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THE USE OF SCATTERING PARAMETERS IN THE DESIGN OF AMPLIFIERS OF SUBNANOSECOND RISETIME

Rosser S. Wilson

March 1972

SUMMARY

This paper discusses techniques useful in the design of wide-band linear amplifiers intended for pulse operation. A suitable model for high-frequency transistors is given, and use is made of the scattering characterization and of a special computer program to deduce element values in the model.

The design of a fast risetime series-shunt pair and series-series triple is carried out, and computer simulations are used to predict performance of the actual designs inclusive of all parasitic reactances consequent upon physical realization of the circuits.
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CHAPTER 1. INTRODUCTION

The need often arises in experiments in high-energy physics for an amplifier possessing moderate gain and having rise-times in the subnano-second region. Good d-c stability and zero offset between input and output, as well as favourable noise performance are additional characteristics desirable in such an amplifier; however, since in this paper the primary concern is with techniques of realizing the necessary transient response, suffice it to say that the d-c requirements can be met using the operational-amplifier stabilizing technique described elsewhere.\(^{(1)}\) The topic of noise minimization is an extensive one in its own right, and could provide the basis for further research.

The minimal specification to be satisfied by the circuit in respect of transient performance may be stated concisely thus:

(i) Rise-time $\leq 500$ psec.

(ii) Overshoot on step-function input $\leq 10\%$.

(iii) Gain of 20 dB (X10) in a 50 $\Omega$ system.

It is evident that the amplifier must be capable of passing frequencies approaching one gigahertz. At the other end of the spectrum, a passband extending down to d-c is desirable from the aspect of reducing tilt on wide input pulses, and of simplifying the application of operational-amplifier d-c stabilizing methods.
The low-frequency requirement is easily satisfied by the use of direct coupling within the amplifier; however, the provision of the requisite high-frequency response is a considerably more difficult problem, and requires in addition to the use of transistors having suitably high $f_T$ figures the application of stripline and thin-film construction techniques in order to reduce to a minimum the effects upon circuit performance of parasitic reactances and transversal delay around the feedback loop.

Two amplifier designs are considered in this paper, and in both series feedback is used at the input in order to raise the input impedance of the circuit. In this way an approximation can be attained to a 50 Ω input impedance through use, at the base of the input transistor, of a 50 Ω resistor to ground. Moreover, earlier work (2) has shown that the two configurations chosen for treatment here, namely the series-shunt pair and series-series triple, are optimal from the standpoint of maximum gain-bandwidth product in comparison with alternate two- and three-stage arrangements.

A preliminary to the design of any amplifier is the determination of suitable models of the active devices intended for use therein. Such models may be deduced directly from physical reasoning provided accurate data are available regarding device construction. Frequently this information is proprietary and hence is not readily available; even if it were, manufacturing processes are often sufficiently ill-defined as to make rather inaccurate a device model determined solely on the basis of a device processing schedule.

The approach to transistor characterization adopted in this paper relies upon physical reasoning only to deduce the topology of the equivalent circuit, and in this way is relieved of dependence upon details of transistor fabrication. Direct measurement of the two-port
parameters of the transistor, in conjunction with the use of a digital computer program devised especially for the task, then enables values to be assigned to the elements of the model. This method has the virtue of reflecting the actual characteristics of the device, and an arbitrary increase in model accuracy is possible simply by incorporating into the model those additional elements necessary to deal with effects not accounted for in more elementary versions.

The two-port measurements necessary for the characterization of the transistors used in the amplifier were done using scattering parameters in order to circumvent the need for accurate short- or open-circuits of device ports necessary for the determination of the more common Y- or Z-parameters. The difficulty at high frequencies of attaining these states can lead to significant inaccuracies in measurement.

Included in this paper is a complete description of the various design and modelling techniques already described in brief above. Chapter 2 is devoted to a brief introduction to the scattering matrix and to the development of formulae useful in the remainder of the report. In Chapter 3, the theoretical aspects of automatic modelling are presented, along with a short description of a computer program written to realise the theory. The results of automated modelling of a high-frequency chip transistor are contained in Chapter 4. Two amplifiers were designed using the transistor characterized in Chapter 4; the design of a series-shunt feedback pair forms the subject of Chapter 5. Chapter 6 discusses the design of a series-series feedback triple; this particular design was accomplished using a variant of the optimization program.
dealt with in Chapter 3. Chapter 7 concludes the report with a discussion of important results from the preceding chapters and with suggestions for further work. Appendices give details of matters considered too specialized for more than preliminary mention in other parts of the paper.
CHAPTER 2. ELEMENTARY THEORY OF THE SCATTERING MATRIX

The scattering parameter method of two-port characterization was devised originally in connection with microwave network theory, where the description of two-port behaviour in terms of incident and reflected wave quantities is a physically realistic approach lending itself to easy visualization.

In measurements upon active devices at high frequencies, the use of scattering parameters is further commended by the fact that at these frequencies, the open- and short-circuits requisite to measurement of the Y- or Z-parameters are exceedingly difficult to attain due to the presence of spurious reactances. Such parasitics can also render an active device unstable and susceptible to free oscillation when the more conventional two-port characterizations are employed. These and attendant problems are greatly reduced in the scattering formulation, for in this, the two ports of the device under test are always terminated in a constant and precisely known reference impedance.

The scattering formulation commonly used, and that to be presented in this chapter, is carried out in terms of a resistive reference immittance. The scattering formalism is completely general, however, and can accommodate complex reference immittances. (3)

Shown in Fig. (2-1) are the basic definitions to be employed in the ensuing discussion which, although it will be given on the basis of a two-port configuration, is easily extensible to the general n-port case.
(a) Calculation of $S_{11}(j\omega), S_{21}(j\omega)$

$a_2(j\omega) = 0$

(b) Calculation of $S_{12}(j\omega), S_{22}(j\omega)$

$a_1(j\omega) = 0$

Fig. 2-1
The customary definition of the basis-free normalized scattering matrix is contingent upon the introduction of an hypothetical pair of vector waves comprising the wave \( a(j\omega) \) incident upon the two-port, and its reflected counterpart \( b(j\omega) \). The scattering matrix \( S(j\omega) \) relates the incident and reflected waves according to the formula: (4)

\[
b(j\omega) = S(j\omega) \ a(j\omega), \tag{2-1}
\]

where,

\[
a(j\omega) = \frac{1}{2} R^{-\frac{1}{2}} \left( V(j\omega) + RI(j\omega) \right), \tag{2-2}
\]

and,

\[
b(j\omega) = \frac{1}{2} R^{-\frac{1}{2}} \left( V(j\omega) - RI(j\omega) \right). \tag{2-3}
\]

By \( R \) is denoted the diagonal matrix of reference normalizing resistances, namely,

\[
R = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix},
\]

and \( V(j\omega) \) and \( I(j\omega) \) are the vectors of port voltages and currents, respectively.

It is apparent from these definitions that a scattering-matrix specification of a network must necessarily include mention of the reference impedances under which the characterization was done.
The evaluation of the individual entries in the scattering matrix is easily undertaken through the expansion of Eq. (2-1) according to Eq. (2-4),

\[
\begin{bmatrix}
  b_1(j\omega) \\
  b_2(j\omega)
\end{bmatrix} =
\begin{bmatrix}
  s_{11}(j\omega) & s_{12}(j\omega) \\
  s_{21}(j\omega) & s_{22}(j\omega)
\end{bmatrix}
\begin{bmatrix}
  a_1(j\omega) \\
  a_2(j\omega)
\end{bmatrix},
\]  

(2-4)

whence it is evident that the following relations are obeyed:

\[
s_{11}(j\omega) = \frac{b_1(j\omega)}{a_1(j\omega)} \quad a_2(j\omega) = 0 \tag{2-5a}
\]

\[
s_{12}(j\omega) = \frac{b_1(j\omega)}{a_2(j\omega)} \quad a_1(j\omega) = 0 \tag{2-5b}
\]

\[
s_{21}(j\omega) = \frac{b_2(j\omega)}{a_1(j\omega)} \quad a_2(j\omega) = 0 \tag{2-5c}
\]

\[
s_{22}(j\omega) = \frac{b_2(j\omega)}{a_2(j\omega)} \quad a_1(j\omega) = 0 \tag{2-5d}
\]

Achievement of the condition \( a_k(j\omega) = 0 \) is readily attained by removing any excitation from the k-th port, whereupon \( V_k(j\omega) = r_k I_k(j\omega) \) and so, according to Eq. (2-2), \( a_k(j\omega) \) will vanish. Figure (2-1) gives a summary of the actual use of Eqs. (2-5a) to (2-5d).
In practice, scattering parameters are measured using the instrumentation arrangement shown in schematic form in Fig. (2-2). The dual-directional couplers of that figure are passive devices giving at their two output terminals a sample of the wave traversing them in the direction of the associated arrow.

The \( a(j\omega) \) and \( b(j\omega) \) vectors are defined as the wave vectors existing at the terminals of the network under consideration. In a practical situation, where the points of evaluation of the incident and reflected waves are located some distance from the network ports, the following relation exists between \( \hat{a}(j\omega) \), the measured quantity corresponding to \( a(j\omega) \), and \( a(j\omega) \) itself,

\[
\begin{bmatrix} \hat{a}_1(j\omega) \\ \hat{a}_2(j\omega) \end{bmatrix} = \begin{bmatrix} K_1 e^{-j\omega T_1} & 0 \\ 0 & K_2 e^{-j\omega T_2} \end{bmatrix} \begin{bmatrix} a_1(j\omega) \\ a_2(j\omega) \end{bmatrix}.
\]

A similar formula holds true in respect of \( \hat{b}(j\omega) \) and its measured counterpart \( \hat{b}(j\omega) \),

\[
\begin{bmatrix} \hat{b}_1(j\omega) \\ \hat{b}_2(j\omega) \end{bmatrix} = \begin{bmatrix} K_1 e^{-j\omega T_1} & 0 \\ 0 & K_2 e^{-j\omega T_2} \end{bmatrix} \begin{bmatrix} b_1(j\omega) \\ b_2(j\omega) \end{bmatrix}.
\]

The constants \( K_1 \) and \( K_2 \) in these two equations account for the attenuation between the waves appearing at the network terminals, and the corresponding samples thereof provided at the directional couplers.
Note that a uniform system impedance of $Z_0$ ohms is assumed.

Fig. 2-2
The linear phase (delay) operator exponentials simulate the effects of propagation delay through the directional couplers and transmission lines, and for simplicity the transmission lines are assumed loss-less.

It is usual in practice to arrange that the channels associated with the two network ports are symmetric, whereupon $K_1 = K_2$ and $\tau_1 = \tau_2$ since then attenuation and delay in the paths are equal. For this commonly encountered case, Eqs. (2-5a) to (2-5d) can be rewritten as:

\begin{align*}
    s_{11}(j\omega) &= \frac{\hat{b}_1(j\omega)}{\hat{a}_1(j\omega)} \left| a_2(j\omega) = a_2(j\omega) = 0 \right. \\
    s_{12}(j\omega) &= \frac{\hat{b}_1(j\omega)}{\hat{a}_2(j\omega)} \left| \hat{a}_1(j\omega) = a_1(j\omega) = 0 \right. \\
    s_{21}(j\omega) &= \frac{\hat{b}_2(j\omega)}{\hat{a}_1(j\omega)} \left| \hat{a}_2(j\omega) = a_2(j\omega) = 0 \right. \\
    s_{22}(j\omega) &= \frac{\hat{b}_2(j\omega)}{\hat{a}_2(j\omega)} \left| \hat{a}_1(j\omega) = a_1(j\omega) = 0 \right.
\end{align*}

Equations (2-8a) to (2-8d) provide the necessary relations for direct experimental determination of the scattering matrix of an arbitrary network. They were obtained by substitution into Eq. (2-4) of Eqs. (2-6) and (2-7).
For the case of non-symmetrical channels, it is easy to show that,

\begin{align*}
  s_{11}(j\omega) &= \frac{\hat{b}_1(j\omega)}{\hat{a}_1(j\omega)} \bigg|_{\hat{a}_2(j\omega) = a_2(j\omega) = 0} = a_2(j\omega) = 0 \quad (2-9a) \\
  s_{12}(j\omega) &= \frac{K_2}{K_1} e^{j\omega(\tau_2 - \tau_1)} \frac{\hat{b}_1(j\omega)}{\hat{a}_2(j\omega)} \bigg|_{\hat{a}_1(j\omega) = a_1(j\omega) = 0} = a_1(j\omega) = 0 \quad (2-9b) \\
  s_{21}(j\omega) &= \frac{K_1}{K_2} e^{j\omega(\tau_1 - \tau_2)} \frac{\hat{b}_2(j\omega)}{\hat{a}_1(j\omega)} \bigg|_{\hat{a}_2(j\omega) = a_2(j\omega) = 0} = a_2(j\omega) = 0 \quad (2-9c) \\
  s_{22}(j\omega) &= \frac{\hat{b}_2(j\omega)}{a_2(j\omega)} \bigg|_{\hat{a}_1(j\omega) = a_2(j\omega) = 0} = a_1(j\omega) = 0. \quad (2-9d)
\end{align*}

In many applications of the scattering parameters to network analysis and synthesis it is necessary, by using standard techniques of analysis, to be able to compute the scattering matrix for a given network whose topology and element values are specified. Towards this end, it is useful to deduce formulae relating the scattering parameters to network port voltages and currents.

Presuming the desirability of employing nodal analysis to accomplish the task of analytic determination of the scattering matrix, it is clear that the preferred excitation and response bases are currents and node-pair potential differences, respectively.
Treating first the case of $s_{11}(j\omega)$ and with reference to Fig. (2-1a), substitution into Eq. (2-5a) of the appropriate scalar version of Eqs. (2-2) and (2-3) yields,

$$s_{11}(j\omega) = \frac{V_1(j\omega) - r_1I_1(j\omega)}{V_1(j\omega) + r_1I_1(j\omega)}.$$  (2-10)

The variable $I_1(j\omega)$ can be eliminated from this expression to give a dependence upon the excitation current source $J_1(j\omega)$ and port voltage $V_1(j\omega)$ by noting that,

$$I_1(j\omega) = J_1(j\omega) - \frac{V_1(j\omega)}{r_1},$$  (2-11)

whence substitution of this into Eq. (2-10) gives the required formula, namely,

$$s_{11}(j\omega) = \left(\frac{2}{r_1}\right)\left(\frac{V_1(j\omega)}{J_1(j\omega)}\right) - 1.$$  (2-12)

In the case of $s_{21}(j\omega)$, a similar development commences with a substitution into Eq. (2-5b) from Eqs. (2-2) and (2-3) to give,

$$s_{21}(j\omega) = \frac{V_2(j\omega) - r_2I_2(j\omega)}{V_1(j\omega) + r_1I_1(j\omega)}.$$  (2-13)
Since in addition to Eq. (2-11), the following relation is also valid,

\[ I_2(j\omega) = \frac{V_2(j\omega)}{r_2}, \tag{2-14} \]

these can be inserted into Eq. (2-13) to yield,

\[ s_{21}(j\omega) = \left( \frac{2}{r_1} \right) \left( \frac{V_2(j\omega)}{J_1(j\omega)} \right), \tag{2-15} \]

In similar fashion, the two remaining scattering parameters can be determined as follows:

\[ s_{12}(j\omega) = \left( \frac{2}{r_1} \right) \left( \frac{V_1(j\omega)}{J_2(j\omega)} \right), \tag{2-16} \]

and,

\[ s_{22}(j\omega) = \left( \frac{2}{r_2} \right) \left( \frac{V_2(j\omega)}{J_2(j\omega)} \right) - 1, \tag{2-17} \]

where reference is now to Fig. (2-1b).

Through the applications of these four expressions, the scattering matrix may readily be obtained using a nodal analysis.

The description of the scattering matrix given in this chapter has been brief and has been limited to the derivation of results useful in subsequent sections of the paper. For a more detailed exposition of the subject, the reader is referred to the references.
CHAPTER 3. THE THEORY OF AUTOMATED MODELLING USING THE SCATTERING MATRIX

The objective of the automatic modelling process is, in most general terms, the determination of element values in a pre-specified network (model) \(_N\) such that the scattering parameters of this circuit agree optimally in some sense with a given set of scattering matrices taken at a sequence of \(\ell\) frequencies \(\omega_1, \omega_2, \ldots, \omega_i, \ldots, \omega_{\ell} \) in the band wherein optimal agreement is sought. The problem can be stated mathematically as one of finding the minimum of an error function whose magnitude is a function of the discrepancy existing between a given set of scattering matrices \(S^G(\omega_1), S^G(\omega_2), \ldots, S^G(\omega_i), \ldots, S^G(\omega_\ell)\), and those calculated from \(N\), \(S^N(\omega_1,x), S^N(\omega_2,x), \ldots, S^N(\omega_i,x), \ldots, S^N(\omega_\ell,x)\). The functional dependence upon the vector \(x = (x_1, x_2, \ldots, x_m)^T\) of variable elements in \(N\) of the \(S^N\) accounts for the fact that variation of the \(S^N\) can be accomplished through changing the values of prescribed elements in \(N\).

Although alternative choices are certainly possible, the error function adopted in this instance is of the weighted sum-of-squares variety, and is expressible in the space of elements subject to variation as,

\[
\varepsilon(x) = \sum_i \sum_{p,q} W^2_{pq} \left[ \left( \text{Re} \left\{ s^N_{pq}(\omega_i, x) \right\} - \text{Re} \left\{ s^G_{pq}(\omega_i) \right\} \right)^2 \\
+ \left( \text{Im} \left\{ s^N_{pq}(\omega_i, x) \right\} - \text{Im} \left\{ s^G_{pq}(\omega_i) \right\} \right)^2 \right].
\]

(3-1)
The index of summation \( i \) in Eq. (3-1) is taken over all frequencies at which \( S^G \) are given, the indices \( p \) and \( q \) being used to signify summation over all four entries in the \( 2 \times 2 \) scattering matrices. A weight vector \( \mathbf{W} \) is also included in the formulation of the error function in order to enable preference to be given in the fitting process to one or more entries in the scattering matrix. Note in passing that \( \epsilon(x) \) of Eq. (3-1) is a non-negative scalar possessing the property of vanishing when the \( S^N \) and \( S^G \) agree for all \( \omega_i \).

It transpires that the most efficient schemes of functional minimization require both the function and its gradient. Differentiation of Eq. (3-1) yields for the gradient of \( \epsilon(x) \) with respect to \( x \),

\[
\frac{\partial \epsilon(x)}{\partial x} = \sum_i \sum_{p,q} 2 \mathbf{W}_{pq} \left[ \left( \text{Re} \left\{ \frac{\partial S^N_{pq}(j\omega_i,x)}{\partial x} \right\} - \text{Re} \left\{ S^G_{pq}(j\omega_i) \right\} \right) + \left( \text{Im} \left\{ \frac{\partial S^N_{pq}(j\omega_i,x)}{\partial x} \right\} - \text{Im} \left\{ S^G_{pq}(j\omega_i) \right\} \right) \right].
\]  

(3-2)

The notation used here for the gradient is rather informal, and is expandible as, \( \partial \epsilon(x)/\partial \mathbf{x} = \left( \frac{\partial \epsilon(x)}{\partial x_1}, \frac{\partial \epsilon(x)}{\partial x_2}, \ldots, \frac{\partial \epsilon(x)}{\partial x_m} \right)^T \). Apparently, the determination of \( \epsilon(x) \) and \( \partial \epsilon(x)/\partial \mathbf{x} \) reduces to the problem of calculating for \( N \) its scattering parameters at all \( \omega_i \), and of finding at these same frequencies the gradients of all four entries in the calculated scattering matrices \( S^N \).
The former task is easily accomplished via nodal analysis in the complex frequency domain and use of Eqs. (2-12) and (2-15) to (2-17); at the i-th frequency $\omega_i$, nodal analysis is done upon network $N$ augmented by the normalizing resistances for which the $S^G$ are specified, and excited at its input port with a current source of intensity $J_1(j\omega_i)$ according to Fig. (2-1a). Solution is made for $V_1(j\omega_i)$ and $V_2(j\omega_i)$ of that figure, whereupon application of Eqs. (2-12) and (2-15) permits direct computation of $s_{11}^N(j\omega_i,x)$ and $s_{21}^N(j\omega_i,x)$. A similar development using the excitation current source in the position of Fig. (2-1b) in conjunction with Eqs. (2-16) and (2-17) enables $s_{12}^N(j\omega_i,x)$ and $s_{22}^N(j\omega_i,x)$ to be found.

The problem of calculating the gradients of the two-port scattering parameters can be recast into one of finding the gradients of the terminal voltages of network $N$ to which appropriate excitation has been applied. Differentiation of Eqs. (2-12) and (2-15) then yields,

$$\frac{\partial s_{11}^N(j\omega_i,x)}{\partial x} = \left( \frac{2}{J_1(j\omega_i) r_1} \right) \frac{\partial V_1(j\omega_i,x)}{\partial x}, \quad (3-3)$$

$$\frac{\partial s_{21}^N(j\omega_i,x)}{\partial x} = \left( \frac{2}{J_1(j\omega_i) r_1} \right) \frac{\partial V_2(j\omega_i,x)}{\partial x}. \quad (3-4)$$

For these two equations, the exciting source for $N$ is a current source of value $J(j\omega_i)$ connected at the input port of $N$ as in Fig. (2-1a). For Eqs. (3-5) and (3-6), the source is in the location of Fig. (2-1b), and the differentiation is done upon Eqs. (2-16) and (2-17).
The voltages \( V_1(\omega, x) \) and \( V_2(\omega, x) \) are dependent upon the variable elements \( x_1, x_2, \ldots, x_m \) in \( N \), hence their functional dependence upon \( x \).

Calculation of the derivatives on the right-hand side of Eqs. (3-3) to (3-6) in the numerically precise form required by the functional minimization algorithm is greatly facilitated through application of the concept of the adjoint network.\(^{5,6,7}\)

The adjoint \( \hat{N} \) of \( N \) possesses a topology identical to that of \( N \). All conductance, capacitance, and reciprocal inductance branches in \( N \) are associated in \( \hat{N} \) with identical branches; voltage-controlled current sources in \( N \) are associated with similar elements in the adjoint, however the roles of controlled and controlling branches are reversed in \( \hat{N} \).

These definitions are illustrated in Fig. (3-1), along with other useful terminology. Essential to the simplification of the subsequent exposition is the introduction of the augmented networks \( \hat{N} \) and \( \hat{N}^\ast \) connected with \( N \) and \( \hat{N} \); the augmented network is simply the original network with the scattering normalizing conductances (reciprocal normalizing resistances) connected across its two ports. It is essential to note that the port voltages of \( \hat{N} \) and \( N \), and of \( \hat{N}^\ast \) and \( \hat{N} \) are identical.
\[ I_{\text{ref}}(j\omega, 2) + I_{\text{sat}}(j\omega, 2) \]

Note: \( q_i = 1/r_i \)

Fig. 3-1
Given the augmented networks \( \hat{N} \) and \( \hat{N}^* \), the calculation of the voltage derivatives in Eqs. (3-3) to (3-6) becomes an exercise in the application of Tellegen's theorem. \(^{8}\) Using the general notation \( I_B(j\omega_1, x) \), \( V_B(j\omega_1, x) \) for branch currents and voltages in \( \hat{N} \), and \( I_B(j\omega_1, x) \), \( V_B(j\omega_1, x) \) for their analogues in \( \hat{N}^* \), Tellegen's theorem states that since \( \hat{N} \) and \( \hat{N}^* \) are topologically equivalent,

\[
\left[ \sum_B V_B(j\omega_1, x) \Phi_B(j\omega_1, x) \right] - V_1(j\omega_1, x) \hat{J}_1(j\omega_1) = 0, \quad (3-7)
\]

and,

\[
\left[ \sum_B I_B(j\omega_1, x) \Psi_B(j\omega_1, x) \right] - J_1(j\omega_1) \Psi_1(j\omega_1, x) = 0. \quad (3-8)
\]

Summation is done over all branches in \( \hat{N} \) and \( \hat{N}^* \), and both networks are assumed to be driven by constant current sources according to Fig. (3-2a).

Now, an infinitesimal perturbation in one of the variable elements of \( \hat{N} \) leads to changes in the branch voltages and currents in \( \hat{N}^* \); the perturbation is symbolically accomplished by altering the vector \( x \) by an infinitesimal amount \( \delta x \) in the case of \( \hat{N} \) so that,

\[
\left[ \sum_B V_B(j\omega_1, x + \delta x) \Phi_B(j\omega_1, x) \right] - V_1(j\omega_1, x + \delta x) \hat{J}_1(j\omega_1) = 0, \quad (3-9)
\]

and,

\[
\left[ \sum_B I_B(j\omega_1, x + \delta x) \Psi_B(j\omega_1, x) \right] - J_1(j\omega_1) \Psi_1(j\omega_1, x) = 0. \quad (3-10)
\]
(a) Calculation of $\partial S_{11}/\partial x$

(b) Calculation of $\partial S_{22}/\partial x$

(c) Calculation of $\partial S_{21}/\partial x$

(d) Calculation of $\partial S_{22}/\partial x$

Fig. 3-2
Subtraction of Eq. (3-7) from Eq. (3-9), followed by subtraction from the resulting expression of the difference between Eqs. (3-8) and (3-10) yields,

\[
\sum_B \left\{ \Phi_B(j\omega_1,x) \left( V_B(j\omega_1,x+\delta x) - V_B(j\omega_1,x) \right) 
- \Psi_B(j\omega_1,x) \left( I_B(j\omega_1,x+\delta x) - I_B(j\omega_1,x) \right) \right\} 
- \left( V_1(j\omega_1,x+\delta x) - V_1(j\omega_1,x) \right) \hat{J}_1(j\omega_1) = 0. \tag{3-11}
\]

Assume now that the perturbed element is a conductance in the K-th position of x so that in \( \hat{N} \), \( x^+ \delta x^G \) where, \( \delta x^G \) denotes the vector \( (0,0,\ldots,\delta x^G_K,\ldots,0)^T \), and the superscript \( G \) signifies treatment of a conductance element. The branch relations for the element \( x^G_K \) are given by Eqs. (3-12) and (3-13) for the case of unperturbed and perturbed \( \hat{N} \), respectively.

\[
I_{x^G_K}(j\omega_1,x) = x^G_K V_{x^G_K}(j\omega_1,x) \tag{3-12}
\]

\[
I_{x^G_K}(j\omega_1,x+\delta x^G) = (x^G_K + \delta x^G_K) V_{x^G_K}(j\omega_1,x+\delta x^G) \tag{3-13}
\]

In \( \hat{N} \), the branch equation,

\[
\Phi_{x^G_K}(j\omega_1,x) = x^G_K \Psi_{x^G_K}(j\omega_1,x) \tag{3-14}
\]

will be satisfied. The subscript scheme on voltage and current
variables denotes quantities appropriate to the branch in $\hat{N}^*$ and $\hat{N}^*$ containing $x^G_K$. Substitution of Eqs. (3-12) to (3-14) into Eq. (3-11) leads directly to,

$$
- \psi_{xK}(j\omega_1, x) V_{xK}(j\omega_1, x + \delta x^G) \delta x^G_K
$$

$$
= J_1(j\omega_1) V_1(j\omega_1, x + \delta x^G) - V_1(j\omega_1, x). \quad (3-15)
$$

By considering the branch relations for the other unperturbed elements in $\hat{N}$ and their correspondents in $\hat{N}^*$ it can readily be verified that all terms on the left-hand side of Eq. (3-11) vanish, leaving only the term due to the branches of $\hat{N}^*$ and $\hat{N}^*$ containing $x^G_K$. Taking the limit as $\delta x^G_K \to 0$ and $\delta x^G \to 0$ yields upon slight rearrangement of Eq. (3-15),

$$
\frac{\partial V_1(j\omega_1, x)}{\partial x^G_K} = \frac{\psi_{xK}(j\omega_1, x) V_{xK}(j\omega_1, x)}{J_1(j\omega_1)}, \quad (3-16)
$$

which enables direct computation of those elements of vector $\partial V_1(j\omega_1, x)/\partial x_K$ arising from conductances.

The gradient terms arising from voltage-controlled current sources are derived using the two-branch representation for that element shown in Fig. (3-1). The canonical form chosen for this element is one providing both a transconductance term $\tau$ and a linear phase (delay) operator $\tau$.

Considering now that position $k$ of the vector $x$ contains a delay term $x^\tau_K$ associated with a voltage-controlled current source, and employing notation similar to that used previously in deriving the gradient
terms arising from conductances, the branch relations of the element in question can be written as,

\[ I_{V_{DI}}(j\omega, x) = g_m e^{-j\omega x} V_{VCI}(j\omega, x) , \quad (3-17) \]

and

\[ I_{VCI}(j\omega, x) = 0 \quad (3-18) \]

in the unperturbed \( N \). A perturbation in \( N \) of \( x + \delta x \), with \( \delta x = (0,0,...,\delta x, ...,0)^T \) causes \( g_m e^{-j\omega x} + g_m e^{-j\omega x} + \delta \left( g_m e^{-j\omega x} \right) \), and therefore yields,

\[ I_{V_{DI}}(j\omega, x+\delta x) = \left[ g_m e^{-j\omega x} + \delta \left( g_m e^{-j\omega x} \right) \right] \times V_{VCI}(j\omega, x+\delta x) , \quad (3-19) \]

and,

\[ I_{VCI}(j\omega, x+\delta x) = 0 . \quad (3-20) \]
Since in the adjoint $N^*$, no perturbation is done,

$$\phi_{\text{VCI}}_{x^T_k}(j\omega_i, x) = g m e^{-j\omega_i x^T_k} \psi_{\text{VDI}}_{x^T_k}(j\omega_i, x), \quad (3-21)$$

and,

$$\phi_{\text{VDI}}_{x^T_k}(j\omega_i, x) = 0 \quad (3-22)$$

The foregoing six equations, Eqs. (3-17) to (3-22), can be inserted directly into Eq. (3-11) with the consequence that,

$$-\psi_{\text{VDI}}_{x^T_k}(j\omega_i, x) V_{\text{VCI}}_{x^T_k}(j\omega_i, x + \delta x^T) g m e^{-j\omega_i x^T_k} \bigg[ V_1(j\omega_i, x + \delta x^T) - V_1(j\omega_i, x) \bigg] \quad (3-23)$$

By taking the limit as $\delta x^T_k \rightarrow 0$ and $\delta x^T \rightarrow 0$, using the identity,

$$\delta \left( g m e^{-j\omega_i x^T_k} \right) = \delta x^T_k \delta \left( g m e^{-j\omega_i x^T_k} \right) / \delta x^T_k,$$

and expanding the complex exponential according to Euler's theorem, it can readily be shown that,

$$\frac{\partial V_1(j\omega_i, x)}{\partial x^T_k} = g m \omega_i \left( \sin \omega_i x^T_k + j \cos \omega_i x^T_k \right) \psi_{\text{VDI}}_{x^T_k}(j\omega_i, x) x V_{\text{VCI}}_{x^T_k}(j\omega_i, x). \quad (3-24)$$
It is now possible to deduce the contributions to the gradient of the scattering parameters arising from each type of element in $N$. Substituting Eq. (3-16) into Eq. (3-3) yields, for conductances,

$$\frac{\partial s_{11}^N(j\omega_1,x)}{\partial x_G^K} = \left(\frac{2}{r_1 J_1(j\omega_1) \hat{J}_1(j\omega_1)}\right) \psi_{X_G^K}(j\omega_1,x) V_{X_G^K}(j\omega_1,x). \quad (3-25)$$

For the remaining types of passive elements, it is easy also to show that, for capacitors,

$$\frac{\partial s_{11}^N(j\omega_1,x)}{\partial x_C^K} = \left(\frac{j2\omega_1}{r_1 J_1(j\omega_1) J_1(j\omega_1)}\right) \psi_{X_C^K}(j\omega_1,x) V_{X_C^K}(j\omega_1,x), \quad (3-26)$$

and for reciprocal inductances,

$$\frac{\partial s_{11}^N(j\omega_1,x)}{\partial x_L^K} = \left(\frac{2}{j\omega_1 r_1 J_1(j\omega_1) \hat{J}_1(j\omega_1)}\right) \psi_{X_L^K}(j\omega_1,x) V_{X_L^K}(j\omega_1,x). \quad (3-27)$$

For the two parameters characterizing the voltage-controlled current source, the gradient terms taken in respect of the delay $\tau$ are given by substitution of Eq. (3-24) into Eq. (3-3) to yield,

$$\frac{\partial s_{11}^N(j\omega_1,x)}{\partial x_T^K} = \left(\frac{2 \gamma m \omega_1}{r_1 J_1(j\omega_1) \hat{J}_1(j\omega_1)}\right) \left(\sin \omega_1 x_T^K + j\cos \omega_1 x_T^K\right)$$

$$\times \psi_{VDI_{X_T^K}}(j\omega_1,x) V_{VDI_{X_T^K}}(j\omega_1,x), \quad (3-28)$$
and for the gradient with respect to the transconductance \( g_m \), it can be shown that,

\[
\frac{\partial s_{11}^N(j\omega_1,x)}{\partial x_K^{g_m}} = \frac{-2}{r_1J_1(j\omega_1) J_1(j\omega_1)} \left( \cos \omega_1 \tau - j\sin \omega_1 \tau \right)
\]

\[
x VDI_{x_K^{g_m}}(j\omega_1,x) V_{ICI_{x_K^{g_m}}}(j\omega_1,x).
\]  

(3-29)

The formulae given in Eqs. (3-25) to (3-29) are applicable in their stated form for calculation of the gradients of the additional three entries in the scattering matrix of \( N \); only the subscripts on the normalizing resistor and scattering variables need be changed, and the excitation modes in \( N^* \) and \( \hat{N}^* \) for the three remaining cases are illustrated in Fig. (3-2b) to (3-2d).

All gradient-based multidimensional minimization algorithms endeavour to reduce the problem of finding the local minimum of a function to one of successive linear searches. The means employed in determining the direction of search, as well as the actual realization of the search, account in large measure for the proliferation of computation techniques available today to solve the problem of functional minimization.

The method adopted in this work is due to R. Fletcher,\(^{(9)}\) and represents a significant improvement over earlier versions of the variable metric algorithm.\(^{(10,11)}\) Although in this new algorithm, a linear search is done at every iteration, it is not necessary as in
the Fletcher-Powell procedure that it locate a minimum along the line of search; rather, only at least an arbitrary prespecified decrease in the error function undergoing minimization is sought.

A defining property of all variable-metric algorithms is the retention of an approximation $H^{-1}(x)$ to the inverse Hessian matrix of second derivatives of the error function; the approximation is improved as the minimum is approached, and converges at the minimum to the true inverse Hessian $H^{-1}(x)$. To commence the theoretical exposition of the method, let the error function $\varepsilon(x)$ of Eq. (3-1) be expanded into a multivariate Taylor's series about a point $x_p$, and let only three terms of the series be retained so that

$$\varepsilon(x) = \varepsilon(x_p) + (x-x_p)^T \frac{\partial \varepsilon(x_p)}{\partial x} + \frac{1}{2} (x-x_p)^T H(x_p) (x-x_p). \quad (3-30)$$

In order that Eq. (3-30) be valid, it is necessary that $\varepsilon(x)$ be twice differentiable and that its Hessian matrix $H(x)$ be non-singular for all $x$ in a sufficiently large neighborhood of $x_p$. Each entry $h_{ij}$ in $H(x)$ is defined according to convention as

$$h_{ij} = \frac{\partial^2 \varepsilon(x)}{\partial x_i \partial x_j}. \quad (3-31)$$

Setting the gradient of the right-hand side of Eq. (3-30) equal to zero at the minimum $x_0$ of $\varepsilon(x)$, since there the gradient vanishes, enables definition of the Newton-Raphson formula,

$$x_0 = x - H^{-1}(x) \frac{\partial \varepsilon(x)}{\partial x}. \quad (3-31)$$
This formula enables calculation of $x_0$ in one step provided Eq. (3-29) holds true exactly, and that $H^{-1}(x)$ and $\partial \epsilon(x)/\partial x$ are calculable at an initial point $x$. In practice, however, $\epsilon(x)$ will not usually be a positive-definite quadratic form, hence Eq. (3-30) will only be an approximation, and so an iterative technique must be used based upon Eq. (3-31). Let $K$ be the iteration counter, then the iteration is given by,

$$x_K = x_{K-1} - H^{-1}(x_{K-1}) \frac{\partial \epsilon(x_{K-1})}{\partial x}.$$  \hspace{1cm} (3-32)

Most functions approximate to quadratic form in the vicinity of their minima, hence rapid convergence of Eq. (3-32) in the sense that $x_K \rightarrow x_0$ as $K$ becomes large is generally assured near the minima of $\epsilon(x)$. This fact demonstrates the advantage of using a good initial estimate in Eq. (3-32) for $x$; a poor estimate can substantially impair convergence.

Direct use of the iteration Eq. (3-32) requires knowledge of the gradient and Hessian of $\epsilon(x)$; the former vector is computable analytically using the technique of the adjoint network; the inverse Hessian can usually be estimated to good accuracy by successively improving an initial positive-definite matrix which is commonly set to the unit matrix $I$.

In order to simplify the subsequent formulae, introduce now the notation,

$$\gamma = \frac{\partial \epsilon(x_K)}{\partial x} - \frac{\partial \epsilon(x_{K-1})}{\partial x} \text{ and, } \xi = x_K - x_{K-1},$$
where \( \xi \) is taken as a scalar multiple \( \beta \) of a direction of linear search chosen in analogy with Eq. (3-32) so that,

\[
\xi = -\beta \hat{H}^{-1}(x_{K-1}) \frac{\partial \epsilon(x_{K-1})}{\partial x}.
\]  

(3-33)

In order to force a sufficient decrease in \( \epsilon(x) \) on an iteration, it suffices to require that,

\[
0 < \left( \epsilon(x_{K-1}) - \epsilon(x_K) \right) \frac{\partial \epsilon(x_K)}{\partial x} \ll 1.
\]  

(3-34)

Inequality (3-34) can ultimately be satisfied by trying values of \( \beta = 1, w, w^2, \cdots \) with \( 0 < w < 1 \), however, the empirical device was adopted of fitting a cubic interpolating polynomial\(^{(14)}\) to the error function if \( \beta = 1 \) does not satisfy the criterion of Eq. (3-34). By re-evaluating \( \epsilon(x) \) and its gradient at the minimum of the interpolating polynomial, an estimate of the minimum of \( \epsilon(x) \) and its gradient there along the direction of search, Eq. (3-33), may be obtained; this strategy injects an element of independence into the successive \( \xi \), so improving the approximation \( \hat{H}^{-1}(x_K) \) to \( H^{-1}(x_K) \). Experience has shown that usually the choice \( \beta = 1 \) will be taken, hence the efficiency of the algorithm in requiring an average of very nearly one function and gradient evaluation per iteration.
The formula used in updating the matrix \( \hat{H}^{-1}(x) \) are two, namely,

\[
\hat{H}_o^{-1}(x_K) = \hat{H}^{-1}(x_{K-1}) + \frac{\xi \xi^T}{\xi^T \gamma} - \frac{\hat{H}^{-1}(x_{K-1}) \gamma \gamma^T \hat{H}^{-1}(x_{K-1})}{\gamma^T \hat{H}^{-1}(x_{K-1}) \gamma}
\]

(3-35)

and,

\[
\hat{H}_1^{-1}(x_K) = \left( I - \frac{\xi \gamma^T}{\xi^T \gamma} \right) \hat{H}^{-1}(x_{K-1}) \left( I - \frac{\gamma \xi^T}{\xi^T \gamma} \right) + \frac{\xi \xi^T}{\xi^T \gamma}.
\]

(3-36)

Equation (3-35) is that deduced by Fletcher and Powell;\(^{(15)}\) Eq. (3-36) is derived from it by observing that Eq. (3-35) obeys the equality \( \hat{H}^{-1}(x_K)\gamma = \xi \). Then, solving Eq. (3-35) for an updating relation for \( \hat{H}(x_K) \) which will hence force \( \hat{H}(x_K)\xi = \gamma \), and interchanging \( \gamma \) and \( \xi \) in the resulting expression leads directly to Eq. (3-36).

Both Eqs. (3-35) and (3-36) are the extreme members of a general class of updating formula generated by the scalar parameter \( \phi \) according to the convex relation,

\[
\hat{H}_\phi^{-1}(x_K) = (1-\phi) \hat{H}_o^{-1}(x_K) + \phi \hat{H}_1^{-1}(x_K).
\]

(3-37)

Fletcher proves in his description of the updating method\(^{(16)}\) that members of the class given by Eq. (3-37) possess the property that through their use, convergence in a certain sense of \( \hat{H}^{-1} \) to \( H^{-1} \) is assured through tendancy of the eigenvalues of \( \hat{H}^{-1} \) to those of \( H^{-1} \).
In order to deduce a criterion for the choice on a particular iteration of using Eq. (3-35) or (3-36) to accomplish the updating of $\hat{H}^{-1}$, it is useful to examine the relation, (17)

\[
\hat{H}^{-1}(x_K) = \hat{H}^{-1}(x_{K-1}) + \frac{\left(\xi - \hat{H}^{-1}(x_{K-1})\gamma\right)^T \left(\xi - \hat{H}^{-1}(x_{K-1})\gamma\right)}{\gamma^T (\xi - \hat{H}^{-1}(x_{K-1})\gamma)}.
\] (3-38)

This formula is in fact a member of the class governed by Eq. (3-37), but with a parameter $\hat{\varphi}$ corresponding to $\varphi$ being given by,

\[
\hat{\varphi} = \frac{\xi^T \gamma}{\xi^T \gamma - \gamma^T \hat{H}^{-1}(x_{K-1})\gamma}.
\] (3-39)

For $\xi^T \gamma$ strictly positive, a property guaranteed by Eq. (3-34), and with $\hat{H}^{-1}$ positive definite, it is demonstrable that $\hat{\varphi}$ cannot lie in the closed interval $[0,1]$. Now, if $\xi^T \gamma > \gamma^T \hat{H}^{-1}(x_{K-1})\gamma$, then $\hat{\varphi} > 1$ hence use of Eq. (3-36) is proper, whereas if $\xi^T \gamma < \gamma^T \hat{H}^{-1}(x_{K-1})\gamma$, then $\hat{\varphi} < 0$ and so Eq. (3-35) should be used for updating $\hat{H}^{-1}$.

A means is needed of sensing when the minimum of $\varepsilon(x)$ has been determined, hence use is made of the fact that there the Euclidean norm of the gradient vector vanishes so that at the minimum $x_0$, $||\delta(x_0)/\Delta x|| = 0$. In practice, convergence is assumed when $||\delta(x_0)/\Delta x|| \leq \varepsilon$, where $\varepsilon$ is a small number, and where $x_0$ denotes the approximation to the minimum $x_0$ of $\varepsilon(x)$. 
As is the case with most multidimensional minimization algorithms, convergence occurs most rapidly when the surface being traversed is free of elongated contours. In an effort to achieve this condition, the gradient vector is scaled to unity at commencement of minimization. By so doing, it was found that as much as a fivefold decrease in number of iterations required for convergence was attained in comparison to the unscaled case. Let \( \mathbf{Z} \) be the square matrix having scale factors on the diagonal and zeroes elsewhere, and let \( \mathbf{x}' \) represent the vector of variable parameters so scaled as to cause the initial gradient vector to be unity. Then, writing,

\[
\mathbf{x}' = \mathbf{Z}\mathbf{x}, \quad (3-40)
\]

differentiation of both sides of this equation followed by application of the chain-rule yields,

\[
\frac{\partial \varepsilon(\mathbf{x}')}{\partial \mathbf{x}'} = \mathbf{Z}^{-1} \frac{\partial \varepsilon(\mathbf{x})}{\partial \mathbf{x}}. \quad (3-41)
\]

The matrix \( \mathbf{Z} \) is determined so that the initial gradient vector, \( \frac{\partial \varepsilon(\mathbf{x}')}{\partial \mathbf{x}'} \), becomes unity, whence,

\[
\mathbf{Z} = \frac{1}{\frac{\partial \varepsilon(\mathbf{x})}{\partial \mathbf{x}}}. \quad (3-42)
\]

The theory described in the preceding portions of this chapter has been translated into Fortran IV code, and forms the basis of a digital computer program especially designed for automated modelling.
A full listing of the code forms the subject of Appendix B, and contained in Appendix A are instructions for use of the program.

In order to maximize the efficiency of the error-function and gradient computation sections of the program, use has been made of sparse-matrix techniques in the solution of nodal equations. The non-zero structure of the nodal admittance matrix is recorded by a pointer system, and the nodal equations are then reordered by an algorithm (18) which seeks to minimize the number of additional non-zero entries introduced into the admittance matrix by the process of decomposing it into the product of a lower triangular matrix having ones on the diagonal and zeroes above, and an upper triangular matrix having zeroes above the diagonal. (19, 20) From these two matrices, a sequence of forward substitution followed by back substitution is employed to solve for nodal potentials. Further computational efficiency is realized by making use of the fact that the nodal admittance matrix of the adjoint network is the transpose of that for the original network, hence only one decomposition of the admittance matrix is required per frequency point. (21)

Storage space is economized in the minimization routine by storing only the upper portion of the inverse hessian matrix; this is possible owing to the numerical symmetry of the matrix.

Figure (3-3) gives a simple flow-chart of program operation. Input data cards are read in free-format by a routine (22) called by the main program, which then passes control to the sparse-matrix and
pointer set-up subprograms, whereupon the minimization routine is
invoked and repeatedly calls a subroutine to evaluate the error func-
tion and its gradient. Final element values are printed upon con-
clusion of the minimization, and execution is terminated. Error traps
are incorporated to catch elementary faults in input data, thereby
avoiding possible wastes of program time arising from faulty or improp-
erly specified parameters.
Program "Mmodel"

Data input \[\rightarrow\] Control \[\rightarrow\] Data output

Input data cards \[\rightarrow\] Free-format card interpreter "PARSE" \[\rightarrow\] Sparse matrix setup "SETUP" "ORD" INDEX

Load 'Y'matrix \[\rightarrow\] LOADY'

LU decomposition \[\rightarrow\] DECOMP'

Network solution \[\rightarrow\] SOLVE'

Adjoint solution \[\rightarrow\] SOLVEA'

Function minimisation routine \[\rightarrow\] MINIMUM'

Gradient \[\rightarrow\] SCALE'

Printed optimum values

Fig. 3-3
CHAPTER 4. MODELLING OF BIPOLAR JUNCTION TRANSISTORS IN THE SMALL-SIGNAL REGIME

The use of modelling techniques in the design of circuits employing bipolar junction transistors is dictated by the necessity of obtaining an analytically tractable representation of a device characterized by the solutions to rather complex partial differential equations. The actual choice of model configuration and degree of complexity is a compromise between several factors. In particular, the complexity of the model is heavily dependent upon the accuracy expected of it, whereas its configuration is governed by the network structure required to simulate the physical phenomena responsible for transistor operation. These two considerations are, of course, interdependent, so that the optimal equivalent circuit would be one so contrived as to yield maximal accuracy and yet require a minimum number of elements in its realization.

It is convenient in devising a model for the bipolar junction transistor or, for that matter, for any physically realized device, to partition the model into two components.

The primary or intrinsic portion of the model accounts for the physical processes directly responsible for device operation. Additional to this are those model elements necessary to describe spurious parasitic effects external to the active region of the device.

The model adopted to characterize the bipolar junction transistor for the purposes of the work described in this paper is the 'hybrid-pi' configuration to which a delay operator $e^{-j\omega T}$ has been added to the
voltage-controlled current source in order to account for the effects of phase-shift through the device caused by poles non-dominant relative to that causing the initial 20 dB/decade fall in forward current transfer ratio. Since the effective loads imposed upon the device by the type of circuits treated here are of the order of 100 Ω, and whereas the device output resistance is commonly of the order of thousands of ohms, no account has been taken of output resistance arising from base-width modulation effects. The important extrinsic elements of the model are the two inductances in the 'base' and 'emitter' leads of the hybrid-pi circuit; these reactances are included to simulate the inductive effects of the two bonding leads necessary to make connection between the transistor chip being characterized and the transmission lines of the test fixture. Figure (4-1) shows the complete model.

For the hybrid-pi model, certain of the elements can be estimated on the basis of commonly available information, as follows: (23)

\[ g_m = \frac{q|I_c|}{kT} \]
\[ r_\pi = \frac{\beta_F(o)}{g_m} \]
\[ c_\pi = \frac{g_m}{2\pi f_T} - C_u \]

In the above equations, \( I_c \) represents the quiescent device collector current, \( \beta_F(o) \) the static forward current transfer ratio, and \( f_T \) the beta transition frequency. The physical constants \( q, k, \) and \( T \) denote, respectively the electronic charge, Boltzmann's constant, and Kelvin temperature. The remaining elements, \( C_\pi, \tau, \) and \( r_b \) are most readily estimated either by direct measurements upon the device or by physical reasoning based upon an intimate knowledge of the device construction, but in this work an initial guess was made of these values on the basis of previous experience with high-frequency devices.
Fig. 4-1
The transistor chip type HPA 35820-A, (24) manufactured by the Hewlett-Packard Company Microwave Division, was chosen for the amplifier described here for the reasons to be outlined in Chapter 5, and the operating point was selected to be \( I_c = 10 \, \text{mA} \) and \( V_{CE} \approx 5 \, \text{V} \). On this basis, estimates can be calculated for \( g_m, r_m \), and \( C_m \) using the knowledge that \( \beta_F(0) \approx 35 \) and \( f_T \approx 4 \, \text{GHz} \). Therefore, \( g_m \approx 0.38 \, \Omega^{-1}, r_m \approx 100 \, \Omega, \) and \( C_m \approx 12 \, \text{pF} \). Guesses were taken of \( C_m = 0.6 \, \text{pF}, \tau = 50 \, \text{psec}, \) and \( r_b \approx 15 \, \Omega \). The modelling program refined these values to \( g_m = 0.3458 \, \Omega, r_m = 96.45 \, \Omega, \)
\( C_m = 14.87 \, \text{pF}, C_m = 0.5253 \, \text{pF}, \tau = 20.33 \, \text{psec}, \) and \( r_b = 9.46 \, \Omega \).

A complete set of scattering data were taken on an actual 35820-A chip in the common-emitter configuration using a Hewlett-Packard Type 8745A scattering parameter test set, Type 8405A Vector voltmeter, and Type 11600B transistor test fixture. (25) This equipment provides a convenient realization of the basic measurement arrangement of Fig. (2-2). Data were taken at an operating point of \( 10 \, \text{mA} \) and for \( V_{CE} \) of \(+3, +5, \) and \(+8 \, \text{V} \) at frequencies \( 50, 75, 100, 200, 400, 600, \) and \( 800 \, \text{MHz} \); the data taken for \( V_{CE} = +5 \, \text{V} \) are plotted in polar form in Figs. (4-2) through (4-5).

In order to single out the two parasitic inductances of Fig. (4-1) from further consideration during the modelling process, a set of scattering matrices were measured at frequencies \( 200, 400, 600, 800, \) and \( 1000 \, \text{MHz} \) for the chip-to-test-fixture bonding leads. These data were provided to the automated modelling program together with estimates of \( 1.5 \, \text{nH} \) for each inductance, a topological description of the 'L' of inductances resulting after the transistor chip had been short-circuited, and a set of weights set uniformly to unity. The refined values returned by the program were \( L_E = 1.345 \, \text{nH} \) and \( L_B = 1.763 \, \text{nH} \).
Fig. 4-2
Fig. 4-3

-42-
Fig. 4-4
Fig. 4-5
The actual modelling of the chip was accomplished by entering the
topology of the complete circuit of Fig. (4-1) into the modelling pro-
gram. Also included were the estimates already given for the values
of the elements in the hybrid-pi model and the values calculated pre-
viously by the program for $L_E$ and $L_B$. The optimization was done simulta-
neously on the parameters $r_b'$, $r_\pi$, $c_\pi$, $c_\mu$, $g_\mu$, and $r$. Since $s_{11}$ and
$s_{21}$ are most important in untuned wideband amplifier work, weight
factors of 15, 10, 1, and 4 were chosen for $s_{11}, s_{12}, s_{21}$, and $s_{22}$,
respectively. The parameter values in the hybrid-pi model resulting
from use of the program are plotted in Figs. (4-6) through (4-11) for
the device operating point $I_C = 10$ mA and $V_{CC} = 3, 5,$ and $8$ V.

Two tests were done on the model determined in this way for the
35820-A transistor chip in an effort to check its validity. The first
was to see if certain trends, predictable on the basis of qualitative
physical reasoning, could be discerned in the model parameters. Of
especial interest in this respect are the plots of $r_b'$ and $c_\mu$ versus
$V_{CE}$ given in Figs. (4-6) and (4-9). Evident in the latter plot is the
decrease of $c_\mu$ with increasing collector-base voltage, an effect pre-
dicted by the third-power law relating graded junction depletion-layer
capacitance to junction reverse bias. Apparent also in the plot of
$r_b'$ versus $V_{CE}$ is the rise of $r_b'$ with increasing $V_{CE}$ due to the fact
that base-width narrows with increasing $|V_{CE}|$. Base-width shrinkage
causes the rise in $r_b'$ owing to the consequent lengthening of the path
from the base contact pad to the active region of the base.
Fig. 4-6
Fig. 4-7
Fig. 4-8
Fig. 4-9

The graph illustrates the relationship between $V_{CE}$ (volts) and $C_{f\mu}$ (pF). As $V_{CE}$ increases from 3.0 to 8.0 volts, $C_{f\mu}$ decreases from approximately 0.7 to 0.4 pF.
Fig. 4-10
The second, and by far the more dramatic, verification made of the efficacy of the model, was to work backwards and calculate the scattering parameters of the equivalent circuit of Fig. (4-1) and compare these with those measured on the actual device. This was done for all three values of $V_{CE}$ used in biasing the transistor, and the results in the three cases were roughly equivalent; those for the case of $V_{CE} = 5$ V are plotted in Figs. (4-2) through (4-5) along with the experimentally determined data, and it will be seen that the fit is excellent. The effect of the predominant weighting accorded to $s_{11}$ and $s_{21}$ is apparent in the virtually perfect fit of these two parameters at the expense of slight inaccuracies in the fits of $s_{12}$ and $s_{22}$.

It seems likely that with improvements in experimental procedure the fit of the model could be improved, since under measurement techniques more refined than those used here, the effects of spurious reactances could be greatly reduced. In any event, all the benefits hoped for in automated modelling have been realized, and it should be obvious that the techniques used here can be readily extended to the characterization of other linear elements on the basis of scattering data.
CHAPTER 5. DESIGN OF A FAST-RISE SERIES-SHUNT PAIR

The series-shunt feedback pair to be dealt with in this chapter has been designed with a view towards satisfying the risetime ($\leq 500$ ps) and gain (20 dB) requirements set down in Chapter 1. Additionally, it is desirable that the amplifier possess the ability to operate within the context of a 50 $\Omega$ system, and to this end a receiving-end termination of appropriate value has been incorporated into the design. The circuit of the amplifier is the subject of Fig. 5-1.

Hewlett-Packard Type HPA 35820-A NPN transistors were chosen for this design because they possess a unique combination of several highly desirable characteristics, namely, a large gain-bandwidth product $f_T$ of 4 GHz, the capability of handling substantial collector current of up to 45 mA, and an extremely low base spreading resistance of approximately 10 $\Omega$. Another transistor, the Motorola MM-4049, was investigated as a possible candidate for a PNP complement to the HPA 35820-A, but it was found to suffer the severe disadvantage of having a base spreading resistance in excess of 80 $\Omega$. The gain-bandwidth product is optimum in the HPA 35820-A for collector currents of 10-20 mA, hence the biasing arrangement employed in the amplifier is designed to insure standing currents in both transistors of 10 mA. In the case of the output transistor, since the amplifier works into 50 $\Omega$ cable, a maximum permissible output transition from 0 V to -0.5 V requires a 10 mA increase in collector current, and it is apparent that provided this limit on output swing is not exceeded, the second transistor operates in a
state guaranteeing maximum $f_T$. Small-signal fluctuations in collector current of the first transistor are considerably less than those in the output stage, so that the first stage is also optimally biased with a 10 mA collector current. A quiescent value of about 5 V was arbitrarily set for the $V_{CE}$ of both transistors in order to effect a compromise between power dissipation on the one hand, and the desirability of minimizing the $V_{CB}$-dependent collector-base depletion layer capacitance, on the other.

Under these operating conditions, the small-signal model of the transistors is that found by the method of Chapter 4, and use of this equivalent circuit in the appropriate locations in Fig. (5-1) enables the entire amplifier to be characterized for small-signal operation as shown in Fig. (5-2). Analysis of the resulting amplifier model is best undertaken with the aid of a digital computer; use thereof enables many of the approximations essential to manual analysis to be dispensed with, giving a consequent increase of accuracy. Since the ultimate criteria of amplifier performance are to be assessed in the time-domain, the pole-zero approach was adopted for the analysis owing to the intimate relation existing between complex-plane singularities and transient behaviour. Initially, the pole-zero computation program, "Frank"(26) was used for these calculations, and no account was taken of parasitic elements or of the excess-phase of transistors. Therefore, these elements are not shown in Fig. (5-2), but they will be re-introduced later in the design process, so that the final circuit simulations are an accurate reflection of true circuit performance.
Fig. 5-1
Fig. 5-2
The resistor $R_s$ of Fig. (5-2) replaces the transmission-line characteristic impedance and the line terminating resistor of Fig. (5-1) in accordance with the relation,

$$ R_s = R_0 R_T/(R_0 + R_T). $$

(5-1)

Capacitor $C_{E1}$ is incorporated into the circuit for reasons of frequency-response compensation, and a discussion will be given later regarding the determination of an appropriate value for it.

The series-shunt feedback configuration may be regarded as an interconnection of two two-port networks; the partition of the overall circuit into a forward gain path $a_V(s)$ and a feedback network $f(s)$ is shown symbolically in the Fig. (5-2). It should be apparent that the hybrid-basis two-port matrices are the appropriate ones to use in this situation. Identification of $a(s)$ and $f(s)$ is therefore easily made, and the circuits pertinent to the computation of these transfer functions are given as Fig. (5-3a) and (5-3b), respectively.

Using the pole-zero analysis program "Frank", the open-loop gain transfer function $a_V(s)$ of the amplifier was found to be,

$$ a_V(s) = \frac{V_o(s)}{E_s(s)} = \frac{4}{\prod_{k=1}^{4} \left(1 - \frac{s}{Z_{ak}}\right)} \cdot \frac{6}{\prod_{i=1}^{6} \left(1 - \frac{s}{P_{ai}}\right)} \cdot a_V(0) $$

(5-2)
Fig. 5-3

(a) Circuit for open-loop gain calculations

(b) Circuit for feedback network transfer function
where,
\[ p_{a1} = -43 \times 10^7, \quad z_{a1} = -82.2 \times 10^8, \]
\[ p_{a2} = -73.76 \times 10^8 - j 62.2 \times 10^8, \quad z_{a2} = -85.88 \times 10^8, \]
\[ p_{a3} = -73.76 \times 10^8 + j 62.2 \times 10^8, \quad z_{a3} = 22.0 \times 10^{10}, \]
\[ p_{a4} = -80.9 \times 10^8, \quad z_{a4} = 65.83 \times 10^{10}, \]
\[ p_{a5} = -73.95 \times 10^9, \quad \text{and,} \]
\[ p_{a6} = -25.24 \times 10^{10}, \quad a_V(o) = 67.1. \]

For practical purposes, since the response of the amplifier is of interest only for frequencies below about two gigahertz, or 12.56 x 10^9 rad/sec, a simplification of Eq. (5-2) can be made on the basis of retaining only those singularities dominant in their influence upon \( a_V(s) \) in that band. Therefore, to good approximation, Eq. (5-2) can be recast as,

\[ a_V(s) = \frac{\prod_{k=1}^{2} \left(1 - s/z_{a_k}\right)}{\prod_{i=1}^{4} \left(1 - s/p_{a_i}\right)} \cdot a_V(o). \quad (5-3) \]

The poles and zeroes in this equation are identical to their correspondants in Eq. (5-2).

The transfer function of the feedback network is easily calculable analytically through consideration of the circuit of Fig. (5-3b), whence it is seen to be a single-pole function of form,

\[ f(s) = \frac{V_f(s)}{V_o(s)} = \frac{f(o)}{1 - s/p_f}, \quad (5-4) \]
where \( f(o) = -R_E/R_{E1}R_F \) and \( P_f = -(R_{E1}R_F)/(C_{E1}R_{E1}R_F) \). For the values of \( R_{E1} \), \( R_F \) and \( C_{E1} \) used here, \( P_f = 82.6 \times 10^8 \). The standard feedback formula, namely

\[
A_v(s) = \frac{a_v(s)}{1-a_v(s)f(s)} \tag{5-5}
\]

can now be applied to relate closed-loop gain \( A_v(s) \) to the amplifier and feedback network transfer functions.

Proceeding first in the low-frequency case in the limit as \( s \to 0 \), substitution into Eq. (5-5) of the low-frequency analogues of Eqs. (5-3) and (5-4) and solution of the resulting formula for \( R_F \) enables Eq. (5-6) to be deduced.

\[
R_F = R_{E1} \frac{a_v(o) - A_v(o) - a_v(o) A_v(o)}{A_v(o) - a_v(o)} \tag{5-6}
\]

Through the intermediary of this relation, a value of \( R_F \) can be calculated appropriate to give the 20 dB gain figure \( A_v(o) = 10 \). Note that this calculation must be done iteratively, for in fact \( a_v(o) \) is dependent upon the value of \( R_F \) as is evident from consideration of Fig. (5-3a). The iterative cycle commenced with an estimate for \( R_F \) of 150 \( \Omega \), whereupon convergence of the process led to a value for \( R_F \) of 170 \( \Omega \).

Analytic determination of a suitable value for \( C_{E1} \) would commence with multiplication of Eq. (5-5) by the Laplace transform of the Heaviside unit step function, followed by expansion of the resultant product
into partial fractions and the application of the Laplace inversion integral to effect a transformation of the response into the time-domain. From the expression obtained in this way, formulae can be deduced relating $C_{El}$ to rise-time and overshoot. The ultimate objective is minimization of rise-time to a degree commensurate with the restriction that overshoot not exceed ten per cent.

The algebra involved in these computations is exceptionally lengthy and involves in several instances the solution of complicated transcendental equations, a task most efficiently done iteratively using a computer. For these reasons, it was decided to use the computer from the start and to adopt the rather empirical approach of variation of $C_{El}$ followed by pole-zero analysis of the closed-loop model of Fig. (5-2).

Direct substitution into Eq. (5-5) of Eqs. (5-3) and (5-4) and algebraic rearrangement yields,

$$A_v(s) = \frac{a_v(o) \prod_{k=1}^{2} \left(1 - \frac{s}{za_k}\right)}{\prod_{i=1}^{4} \left(1 - \frac{s}{pa_i}\right)} \frac{f(o)}{1 - a_v(o) \prod_{k=1}^{2} \left(1 - \frac{s}{za_k}\right) \prod_{i=1}^{4} \left(1 - \frac{s}{pa_i}\right) \left(1 - \frac{s}{p_f}\right)}$$

$$= \frac{a_v(o) \left(1 - \frac{s}{p_f}\right) \prod_{k=1}^{2} \left(1 - \frac{s}{za_k}\right)}{\left(1 - \frac{s}{p_f}\right) \prod_{i=1}^{4} \left(1 - \frac{s}{pa_i}\right) - a_v(o)f(o) \prod_{k=1}^{4} \left(1 - \frac{s}{za_k}\right)}$$
Equation (5-7) indicates the fact that by variation of $C_E$, and hence of $P_f$, the position of the closed-loop poles of the amplifier can be altered. It is notable that besides appearing in both numerator and denominator of Eq. (5-7) through $P_f$, capacitor $C_E$ also gives rise to zero $Z_{a1}$ in the open-loop amplifier response $a_V(s)$ because of its interaction with the equivalent emitter resistance $R_E R_P/(R_E + R_P)$ of the input transistor. This is an elementary consequence of the fact that a common-emitter stage with a simple parallel R-C network in its emitter lead has a zero lying at $1/RC$.

A plot of the trajectory of the roots of the denominator of Eq. (5-7) as $a_V(o)f(o)$ is varied (27) is the subject of Fig. (5-4). The circles and x's represent zeroes and poles, respectively, of the quantity $a_V(s)f(s)$, whereas the boxes denote the location of the closed-loop zeroes of $1+a_V(s)f(s)$, which are hence the poles of $A_V(s)$.

Under influence of the quantity $a_V(o)f(o)$, the complex pair of poles of $a_V(s)$, $P_{a2}$ and $P_{a3}$, is displaced to the location $P_{A1} = -43.3 \times 10^8 - j 48.2 \times 10^8$, $P_{A2} = -43.3 \times 10^8 + j 48.2 \times 10^8$, whereas $P_{a1}$ is forced out to the location $P_{A3} = -66.2 \times 10^8$. The remaining poles $P_{a4}$ and $P_f$ completely subsume zeroes $Z_{a1}$ and $Z_{a2}$, and so cancel these same quantities in the numerator of Eq. (5-7). Thus, the closed-loop response of the circuit is governed by a constellation of three poles and one zero, as shown in Fig. (5-5), hence it is possible to write for the dominant effects in the closed-loop response the expression,

$$A_V(s) = \frac{A_V(o)}{\prod_{k=1}^2 \left(1 - \frac{s}{P_A k}\right)}.$$ (5-8)
Virtual cancellation

Not to scale
$C_{EI} = 8$ pf

Fig. 5-4
$\alpha: C_{E_1} = 6 \text{ pf}$
$
\beta: C_{E_1} = 8 \text{ pf}$
$
\gamma: C_{E_1} = 10 \text{ pf}$

Fig. 5-5
The value of \( C_{E1} \) adopted in the design of \( C_{E1} = 8 \) pF is that required to place the closed loop complex poles of \( A_v(s) \) on nearly the 45° radials; this positioning yields optimal transient response. Larger values of \( C_{E1} \) cause undesirable post- and pre-transition oscillation in the transient response, whereas smaller values leave opportunity for rise-time improvement through the introduction of a tolerable amount of overshoot in the response.

Shown also in Fig. (5-5) are the closed-loop singularity locations for values of \( C_{E1} = 6 \) pF and \( C_{E1} = 10 \) pF; the coordinates of these singularities are given explicitly in Table (5-1).

<table>
<thead>
<tr>
<th>( C_{E1} )</th>
<th>( A_v(0) )</th>
<th>( P_{A1} )</th>
<th>( P_{A2} )</th>
<th>( P_{A3} )</th>
<th>( Z_{A1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 pF</td>
<td>9.8</td>
<td>-48.9x10^8 - j39.4x10^8</td>
<td>-48.9x10^8 + j39.4x10^8</td>
<td>-92.5x10^8</td>
<td>-11.3x10^9</td>
</tr>
<tr>
<td>8 pF</td>
<td>9.8</td>
<td>-43.3x10^8 - j48.2x10^8</td>
<td>-43.3x10^8 + j48.2x10^8</td>
<td>-66.2x10^8</td>
<td>-84.2x10^8</td>
</tr>
<tr>
<td>10 pF</td>
<td>9.8</td>
<td>-35.9x10^8 - j51.0x10^8</td>
<td>-35.9x10^8 + j51.0x10^8</td>
<td>-57.7x10^8</td>
<td>-67.2x10^8</td>
</tr>
</tbody>
</table>

TABLE 5-1. Dominant Closed-Loop Series-Shunt Pair Singularities.

The onset of ringing in the transient response for \( C_{E1} > 8 \) pF is reflected in the entries of Table (5-1) by the increasing radial angle of the complex pole pair \( P_{A1}, P_{A2} \) for increasing \( C_{E1} \); the angle exceeds 45° for \( C_{E1} > 8 \) pF.

In order to predict in more refined fashion the actual transient response of the amplifier, account must be taken of the physical realization of the circuit. This requires estimates to be obtained for
parasitic effects, so that the final simulation of amplifier performance more closely conforms to physical reality than the idealized analyses just made.

The need for extreme miniaturization of a feedback amplifier possessing subnanosecond risetime is manifest from consideration of the fact that the velocity of propagation of electromagnetic waves in free space is $3 \times 10^{10}$ cm/s; at this velocity, a path length of 36 cm would contribute a $360^\circ$ phase shift at a 1 GHz. This is a liberal figure, for in most media, the propagation velocity is somewhat less than that in free space. It should be apparent from these figures that at microwave frequencies, the propagation delay of signals around its feedback loop can render an amplifier unstable. For this reason, it is essential that the physical length of the loop be maintained at an absolute minimum.

Parasitic reactances also pose a problem at frequencies in the gigahertz region, and these can be dealt with in either of two ways. A common approach is the incorporation of the parasitics into transmission lines. An alternative to this technique, and one more suited to the demands of feedback circuitry, where size is of paramount importance, is the minimization of spurious reactances through reduction of circuit size.

Fortunately, both these problems can be abated simultaneously through application to circuit construction of hybrid thin-film technology. Therefore, the amplifier described herein was fabricated upon a
low-loss sapphire substrate 1.0 inch square by 0.025 inch thick. Resistors were formed from a nichrome film deposited to a thickness giving 50 Ω/square resistivity, and a pattern of pure gold was used to provide conductive interconnections between elements.

Microstrip transmission lines of 50 Ω characteristic impedance provide the input and output signal feeds, and in this way irregularities in the signal path are kept to an absolute minimum. The requisite conductor width required to realize this value of impedance was determined to be 0.025 inch using data published elsewhere, in conjunction with the fact that for the sapphire substrate, εᵣ ≈ 9.8.

Shown in Fig. (5-6) is a diagram of the layout adopted for the amplifier. Connections to the microstrip lines were made using stripline launchers which provide a smooth transition from coaxial cable to microstrip. All supply rails are bypassed to the extensive ground-plane using ceramic chip capacitors; in this way, low impedance levels are maintained on the supply rails from dc to well into the microwave region. A large-area contact is provided by the test fixture to the ground-plane, and fine wires were used to convey supply potentials to the pads provided on the substrate for that purpose.

In spite of the reduction of parasitics afforded by hybrid construction, spurious reactances still exist, and in an accurate simulation of the amplifier, they must be taken into account.
Connexion schedule

A  -0.86 V.
B  -5.7 V.
C  +20 V.
D  -20 V.
E  Ground (0 V.)
F  Input
G  Output

Not to scale:

- Gold
- Nichrome

Fig. 5-6
Parasitic capacitance is not, as might perhaps first be expected, solely a function of resistor and pad area and substrate thickness. Rather, it is a complex function of resistor pattern and material properties of the metal out of which the resistances are fabricated. For the amplifier described here, estimation of these capacitances was made on the basis of earlier work. (30)

Device bonding wires contribute parasitic inductance, and appropriate estimates of values can be done in a straightforward fashion using the fact that the leads are an average 0.03 inch long and 0.0007 inch in diameter. These parasitics manifest themselves in actuality as distributed effects, and the lumped-element approximations used here are only adopted out of the necessity of maintaining a tractable model.

The level-shifting zener diode of Fig. (5-1) can be modelled by the parallel combination of its 18 $\Omega$ dynamic resistance shunted by a 500 pF capacitor.

A difficulty in using existing transient analysis programs in the simulation of high-frequency circuits arises from their inability to accept delay operators directly. Consequently, it becomes necessary to formulate an adequate approximation to the delayed voltage-controlled current sources in the models of the transistors if account is to be taken of excess-phase effects.

A digression will therefore be made at this point to develop an adequate equivalent to the delayed voltage-controlled current source representing that element accurately over the spectrum of frequencies at which the overall amplifier is expected to operate.
Let the complex exponential operator be expanded into Mac Laurin's series, and retain only the first two terms of the expansion, whereupon

\[ e^{-j\omega T} \approx 1 - j\omega T. \]  

(5-9)

The rational approximation of Eq. (5-9) possesses a phase characteristic falling in asymptotic fashion from zero to \(-\pi/2\) at high frequencies, however, it possesses the disadvantage of having considerable magnitude error. For example, a single-pole R-C network having the phase response of the right-hand side of Eq. (5-9) for \(T = 20\) psec causes nearly a 10\% magnitude error at 1 GHz, and for accurate simulation, this is considered too high.

A superior candidate is a cascade of one or more lossless \(\pi\)-sections of the form of Fig. (5-7); the appropriate formulae for these sections can be deduced in terms of image parameters, and the characteristic impedance shown to be,

\[ Z_0 = \sqrt{\frac{L}{C}} \frac{1}{\sqrt{1 - \left(\frac{\omega}{\omega_0}\right)^2}} \]  

(5-10)

where the parameter \(\omega_0\) is given by,

\[ \omega_0 = \frac{2}{\sqrt{LC}} \]  

(5-11)
Fig. 5-7

\[ \frac{0.2V}{0.2NH} \]

\[ \frac{Z_j}{10\Omega} \]

\[ \frac{Z_j}{1pf} \]

\[ \frac{C/2}{1pf} \]

\[ \frac{Q/2}{1pf} \]

\[ Q/2 \]

\[ V \]

\[ + \]

\[ - \]
It can be shown also, that provided the input and output ports of the π-section are terminated in an impedance $Z_0$, then the delay through the section is given by, \(^{31}\)

\[
\tau_s = \frac{\sqrt{LC}}{\sqrt{1 - \left(\frac{\omega}{\omega_0}\right)^2}},
\]

provided $\omega \leq \omega_0$.

It can easily be verified that values of $C = 2 \text{ pF}$, and $L = 0.2 \text{ nH}$ yield $f_0 = \omega_0/2\pi = 16 \text{ GHz}$. In this case, for $f \leq 2 \text{ GHz}$, $\omega/\omega_0 \leq 0.13$, whence $\sqrt{1 - \left(\frac{\omega}{\omega_0}\right)^2} \approx 1$, and $\tau_s \approx 20 \text{ psec}$ and $Z_0 = 10 \Omega$.

Figure (5-7) shows the discrete approximate equivalent of the ideal delayed source, and plotted in Fig. (5-8) are the phase and magnitude curves of the approximating network along with those of the ideal element.

Note the excellence of the agreement, which attests to the validity of the approximation.

All parasitic elements can now be incorporated into the basic equivalent circuit of Fig. (5-2) to yield the circuit given in Fig. (5-9). Although not shown explicitly in that illustration, all voltage-controlled current sources are replaced by the equivalent network of Fig. (5-7) for purposes of computer transient-response simulation using the "Cancer" Program. \(^{32}\)
Fig. 5-8
Fig. 5-9
The results of that simulation, which was carried out with a step-function input to the amplifier equivalent circuit, are plotted in Fig. (5-10). From that graph, the 10% to 90% risetime on the output is seen to be 330 psec, and the overshoot is 9%. These figures are well within the design bounds stated at the outset of this chapter.

In order experimentally to assess the transient response of the actual amplifier a Hewlett-Packard Type 141A oscilloscope equipped with the Type 1425A sampling time base and Type 1411 sampling amplifier was employed. The sampling head used is specified for a risetime < 28 psec, so that in the measurements to be described, the effects of oscilloscope response can essentially be neglected.

A step transition input to the amplifier was generated by a Hewlett-Packard Type 1105A pulser operating into a Type 1106A tunnel-diode mount. Illustrated in Fig. (5-11a) is a photograph of the output of the tunnel-diode pulser after attenuation sufficient to reduce the pulse amplitude to a 10 mV transition. It is evident from that figure that the testing waveform in fact possesses a finite risetime of about 60 psec, hence a small correction must be made for this fact when evaluating the output of the amplifier in the calculation of amplifier risetime. Figs (5-11b,c) show the amplifier output corresponding to the input of Fig. (5-11a). The risetime of the output is about 400 psec, with an overshoot of 10%. These figures agree excellently with the values of risetime ≈ 370 psec and overshoot ≈ 10% as computed using program "Cancer" for the model of Fig. (5-9) driven with a pulse possessing a linear front edge of 60 psec risetime.
FIG. 5-11 SMALL-SIGNAL STEP RESPONSE OF THE SERIES-SHUNT PAIR
Although the amplifier was designed so as to pass negative-going pulses, the use of the positive-going pulse employed in these tests was dictated by the unavailability of a negative-going pulse generator of adequately fast risetime.

To ensure a proper transient response of the amplifier, it is necessary properly to terminate the driving coaxial cable at the amplifier input for minimum reflections in the cable. This is especially important when the sending end is unterminated, as would be the case in a cascade of amplifiers of the type described here.

The input impedance at low frequencies is virtually that of the 50 Ω terminating resistor, since the series feedback at the input effectively increases the open-loop input impedance by a factor of \(1 + a(o)f(o)\). The magnitude of this quantity decreases with increasing frequency, hence so also does the impedance looking into the first stage, and with it the input impedance of the entire circuit.

The scattering parameter approach provides the most convenient vehicle of input impedance measurement, and from the discussion of Chapter 2, it follows that an ideal input termination corresponds, for identical system and normalizing impedances, to a value for \(s_{11}\) of zero.

The magnitude and phase of \(s_{11}\) were calculated for the model of Fig. (5-9) using the A-C analysis capability of the "Cancer" program in conjunction with a small additional program designed to calculate the scattering parameter \(s_{11}\) from the impedance looking into the amplifier.
Measurements were made in the laboratory on the actual circuit, and the data obtained are plotted alongside the analytically determined values of $s_{11}$ in Fig. (5-12).

The forward scattering parameter $s_{21}$ is also of interest, insofar as it gives a measure of the frequency response of the amplifier, hence there are plotted in Fig. (5-13) both analytically determined and experimentally measured values of this parameter. Once again, agreement is excellent between theory and practice.

Hitherto, all measurements done on the amplifier have been of 'small-signal' nature. Even the theoretical analysis has been carried out on the small-signal basis, since linearization of the inherently non-linear phenomena responsible for transistor action demands that only small perturbations about a quiescent operating point be considered. Extension of the analysis to the large-signal case would require use of a full non-linear model for the transistors.

The situation frequently arises, however, that in nuclear pulse amplifiers high-amplitude pulses will be encountered that do not satisfy the small-signal assumption, and so it is instructive at least experimentally to examine the resultant response of the amplifier. The experimental arrangement employed for lack of better equipment a Hewlett-Packard pulse generator Type 215-A having a risetime approximating to 1 nanosecond to generate a large-scale negative-going excursion at the amplifier input. Figure (5-14) shows some of the results of this test. A more detailed interpretation of large-signal response results requires a pulse generator of faster risetime than that used to produce Fig. (5-14).
Any amplifier other than the hypothetical ideal amplifier will contribute noise to the signal passing through it in addition to the intrinsic noise of the signal source. For signals of moderate to large amplitudes, the effect of amplifier noise may be negligible. However, noise performance of the amplifier may in some instances be of importance, for example, if it is to be used to preamplify very low-level signals.

A common measure of the noise behaviour of an amplifier is given by its noise figure \( F \), which is defined as the quotient of input signal-to-noise ratio to output signal-to-noise ratio thus, \(^{(33)}\)

\[
F = \frac{P_{si}/P_{ni}}{P_{so}/P_{no}}, \quad (5-13)
\]

where \( P_{si} \) and \( P_{ni} \) are the input signal power and input signal noise power, respectively, and \( P_{so} \) and \( P_{no} \) are the corresponding quantities for the signals at the amplifier output. The noise figure is proportional to the decadic logarithm of according to the relation, \(^{(5-14)}\)

\[
N = 10 \log_{10} F \text{ dB}. \quad (5-14)
\]
Fig. 5-12
Fig. 5-13
FIG. 5-14a AMPLIFIER INPUT WAVEFORM
1 nsec/div.
20 mV/div.

FIG. 5-14b AMPLIFIER OUTPUT VIA 20 dB ATTENUATOR
1 nsec/div.
20 mV/div.

FIG. 5-14 LARGE-SIGNAL RESPONSE OF THE SERIES-SHUNT PAIR
Spot-noise measurements were taken for $N$ at selected points within the passband of the amplifier using a white noise generator and microwave receiver equipped with a 100 KHz notch filter, and the data obtained thereby are plotted in Fig. (5-15).

As often useful alternative index of amplifier noise behaviour is given by its input-referred noise, a quantity deducible from $N$ if Eq. (5-13) is written in its equivalent form,

$$F = \frac{\overline{e_{si}^2}}{\overline{e_{ni}^2}} \cdot \frac{\overline{e_{no}^2}}{\overline{e_{so}^2}},$$

(5-15)

where $F$ is now expressed in terms of mean-square noise voltages. If $A_V$ denotes amplifier voltage gain, Eq. (5-15) becomes, since

$$\overline{e_{so}^2} = A_V \overline{e_{si}^2},$$

$$F = \frac{1}{A_V} \frac{\overline{e_{no}^2}}{\overline{e_{ni}^2}}.$$  

(5-16)

Let a new variable $\overline{e_{ai}^2}$ be defined to represent the noise generated in the amplifier itself and referenced to the input, so that,

$$\overline{e_{no}^2} = \overline{e_{ni}^2} A_V + \overline{e_{ai}^2} A_V.$$  

(5-17)
Fig. 5-15

Noise figure (dB)

Frequency (MHz)

XBL724 - 2788
Since the amplifier input is assumed to be terminated properly by the resistance $R_T$ of 50 $\Omega$, thermal noise generated within $R_T$ comprises $\overline{e_{ni}^2}$ according to the Nyquist formula,

$$\overline{e_{ni}^2} = 4kT R_T \Delta f,$$  \hspace{1cm} (5-18)

where $T$ is the temperature in degrees Kelvin, and $k$ is the Boltzmann constant. The thermal noise is considered over a frequency interval $\Delta f$ in width. Substitution into Eq. (5-16) of Eqs. (5-17) and (5-18) then yields after rearrangement,

$$\overline{e_{ai}^2} = (F-1) (4kT R_T \Delta f).$$ \hspace{1cm} (5-19)

For an example of the use of Eq. (5-14), $F$ at 400 MHz is, from Eq. (5-19) and Fig. (5-15), $F \approx 1.5$. Assuming room temperature operation of the amplifier input terminating resistor Eq. (5-19) works out to a value of $\overline{e_{ai}^2} \approx 40 \times 10^{-20}$ V$^2$/Hz.

The noise figures found for this amplifier at midband are about 3.5 dB. This value could be improved upon by selecting a lower collector current for the transistors, since under the assumption of constant $\beta_{dc}$ versus $I_C$ in the contemplated range of $I_C$, the high-frequency shot noise generated in the collector and base regions is proportional to $I_C$.\(^{34}\) Noise figure will increase over the midband value at the low end of the amplifier passband due to the onset there of the effects of colored flicker noise.
CHAPTER 6. DESIGN OF A FAST RISE SERIES-SERIES TRIPLE

The series-series triple feedback configuration offers the very substantial advantage over the series-shunt pair just treated of a much higher potentially available loop gain. This in turn leads to a superior gain stability than that possible with the feedback pair. This advantage is not obtained without penalty, however, for owing to the more complex singularity pattern of the triple, it is correspondingly more difficult to compensate for a given transient behaviour than the series-shunt pair. Another difficulty with the feedback triple manifest at high frequencies, is that its increased circuit complexity leads to increased parasitic effects when compared with the simpler series-shunt feedback pair. These factors, in addition to the excess phase delay contributed by the additional transistor of the triple, tend to destabilize the circuit.

Shown in Fig. (6-1) is the full electrical diagram of a series-series triple designed with a view towards meeting the elementary specification set down in Chapter 1; it can easily be verified from the figure that each of the three transistors passes a standing current of 10 mA and sustains $V_{CE} = 5$ V. In order that the output transistor operate at all times at the 10 to 20 mA level of $I_C$ necessary for optimal $f_T$, the output of the amplifier is constrained to be in the range 0 to -0.5 V.
Fig. 6-1
A drawing of the physical layout adopted for the circuit is illustrated in Fig. (6-2). Figure (6-3) shows the full small-signal equivalent circuit, using the hybrid pi model deduced in Chapter 4 for the HPA 35820-A, and including all the attendant parasitic effects. The resistor $R_s$ in series with the input voltage source $E_s$ is defined identically to its counterpart of the preceding chapter.

As in the analysis done for the series-shunt pair, the low-frequency amplifier characteristics are first determined. For the low-frequency case, the series-series triple can be partitioned into its forward gain path and feedback network by opening all capacitors and short-circuiting all inductors of Fig. (6-3), and by noting that the open-circuit impedance matrix is the appropriate two-port formulation for the case of series-series feedback. The result of this operation is shown in Fig. (6-4), whence it is apparent that provided the forward alpha of the output transistor is near unity, the feedback function at d-c can be written as,

$$f(o) = \frac{R_{E1} R_{E3}}{R_L (R_{E1} + R_{E3} + R_F)}$$  \hspace{1cm} (6-1)

This formula relates the voltage fed back to the amplifier input, to the voltage developed at the amplifier output node. Now, the amplifier closed-loop d-c gain $A_V(o)$ is related to the d-c open-loop gain $a_V(o)$ and to $f(o)$ by the equation,

$$A_V(o) = \frac{a_V(o)}{1 - a_V(o) f(o)}.$$  \hspace{1cm} (6-2)
Fig. 6-2

Connexion schedule
A  - 6 V.
B  + 12 V.
C  - 20 V.
D  - 2.5 V.
E  Ground
F  Input
G  Output
H  -10.7+ V.

Not to scale

Gold
Nichrome
Fig. 6-3
(a) d-c open-loop gain calculation

(b) d-c Feedback function calculation

Fig. 6-4

XBL724-2800
Substitution into Eq. (6-2) of Eq. (6-1) followed by algebraic rearrangement yields an expression for the value of feedback resistor $R_F$ in terms of other circuit parameters, as follows,

$$R_F = \frac{R_L \left[ a_V(o) - A_V(o) \right] \left[ R_{E1} + R_{E3} \right] - A_V(o) a_V(o) R_{E1} R_{E3}}{R_L \left[ a_V(o) - a_V(o) \right]}.$$  \hspace{1cm} (6-3)

Several principal factors enter into the choice of values for the emitter resistances $R_{E1}$ and $R_{E3}$. In order to secure a large closed-loop bandwidth in the triple, it is desirable to locate the second and third dominant poles as far from the origin of the complex phase as possible. This requires broadbanding of the first and third stages of the amplifier, a condition achieved by using large values for $R_{E1}$ and $R_{E3}$. On the other hand, high d-c loop gain in the triple, with the consequent advantage of increased desensitivity, demands low values for the two emitter resistances. An additional factor favouring choice of low values for $R_{E1}$ and $R_{E3}$ arises out of the practical consideration that any compensating capacitor used across $R_F$ must have a reasonably large value in order to be physically realizable. Arguing on a first-order basis, it is therefore desirable that $R_F$, hence also $R_{E1}$ and $R_{E2}$, be of low value.

These several conflicting requirements are resolved in the particular selection made for the emitter resistors of $R_{E1} = R_{E2} = 33 \, \Omega$. Once this choice has been taken, Eq. (6-3) must be solved iteratively for the requisite $A_V(o)$ of -10 since $a_V(o)$ depends upon the value of $R_F$ as is apparent from consideration of Fig. (6-3a). Commencement of the iteration
with a value of $R_F = 150 \, \Omega$ led after two cycles to a value of $R_F = 175 \, \Omega$, at which $a_v(0)$ was calculated to be -158.

With the d-c characteristics of the amplifier thus determined in closed-loop, it remains to take up the more difficult topic of the arrangement of suitable compensation to provide the fastest risetime on the output waveform, for a step-function input to the amplifier, consistent with an overshoot not exceeding ten per cent.

Although in the design of the series-shunt pair considered in Chapter 5, a pole-zero analysis was used owing to the intimate connection existing between this representation and time-domain behaviour, it was decided here to use optimization techniques to compensate the circuit. This approach enables direct use to be made of scattering parameters, and provides a link between the methods used for transistor characterization, and the design of the final amplifier. The disadvantage of the method is, of course, that an immediate and readily apparent correlation does not exist between transient response and the scattering parameter representation. Consequently, it was sought only to ensure small peaking and maximum break frequency for $|s_{21}|$ in the hope that this would yield adequate step response.

The program discussed in Chapter 3 was used as the vehicle for optimization; a modification was incorporated into the program to enable the optimization to be carried out with respect only to the magnitude of the forward scattering parameter $s_{21}$. 
In particular, the error function of Eq. (3-1) was reformulated as,

$$\varepsilon(x) = \sum_i \left| s_{21}^A(j\omega_1, x) \right| - \left| s_{21}^G(j\omega_1) \right|^2,$$

where,

$$\left| s_{21}^A(j\omega_1, x) \right| = \left[ \text{Re} \left\{ s_{21}^A(j\omega_1, x) \right\} \right]^2 + \left[ \text{Im} \left\{ s_{21}^A(j\omega_1, x) \right\} \right]^2.$$

The superscript $G$ signifies a given parameter to which fit is sought, and superscript $A$ denotes a quantity calculated from the amplifier circuit of Fig. (6-3). The gradient vector corresponding to the error function of Eq. (6-4) can be computed in terms of $\partial s_{21}^A(j\omega_1, x)/\partial x$ as,

$$\frac{\partial \varepsilon(x)}{\partial x} = \sum_i 2 \left[ \left| s_{21}^A(j\omega_1, x) \right| - \left| s_{21}^G(j\omega_1) \right| \right] \frac{1}{\left| s_{21}^A(j\omega_1, x) \right|} \left[ \text{Re} \left\{ s_{21}^A(j\omega_1, x) \right\} \right].$$

$$\text{Re} \left\{ \frac{\partial s_{21}^A(j\omega_1, x)}{\partial x} \right\} + \text{Im} \left\{ s_{21}^A(j\omega_1, x) \right\} \cdot \text{Im} \left[ \frac{\partial s_{21}^A(j\omega_1, x)}{\partial x} \right].$$

The capacitor $C_F$, located across the feedback resistor $R_F$, was selected as the sole compensating element, and the optimization routine was employed to locate optimally the phantom zero and the inevitable higher-order pole introduced by $C_F$. No application was made of 'pole-splitting' techniques (35) within the amplifier forward gain path, since these would require the use of a capacitor of impractically small value.
The form chosen for the $|s_{21}^G|$ was that of a constant 20 dB out to 1.2 GHz, after which a fixed 60 dB/decade rolloff was followed. The slope used for the rolloff of 60 dB/decade was chosen on the basis of a three pole transfer function for $A_V(s)$. Thus, with the mode of compensation used here, the closed loop gain $A_V(s)$ will also possess three poles, which in the limit yields an asymptotic approach of $|A_V(j\omega)|$ to the -60 dB/decade line. The optimization was done on the full model of Fig. (6-3), thus all parasitics are accounted for from the outset.

Figure (6-5) shows the form of $|s_{21}|$ calculated from the optimized circuit for the optimal value of $C_F = 1.6 \, \text{pF}$. Shown also in Fig. (6-5) is a plot of the phase characteristics of $s_{21}$ of the optimized amplifier. Figure (6-6) shows the input reflection parameter $s_{11}$ in both phase and magnitude, as computed from the optimized circuit. The absence of severe peaking in the plot of $|s_{21}|$ implies a transient response having tolerable overshoot, whilst the plot of $s_{11}$ shows that the input impedance of the amplifier is maintained reasonably constant at $50 \, \Omega$ up to nearly 1 GHz.

In order to assess the transient response of the amplifier, the model of Fig. (6-3) was entered into the "Cancer" program along with the approximation developed in the preceding chapter for the delayed voltage-controlled current sources. The results of this simulation are plotted in Fig. (6-7), whence the figures of risetime $\approx 270$ psec and overshoot $\approx 8\%$ are calculable.
Fig. 6-5
Fig. 6-6
Actual test of the amplifier was made using the equipment employed for evaluation of the series-shunt pair described in Chapter 5. Figure (6-8a) shows the amplifier driving waveform used in the test, and shown in the Figs. (6-8b,c) is the resulting output. The output waveform possesses a risetime of approximately 320 psec with maximum overshoot of about 13%.

The risetime of the experimental device is about 16% slower than the predicted risetime, and the overshoot is about 5% higher. These discrepancies are certainly within the range of acceptability, since estimation of all circuit parasitics was made on a simple first-order basis, and since the passive components are not at their exact values due to fabrication tolerances.

The errors notwithstanding, the response of the amplifier meets for all practical purposes the specification laid down for it in Chapter 1. The overshoot can easily be brought down to less than 10% simply by increasing slightly the value of $C_F$. This procedure would, of course, increase the risetime, but an adequate safety margin of 180 psec exists before the limiting value quoted in the specification of 500 psec is reached.

Scattering parameter evaluations were also made on the amplifier for the input scattering quantity $s_{11}$, and the forward scattering parameter $s_{21}$. The results of these measurements are plotted in Fig. (6-6) and (6-5), respectively, alongside the analytically determined parameters. Agreement is good, and the slightly lower frequency rolloff of $s_{21}$ in the experimental case may be taken as indication of the observed fact that the step response of the actual amplifier is a little slower than predicted.
FIG. 6-8a AMPLIFIER INPUT WAVEFORM
2mV/div.
50 psec/div.

FIG. 6-8b AMPLIFIER OUTPUT WAVEFORM
VIA 20dB ATTENUATOR
2mV/div.
1nsec/div.

FIG. 6-8c AMPLIFIER OUTPUT WAVEFORM
VIA 20 dB ATTENUATOR
2mV/div.
400 psec/div.

FIG. 6-8 SMALL-SIGNAL RESPONSE OF THE SERIES-SERIES TRIPLE.
As was done in Chapter 5 for the series-shunt pair, it is of interest to make an experimental assessment of the large-signal behaviour of the series-series triple. To this end, the amplifier was driven with a variable amplitude version of the same pulse used to evaluate its small-signal response, and risetime measurements were made on the output. Figure (6-9) gives a pictorial example of the large-signal response of the amplifier; the upper photograph shows the input waveform, and shown in the lower photograph is the associated amplifier output. It is apparent that even with an 800 mV amplitude on the output waveform, the risetime and overshoot are impaired only slightly over the small-signal case. A graphical presentation of the large-signal transient-response measurements is the subject of Fig. (6-10), and it will be noted that in all cases, the risetime at the output is held appreciably below the design limit of 500 psec. This very high slew rate is a consequence of the fact that the design of the amplifier incorporates no large capacitors which have to be charged through high impedance sources, and which would hence limit the speed of the output. Slew rate can be calculated directly from Fig. (6-10) for any value of output level by taking the ratio of output risetime, to 80% of the total output voltage excursion. A typical value for slew rate is that for an output transition amplitude of 500 mV, at which an elementary calculation shows that $t_{\text{slew}} \approx 1 \text{ V/ns}$. The mechanism responsible for risetime degradation in the large signal case is most probably the effect of transistor non-linearities, especially those in the output device, which in large signal cases is required to operate over a nearly 3:1 variation in $I_c$, with
a rather lesser change in $V_{CE}$. Quantitative evaluation of these effects requires analysis of the amplifier to be done using the full nonlinear model of the constituent transistors.

Spot measurements were taken of the noise figure $N$ of the series-series triple, and the results plotted in Fig. (6-11) where it is seen that a midband noise-figure of about 4 dB is attained. Corresponding to this, from Eq. (5-19), is an input-referred noise voltage of $\frac{e^2}{a_i} \approx 47 \times 10^{-20} \text{ V}^2/\text{Hz}$. As was the case with the series-shunt pair noise figure, $N$ can be expected to increase for low frequencies due to the effects of flicker noise in the transistors. Similarly, noise figure increases at the high-frequency end of the amplifier passband due to rolloff of amplifier gain, and the increase of transistor $1/f^2$ noise.
FIG. 6-9a AMPLIFIER INPUT WAVEFORM
20 mV/div.
50 psec/div.

FIG. 6-9b AMPLIFIER OUTPUT WAVEFORM VIA 20 dB ATTENUATOR
20 mV/div.
1 nsec/div.

FIG. 6-9c AMPLIFIER OUTPUT WAVEFORM VIA 20 dB ATTENUATOR
20 mV/div.
400 psec/div.

FIG. 6-9 LARGE-SIGNAL RESPONSE OF THE SERIES-SERIES TRIPLE.
Fig. 6-10
Fig. 6-11
CHAPTER 7. CONCLUSION

The efficient design and accurate prediction of performance of linear pulse amplifiers of subnanosecond risetime requires, in addition to standard design and measurement practice, the use of scattering parameters and of the computer.

The use of scattering parameters is especially necessary in the small-signal characterization of high-frequency transistors owing to the difficulty of obtaining reliable data by the more usual approaches of Y- or Z-matrix measurement, or through the application to the device of a series of measurements designed to single out for evaluation a particular element of the small-signal model. The accurate interpretation of scattering data for a particular device is greatly aided by the use of computer programs designed to fit an arbitrary model to the measured data. That this operation can be done efficiently can be verified by noting that one complete characterization of the type discussed in Chapter 4 took 2.07 seconds of central processor time on a CDC 6600 machine.

In the actual design of amplifier circuits intended for high-frequency operation, the use of the computer is essential, for it enables full account to be taken of all spurious reactances consequent to the physical realization of any circuit. Computer aided analysis is further commended if hybrid or integrated circuit methods of circuit construction are contemplated due to the difficulties attendant upon prototype alteration.
Upon conclusion of the amplifier design and construction, scattering parameters provide a convenient vehicle of performance evaluation in the frequency domain, and are useful directly if it is desired that the amplifier be designed to meet a frequency-domain response specification.

The excellent agreement observed in this work between theoretical and experimental data confirms that the approaches described herein are valid, and gives confidence that they can be applied in future work in linear wideband circuit design and yield accurate results.

Of the two feedback configurations treated in this paper, the series-series triple appears theoretically to have a definite superiority over the series-shunt pair. This is a consequence of the improved risetime of the triple, a prediction which was also borne out experimentally. An ancillary benefit of the triple lies in its possession of a potentially higher loop gain than that of the pair; this leads to an increased densensitivity of the triple to parameter variations in comparison with that of the feedback pair.

In respect of noise performance, both the series-shunt pair and series-series triple appear from an experimental standpoint to be about equal. The accuracy of the noise measurements made on the amplifiers was limited by that of the test equipment used; and a more detailed comparison of amplifier noise figure awaits measurements of higher accuracy and greater frequency range than those made here.
The performance of these amplifiers is to an extent limited by the capabilities of the apparatus and technology used in their fabrication, and also by the fact that the designs were carried out as an investigation into the feasibility of attaining subnanosecond risetime feedback amplifiers rather than with the intention of designing a circuit for general use. Suggestions for a number of improvements in, and additions to, the original designs can therefore be made.

In regard to device fabrication, there appears to be no bar to reducing by a factor of, say, two, the size of the active area of the circuit. Such a reduction would yield further decreases in magnitude of the parasitic elements. The effects of a large emitter inductance can be serious, for it leads to degenerative local series feedback which increases with frequency; this causes the stage gain to fall off at a faster rate than would be the case were the emitter inductance absent. It has been found, however, by means of repeated computer aided analysis, that only a small ($\approx 20$ psec in the series-series triple) theoretical improvement in circuit risetime is possible by reducing to zero the emitter inductance parasitics in the circuits described here. This degree of improvement is not significant for the order of magnitude of risetime possessed by these circuits, however, as risetimes are improved, the effects of lead inductance may increase in dominance.
As a safety precaution in a user-oriented design, it would seem advisable to incorporate into the output stage some sort of output current limiting in order to avoid catastrophic failure of the output stage due to excessive current drain. For the HPA 35820-A transistor used in the amplifiers described herein, 45 mA is quoted as the maximum safe collector current. Hence, output stage current limiting could reasonably be set to limit at somewhat above 30 mA, a value corresponding to more than one volt output into 50 ohms, assuming a quiescent emitter current of 10 mA.

As a final modification, it would be of utility to incorporate d-c baseline stabilization into the amplifier. Especially at low pulse duty-cycle rates, an efficient way to accomplish this is through the use of an ancillary circuit which integrates the input and output of the wideband amplifier and furnishes a d-c error voltage to the wideband amplifier of such a polarity as to cause the amplifier output to track the input d-c level. High loop gain in the correction circuit can be attained by using a standard integrated operational amplifier.

Future work along the lines of circuit realization to provide extremely high-speed amplification will most probably necessitate the abandonment of multi-stage feedback configurations due to the inevitable problems in them occasioned by loop length and the consequent destabilizing effect of the transversal delay introduced thereby. Moreover, the parasitics in the circuit tend to introduce additional lags over and above those due to dominant singularities; these additional phase shifts reduce the loop phase margin, thereby increasing the difficulty of applying overall feedback around several stages.
A more profitable approach for amplifiers intended to have risetime of the order of say 100 psec, would be the use of a cascade of single stages, perhaps with local feedback. The use of impedance level mismatching between adjacent stages has the additional virtue of permitting an overall amplifier pole-zero pattern to be synthesized by appealing to each stage in turn since, at least to first order, mismatching would ensure response independence of an intermediate stage from its predecessor and successor.

Transistors are now available with $f_T$ figures of the order of 6 GHz, and although these devices presently suffer the limitation of a restricted range of operating conditions, this disadvantage may be remedied in future.\(^{(36)}\) An alternate route to the direct use of high $f_T$ devices might be the application of novel circuit techniques; for example, the 'f\(_T\)-doubler' circuit proposed recently\(^{(37)}\) by workers at the Tektronix Company is capable of a theoretical doubling of the $f_T$ of the constituent transistors.

The development of higher speed amplifiers will also necessitate the tighter control over parasitic elements, and the more universal application of transmission-line technique than has been used in the circuits described in this paper.
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17. ibid, pg. 319.


22. Computer Program "ROHRER-X" written in Fortran IV for general circuit analysis in dc and ac steady-state conditions.

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24. Hewlett-Packard Company, HPA 35820-A Transistor Specifications. (Microwave Division.)

25. Hewlett-Packard Company, Instruction Handbooks for the following instruments:
   (i) Type 8405A - Vector Voltmeter
   (ii) Type 8745A - Scattering Parameter Test Set
   (iii) Type 11600B - Transistor Test Fixture


References (Continued)


36. See for example, Avantek Company, Type AT-240-B. Avantek Company, 2981 Copper Road, Santa Clara, California 95051.

APPENDICES
APPENDIX A. USE OF THE AUTOMATED MODELLING PROGRAM

Data input to the program is via punched cards; in order to facilitate the task of entering data, the cards are read by the program under a 'free' format wherein fields on a card are separated by a delimiter 'd' which may be a comma or one or more spaces. Numerical information may be given in fixed-point, floating, or exponential form, and any card may be continued an arbitrary number of times by punching as the first character on each continuation card an asterisk. Batch-processing is used, so that any number of data decks may be processed consecutively by the program.

What follows is a description of the various cards required by the program.

[A] TITLE CARD

The title card must be the first card in each data deck, and it may contain arbitrary alphanumeric information descriptive of the problem being done, for example, MODEL OF HPA 35820-A TRANSISTOR AT IC = 10 MA, VCE = 5 V.

[B] SCATTERING PARAMETER CARD

Up to sixty scattering parameter cards may be used. These cards contain the scattering matrices to which the circuit undergoing optimization is to be fitted by the program. Their general form is:
S d FREQUENCY IN Hz d MAGNITUDE S₁₁ d ANGLE S₁₁ d
MAGNITUDE S₁₂ d ANGLE S₁₂ d MAGNITUDE S₂₁ d ANGLE S₂₁ d
MAGNITUDE S₂₂ d ANGLE S₂₂

[C] WEIGHT-FACTOR CARD
This card contains the weighting factors to be used in the error function and also the normalizing resistance under which the scattering parameters of [B], above, are taken. This card is of general form:

WEIGHT d WEIGHT S₁₁ d WEIGHT S₁₂ d WEIGHT S₂₁ d
WEIGHT S₂₂ d NORMALIZING RESISTANCE

One weight card must be included with each problem.

[D] ELEMENT CARDS
These cards describe the elements of the circuit in terms of their topological position and value. The nodes of the circuit are assumed to have been assigned integer numbers with the sole restrictions that the datum node must be numbered zero, and that no number must be used more than once. Element cards are of four classes, as follows:

(i) Resistors
NAME OF UP TO SEVEN CHARACTERS BEGINNING WITH R d
ESTIMATED OR TRUE VALUE d + NODE d - NODE.
(ii) Capacitors

NAME OF UP TO SEVEN CHARACTERS BEGINNING WITH C d
ESTIMATED OR TRUE VALUE d + NODE d - NODE.

(iii) Inductors

NAME OF UP TO SEVEN CHARACTERS BEGINNING WITH L d
ESTIMATED OR TRUE VALUE d + NODE d - NODE.

(iv) Voltage-Controlled Current Sources

NAME OF UP TO SEVEN CHARACTERS BEGINNING WITH GM d
ESTIMATED OR TRUE VALUE OF TRANSCONDUCTANCE d. NAME OF UP
TO SEVEN CHARACTERS BEGINNING WITH TO d ESTIMATED OR TRUE
VALUE OF DELAY d + NODE d - NODE d + CONTROL NODE d - CONTROL
NODE.

[E] PORT CARDS

Two port cards must accompany each problem; that for the input
port is as follows:

INPORT d + NODE d - NODE.

while that describing the output port has the form:

OUTPORT d + NODE d - NODE.

[F] OPTIMIZE CARD

This card describes by name the circuit parameters subject to
variation by the program in the fitting process. This card is of the
following form:
.OPT d MAXIMUM PERMISSIBLE NUMBER OF ITERATIONS IN THE OPTIMIZATION d PRINT INTERVAL IN THE OPTIMIZATION d NAME OF FIRST PARAMETER TO BE OPTIMIZED d NAME OF SECOND PARAMETER TO BE OPTIMIZED d -----

Up to 30 parameters may be optimized simultaneously.

[G] END CARD

This card must be the last card in each problem, and has the form:

.END

An example circuit and data deck are illustrated in Fig. A-1.
Problem:
Calculate CMU, GM, and so on so as to give best agreement of circuit scattering parameters and measured scattering matrices.

Program Input Data:

Test Example
Weight: 1, 1, 1, 1, 50
RB, 10, 01, 02
RPI, 100, 02, 00
CPI, 120, 02, 00
CMU, 0.50, 02, 03
INPUT, 01, 00
GM, 10, 38, TO, 30, 03, 00, 02, 00
OUTPUT, 03, 00
.OPT, 300, 10, GM, CMU, TO
S, .2E-4, .3, -.95, .034, 69, 51.2, 123, .69, -.26
S, 4.5E-4, 22, -.165, .052, 68, 6.5, 100, .55, -.25
S, .0006, .22, -.176, .072, 68, 4.4, 87, .50, -.26
.END

Note that up to 300 iterations will be made. That print-outs of program status will be taken every 10 iterations, and that all scattering parameter weights have been set to unity.

Fig. A-1
APPENDIX B.

Given in this appendix is a full Fortran listing of the program "MODEL". In its present form, it is suitable for use on any of the CDC 6400, 6600, or 7600 computers; minor alterations may be necessary to suit the program for use on other machines. In particular, it may be found necessary on computers having smaller word sizes than the CDC machines to use double-precision in the matrix solution and decomposition routines, and in the optimization subprogram in order to avoid numerical difficulties arising from rounding errors.
PROGRAM MMODEL (INPUT, OUTPUT, TAPE5=INPUT)

R. WILSON. VERSION WITHOUT POST-OPTIMISATION SCATTERING PARAMETER CALCULATION.

*******************************************************************************

MMODEL is a computer program to determine the values of prescribed elements of an R-L-C-G-M-T0 (delay) network required to realise the best fit of the network to a given set of two-port responses. The desired response is furnished to the program in form of a series of up to sixty sets of two-port scattering parameters taken at arbitrary frequencies over the spectrum in which best fit of the network to the given data is sought. The incorporation into this program of provision for simulating delay time and for accepting scattering parameters makes it particularly useful in the modelling of microwave devices in the small-signal region of operation.

PROGRAM CAPACITY IS AS FOLLOWS.

  30 NODES, NOT INCLUDING DATUM.
  120 ELEMENTS.
  R, L, C, AND GM/TO ARE ACCEPTED AS ELEMENTS.
  30 OPTIMISABLE PARAMETERS.
  ZERO-VALUED ELEMENTS ARE NOT PERMITTED.

*******************************************************************************

THIS IS THE CONTROLLING PROGRAM.

COMMON DEclarations.

COMMON /KILL/ NOGO, JEOF
COMMON /OPTIMUM/ WEIGHT(4), JLOCX(30), IDOPT(30), NOPT, NPRINT,
  1 NITERX(30), SCALEX(30)
COMPLEX YDATUM, YNL, VDATUM, VN
COMMON /MATRIX/ YDATUM, YNL(300), VDATUM, VN(30), MATLOC(48R)
COMPLEX PORT
COMMON /PORTDAT/ PORT(60), OMEGA(60), NPOINTS, RNORM, LOCPR1,
  1 LOCPR2
COMMON /NETDATA/ NAME(122), VALUE(122), LOCAL(122), LOCATE(5),
  1 NODPLC(484), NUMNOD
COMMON /CARD/ FIELD(32), NUMFLD, IO, IHALT, ICARD1
DIMENSION TEMP(648)
EQUIVALENCE (TEMP0T(1,1), YNL(1))
DIMENSION TITLE(R)
DIMENSION IDNO(120), JNODE(30)
EQUIVALENCE (IDNO(1), MATLOC(1)), (JNODE(1), MATLOC(121))
DATA ISHIFT/100000000000B/
DATA ILET/228/
DATA ILET/038/
DATA ILET/148/
DATA ILETG/078/
DATA ILETT/248/

C CALCULATE CONSTANTS AND INITIALISE.
PROGRAM MMODEL (INPUT, OUTPUT, TAPE5=INPUT)

R. WILSON, VERSION WITHOUT POST- OPTIMISATION SCATTERING PARAMETER CALCULATION.

#MODEL IS A COMPUTER PROGRAM TO DETERMINE THE VALUES OF PRESCRIBED ELEMENTS OF AN R-L-C-GM-TO (DELAY) NETWORK REQUIRED TO REALISE THE BEST FIT OF THE NETWORK TO A GIVEN SET OF TWO-PORT RESPONSES. THE DESIRED RESPONSE IS FURNISHED TO THE PROGRAM IN FORM OF A SERIES OF UP TO SIXTY SETS OF TWO-PORT SCATTERING PARAMETERS TAKEN AT ARBITRARY FREQUENCIES OVER THE SPECTRUM IN WHICH BEST FIT OF THE NETWORK TO THE GIVEN DATA IS SOUGHT. THE INCORPORATION INTO THIS PROGRAM OF PROVISION FOR SIMULATING DELAY TIME AND FOR ACCEPTING SCATTERING PARAMETERS MAKES IT PARTICULARLY USEFUL IN THE MODELLING OF MICROWAVE DEVICES IN THE SMALL-SIGNAL REGION OF OPERATION.

PROGRAM CAPACITY IS AS FOLLOWS.

30 NODES, NOT INCLUDING DATUM;
120 ELEMENTS;
R, L, G, AND GM/TO ARE ACCEPTED AS ELEMENTS;
30 OPTIMISABLE PARAMETERS;
ZERO-VALUED ELEMENTS ARE NOT PERMITTED.

THIS IS THE CONTROLLING PROGRAM.

COMMON DECLARATIONS.

COMMON /KILL/ NOGO+JEOFI
COMMON /OPTIMUM/ WEIGHT(4),JLOCX(30),IDOPT(30),NOPT+NPRIV,
1 NITER+X(30),SCALEX(30)
COMPLEX YDATOM+YNL+YDATUM+VN
COMMON /MATRIX/ YDATOM+YNL(300)+YDATUM+VN(30)+MATLOC(484)
COMPLEX PORT
COMMON /PORTDAT/ PORT(60+4)+OMEGA(60)+NPONTS+RNORM+LOCPR(1)
1 LOCPR2
COMMON /NETDAT/ NAME(122+2),VALUE(122+2),LOCAL(122)+LOCATE(5),
1 NODLC(484)+NUMMOD
COMMON /CARD/ FIELD(32)+NUMFLD+ID+HALT+ICARD
DIMENSION TEMPORT(64+8)
EQUIVALENCE (TEMPORT(1+1),YNL(11))
DIMENSION TITLE(16)
DIMENSION JNODE(120)+NUMNODE(30)
EQUIVALENCE (IDNO(1)+MATLOC(11)+JNODE(1)+MATLOC(121))
DATA ISHIFT/1000000000000000/
DATA ILETR/228/
DATA ILETC/038/
DATA ILETL/148/
DATA ILGT/078/
DATA ILGT/248/

CALCULATE CONSTANTS AND INITIALISE.
C
TW0PI=8.0*ATAN2(1.0+1e0)
RADIUS=TW0PI/360.0

BEGIN A JOB.

15 CALL SECOND(TIME1)

NPOINTS=0
NUMNODE=0
JE0F=0
NOSTOP=0
IPORT1=0
IPORT2=0
NUMEL=0
WEIGHT=0
NUMOPT=0
NOGO=0

READ JOB TITLE CARD.

READ (5,20) (ITITLE(I),I=1,8)
20 FORMAT (8R10)
23 IF (EOF.EQ.5) 23,24
24 CALL EXIT
27 CALL EXIT
24 PRINT 25, (ITITLE(I),I=1,8)
25 FORMAT (1H6,E1X,120(1H0))///1X,8R10,20X,*•~ MODEL ••••
1/2X,120(1H0))///7X,*— INPUT DATA ——//)

BEGIN, GET A CARD.

ICARD1=1
30 CALL PARSE
40 IF (JE0F.EQ.1) GO TO 2000
42 IF (IHALT.EQ.0) GO TO 31
43 NOGO=1
44 GO TO 30

CHECK FOR CONTROL OR ELEMENT CARD.

31 IF (ID.EQ.8) GO TO 135

ELEMENT CARD.

51 IF (ID.EQ.4) GO TO 50
53 NUMEL=NUMEL+1
54 IF (NUMEL.LT.120) GO TO 40
56 PRINT 35
35 FORMAT (//5X,*——— MORE THAN 120 ELEMENTS//)
62 NOGO=1
63 GO TO 30
64 40 IF (ID.EQ.4) GO TO 45

PASSIVE ELEMENTS.

66 IP0INT=3
67 IST0P=2
70 GO TO 65
C VOLTAGE CONTROLLED CURRENT SOURCES.

71 45 IPOINT=5
72 lSTOP=6
73 GO TO 65
74 50 IF (ID.GT.6) GO TO 105

C PORTS.

100 IP0INT=2
100 STOP=2
101 IF (ID.EQ.6) GO TO 52
104 IPORT1=IPORT1+1
105 IF (IPORT1.LE.1) GO TO 65
107 NOGO=1
110 PRINT 51
51 FORMAT (/5X,'--------- MORE THAN 1 INPUT PORT//')
113 GO TO 30
114 52 IPORT2=IPORT2+1
116 IF (IPORT2.LE.1) GO TO 65
117 NOGO=1
120 PRINT 53
53 FORMAT (/5X,'--------- MORE THAN 1 OUTPUT PORT//')
124 GO TO 30

C SET NODE NUMBERS AND RENUMBER NODES SEQUENTIALLY.

125 65 IF ((IPOINT*ISTOP-1).NE.NUMFLD) GO TO 900
131 LOC1=NOSTOP+1
132 DO 85 I=1,ISTOP
134 NODE=FIELD(IPOINT)
135 IF (NODE.EQ.0) GO TO 80
137 IF (NUMNOO.EQ.0) GO TO 75
140 DO 70 J=1,NUMNOO
141 IF (NODE.NE.JUNODE(J)) GO TO 70
143 NODE=J
144 GO TO 80
144 70 CONTINUE
147 75 NUMNOO=NUMNOO+1
151 JUNODE(NUMNOO)=NODE
153 NODE=NUMNOO
155 NODPLC(NOSTOP)=NODE
157 IPOINT=IPOINT+1
160 85 CONTINUE
162 GO TO (90,90,95,100,100) ID

C SET ELEMENT VALUES AND LOAD ARRAYS.

C PASSIVE ELEMENTS.

176 90 NAME(NUMEL)=FIELD(1)
177 LOCAL(NUMEL)=LOC1
180 VALUE(NUMEL)=FIELD(2)
182 IGN0(NUMEL)=ID
184 GO TO 30
VOLTAGE CONTROLLED CURRENT SOURCES.

95 NAME(NUMEL+1)=FIELD(1)
206 VALUE(NUMEL+1)=FIELD(2)
211 NAME(NUMEL+2)=FIELD(3)
212 VALUE(NUMEL+2)=FIELD(4)
214 LOCAL(NUMEL)=LOC1
215 IDNO(NUMEL)=ID
217 GO TO 30

PORTS.

100 IF (ID.EQ.5) LOCPR1=LOC1
220 IF (ID.EQ.6) LOCPR2=LOC1
230 GO TO 30
231 105 IF (ID.GT.7) GO TO 115

WEIGHTING FACTORS AND NORMALISING RESISTANCE CARD.

IWEIGHT=1
235 IF (NUMFLD.NE.6) GO TO 900
240 RNORM=FIELD(6)
241 DO 110 I=1,6
246 110 WEIGHT(I)=FIELD(I+1)
250 GO TO 30

PORT FREQUENCY RESPONSE DATA.

115 IF (NUMFLD.NE.10) GO TO 900
250 NPOINTS=NPOINTS+1
254 IF (NPOINTS.LE.60) GO TO 125
256 NOGO=1
257 PRINT 120
120 FORMAT ('/SX,*---------- MORE THAN 60 DATA POINTS//')
262 GO TO 30
263 125 OMEGA(NPOINTS)=FIELD(2)
265 DO 130 I=1,8
275 130 TEMPORT(NPOINTS,I)=FIELD(I+2)
277 GO TO 30

CONTROL CARDS.

135 IF (ID.EQ.10) GO TO 155

OPTIMISATION CARD.

302 IF (NUMFLD.LT.3) GO TO 900
305 NUMOPT=1
306 145 DO 150 I=3,NUMFLD
314 150 IDOPT(I-2)=FIELD(I)
316 NOPT=NUMFLD-2
320 NPRINT=FIELD(2)
321 NITER=FIELD(1)
323 GO TO 30
324 900 NOGO=1
325 PRINT 905
329 905 FORMAT ('/SX,*---------- ERROR IN ABOVE CARD//')
331 GO TO 30
ENTRY INTO THE FOLLOWING PART OF THE PROGRAM IMPLIES THAT AN 
END CARD HAS BEEN ENCOUNTERED.

155 IF (NOGO.EQ.0) GO TO 170

ERROR MESSAGES.

165 FORMAT (///10X.-- ERRORS IN INPUT DATA, EXECUTION HALTED --*//)
GO TO 2000
170 IF (NUMEL.GT.0) GO TO 176
NOGO=1
PRINT 175
175 FORMAT (///10X.-- CIRCUIT HAS NO ELEMENTS, EXECUTION HALTED --/*)

176 IF (NUMOPT.GT.0) GO TO 180
NOGO=1
PRINT 177
177 FORMAT (///10X.-- NO OPTIMISATION REQUESTS, EXECUTION HALTED --/*)

180 IF (IPORT1.EQ.1) GO TO 186
NOGO=1
PRINT 185
185 FORMAT (///10X.-- CIRCUIT HAS NO INPUT PORT, EXECUTION HALTED --/*)

186 IF (NPONTS.GT.0) GO TO 188
NOGO=1
PRINT 187
187 FORMAT (///10X.-- NO SCATTERING PARAMETER DATA, EXECUTION HALTED ---/*)

188 IF (IPORT2.EQ.1) GO TO 190
NOGO=1
PRINT 189
189 FORMAT (///10X.-- CIRCUIT HAS NO OUTPUT PORT, EXECUTION HALTED ---/*)

190 IF (IWEIGHT.EQ.1) GO TO 200
NOGO=1
PRINT 191
191 FORMAT (///10X.-- NO WEIGHT CARD, EXECUTION HALTED ---/*)

200 IF (NOGO.EQ.1) GO TO 2000

PROCESS TWO-PORT DATA.

PRINT OUT TWO-PORT DATA.

PRINT 205
205 FORMAT (///10X.-- SCATTERING PARAMETER DATA ---//%X,*FREQ*,7X 
1*MAG S11*,7X,*ANG S11*,7X,*MAG S12*,7X,*ANG S12*,7X,*MAG S21*,7X, 
2*ANG S21*,7X,*MAG S22*,7X,*ANG S22*)
DO 215 I=1,NPOINTS
PRINT 210, OMEGA(I), (TEMPORI(I,J),J=1,8)
210 FORMAT (///5X,E10.3,8(4X,E10.3))
215 CONTINUE

CONVERT PORT DATA FROM POLAR TO RECTANGULAR COMPLEX FORM.
DO 220 I=1,NPOINTS
445 OMEGA(I)=W2*OMEGA(I)
447 DO 216 J=1,4
451 ANG=RADIANS*TEMPRT(I*2*J)
455 XMAG=TEMPRT(I*2*J-1)
460 216 PORT(I*J)=CMPLX(XMAG*COS(ANG),XMAG*SIN(ANG))
477 CONTINUE

C PRINT PORTS AND WEIGHTING FACTORS.
C
502 PRINT 225
506 225 FORMAT (/4X,38X-TWO PORT DATA -/-20X,**NODE*,7X,**NODE**/)
507 KPNOE=NODEPLC(LOCPTI)
508 KNODE=NODEPLC(LOCPTI+1)
511 IF (KPNOE.EQ.0) GO TO 530
512 KPNOE=JUNODE(KPNOE)
513 IF (KNODE.EQ.0) GO TO 235
514 KPNOE=JUNODE(KNODt)
515 235 PRINT 240,KPNOE,KNODE
516 FORMAT (/5X,**INPUT PORT**,5X,15,7X,15)
517 KPNOE=NODEPLC(LOCPT2)
518 KNODE=NODEPLC(LOCPT2+1)
521 IF (KPNOE.EQ.0) GO TO 245
522 KPNOE=JUNODE(KPNOE)
523 IF (KNODE.EQ.0) GO TO 250
524 KPNOE=JUNODE(KNODt)
525 245 PRINT 251, KPNOE, KNODt
526 251 FORMAT (/5X,**OUTPUT PORT**,4X,15,7X,15)
527 PRINT 255, (WEIGHT(I)*I=1,4)
528 255 FORMAT (/5X,**INPUT**,4X,15,7X,15)
529 256 FORMAT (/5X,**NORMALISING RESISTANCE = *E10.3)

C PROCESS WEIGHTING FACTORS.
C
561 DO 257 I=1,4
566 257 WEIGHT(I)=WEIGHT(I)**2
C PROCESS CIRCUIT ELEMENT DATA.
C
SORT ELEMENTS INTO THE ORDER R, C, L, SM/TO, AND ESTABLISH
THE LOCATE ARRAY.

570 LOCATE(I)=1
570 NPLACE=1
571 DO 266 I=1,4
572 IF (NPLACE.GT.NUMEL) GO TO 265
573 NPLACE=NPLACE+1
574 DO 660 J=NPLACE,NUMEL
575 IF (IDNO(J),I.EQ.1) GO TO 260
576 IF (I.EQ.1,NPLACE) GO TO 259
C EXCHANGE ELEMENTS.
C
601 TEMP=NAME(J+1)
610 NAME(J+1)=NAME(NPLACE+1)
611 NAME(NPLACE+1)=TEMP
NTEMP=NAME(J,2)
NAME(J,2)=NAME(NPLACE+2)
NAME(NPLACE+2)=NTEMP
NTEMP=LOCAL(J)
LOCAL(J)=LOCAL(NPLACE)
LOCAL(NPLACE)=NTEMP
NTEMP=IDNO(J)
IDNO(J)=IDNO(NPLACE)
IDNO(NPLACE)=NTEMP
TEMP=VALUE(J,1)
VALUE(J,1)=VALUE(NPLACE+1)
VALUE(NPLACE+1)=TEMP
TEMP=VALUE(J,2)
VALUE(J,2)=VALUE(NPLACE+2)
VALUE(NPLACE+2)=TEMP
NPLACE=NPLACE+1
CONTINUE

PRINT ELEMENT TABULATION.
PRINT 270
270 FORMAT (////////40X,*-- CIRCUIT SUMMARY -*-///)
PASIVE ELEMENTS.

ISTART=1
ISTOP=LOCATE(4)-1
IF (ISTOP.LT.ISTART) GO TO 300
PRINT 275
275 FORMAT (////////40X,*-- PASIVE ELEMENTS -*-///
1NODE*,15X,*VALUE*,15X,*VALUE*,15X,*NODE*)
DO 295 I=ISTART,ISTOP
LOC1=LOCAL(I)
KNOODE=NODPLC(LOC1)
ICNODE=NODPLC(LOC1+1)
IF (KNOODE.EQ.0) GO TO 280
IF (ICNODE.EQ.0) GO TO 285
KNOODE=JUNODE(KNOODE)
ICNODE=JUNODE(ICNODE)
PRINT 285
285 FORMAT (///////40X,*NAME*,8X,*VALUE*,8X,*NAME*,8X,*VALUE*,8X,*NAME*,8X,*VALUE*,8X,*NAME*,8X,*VALUE*)
DO J35 I=ISTART,ISTOP
LOC1=LOCAL(I)

VOLTAGE-CONTROLLED CURRENT SOURCES.

ISTART=LOCATE(4)
ISTOP=LOCATE(5)-1
IF (ISTOP.LT.ISTART) GO TO 338
PRINT 305
305 FORMAT (///////////40X,*-- VOLTAGE CONTROLLED CURRENT SOURCES -*-///
1M*,8X,*NAME*,10X,*DELAY*,5X,*DELAY*,6X,*NODE*,3X,*NOODE*,3X,*NODE*,3X,*NODE*,3X,*NODE*,3X,*NODE*)
DO 335 I=ISTART,ISTOP
LOC1=LOCAL(I)
INTEGRATE SCATTERING-PARAMETERS NORMALISING RESISTANCE INTO STACKS.

C
338 ISTART=LOCATE(2)
339 ISTOP=LOCATE(5)-1
340 IF (ISTOP.LT.ISTART) GO TO 3005
341 K=ISTOP
342 J=ISTOP+2
343 DO 3000 I=ISTART,ISTOP
C MOVE ELEMENTS DOWN BY TWO LOCATIONS IN STACK.
C
3000 CONTINUE
C AMEND LOCATE ARRAY TO REFLECT NEW STATUS OF STACK.
C
3005 DO 310 I=2,5
310 LOCATE(I)=LOCATE(I)+2
C INSERT RNORM ACROSS INPUT AND OUTPUT PORTS.
C
316 I=LOCATE(2)-1
317 VALUE(I-1)=RNORM
318 LOCAL(I)=LOCPRT1
319 I=I-1
320 VALUE(I-1)=RNORM
321 LOCAL(I)=LOCPRT2
C INVERT RESISTANCES AND INDUCTANCES.
C
336 ISTART=1
337 ISTOP=LOCATE(2)-1
338 DO 337 I=ISTART,ISTOP
339 VALUE(I-1)=1.0/VALUE(I-1)
340 ISTART=LOCATE(3)
ISTOP = LOCATE(4) - 1
IF (ISTOP .LT. ISTAR) GO TO 340
DO 339 I = ISTART, ISTOP
339 VALUE(I+1) = VALUE(I+1) / VALUE(I+1)

C Establish Admittance Matrix Sparse Pointer System.
C
340 CALL SETUP
IF (VUGO.EQ.1) GO TO 2000

C Process Optimisation Requests.
C
PRINT 344
344 FORMAT (//////////40X,* OPTIMISATION SUMMARY ----*)
C Establish Optimisation Pointer System.
C
PRINT 345, NITER, NPRINT
345 FORMAT (//////////5X,*A MAXIMUM OF **I4** ITERATIONS WILL BE MADE**//5X, 
*PRINT-OUTS WILL BE MADE EVERY **I4** ITERATIONS**//5X,**-- ELEMENT 
25 TO BE OPTIMISED ----*)
DO 390 I = 1, NOPT
NAM = IDOPT(I)
NAMLET = NAM / ISHIFT
PRINT 346, NAM
346 FORMAT (//5X,R7)
IF (NAMLET .NE. ILETR) GO TO 350
NPLACE = 1
ISTART = LOCATE(2) - 3
IDOPT(I) = 1
GO TO 370
350 IF (NAMLET .NE. ILETC) GO TO 355
NPLACE = 1
ISTART = LOCATE(2)
ISTOP = LOCATE(3) - 1
IDOPT(I) = 2
GO TO 370
355 IF (NAMLET .NE. ILETL) GO TO 360
NPLACE = 1
ISTART = LOCATE(3)
ISTOP = LOCATE(4) - 1
IDOPT(I) = 3
GO TO 370
360 IF (NAMLET .NE. ILETG) GO TO 365
NPLACE = 1
ISTART = LOCATE(4)
ISTOP = LOCATE(5) - 1
IDOPT(I) = 4
GO TO 370
365 IF (NAMLET .NE. ILET! ) GO TO 380
NPLACE = 2
ISTART = LOCATE(4)
ISTOP = LOCATE(5) - 1
IDOPT(I) = 5
GO TO 370
370 IF (ISTART .GT. ISTOP) GO TO 380
DO 375 J = ISTART, ISTOP
375 IF (NAM .NE. NAME(J, NPLACE)) GO TO 375
1241     JLOCX(I)=J
1243     GO TO 390
1243     375 CONTINUE
1246     380 PRINT 385
1246     385 FORMAT (/5X, *---------- ERROR IN ABOVE ELEMENT SPECIFICATION*)
1252     NOGO=1
1253     390 CONTINUE
1256     IF (NOGO.EQ.0) GO TO 393
1257     PRINT 392
1257     392 FORMAT (/5X, *-- ERRORS IN OPTIMISATION REQUESTS, EXECUTION HALTED *)
1262     GO TO 2000
1263     C
1263     LOAD INITIAL PARAMETER VALUES INTO X.*
1271     393 DO 394 J=1, NOPT
1272     394 JLOC=JLOCX(J)
1273     IF (I=OPT(J).EQ.5) NPLACE=2
1276     394 X(J)=VALUE(JLOC, NPLACE)
1303     PRINT 395
1303     395 FORMAT (/40X, *-- OPTIMISATION RESULTS --*/*")
1306     CALL THE OPTIMISATION ROUTINE.
1307     PRINT 405
1307     405 FORMAT (/40X, *-- FINAL CIRCUIT ELEMENT VALUES --*/*")
1313     PASSIVE ELEMENTS.
1315     DO 410 I=1,3
1316     410 NAME(I)=NAME(I)
1318     VAL=VALUE(I,1)
1320     IF (I.EQ.1) ISTOP=ISTOP=2
1323     IF (ISTOP.LT.100) GO TO 410
1325     DO 407 J=ISTART, ISTOP
1328     407 NAME(J)+VALUE(I,1)+VAL
1330     406 FORMAT (/5X,R7.0 = *E10.3)
1332     PRINT 406, NAME(J), VAL
1334     406 FORMAT (/5X,R7.0 = *E10.3)
1353     407 CONTINUE
1356     410 CONTINUE
1360     VOLTAGE CONTROLLED CURRENT SOURCES.
1361     ISTART=LOCATE(4)
1363     ISTOP=LOCATE(5)+1
1366     415 I=ISTART, ISTOP
1368     415 PRINT 420, NAME(I)+VALUE(I)+NAME(I)+VALUE(I)
1370     420 FORMAT (/5X,R7.0 = *E10.3, 10X, R7.0 = *E10.3)
1387     EXIT.
```
2000 CALL SECOND(TIME2)
ET=TIME2-TIME1
PRINT 2005, ET
2005 FORMAT (///////5X,******** TOTAL JOB TIME = •*F10.5,* SECONDS*)
GO TO 15
END

PROGRAM LENGTH INCLUDING I/O BUFFERS
003230

FUNCTION ASSIGNMENTS

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<tr>
<th>STATEMENT</th>
<th>ASSIGNMENTS</th>
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BLOCK NAMES AND LENGTHS
KILL   000002 OPTIMUM- 001777 MATRIX   002200 PORTDAT   001040

VARIABLE ASSIGNMENTS
ANG    002141 ET    002161 FIELD    00000506 I    002133
ICARD1 00043S06 ID    00041506 IDND    001230503 IDOPT    00042504
IHALT  00042S06 ILETG    002116 ILETG    002120 ILETG    002117
ILETP   002115 ILETG    002121 IPOINT    002134 IPOINT    002126
IPORT2 002127 ISHIFT    002114 ISTART    002150 ISTOP    002135
ITITLE 002104 IWEIGHT    002131 J    002140 JENF    000001501
JLOC   002156 JLOCX    000004502 JUNODE    00142503 K    002153
KNODE* 002152 KNODE    002144 KPCNODE    002151 KPCNODE    002143
LOCAL  000750505 LOCATE    001142505 LOPRT1    001035045 LOPRT2    001037504
LOC1   002136 MATLOC    001230503 VAR    002154 NAME    00000505
NAMLET 002155 NITER    000102502 NODE    002137 NODPLC    001147505
NOGO   000000501 NOPT    00100502 VOSTOP    002125 NPLACE    002145
```
SUBROUTINE PARSE

ROUTINE TO INTERPRET A CARD READ UNDER ALPHANUMERIC FORMAT.

ELEMENT CARD

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<th>ID NUMBER</th>
<th>ELEMENT TYPE</th>
<th>CARD CODE</th>
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<tr>
<td>1</td>
<td>RESISTOR</td>
<td>R (228)</td>
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<td>CAPACITOR</td>
<td>C (038)</td>
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<td>3</td>
<td>INDUCTOR</td>
<td>L (148)</td>
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<td>4</td>
<td>VOLTAGE (V)</td>
<td>G (078)</td>
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<tr>
<td></td>
<td>AND TO</td>
<td></td>
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<tr>
<td>5</td>
<td>INPUT PORT</td>
<td>I (118)</td>
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<td>6</td>
<td>OUTPUT PORT</td>
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<td>7</td>
<td>WEIGHTS</td>
<td>W (278)</td>
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<td>8</td>
<td>S-PARAMETERS</td>
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CONTROL CARD

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<th>ID NUMBER</th>
<th>OPERATION</th>
<th>CARD CODE</th>
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<td>9</td>
<td>OPTIMISE</td>
<td>*OP (17208)</td>
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<tr>
<td>10</td>
<td>END</td>
<td>*EN (05168)</td>
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</table>

COMMON DECLARATIONS.

COMMON /KILL/, NOGO, JEOF;
COMMON /CARD/, FIELD(32), NUMFLD, ID, IHALT, ICARD1
DIMENSION IDEL(8), FIELD(10)
DATA IDEL /228, 038, 148, 079, 118, 178, 278, 238/;
DATA ISHIFT/1008/;
DATA JSHIFT/1000000000000008/;
DATA IBLNK/55555555555555555555/;
DATA ISPACE, IASTK, IPOK, ICOMMA, IZERO, IPPLUS, IMINUS, ILETE/559, 478, 1573, 568, 338, 458, 468, 059/;
DATA JOPT/17208/;
DATA JEND/05168/;

INITIALISE.

NUMFLD=0
ID=0
IHALT=0
IF (ICARD1.EQ.0) GO TO 20

READ A CARD.

400 READ (5,401) (IFIELD(I), I=1,10)
401 FORMAT (10B8)
12 IF (EOF.EQ.5) 405,410
15 405 PRINT 406
406 FORMAT (7/5X,**-- INCOMPLETE INPUT DATA. PROGRAM EXECUTION TERMINAT
IEND --*//)
JEOF=1
RETURN
C REMOVE BLANK FIELDS.
C
^10 ISTOP=10
^15 IF (IFIELD(ISTOP).NE.1BLNK) GO TO 20
ISTOP=ISTOP-1
30 IF (ISTOP.LT.1) GO TO 400
31 GO TO 415
32 420 IF (ICARD1.EQ.0) GO TO 430
33 ICARD1=0
34 GO TO 20
35 430 NOFLO=0
36 JUMP=15
37 GO TO 301
C CHECK FOR PRESENCE OF CONTINUE CARD.
C
^35 IF (ICHAR,EQ.ISPACE) GO TO 300
37 IF (ICHAR,NE.IASTK) RETURN
38 PRINT 500, (IFIELD(I)+1,ISTOP)
39 IF (IHALT,EQ.1) GO TO 420
40 GO TO 55
C FIRST PASS.
C
55 PRINT 500, (IFIELD(I)+1,ISTOP)
60 NOFLO=0
61 JUMP=1
62 GO TO 301
67 25 IF (ICHAR,EQ.ISPACE) GO TO 300
71 IF (ICHAR,EQ.IPER) GO TO 31
C ELEMENT CARD. GET ID.
C
73 DO 30 I=1,8
74 IF (ICHAR,NE.IDEL(I)) GO TO 30
76 ID=1
77 GO TO 95
77 30 CONTINUE
80 GO TO 50
C CONTROL CARD. PACK LEADING PERIOD AND FIND ID.
C
82 31 JUMP=14
83 GO TO 300
84 32 JCHAR=ICHAR
85 JUMP=2
86 GO TO 300
87 35 JCHAR=JCHAR*ISHIFT*ICHAR
88 IF (ICHAR,NE.IOPT) GO TO 36
90 ID=9
91 JUMP=3
92 GO TO 300
96 40 IF (ICHAR,EQ.ISPACE) GO TO 55
100 IF (ICHAR,EQ.ICOMM) GO TO 61
GO TO 300
36 IF (ICHAR.NE.JEND) GO TO 50
124 ID=10
125 RETURN
C UNIDENTIFIABLE CARD ERROR MESSAGE.
C 50 IHALT=1
127 PRINT 51
133 FORMAT (/S5X,*-------- ABOVE CARD IS UNIDENTIFIABLE*/*)
GO TO 400
C THIS LOOP IS USED FOR ALL PASSES OTHER THAN THE FIRST.
C IF LAST FIELD WAS ENDED BY A SPACE, IGNORE SPACES AND FIRST COMMA.
C 55 JUMP=4
134 GO TO 300
135 IF (NOFLD.GT.ISTOP) GO TO 400
142 IF (ICHAR.EQ.ISPACE) GO TO 300
144 IF (ICHAR.NE.ICOMMA) GO TO 65
C IF LAST FIELD WAS ENDED BY A COMMA, IGNORE ONLY SPACES.
C 61 IF (NUMFLD.EQ.32) GO TO 400
147 JUMP=5
150 GO TO 300
151 IF (NOFLD.GT.ISTOP) GO TO 400
155 IF (ICHAR.EQ.ISPACE) GO TO 300
157 IF (ICHAR.NE.ICOMMA) GO TO 65
C TWO COMMAS SEPARATED BY SPACES DENOTES A ZERO-VALED FIELD.
C 160 NUMFLD=NUMFLD+1
162 FIELD(NUMFLD)=0.0
163 GO TO 61
C CHECK WHETHER NUMBER OR NAME.
C 65 IF (ICHAR.GE.IZERO) GO TO 125
C NAME FIELD.
C 95 IF (NUMFLD.EQ.32) GO TO 212
171 NUMFLD=NUMFLD+1
172 NAME=0
173 ICW=0
173 ICOUNT=1
175 ICHK=NAME*ISHIFT*ICHAR
200 ICOUNT=ICOUNT+1
202 IF (ICOUNT.GT.7) GO TO 110
205 JUMP=6
206 GO TO 300
206 IF (ICHAR.EQ.ISPACE) GO TO 118
210 IF (ICHAR.EQ.ICOMMA) GO TO 117
212 GO TO 100
C FINISH THE FIELD.
212 110 FIELD(NUMFLD)=NAM
215 JUMP=7
216 GO TO 300
216 115 IF (ICHAR.EQ.ISPACE) GO TO 55
220 IF (ICHAR.EQ.ICOMMA) GO TO 61
222 GO TO 300
C PACK THE FIELD WITH BLANKS.
222 117 ICHK=1
223 118 DO 120 I=ICOUNT,7
231 120 NAME=NAM*ISHIFT*ISPACE
234 121 FIELD(NUMFLD)=NAM
236 IF (ICHK) 55,55,61
C NUMBER FIELD.
240 125 IF (NUMFLD,EQ.32) GO TO 212
242 ICHK=0
242 NUMFLD=NUMFLD+1
244 NUM=0
245 IEXP=0
245 ISET=0
C CHECK FOR A SIGN.
246 ISIN=0
246 IF (ICHAR.EQ.IPLUS) ISIN=1
252 IF (ICHAR.EQ.IMINUS) ISIN=-1
255 IF (ISIN.NE.0) GO TO 130
256 ISIN=1
257 JUMP=9
260 GO TO 138
261 130 JUMP=8
262 GO TO 300
263 135 IF (ICHAR.EQ.ISPACE) GO TO 300
265 JUMP=9
266 136 IF (ICHAR.EQ.ICOMMA) GO TO 205
270 IDIGIT=ICHAR-IZERO
271 IF (IDIGIT.LT.0) GO TO 140
273 IF (IDIGIT.GT.9) GO TO 140
276 NUM=NUM*10+IDIGIT
277 IF (ISET.EQ.0) GO TO 300
301 IEXP=IEXP-1
302 GO TO 300
C CHARACTER IS NOT A DIGIT.
C CHECK FOR SPACE, DECIMAL POINT, E-NOTATION.
303 140 IF (ICHAR.EQ.ISPACE) GO TO 206
305 IF (ICHAR.EQ.IPER) GO TO 155
307 IF (ICHAR.EQ.ILETE) GO TO 160
310 GO TO 210
C DECIMAL POINT.

311 155 IF (ISET.EQ.1) GO TO 210
313    ISET=1
314    GO TO 300

C FORMAT.

315 160 ITEMP=0
316    JUMP=11
317    GO TO 300
320 165 IF (ICHAR.EQ.ISPACE) GO TO 300
322    IEKSIN=0
322    IF (ICHAR.EQ.IPLUS) IEKSIN=1
325    IF (ICHAR.EQ.IMINUS) IEKSIN=-1
330    IF (IEKSIN.NE.0) GO TO 170
331    IEKSIN=1
332    JUMP=13
333    GO TO 178
334 170 JUMP=12
335    GO TO 300
336 175 IF (ICHAR.EQ.ISPACE) GO TO 300
340    JUMP=13
341 178 IF (ICHAR.EQ.ICOMMA) GO TO 179
343    IF (ICHAR.EQ.ISPACE) GO TO 180
345    IDIGIT=ICHAR=ZERO
346    IF (IDIGIT.GT.9) GO TO 210
351    ITEMP=ITEMP*10+IDIGIT
353    GO TO 300
353 179 ICHK=1
354 180 IEXP=IEXP*IEKSIN*ITEMP
357    GO TO 206

C ASSEMBLE THE NUMBER.

360 205 ICHK=1
361 206 IF (NUM) 207*209*207
362 207 IF (IEXP.GT.40) GO TO 210
366    IF (IEXP.LT.-40) GO TO 209
367    IF (IEXP.EQ.0) GO TO 208
370    VAL=NUM*10.0**IEXP
374    IF (VAL.GT.1.0E20) GO TO 210
402    IF (VAL.LT.1.0E-40) GO TO 209
404    FFIELD(NUMFLD)=ISIN*VAL
406 208 FFIELD(NUMFLD)=ISIN*NUM
412    IF (ICHK) 55,55,61
414 209 FFIELD(NUMFLD)=0.0
416    IF (ICHK) 55,55,61

C ERROR MESSAGES.

420 210 IHALT=1
421    PRINT 211, NUMFLD
427    GO TO 400
430 212 IHALT=1
431    PRINT 213


```
213 FORMAT (//5X,*--------- ABOVE CARD HAS MORE THAN 32 FIELDS*)
GO TO 490
C LOCAL SUBROUTINE TO GET NEXT CHARACTER, THE POINT OF RETURN IS
C DETERMINED BY #JUMP#
C
300 IFDPOS=IFDPOS+1
440 IF (IFDPOS.LT.9) GO TO 305
301 NOFLD=NOFLD+1
444 IF (NOFLD.GT.ISTOP) GO TO 310
447 IFLD=FIELD(NOFLD)
450 IFDPOS=1
451 IFDLD=(IFLD-ICARJ*JSHIFT)*JSHIFT
310 ICAP=ISPACE
315 GO TO (25.35,40,60,62,105,115,135,138,210,165,175,178,32,
1435), JUMP
500 FORMAT (5X,10RB)
END

SUBPROGRAM LENGTH
000653

FUNCTION ASSIGNMENTS

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SUBROUTINE SETUP
SUBROUTINE TO ESTABLISH THE SPARSE-MATRIX POINTER SYSTEM.

COMMON DECLARATIONS.

COMMON /KILL/ NNGO, JEOF
COMPLEX YDATUM, YNL, VDATUM, VN
COMMON /MATRIX/ YDATUM, YNL(300), VDATUM, VN(30), MATLOC(488)
DIMENSION KNODES(4)
COMMON /NETDAT/ NAME(122, 2), VALUE(122, 2), LOCAL(122), LOCATE(5)
COMMON /POINT/ IORDER(30), JMNODE(30), IUC(300), IUR(31), MEMO(300)
1 NODPLC(484), NUMNOD

COMMON /POINTS/ IORDER(30), JMNODE(30), IUC(300), IUR(31), MEMO(300),
1 IUS, ILS, LASTUT, LASTLT
DIMENSION ITABLE(30, 5)
EQUIVALENCE (ITABLE(I,J), MATLOC(I))
DATA IUS/30/
DATA ILS/165/
DATA MIRROR/135/

SET UP NODE TABLE.

DO 5 I=1, NUMNOD
   5 ITABLE(I, 1) = 0
   DO 10 ID = 1, 4
   10 ISTART = LOCATE(ID)
   11 ISTOP = LOCATE(ID+1) - 1
   12 IF (ISTART.GT.ISTOP) GO TO 50
   13 NNODE = 2
   14 IF (ID.EQ.4) NNODE = 4
   15 DO 20 I = ISTART, ISTOP
   20 LOC = LOCAL(I)
   21 JSTOP = LOC + NNODE - 1
   22 DO 30 J = 2, JSTOP
   23 ITABLE(I, J) = ITABLE(I, 1) + JSTOP
   24 ITABLE(I, J) = LOCAL(I)
   25 IF (ITABLE(I, J).GE.LOCATE(4)) 
      102 KNOSTP = 4
   26 DO 70 J = 2, ISTOP
   27 IENJM = ITABLE(I + 1, J)
   28 LOC = LOCAL(IENJM)
   29 KNOSTP = 2
   30 IF (IENJM.GE.LOCATE(4)) 
      102 KNOSTP = 4
   31 DO 100 J = 1, KNOSTP
   32 LOC1 = LOC + J - 1
   33 NODE = NODPLC(LOC1)

SET UP IJC AND IUR POINTERS.
THE POINTERS ARE SET UP TO PRESERVE MAXIMUM SPARSITY OF THE MATRIX.
114  KNOODES(L)=NODE
115  1025 CONTINUE
C
C NOW SET THE POINTER, IUR POINTS TO THE LOCATION OF THE FIRST
C CROSS-TERM FOR THAT NODE, IUR IS THE CROSS-TERM NODE.
C
117  DO 1400 K=1,KNOSTP
118    NODEC=KNOODES(K)
119    IF (NODEC.EQ.0) GO TO 1400
120    IF (NODEC.EQ.1) GO TO 1400
121    ISTART=IUR(I)
122    IF (IUCEND.LT.ISTART) GO TO 1320
123    DO 1310 L=ISTART,IUCEND
124      IF (NODEC.EQ.IUC(L)) GO TO 1400
125  1310  IUCEND=IUCEND+1
126      IF (IUCEND.EQ.271) GO TO 1330
127      NOG=1
128      PRINT 1321
129    1321 FORMAT ('///10X,*** MATRICES OVERFLOW TYPE 1, EXECUTION Halted ---///)
130    RETURN
131  1330  IUC(IUCEND)=NODEC
132  1400  CONTINUE
133  1800 CONTINUE
134  2000 CONTINUE
135  IUR(NUMNOD+1)=IUCEND+1
C
C MAKE OPTIMAL REORDERING OF NODAL ADMITTANCE MATRIX.
C
136    CALL OPTORD
137    IF (NOGO.EQ.1) RETURN
C
C ESTABLISH MATRIX LOCATIONS.
C
138    IFIND=0
139    ISTOP=LOCATE(4)=1
140    IF (ISTOP.LT.1) GO TO 130
141    DO 120 I=1,ISTOP
142      LOC=LOCAL(I)
143      NODE1=NODPLC(LOC)
144      NODE2=NODPLC(LOC+1)
145      IF (NODE1.EQ.NODE2) GO TO 108
146      MATLOC(IFIND+1)=NODE1
147      MATLOC(IFIND+2)=NODE2
148      IF (NODE1.EQ.0) GO TO 110
149      IF (NODE2.EQ.0) GO TO 110
150      ISPOT=INDEX(NODE1,NODE2)
151      MATLOC(IFIND+3)=ISPOT
152      IF (ISPOT.GT.ILS) GO TO 106
153      MATLOC(IFIND+4)=ISPOT+MIRROR
154      IFIND=IFIND+4
155      GO TO 120
156      106  MATLOC(IFIND+4)=ISPOT-MIRROR
157      IFIND=IFIND+4
158      GO TO 120
159      108  MATLOC(IFIND+1)=0
MATLOC(IFIND+2)=0
MATLOC(IFIND+3)=0
MATLOC(IFIND+4)=0
IFIND=IFIND+4
CONTINUE
C VOLTAGE CONTROLLED CURRENT SOURCES.
C
ISTART=LOCATE(4)
ISTOP=LOCATE(S)-1
IF (ISTART.GT.ISTOP) GO TO 160
DO 150 I=ISTART,ISTOP
LOC=LOCAL(I)
NODE1=NOOPLC(LOC)
NODE2=NOOPLC(LOC+1)
NODE3=NOOPLC(LOC+2)
NODE4=NOOPLC(LOC+3)
MATLOC(IFIND+1)=INDEX(NODE1,NODE3)
MATLOC(IFIND+2)=INDEX(NODE1,NODE4)
MATLOC(IFIND+3)=INDEX(NODE2,NODE3)
MATLOC(IFIND+4)=INDEX(NODE2,NODE4)
IFIND=IFIND+4
CONTINUE
C SET UP THE ARRAY #MEMO#
C
LE=NUMNOD-1
ISQ=0
DO 171 L=1,LE
IS=IUR(L)
IE=IUR(L+1)-1
IF (IE.LT.IS) GO TO 171
DO 170 IL=IS,IE
IO=IUC(IL)
J=IORDER(IO)
DO 165 IU=IS,IE
JO=IUC(IU)
J=IORDER(JO)
J=INDEX(I+J)
ISQ=ISQ+1
IF (ISQ.GT.300) GO TO 180
MEW0(ISQ)=J
CONTINUE
CONTINUE
CONTINUE
CONTINUE
LASTUT=IUS+IJR(NUMNOD+1)-1
LASTLT=ILS+LASTUT
RETURN
NOGO=1
PRINT 185
FORMAT ('\\10X*** MATRIX OVERFLOW TYPE 2, EXECUTION HALTED ***\\')
RETURN
END

SUBPROGRAM _LENGTH
SUBROUTINE OPTORD

C RENUMBERS THE ROWS OF A STRUCTURALLY SYMMETRIC SPARSE MATRIX IN
C ORDER TO MINIMISE THE NUMBER OF MAJOR ARITHMETIC OPERATIONS REQUIRED
C TO DECOMPOSE IT INTO LU FORM.

C COMMON DECLARATIONS.

COMMON /KILL/ NLOG, JEOF;
COMMON /NETDAT/ NAME(122*2), VALUE(122*2), LOCAL(122), LOCATE(5),
1 NODPLC(484), NUMNOD
COMMON /POINTS/ IORDER(30), JNODE(30), ICT(300), IUR(31), MEMO(300),
1 MISC*ILS, LASTUT, LASTLT
DIMENSION ILD(30), ILR(30), NUMOFF(30), ITA(30), IFILL(30)
EQUIVALENCE (NUMOFF(1), ILD(1), MEMO(1))
EQUIVALENCE (ITA(1), ILR(1), MEMO(31))
EQUIVALENCE (IFILL(1), MEMO(61))

C NN=NUMNOD+1
3 LE=NUMNOD-1

C ASCERTAIN THE NUMBER OF OFF-DIAGONAL ELEMENTS IN EACH ROW.
C
DO 1 J=1, NUMNOD
1 IORDER(J)=J
JNODE(J)=J
1 NUMOFF(J)=IUR(J+1)-IUR(J)
IF (LE.EQ.0) GO TO 110

C FIND ROWS WITH ONLY ONE OFF-DIAGONAL ELEMENT.
C
LOAD=1
1 IRO=1
16 IF (IRO.LT.LOAD) IRO=LOAD
23 IR=TORDER(IRO)
25 NU=1
26 IF (NUMOFF(IR)-1) 18, 20, 17
31 IRO=IRO+1
33 IF (IRO=NUMNOD) 16, 16, 40
35 NU=0

C LOAD THE NEWLY SELECTED ROW.
C
20 IITEMP=IORDER(LOAD)
40 IROT=JNODE(IR)
42 IF (IROT.GT.NUMNOD) GO TO 21
IF (IROT.LT.LOAD) GO TO 21
46 IROEH(IROT)=ITEMP
50 JNODE(ITEMP)=IROT
51 IORDER(LOAD)=IR
53 JNODE(IR)=LOAD
54 LOAD=LOAD+1
55 IF (LOAD=NUMNOD) 21, 110, 110

C SUBTRACT THE COLUMN ELEMENTS OF THE ROW JUST LOADED
C FROM THE RESPECTIVE ROWS IN WHICH THE TERM ALSO APPEARS.
C
IF (WM > 350, 23
   ICT = ICT + 1
   IF (ICT = ICS) 24, 24, 30
   GO TO 16
C
   CALCULATE THE FILL-IN OF EACH REMAINING ROW IF IT WERE TO
   BE LOADED NEXT. A FILL-IN OCCURS AT THE INTERSECTION OF EACH OFF-
   DIAGONAL ELEMENT ROW AND COLUMN. FOR LOADING A ROW, THE POINTER IS
   SET IN FOR THE FILL-INS.
C
   40 IPUTIN = 0
   41 IROF = LOAD
   42 IR = IORDER (IROF)
      IFILL (IROF) = 0
      ICS = IUR (IR + 1) - 1
      NUM = 0
      ICT = IUR (IR)
      43 IC = IUC (ICT)
      IF (JMNODE [IC] = LOAD) 45, 44, 44
      NUM = NUM + 1
      44 ITA (NUM) = IC
      ICT = ICT + 1
      IF (ICT = ICS) 43, 43, 46
C
   DETERMINE FILL-INS OF REMAINING ROWS.
C
   46 NE = NUM - 1
   47 IF (I > NE) GO TO 59
      IRT = ITA (I)
      J = I + 1
      48 IF (J > NUM) GO TO 56
      IC = ITA (J)
      ICS = IUR (IRT + 1) - 1
      ICT = IUR (IRT)
      51 IF (IUC (ICT) .EQ. IC) GO TO 55
      ICT = ICT + 1
      IF (ICT = ICS) 51, 51, 53
      53 IFILL (IROF) = IFILL (IROF) + 1
      IF (IPUTIN .EQ. 0) GO TO 55
C
   ADJUST IJC AND IUR.
C
   57 IDOWN = IUR (NN) - IUR (IRT + 1)
   DO 155 L = 1, IDOWN
      206 K = IUR (NN) - L
      207 IUC (K + 1) = IUC (K)
      213 IUC (K) = IC
      NUMOFF (IRT) = NUMOFF (IRT) + 1
      JS = IRT + 1
      217 DO 156 L = JS, NN
      223 156 IUR (L) = IUR (L) + 1
IF (IUR(NN), GT, 300) GO TO 350
IDOWN = IUR(NN) - IUR(IC + 1)
DO 157 L = 1, IDOWN
K = IUR(NN) - L
157  IUC(K + 1) = IUC(K)
     IUC(K) = IRT
     NUMOFF(IC) = NUMOFF(IC) + 1
     JS = IC + 1
     DO 158 L = JS, NN
158  IUR(L) = IUR(L) + 1
     JS = IC + 1
     GO TO 48
     56 I = I + 1
     GO TO 47
IF (IPUTIN, EQ, 0) GO TO 60
IPUTIN = 0
GO TO 20
IF (IFILL(IROF), GT, 0) 30 TO 61
GO TO 20
IF (IFILL(LOAD1), NE, IPUTIN) GO TO 80
GO TO 41
C SEARCH FOR THE ROW THAT WILL CAUSE THE MINIMUM NUMBER OF
C FILL-INS. IF TWO OR MORE ROWS SATISFY THIS CONDITION, SELECT
C THE ROW WITH THE GREATEST NUMBER OF OFF-DIAGONAL ELEMENTS.
DO 75 J = LOAD + LE
    ITEST = IORDER(J)
    KS = J + 1
    DO 74 K = KS, NUMNOD
    IF (IFILL(K), GE, IFILL(J)) GO TO 74
    IR = IORDER(K)
    IORDER(K) = ITEST
    JNODE(I) = K
    IORDER(J) = IR
    JNODE(IR) = J
    ITEST = IR
    IR = IORD1(K)
    IF (IREQ1(K), LE, IREQ1(I)) GO TO 78
    CONTINUE
    IF (IFILL(LOAD1), NE, IFILL(J)) GO TO 76
CONTINUE
75 CONTINUE
76 KE = J + 1
    ITEST = IORDER(LOAD1)
    KS = LOAD1 + 1
    IF (KE, LT, KS) GO TO 80
    DO 58 K = KS, KE
    IR = IORDER(K)
    IF (NUMOFF(IR), LE, NUMOFF(I)) GO TO 78
    IORD1(K) = ITEST
    JNODE(I) = K
    IORDER(LOAD1) = IR
    JNODE(IR) = LOAD
    ITEST = IR
CONTINUE
78 CONTINUE
80 IPUTIN = 1
GO TO 41
SUBPROGRAM LENGTH
000165

FUNCTION ASSIGNMENTS

STATEMENT ASSIGNMENTS
1  =  000041  2  =  000044  4  =  000020  5  =  000064
6  =  000113  7  =  000116  10  =  000137

BLOCK NAMES AND LENGTHS
NETDAT  =  002114  POINTS  =  001267  MATHIX  =  002200

VARIABLE ASSIGNMENTS
I  =  000060  IC  =  000163  IORDER  =  00000502  IU  =  000161
IUC  =  000074502  IUE  =  000162  IUR  =  000050502  JB  =  000164
JMNODE  =  000036502  L  =  000157  LE  =  000166  LOCAL  =  000750501
LOCATE  =  001142501  MATLOC  =  001230503  MEMO  =  0000750502  NAME  =  00000501
NOPLC  =  001147501  NUMNOD  =  002113501  U  =  000076503  UL  =  000514503
VALUE  =  00364501  VDATUM  =  001132503  VN  =  001134503  YDATUM  =  00000503
YNL  =  000002503

START OF CONSTANTS
000144

START OF TEMPORARIES
000145

START OF INDIRECTS
000147

UNUSED COMPILER SPACE
024500
SUBROUTINE SOLVEA
C
ROUTINE TO PERFORM ADJOINT FORWARD AND BACKWARD SUBSTITUTION
ON THE DECOMPOSED NODAL ADMITTANCE EQUATIONS.
C
COMMON DECLARATIONS.
C
COMMON /NETDAT/ NAME(122,2), VALUE(122,2), LOCAL(122), LOCATE(5),
1 NODEPLC(4,44), NUMNODE
COMMON /POINTS/ IORDER(30), JMNODE(30), IUR(31), MEMO(300),
1 IUUSILS, LASTUT, LASTLT
COMPLEX YDATUM, YNL, VDATUM, VN
COMMON /MATRIX/ YDATUM, YNL, VDATUM, VN(30), VATLOC(498)
COMPLEX U, UL
DIMENSION U(135), UL(135), VN(135)
EQUIVALENCE (U(1), YNL(31)), (UL(1), YNL(166))
C
LE=NUMNODE-1
3 IF (LE.GT.0) GO TO 4
C
ACCOUNT FOR CIRCUITS HAVING ONLY ONE NODE.
C
5 L=IORDER(I)
5 VN(L)=VN(L)/YNL(L)
17 RETURN
C
FORWARD SUBSTITUTION.
C
17 DO 5 I=1, LE
21 L=IORDER(I)
24 VN(L)=VN(L)/YNL(L)
35 IU=IUR(I)
36 IUE=IUH(I)+1
40 IF (IUE-IUE) 2, 2, 5
43 IC=IJC(IU)
45 IC=IORDER(IC)
46 VN(IC)=VN(IC)-U(IU)*VN(L)
61 IU=IU+1
62 GO TO 1
63 5 CONTINUE
C
BACKWARD SUBSTITUTION.
C
66 L=IORDER(NUMNODE)
67 VN(L)=VN(L)/YNL(L)
101 DO 10 I=1, LE
102 L=IORDER(NUMNODE-I)
105 IU=IUR(NUMNODE-I)
106 IUE=IUR(NUMNODE-I)+1
112 IF (IUE-IUE) 7, 7, 10
117 JB=IJC(IU)
120 JB=IORDER(JB)
122 VN(L)=VN(L)-UL(IU)*VN(JB)
134 IU=IU+1
136 GO TO 6
136 10 CONTINUE
141 RETURN
SUBROUTINE SOLVE

ROUTINE TO PERFORM FORWARD AND BACKWARD SUBSTITUTION ON THE DECOMPOSED VODAL ADMITTANCE EQUATIONS.

COMMON DECLARATIONS.

COMMON /NETDAT/ NAME(122,2), VALUE(122,2), LOCAL(122), LOCATE(5),
1 NODPLC(484), NUMNOD
COMMON /POINTS/, IORDER(30), JMNODE(30), IUC(300), IUR(31), MEMO(300),
1 IUS, IL, LASTUT, LASTLT
COMPLEX YDATUM, YNL, VDATUM, VN
COMMON /MATRIX/, YDATUM, YNL(300), VDATUM, VN(30), MATLOC(488)
COMPLEX UL
DIMENSION U(135), UL(135)
EQUIVALENCE (U(1), YNL(3)), (UL(1), YNL(166))

LE=NUMNOD-1
3 IF (LE.GT.0) GO TO 5

ACCOUNT FOR CIRCUITS HAVING ONLY ONE NODE.

L=IORDER(1)
5 VN(L)=YNL(L)/YNL(L)
17 RETURN

FORWARD SUBSTITUTION.

5 DO 20 J=1,LE
21 JS=IUR(J)
22 JES=IUR(J+1)-1
24 IF (JE.LT.JS) GO TO 20
27 L=IORDER(J)
30 DO 15 IL=JS,JE
35 IU=IUC(IL)
40 I=IORDER(IU)
43 VN(J)=VN(J)-UL(IL)*VN(L)
61 20 CONTINUE

BACKWARD SUBSTITUTION.

64 L=IORDER(NUMNOD)
65 VN(L)=VN(L)/YNL(L)
77 DO 100 I=1,LE
100 L=IORDER(NUMNOD-I)
103 JS=IUR(NUMNOD-I)
104 JES=IUR(NUMNOD-I+1)-1
107 IF (JE.LT.JS) GO TO 100
112 DO 35 IU=JS,JE
122 J0=IUC(IU)
123 J=IORDER(JO)
125 35 VN(J)=VN(J)-U(IU)*VN(J)
142 40 VN(L)=VN(L)/YNL(L)
155 RETURN
104  26 PRINT 27
27 FORMAT (//5X*UNDERFLOW DURING LU DECOMPOSITION OF ADMITTANCE MATR
1IX, EXECUTION TERMINATED*)
110  N=1, RETURN
112  END

SUBPROGRAM LENGTH
000162

FUNCTION ASSIGNMENTS

STATEMENT ASSIGNMENTS
21 = 000020  22 = 000041  25 = 000162  26 = 000105
27 = 000117

BLOCK NAMES AND LENGTHS
KILL = 000002  NETDAT = 002114  POINTS = 001267  MATRIX = 002200

VARIABLE ASSIGNMENTS
EPS = 000150  IE = 000156  IL = 000157  IORDER = 000000503
IS = 000155  ISQ = 000151  IU = 000140  IUC = 000074503
IUH = 00550503  J = 000161  J MNODE = 000036503  KK = 000153
L = 000154  LE = 000152  LOCAL = 00750502  LOCATE = 001142502
MATLOC = 001230504  MEMO = 00607503  NAME = 000000502  NODELC = 001147502
NIGO = 000000501  NUMMOD = 02113502  U = 00076504  UL = 00514504
VALUE = 00364502  VDATUM = 001132504  VN = 001134504  YDATUM = 000000504
YNL = 00002504  YPARTS = 000002504

START OF CONSTANTS
000115

START OF TEMPORARIES
000131

START OF INDIRECTS
000140

UNUSED COMPILER SPACE
024600
SUBROUTINE DECOMP

C PERFORMS LU DECOMPOSITION ON THE COMPLEX ADMITANCE MATRIX.
C
C YNL = COMPLEX SPARSE MATRIX OF SYMMETRIC STRUCTURE.
C IUR(L) = LOCATION IN U 1ST NON-ZERO ELEMENT IN ROW L.
C IUC(IU) = REORDERED COLUMN NUMBER FOR ELEMENT IU IN U.
C ORDER(I) = STORED IN THIS ARRAY ARE THE USER-SPECIFIED NODE NUMBERS
C APPEARING IN THE ORDER IN WHICH DECOMPOSITION WILL PROCEED. AN LU
C DECOMPOSITION OF THE REARRANGED MATRIX IS THEN DONE TAKING ADVANTAGE
C OF SPARSITY.
C ID = REORDERED MATRIX ROW NUMBER.
C JO = REORDERED MATRIX COLUMN NUMBER.
C IL = U ARRAY INDEX NUMBER.
C IU = U ARRAY INDEX NUMBER.
C J = ORIGINAL ADMITANCE MATRIX COLUMN NUMBER.
C I = ORIGINAL ADMITANCE MATRIX ROW NUMBER.
C
C COMMON DECLARATIONS.
C
COMMON /KILL/ NOGO*JEQF,
COMMON /NETDAT/ NAME(122,2),VALUE(122,2),LOCAL(122),LOCATE(5),
1 NOPLC(484),NUMNOO
COMMON /POINTS/ ID ORDER(30),JNODE(30),IUC(300),IUR(31),MEMO(300),
1 IU,ILS,LASTLT,LASTL
COMMON YDATU(30),VN,
COMMON /MATRIX/ YDATU(YNL(300),VDATU(VN(30)),MATLOC(488)
DIMENSION YPARTS(2,300)
EQUIVALENCE (YPARTS(1,1),YNL(1))
COMPLEX UL,
DIMENSION U(135),UL(135)
EQUIVALENCE (UL(1),YNL(31),(UL(1),YNL(166))
DATA EPS/1.0E-20/

C BEGIN LU DECOMPOSITION.
C
ISq=0
2  LE=NUMNOO+1
4  KK=ORDER(NUMNOO)
5  IF (ABS(YPARTS(1,1,1))*ABS(YPARTS(2,1,1)))*LE.EQ.EPS) GO TO 26
16  IF (LE.EQ.0) RETURN
21  DO 25 L=1,LE
22  IS=IUR(L)
24  IF (IE.LT.IS) GO TO 25
27  KK=ORDER(L)
30  IF (ABS(YPARTS(1,1,1))*ABS(YPARTS(2,1,1)))*LE.EQ.EPS) GO TO 26
40  DO 24 IL=IS,IE
42  UL(IL)=UL(IL)/YNL(KK)
53  DO 23 IU=IS,IE
56  ISq=ISq+1
61  J=MEMO(ISq)
65  YNL(J)=YNL(J)-UL(IL)*U(IU)
75  CONTINUE
86  CONTINUE
99  CONTINUE
104  RETURN
LOC4 = MATLOC(IFIND+4)
IFND = IFIND+4
YPARTS(2,LOC1) = YPARTS(2,LOC1) + VAL
YPARTS(2,LOC2) = YPARTS(2,LOC2) + VAL
YPARTS(2,LOC3) = YPARTS(2,LOC3) - VAL
YPARTS(2,LOC4) = YPARTS(2,LOC4) - VAL
30 CONTINUE

C LOAD RECIPROCAL INDUCTANCES.

40 ISTART = LOCATE(3)
ISTOP = LOCATE(4) - 1
IF (ISTART.GT.ISTOP) GO TO 60
10 50 I = ISTART, ISTOP
20 VAL = VALUE(I+1)/FREQ
LOC1 = MATLOC(IFIND+1)
LOC2 = MATLOC(IFIND+2)
LOC3 = MATLOC(IFIND+3)
LOC4 = MATLOC(IFIND+4)
IFND = IFND+4
YPARTS(2,LOC1) = YPARTS(2,LOC1) + VAL
YPARTS(2,LOC2) = YPARTS(2,LOC2) + VAL
YPARTS(2,LOC3) = YPARTS(2,LOC3) - VAL
YPARTS(2,LOC4) = YPARTS(2,LOC4) - VAL
50 CONTINUE

C LOAD VOLTAGE-CONTROLLED CURRENT SOURCES.

60 ISTART = LOCATE(4)
ISTOP = LOCATE(5) - 1
IF (ISTART.GT.ISTOP) RETURN
10 50 I = ISTART, ISTOP
20 I = ISTART, ISTOP
Z = FREQ*VALUE(I+1)
VALREAL = VALUE(I+1)*COS(Z)
VALIMAG = -VALUE(I+1)*SIN(Z)
LOC1 = MATLOC(IFIND+1)
LOC2 = MATLOC(IFIND+2)
LOC3 = MATLOC(IFIND+3)
LOC4 = MATLOC(IFIND+4)
IFND = IFND+4
YPARTS(1,LOC1) = YPARTS(1,LOC1) + VALREAL
YPARTS(2,LOC1) = YPARTS(2,LOC1) + VALIMAG
YPARTS(1,LOC2) = YPARTS(1,LOC2) - VALREAL
YPARTS(2,LOC2) = YPARTS(2,LOC2) - VALIMAG
YPARTS(1,LOC3) = YPARTS(1,LOC3) - VALREAL
YPARTS(2,LOC3) = YPARTS(2,LOC3) - VALIMAG
YPARTS(1,LOC4) = YPARTS(1,LOC4) - VALREAL
YPARTS(2,LOC4) = YPARTS(2,LOC4) - VALIMAG
70 CONTINUE
60 RETURN
END

SUBPROGRAM LENGTH
000434

FUNCTION ASSIGNMENTS
SUBROUTINE LOADY(FREQ)
LOADS THE COMPLEX ADMIITANCE MATRIX.

COMMON DECLARATIONS.

COMMON /NETDAT/ NAME(122:2), VALUE(122:2), LOCAL(122), LOCATE(5),
1 NODPLC(484), NUMNOD
COMMON /POINTS/ IDNODE(30), JHNODE(30), IUC(300), IUR(31), MEMO(300),
1 IUS, ILS, LASTUT, LASTLT
COMMON YDATUM, YNL, VDATUM, VN
COMMON /MATRIX/ YDATA(M, YNL), YN(VN(30), MATLOC(488)
DIMENSION YPARTS(2:300)
EQUIVALENCE (YPARTS(1:1), YNL(1))

IFIND=0

INITIALISE ADMITTANCE MATRIX TO ZERO.

YDATUM=CMPLX(0, 0)
DO 99 I=1, NUMNOD
YPARTS(I, I)=0
99 CONTINUE

YPARTS(I, I)=0
DO 999 I=IUS, LASTUT
YPARTS(I, I)=0
999 CONTINUE

IFIND=0

LOAD CONDUCTANCES.

ISTOP=LOCATE(2)-1
IF (ISTOP.LT.1) GO TO 20
DO 10 I=1, ISTOP
VAL=VALUE(I, I)
LOC1=MATLOC(IFIND+1)
LOC2=MATLOC(IFIND+2)
LOC3=MATLOC(IFIND+3)
LOC4=MATLOC(IFIND+4)
IFIND=IFIND+4
YPARTS(I, LOC1)=YPARTS(I, LOC1)+VAL
YPARTS(I, LOC2)=YPARTS(I, LOC2)+VAL
YPARTS(I, LOC3)=YPARTS(I, LOC3)-VAL
YPARTS(I, LOC4)=YPARTS(I, LOC4)-VAL
10 CONTINUE

LOAD CAPACITANCES.

ISTART=LOCATE(2)
ISTOP=LOCATE(3)-1
IF (ISTART.GT.ISTOP) GO TO 40
DO 140 I=ISTART, ISTOP
VAL=VALUE(I, I)*FREQ
LOC1=MATLOC(IFIND+1)
LOC2=MATLOC(IFIND+2)
LOC3=MATLOC(IFIND+3)
140 CONTINUE
SUBROUTINE SCALE(Grad)

SUBROUTINE SCALES PARAMETERS BEING OPTIMISED SO AS TO
IMPRESS CONVERGENCE OF THE MINIMISATION ROUTINE. SCALING IS DONE
TO NORMALISE TO UNITY THE GRADIENT VECTOR AT THE COMMENCEMENT
OF OPTIMISATION.

COMMON DECLARATIONS.

COMMON /NETDAT/ NAME(122,2), VALUE(122,2), LOCAL(122), LOCATE(5),
1 NOPLC(444), NUMNOO
COMMON /OPTIMUM/ WEIGHT(4), JLOCX(30), IDOPT(30), NOPT, NPRINT,
1 NITER, X(30), SCALEX(30)
DIMENSION GRAD(30), DATA EPS/1.0E-10/

VERIFY THAT SCALING IS POSSIBLE. IF NOT, SET SCALE TO UNITY.

DO 5 I=1,NOPT
   IF (ABS(Grad(I)) GT EPS) GO TO 5
   PRINT 2
   2 FORMAT (//5X, "AT LEAST ONE COMPONENT OF THE INITIAL UNSCALED GRADIENT VECTOR IS TOO SMALL TO PERMIT SCALING"//5X, "SCALES WILL THEREFORE BE SET TO UNITY")
   DO 3 J=1,NOPT
   3 SCALEX(J)=1.0
   RETURN
   5 CONTINUE

COMPUTE VECTOR OF SCALE FACTORS.

DO 10 I=1,NOPT
   Z=Grad(I)
   X(I)=X(I)*Z
   SCALEX(I)=1.0/Z
   10 GRAD(I)=1.0/Z
   RETURN

UNSCALE X-VCTOR AND UPDATE ELEMENT STACKS.

ENTRY DESCALE
DO 20 I=1,NOPT
   J=1
   IF (IDOPT(I), EQ, 5) J=2
   20 VALUE (LOC*, J) = X(I) * SCALEX(I)
END

RESCALE THE GRADIENT VECTOR.

ENTRY RESCALE
DO 30 I=1,NOPT
   GRAD(I)=GRAD(I) * SCALEX(I)
   30 RETURN
END
563  DGx=DGX*DGDG
565  DGW=DGDG
566  145  IJ=0
567  DO 150  I=1,NOPT
572  W=DELX(I)/DGDX
573  Z=WORK(I)/DGDG
576  DO 150  J=1,NOPT
605  \#IJ=IJ+1
606  150  HESS(IJ)=HESS(IJ)*\#DELX(J)-Z*WORK(J)

\*C INCREMENT THE ITERATION COUNTERS.
\*C
616  ITN=ITN+1
620  NFIRST=NFIRST+1
621  GO TO 20.
621  END

SUBPROGRAM LENGTH
002243

FUNCTION ASSIGNMENTS

STATEMENT ASSIGNMENTS
10 - 000626  11 - 00022  15 - 000040  20 - 000045
30 - 000722  45 - 00177  50 - 000677  51 - 000214
52 - 000715  55 - 000227  60 - 000730  65 - 000241
70 - 002571  71 - 000750  72 - 000274  84 - 000401
85 - 000771  90 - 00141  91 - 000440  115 - 000444
119 - 000466  125 - 00522  145 - 00567  500 - 000635
505 - 000650  510 - 000973

BLOCK NAMES AND LENGTHS
KILL - 000002  NETDAT - 002114  OPTIMUM - 000177

VARIABLE ASSIGNMENTS
ALFA - 001101  BOUND - 002234  BOUND - 002212  DLG - 001175
DELX - 001173  DGX - 002241  DGDG - 002242  ERNEW - 002236
EROR - 002213  GUX - 002225  GG - 002224  GDX - 002237
GPDX - 002240  GRAD - 001043  HESS - 001271  I - 002222
IOPT - 00042503  IFIRST - 002322  II - 002227  IJ - 002221
ITN - 002216  J - 002223  JLOCX - 00004503  K - 000231
KPRINT - 002215  LIMIT - 002233  LOCAL - 000750502  LOCATE - 001142502
NAME - 00000502  NFCALL - 002214  NFIRST - 002217  NTER - 000012503
NOPLC - 00147502  NOGO - 00000501  NOPT - 000140503  NPRINT - 00101503
SCALEX - 00014503  STEP - 002220  VALUE - 000364502  W - 002235
WEIGHT - 00000503  WORK - 001233  X - 000103503  Z - 002226
ZZ - 002230

START OF CONSTANTS
000624

START OF TEMPORARIES
001017

START OF INJRECTS
001035
IF (N.FIRST.GT.NOPT).AND.((GPDX.LT.0.0)) GO TO 84
IF ((IFIRST.EQ.0).AND.((GPDX.GT.0.0)).OR.(ERRENEW.GT.ERROR))
1 GO TO 90
ZB=BOUND
GO TO 91
84 IT=IT+1
85 FORMAT (/5X,*ULTRA-QUADRATIC BEHAVIOUR DETECTED IN THE ERROR
FUNCTION ON THE */4,*/TH ITERATION*/5X,*REVERSION TO STEEPEST DESCEN
2T NECESSARY*)
GO TO 11
C C INTERPOLATE CUBICALLY TO ESTIMATE A STEP-SIZE IF A STEP-SIZE
C IF ONE DID NOT LEAD TO A SUFFICIENT DECREASE IN THE ERROR FUNCTION,
C AND IF THE INTERVAL BOUNDARY CONDITIONS ARE CORRECT.
C
90 ZB3.0*(ERROR-ERRNEW)+3*GPDX+GOUX
91=0.5*GPDX/ZZ*ATANH(ZZ)
24 ZB1.0-(GPDX+ZZ)/(DGDX+2.0*W)
32 IF (ZB.LT.BOUND) ZB=BOUND
36 IFIRST=1
37 IT=IT+1
91 BOUND=BOUND+0.1
44 LIMIT=LIMIT+1
46 GO TO 70
43 ERROR=ERRNEW
45 DO 116 I=1,NOPT
116 GRAD(I)=WORK(I)
46 IF (DGDX.GT.0.0) GO TO 119
48 GDX=GPDX
49 ZZ=A.0
50 BOUND=BOUND+0.1
51 LIMIT=LIMIT+1
53 IFIRST=0
54 GO TO 70
55 STEP=Z disc.2.0*STEP
56 IF (GPDX.LT.0.5*GDX) STEP=2.0*STEP
C C UPDATE THE INVERSE HESSIAN MATRIX.
C
135 DO 135 I=1,NOPT
135 Z=0.0
137 I=I+1
501 IF (I.EQ.1) GO TO 125
503 I=I-1
504 DO 120 J=1,II
120=Z+HESS(IJ)*DELG(J)
513 120 I=J+NOPT=J
521 125 DO 130 J=1,NOPT
530 Z=Z+HESS(IJ)*DELG(J)
532 130 I=J+1
535 UGMOU+DGMDG+Z*DELG(I)
540 WORK(I)=Z
543 IF (DGMDG.LT.0.0) UGMOU=DGDX*0.01
546 IF (DGDX.LT.DGMDG) GO TO 145
551 W=1.0*UHMMDG/DGDX
553 DO 140 I=1,NOPT
560 DELX(I)=DELX(I)-WORK(I)
C CHECK FOR ROUND-OFF ERRORS OR SINGULAR HESSIAN.
C SHOULD THESE CONDITIONS OCCUR, THE SEARCH WILL BE IN AN ASCENT
C DIRECTION.

55 IF (GDX.LT.0.0) GO TO 65
ITN=ITN+1
PRINT 65, ITN
60 FORMAT (//5X,*ROUND-OFF ERRORS OR SINGULAR HESSIAN ON *I*,*I*-TH IT
ERATION. REVERSION TO STEEPEST DESCENT NECESSARY*)
GO TO 11

C LINEAR SEARCH.
C
C ESTIMATE A SUITABLE STEP SIZE.

65 ZZ=1.0
240 IF (INST=0
LIMIT=0
BOUND=0.1
244 IF (NINST.LT.NOPT) ZZ=STEP
250 W=-X.O*ERROR/GDX
252 IF (W.LT.ZZ) ZZ=W
256 IF (LIMIT.LT.10) GO TO 72
261 ITN=ITN+1
262 PRINT 71, ITN, ERRNEW
71 FORMAT (//5X,*INTERPOLATION LIMIT IN LINEAR SEARCH ENCOUNTERED ON
1THE *I*,*I*-TH ITERATION. EXECUTION TERMINATED*//5X,*FINAL VALUE OF
2 THE ERROR = *E10.*)
RETURN

72 GDXX=GDX*ZZ
273 DO 75 I=1,NOPT
275 Z=ALFA(I)*ZZ
280 DELX(I)=Z
285 X(I)=X(I)+Z
300 CALL DESCALE(WORK)
311 CALL ERRFCN(WORK,ERRNEW)
313 IF (NOMO.EQ.1) RETURN
316 CALL HESCALE(WORK)
320 NFCALL=NFCALL+1
322 GPDX=0.0
333 DO 81 I=1,NOPT
335 GPDX=GPDX+WORK(I)*DELX(I)
338 DO 81 GPDXX=GPDXX+DELX(I)
340 DO 81 GPDXX=GPDXX-0.5
350 CALL HESCALE(WORK)
360 IF (ERROR.GT.(ERRNEW-1.0E-04*GDXX)) GO TO 115
DO 81 I=1,NOPT
81 X(I)=X(I)-DELX(I)
C CHECK FOR ULTRA-QUADRATIC BEHAVIOUR IN THE ERROR FUNCTION PROVIDED.
C AT LEAST NOPT ITERATIONS HAVE BEEN EXECUTED SINCE THE LAST
C REVERSION TO STEEPEST DESCENT.
35 IF (I.EQ.J) HESS(IJ) = 1.0
37 IJ = IJ + 1
C
C COMPUTE SOME USEFUL QUANTITIES.
C
44 GG = 0.0
45 GDX = 0.0
46 DO 40 I = 1, NOPT
47 Z = 0.
48 I = I - 1
51 IF (I.EQ.1) GO TO 30
54 DO 25 J = 1, II
53 II = II - 1
63 Z = Z - HESS(IJ) * GRAD(J)
65 IJ = IJ + NOPT - J
70 DO 35 J = 1, II
70 Z = Z - HESS(IJ) * GRAD(J)
100 IJ = IJ + 1
106 GG = GG + GRAD(I) * 2
110 ALFA(I) = Z
111 40 GDX = GDX + GRAD(I) * Z
115 GG = SQRT(GG)
C
C MAKE A PRINT-OUT IF APPROPRIATE.
C
116 IF (KPRINT .LT. NPRINT) 30 TO 45
121 KPRINT = 0
122 PRINT 500, ITN
500 FORMAT (15X, '--- STATUS OF THE MINIMISATION ROUTINE AFTER
ITHE **I4**-TH ITERATION ------
127 PRINT 510, ERROR, ZG + NCALL
510 FORMAT (15X, 'CALLS TO ERRFCN = **I4**/5X, 'PARAMETER NAME**/10X, 'VECTOR ENTRY
2*10X, 'VECTOR ENTRY**/10X, 'SCALE FACTOR**/10X)
141 DO 515 I = 1, NOPT
144 J = JLOC(I)
146 IF (IDOPT(I), EQ, 5) K = 2
151 PRINT 510, NAME(J, K), X(I), GRAD(I), SCALEX(I)
173 510 CONTINUE
176 45 KPRINT = KPRINT + 1
C
C CHECK CONVERGENCE.
C
200 IF (GG.GT. CONVTOL) GO TO 51
203 PRINT 50, ITN, ERROR
50 FORMAT (15X, 'CONVERGENCE ATTAINED IN **I4** ITERATIONS OF THE MIN
IMISATION ROUTINE**/15X, 'FINAL VALUE OF THE ERROR = **E10.3)
212 RETURN
C
C CHECK IF NUMBER OF ITERATIONS EXCEEDS NITER.
C
213 IF (ITN .LE. NITER) GO TO 55
216 PRINT 52, NITER, ERROR
52 FORMAT (15X, 'NO CONVERGENCE IN **I4** ITERATIONS, EXECUTION MALTE
1D, 'FINAL VALUE OF THE ERROR = **E10.3)
225 RETURN
SUBROUTINE MINIMUM

SUBROUTINE TO FIND THE MINIMUM OF THE ERROR FUNCTION USING THE
DUAL-UPDATE VARIABLE METRIC ALGORITHM OF R. FLETCHER.
THE METHOD IS FULLY DESCRIBED BY FLETCHER IN, "A NEW APPROACH
TO VARIABLE METRIC ALGORITHMS", THE COMPUTER JOURNAL, VOLUME 13,
NUMBER 3, AUGUST, 1970.

ARRAY IDENTITIES ARE AS FOLLOWS:
X VECTOR OF PARAMETERS BEING OPTIMISED
DELX STORES THE CHANGE IN THE X VECTOR BETWEEN LAST AND
PRESENT ITERATIONS
GRAD VECTOR OF GRADIENT OF PARAMETERS BEING OPTIMISED
DELG STORES THE CHANGE IN THE GRADIENT VECTOR BETWEEN LAST AND
PRESENT ITERATIONS
HESS STORES THE UPPER TRIANGULAR PORTION OF THE INVERSE HESSIAN
MATRIX BY ROWS
ALFA STORES THE TRIAL STEP-SIZE FOR THE LINEAR SEARCH
WORK WORKING SPACE

COMMON DECLARATIONS:
COMMON /KILL/ NGO, EIOFI
COMMON /NETDAT/ NAME (122*2), VALUE (122*2), LOCAL (122), LOCAL (5),
1 NODPLC (484), NUMMOD
COMMON /OPTIMUM/ WEIGHT (4), JLOCX (30), IDOPT (30), NOPT, NPRINT,
1 NITER (30), SCALEX (30)
DIMENSION GRAD (30), ALFA (30), DELX (30), DELG (30), WORK (30), HESS (465)

SET CONVERGENCE CRITERION:
DATA CONVTOL / .00002 /

INITIAL FUNCTION EVALUATION:
CALL ERRFCN (GRAD, ERROR)
IF (NGO.EQ.1) RETURN
CALL SCALEIGHAD
PRINT 10, ERROR
10 FORMAT (/ 5X, * INITIAL VALUE OF THE ERROR = *E10.3)

INITIALISE:
NFCALL=1
KPRINT=0
IN=0
NFIRST=0
STEP=1.0

SET INVERSE HESSIAN TO THE UNIT MATRIX.
I J=1
DO 15 I=1, NOPT
15 DO 15 J=1, NOPT
          HESS (IJ) = 0.0
c 25 psi = psi * crfreq
313 go to 40
c contributions of transcondjctance of voltage controlled
c current sources.
c
314 arg = freq * valueloc1,2)
316 psi = psi * complex(cos(arg) - sin(arg))
333 go to 40

c contributions of delay of voltage controlled current sources.

335 arg = freq * value(loc1,2)
337 gomeg = -freq * value(loc1,1)
340 psi = psi * complex(gomeg * sin(arg) + gomeg * cos(arg))
366 sdel(ii,icol) = sdel(ii,icol) * psi
374 sdel(ii,jcol) = sdel(ii,jcol) * psi
402 45 continue
404 icol = 3
405 jcol = 4
406 1000 continue
410 s(1) = s(1) - 1.0
414 s(4) = s(4) - 1.0

c iterate over two-port parameters.

c do 500 k = 1,4

c form the error function.

c 426 wgt = eight(k)
426 diff = s(k) - port(i,k)
433 diffre = real(diff)
434 diffim = imag(diff)
436 error = error + wgt * (diffre * 2 + diffim * 2).

c form the gradient of the error function.

c 442 do 400 j = 1, nopt
452 grad(j) = grad(j) + wgt * (diffre * real(sdel(j,k)) +
1 diffim * imag(sdel(j,k)))
457 400 continue
462 500 continue
464 5000 continue
466 rinv = 2.0 * rinv
470 do 900 i = 1, nopt
474 900 grad(i) = grad(i) * rinv
476 return
476 end

subprogram length
001550

function assignments
JNODE2=NODE2
IF (K.EQ.2) .OR. (K.EQ.4) JNODE2=NODE4
C SET NODE VOLTAGE VECTOR TO ZERO.
C DO 5 II=1,NUMNOD
  106 VPART(1,II)=0.0
  107 5 VPART(2,II)=0.0
C ESTABLISH EXCITATION.
C VPART(1,JNODE1)=-1.0
  116 VPART(1,JNODE2)=+1.0
  120 VDAT JM=CMPLX(0.0,0.0)
  122 IF (K.GT.2) GO TO 15
C PROCESS THE ORIGINAL NETWORK.
C CALL SOLVE
C SOLVE FOR SCATTERING PARAMETERS.
C S(NCOL)=RINV*(VN(NODE2)-VN(NODE1))
  150 S(MCOL)=RINV*(VN(NODE4)-VN(NODE3))
C ACCOUNT FOR CONTRIBUTION TO THE GRADIENT OF THE NETWORK.
C DO 10 II=1,NOPT
  204 LOC=JLOCX(II)
  205 LOC=LOCAL(LOC)
  207 IF (IDOPT(II).GT.3) LOC=LOC+2
  213 NODEX1=NODPLC(LOC)
  215 NODEX2=NODPLC(LOC+1)
  220 V=VN(NODEX2)-VN(NODEX1)
  231 SDEL(II,NCOL)=V
  233 10 SDEL(II,MCOL)=V
  237 NCOL=2
  240 MCOL=4
  241 GO TO 1000
C ACCOUNT FOR CONTRIBUTIONS TO THE GRADIENT OF THE ADJOINT NETWORK.
C 15 CALL SOLVEA
  244 DO *5 II=1,NOPT
  250 LOC=JLOCX(II)
  251 LOC=LOCAL(LOC)
  252 NODEX1=NODPLC(LOC)
  254 NODEX2=NODPLC(LOC+1)
  256 PSI=VN(NODEX1)-VN(NODEX2)
  264 IF=IDOPT(II)
  266 GO TO (40,20,25,30,35,10)
C CONTRIBUTIONS OF CAPACITORS.
C 20 PSI=PSI*CFREQ
  304 GO TO 40
C CONTRIBUTIONS OF RECIPROCAL INDUCTANCES.
SUBROUTINE ERRFCN(GRAD,ERROR)

C SUBROUTINE TO COMPUTE THE ERROR FUNCTION AND ITS GRADIENT
C IN THE SPACE OF PARAMETERS BEING OPTIMISED. THE ERROR FUNCTION
C IS OF THE WEIGHTED SUM-OF-SQUARES TYPE.

COMMON DECLARATIONS.

COMMON /KILL/ N0GO*JEQF,
COMMON /NETDAT/ NAME(122+2),VALUE(122+2),LOCAL(122),LOCATE(5),
1 NODPLC(484),NUMNOO,
COMMON /OPTIMUM/ WEIGHT(4),JLOCX(30),1DNOPT(30),NOPT,NPRV,N,
1 NTERX(30),SCALEA(30)
COMMON PORT
COMMON /PORTDAT/ PORT(60+4),OMEGA(60),NPOINTS,RNORM,LOCPR1,
1 LOCPR2
COMMON YDATUM,YNL,VDATUM,VN
COMMON /MATRIX/ YDATUM,YNL(300),VDATUM,VN(300),MATLOC(488)
DIMENSION VNPART(2+30)
EQUIVALENCE (VNPART(1+1),VN(1))
COMPLEX CFREQ,CRFREQ,V,PSI,DIFF
COMPLEX S,DEL
DIMENSION S(4),SDEL(60+4),GRAD(30)

INITIALISE.

ERROR=0.0
DO 2 I=1,NOPT
2 GRAD(I)=0.0
NODE1=NODPLC(LOCPR1),
NODE2=NODPLC(LOCPR1+1)
NODE3=NODPLC(LOCPR2)
NODE4=NODPLC(LOCPR2+1)
RINV=2.0/RNORM

ITERATE OVER FREQUENCY.

DO 5000 I=1,NPOINTS
30 CFREQ=CMPLX(V,0,FREQ)
31 CRFREQ=CMPLX(V,0,-1.0/FREQ)

LOAD THE ADMITTANCE MATRIX AND DECOMPOSE IT INTO LU FORM.

CALL LOADY(FREQ)
CALL DECOMP
IF (N0GO.EQ.1) RETURN

CALCULATE THE TWO-PORT SCATTERING PARAMETERS AND THEIR GRADIENTS.

NCol=1
MCOL=3
ICOL=1
JCOL=2
DO 1000 K=1,K
JNODE1=NODE1
IF ((K.EQ.2),OR,(K.EQ.4)) JNODE1=NODE3
FUNCTION INDEX(NODE1, NODE2)
C ROUTINE TO FIND MATRIX LOCATION GIVEN NODE SPECIFICATION.
C
C COMMON DECLARATIONS.
C
COMMON /NETDAT/ NAME(122*2), VALUE(122*2), LOCAL(122), LOCATE(5),
1 NODELC(484), NUMNODE
COMMON /POINTS/ IORDER(30), JMNODE(30), IUC(300), IUR(31), MEMO(300),
1 IUS, ILS, LASTUT, LASTLT
C
IF (NODE1.EQ.0) GO TO 15
IF (NODE2.EQ.0) GO TO 15
IF (NODE1.NE.NODE2) GO TO 1
INDEX=NODE1
RETURN
10 IO=JMNODE(NODE1)
10 JO=JMNODE(NODE2)
13 ISTART=IUS
15 IF (JO.GT.IO) GO TO 4
20 ISTART=ILS
21 ITEMP=JO
22 JO=ITEMP
23 IF (JO.LE.NUMNODE) GO TO 8
26 ITEMP=IO
27 IO=JO
30 JO=ITEMP
32 JS=IUR(IO)
33 JE=IUR(IO+1)-1
35 IF (JE.LT.JS) GO TO 15
40 DO 10 L=JS,JE
41 IF (IUC(L).NE.JO) GO TO 10
43 INDEX=ISTART+L
45 RETURN
45 CONTINUE
50 INDEX=0
52 RETURN
END

SUBPROGRAM LENGTH
000076

FUNCTION ASSIGNMENTS

STATEMENT ASSIGNMENTS
1 - 000011 4 - 000024 8 - 000032 10 - 000046
15 - 000051

BLOCK NAMES AND LENGTHS
NETDAT - 002114 POINTS - 001267

VARIABLE ASSIGNMENTS
ILS - 0J1264S02 INDEX - 000066 IO - 000067 IORDER - 000005S02
ISTART - 000071 ITEMP - 000072 IUC - 000074S02 IUR - 000550S02
NODE RENUMBERING IS COMPLETE. ILC AND ILR ARE NOW SET UP
ACCORDING TO THE RE-ORDERED NODE NUMBERS.

C        110 J=0
C        400 DO 90 L=1,NUMNOD
C        401 ILC(L)=J+1
C        403 IN=ORDER(L)
C        406 KS=IUR(IN)
C        407 KE=IUR(IN+1)=1
C        411 IF (KS.GT.KE) GO TO 90
C        413 DO AR K=KS+KE
C        417 IN=IJC(K)
C        427 IRN=JMNOD(IN)
C        430 IF (IRN.LT.L) GO TO 89
C        431 J=J+1
C        434 ILR(J)=IRN
C        434 89 CONTINUE
C        437 90 CONTINUE
C        442 IUTOT=J
C        443 ILC(NUMNOD+1)=J+1
C        445 DO 96 J=1,NN
C        456 96 IUR(J)=ILC(J)
C        459 IF (IUTOT.EQ.0) RETURN
C        464 DO 97 J=1,IUTOT
C        463 97 IJC(J)=ILR(J)
C        465 RETURN

C STOP RENUMBERING IN EVENT OF ERROR AND PRINT APPROPRIATE MESSAGE.
C
C        465 350 NOGO=1
C        466 PRINT 351
C        351 FORMAT (///10X,--- MATRIX OVERFLOW TYPE 3, EXECUTION HALTED ---/)
C        472 RETURN
C        473 END

SUBPROGRAM LENGTH
000566

FUNCTION ASSIGNMENTS

STATEMENT ASSIGNMENTS
15 - 000027 16 - 000021 17 - 000032 18 - 000036
20 - 000037 21 - 000060 23 - 000042 24 - 000067
25 - 000074 28 - 000077 30 - 000133 40 - 000105
41 - 000106 42 - 000110 43 - 000135 47 - 000140 48 - 000147
45 - 000131 46 - 000135 53 - 000167 55 - 000245 56 - 000267
51 - 000161 53 - 000167 55 - 000245 70 - 000302
59 - 000271 60 - 000273 61 - 000276 70 - 000302
74 - 000324 76 - 000346 78 - 000374 80 - 000377
89 - 000435 90 - 000440 110 - 000471 350 - 000466
351 - 000500

BLOCK NAMES AND LENGTHS
KILL - 000002 NETDAT - 002114 POINTS - 001247
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