\eta}_\Lambda (\Xi), \gamma_\Lambda (\Xi)) = \frac{1}{m_\Lambda} (\hat{p}_\Lambda (\Xi), E_\Lambda (\Xi)) \) refer to the \( \Xi \) in the lab frame and to the \( \Lambda \) in the \( \Xi \) rest frame respectively. The \( \Lambda \) detection efficiency is approximately of the form (if \( \Lambda \) dip angle is ignored)

\[ \exp \left[ -\eta_{\min} m_\Lambda / p_\Lambda (\text{lab}) \right] \approx \eta_\Lambda \]

\[ \approx 1 - \frac{\eta_{\min} (\text{lab})}{\eta_\Lambda (\text{lab})} \left[ 1 - \frac{\beta_\Lambda (\Xi)}{\beta_\Xi (\text{lab})} \cos \theta_\Lambda ' (\Xi) \right] \]

where \( \eta_{\min} \) is an effective short-\( \Lambda \) cutoff length and \( \tau_\Lambda \) is the \( \Lambda \) lifetime. The overall loss rate \( (\sim \eta_{\min} / N_\Lambda (\text{lab}) \cdot \tau_\Lambda) \) is of the order of 2% for K-72 events and most likely smaller for K-63 events; the quantity \( \beta_\Lambda (\Xi)/\beta_\Xi (\text{lab}) \) is typically of the order of 0.15. Hence the expected asymmetry in \( \cos \theta_\Lambda ' (\Xi) \) is of the order of 0.003 (to be compared with a statistical error of 0.011). We conclude that the anisotropy in \( \cos \theta_\Lambda ' (\Xi) \) caused by the loss of short \( \Lambda \) is negligible in comparison with the anisotropy resulting from...
other causes (e.g., small angle $\Xi^-$ decay), and may be neglected in the analysis of $\Xi^-$ decay. We have verified that application of a length cutoff for short $\Xi^-$ and/or $\Lambda$ (with appropriate weighting) does in fact produce a negligible change in measured values of $\Xi^-$ decay parameters.

3. Escape Losses.

Due to the escape from the chamber prior to decay, one expects a preferential loss of forward-produced $\Xi^-$ and of events having large values of $\cos \theta_\Lambda' (\Xi)$. The loss of high-momentum $\Xi^-$ can affect the $\Xi^-$ production distribution (Fig. A-2) but not the $\Xi^-$ decay distribution (Fig. A-3).

By a crude calculation taking into account the spatial distribution of the beam and the distribution of $\Lambda$ lab momenta, we estimate the asymmetry in $\cos \theta_\Lambda' (\Xi)$ resulting from the loss of high-energy $\Lambda$ to be of the order of 0.002 (to be compared with a statistical error of 0.011). The effect is comparable with that from the loss of short $\Lambda$ and of opposite sign, and likewise may safely be neglected in the analysis of $\Xi^-$ decay.

4. Precession of $\Xi$ and $\Lambda$ polarization.

As a function of time $t$, the precession of the polarization vector $\mathbf{P}(t)$ of a particle in a magnetic field $\mathbf{H}$ is described by

$$\mathbf{\dot{P}}(t) = \frac{d\mathbf{P}(t)}{dt} = \mathbf{\Omega}(t) \times \mathbf{P}(t) .$$
(Here \( \hat{\mathbf{P}} \) is the polarization three-vector defined in the particle's rest frame, and \( t \) is measured in the lab frame.) As discussed by Simmons, the effective angular precession velocity \( \hat{\omega}(t) \) may be considered as consisting of two terms:

\[
\hat{\omega}(t) = \hat{\omega}_{\text{Larmor}} + \hat{\omega}_{\text{Thomas}}(t)
\]

(67)

where

\[
\hat{\omega}_{\text{Larmor}} = \frac{-\mu}{J} \frac{e_p}{2m_p c} \hat{H}
\]

(68)

represents the Larmor precession of a particle at rest, and

\[
\hat{\omega}_{\text{Thomas}}(t) = \frac{\gamma^{-1}}{v^2} [\hat{v}(t) \times \hat{\mathbf{v}}(t)],
\]

(69)

called the Thomas precession, is a relativistic effect caused by the acceleration of the particle (if charged) in the magnetic field. Here

\[
u = \text{magnetic moment in units of the Bohr nuclear magneton } e_p \frac{h}{2m_p c} \quad (e_p \text{ and } m_p \text{ are the proton charge and mass}),
\]

\[
J = \text{spin in units of } h,
\]

\[
\gamma = (\text{total lab energy/rest mass}), \text{ and }
\]

\[
\hat{v}(t) = \text{particle velocity in the lab frame, described by } \hat{v}(t) = \frac{e}{\gamma m c} \hat{v}(t) \times \hat{H},
\]

where \( e \) and \( m \) are the (signed) charge and mass of the particle in question. Hence
\[ \hat{\omega}(t) = C_1 H + C_2 \left[ \hat{H} \cdot \hat{v}(t) \right] \hat{v}(t) , \]  
where

\[ \begin{align*}
C_1 &= \left[ \frac{-\mu}{2J} + \frac{e_p}{m_p c} \right] H + \frac{\gamma-1}{\gamma} \frac{e_p}{m_c} H, \\
C_2 &= \left( \frac{\gamma-1}{\gamma} \right) \frac{e_p}{m_c} H.
\end{align*} \]

Assuming for magnetic moments the mass-corrected SU(3) values \( \mu_{\Xi^-} = -0.66, \mu_{\Xi^o} = -1.32, \) and \( \mu_{\Lambda} = -0.78, \) one obtains

for \( \Xi^- \):
\[ \begin{align*}
C_1 &= \left[ 0.66 - 0.71 \frac{\gamma-1}{\gamma} \right] \frac{e_p}{m_p c} H, \\
C_2 &= \left[ 0.71 \frac{\gamma-1}{\gamma} \right] \frac{e_p}{m_p c} H.
\end{align*} \]

for \( \Lambda \):
\[ \begin{align*}
C_1 &= \left[ 0.78 \right] \frac{e_p}{m_p c} H; \quad C_2 = 0.
\end{align*} \]

where \( \frac{e_p}{m_p c} H = 9.58 \times 10^3 \text{ sec}^{-1} \text{ gauss}^{-1} \times 17.9 \text{ k gauss} \)
\[ = 1.71 \times 10^8 \text{ sec}^{-1}. \]

Precession angles prior to decay are small (of the order of 10° or less), so to a good approximation we may ignore the variation of \( \hat{v}(t) \) as a function of time; during a time interval \( dt = \gamma d\tau \) (where \( \tau \) is proper time), \( \hat{P}(t) \) changes by approximately

\[ \frac{d\hat{P}}{\hat{v}} = C_1 \hat{H} X \hat{P}(0) + C_2 \left[ \hat{H} \cdot \hat{v}(0) \right] \hat{v}(0) X \hat{P}(0) \text{ dt} , \]

where \( \hat{v}(t) \) and \( \hat{P}(t) \) are evaluated at \( t = 0 \), the instant of \( \Xi \) production or decay, for \( \Xi \) and \( \Lambda \), respectively.

(1) \textbf{Precession of } \( \Xi^- \text{ polarization.} \)

At time \( t = 0 \), the \( \Xi \) have direction \( \hat{v}(0) = \hat{E} \), and
polarization \( \hat{P}(0) = \hat{P}_E = P_0 \hat{n} \) along the production normal \( \hat{n} = (\hat{K} \times \hat{E}) \). (As in Fig. IV-3, \( \hat{K}, \hat{E}, \) and \( \hat{n} \) are defined in the production c.m.; \( \hat{H} \) is defined in the lab frame.)

After an interval \( dt = \gamma d\tau \), the \( \hat{E} \) will have acquired a longitudinal polarization component (relative to axes defined at production) given by

\[
d\hat{P}_E \cdot \hat{E} = P_0 [C_1(\hat{H} \times \hat{n} \cdot \hat{E}) + C_2(\hat{H} \cdot \hat{E})(\hat{E} \times \hat{n} \cdot \hat{E})] \gamma d\tau
\]

\[
= P_0 C_1(\hat{H} \times \hat{n} \cdot \hat{E}) \gamma d\tau
\]

\[
= P_0 C_1 \cos \theta_\Sigma(c.m.) \cos \phi_\Sigma(c.m.) \gamma d\tau ,
\]

and a component along \( (\hat{E} \times \hat{n}) = (\hat{E} \times \hat{n}) \) given by

\[
d\hat{P}_E \cdot (\hat{E} \times \hat{n}) = P_0 [C_1(\hat{H} \times \hat{n}) \cdot (\hat{E} \times \hat{n}) + C_2(\hat{H} \cdot \hat{E})(\hat{E} \times \hat{n}) \cdot (\hat{E} \times \hat{n})] \gamma d\tau
\]

\[
= P_0 (C_1 + C_2) (\hat{H} \cdot \hat{E}) \gamma d\tau
\]

\[
= P_0 (C_1 + C_2) \sin \theta_\Sigma(c.m.) \cos \phi_\Sigma(c.m.) \gamma d\tau ,
\]

where \( \theta_\Sigma(c.m.) \) and \( \phi_\Sigma(c.m.) \) define the direction of the \( \hat{E} \) in the production c.m., as illustrated in Fig. A-1.

As the production of events is uniform about the beam axis, the average precession angles

\[
\left< \frac{d\hat{P}_E \cdot \hat{E}}{P_0} \right> \quad \text{and} \quad \left< \frac{d\hat{P}_E \cdot (\hat{E} \times \hat{n})}{P_0} \right>
\]

are zero.
Hence the precession of the $\Xi^-$ polarization cannot produce an anisotropy in $\hat{\Lambda} \cdot \hat{k}$ or $\hat{\Lambda} \cdot \Xi^-$. From observed distributions of $\gamma$ and $\theta_{\Xi^-}^{(c.m.)}$ we estimate the rms angles

$$\left\langle \left[ \frac{d(\hat{P}_\Xi^- \cdot \Xi^-)}{P_0} \right]^2 \right\rangle^{1/2} \approx 1.5^\circ \quad (78)$$

$$\left\langle \left[ \frac{d(\hat{P}_\Xi^- \cdot \Xi^- n)}{P_0} \right]^2 \right\rangle^{1/2} \approx 1.8^\circ$$

The net effect is a negligible ($\approx 0.05\%$) decrease in the $\hat{n}$-component of $\Xi^-$ polarization, which is the only component considered in the distribution function $[\text{Eq.}(35)]$. We conclude that the precession of $\Xi^-$ polarization cannot bias our measurement of $\Xi^-$ decay parameters.

(ii) Precession of $\Lambda$ polarization.

At $t = 0$, the instant of $\Xi$ decay, the $\Lambda$ have direction $\hat{v}(0) = \hat{\Lambda}$ and polarization $\hat{P}(0) = \hat{P}_\Lambda$ specified by helicity components $\hat{P}_\Lambda \cdot \hat{x}$, $\hat{P}_\Lambda \cdot \hat{y}$ (and $\hat{y}$ are now defined as in Fig. IV-1). After a time interval $d\tau = \gamma d\tau$, a $\Lambda$ initially having a longitudinal polarization $\hat{P}_\Lambda = P_0 \hat{\Lambda}$ will have acquired transverse polarization components given by

$$d\hat{P}_\Lambda \cdot \{\hat{x}\} = P_0 C_1 [\hat{\Lambda} \cdot \hat{x} \gamma d\tau]$$

$$= P_0 C_1 \left\{ \frac{\sin \phi_\Lambda \cos \phi_\Xi^{(c.m.)}}{-\sin \theta_\Lambda \sin \phi_\Xi^{(c.m.)} + \cos \theta_\Lambda \cos \phi_\Lambda \cos \phi_\Xi^{(c.m.)}} \right\} \gamma d\tau,$$

where $(\theta_\Lambda, \phi_\Lambda)$, corresponding to $(\theta, \phi)$ of Fig. IV-1, describe the $\Lambda$ direction in the $\Xi$ rest frame. (Terms proportional
to $C_2$ vanish for neutral particles.) Similarly, a $\Lambda$ initially having polarization $P_0^x$ will have acquired a $y$-component given by
\[
\frac{dP_0^y}{d\gamma} = P_0 C_1 [\hat{H} \cdot \hat{x} \cdot \hat{y}] \gamma d\gamma
\]
\[
= P_0 C_1 [-\sin \theta_\Lambda \cos \phi_\Lambda \cos \phi_\Xi^{(c.m.)} - \cos \theta_\Lambda \sin \phi_\Xi^{(c.m.)}] \gamma d\gamma.
\]
(With signs reversed represent the precession of $x$- and $y$-components onto the $\Lambda$-axis, and of the $y$-component onto the $x$-axis.) Each of the above expressions averages to zero upon integration over $\phi_\Xi^{(c.m.)}$; were this not the case, the precession described by Eqs. (79) and (80) could result in a biased determination of $\alpha_\Xi$ and/or $J_\Xi$, and $\phi_\Xi$, respectively. Averaging each of the expressions over the observed $\gamma$ and $(\theta_\Lambda, \phi_\Lambda)$ distributions, we estimate the rms angles as
\[
\left\langle \left\{ \frac{d(P_0^\Lambda) \cdot \hat{x}}{P_0^x} \right\}^2 \right\rangle^{1/2} \approx 2.9^o;
\]
\[
\left\langle \left\{ \frac{d(P_0^\Lambda) \cdot \hat{y}}{P_0^y} \right\}^2 \right\rangle^{1/2} \approx 3.7^o;
\]
\[
\left\langle \left\{ \frac{d(P_0^x) \cdot \hat{y}}{P_0^x} \right\}^2 \right\rangle^{1/2} \approx 3.3^o.
\]
The net effect is a slight uncertainty in the true direction of the $\Lambda$ polarization vector, which is not of such a nature as to bias our measurement of the $\Xi^-$ decay parameters.
FOOTNOTES AND REFERENCES

* Work performed under the auspices of the U.S. Atomic Energy Commission.
† Present address: CEN-Saclay, B.P. No. 2, Gif-sur-Yvette, France.
‡ Present address: Physics Department, University of Massachusetts, Amherst, Massachusetts.


6. Janice Button-Shafer and Deane W. Merrill, Spin and Decay Parameters of the $\Xi^-$ Hyperon, UCRL-11884, Jan. 1965.


14. Deane W. Merrill, Design of the K-63 Beam Using an Analog Computer, Alvarez Group Physics Memo 519 (Rev.), August 1964; (next page)

S. Flatté, S. Chung, L. Hardy, and R. Hess, K-63: Changing the Incident K⁻ Momentum from 2.7 GeV/c to 2.1 GeV/c, Alvarez Group Physics Memo 524, August 1964.


17. Compelling interest in 1.7 BeV/c K⁻ (especially by Shafer) prompted realignment of the 2.1 BeV/c beam channel, which yielded good K⁻ flux at 1.7 BeV/c.

18. Events containing $\pi^-$ (event-types 12, 72, and 74) were measured exclusively on Franckensteins; events containing $\pi^0$ (types 32, 40, and 42) were measured on SMP's as well as Franckensteins.


20. Knowledge of the final state governs only the division of events into arbitrary subsamples (see Table III-2). Inclusion of the ambiguous events can bias
measured values of the $E^-$ polarization but not of the $E^-$ decay parameters $a_{E^-}$ and $\Phi_{E^-}$.


32. S. M. Berman and M. Jacob, Spin and Parity Analysis in Two-Step Decay Processes, Stanford Linear Accelerator Center Report SLAC-43 (Stanford University, Stanford, California, May 1965).
35. Consider a momentum four-vector \((\vec{p}, E)\) in frame 1, with frame 2 having velocity \(\vec{v}\) relative to frame 1. Mathematically, the direct Lorentz transformation corresponds to (i) a rotation to align \(\vec{v}\) with the z-axis; (ii) a Lorentz transformation along the z-axis \((p_x\) and \(p_y\) are unchanged); (iii) the inverse of the rotation in (1).


39. Our definition of the $\mathcal{O}_{m_1 m_2}^J$ functions corresponds to that of Ref. 41.

40. See Ref. 30, Eqs. (3), (11), and (38).


42. This is similar to the distribution $I(0)$ for a "formation" resonance. See J. Button-Shafer, Phys. Rev. 150, 1308 (1966).

43. Cf. Eq. 24 of Ref. 30. (Zemach in Ref. 31 also presents a similar relation, though he utilizes different language.)


46. See Appendix E of Ref. 21.
47. Given $N$ subsamples having $|t_{10}^{\text{actual}}| \lesssim t_{10}^{\text{max}}$, we expect half of these, on the average, to yield experimental values $|t_{10}^{\text{obs}}| > t_{10}^{\text{max}}$, thereby producing a decrease of $\sim N/4$ in $\ln L$ when the constraint is applied. Our data are consistent with $J = 3/2$, with $\sim 30$ of the 47 subsamples having $|t_{10}^{\text{actual}}| \lesssim t_{10}^{\text{max}}$.

48. We define the $S$ and $P$ amplitudes as

$$S = |S| e^{i\Delta S} = \rho_S e^{i\delta_S}$$

$$P = |P| e^{i\Delta P} = \rho_P e^{i\delta_P}$$

where $\delta_S$ and $\delta_P$ are final-state $\Lambda\pi$ scattering phase shifts. Invariance under $T$ ($C$) requires that $\rho_S$ and $\rho_P$ be relatively real (imaginary). As no spin-$1/2$ $\Lambda\pi$ resonances are known in the vicinity of 1320 MeV, $\delta_S$ and $\delta_P$ are expected to be small.


54. See N. Cabibbo, p. 29, Proceedings of the XIIIth International Conference on High-Energy Physics, Berkeley, California, September, 1966 (University of California Press, Berkeley, California), and especially Appendix B by J. P. Berge, for world-average results, which include those presented here.


56. World-average values are tentative values from a forthcoming compilation by Arthur H. Rosenfeld; Angela Barbaro-Galtieri, Janos Kirz, William J. Podolsky, Matts Roos, William J. Willis, and Charles Wohl, Data on Elementary Particles and Resonant States, UCRL-8030 (Rev.), August 1966.

58. For a discussion of the statistical treatment (suggested by N. Byers), see Appendix C of Ref. 6.


60. Like the bubble-chamber coordinate system \((x_{ch}, y_{ch}, z_{ch})\), the systems \((x, y, z)\) and \((x', y', z')\) are left-handed in real space; i.e., \(\hat{x}(x'), \hat{y}(y'), \) and \(\hat{z}(z')\) lie roughly along \(\hat{z}_{ch}, \hat{x}_{ch}, \) and \(\hat{y}_{ch}\), respectively. For purposes of this discussion we assume the magnetic field to point along \(-\hat{z}_{ch}\) (which is not really the case) so that negative particles curve toward \(+x_{ch}\), as desired.

61. Here \(\vec{p}_E^{(lab)}\) is measured at the \(\Xi\) decay vertex.

62. For most events, \(\cos \theta'_\Lambda(\Xi)\) is more nearly equivalent to \((\hat{\Lambda} \cdot \hat{K})\) than to \((\hat{\Lambda} \cdot \hat{\Xi})\), since \(\vec{p}_E^{(lab)}\) is more nearly parallel to \(\hat{K} = \vec{p}_K^{(c.m.)}\) than to \(\hat{\Xi} = \vec{p}_\Xi^{(c.m.)}\).

Nevertheless, due to the forward \((\hat{\Xi} \cdot \hat{K} \propto +1)\) peak in the \(\Xi\) production distribution, \(\cos \theta'_\Lambda(\Xi)\) is roughly equivalent to \((\hat{\Lambda} \cdot \hat{\Xi})\) as well as to \((\hat{\Lambda} \cdot \hat{K})\).

63. The observed distribution of \(p_\Xi^{(lab)}\) extends from 0.5 to 3.0 BeV/c, with a mean value near 1.7 BeV/c. Approximately 80% of the events lie between the representative values 1.0 and 2.4 BeV/c.

64. See Appendix B of Ref. 21.

66. For $E^-$, Eq. (70) expresses the rate of precision relative to fixed axes defined at the $E^-$ production vertex, which we have used. Relative to axes rotating with the $E^-$ momentum, the $\hat{H}$-component of $\tilde{\omega}(t)$ is $C_i' = C_i - \omega_{cy}$, where $\omega_{cy}$, the cyclotron frequency, is $-eH/\gamma mc$. For the case of protons, $C_i'$ reduces to the familiar form $\gamma(\mu-1)\omega_{cy}$.

67. M. A. B. Beg and A. Pais, Phys. Rev. 137, B1514 (1965). The experimental value of $\mu_A$ is $-0.73 \pm 0.17$. [See D. A. Hill and K. K. Li, Phys. Rev. Letters 15, 85 (1965).]
FIGURE CAPTIONS

Fig. II-1. K-63 Beam (top view). S1 and S2 are electrostatic separators; M1, M2, M3, and M4 are bending magnets; Q1, Q2, ..., Q13 are quadrupoles.

Fig. II-2. K-63 beam profile in (a) vertical plane and (b) horizontal plane. The y-axis (beam direction) is compressed by a factor of 80 relative to x and z, and effects of bending magnets have been ignored.

Fig. II-3. Schematic drawing of first mass slit: (a) seen end view; (b) top view of lower jaw. The y-axis (beam direction) is compressed by a factor of 6 relative to x and z. High- and low-momentum K^- (2% above and below the nominal momentum) are focused at A and B respectively; π^- are focused ~ 1/8" above the K^- image and pass through the uranium upper jaw of the slit.

Fig. IV-1. Diagram of Ξ production and decay via the sequence (A) K^-+p→Ξ+K; (B) Ξ→Λ+π; (C) Λ→p+π. The Ξ production angle Θ is defined in the production c.m. frame (A); angles θ and φ describe Λ decay in Ξ rest frame (B); Λ polarization is described with reference to axes x and y, illustrated in the blowup of system (B). Momentum four-vectors of Ξ, Λ, and p are obtained from measured lab momenta.
via successive Lorentz transformations through frames (A), (B), and (C).

Fig. V-1. Correlation between $\alpha_{\Lambda}$ and $\alpha_{E^-}$ for combined K-72 and K-63 sample. With no external information on $\alpha_{\Lambda}$, the best fit to the data yields $\alpha_{\Lambda} = 0.698 \pm 0.069$, $\alpha_{E^-} = -0.381 \pm 0.045$ (standard-deviation ellipse centered at A). With an additional factor of $\exp[\frac{-1}{2}(\alpha_{\Lambda} - 0.62)^2/(0.07)^2]$ in the likelihood, the best fit yields $\alpha_{\Lambda} = 0.657 \pm 0.047$, $\alpha_{E^-} = -0.394 \pm 0.041$ (ellipse centered at B). Projections shown represent quoted $\alpha_{\Lambda}$ and $\alpha_{E^-}$ values with errors.

Fig. V-2. Distributions of $(p \cdot \hat{A})$ for 3431 $E^-$ events. (See Table III-1.) K-72 events are shaded. The theoretical curve is proportional to $1 + \alpha_{\Lambda} \alpha_{E^-} (p \cdot \hat{A})$, where $\alpha_{\Lambda} \alpha_{E^-} = -0.28$.

Fig. V-3. Decay distributions of 3431 $E^-$ events (K-72 and K-63 data combined). The theoretical curves correspond to the best fit with $\alpha_{\Lambda}$ free, namely $\alpha_{\Lambda} = 0.698 \pm 0.069$, $\alpha_{E^-} = -0.381 \pm 0.045$, $\phi_{E^-} = 10.0^0 \pm 8.9^0$, $\langle p_{E^-} \rangle = 0.26 \pm 0.04$. Most of the (very weak) evidence for $\phi_{E^-}$ or $\beta_{E^-} > 0$ comes from 179 $E^-K^+$ events at 2.45 and 2.55 BeV/c, having $\langle p_{E^-} \rangle = 0.90 \pm 0.19$ and $\sum_{i=1}^{179} (p \cdot \hat{y})_i = 12.3 \pm 7.7$ and yielding $\phi_{E^-} = 30^0 \pm 18^0$ when fitted separately. All other subsamples have smaller average polariz-
ations and yield values of $\Phi\pi$ consistent with zero.

Fig. V-4. Dependence of $w = \ln \mathcal{L}$ upon assumed spin factor $(2J+1)$, for 3278-event $\Xi^-$ sample. The decrease of 3.02 in $\ln \mathcal{L}$ indicates discrimination against the $J = 3/2$ hypothesis by 2.45 standard deviations. The additional decrease of 7.03, due to violation of the $J = 3/2$ density matrix constraint, is not statistically significant.

Fig. V-5. Measured values of $X = \Delta \ln \mathcal{L}$ for (a) 15 $\Xi$ subsamples; and (b) 75 Monte Carlo samples generated assuming $J = 1/2$, $\lambda = 0.62$, $\alpha = -0.40$, $\Phi = 0$, $t_{10} = 0$. The curve shown in (b) represents $\bar{X} = 0$, $\sigma_X = 0.27$.

Fig. V-6. Measured values of $X = \Delta \ln \mathcal{L}$ for (a) 75 Monte Carlo samples generated assuming $J = 1/2$, $\lambda = 0.62$, $\alpha = -0.40$, $\Phi = 0$, $t_{10} = t_{10}^{\text{max}} = 0.57$; and (b) 75 Monte Carlo samples having $J = 3/2$, $\lambda = 0.62$, $\alpha = -0.40$, $\Phi = 0$, $t_{10} = t_{10}^{\text{max}} = 0.43$. Curves shown represent (a) $\bar{X} = 1.66$, $\sigma_X = 1.93$; (b) $\bar{X} = -0.67$, $\sigma_X = 1.07$.

Fig. V-7. Scatter plot and projections for $\Xi\pi$ mass-squared vs. $\Xi \pi \pi$ mass-squared for 164 $\Xi K \pi \pi$ events from the K-63 run.

Fig. V-8. Scatter plot of events having $\Xi \pi \pi$ and $\Xi \pi_2$
masses corresponding to the $\pi^*(1817)$ and $\pi^*(1530)$.
For removal of events with $K\pi$ mass $\lesssim K^*$ mass.

Fig. V-9. Two decay-angle distributions for events which
qualify for $\pi^*(1817) \rightarrow \pi^*(1530) \rightarrow \pi$. In plot (a)
any significant amount of $|\cos \theta|^2$ contribution would
demand that the spin of the $\pi^*(1817)$ be 3/2 or
higher.

Fig. VI-1. Comparison of decay-parameter results from
K-72 and K-63 data with those from previous experi­
ments. (See Table VI-1.)

Fig. VI-2. Representations of the Lee SU$_3$ triangle (for
$\pi$, $\Lambda$, and $\Sigma$ decay amplitudes) and the $|\Delta I| = 1/2$
triangle (for $\Sigma$ decay amplitudes). Experimental
results used are new world averages which include
the results of this report. (See Table VI-2.)

Fig. VI-3. Values of $X = \Delta \ln \chi^2$ vs. $|t_{10}\left(3/2\right)|$
for 45
$\pi^-$ subsamples and 7 $\pi^0$ subsamples. Dashed curves
represent the expected range of $X (\bar{X} + \sigma_X)$ assuming
$J = 1/2$ and $J = 3/2$, as a function of $|t_{10}^{1/2}|$.
The solid curve represents the expected distribution
in $|t_{10}^{1/2}|$ (due to the measurement errors alone)
if all subsamples have zero polarization. Points
(a), (b), and (c) designate $\langle t_{10} \rangle^{1/2}$ from
$\sigma_X^{1/2}$ and $\sigma_{t_{10}}^{1/2} = 0.22$. 

$\langle t_{10} \rangle^{1/2} = \langle t_{10} \rangle^{1/2}$
Fig. A-1. Distributions in $\hat{A} \cdot \hat{K}$ and $\hat{A} \cdot \hat{\Xi}^-$ for the $\Xi^-$ events of Table III-2. (Directions defined in the $\Xi$ rest frame.)

Fig. A-2. Scatter plot (and projection) in $\phi_\Xi$ and $\cos \theta_\Xi$ production parameters for the $\Xi^-$ events of Table III-1.

Fig. A-3. Scatter plot (and projection) in decay angles describing the $\Lambda$ direction in the $\Xi$ rest frame, for the $\Xi^-$ events of Table III-1.
Fig. II-2
(a) 

![Diagram](a)

(b) 

![Diagram](b)

Fig. II-3
\[ n = K \times \vec{\varepsilon} = \vec{Z} \]

\[ \vec{x} = \Lambda \times (\Lambda x n) \]

\[ \vec{y} = \hat{n} \times \Lambda \]

Fig. IV-1
Fig. V-1
$N$

$\Sigma (\hat{\rho} \hat{\lambda}_i)$

$\Sigma (\hat{\rho} \hat{\chi}_i)$

$\hat{\Lambda} \cdot \hat{n} = \cos \theta$

Fig. V-3
Fig. V-4

Independent information used in fit

\[
\begin{array}{ccc}
\alpha_\Lambda & \alpha_\Lambda \alpha_\Xi \\
\hline
\text{free} & \text{free} \\
0.62 \pm 0.07 & 0.62 \pm 0.07 & -0.321 \pm 0.048
\end{array}
\]

\(2J + 1 = 2.0^{+0.45}_{-0.40}\)
Fig. V-5
(a) Spin 1/2, 100% polarized

(b) Spin 3/2, 100% polarized

$X = \Delta \ln \mathcal{L}$

Fig. V-6
Fig. V-7
(● o) K⁻ K⁺ π⁺ π⁻ (23 events)
(▲ △) K⁻ K⁰ π⁺ π⁰ (13 events)
(■ ■) K⁰ K⁺ π⁺ π⁻ (5 events)

21 K⁺⁰ events (● ▲ ■), removed
8 Doubly weighted events (● ▲ ■)
12 Singly weighted events (o ▲ ■)

C.m. energy (BeV)

Cos α = (1530) • (1817)

Fig. V-8
Fig. V-9
<table>
<thead>
<tr>
<th>Lab</th>
<th>Events</th>
</tr>
</thead>
<tbody>
<tr>
<td>LRL (K-63)</td>
<td>2529</td>
</tr>
<tr>
<td>LRL (K-72)</td>
<td>1004</td>
</tr>
<tr>
<td>BNL+S</td>
<td>700</td>
</tr>
<tr>
<td>Ep⁺d</td>
<td>517</td>
</tr>
<tr>
<td>UCLA</td>
<td>356</td>
</tr>
<tr>
<td>CERN</td>
<td>62</td>
</tr>
</tbody>
</table>

![Graph](image)

**Fig. VI-1**
Fig. VI-2
Fig. VI-3
Fig. A-1
Fig. A-2
Before losses (~2980 events)

After losses (2529 events)

\[ \cos \theta_{\Delta}^{(H)} = -\cos \theta_{\pi}^{(H)} \]

Fig. A-3
This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.