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ESTIMATES OF A MORE GENERAL MODEL
OF CONSUMER CHOICE IN THE HOUSING MARKET
BY
JOHN M. QUIGLEY

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Estimates of a More General Model of Consumer Choice in the Housing Market

by

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Revised
December 1981

This analysis benefited from discussions with Daniel McFadden about the general model and with William Apgar about the sources of data. Neither has any responsibility for errors or interpretations. Programming assistance was provided by Paul Pfleiderer and James Trask, and financial support by the National Science Foundation.
I. Introduction

The multinomial logit model is an appealing representation of the choice processes of consumers in the housing market. Its appeal for empirical analysis of household behavior is based upon two salient features of the housing commodity: the dimensionality of the bundle of residential services, and the joint pricing of these residential attributes.

In any local market, most of the housing services consumed in a market run are provided by the existing stock. Since this stock is expensive to modify, the spatial pattern of prices in temporary equilibrium, such that suppliers earn identical normal profits and consumers of identical incomes enjoy equal satisfaction, is likely to include substantial positive or negative quasi rents to particular configurations of housing components at specific locations. Except in the longest of long runs, the temporary equilibrium pattern of housing prices is likely to be quite complex indeed.

Thus it is natural to view consumer choice as the selection of a single discrete dwelling unit out of a large number of alternatives indexed by their characteristics and prices. In selecting a particular dwelling unit, households jointly choose a vector of characteristics and a rental payment. In contrast to consumer choice in most markets, housing choices involve the joint selection of quantities and prices by demanders.

During the past few years, there have been a number of studies applying the multinomial logit model to observations on the choices made by housing consumers. This paper begins by reviewing briefly the assumptions and maintained hypotheses of the housing market models estimated by Ellickson (1977, 1981), Case (1981), Kain and Apgar (1977), Lerman (1977, 1979), Quigley (1973, 1976), and Williams (1979), among others. We then consider
several extensions of the basic theoretical model, extensions proposed by Domenchich and McFadden (1975) and McFadden (1977). On the basis of these results, the parameters of a more general model of housing choice are estimated. The empirical analysis provides a comparison between the results of the more general model and those obtained from the class of "traditional" models of housing choice. The specific comparison uses my own paper, written in 1973, as the horrible example.

II. The Traditional Logit Model Applied to Housing Choice

Call the set of all dwellings D with J members (j = 1,2,...,J). When a consumer chooses a dwelling unit i out of D, he also selects a set of neighborhood and public service amenities and a journey to work (X_j), as well as a price, (i.e., a monthly rent or purchase price R_i). Consumers of income y have preferences over the set of public service-amenity packages, housing characteristics, and other goods, y - R_i.

Assume the utility function for households consists of a systematic component V and an additive stochastic component ε

1) \( U[X_i, y - R_i] = V(i) + \epsilon(i) \)

Assumptions (maintained hypotheses) about the form of the stochastic component of the utility function permit probability statements about the choice of any specific dwelling to be made. In particular, as is well known, McFadden [1974] demonstrated that: if it is assumed that the stochastic terms are independently and identically distributed according to the Weibull distribution\(^1\), then the form of the probability statement is

2) \( p(i) = \text{prob}[U(i) > U(j)] = e^{V(i)} / \sum_j e^{V(j)} \)

for all \( j, j \neq i \)

\(^1\) (n-1) \( \text{prob}[\epsilon(i) \leq A] = \exp[-\exp(-A)] \) for all i
Equation (2) is a well behaved probability statement with values bounded by 0 and 1. The probability of choosing any dwelling unit depends upon the characteristics of all dwellings in the choice set. Equation (2) is estimated by maximizing a log likelihood function of the form

\[ 3) \ log L = \frac{1}{K} \sum_{k} \log \left( \frac{e^{V(i)}}{\sum_{j} e^{V(j)}} \right) \]

for a sample size of K observations on choices i and on available alternatives j.

Finally, if it can be assumed that the systematic component of the utility function is linear in its parameters, McFadden has shown that the likelihood function (3) is concave, and the parameters are unique up to a factor of proportionality. For the problem of housing choice, a linear relationship (another maintained hypothesis)

\[ 4) \ V(i) = \alpha X_i + \beta (y - R_i) \]

renders the parameters of the model, \( \alpha \) and \( \beta \), estimable. This maintained hypothesis is, of course, rather innocuous; any non-linear function can be approximated by one linear in its parameters.

Under the maintained hypotheses of (1), (2) and (4), estimates of the discrete model of housing choice have been presented by a number of researchers.

It is worth pointing out two serious limitations shared by all these analyses of consumer behavior in the housing market—one conceptual limitation, and one practical limitation which compromises existing empirical work.
First, according to equation (2), the odds of choosing housing unit \( m \) relative to \( n \) are independent of the characteristics of all other alternatives available to consumers. This maintained hypothesis, the so-called independence of irrelevant alternatives (IIA), is simply not testable within the traditional model. The assumption is surely inappropriate in many situations involving the choice of housing and neighborhood characteristics.

Second, there is a real practical problem in maximizing the log likelihood function (3). Clearly, the theoretical problem solved by consumers in the marketplace is the selection of one specific dwelling unit out of the large number of alternative dwellings (D) actually available on the market. However, for an economist to maximize the likelihood function (3) for any sample of consumers of size \( K \), it is necessary to make the set of alternatives "small enough" somehow to render an iterative solution procedure computationally feasible. Note that this latter complication did not arise at all in the original applications of the multinomial logistic model to model choice. Inherently, there are a relatively small number of available transport modes; in constrast, however, there are a large number of potential dwelling units available for occupancy by housing consumers.

Typically, analysts have "solved" this problem in an ad hoc way. They have represented the heterogeneity of the housing, neighborhood and public services available to consumers by a small number of "types" of residential housing, e.g., specified components of the bundle of housing services at particular values. Thus, in practical estimation, the choice problem has been defined as the selection of one "housing type" out of an arbitrarily defined set of alternatives.
Table 1 indicates the ad hoc definitions of the choice set employed by analysts in estimating the logit model of consumer choice in the housing market.\(^2\)

Quigley (1973, 1976) formulated the consumer's problem as the choice of one specific type of housing out of a set of 18 types of rental units, based upon dwelling units characteristics alone. Case (1981) considers the choice among nine types of housing. Kain and Apgar (1977) and Williams (1979) considered the problem of choosing one housing type out of 50 (based upon 10 classifications of dwelling unit characteristics and 5 classifications of neighborhood). In contrast, Lerman's (1977, 1979) formulation investigates the household's choice of a census tract for residence, rather than the choice of a dwelling. In this formulation, alternatives are described by the average characteristics of housing in different tracts. The data consist of an observation on the census tract chosen by each household and observations on each rejected alternative in

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\(^2\) Ellickson's (1977, 1981) analysis is somewhat different from the other studies noted in Table 1. He estimates the parameters of consumers' bid rent functions directly rather than indirectly through utility parameters. This "reverse logit" analysis estimates the conditional probability that a dwelling will be occupied by a household of given characteristics.

This approach, however, does not avoid the problem of arbitrary definition of the discrete alternative. Ellickson defines households by 11 discrete categories (race, income, and family size groups) for his empirical analysis.
Table 1

Definitions of Housing Alternatives
Used in Multinomial Logit Analyses of Housing Choice

<table>
<thead>
<tr>
<th>Author</th>
<th>Number of Housing Alternatives</th>
<th>Definition of Housing Alternatives</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kain and Apgar</td>
<td>50 housing types</td>
<td>Single family: 2 density classes by 3 size classes by 5 neighborhood categories. Multi family: 2 density classes by 2 size classes by 5 neighborhood classes.</td>
</tr>
<tr>
<td>(1977)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Williams</td>
<td>50 rental types</td>
<td>Ten combinations of lotsize, structure type, and number of bedrooms by 5 neighborhood classes.</td>
</tr>
<tr>
<td>(1979)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lerman</td>
<td>145 census tracts</td>
<td>Average census characteristics of housing units in each tract.</td>
</tr>
<tr>
<td>(1977, 1979)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case (1981)</td>
<td>9 housing types</td>
<td>Owner occupied: 3 size classes. Rental units: 3 size classes by 3 structure classes.</td>
</tr>
<tr>
<td>Quigley (1976)</td>
<td>18 rental types</td>
<td>3 structure types by 3 size classes by 2 age classes.</td>
</tr>
</tbody>
</table>
the urban area. Thus Lerman's analysis avoids the problem of defining himself arbitrary "types" of residential housing, but only by relying upon the averaged census characteristics of housing units (themselves a somewhat arbitrary aggregation of alternatives). Lerman's empirical work analyzes the choices made by 177 Washington, D.C. households. Thus the iterative procedure for maximizing the likelihood function is based upon a data set consisting of more than 25,000 observations on consumer choices among available census tracts!

III. Applying the Extended Logit Model to Housing Choice

The bundle of services jointly consumed by the selection of a dwelling unit can be partitioned into (at least) two components: those that vary by dwellings within neighborhoods (or census tracts or towns) \( X_1 \); and those that are constant for dwellings within neighborhoods but vary across neighborhoods (or census tracts or towns) \( X_2 \). The size or condition of a dwelling unit or its price are examples of the first component; the quality of local schools or the racial composition of the neighborhood are examples of the second.\(^3\)

\(^3\) For notational convenience only, consider the choices made by households of the same income. Thus \((y - R_i)\) can be treated as one component of the vector of characteristics \(X_{1i}\) that vary across dwelling units.
From (2), under the usual assumptions, the probability of choice of neighborhood \( (n) \) and dwelling unit \( (i) \) is

\[
p(i,n) = e^{\alpha_1X_{1i} + \alpha_2X_{2n}} / \sum_{j,k} e^{\alpha_1X_{1j} + \alpha_2X_{2k}}
\]

As with any joint probability statement, (5) can be decomposed into a marginal and a conditional probability statement. If \( \sigma = 1 \), then

\[
(6a) \ p(i|n) = e^{\alpha_1X_{1i}/(1-\sigma)} / e^{I_n}
\]

\[
(6b) \ p(n) = e^{\alpha_2X_{2n} + (1-\sigma)I_n} / \sum_{k} e^{\alpha_2X_{2k} + (1-\sigma)I_k}
\]

\[
(6c) \ I_n = \log \sum_{i \in n} e^{\alpha_1X_{1i}/(1-\sigma)}
\]

is arithmetically identical to (5). The parameters of the choice model can be estimated directly by using (5) or sequentially. The latter procedure involves estimating \( \alpha_1 \) from (6a), using the parameters to calculate \( I_n \) for each neighborhood (or census tract or town) and then estimating \( \alpha_2 \) from (6b). The sequential approach "involves some loss of efficiency relative to direct estimation of the joint choice model" (McFadden, 1977); it is, however, merely an alternative way of approaching the same problem.

The same sequential approach can of course be used to estimate the value of \((1-\sigma)\), as well as \(\alpha_1\) and \(\alpha_2\) and thus to test whether \(\sigma\) is different from zero. McFadden and Domenichich (1975) have shown that the joint probability function consistent with (6) is of the form

\[
p(i,n) = \left[ e^{V(i,n)/(1-\sigma)} \right] \left[ \sum_{j \in n} e^{V(j,n)/(1-\sigma)} \right]^{-\sigma} / \left[ \sum_{k \in n} e^{V(j,k)/(1-\sigma)} \right]^{(1-\sigma)}
\]

This is a direct generalization of the traditional problem of joint choice. If \(\sigma\) is indeed equal to zero, then equation (7) reduces to (2),
that is, to a choice model with the IIA property. If \( \sigma \) is equal to one, then from (6b), the choice of neighborhood depends only upon neighborhood attributes, i.e., all housing units within a neighborhood are viewed as identical. Thus sequential estimation of equation (6) provides a direct statistical test of the degree of independence of irrelevant alternatives.

Now consider the problem of estimating the theoretically correct choice model—the selection of one dwelling unit out of a large number of discrete alternatives. Estimating the choice model using as observations the entire set of metropolitan housing alternatives facing each consumer is clearly out of the question. If, however, for each consumer we select a subset of alternatives, \( d \), and observe the consumer's choice among elements in this subset, then it may be possible to derive consistent estimates of the theoretically correct choice model. In particular, suppose \( f(\{d\}_i) \) is the sampling rule for obtaining subset \( d \), conditional upon the observed choice of dwelling unit \( i \). McFadden (1977) has shown that if the sampling rule has certain weak properties, i.e.,

\[
\text{(8) if } f(\{d\}_i) > 0 \text{, then } f(\{d\}_i) > 0,
\]

then maximization of the modified likelihood function

\[
\text{(9) } \log L = \frac{1}{K} \sum_{k} \log [e^{V(i) + \log f(\{d\}_i)}/\sum_{j\in d} e^{V(j)} + \log (d\{d\}_i)]
\]

yields consistent estimates of the parameters of the choice function. Equation (8) specifies a sampling rule with the property that if rejected alternative \( j \) is assigned to the subset \( d \), then it is logically possible that \( j \) could have been the observed choice. Under these conditions, McFadden's result indicates that the likelihood function need only be modified to take into account the sampling rule for selecting \( d \). Thus
it is possible to estimate, consistently at least, the parameters of the model which views households as choosing one specific dwelling unit out of the entire set of available units in the metropolitan housing market.

IV. Empirical Application

In a paper presented in 1973 and published in 1976 Quigley used the multinomial logistic model to investigate the housing choice decisions of households in the Pittsburgh metropolitan housing market. The basic data were drawn from a home interview survey of 24,626 households conducted in 1967. Separately for 30 income and family size classifications of white renter households who had recently moved into their dwellings, the parameters of household choice among discrete types of residential housing were estimated according to the procedure described in equations (1) through (4). As noted in Table 1, for each of these groups of households, the heterogeneity of housing alternatives was represented by 18 "types" of residential housing: three structure types (single family units, row houses or common wall units, and apartments); three size classes (1, 2, and 3-or-more bedrooms); and two quality or age classes (units built before 1930 and those constructed after 1930). In addition, the average occupancy cost for each type of housing facing each consumer was estimated. For each housing type, the occupancy cost was computed for each household as the minimum of the sum of gross rental payments and the journey to work cost to the household's workplace in a set of 333 residence sites (zones) in the metropolitan area.  

4 E.g., for housing type $i$ the occupancy cost for a household with workplace $j$ was computed as $\min (P_{ik} + T_{jk}w)$ where $P_{ik}$ is the rent for type $i$ in zone $k$, $T_{jk}$ is (monthly) transport time from $j$ to $k$, and $w$ is the wage rate. This occupancy cost has subsequently been called the "gross price" of housing (see Ingram, 1979).
Thus for the maximization of the likelihood function, the $x$ vector consists of variables representing structure type, age, interior size and occupancy costs. The data consist of observations on the type of residential housing chosen and the 17 rejected alternatives for each household in a given classification.

Table 2 is reproduced from that study. The table presents the coefficients of the choice model, estimated separately for middle income households of 1, 2, 3, 4 and 5-or-more members. For any household class, ratios of the coefficients are interpreted as the marginal rates of substitution between housing characteristics; the bid rents of consumers can be obtained by inverting the utility function.

Note the limitations of this approach. First, the "independence of irrelevant alternatives" is a maintained hypothesis for this analysis. For example, the log odds of choice between new-single-family-two-room-units and old-row-house-three-room-units is assumed to be independent of the availability of new-single-family-three-room-units in the metropolitan housing market. Second, representation of the heterogeneity of housing by 18 types is an extremely strong assumption. However, modelling a "realistic" choice process by increasing the dimensionality of housing (to include, say neighborhood attributes) would have increased the estimation cost more than proportionately.

The discussion in Section II indicates that a more general model which avoids these two assumptions can be estimated for reasonable cost.

Suppose for each consumer who faces the choice of selecting one dwelling out of the metropolitan housing stock (D) we select a subset of dwellings $d$ according to the following sampling rule: include in the subset of
Table 1: Asymptotic t-statistics in parentheses

<table>
<thead>
<tr>
<th>(12.16)</th>
<th>(6.61)</th>
<th>(3.89)</th>
<th>(13.46)</th>
<th>(7.33)</th>
<th>(7.53)</th>
<th>273</th>
<th>+5</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>-6.116</td>
<td>-0.883</td>
<td>0.113</td>
<td>-2.203</td>
<td>-1.953</td>
<td>194</td>
<td>4</td>
</tr>
<tr>
<td>0100</td>
<td>-4.149</td>
<td>-2.738</td>
<td>-3.833</td>
<td>-2.693</td>
<td>-2.693</td>
<td>223</td>
<td>2</td>
</tr>
<tr>
<td>(19.89)</td>
<td>(5.87)</td>
<td>(3.03)</td>
<td>(10.72)</td>
<td>(3.67)</td>
<td>(3.67)</td>
<td>162</td>
<td>1</td>
</tr>
<tr>
<td>0500</td>
<td>-4.465</td>
<td>-4.383</td>
<td>-2.302</td>
<td>-2.302</td>
<td>-2.302</td>
<td>223</td>
<td>1</td>
</tr>
<tr>
<td>(6.94)</td>
<td>(0.20)</td>
<td>(0.69)</td>
<td>(1.20)</td>
<td>(1.20)</td>
<td>(1.20)</td>
<td>535</td>
<td>1</td>
</tr>
<tr>
<td>0600</td>
<td>-1.888</td>
<td>-3.049</td>
<td>-2.349</td>
<td>-2.349</td>
<td>-2.349</td>
<td>150</td>
<td>1</td>
</tr>
</tbody>
</table>

---

<table>
<thead>
<tr>
<th>Stock (β)</th>
<th>Price (ACR)</th>
<th>Average Relative Size</th>
<th>Relative Average Sales</th>
<th>APY (APM)</th>
<th>CV (W)</th>
<th>Family Number of Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>57</td>
<td>78</td>
<td>90</td>
<td>12</td>
<td>34</td>
<td>56</td>
<td>78</td>
</tr>
</tbody>
</table>

Model by Family Size for Income Class $5,000-6,999

Estimated Coefficients of the Multinomial Logistic

Table 2
alternatives d the observed choice and a number of rejected alternatives obtained by considering each element in D sequentially and including it with probability \( p \). Under these conditions the probability of d, \( f(d|_i) \), depends only on the number of elements it contains, according to the binomial formula. Obviously, if any rejected alternative j appears in the subset d, it has the logical possibility of being the observed choice i from that set. This particular sampling rule implies a somewhat stronger condition, however. Since d always contains the chosen alternative i and some number of rejected alternatives selected with equal probability, then the probability of any set d depends only upon its number of elements.

In this special case, termed the "uniform conditioning property" by McFadden (1977), the terms containing \( \log f(d) \) in the numerator and denominator of equation (9) simply cancel out. Maximization of the simple likelihood function based upon a sample size \( K \) of observations on choices i out of subsets d yields consistent estimates of the parameters estimated by maximizing the likelihood function based on \( K \) observations on choices i out of D, the entire set of alternatives.

We now use this result to estimate the more general model of housing choice for one classification of consumers, a subset of the 224 observations on white renter households with three members reported in Table 2. The subset consists of those 131 households with one full time worker and which had moved into their present unit within the past year. From the survey data on the rental dwelling units in the metropolitan housing market (D), a subset (d) of alternatives was selected for each individual. For each household the subset consists of its chosen dwelling unit and
a number of rejected dwelling units obtained by including each other rental unit in the sample with some probability \( p \), chosen so that the subset of alternatives for each household would contain about five members. For each household, it was possible to estimate the journey to work time by auto and public transit for each dwelling unit in its subset. In addition the average characteristics of the census tracts containing each sampled unit were obtained. Finally, it was possible to obtain information about the local spending of the jurisdiction servicing each dwelling unit.

Table 3 presents summary data on the alternatives chosen by these households and on the subset of rejected alternatives drawn for each household. The dwelling units chosen by these households are more likely to be single family units and less likely to be apartments than those units not selected. On average, the 131 chosen units are slightly smaller, in slightly better condition, and slightly more expensive than those in the sample of 524 rejected alternatives.

The dwelling units chosen by these households are considerably more accessible to their worksites than the sample of rejected alternatives. The average difference in accessibility is more than 20 minutes (one way) by auto and almost 30 minutes (one way) by public transit.

The differences in average census tract characteristics suggest that this group of households chooses housing in better neighborhoods--their chosen dwelling units tend to be in neighborhoods where homeownership rates are higher (61\% vs. 52\%), where vacancy rates are lower (3.2\% vs. 5.5\%), and those with a smaller minority population (2.7\% black vs. 10.2\%) than in the neighborhoods containing the rejected units.
Table 3
Sample Characteristics of Chosen Dwellings
and a Sample of Rejected Units for 131 Renter Households

<table>
<thead>
<tr>
<th>Dwelling Characteristics: a</th>
<th>Chosen Alternatives</th>
<th>Sample of Rejected Alternatives</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Central City</td>
<td>Suburbs</td>
</tr>
<tr>
<td><strong>Common Wall</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Units</td>
<td>0.464</td>
<td>0.418</td>
</tr>
<tr>
<td>Apartment Units</td>
<td>0.357</td>
<td>0.233</td>
</tr>
<tr>
<td>Number of Bedrooms</td>
<td>1.821</td>
<td>1.961</td>
</tr>
<tr>
<td>Number of Bathrooms</td>
<td>1.000</td>
<td>1.039</td>
</tr>
<tr>
<td>Condition (1=bad)</td>
<td>0.036</td>
<td>0.010</td>
</tr>
<tr>
<td>Age (years)</td>
<td>42.04</td>
<td>37.77</td>
</tr>
<tr>
<td>Monthly Rent</td>
<td>$82.11</td>
<td>$60.61</td>
</tr>
<tr>
<td>Auto Journey to Work (min.)</td>
<td>27.36</td>
<td>24.46</td>
</tr>
<tr>
<td>Transit Journey to Work</td>
<td>40.71</td>
<td>56.17</td>
</tr>
<tr>
<td>Occupancy Cost</td>
<td>$151.80</td>
<td>$122.90</td>
</tr>
<tr>
<td>Census Tract Characteristics b</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Proportion Vacant</td>
<td>0.036</td>
<td>0.030</td>
</tr>
<tr>
<td>Proportion Homeowner</td>
<td>0.561</td>
<td>0.629</td>
</tr>
<tr>
<td>Proportion Lacking Plumbing</td>
<td>0.039</td>
<td>0.032</td>
</tr>
<tr>
<td>Median Rent</td>
<td>$109.60</td>
<td>$101.80</td>
</tr>
<tr>
<td>Proportion Black</td>
<td>0.018</td>
<td>0.029</td>
</tr>
<tr>
<td>Public Sector Characteristics c</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Per Capita Spending</td>
<td>$3577.00</td>
<td>$3884.00</td>
</tr>
<tr>
<td>School Spending Per Student</td>
<td>$795.70</td>
<td>$664.20</td>
</tr>
<tr>
<td>Student Teacher Ratio</td>
<td>19.46</td>
<td>19.87</td>
</tr>
<tr>
<td>Percent Non-White in School</td>
<td>38.21</td>
<td>6.06</td>
</tr>
<tr>
<td>Number of Cases</td>
<td>28</td>
<td>103</td>
</tr>
</tbody>
</table>

b. Source: 1970 Census of Population and Housing
c. Source: See Quigley, Trask and Trask [1977]
There are some differences in average public sector characteristics between the chosen and rejected units. The chosen units tend to be in jurisdictions with slightly lower per capita spending and slightly lower school spending per pupil, but in jurisdictions where a larger fraction of public school students are white.

Twenty-eight of the 131 chosen dwelling units are located in the central city of Pittsburgh. Of the sample of 524 rejected units, 186 are located in the central city. Table 3 also compares the average characteristics of chosen and rejected units separately for central city and suburban locations. Conditional upon the choice of a central city unit, the size of the chosen dwelling is slightly larger; it is in better condition, is more accessible, and is more expensive than the average rejected unit. Conditional upon a suburban choice, the chosen unit is slightly smaller, but the other comparisons of dwelling characteristics are similar.

Table 4 presents estimates of the multinomial logit model based on this body of information describing the housing choices of recent movers. The first column of the table reports the results based only upon the housing market information used in the previous paper, that is, the structure type, size, and the age class of the chosen and rejected alternatives. In addition, the occupancy cost or "gross price" of the units is included.\(^5\)

\(^5\) This variable is constructed to be as close as possible to the occupancy cost measure used in the earlier analysis. For each dwelling unit in the set, it is defined as the sum of the monthly rent and monthly commuting costs. Commuting costs are computed from auto commute times using assumed out of pocket costs and assuming that commute time is valued at the (average) wage rate. This procedure is described somewhat exhaustively in Quigley (1972).
Table 4

Alternative Specifications of Expanded Choice Model for 131 Pittsburgh Households

<table>
<thead>
<tr>
<th>Variable</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structure type</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>common wall</td>
<td>-1.033</td>
<td>-.056</td>
<td>-.418</td>
<td></td>
</tr>
<tr>
<td>(2.10)</td>
<td>(0.17)</td>
<td>(1.14)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>apartment</td>
<td>-0.637</td>
<td>-.653</td>
<td>-.408</td>
<td></td>
</tr>
<tr>
<td>(2.11)</td>
<td>(2.01)</td>
<td>(1.13)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Size</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>number of bedrooms</td>
<td>0.830</td>
<td>.360</td>
<td>.381</td>
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</tr>
<tr>
<td>(4.16)</td>
<td>(2.06)</td>
<td>(1.89)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>number of bathrooms</td>
<td>1.621</td>
<td>1.742</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1.31)</td>
<td>(1.09)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Condition</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>age class</td>
<td>-2.400</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>(3.10)</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>age (years) x 100</td>
<td></td>
<td>-.410</td>
<td>.257</td>
<td></td>
</tr>
<tr>
<td>(4.93)</td>
<td>(0.26)</td>
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<tr>
<td>condition</td>
<td></td>
<td>-.134</td>
<td>-.054</td>
<td></td>
</tr>
<tr>
<td>(1 = bad)</td>
<td>(0.57)</td>
<td>(0.21)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost</td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>occupancy cost (gross price)</td>
<td>-2.631</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2.61)</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>rent (x 10)</td>
<td></td>
<td>-.831</td>
<td>-.995</td>
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<tr>
<td>(2.06)</td>
<td>(1.90)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>commute time (hours per month)</td>
<td></td>
<td>-.096</td>
<td>-.106</td>
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</tr>
<tr>
<td>(8.31)</td>
<td>(7.75)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Neighborhood - Town</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>home ownership rate</td>
<td>2.379</td>
<td></td>
<td>3.505</td>
<td></td>
</tr>
<tr>
<td>(2.63)</td>
<td></td>
<td>(1.53)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>proportion black</td>
<td>-2.742</td>
<td></td>
<td>-9.688</td>
<td></td>
</tr>
<tr>
<td>(2.10)</td>
<td></td>
<td>(2.51)</td>
<td></td>
<td></td>
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<tr>
<td>school expenditure/student</td>
<td>.170</td>
<td></td>
<td>-.803</td>
<td></td>
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<tr>
<td>(0.94)</td>
<td></td>
<td>(1.10)</td>
<td></td>
<td></td>
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<tr>
<td>proportion black in elementary school</td>
<td></td>
<td>-.029</td>
<td>-.035</td>
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<tr>
<td>(2.74)</td>
<td></td>
<td>(2.36)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(1 - \sigma)                             |       |       | 1.034 |       |
| (3.67)                                  |       |       |       |       |

-2 \log (L/L_0)                           | 86.2  | 166.1 | 208.0 | 121.7 |

Notes:
  a. asymptotic t ratios in parentheses
Despite the differences in the sample of households and in the choice set, the coefficients are qualitatively similar to those reported in Table 2 for the same group of households.

Column 2 reports the results when the model is expanded to exploit the fact that it has been estimated from a sample of dwelling units, rather than from an exhaustive population of housing "types."

The choice model is expanded to include the age and condition of the dwelling and the number of bathrooms. Column 3 expands the model to include measures of the characteristics of neighborhoods and the public services consumed jointly with housing attributes. When the analysis is conducted using a sample of actual dwellings as the alternatives, a major advantage is immediately apparent: it is now possible to include separately the monthly rent commanded by each dwelling and the accessibility of each dwelling to the workplace of specific consumers. When the occupancy cost measure is replaced by monthly rent and the accessibility (auto commute time) of each dwelling to the worksites of consumers, both coefficients are significantly different from zero. Isolating the separate effects of rental expenditures and commuting times on utility levels permits a direct analysis of the bid rents of consumers for housing attributes and permits an investigation of the tradeoff households make between the journey to work and the out of pocket costs of housing. For example, from column 2, households are willing to bid .0831 in additional rent to save .096 hours in monthly commuting times. The value of travel time estimated from the residential location decisions of consumers is thus $1.16 per hour, on average, or about 39 percent of the hourly wage (i.e., $6000 annual income, 2000 hours of work). A similar comparison based upon the expanded model in column 3 yields a value of travel time equal to roughly 36 percent of
the (average) pre-tax wage rate. These estimates of the value of commuting time are rather similar to those estimated in studies which have considered the implicit value of travel time reflected in the modal choice decisions of commuters.

From column 2, the results indicate that this class of consumers is willing to bid $4.33 per month (or about 7 percent of their average rent) for an additional bedroom; they are willing to pay $7.86 more to enjoy the same housing services in a single detached rental unit rather than a higher density apartment. From column 3, the comparisons are $3.82 and $4.10 respectively. When the model is expanded to include the neighborhood and public services jointly consumed with housing services, the home ownership rate and the racial composition of the neighborhood are both significant. Interestingly, the variable measuring the age of the dwelling unit, highly significant in the models which do not include measures of the neighborhood environment, is totally insignificant. Apparently, given the historical pattern of development in the metropolitan area, the age of the dwelling itself is a good proxy for the condition of the neighborhood; given the characteristics of the neighborhood, however, the age of the structure itself is not associated with better or more desirable dwellings.

The fourth column of Table 4 tests the IIA hypothesis in one particular form. For 116 out of the 131 renter households, the choice based sample selected by the binomial process included at least one dwelling in the central city and one in the suburban housing market. Thus we can analyze the marginal choice of central city or suburban neighborhood and the conditional choice of housing attributes, given central city or suburban neighborhood. In this analysis, the inclusive value is estimated from
the conditional choice model reported in column 2.\(^6\)
The point estimate of \((1 - \sigma)\) is 1.034, and the t-ratio for the hypothesis that \(\sigma\) is significantly different from zero is 0.12. Thus the independence of irrelevant alternatives hypothesis cannot be rejected for the choice of central city or suburban location and the marginal choice of housing attributes. The characteristics of dwelling units do affect the choice of central city or suburban location.\(^7\)

V. Conclusion

This paper presents some crude and preliminary estimates of a class of housing choice models which is considerably more general statistically than those previously estimated. More important, however, the economic model is considerably more faithful to the appealing theoretical notions: that households choose a single dwelling unit out of a large number of alternatives, described by housing, neighborhood, public service characteristics and prices; and that, in considering the choice of particular dwelling unit characteristics, neighborhood characteristics, and public service packages, there may be a greater or lesser degree of independence among some of these components but that the independence of irrelevant alternatives is an extreme case.

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6. For each of the 116 households whose choice based sample included both central city and suburban dwellings, the inclusive value for the central city (suburbs) is computed as \(\log \frac{N \sum \alpha_i}{m} \) where \(m\) is the number of elements in the assigned choice set in the central city (suburbs) and \(N\) is the number of rental units in the central city (suburbs).

7. Stated another way, the estimate of \(\sigma\) suggests that dwelling units are not viewed as identical within the central city or suburbs by consumers
Of particular interest to urban economists is the application of such models to analyzing the housing price-transportation cost tradeoffs made by households and their bid rents for the attributes of housing services. On theoretical grounds, these issues are central to understanding residential location and urban spatial structure.

7. (continued)

in their choice of residential location. A more useful test of this hypothesis would be: the choice of dwellings given neighborhood (census tract); the choice of neighborhood given town; and the marginal choice of town.
References


10. John M. Quigley, "Residential Location with Multiple Workplaces and a Heterogeneous Housing Stock," Harvard University Program on Regional and Urban Economics, Discussion Paper No. 80, September 1972.


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