Lawrence Berkeley National Laboratory
Recent Work

Title
KINETIC ENERGY CONSIDERATIONS AND THE VACUUM

Permalink
https://escholarship.org/uc/item/1qb6m80c

Author
Eylon, Y.

Publication Date
1977-07-01
KINETIC ENERGY CONSIDERATIONS AND THE VACUUM

For Reference
Not to be taken from this room

Y. Eylon and E. Rabinovici

July 11, 1977

Prepared for the U. S. Energy Research and Development Administration under Contract W-7405-ENG-48
DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.
KINETIC ENERGY CONSIDERATIONS AND THE VACUUM

Y. Eylon
and
E. Rabinovici

Lawrence Berkeley Laboratory
University of California, Berkeley, California 94720

July 11, 1977

ABSTRACT

The suppression of infinite kinetic-energy trajectories is related to the topological classification in non abelian gauge theories.

The quantum-mechanical states of an SU(2) gauge theory in the $A_0 = 0$ gauge have been recently considered. The potential energy of a configuration $A_1(x)$ (at a given time) is given by $V[A] = \int d^3x \text{Tr} F_{ij}^2$. The configurations that correspond to the classical vacuum satisfy $V[A] = 0$, and are given by $A_1(x) = g^{-1}(x) g(\Omega(x))$, where $g(x)$ is any continuous mapping from 3-space to the group. For simplicity we consider only such mappings for which the following limit exists: $\lim_{r \to 0} g(r, \Omega) = g(\Omega)$. The manifold of these vacuum configurations is simply connected. This means that $V[A]$ has one continuous minima valley (unlike the double-well potential in quantum mechanics with one degree of freedom, in which there are two discrete minima which communicate only through the tunneling effect.)

This work is supported by the U. S. Energy Research and Development Administration under the auspices of the Division of Physical Research.

Jackiw and Rebbi postulated the condition $g(\Omega) = 1$. As a result, the vacuum manifold is decomposed into disconnected sections, which they called the $|\pi>$ vacua. The purpose of this comment is to show that this condition is a consequence of kinetic energy considerations.

We first recall that among all the functions $q(\tau)$ which satisfy $q(t_0) = q_0$ and $q(t_1) = q_1$, the minimal value of the "kinetic energy" $T = \int_{t_0}^{t_1} dt (\frac{dq}{dt})^2$, is achieved by the linear interpolating function (This is equivalent to minimizing the action of a free particle). The minimal value of $T$ is $T_{\text{min}} = (q_1 - q_0)^2/(t_1 - t_0)$.

In the $A_0 = 0$ gauge, $T = \int_{t_0}^{t_1} dt \int d^3x \sum_{i,j} \left( \frac{\partial}{\partial \tau} A_i^a(x, \tau) \right)^2$.

Since $T$ does not couple different points in ordinary ($x$) space, its minimal value under the constraints $A_i^a(x, t_0) = A_i^a(x, t_1)$ is $T_{\text{min}} = 1/(t_1 - t_0) \int d^3x \sum_{i,j} \left( A_i^a(x) - A_{i,j}^a(x) \right)^2$.

This expression is finite if $A_i^a(x) - A_{i,j}^a(x) \sim 1/r^{3/2} + \epsilon$. Suppose $A_{i,0}^a$ and $A_{i,1}^a$ are vacuum configurations, which are derived from the mappings $g(x)$ and $g'(x)$ respectively (with the corresponding limits $g(\Omega)$ and $g'(\Omega)$). Unless $g'(\Omega) = g\tilde{g}(\Omega)$ where $\tilde{g}$ is a constant element of the group, the two configurations are different, and their difference goes like $1/r$. Therefore, $T_{\text{min}} = \infty$, and there will be no communication between these two vacuum configurations in the functional integral formalism. Consequently, if we start at $t = t_0$ with some vacuum configuration with a given $g(\Omega)$, we can ignore at any future time all vacuum configurations which correspond to different $g(\Omega)$. In particular, if we start with the Jackiw-Rebbi
condition, \( g(\Omega) = \text{const.} \) we can ignore all other vacuum configurations. At that point one can use the result that the mappings \( g(x) \) with \( g(\Omega) = \text{const.} \) fall into homotopy classes, labelled by the integers. We thus see that the classification of vacuum configurations into the \( n \)-classes, advocated by Jackiw and Rebbi, rises in a natural way.

Let us now consider the mappings \( g(x) = \exp[-i\alpha(r) \vec{\tau} \cdot \vec{x}/2] \) where \( \vec{\tau} = \vec{\tau}^1/2 \) and \( \vec{\tau}^1/2 \) are the \( (2 \times 2) \) generators of the SU(2) group. To get a continuous mapping at \( x = 0 \) we set \( \alpha(0) = 0 \). If \( \alpha(\infty) = 2\pi n \), the corresponding mapping, \( g_n(x) \), is in the \( n \) class (with \( g_n(\Omega) = (-1)^n \)). As an illustration to our arguments we construct a trajectory \( A_1^n(x,t) \), which interpolates the \( n = 0 \) configuration (namely, \( g^{-1}_0 g_0 \)) at \( t = t_0 \) with the \( n = 1 \) configuration at \( t = t_1 \) through the valley (namely, \( V[A_1^n(x,t)] = 0 \) at each \( t \). Note that any such trajectory cannot satisfy the classical equations of motion). For example, at each \( t \) we take \( \alpha(r;t) = y(t)\alpha(r) \) with \( y(t_0) = 0 \), \( y(t_1) = 1 \) and \( \alpha(0) = 0 \), \( \alpha(\infty) = 2\pi \), and then set \( A_1^n(x,t) = g^{-1} (x;t) g(x;t) \). However, since we change \( g(\Omega) \) with time, the kinetic energy \( T \) must be infinite, and a direct calculation gives

\[
T = 2\pi \int_{t_0}^{t_1} dt \left( \frac{dy}{dt} \right)^2 2 \int_0^\infty dr \left( \frac{d\alpha}{dr} \right)^2 r^2 + 2\alpha^2 \Rightarrow \infty
\]

We see that any trajectory which interpolates the \( n = 0 \) and the \( n = 1 \) configurations with a finite kinetic energy must cross the potential barrier. The instanton\(^6\) is an example of such a trajectory, which has a finite action.

We wish to thank S. Coleman and S. Mandelstam for helpful discussions.

REFERENCES

This report was done with support from the United States Energy Research and Development Administration. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the United States Energy Research and Development Administration.