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From strings to the MSSM

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Abstract We review recent progress in embedding the supersymmetric standard model into string theory. We discuss how, with the incorporation of certain aspects of grand unification, a search strategy can be developed that allows one to efficiently find rather large numbers of promising string vacua. Global string-derived models with the following features are discussed: (i) exact MSSM spectrum below the unification scale; (ii) $R$-parity; (iii) hierarchical Yukawa couplings with non-trivial mixing; (iv) solution to the $\mu$ problem; (v) see-saw suppressed neutrino masses.

1 Introduction

Attempts to relate superstring theory to models of elementary particle physics do not yet provide us with a clear solution. While string theory contains everything which is needed to describe the real world, i.e. gravity, gauge interactions and chiral matter, the ‘details’ appear not quite to fit observation: string theory predicts ten rather than four space-time dimensions, $N=4$ or $N=8$ instead of $N=0$ (or $N=1$) supersymmetry and gauge groups as large as $E_8 \times E_8$ or $SO(32)$ instead of the standard model (SM) gauge group,

$$G_{\text{SM}} = SU(3)_C \times SU(2)_L \times U(1)_Y.$$  \hspace{1cm} (1)

It is generally expected that these discrepancies can be eliminated by compactifying six spatial dimensions on suitable manifolds. Different shapes of compact space will lead to different phenomenologies. The standard model fields will then be interpreted as (almost) massless vibration modes of internal space. One might think that one would just have to ‘dial’ the right compact space to obtain the standard model. Unfortunately this is not the case: since string theory is rather restrictive, even when one is allowed to choose the internal manifold at will it is non-trivial to obtain something that looks like the standard model at low energies. Stringy consistency conditions often enforce the presence of unwanted states in the low-energy spectrum and/or lead to unrealistic couplings. How can we find realistic string compactifications then? A blind scan is hopeless since string theory equations have many solutions, i.e. there is a huge number of 4D vacua (the so-called string theory landscape), which cannot be analyzed systematically. To obtain predictive string models one therefore has to develop a suitable strategy.

In this paper we explain how low-energy observations allow us to draw some conclusions on the generic properties of compact space. We know that the standard model gauge group is unbroken above the electroweak scale and that most of the Yukawa couplings are hierarchically small. We shall argue that these facts allow us to conclude that we live close to a special (‘symmetry-enhanced’) point in moduli space. Furthermore, the successful aspects of grand unified theories (GUTs) indicate that $SO(10)$ structures represent an important ingredient of the stringy completion of the SM. From these considerations we infer that the heterotic string \cite{1,2} compactified on an orbifold \cite{3–12} can be a good starting point for the task of embedding the SM into string theory. First, the orbifold point denotes a symmetry-enhanced point in moduli space (away from it the gauge symmetry gets reduced). Second, the gauge group $E_8$, which is specific to the heterotic string, contains an $SO(10)$ subgroup as well as the 16-dimensional $SO(10)$ spinor in the coset $E_8/ SO(10)$, which unifies one family of quarks and leptons. Orbifold compactifications have a long history (see \cite{13,14} for earlier reviews); the focus of this paper is on reviewing recent developments which allow one, by incorporating certain aspects of grand unification, to construct phenomenologically attractive string models. (For reviews of alternative
approaches leading to interesting models see e.g. [15–17].) In the emerging scheme, low-energy supersymmetry plays a key role.

2 Supersymmetry and grand unified structures

As is well known, the concept of grand unification allows us to simplify the observed matter content of the standard model. One generation of standard model matter, transforming as
generation = (3, 2)_{1/6} + (3, 1)_{-2/3} + (3, 1)_{1/3}
 + (1, 2)_{-1/2} + (1, 1)_{1/2}
under $G_{\text{SM}}$, can be combined into two SU(5) representations [18],

$10 = (3, 2)_{1/6} \oplus (3, 1)_{-2/3} \oplus (1, 1)_{1/3}$, \hspace{1cm} (3a)

$\bar{5} = (\bar{3}, 1)_{1/3} \oplus (1, 2)_{-1/2}$, \hspace{1cm} (3b)

These two irreducible representations, together with the right-handed neutrino, form a 16-dimensional spinor representation of SO(10) (16-plet) [19, 20], i.e.

$16 = 10 \oplus \bar{5} \oplus 1$

$= (3, 2)_{1/6} \oplus (3, 1)_{-2/3} \oplus (3, 1)_{1/3}
 + (1, 2)_{-1/2} \oplus (1, 1)_{1/2} \oplus (1, 1)_{0}$. \hspace{1cm} (4)

These facts represent some of the most compelling evidence for grand unification.

The paradigm of grand unified theories (GUTs) gets supported by the observation that gauge couplings unify in the minimal supersymmetric extension of the standard model (MSSM) [21] at the GUT scale $M_{\text{GUT}} = \text{few} \times 10^{16}$ GeV. This scale seems to play a role also in neutrino physics, where the see-saw [22] appears to be the most plausible explanation for the smallness of neutrino masses. That is, (weak scale)$^2 / M_{\text{GUT}}$ gives roughly the right scale for the observed mass squared differences. Together with other indirect evidence for supersymmetry, such as a compelling particle physics candidate for the observed cold dark matter and experimental hints for a light Higgs boson, this leads to the following popular picture for physics beyond the standard model: above a scale of the order TeV, the world becomes supersymmetric. The particle content of the supersymmetric standard model yields an adequate description up to $M_{\text{GUT}}$, where gauge couplings meet. At this scale, new physics appears.

In the scheme of traditional grand unification, this would be a gauge theory based on a unified (GUT) group. These so-called SUSY GUTs are very popular, mostly because of the following nice properties:

- Unified multiplets, in particular spinors of SO(10)
- Gauge coupling unification
- GUTs yield a reasonable prediction for the see-saw scale
- Yukawa unification, i.e. the $\tau$ and $b$ masses enjoy approximate unification at the GUT scale, and for suitable $\tan \beta$ also the top mass may be unified

Arguably, it is hard to believe that these relations are just accidents. However, the scheme of 4D grand unification exhibits certain problematic features:

- The doublet–triplet splitting problem: in a four-dimensional GUT theory, the particle content has to respect the GUT symmetry. This applies, in particular, to the Higgs fields. However, the smallest SO(10) representation containing the MSSM Higgs doublets (or the SM Higgs) is the 10-plet, which decomposes as

$10 = (1, 2)_{1/2} \oplus (1, 2)_{-1/2} \oplus (3, 1)_{-1/3} \ominus (3, 1)_{1/3}$

under SO(10) $\rightarrow G_{\text{SM}}$. That is, requiring the existence of Higgs doublets $(1, 2)_{\pm 1/2}$ leads necessarily also to color triplets. However, there exist rather uncomfortable lower bounds on the mass of these triplets; for instance, in the context of SO(10) SUSY GUTs, a lower bound of about the Planck scale has been reported [23]. Although this problem may be solved [24–26], the complexity of the known solutions casts some shadow on the scheme of 4D grand unification.

- While third generation fermion masses seem to comply with grand unification [27], the GUT fermion mass relations are challenged by observation. (There is also a tension between precision unification of the third family masses and FCNC constraints [28].)

- Breaking of the GUT symmetry requires Higgs fields in large representations. It is hard, if not impossible, to get these out of string theory, which is the most promising candidate for the description of all forces [29].

These are perhaps the greatest problems of the traditional scheme of grand unification. This raises the question whether one may modify the scheme in such a way that (1–4) are retained while (1–3) are avoided.

The answer to this question is affirmative: extra dimensions (with size of the order of $M_{\text{GUT}}$) allow one to solve these problems. In fact, it has been pointed out in the context of string theory that such schemes allow for GUT symmetry breaking without the need for large representations and for successful doublet triplet splitting [6, 30, 31]. Simplified versions of the stringy mechanism have been discussed in the context of what is known as ‘orbifold GUTs’ [32–39] (for a review, see e.g. [40]). From these one gains a geometric intuition on certain lower-dimensional building blocks which facilitate the construction of promising models exhibiting the appealing features of GUTs while avoiding most of their problems (for a recent review pursuing a
bottom–up approach going from the SM via GUTs and orbifold GUTs to strings see [41]). In what follows, we proceed as follows: we first review the heterotic string compactified on orbifolds, introducing the concepts of ‘gauge group topographies’ and ‘local GUTs’. We shall show how they can be used to define a search strategy for obtaining promising models. Finally, we comment on how orbifold GUTs can be derived from string orbifolds.

3 Orbifold compactifications

Having discussed the virtues and problems of grand unification, we continue by pursuing a more ‘top–down’ approach. We aim at embedding the standard model into string theory. In our task, we take the positive features of SUSY GUTs (1–4) seriously and therefore seek string models where they emerge. Grand unified structures seem to require the SO(10) gauge group as well as 16-plets. Among the known perturbative string theories, only the heterotic string does possess these ingredients. Hence, we shall concentrate on compactifications of the heterotic string. We also saw that (an approximate) $N = 1$ supersymmetry is crucial. The requirement of $N = 1$ supersymmetry restricts the compact space: it has to be such that the holonomy group fits into an SU(3). In the case of a smooth compactification manifold this means that the 6D space has to be of the Calabi–Yau type [42]. But string theory is also well defined and regular on non-smooth compactifications, in particular on orbifolds [3, 4], which turn out to be easier to deal with. So our strategy will be to first focus on heterotic orbifolds; later, in Sect. 6, we shall comment on the relation between these constructions and Calabi–Yau compactifications.

Rather than discussing orbifolds in general, we shall focus on a specific example—the $\mathbb{Z}_6 - \mathbb{Z}_3 \times \mathbb{Z}_2$ orbifold. Here, in a first step, six dimensions get compactified on a torus $\mathbb{T}^6$ that enjoys a $\mathbb{Z}_3 \times \mathbb{Z}_2$ discrete symmetry. One example for such a torus is

$$\mathbb{T}^6 = \mathbb{T}_{G_2}^2 \times \mathbb{T}_{SU(3)}^2 \times \mathbb{T}_{SO(4)}^2.$$  

The subscripts represent the Lie algebra fixing the geometrical relations of the lattice vectors defining the corresponding torus. For instance, $\mathbb{T}^2_{G_2}$ emerges as $\mathbb{R}^2 / \mathbb{Z}_{G_2}$, that is, two points of the two-dimensional plane are identified if they differ by an integer linear combination of lattice vectors whereby the basis vectors enjoy the same geometrical relations as the roots of $G_2$, i.e. enclose 150° and have a length ratio $1/\sqrt{3}$ (cf. Fig. 1).

The orbifold emerges by modding out the discrete symmetry of the torus,

$$\text{orbifold} = \mathbb{T}^6 / \mathbb{Z}_6,$$

where the $\mathbb{Z}_6$ operation $\theta$ acts as a simultaneous rotation by 60°, 120° and 180° on the $G_2$, SU(3) and SO(4) tori, respectively. This action is not free, i.e. there appear fixed points, which get mapped onto themselves under $\theta$. It is this orbifold action that breaks $N = 4$ supersymmetry in 4D, which one would obtain from a torus compactification, down to $N = 1$.

Apart from the geometrical properties discussed above, to build an orbifold model one has to specify the gauge embedding. That is, the geometrical operation $\theta$ is to be associated to an operation in the $E_8 \times E_8$ gauge degrees of freedom. In the bosonic formulation we are using here, this action is encoded in a 16-dimensional vector $V$, called the shift vector. This breaks the gauge symmetry from $E_8 \times E_8$ to a subgroup, as we shall discuss shortly.

In the heterotic string compactified on an orbifold, there are two types of closed strings. Firstly, there are ordinary closed strings which are free to move in the ten-dimensional bulk, called untwisted strings. In addition, there appear new closed strings, which only close after the action of $\mathbb{Z}_6$ and are attached to the fixed points. These strings are called twisted strings. Hence, we distinguish between different sectors, according to the rotation $\theta^k$ necessary to close the string. Untwisted strings ($k = 0$) comprise the untwisted sectors, denoted by $U_i$, where $i = 1, 2, 3$ refers to the three two-dimensional planes of the compact space. Twisted strings ($k = 1, \ldots, 5$) give rise to the twisted sectors $T_i$.

3.1 Simple ways of envisaging orbifolds

Many important aspects of 6D orbifolds can be understood from considering lower-dimensional versions. We start with $\mathbb{T}^2 / \mathbb{Z}_2$, which emerges by dividing the torus $\mathbb{T}^2$ by a point reflection symmetry. The torus is defined by two (linearly independent) lattice vectors $e_1$ and $e_2$ spanning the fundamental domain. The $\mathbb{Z}_2$ then acts as a reflection (or, equivalently, 180° rotation) about an arbitrary lattice node which one could call the ‘origin’. Certain points are mapped under the orbifold action onto themselves (up to lattice translations); these are the orbifold fixed points. After identifying points in the fundamental domain of the torus that are related by the $\mathbb{Z}_2$ orbifold action, one arrives at the fundamental domain of the orbifold. By folding the fundamental domain along the line connecting the upper two fixed points and gluing the adjacent edges together, one arrives at a ravioli- [13] or pillow- [43] like object which is flat everywhere except for the corners, i.e. the orbifold fixed points, at which curvature and gauge field strength terms are localized.

---

1Note that if the holonomy group fits additionally into an $\text{SU}(2) \subset \text{SU}(3)$, one retains at least $N = 2$ supersymmetry.
3.2 Gauge group topography

Gauge theories in higher dimensions can exhibit a feature which we would like to refer to as ‘gauge group topography’. That is, different gauge groups can be realized at different points in internal space. The prime example is orbifolds with non-trivial gauge embedding and discrete Wilson lines [5]. Here, the 10D gauge group $E_8 \times E_8$ gets broken to different subgroups at different orbifold fixed points. Figure 3 illustrates this situation in a two-dimensional $Z_2$ orbifold. In non-prime orbifolds the situation gets slightly richer in that there appear fixed planes which are endowed with different gauge groups. This has lead to the notion ‘heterotic brane world’ [45].

In this review we refrain from giving a detailed, technical description on how this works. All we need for the subsequent discussion is to understand how the local gauge groups emerge. They are comprised out of the gauge bosons corresponding to generators that fulfill certain (local) invariance conditions, which can be recast schematically as

$$\text{generator} \cdot \text{local shift} = 0 \mod 1.$$  \hspace{1cm} (8)

The local shift [46, 47] introduced here depends on the fixed point [5]. Since the invariance conditions can be different at different fixed points, different gauge groups can live at the various fixed points. For example, at the fixed points of the first twisted sector $T_1$, one finds

$$V_{\text{local}} = V + \text{Wilson lines} \leftrightarrow G_{\text{local}},$$  \hspace{1cm} (9)

where the ‘Wilson lines’ term depends on the specific fixed point. This is illustrated in Fig. 4. In the field-theoretic description, one would say that the local gauge groups are made up from the generators for which the corresponding gauge boson has a non-vanishing profile at the fixed
point. This also means that the coset, \((E_8 \times E_8)/G_{\text{local}}\) with \(G_{\text{local}}\) denoting the local gauge group, comprises generators where the gauge bosons’ profile has to vanish at the fixed point. From this point of view it is also clear what the massless gauge interactions in 4D are: they are mediated by the bosons with flat profile, i.e. which fulfill the invariance conditions \((8)\) everywhere. In other words, the 4D gauge group emerges as the intersection of local gauge groups in \(E_8 \times E_8\).

A simple way to think of this is by noting that gauge bosons are bulk fields, and therefore feel what is going on at all positions in internal space. This is different for localized fields, i.e. twisted matter, as we shall discuss shortly in Sect. 3.3.

As already said, we aim at deriving the standard model from heterotic orbifolds. Since the 4D gauge group is the intersection of the local gauge groups at the fixed points, this implies that, when the gauge group topography is non-trivial, the local gauge groups at the fixed points have to be larger than the standard model, \(G_{\text{local}} \supset G_{\text{SM}}\). This then leads to the picture of ‘local grand unification’ \([48]\).

### 3.3 Local grand unification

From the requirement \(G_{\text{local}} \supset G_{\text{SM}}\) we see immediately that the local gauge groups \(G_{\text{local}}\) can be groups that have been discussed in the context of grand unification, such as the Pati–Salam group \(G_{\text{PS}} = SU(4) \times SU(2)_L \times SU(2)_R\), SU(5), and SO(10). Recalling that fields confined to a region with certain gauge symmetry have to furnish complete representations of this symmetry, we see that matter fields localized in a region (or at a fixed point) with GUT symmetry appear as GUT multiplets in the low-energy theory although the gauge symmetry of the low-energy theory will be smaller. Roughly speaking, brane fields only feel what is going on at where they live and do not care about what is going on elsewhere. This statement applies, in particular, to matter living in regions with SO(10) symmetry: it will furnish complete SO(10) representations, i.e. a 16-plet will give rise to a complete family of quarks and leptons. In other words, incomplete families cannot appear at these points nor do parts of these families get projected out.

On the other hand, matter fields can also come from the bulk, i.e. the untwisted sector. Such fields do, like the gauge bosons, feel the symmetry breaking at every fixed point, and thus appear in split multiplets of the various local gauge groups. Together with what we have discussed in the previous paragraph, this provides a very simple scheme allowing us to understand the simultaneous existence of complete and split GUT multiplets in Nature.

Based on these observations, we shall below develop a strategy to construct particle physics models in string theory, i.e. models with three chiral generations plus one pair of Higgs bosons below the compactification scale. The most obvious possibility is to arrange things in such a way that there are three 16-plets living in regions with SO(10) symmetry plus a pair of Higgs multiplets coming from the bulk. That is, we are going to look at models in which there are orbifold fixed points at which \(E_8\) gets broken to SO(10) and which give rise to localized 16-plets. The simplest (and therefore best studied) setting, the \(Z_3\) orbifold, fails to satisfy this criterion \([49]\). As we shall see later in Sect. 4, the situation improves when one turns to more complicated settings, such as the \(Z_6\) orbifold. Before presenting this, we need to discuss certain aspects of the structure of orbifold vacua.

### 3.4 Orbifold vacua

As we have seen, an orbifold is defined by its geometry and its gauge embedding. That is, given the geometry, shift and Wilson lines, the orbifold model is, in a certain way, fixed: the massless spectrum as well as the couplings between the various states are determined and calculable. However, as we shall explain now, this does not fix the phenomenological properties of the model completely. Rather, a given orbifold admits many vacua. The orbifold point, i.e. the point in field space where all massless charged fields have zero vacuum expectation values turns out not to correspond to a vacuum in most of the models with non-standard embedding. Generically one of the \(U(1)\) factors appears anomalous. This means that, at one loop, a Fayet–Iliopoulos (FI) \(D\)-term is induced \([50]\). Hence the orbifold point can be thought of as a saddle point in field space. To cancel the FI term, certain fields with charge opposite to the trace of the ‘anomalous’ \(U(1)\) have to attain vacuum expectation values. In supersymmetric vacua, these vacuum expectation values need to be consistent with zero \(F\)- and (other) \(D\)-terms. The vacuum space of orbifolds can be analyzed field-theoretically. It is well known that a supersymmetric gauge theory with generic superpotential has supersymmetric vacua with \(F_l = D_a = 0\) \([51]\) (otherwise it would not be so hard to build field-theoretic examples which spontaneously break supersymmetry). To see this, recall that solutions to the \(F\)-equations still admit complex-
4.3 Different types of supersymmetric vacua of an orbifold model

In conclusion, a given orbifold model has several branches of vacua with different properties (Fig. 5). The orbifold point is a point in field space around which the theory can be expanded, but does in general not correspond to a vacuum configuration. As long as the field VEVs are not too large, one might arguably retain control over the theory. Potential obstacles have been identified (see, e.g., [54]), and more work in this direction will be needed. But it is also clear that certain features, such as the chiral spectrum, will not be affected by moving in moduli space.

Before turning to the search for realistic vacua in orbifold models, let us remark that to figure out how many vacua of a given dimension exist in an orbifold model would represent an important piece of information in the context of the landscape discussion [55]. We shall not attempt to perform this analysis here, but for the subsequent discussion it is important to keep in mind that counting orbifold models should not be confused with counting string vacua; the actual number of orbifold vacua is certainly larger than the number of models by many orders of magnitude. We shall also discuss later in some detail (Sect. 6) that one cannot strictly distinguish between orbifold vacua and Calabi–Yau vacua: giving vacuum expectation values to certain (twisted) fields corresponds to blowing up the orbifold, i.e. smoothening out the singularities associated to the orbifold fixed points.

3.5 Properties of the effective field theories derived from orbifolds

Given the field content of the orbifold, the next step is to study interactions of the theory. Couplings on orbifolds are governed by certain selection rules [44, 56]. From the effective field theory perspective, these rules are

- Gauge invariance
- (Non-Abelian) discrete symmetries, and
- Discrete $R$ symmetries

The selection rules for the $Z_6$-II orbifold are summarized in [47] (a careful discussion of the space-group rule is pre-
sented in [57]). They govern the couplings at the orbifold point.

If one, as discussed above, moves away from the orbifold point in moduli space, one obtains further, effective couplings, induced by the fields that attain VEVs, which we call \( s_i \) in what follows. The emerging picture is very similar to the Froggatt–Nielsen scheme [58], i.e. one has different couplings arising at different orders in the \( s_i \) fields. As long as one stays close to the orbifold point, the VEVs \( (s_i) \) are small in string units, and the effective couplings exhibit a hierarchical structure. This is perhaps the most plausible way of generating hierarchical (Yukawa) couplings: there are strong arguments by ‘t Hooft that hierarchically small parameters \( y \) are only natural if in the limit where \( y \rightarrow 0 \) a new symmetry appears [59]. Orbifolds have this property: the orbifold point has many symmetries some of which get broken by the \( (s_i) \) VEVs, and get restored as \( (s_i) \rightarrow 0 \). The fact that there is a huge literature on rather compelling models in which realistic Yukawa couplings are explained in this way (cf. e.g. [60, 61]) gives further credit to this picture. A specific example of an orbifold that leads to hierarchical couplings will be discussed in Sect. 4.4. In conclusion, from the fact that the Yukawa couplings observed in Nature exhibit (strong) hierarchies one might conclude that we live somewhere not too far from an orbifold point in field space.

The main lesson is that, once the geometry and gauge embedding are fixed, the model at the orbifold point is completely determined. In particular, unlike in the field-theoretic constructions, the spectrum is fixed by the compactification and one cannot ‘put by hand’ representations and/or couplings. For configurations close to the orbifold point, one can use perturbation theory in order to calculate the effective couplings.

4 Approaching the MSSM

In the previous sections we have seen that orbifold compactifications provide us with everything needed for deriving the standard model from string theory: gauge groups smaller than \( E_8 \times E_8 \) and chiral matter. We have also explained that, by exploiting the gauge group topography of orbifolds, one can set up models of ‘local grand unification’ which can explain the appearance of larger representations while the gauge group is broken to smaller pieces. A crucial ingredient in this scheme is \( 16 \)-plets localized at fixed points with \( SO(10) \) symmetry.

In the \( Z_6\)-II orbifold, there are two (local) shifts that produce a local \( SO(10) \) GUTs with local \( 16 \)-plets [62],

\[
V^{SO(10),1} = \left( \begin{array}{c}
\frac{1}{5} \\
\frac{1}{5} \\
0, 0, 0, 0, 0
\end{array} \right),
V^{SO(10),2} = \left( \begin{array}{c}
\frac{1}{5} \\
\frac{1}{5} \\
0, 0, 0, 0, 0
\end{array} \right).
\]

(11)

Our strategy will be to construct models based on the \( SO(10) \) shifts and see to which extent they can be consistent with particle physics. More precisely, we shall conduct a search for models with vacua which can give rise to the MSSM at energies below \( M_{GUT} \) within this set of orbifolds.

4.1 String derived orbifold GUTs

The shift \( V^{SO(10),2} \) has been used in the first string-derived orbifold GUT models [63, 64]. There, various models with Pati–Salam symmetry \( G_{PS} \) at the orbifold point were constructed. These models exhibit three chiral generations as well as the Higgs fields, required to break \( G_{PS} \rightarrow G_{SM} \), and the electroweak Higgs doublets. There are also further interesting features such as order one Yukawa couplings for the third generation and a \( D_4 \) flavor symmetry for the two light generations. It has also been attempted to use the shift \( V^{SO(10),2} \) in a model with standard model gauge symmetry where three generations arise from three \( 16 \)-plets localized at \( SO(10) \) fixed points [65]. However, all of the models of [63–65] have a common problem: the appearance of exotic states at low energies.

In fact, we have seen that, in order to get \( E_6 \) broken to the standard model, we need fixed points with a symmetry that is not \( SO(10) \supset G_{SM} \). These fixed points will carry twisted states, giving rise to non-SM matter. The only situation in which these exotics can be harmless is when they are vector-like with respect to \( G_{SM} \): in this case, certain SM singlet fields might attain VEVs which, provided that suitable couplings exist, lead to mass terms for the exotics. To show that the required couplings and singlets exist is of course not sufficient; one must also verify that the desired singlet VEVs lie at (local) minima of the potential. In the models of [63, 64] it was found that the required couplings do not exist.\(^3\)

In the simple models where all three generations originate from \( 16 \)-plets localized at \( SO(10) \) fixed points, in particular in the one presented in [65], it is found that if one insists on hypercharge being normalized as in GUTs, there are always chiral exotics [47, 66]. This statement applies to \( Z_{N \leq 3} \) orbifolds. One is hence lead to consider schemes in which at least one family does not come from a localized \( 16 \)-plet.

In summary, the first string-derived orbifold GUTs have many interesting and appealing features. The main drawback is the appearance of unwanted states. We would like to stress that constraints from the model to be global (rather than local) has important consequences for the phenomenological viability of string compactifications. If we were to neglect the states from certain fixed points, it would be easy to obtain constructions without unwanted exotics.

\(^3\)By now it has become clear that, with the correct selection rules [57], the couplings do exist. It might therefore be worthwhile to revisit the models.
Recently there has been progress in \( F \)-theory compactifications of string theory \([67-69]\). As of now, only local models have been discussed. It remains to be seen if this approach allows one to build globally consistent and simultaneously phenomenologically viable models. This is an interesting question because these constructions are related to D-brane models where getting the exact MSSM spectrum appears to be notoriously problematic. On the other hand, some constructions in this scheme \([67]\) are dual to heterotic compactifications, where models with the exact MSSM spectrum can indeed be obtained, as we shall now discuss.

4.2 MSSM from the heterotic string

There are \( \mathbb{Z}_6 \)-II models which can indeed exhibit the exact MSSM spectrum below \( M_{\text{GUT}} \); an example has been presented in \([47, 70]\). It is based on \( V^{\text{SO}(10)} \); the Wilson lines are chosen such that

\[
\text{gauge group} = (G_{\text{SM}} \subset \text{SO}(10)) \times \text{extra factors} \quad (12)
\]

and

\[
\text{spectrum} = 3 \times \text{generation}
\]

\[+ \text{vector-like with respect to } G_{\text{SM}} \quad (13)\]

at the orbifold point. It has been demonstrated that the exotic states can be decoupled.

(i) There exist couplings of the form

\[
x_j \bar{x}_k \prod_i s_i, \quad (14)
\]

where \( x_j \) and \( \bar{x}_k \) denote the vector-like exotics and \( s_i \) are standard model singlets.

(ii) It can be shown that the \( s_i \) VEVs are consistent with supersymmetry (as discussed in Sect. 3.4, one essentially has to check that the \( s_i \) VEVs are consistent with vanishing \( D \)-terms).

In this model, two generations stem from localized 16plets, while the third generation comes from states from the untwisted or higher twisted sectors, i.e. fields that live in the bulk or at orbifold fixed planes.

This model has been analyzed in detail in \([47, 71]\), where the advantages of the local GUT representations have been exploited and the (higher-dimensional) anomaly constraints have been checked. We shall refrain from reviewing this discussion here in detail since below we shall discuss a very similar model that shares many important properties with the one considered here.

4.3 The heterotic mini-landscape

The scheme of local grand unification can be utilized in the search for models with realistic features. Given the model discussed above (Sect. 4.2), one might wonder whether there are more models with similar properties. In what follows, we discuss the results of a scan over \( \mathbb{Z}_6 \)-II orbifolds with local SO(10) GUT structures. For each of the SO(10) shifts \((11)\), the following steps were performed:

➀ Generate Wilson lines \( W_3 \) and \( W_2 \).

One of the SO(10) shifts \((11)\) is chosen and all two Wilson line models are constructed using the methods described in \([49]\) and in the appendix of \([72]\). These models can be separated into two cases: either these two Wilson lines are of order two \((W_2 \text{ and } W_2')\) or one Wilson line is of order three \((W_3)\) and one of order two \((W_2)\).

➁ Identify “inequivalent” models.

As is well-known, different looking Wilson lines can still lead to equivalent models because they are related by Weyl reflections and/or lattice translations. However, as the Weyl group is huge, it would be extremely time consuming to probe whether two Wilson lines are equivalent or not. In our computations, we therefore adopt a rather pragmatic notion of equivalence: two models are taken to be “equivalent” if their spectra coincide.\(^5\) This underestimates the true number of models.

➃ Select models with non-anomalous \( G_{\text{SM}} \subset \text{SU}(5) \subset \text{SO}(10) \).

We select models with SU(3) \( \times \) SU(2) that fit into the local SO(10). There will always be an SU(5) subgroup of SO(10) that contains SU(3) \( \times \) SU(2). Since (Abelian) \( \mathbb{Z}_N \) orbifolds do not reduce the rank, all \( U(1) \) factors survive orbifolding, and hence the selected models will have \( G_{\text{SM}} \subset \text{SU}(5) \subset \text{SO}(10) \).

➄ Select models with three net \((3, 2)\).

Having identified \( SU(3)_C \times SU(2)_L \subset G_{\text{SM}} \) in the previous step, we project now on models that have a net number of three \((3, 2)\) (quark doublets). Here, a net number means that we also allow for situations where the spectrum contains four \((3, 2)\) plus one \((\bar{3}, 2)\), for example. At this level, this amounts to requiring three families.

➅ Select models with non-anomalous \( U(1)_Y \subset SU(5) \).

This ensures that the hypercharge chosen previously in step 4.3 is non-anomalous. Technically, this is achieved by demanding that the respective generators are orthogonal, i.e. \( ty \cdot t_{\text{anom}} = 0 \). A non-anomalous hypercharge is

\(^4\) A comment for experts is in order: the \( \mathbb{Z}_6 \)-II orbifold does not allow for (generalized) discrete torsion \([73, 74]\) (cf. \([75]\)). Hence the ansatz of \([49, 72]\) is sufficient to construct all different models. It is, however, clear that, once one changes the geometry, for instance, once one goes to non-factorizable lattices \([76-78]\), new models will arise.

\(^5\) In practice, we compare non-Abelian representations and the number of non-Abelian singlets.
necessary, because an anomalous one would be broken at the high scale due to the presence of the FI $D$-term (cf. Sect. 3.4), resulting in electroweak symmetry breaking at a high-energy scale.

⑥ Select models with net three SM families + Higgses + vector-like.

In the last step models are selected which have the chiral matter content of the MSSM, i.e. three generations of quarks and leptons. Additionally, the models are allowed to have vector-like exotics. In order for some exotics to be vector-like they either have to form real representations or they have to come in pairs of some representations plus their complex conjugates. Then, it is in principle possible to write down a mass term for these exotics with a very high mass such that the exotics decouple from the low-energy effective theory. Note, however, that the couplings in the superpotential relevant for the mass terms cannot be put in by hand, but they have to be derived from string theory, as discussed in Sect. 3.5.

It turns out that in these models almost 1% has the MSSM spectrum plus vector-like exotics (Table 1).

The main conclusion that one can draw from these statistics is that heterotic orbifolds with local SO(10) structures are a particularly "fertile" scheme for producing models that are close to the MSSM. (The question "how close?" will be studied in the next subsection.) To see this, let us compare our statistics with other MSSM searches in the literature. In certain types of intersecting D-brane models, it was found that the probability of obtaining the SM gauge group and three generations of quarks and leptons, while allowing for chiral exotics, is less than $10^{-9}$ [79, 80]. The criterion which comes closest to the requirements imposed in [79, 80] is ⑥. We find that within our sample the corresponding probability is 6%. In [81, 82], orientifolds of Gepner models were scanned for chiral MSSM matter spectra, and it was found that the fraction of such models is $4 \times 10^{-14}$. These constructions contain the MSSM matter spectrum plus, in general, vector-like exotics. This is most similar to step ⑥ in our analysis where we find 218 models out of a total of $3 \times 10^4$ or 0.7%. Additionally, approximately 0.6% of our models have the MSSM spectrum at low energies with all vector-like exotics decoupling along $D$-flat directions. Note also that in all of our 218 models hypercharge is normalized as in standard GUTs and thus consistent with gauge coupling unification. Let us also remark that all such models are very similar to the one discussed in Sect. 4.2; in particular, they exhibit a $2+1$ family structure, i.e. there are two very similar families coming from localized 16-plets living at SO(10) fixed points plus one very different family scattered over the bulk and fixed planes. Note also that the identification of the third family is not straightforward as there is a mixing with vector-like states carrying SM quantum numbers.

We would like to remark that our GUT-based strategy to determine the hypercharge is, of course, not unique. One could instead express $U(1)_Y$ as an arbitrary linear combination of all $U(1)$’s (not only of those embedded in the local GUT symmetry), such that it gives the correct values of hypercharge to the MSSM particles. This approach was followed in [83]. The authors of [83] find that the majority of the models at step ⑥ allow for a definition of a non-anomalous $U(1)_Y$. However, only in 12% of those models, hypercharge is in harmony with coupling unification. That means, in particular, that even in a more general scheme, relaxing the demand $U(1)_Y \subset SU(5)$, (almost) only those 223 models at step ⑥ of our search meet all the phenomenological properties we require.

Given these models, we study their properties. We analyze the question whether the appearance of MSSM gauge group and spectrum is correlated to other properties of the model. To be specific, we study the properties of the so-called hidden sector.

It is clear that the pure MSSM does not represent a complete setup; one has to amend it by a sector that is responsible for supersymmetry breakdown, which is usually called the ‘hidden sector’ [84]. This is because settings in which supersymmetry is broken by auxiliary VEVs of MSSM fields are challenged by observation, if not ruled out. Arguably, the most appealing types of hidden sectors are those in which the scale of supersymmetry breakdown is explained by dimensional transmutation, as in the scheme of gaugino condensation [85–88]. The gravitino mass, setting the scale of MSSM soft supersymmetry breaking terms, is

$$m_{3/2} \sim \frac{\Lambda^3}{M_{Pl}^2},$$

(15)

where the gaugino condensation scale $\Lambda \equiv \langle \lambda\lambda \rangle^{1/3}$ is given by the renormalization group (RG) invariant scale of the condensing gauge group,

$$\Lambda \sim M_{GUT} \exp \left( -\frac{1}{2\beta} \frac{1}{g^2(M_{GUT})} \right).$$

(16)

Here $\beta$ denotes the $\beta$-function coefficient, which depends on the gauge group and the matter content.

<table>
<thead>
<tr>
<th>Criterion</th>
<th>$\psi_{SO(10),1}$</th>
<th>$\psi_{SO(10),2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>② Inequivalent models with 2 Wilson lines</td>
<td>22,000</td>
<td>7,800</td>
</tr>
<tr>
<td>③ SM gauge group $\subset SU(5) \subset SO(10)$</td>
<td>3,563</td>
<td>1,163</td>
</tr>
<tr>
<td>④ 3 net (3, 2)</td>
<td>1,170</td>
<td>492</td>
</tr>
<tr>
<td>⑤ Non-anomalous $U(1)_Y \subset SU(5)$</td>
<td>528</td>
<td>234</td>
</tr>
<tr>
<td>⑥ Spectrum = 3 generations + vector-like</td>
<td>128</td>
<td>90</td>
</tr>
</tbody>
</table>

Table 1 Statistics of $Z_6$-II orbifolds based on the SO(10) shifts (11) with two Wilson lines
In string theory, the question of supersymmetry breakdown is usually closely related to the mechanism of moduli stabilization. A condensing gauge group often leads to the dilaton run-away problem [89]. This problem can be avoided in various ways. Here we consider the scheme of Kähler stabilization where the tree level Kähler potential of the dilaton gets amended by a non-perturbative correction, $K = -\ln(S + \bar{S}) + \Delta K_{\text{np}}$. The form of this correction has been studied in [90–92] (for a review see [93]). With a favorable choice of the parameters, the dilaton can be stabilized at a realistic value of $\text{Re} S \approx 2$. In this review, we do not discuss how plausibly the parameters will fall into favorable ranges; we shall just assume that there exists a mechanism that successfully stabilizes the dilaton while breaking supersymmetry,

$$F_S \sim \frac{\Lambda^3}{M_{\text{Pl}}},$$

which is in agreement with relation (15). Note that in this scheme, as a consequence of $T$-duality invariance of the non-perturbative superpotential terms, the geometric moduli, in particular the Kähler (or volume) moduli get fixed as well [94, 95].

In string models, of course, we cannot ‘invent’ hidden sectors, but we have to live with what strings give us. The non-Abelian subgroups of the second $E_8$ factors in the 218 models with chiral MSSM spectrum (last line in Table 1) were analyzed in [96]. The result is illustrated in Fig. 6. We see that there is a statistical preference for SU($N$) and SO(2$N$) groups with $N$ ranging between 4 and 6. By calculating the corresponding $\beta$-functions, we obtain an estimate on the scale of $\Lambda$ (16) (Fig. 7). This estimate is very rough; it neglects, in particular, string threshold corrections [97–99].

It is, nevertheless, remarkable that the distribution shows a clear statistical preference for intermediate scale supersymmetry breaking. This might be interpreted as a top–down motivation for low-energy supersymmetry [96].

4.4 How good are the mini-landscape models?

Having obtained about 200 models with exact MSSM spectra, we now discuss their phenomenological properties. There are two kinds of questions that we shall address.

1. Can we reproduce/accommodate all the features of the MSSM?
2. Does string theory give us even more, i.e. are there stringy mechanisms at work that help to solve some of the MSSM puzzles (such as the $\mu$, strong CP and MSSM fine tuning problems)?

In what follows we shall answer the questions as far as possible. We shall base our discussion on a specific model—‘benchmark’ model 1A presented in [57]—yet the features discussed are shared with many models of the mini-landscape.

As explained before, once the shift and Wilson lines are specified, the spectrum of the model is fixed. The gauge group of the benchmark model is

$$G_{\text{SM}} \times U(1)_{B-L} \times [\text{SU}(4) \times \text{SU}(2)] \times U(1)_{\text{anom}} \times U(1)^6.$$  

The quantum numbers of the massless states with respect to SU(3) × SU(2) × [SU(4) × SU(2)] are shown in Table 2. Here we list also the $B-L$ (with $B$ and $L$ denoting baryon and lepton number, respectively). This $B-L$ symmetry is related, but not identical, to the canonical $B-L$ embedded in the local SO(10). Indeed, the canonical $B-L$ symmetry turns out to be anomalous, i.e. the corresponding generator $t_{B-L}^{\text{SO}(10)}$ is not orthogonal to the generator of the anomalous U(1). It is, however, possible to define $U(1)_{B-L}$ as a linear combination of $t_{B-L}^{\text{SO}(10)}$ with the other $U(1)$ generators in such a way that

- The charges of the members of the 16-plets are standard
• The spectrum is three chiral generations plus vector-like with respect to $G_{SM} \times U(1)_{B-L}$.

Given these properties, it is clear that, just like in usual GUTs, the $B-L$ symmetry can be used to distinguish Higgs from lepton doublets. The fact that we had to redefine $B-L$ implies that the normalization is different from the one in ordinary GUTs, which leads to the appearance of SM singlet fields with $B-L$ charge $q_{B-L} = \pm 2$ (named $\chi$-plets in Table 2). This has important consequences, as we shall discuss next.

As already mentioned, one $U(1)$ factor appears anomalous. As we have discussed in Sect. 3.4, this means that there is a $D$-term that has to be canceled. It has been checked that this can be achieved by giving VEVs to fields $\tilde{\chi}_i$ which transform trivially under $G_{SM} \times SU(4)$ and whose $B-L$ charge is either 0 or $\pm 2$. As explained in Sect. 3.4, the $F$-term equations can be solved simultaneously; hence we have obtained supersymmetric vacua. Because $B-L$ is broken by two units, these vacua exhibit a matter parity symmetry [101] $Z_2^M$, which has the same phenomenological implications as the usual MSSM $R$-parity: dangerous dimension four proton decay operators do not exist and the lightest supersymmetric particle (LSP), which is known to be an excellent dark matter candidate, is stable.

Giving VEVs to the $\tilde{\chi}$ fields has further crucial implications. As we have seen, the spectrum (Table 2) after compactification, i.e. at the orbifold point, contains, as desired, three generations of MSSM matter as well as (unwanted) vector-like exotics. It turns out that due to the effective $D$-term the $\tilde{\chi}_i$ VEVs the exotics attain masses, i.e. there exist couplings of the form

$$\chi_i \tilde{\chi}_j \cdot \tilde{\chi} \text{ fields,}$$

where $\chi_i$ and $\tilde{\chi}_j$ denote vector-like exotics. The terms (19) give rise to mass terms when the $\tilde{\chi}$ acquire VEVs, as discussed before in Sect. 3.5 and below equation (14). It has been verified that the corresponding mass matrices have full rank. Some of the relevant mass terms appear only at order 6 in the $\tilde{\chi}$ fields. As we have discussed earlier, we implicitly assume that the $\tilde{\chi}$ VEVs are not too large (in string units), so that we have some sort of perturbative control. This raises the question whether mass terms arising at order 6 only are far below the GUT scale. This is not necessarily the case: there are combinatorial factors in the superpotential couplings [54] that partially undo the suppression coming from the high powers in singlets. Further, the coefficients of the couplings have to be calculated; recently general expressions for this purpose have been derived [102], but they have not yet been applied in concrete models.

In principle, one would expect that all vector-like states acquire large masses. So one might lose the pair of Higgs doublets, which is also vector-like with respect to $G_{SM}$. As we shall discuss now, this is not the case. This issue is related to the GUT doublet–triplet splitting and the MSSM $\mu$ problems. In our construction the triplets sitting with the Higgs doublets $h_u,d$ in the same multiplet get projected out by the orbifold action. Let us now have a closer look at the effective $h_u h_d$ mass term. It turns out that the pair $h_u h_d$ is neutral with respect to the selection rules, i.e. whenever a coupling $h_u h_d M$, with $M$ denoting a monomial of fields $M = \phi_1 \cdots \phi_n$, satisfies all selection rules, the coupling $M$ by itself represents an allowed coupling. Further, it was found [57] that (at least at order 6) the global SUSY $F$-term equations are satisfied term by term,

$$\frac{\partial M}{\partial \tilde{\chi}_i} = 0,$$

where $M$ denotes a monomial of standard model singlets $\tilde{\chi}_i$ representing a superpotential term consistent with all selec-

### Table 2

<table>
<thead>
<tr>
<th>#</th>
<th>Irrep</th>
<th>Label</th>
<th>#</th>
<th>Irrep</th>
<th>Label</th>
</tr>
</thead>
<tbody>
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<td>3</td>
<td>(3, 2, 1)_{1/6,1/3}</td>
<td>$q_i$</td>
<td>3</td>
<td>(3, 1, 1)_{1/2,3,−1/3}</td>
<td>$\tilde{u}_i$</td>
</tr>
<tr>
<td>3</td>
<td>(1, 1, 1)_{1/1}</td>
<td>$\tilde{e}_i$</td>
<td>8</td>
<td>(1, 2, 1)_{1/0,1}</td>
<td>$m_i$</td>
</tr>
<tr>
<td>4</td>
<td>(3, 1, 1, 1)_{1/3,−1/3}</td>
<td>$\tilde{d}_i$</td>
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<td>(3, 1, 1)_{1/0,1}</td>
<td>$\tilde{d}_i$</td>
</tr>
<tr>
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<td>$\tilde{\ell}_i$</td>
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<td>(1, 2, 1)_{1/2,1}</td>
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</tr>
<tr>
<td>1</td>
<td>(1, 2, 1, 1)_{−1/2,0}</td>
<td>$h_{d}$</td>
<td>1</td>
<td>(1, 2, 1)_{1/2,0}</td>
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<tr>
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<td>$\tilde{\delta}_i$</td>
<td>6</td>
<td>(3, 1, 1)_{1/3,−2/3}</td>
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</tr>
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<td>14</td>
<td>(1, 1, 1)_{1/2,0}</td>
<td>$s_{i}^{−}$</td>
</tr>
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<td>16</td>
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<td>(1, 1, 1)_{0,−1}</td>
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</tr>
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<td>(1, 1, 1, 2)_{0,−1}</td>
<td>$\nu_i$</td>
</tr>
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<td>10</td>
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<td>$h_{i}$</td>
<td>2</td>
<td>(1, 2, 1, 2)_{0,0}</td>
<td>$h_{i}$</td>
</tr>
<tr>
<td>6</td>
<td>(1, 1, 4, 1)_{0,1}</td>
<td>$f_{i}$</td>
<td>6</td>
<td>(1, 1, 4, 1)_{0,1}</td>
<td>$f_{i}$</td>
</tr>
<tr>
<td>2</td>
<td>(1, 4, 1)_{−1/2,−1}</td>
<td>$\tilde{f}_{i}$</td>
<td>2</td>
<td>(1, 1, 4)_{1/2,1}</td>
<td>$\tilde{f}_{i}$</td>
</tr>
<tr>
<td>4</td>
<td>(1, 1, 1)_{0,±2}</td>
<td>$\chi_i$</td>
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<td>$\chi_{i}^{0}$</td>
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<tr>
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<td>$\tilde{\chi}_i$</td>
<td>2</td>
<td>(3, 1, 1)_{1/6,−2/3}</td>
<td>$\chi_{i}$</td>
</tr>
</tbody>
</table>
tion rules, this implies that also $\mathcal{M}$ vanishes. That is, in supersymmetric vacua where the $F$-terms vanish term by term, all potential superpotential terms are zero as well. Hence, at the perturbative level the $h_u h_d$ mass term is zero.

How can one obtain a $\mu$-term of the right size? In the context of orbifold compactifications of the heterotic string, two solutions to the $\mu$ problem have been proposed:

1. The proposal by Casas and Muñoz [103] where $\mu$ originates from superspotential couplings to the hidden sector, and
2. The proposal by Antoniadis et al. [104] where an effective $\mu$-term gets induced from the Kähler potential (for a recent, field-theoretic discussion along these lines see [105])

It turns out that both solutions can be employed in the model under consideration (simultaneously). For the first solution, one needs that the $\mu$-term be absent in supersymmetric vacua and there has to be a relation between the $\mu$-term and the expectation value of $W$,

$$\mu \sim \langle W \rangle.$$  \hspace{1cm} (21)

The effective $\mu$-term is then generated by hidden sector dynamics, as in the field-theoretic models by Kim et al. [106, 107]. Both criteria are met in the model under consideration. For solution 2 to work, the Higgs fields $h_{u,d}$ have to come from the untwisted sector associated to a $\mathbb{Z}_2$ twisted plane, as happens to be the case in the model. Then the Kähler potential has a favorable form, similar to the one discussed in the Giudice–Masiero mechanism [108]. Altogether we see that all prerequisites for a successful solution to the $\mu$ problem that have been discussed in the literature [103, 104] are present in the model. A more detailed analysis of these issues is under way [109].

Let us now turn to the flavor structure of the model. We are interested in the Yukawa couplings,

$$W_{\text{Yukawa}} = Y_u \bar{q}_f \bar{u}_s h_u + Y_d \bar{q}_f \bar{d}_s h_d + Y_e \bar{\ell}_f \bar{e}_s h_d.$$  \hspace{1cm} (22)

At tree level all Yukawa couplings vanish with the exception of the top coupling, which is predicted to (roughly) coincide with the gauge coupling at the high scale. All other Yukawas appear at higher order, i.e. are proportional to different powers of $\tilde{s}$ fields. Truncating at six singlets, one obtains

$$Y_u = \begin{pmatrix} \tilde{s}_5 & \tilde{s}_5 & \tilde{s}_5 \\ \tilde{s}_5 & \tilde{s}_5 & \tilde{s}_6 \\ \tilde{s}_6 & \tilde{s}_6 & 0 \end{pmatrix}, \quad Y_d = \begin{pmatrix} 0 & \tilde{s}_5 & 0 \\ \tilde{s}_5 & 0 & 0 \\ \tilde{s}_6 & 0 & 0 \end{pmatrix},$$

$$Y_e = \begin{pmatrix} 0 & \tilde{s}_5 & \tilde{s}_6 \\ \tilde{s}_5 & 0 & 0 \\ \tilde{s}_6 & 0 & 0 \end{pmatrix}.$$  \hspace{1cm} (23)

As discussed, one should not take these textures literally. The $\tilde{s}$ entries represent polynomials of a total of 46 different fields, which will have hierarchies between their VEVs. Nevertheless it is remarkable that the Yukawas exhibit the qualitatively right features:

- $\mu > \langle W \rangle$ all other Yukawa couplings;
- Hierarchies between the eigenvalues;
- Non-trivial mixings.

The models of the mini-landscape exhibit a non-Abelian discrete flavor symmetry, $D_4$, for the two light generations [64, 110]. This symmetry is only exact at the orbifold point, and broken in realistic vacua. Nevertheless, using the $D_4$ symmetric situation as a starting point and then adding corrections might have certain advantages when discussing the (supersymmetric) flavor structure [111]. The structure of the soft masses is

$$m^2 = \begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & b \end{pmatrix} + \text{terms proportional to } D_4 \text{ breaking VEVs.}$$  \hspace{1cm} (24)

It is known that such an approximate form of the soft masses makes it possible to avoid the supersymmetric flavor problems.

Let us now have a look at neutrino masses in this model. In order to discuss neutrino masses in string-derived models one has first to clarify what a (right-handed) neutrino is. In supersymmetric MSSM vacua with R-parity this question is answered quite easily: a neutrino is an R-parity odd $G_{\text{SM}}$ singlet. In the model discussed so far, there are 49 neutrinos. Further, as discussed more generally in [112], all ingredients of the see-saw [22] are present in this model (already at order 6 in the standard model singlets):

- The right-handed neutrino mass matrix $M_\nu$ has full rank, and
- Neutrino Yukawa couplings $Y_\nu$ exist such that the effective neutrino mass matrix for the light neutrinos,

$$m_\nu = v_u^2 Y_\nu^T M^{-1} Y_\nu,$$  \hspace{1cm} (25)

has full rank. This feature is again not specific to the model under discussion, we find more generally that MSSM vacua of mini-landscape models have (typically, i.e. in all models of the mini-landscape that we have analyzed so far) see-saw suppressed neutrino masses. We also remark that, due to the large number of neutrinos, the effective neutrino mass operator gets many contributions such that neutrino masses are slightly enhanced against the naive estimate $m_\nu^{\text{naive}} \sim v_u^2/M_{\text{GUT}}$. Let us also mention
Comparison between 4D and string-derived higher-dimensional GUTs

<table>
<thead>
<tr>
<th>Framework</th>
<th>4D SO(10) GUTs</th>
<th>‘Local GUTs’</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spectrum</td>
<td>$3 \times 16 + 10 + 210 + \overline{126} + \cdots$</td>
<td>$3 \times$ generation + Higgses + vector-like exotics</td>
</tr>
<tr>
<td>Masses of extra particles</td>
<td>$\sim M_{GUT}$</td>
<td>$\sim M_{\text{string}} \times \left( \frac{\text{singlet VEV}}{M_{\text{string}}} \right)^0$</td>
</tr>
<tr>
<td>Doublet triplet splitting</td>
<td>Cancellation between different vacuum expectation values necessary</td>
<td>Automatic connection $\mu \leftrightarrow (W) = m_{3/2}$</td>
</tr>
<tr>
<td>GUT symmetry breaking</td>
<td>Appropriate Higgs fields need to be introduced</td>
<td>Encoded in geometry &amp; gauge embedding (choice ‘by hand’)</td>
</tr>
<tr>
<td>Gauge coupling unification</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>See-saw</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>GUT relations for masses</td>
<td>✓</td>
<td>✗</td>
</tr>
<tr>
<td>$R$- or matter parity</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Proton stability</td>
<td>Requires additional symmetries</td>
<td>Problem of dimension five operators not yet solved</td>
</tr>
</tbody>
</table>

that meanwhile the many neutrino scenario has been analyzed in some detail. It has been found that many neutrinos somewhat relax the tension between leptogenesis and supersymmetry [113, 114]. Furthermore, the presence of many neutrinos and their Yukawa couplings have important consequences for supersymmetric lepton flavor violation [114].

Because of the exact $R$-parity, dimension four proton decay operators are absent in this model. However, it was found that both $q q q \ell$ and $\bar{u} \bar{u} d \bar{e}$ appear at order 6 in the SM singlets, and that they are also generated by integrating out the heavy exotics. This leads to dimension five proton decay, which is also a problem in 4D GUTs [23] and might require further (perhaps discrete) symmetries [115]. There exists a large amount of (discrete) symmetries at the orbifold point. The question whether letting some of them survive to low energies could suppress proton decay is currently under investigation. At this point, we are in the position to present a (probably biased) comparison between the usual 4D GUTs and the string-derived higher-dimensional GUTs (see Table 3).

Let us now answer the questions raised at the beginning of the section. First of all, we can indeed qualitatively reproduce the MSSM with all necessary features (matter content, gauge interactions with the correct coupling strengths, hierarchical Yukawa couplings with non-trivial mixing, see-saw suppressed neutrino masses, a hidden sector dynamically explaining soft masses in the right range, etc.). However, we cannot claim to be able to compute the precise numerical values of standard model parameters (such as the electron mass).\(^7\) On the other hand, we have seen that string theory gives us more than just reproducing (known) features. For instance, the fact that the $\mu$ problem is solved automatically is very encouraging, and might not just be an accident. We shall have yet to see whether similar statements can be made with regards to other puzzles such as the strong CP problem.

### 4.5 MSSM-like models from the $\mathbb{Z}_{12}$-I orbifolds

So far we have focused on $\mathbb{Z}_6$-II orbifolds. It turns out that in the $\mathbb{Z}_{12}$-I orbifold geometry promising models can be found as well. The exploration of this geometry started with [116, 117], where models with the flipped SU(5) gauge group, $G_{\delta} = \text{SU}(5)' \times \text{U}(1)_X$, were constructed. The advantage of this gauge symmetry is that one does not need large representations in order to break $G_{\delta}$ to $G_{\text{SM}}$; it is sufficient to let an SU(5) 10 acquire a VEV. The question of $R$-parity has been analyzed in this class of models as well [118]. It was found that one can either have an exact $R$-parity and states beyond the MSSM matter content or remove the exotics and have only an approximate $R$-parity, which might not necessarily be inconsistent with observation. The question of get-

\[^7\]The obstacles to such a computation are well known: one would have to come up with a theory of moduli stabilization that allows one to pre-
ting a phenomenologically viable axion from these constructions has also been addressed [119]. Another model with an effective $R$-parity, but not based on flipped SU(5), has been presented in [120].

4.6 Comments on the SO(32) heterotic string

Early works on orbifold compactifications mainly focused on the $E_8 \times E_8$ heterotic string, leaving the SO(32) version, to a large extent, unexplored. One of the reasons for this is that the coset SO(32)/SO(10) does not contain 16-plets, implying that it is impossible to obtain standard model families from the untwisted sector. Further, the simplest, i.e. $Z_3$, orbifold does not have any SO(10) spinors in its massless spectrum at all. This leads to the expectation that it is difficult to get phenomenologically viable models from the SO(32) theory.

Only recently, the interest in four-dimensional heterotic SO(32) orbifold constructions has been revived [121, 122]. In the case of $Z_N$ orbifolds, a complete classification of gauge embeddings in the absence of background fields has been obtained [72, 123]. Interestingly, that classification has shown that spinors of SO(2n) gauge groups appear rather frequently in the twisted sectors of SO(32) orbifolds. Spinors of SO(10), in particular, are found locally at fixed points of the first twisted sector of many orbifold models. Moreover, that classification reveals that the amount of available SO(32) orbifold models is comparable, at the same level, to that of its more famous brother, the $E_8 \times E_8$ string. From this we conclude that model building based on the heterotic SO(32) string theory might be as interesting as that based on the $E_8 \times E_8$ theory.

Let us also remark that the appearance of spinors in models derived from the SO(32) heterotic string will also be important for completing the understanding of the SO(32) heterotic type I duality in four space-time dimensions. We know that spinors do not appear in the perturbative type I theory.

5 Orbifold GUT limits

Much of the success of the orbifold GUTs is due to the fact that they provide a very simple understanding of the power of gauge symmetry breaking in extra dimensions. Here we discuss a top–down motivation for the orbifold GUT scheme, following [124]. It is well known that there is a (rather small) discrepancy between the scale where the gauge couplings meet, $M_{GUT}$, and the string scale $M_{string} = (\alpha')^{-1/2}$. The heterotic string scale can be defined as the ratio of 10D gauge and gravitational coupling strength, $m_{het} = \frac{\kappa_0}{\kappa_{10}}$ in standard notation (cf. e.g. [124]). It can be interpreted as the mass of the lowest-lying massive string state. The 10D and 4D gauge couplings are related by

$$g_4^2 = \frac{g_{het}^2}{V},$$  \hspace{1cm} (26)

where $V$ denotes the volume of the 6D compact space. On the other hand, the string scale in the heterotic case is fixed, and using $\alpha = g_4^2/4\pi \simeq 1/25$ one has

$$M_{string} = (\alpha')^{-1/2} \simeq 8.6 \times 10^{17} \text{ GeV}$$

while

$$M_{GUT} \simeq (2-3) \times 10^{16} \text{ GeV},$$  \hspace{1cm} (27)

i.e. there is a discrepancy

$$\frac{M_{string}}{M_{GUT}} \sim 30.$$  \hspace{1cm} (28)

We regard it as an encouraging fact that both scales are rather close. This gives further credit to the ‘grand desert’ picture of MSSM gauge coupling unification; in other schemes one typically has to work much harder to obtain similar agreement. Nevertheless, the discrepancy (28) asks for an explanation. The probably most far-reaching proposal is that of Hofava and Witten [125, 126], which we, however, do not discuss in detail here. In a footnote, Witten has pointed out a very simple solution: the GUT scale can be related to a compactification scale if internal space is anisotropic [127, footnote 3]. For an isotropic toroidal compactification one has

$$\frac{\alpha_{GUT}}{2} = \frac{g_{het}^2}{(R m_{het})^6}.$$  \hspace{1cm} (29)

Anisotropic compactification may mitigate the discrepancy between $M_{GUT}$ and $M_{string}$ [124, 127]. For example, if one assumes a toroidal compactification and chooses one radius much larger than the other five, one has

$$\frac{\alpha_{GUT}}{2} = \frac{g_{het}^2}{(R_{large} m_{het}) (R_{small} m_{het})^5}.$$  \hspace{1cm} (30)

This means that for $g_{het} \simeq 1$ and $R_{small} \simeq m_{het}^{-1}$ one can choose $R_{large} m_{het} \sim 50$ or $R_{large} \sim 3 \times 10^{16}$ GeV, i.e. the large radius can be of the order of the GUT scale. Although there might be some obstructions for taking just one radius much larger than the other [124, footnote 7], this result indicates that taking one or two radii much larger than the others can mitigate the discrepancy between $M_{GUT}$ and $M_{string}$.

As discussed in [127], the amelioration resulting from the anisotropy might have to be combined with the strong coupling effect described in [125–127].

Let us remark that recently it has been shown that, using Casimir stabilization, a radius $R \sim M_{GUT}$ appears possible
what miraculous. The third possibility is the one of the orbifold GUT picture, where at \( M_{\text{GUT}} \) one or two extra dimensions open up. Above \( M_{\text{GUT}} \) the running is dominated by the bulk group, which unifies the standard model gauge factors, \( G_{\text{bulk}} \supset G_{\text{SM}} \) (see also [132]). The running is hence universal (lower inlay in Fig. 9). This possibility appears particularly attractive and deserves to be studied in more detail.

6 Orbifolds versus Calabi–Yau compactifications

We have discussed how one can construct MSSM-like models by compactifying on an orbifold and then selecting supersymmetric vacua in which certain (SM singlet) fields attain vacuum expectation values. It is known that giving VEVs to fields localized at orbifold singularities corresponds to blowing up the orbifold, i.e. resolving the orbifold singularities [4, 44, 56, 133–135]. After the blow-up procedure one obtains a Calabi–Yau (CY) manifold (or at least something that looks in some regions like a CY). For instance, blowing up the \( \mathbb{Z}_3 \) orbifold is known to yield the standard CY [4, 44, 56]. In the process of blowing up,

- The gauge symmetry gets reduced
- Pairs of states which are vector-like with respect to the remnant gauge symmetry may get massive

In the CY picture, one does usually not discuss the masses of the states which pair up away from the orbifold point. In a generic point of the moduli space these masses will be of the order of the heterotic string scale. On the other hand, if the blow-up fields are moduli, as often happens to be the case, then there is a smooth interpolation between the orbifold point and a generic CY vacuum. The masses will then depend on ‘how generic’ the point in moduli space is. As long as the VEVs of the blow-up moduli are small, one retains perturbative control over the setting, and one can calculate the masses of the fields as well as other features of the model. On the other hand, there are features that do not depend at which point in moduli space one is sitting; the chiral spectrum of the model being one example.

The blow-up procedure, i.e. the interpolation between the orbifold point and a CY point has been revisited recently in a series of papers [136–140] (see [141] for related work). The focus of these works is the resolution of local singularities \( \mathbb{C}^n / \mathbb{Z}_n \), especially \( \mathbb{C}^3 / \mathbb{Z}_3 \). Very much the same as in the local GUT scheme, one starts with the compact orbifold, say...
and zooms into a given fixed point such that the local model $C^n/Z_n$ is described by the local shift $V_{\text{local}}$ only. This local shift determines the local gauge group and the local untwisted and twisted matter spectrum. Then, the blow-up is induced by a VEV of a single twisted field, the blow-up mode. In order to see the blow-up procedure in detail, the corresponding smooth resolution space $M^3$ of $C^n/Z_n$ is constructed explicitly. $M^3$ is parametrized by a parameter $r$ defining the radius of the resolved singularity, i.e. when $r \to 0$ the resolved space approaches the singular orbifold $C^3/Z_3$ (see Fig. 10 for an illustration). In order to compactify the heterotic string on $M^3$, a specific $U(1)$ bundle corresponding to the local shift $V_{\text{local}}$ is chosen. This choice completely defines the resolved model. Note, however, that the blow-up of an orbifold is not unique. Rather, as already remarked in [4], the orbifold point might be thought of as a junction in moduli space that can be blown up to different smooth spaces. However, the blow-up orbifold model and the resolved model seem to differ in general; for example, there can be two anomalous $U(1)$’s on the resolution in contrast to a single anomalous $U(1)$ on the orbifold. Nevertheless, these discrepancies can be resolved when one carefully analyzes the role of the blow-up mode: on the orbifold side, the standard Green–Schwarz mechanism, involving one single universal axion, is combined with a Higgs mechanism giving rise to the blow-up. On the resolution, this combination is mapped into a Green–Schwarz mechanism involving two axions. These axions are mixtures of the orbifold axion and of the blow-up mode. Therefore, one can achieve a complete matching between the blown-up local orbifold models and the corresponding smooth resolutions. Finally, in [140] it is shown how the local blow-up procedure can be applied to the compact case $T^6/Z_3$, even in the presence of Wilson lines. As an application, the blow-up of an early MSSM candidate constructed from $T^6/Z_3$ with “3 generations + vector-like” [6] is analyzed. It turns out that a full blow-up of all fixed points would destroy the nice properties of this model, e.g. it would break the hypercharge near the string scale. On the other hand, a partial blow-up of some fixed points can retain the MSSM properties. Additionally, since many vector-like exotics have trilinear couplings to the blow-up modes, they become heavy yielding a natural explanation for the decoupling of the vector-like matter. The main lessons for phenomenology from this discussion might be that reasonable particle physics models can be regarded as obtained from partial blow-ups of orbifolds.

Now one might ask why not to compactify directly on a CY rather than starting with the orbifold point and then moving away in moduli space. Indeed, CY compactifications are known to lead to promising models [142–148] (GUT models were derived from $U(N)$ line bundles [149, 150]). It is, in particular, reassuring that models with the exact MSSM spectrum have been found in this context as well.

The couplings in a potentially realistic CY compactification have been calculated and discussed [151]. The qualitative picture is as follows. The Yukawa coupling is either
order 1 or 0; however, it appears to be difficult to obtain hierarchical couplings at a generic point in moduli space. This does not mean that the CY is incapable of reproducing phenomenology, but it might mean that one would have to move close to a special point in moduli space. Whether this point can, at least to some extent, be identified with a configuration close to an orbifold point (or a partial blow-up of an orbifold) represents an interesting question. In fact, in the CY model [147, 151] there is no light Higgs pair at a generic point in moduli space: the $\mu$-term only vanishes if the size of a certain cycle shrinks to zero. A $\mu$-term of order TeV forces us to move to a rather special point in moduli space.

Altogether we see that neither the orbifold point nor the generic CY configuration in moduli space appear to describe observation. We have argued that vacua not too far from the orbifold point exhibit various phenomenologically desirable properties (but, of course, there might be different possibilities). Currently different strategies to explore the phenomenologically interesting region in moduli space are pursued: one can either start from the orbifold point or directly compactify on CY spaces. It might well be that only a combination of both strategies will be ultimately successful.

7 Summary

We have described how gauge symmetry breaking in extra dimensions allows us to resolve some of the most vexing problems in 4D grand unification. We discussed how orbifold GUTs can be derived from heterotic string theory. The resulting constructions have a very simple geometric interpretation. The key aspects of gauge group topography and local grand unification have been described in some detail. With these guidelines it is possible to construct string models with the exact MSSM matter content and $R$-parity. Because they are string derived, these models have certain additional (and surprising) relations which are, as opposed to field-theoretic constructions, not ‘put in by hand’. The most striking features (or ‘stringy surprises’) are

- A statistical preference for low-energy supersymmetry
- A relation between the $\mu$-term and the gravitino mass
- See-saw for free
- $\gamma_1 \gg$ other Yukawa couplings

It is very tempting to believe that these features are not just accidents, but somehow tell us that we are not on a completely wrong track.

We have derived models from string theory which are not immediately ruled out by observation (e.g. due to the prediction of unobserved states). On the other hand, we cannot claim that we have found ‘the’ MSSM (nor do we know that all our constructions are incapable to reproduce ‘the’ MSSM). To obtain quantitative predictions from our constructions (such as on the electron mass) one would have to explore the vacua of the models in much greater detail. As of now, not all necessary tools are available, so it will require a major effort to accomplish this task. But given the progress over the past few years we feel that this effort will be justified.

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