Essays on Monetary Policy, Stock Market and Foreign Exchange Reserve

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To my family for their love
ABSTRACT OF THE DISSERTATION

Essays on Monetary Policy, Stock Market and Foreign Exchange Reserve

by

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Dr. Marcelle Chauvet, Chairperson

Chapter one investigates the asymmetric effects of monetary policy on the U.S. stock market across different monetary policy regimes and stock market phases. It uses a Markov-switching dynamic factor model to date the turning points of each bear market and bull market, and to generate a new stock market movement measure. A time-varying parameter analysis is then used to study the contemporaneous and lead-lag effects of monetary policy on stock returns. The results provide evidences that the monetary policy of changing monetary aggregates has fewer impacts in bear markets than bull markets, but changes in federal funds rate can be more influential in bear markets. The results also indicate that increases in monetary aggregates or reductions in the federal funds rate have positive contemporary impacts on stocks only during the periods in which they are used as the monetary policy intermediate target.

Chapter two (joint with Dr. Marcelle Chauvet) investigates the overall interrelationship between monetary policy and stock returns. Using a Markov-switching dynamic bi-factor model, the chapter extracts a latent factor from monetary variables to
represent changes in monetary policy, and a second latent factor from stock indices to represent stock market movements. These unobserved factors as well as their relationship with each other are estimated simultaneously in a joint nonlinear model from the observable variables. The two factors are allowed to follow different two-state Markov-switching process. The factors are set in a bivariate vector autoregression framework to examine the dynamic relationship. The results indicate that contractionary monetary policy has a negative effect on stock returns, but monetary policy doesn’t respond to changes in stock returns.

Chapter three investigates the reason why Hong Kong’s foreign exchange reserves increase dramatically, and particularly studies the effects of monetary policy. Cointegration test, Granger causality test, and vector error correction model are employed. The results show a negative long-run relationship between money supply and foreign exchange reserve, and no long-run relationship between exchange rate and foreign exchange reserve. The results indicate a low speed of adjustment of the foreign exchange reserve departure from its long-run equilibrium is the reason for its large holdings.
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Chapter 1

The Asymmetric Effects of Monetary Policy on Stock Market

1.1 Introduction

The Federal Reserve has two ultimate objectives for its monetary policy: to support maximum sustainable output and employment, and to maintain stable price level. These two goals are explicitly announced in the 1977 amendment to the Federal Reserve Act. It is stated by mounting literatures on the transmission of monetary policy that the Federal Reserve affects real economy through the financial markets and especially the stock market. For instance, as Bernanke and Kuttner (2005) stated, the effects of monetary policy on macroeconomic objectives are at best indirect and lagged, and the most direct and immediate influence of monetary policy is on the stock market. Many other studies also support the view that monetary policy has an instantaneous and significant impact on stock market (see, for example, Thorbecke, 1997; Patelis, 1997; Lastrapes 1998; Rigobon and Sack, 2004; Farka 2009, among others). Strength or weakness of the stock market can have a substantial impact on real activities such as consumption through the wealth effect and investment through the credit channel. Many believe that in the context of monetary policy management,
the Federal Reserve must view its macroeconomic objectives and stock market sustainability as complementary and consistent goals, to be pursued within an integral policy framework. The commonly accepted wisdom is that expansionary monetary policy measures should have a positive effect on the stock performance.

Given the fact that monetary policy has significant influence on stock market, several cross-section studies have sought to investigate if monetary policy has asymmetric impacts on stock performance according to different firm characteristics such as its size and capital intensity. For example, Ehrmann and Fratzscher (2004) reached the conclusion that small-size firms and financial-constraint firms are more strongly affected by monetary policy.

Some time-series studies (Durham 2003, 2005) showed that the relationship between monetary policy and stock market return is historically unstable and time-varying. However, there is not much done in the literature analyzing how and why the relationship varies over time. Is it possible that the time-varying response of stock return to monetary policy depends on drastic changes in monetary regimes or on the phases of the stock market being in a bull or bear market?

The aim of this chapter is to explore whether the effects of monetary policy on stocks are asymmetric over time depending on the phases of the stock market and the monetary policy regimes from 1970s to present. This topic has gained popularity in the current scenario of expansionary monetary policy and historically high stock price level in the U.S. Understanding the responsiveness of stock market to changes in
monetary policy shed light on the transmission mechanism of monetary policy, since stock market performance plays an important role on real activities through many channels.

Investigating the impact of monetary policy across different stock market phases and monetary policy regimes naturally requires identifying the beginning and end of these phases and regimes. The periods of monetary policy regimes can be defined using the dates on which monetary policy intermediate targets changed, which is well-documented in the Federal Reserve’s history. Yet, agreement on the dates of stock market turning points between bull and bear market regimes is far from unanimous. Moreover, there is no commonly accepted formal definition of bear and bull markets in academic literatures. In the U.S., the National Bureau of Economic Research (NBER) provides business cycle dates that are regarded as official. This dating is obtained by examining the comovement in the switch of several major economic variables.

This chapter uses the NBER’s principle together with Chauvet (1998/1999) classification method to define the business cycle and stock market fluctuation by employing a Markov-switching dynamic factor model to date their turning points. The framework is cast in a state space model, and estimated via Kalman Filter (1960) and Hamilton Filter (1989). The dynamic factor model captures the clustering of shifts between upward and downward tendency of a variety of popular stock indices. The Markov-switching feature reflects the asymmetry of stock movements in terms of
growth rate and volatility, and is able to statistically identify the date of turning points through the smoothed probabilities.

The results show that the model successfully captures all bear markets and bull markets in the sample. Moreover, the model also produces a new composite index that represents the stock market price movements more precisely and broadly. The new composite measure has advantages over existing stock indices, given that they are criticized for their limitation on the coverage of certain types of stocks and stock exchanges. The Markov-switching dynamic factor model also calculates the average durations of bear and bull markets, and the probability of bear and bull market at every time point. These results can be instrumental in assisting investors and policy makers to understand in which state the stock market is and where the stock market will move towards.

In the next step, this chapter uses the proposed new stock market movement index into a time-varying parameter model to explore the dynamic interrelationship between monetary policy and stock performance across different monetary policy regimes and stock market phases. Monetary policy is represented not only by short-term policy interest rate and but also by monetary aggregates to reflect the fact that these two variables have been used as the monetary targets in the Federal Reserve’s history. The lead-lag relationship and contemporaneous relationship are analyzed in two separate time-varying parameter models, which are represented in the state space models, and estimated through the Kalman Filter and maximum likelihood
estimation method. To the best of my knowledge, very few studies investigated this topic in the framework of Markov-switching dynamic factor model and time-varying parameter model. My study can unveil features of their relationship that have not been captured previously.

The results show that the influence of monetary policy on stock return is different across monetary policy regimes which are classified by the monetary policy target changes. The contemporary signaling effect of federal funds rate changes impact the stock market only during periods in which the federal funds rate is used as monetary target by the Federal Reserve. This is also the case for monetary aggregates. That is, monetary aggregates affects stock market positively only during periods in which they are used as monetary targets in 1970s and 1980s. The findings also indicate that a positive predictive relationship between monetary aggregate and stock market occurs during the periods of strong economic growth, but not during the periods of economic recession or slow recovery.

This chapter provides evidence of the asymmetric response of stock return to monetary policy during bear and bull markets. In fact, there is a sharp drop in the correlation between monetary aggregate and stock returns in every bear market, indicating that the influence of expansionary monetary policy through increases in monetary aggregate is much weaker in a bear market, and can even have a negative effect on the stock market. However, an expansionary monetary policy through reduction in short-term policy interest rate is influential in improving stock returns.
The remainder of the chapter is organized as follows. The next section discusses the studies conducted in the past literature. The third section describes the theoretic framework of the relationship between monetary policy and stock movements. The data are described in the fourth section. The fifth section illustrates the Markov-switching dynamic factor model and time-varying parameter model, which are the empirical models applied in this study. The sixth section presents the empirical results. This chapter is concluded in the seventh section. Estimation procedures are discussed in the Appendix.

1.2 Literature Review

1.2.1 Literature on the U.S. Stock Market Regimes

The fundamental understanding of a bull market is a period of substantial and continuous increase of stock prices, and a bear market is a period of substantial and continuous reduction in stock prices. Stock market commentators often define a bull market as a 20% or 25% stock price rise, and a bear market as a 20% or 25% stock price decline. Some financial analysts identify the beginning of a bear market when the 50-day moving average line crosses the 200-day moving average line from the above, and holds below. However, in the academic area, the finance and economics literatures have no commonly accepted definition of bull market and bear market. Several studies provided their own definitions of bull and bear markets, such as Chauvet and Potter (2000, 2001), Pagan and Sossounov (2003), and Chen (2007). For
example, Chen (2007) used a simple Markov-switching model on S&P 500 stock returns to estimate the probabilities of bear market and bull market, and it found that the correlation between the bull market probability and the bull market binary variable constructed by using 20% cutoff line is round 0.7.

1.2.2 Literature on the U.S. Monetary Policy Regimes

According to Meulendyke (2003) and Mishkin (2006), the Federal Reserve’s monetary policy experienced substantial changes over the past four decades. In 1970, Arthur Burns was appointed chairman of Board of Governors of the Federal Reserve, and the Federal Reserve started to use monetary aggregates as intermediate target and federal funds rate as operating target to fight inflation, which was caused by the procyclical monetary policy. The Federal Open Market Committee (FOMC) selected growth rate for monetary aggregate, and chose a federal funds rate that would achieve that desired monetary aggregate growth rate. However, this monetary target policy was unsuccessful in controlling inflation, due to the fact that monetary aggregate target and federal funds rate may conflict with each other. The federal funds rate targeting led to a procyclical monetary policy which raised inflation pressure during the periods of economic expansion in the early 1970s. The economic contraction started from the middle of 1970s was associated with federal funds rate reduction and monetary aggregate growth sharp drop, which in turn made the economic condition even worse. Combined with other inflation factors such as a decrease in oil supply
and a decrease in agriculture products supply, the period of 1970s was mainly featured by stagflation.

In October 1979, Paul Volcker became chairman of Board of Governors of the Federal Reserve. The Federal Reserve’s monetary policy has shifted into a new regime in 1980s. The main goal in this era is to change interest rate to fight serious inflation. The operating target was switched from federal funds rate into nonborrowed reserve and borrowed reserve sequentially. Monetary aggregate still served as the intermediate monetary target. A predetermined target path for nonborrowed reserve and borrowed reserve was based on the objective for the monetary aggregate. The federal funds rate was largely raised in early 1980s and the inflation was successfully controlled. However, this anti-inflation monetary strategy missed most monetary aggregate targets, indicating that monetary aggregate was deemphasized as the monetary target.

When Alan Greenspan was elected as Federal Reserve’s chairman in 1987, the Federal Reserve announced that it would no longer use monetary aggregate as its target. In 2000, legislation amending the Federal Reserve Act officially ceased to require the Federal Reserve to report monetary aggregate target to Congress. Abandoning monetary aggregates as the guide for its monetary policy, the Federal Reserve has restarted to target federal funds rate since early 1990s. Periods in 1990s and 2000s were featured by the clear monetary policy goal in terms of macroeconomic variables, clear operating target which is federal funds rate, without
an explicit intermediate target. By actively and timely changing federal funds rate, the Federal Reserve tried to keep the economy and financial market on track. Ben Bernanke began his tenure in early 2006. The same monetary strategy continued until 2007, when a new and more complicated problem came up. Since 2008, a sufficient injection of bank reserves has brought the federal funds rate fundamentally close to zero, so that the zero lower bound rules out further policy interest rate reduction. The Federal Reserve has to seek alternative nontraditional monetary policy tools to improve the condition of financial market and promote the growth of economy, which are known as quantitative easing and forward guidance.

1.2.3 Literature on General Responsiveness of Stock to Monetary Policy

The responsiveness of stock movements to monetary policy has been a matter of increased concern since 1980s. There is a body of literature investigating this issue. For most of these studies, monetary policy is divided into two main streams: changing the monetary aggregate and changing the policy interest rates. The effects of expansionary monetary policy, such as increasing money supply and reducing policy interest rates, on the stock return are claimed to be positive in these empirical researches. Thorbecke (1997) employed a monthly VAR model for the period from 1967 to 1990 to analyze the link and used the federal funds rate to measure monetary policy. He found that the response of stock returns to a negative one standard deviation shock to the federal funds rate is 0.8%. This empirical finding that a positive
relationship between the expansionary monetary policy of reducing policy interest rate and stock return has been confirmed by Patelis (1997), Lastrapes (1998) and many others.

In a more recent study, Rigobon and Sack (2004) used the policy shocks that take place on certain dates such as the days of FOMC to examine this topic, and documented a positive linkage between expansionary monetary policy and stock movements. In a similar vein, Bernanke and Kuttner (2005) took a more traditional event-study approach, while controlling directly for certain kinds of information jointly influencing monetary policy and stock return. They applied ordinary least squares regressions in an event study, and found that an unexpected 25 basis points decrease in the federal funds target rate is associated with a one percent increase in the stock prices.

But there is not yet a consensus on this conclusion, as several articles provide counter examples on the direction of effects. Cornell (1983) found the link between money supply announcement and asset prices can be either positive or negative, depending on the underlying assumption and hypothesis. He discussed three hypotheses (expected inflation hypothesis, Keynesian hypothesis, and real activity hypothesis) suggested in the previous literature as well as the risk premium hypothesis that he proposed. These results were consistent with those of other studies which have analyzed the relationship between monetary policy and the stock return. Lee (1997), for example, applied rolling regressions to measure the relationship between
short-term interest rate and stock prices, which is measured by the S&P 500 index, indicating an unstable linkage. Another effort along these lines is that of Garg (2008), who conducted empirical research about the effects of changes in federal fund rate on stock prices in different sectors. His work showed that stock prices and interest rate move in the same direction, indicating an expansionary monetary policy of reducing policy interest rates may deteriorate the stock performance. He also gave theoretical explanation for this seemingly surprise result.

There is some dissent on the response of stock market to the monetary policy among the existing literature. The direction of the reaction is impossible to determine ahead. Possible explanations for this dissent are provided in the theoretical framework section of this chapter.

1.2.4 Literature on the Asymmetric Effects of Monetary Policy on Stock Return

Chen (2007) studied the monetary policy’s asymmetric effects on stock returns in different stock market conditions, and found that monetary policy has a larger effect in less booming stock markets and stagnant stock markets. His finding indicated that a contracting monetary policy is more likely to cause a weak stock market. Jansen and Tsai (2010) investigated the asymmetric impact of monetary policy on stock return in bull and bear market during the time period from 1994 to 2005, and showed that the monetary policy shocks in bear market is large, negative, and statistically significant. Kurov (2010) analyzed the stock returns on Federal Open Market Committee (FOMC)
announcement days, and found that monetary policy shocks have strong influence on market participants’ sentiment, and this impact is even stronger in a bear stock market.

Jensen, Mercer and Johnson (1996) suggested that monetary policy regime affects investors’ required return. They found that stock return is higher in tight monetary policy regime than expansionary monetary policy regime. Kaul (1987) showed that the relationship among monetary policy, inflation, and stock return can be either positive or negative depending on whether monetary policy is pro-cyclical or counter-cyclical. Du (2006) supported this conclusion and found that changes in money supply and its consequential inflation can have different effects on stock returns during different monetary policy regimes. The results showed that there was a positive relationship among money supply, inflation and stock return during the period of pro-cyclical monetary policy regime, and this relationship became negative during the period of counter-cyclical monetary policy regime. Laopodis (2013) examined the dynamic relationship between monetary policy and stock market during the three distinct monetary policy regimes of Burns, Volcker and Greenspan since 1970s. It found there was a very weak relationship between monetary policy action via federal funds rate and stock return in 1990s. His chapter provides evidence for asymmetric effects of monetary policy on stock in different regimes of monetary policy and different stock market conditions.
1.3. Theoretical Framework

1.3.1 Theoretical Background of Stock Price Valuation

Recall that the objective of this chapter is to investigate the effects of monetary policy on stock price movements. To do so it is necessary to have a solid understanding of stock price valuation. The most popular theory for the stock valuation is the present value model or discounted cash flow model. This model was well explained by Crowder (2006) and Ioannidis and Kontonikas (2008), among many studies. The intrinsic stock price $P_t$ is valued as the present value of future expected dividends cash flows $D_{t+j}$ of the company and terminal stock price at the last period of holding horizon. Under the assumption of constant discounting rate, the present value model is expressed as follows,

$$P_t = E_t \left[ \sum_{j=1}^{N} \left( \frac{1}{1 + R_t} \right)^j D_{t+j} \right] + E_t \left[ \left( \frac{1}{1 + R_t} \right)^N P_{t+N} \right]$$

where $E_t$ is the conditional expectation operator based on the information available up to time $t$, $N$ is the number of investor’s holding period, $R_t$ is the rate of return to discount the future values. As the stock holding periods $N$ increases to infinity, the second term on the right hand side of the equation vanishes to zero.

$$\lim_{N \to \infty} E_t \left[ \left( \frac{1}{1 + R_t} \right)^N P_{t+N} \right] = 0$$

Therefore, the stock price valuation model can be described as follows

$$P_t = E_t \left[ \sum_{j=1}^{N} \left( \frac{1}{1 + R_t} \right)^j D_{t+j} \right]$$
According to the above theory, the intrinsic stock price is simultaneously determined by two parts: future cash flows and the discounting rate. Therefore, monetary policy can affect stock price through both future cash flows and discounting rate that is linked to interest rate.

1.3.2 Theoretical Background of the Effects of Monetary Policy on Stock Price

The Federal Reserve has several monetary tools available, such as open market operations, discount loans, and required reserves. It also has the ability to set discount rate and federal funds rate target to affect the financial markets and real economic activities. It is widely accepted that all the monetary policy measures can be summarized into two major channels: changes in monetary aggregate and changes in short-term interest rate. These two measures are correlated most of the time, in that a rise of money supply in terms of bank reserves will put downward pressure on the short-term interest rate which clears the reserve market. In other words, an increase in money supply will generate a drop in interest rate. The only exception arise in the case of current zero lower bound interest rate, which already rules out further policy interest rate reduction. Hence, it is appropriate to examine the effect of change in money supply and change in interest rate separately.

It is commonly believed that expansionary monetary policy, considered as a rise in money supply or a reduction in short-term policy interest rate, can drive up the stock price by increasing the future cash flow and decreasing discounting rate.
However, the actual mechanism behind is much more complicated. The impacts of expansionary monetary policy on stock market can be either positive or negative (see Cornell 1983 and Sellin 2001, for example). In addition, the effects of monetary policy through these two channels can reinforce or offset each other.

In general, the response of stock prices to the expansionary monetary policy of reducing interest rate is positive. That is why there exists a long tradition for the Federal Reserve to drop short-term policy interest rates in an attempt to promote the stock market condition. The detailed reasons for the positive linkage are presented as follows. First, it is obvious that a lower interest rate indicates a lower discounting rate, implying a higher present value of future cash flows and hence a higher stock price, given that the future cash flows are constant. Second, when interest rates decrease, saving in banks and investing in bonds or other interest related investment vehicles become less profitable and attractive. Financial market participants switch into stock markets investment, leading to a rise in the demand for stocks. Stock prices go up accordingly. Third, companies with high debt in their balance sheets will benefit when interest rates decrease, resulting in higher net income and higher stock prices. It is also less costly for firms to borrow new loans to fuel their business growth, which will be favorable for firms’ financial situation and stock value growth. Fourth, with lower interest rates, consumers are more willing to borrow to finance big purchases. It would largely affect certain industries such as real estate and automobiles, generating considerable a boost in companies’ revenues and hence their stock prices. Therefore,
lower short-term interest rates generate higher stock prices, and the effect of expansionary monetary policy of reducing interest rates on stock price movements is positive.

However, there are several exceptions to the above situations, leading to a negative linkage between the expansionary monetary policy of reducing interest rate and the stock price movements. First, companies in the certain industries would suffer loss from the reduced interest rate. For example, a lower interest rate will generate a smaller net interest margin - the difference between the interest banks earn on lending money and the interest banks pay to the depositors - for banks. This will cause a decrease in profits and stock prices in banking industry, resulting in a negative relationship between the expansionary monetary policy of reducing interest rate and the stock price. Second, in the portfolio theory elucidated by Cornell (1983), money balance and stocks are considered as two of many assets in the portfolio of investors. Since interest rate measures the opportunity cost of holding money balance, a change in interest rate will affect investors’ decision about the proportion of money to be held in their portfolio. A decrease in interest rate means the opportunity cost of holding money is lower, motivating investors to replace stocks with money. A lower demand for stocks will reduce stock prices, resulting in a negative relationship between the expansionary monetary policy of reducing interest rate and the stock price.

The above positive and negative relationship between the expansionary monetary policy of reducing interest rate and stock price movements may offset each other. The
final linkage can be either positive or negative as stated above, depending on which force dominates the other. Hence, in theory, the ultimate effect of expansionary monetary policy by reducing interest rate can be ambiguous.

More surprising is that the second measure of expansionary monetary policy (increasing money supply) can also have either positive or negative impacts on stock market movements. The following reasons explain the positive effect of expansionary monetary policy of increasing money supply on stock prices. First, the main channel for the Federal Reserve to increase money supply is purchasing bonds and notes issued by government or government-sponsored enterprises through open market operations. By reducing the bond supply, the Federal Reserve drives up bond prices and drops bond yields accordingly. The low bond yields, in turn, reduce the borrowing cost of listed firms who also issue corporate bonds, and hence increase companies’ earnings and stock prices, leading to a positive relationship between money supply and stock prices. Second, a higher money supply allows banks to have more cash for loans. Consumers are easier to borrow to make big purchases, which will contribute to the rise of firms’ revenue and stock prices. At the same time, the firms are easier to get access to loans, which provide the fuel for business expansion and stock price growth. Third, this mechanism is associated with the real activity hypothesis discussed by Cornell (1983). One of the Federal Reserve’s responsibilities is to balance the money demand and the money supply in the economy. An increase in Federal Reserve’s money supply hints at a higher money demand anticipated by the Federal Reserve,
caused by higher anticipated future output. Higher anticipated future output will raise firms’ future revenue and cash flows, leading to higher stock prices. Besides, higher anticipated future output can also tremendously improve investors’ sentiment, which is favorable for stock price growth. Fourth, according to the quantity theory of money (Friedman 1961, 1988; Friedman and Schwartz 1963; Dhakal, Kandil, and Sharma 1993), a change in money supply unbalances the equilibrium position of money in the portfolio of investors with respect to other assets such as stocks. An increase in money supply generates an excess proportion of money in the portfolio, motivating investors to increase the holding of other assets such as stocks. A higher demand for stocks will induce higher stock prices. Therefore, changes in money supply display a positive relationship with stock price.

On the other hand, the expansionary monetary policy of a rise in money supply can also have negative impacts on stock prices, which is supported by Keynesian economists. According to them, the change in money supply only affects the stock market through altering expectations of future monetary policy. The stock market can also perceive the increase in money supply as a reinforcement signal that the economy is entering difficult times and the Federal Reserve is taking measures to help the declining market, which generate a pessimistic sentiment and has a negative effect on market sentiment and stock performance. Additionally, under the Keynesian assumption of sticky price, an increase in money supply will cause the real money balances to rise. Interest rates must drop to produce an offsetting rise in money
demand to clear money market. Interest rate is also considered as the price of money in the money market. An increase in the money supply would reduce the price of money, which lowers the interest rate. Since there is a possible positive relationship between interest rate and stock prices, which is illustrated above, the ultimate effect of an increase in money supply on stock prices is likely to be negative. Lastly, higher money supply will create a higher expected future inflation. Since stock return is considered to be negatively inflation, which is claimed by existing studies (see Nelson, 1976; Fama and Schwert, 1977; Kaul, 1987), stock prices will reduce accordingly due to the high inflation. Given the fact that stock market is forward-looking and reflects market participants expectations about the future state of the economy and future action of monetary authority, a potential high inflation that caused by an increase in money supply is expected to trigger Federal Reserve’s contractive monetary action, leading to a decrease in stock price.

Normally the impact of monetary policy takes some time to take effect due to the monetary policy transmission lag. However, it is possible that forward-looking investors, who price the stocks as the present value of future cash flow, will immediately discount the cash flows, generating a change in stock prices before the actual impact of the new monetary policy on firms’ revenue take place. Due to the above reasons, the effect of expansionary monetary policy on stock movements can’t be determined ahead.
1.3.3 Theoretical Background of the Asymmetric Effects of Monetary Policy on Stock Price

The traditional theory explaining the asymmetric impact of monetary policy on stock price in different stock market conditions and different monetary policy regimes is the agency costs of financial intermediation (Bernanke and Gertler, 1989; Kiyotaki and Moore, 1997). The theory indicates that agency costs result in information asymmetry between firms and financial intermediaries. If there is information asymmetry in the financial markets, agents with information disadvantages behave as they are constraint financially. The degrees of financial constraints are different in different stock market condition and different regimes of monetary policy. Therefore, a monetary policy action can have different effects on stock returns in bull and bear market regimes, as well as in different monetary policy regimes.

1.4 Data

The overall price level of stock market is measured by the stock index. The most popular and influential stock indices in the U.S stock market nowadays are Dow Jones Industry Average, Standard & Poor’s 500, and NASDAQ Composite. Fortune (1998) shows that these stock indices display divergent movements, implying that different stock index represents different segments of the U.S. stock market and contributes different information about the stock market. Dow Jones Industry Average Index has the longest history and is the only price-weighted index. It only covers the largest 30
blue-chip stocks and all the stocks are listed in New York Stock Exchange. S&P 500 is a value-weighted stock index, representing 500 stocks traded in New York Stock Exchange, American Stock Exchange, and NASDAQ stock market. The market value of stocks included in the S&P500 range from large-capitalization to mid-capitalization. NASDAQ Composite covers more than 5000 stocks listed in the NASDAQ exchange. Most of these stocks are considered as technology stocks and small-capitalization stocks. As each stock index measures different stock market segments, it is reasonable to combine all three stock indices to study the overall movements of the U.S. stock market. A major contribution of this chapter is developing a better and broader composite measure for stock market price movements by capturing the clustering in movement of different stock exchanges and stock sectors. This is very distinguished from Chauvet (1998/1999), which uses stock fundamentals such as price earnings ratio and dividend yield to extract a stock market common factor to represent the fluctuations in the stock market.

Interest rate and monetary aggregate are two main measures of the Federal Reserve’s monetary policy. As mentioned in the literature review, both federal funds rate and different measures of monetary aggregates have been used as the monetary policy target in the Federal Reserve’s history. This chapter uses the federal funds rate to represent the short-term policy interest rate. The Federal Reserve directly controls two short-term policy interest rates, which are discount rate and federal funds rate. The discount rate is the short-term interest rate the Federal Reserve charges depositary
institutions for the loans borrowed directly from the Federal Reserve. The federal funds rate is the interest rate set by the Federal Reserve for depositary institutions to charge each other for the short-term loans. As a measurement of interest rate monetary policy, the federal funds rate is more favorable than discount rate. In 2003, the Federal Reserve reformed the discount lending system, and set the discount rate 100 basis point higher than the federal funds rate to penalize the discount borrowing. Discount loan is no longer used regularly by the depository institutions during the normal time. It became the emergency loan of last resort during the financial crisis. The choice of federal funds rate was also supported by Bernanke and Kuttner (2005), who claim that changes in federal funds rate has the most immediate effect on financial markets. On the other hand, this chapter chooses broader measure Divisia M4 and M2 as the representative of monetary aggregate. Divisia M4 is a broad monetary aggregate, containing negotiable money market securities, such as commercial paper, negotiable CDs, and T-bills. Divisia M4's components are modernized to be consistent with current financial innovations and financial market realities.

This study doesn’t distinguish between anticipated and unanticipated changes in money supply and interest rate. The proponents of efficient market hypothesis argue that all the information is already embedded in the stock price, and only the unanticipated changes in money supply and interest rate can affect the stock price. However, the conventional wisdom contends that efficient market hypothesis doesn’t hold in the current stock market, and all available information is not embedded in the
stock price. Therefore, anticipated changes in money supply and interest can also have an impact on stock price movement. Many previous studies show that anticipated changes in money supply and interest rate matter more than unanticipated changes (see Maskay 2007).

The data is measured in monthly frequency and the sample period ranges from March 1971 to November 2012. The data is obtained from the websites of Federal Reserve Bank of St. Louis FRED database, Center for Financial Stability and Yahoo Finance.

1.5. Empirical Models

1.5.1 Empirical Model for the Identification of Bull and Bear Markets

Burns and Mitchell (1946) proposed and Diebold and Rudebusch (1996) stressed two important features for the business cycle of economy: the comovement of the macroeconomic variables and the asymmetry between expansions and recessions. This is also the principle that the National Bureau of Economic Research (NBER) uses to provide the official periods of business cycle and the dates at which the shift of economic phase take place in the United States. In order to date an economic peak, which is the turning points of the transition from an expansion to a recession, the National Bureau of Economic Research seeks for the comovement in the switch of several major economic variables from the upward growth into the decline. The economic trough, which is the turning point of the transition from an expansion phase
to a recession phase, is dated by the National Bureau of Economic Research using the reversed method. The dates of business cycle turning points and its calculation method are widely accepted by the public. These two features – comovement and asymmetry – apply to the fluctuation cycle of stock market as well. First, there exists a comovement of stock prices among stocks in different sectors and different exchanges. The common dynamics of different stock prices can be represented by an unobserved common factor in a dynamic factor model, which reflects the overall movement of the stock market. The dynamic factor model, developed by Geweke (1977), Sargent and Sims (1977), and Stock and Watson (1989, 1991), successfully captures the common underlying source which generates comovements among different variables. The second feature demonstrates that stock market behaves differently during bull market regime versus bear market regime. It is possible that the growth rate or volatility is completely different in different regimes. However, a linear model is not capable to capture this asymmetry in the stock market price dynamics. Hamilton’s (1989) state-dependent Markov switching model is designed to characterize this nonlinearity feature as it allows for switching between different regimes.

Therefore, in order to apply the NBER’s principle to date the turning points of stock market regimes and study the two features inherent in the stock market, which are comovement and asymmetry, the dynamic factor model and the state-dependent Markov-switching model become the natural choice for my research. More
specifically, one aim of this chapter is to combine the dynamic factor model and the state-dependent Markov switching model, and construct a new composite stock market indicator to better represent the overall movements of the U.S. stock market. The Markov-switching dynamic factor model is undertaken in the framework of a state space model, and estimated via Kalman Filter (1960) and Hamilton Filter (1989). The dynamic factor model captures the clustering of shifts of a variety of popular stock indices between their upward tendency and downward tendency. The Markov-switching feature reflects the asymmetry of stock movements in growth and volatility, and is able to statistically identify the dates of turning points using transition probabilities.

Diebold and Rudebusch (1996) proposed a Markov-switching dynamic factor model which encompasses these two features in one model for the first time. However, they did not actually carry out the estimation due to the heavy computational burden. Kim and Yoo (1995) and Chauvet (1998) developed the Markov-switching dynamic factor model and actually undertook the estimation by using the maximum likelihood estimation method to estimate both the dynamic common factor and the regime-switching transition probabilities simultaneously. This chapter follows Chauvet (1998) to assume that the intercept and variance of the common factor is Markov switching between different regimes. Kim and Nelson (1999) provided a detailed summary and an overview, and this chapter uses their algorithm as the main reference.
Markov-switching dynamic factor model is carried out within state-space models. State-space model was originally developed by Kalman (1960), and was applied to solve dynamic problems that involve unobserved state variables. The unobserved dynamic common factor is just one component of the unobserved state vector. State-space models are made up of two equations, which are measurement equation and transition equation. Measurement equation describes the relationship between observed variables and unobserved state variables. Transition equation in the state-space model describes the dynamic relationship between the state variable and its own lagged terms.

The essence of a Markov-switching dynamic factor model is that one unobserved dynamic factor, $f_t$, captures the comovements of a vector of time-series observed variables, $Y_t$, which have higher dimension. The unobserved dynamic factor, which follows an autoregression, has the mean and conditional volatility that are functions of a Markov state variable $S_t$, with the purpose of measuring the potential asymmetries across different stock market regimes in terms of growth rate and volatility. The random variable $S_t$ takes the value of zero or one, and represents the regime of stock market, either bear or bull. The vector of time-series observed variables is also impacted by a vector of idiosyncratic disturbances, $e_t$. These idiosyncratic disturbances capture the special features that are specific to an individual observed variable. The latent factors also follow an autoregressive time series process, which can take the form of either AR(1) or AR(2).
In equations, the Markov-Switching dynamic factor model is presented as following,

\[ \Delta Y_t = \gamma \Delta f_t + \Delta e_t \]

\[ \Delta f_t = \mu_{S_t} + \phi \Delta f_{t-1} + w_t, \quad w_t \sim i.i.d. N(0, \sigma_{\omega, S_t}^2) \]

\[ e_t = \varphi(L)e_{t-1} + \epsilon_t, \quad \epsilon_t \sim i.i.d. N(0, \Omega) \]

\[ \mu_{S_t} = \mu_0 S_t + \mu_1 (1 - S_t), \quad S_t = 0,1 \]

\[ \sigma_{\omega, S_t}^2 = \sigma_{\omega, 0}^2 S_t + \sigma_{\omega, 1}^2 (1 - S_t), \quad S_t = 0,1 \]

where \( L \) is the lag operator and \( \Delta = 1 - L \); \( \Delta f_t \) is a unobserved common factor extracted from major stock indices; \( \gamma \) represents the vector of factor loadings that describes the contribution of each stock index; \( e_t \) denotes the vector idiosyncratic components representing the unique feature of each stock index. The idiosyncratic components are assumed to follow a normal distribution.

In the setting of Markov switching dynamic factor model in this chapter, observed time series are stock indices. This chapter uses these three indices to construct the new composite measure of stock market movements. Let \( Y_t \) be a vector of 3 x 1 observed variables in their log form at time \( t \), which consists of Dow Jones Industry Average Index, S&P 500 Index, and NASDAQ Index in order. Every variable can be decomposed into a common factor and a specific or idiosyncratic component. The common factor captures the simultaneous upward and downward fluctuations of stocks that are widespread in all the stock exchanges and sectors. In other words, a bear market occurs when all the three indices drop significantly at the same time and a
bull market occurs when all the three indices increase simultaneously. If only one index drops and other indices increase or stay the same, this movement will be captured by the idiosyncratic term of that index, rather than by a common unobserved factor.

The Markov switching from one state to another is controlled by the transition probability matrix with element $P_{ij} = p(S_t = j|S_{t-1} = i)$, where $\sum_{j=0}^{1} P_{ij} = 1, i, j = 0, 1$. Besides, $\Delta e_t$ and $w_t$ are assumed to be mutually independent at all lags and leads. $\varphi(L)$ and $\Omega$ are diagonal based on the setting of dynamic factor framework. The common factor $f_t$ and idiosyncratic terms $e_t$ are assumed to be uncorrelated at all lags and leads. The common factor and the idiosyncratic term follow a separate autoregressive process. For the dynamic factor model, it is widely accepted that the common factor follows a AR(1) process. However, the dynamics of the idiosyncratic terms have several possibilities. This chapter estimates two most popular specifications, which are AR(1) and AR(2). The first Markov-switching dynamic factor model (MSDF-Model 1) uses AR(1) for the idiosyncratic terms and the second Markov-switching dynamic factor model (MSDF-Model 2) uses AR(2) for the idiosyncratic terms.

The specific state-space representations for the Markov-switching dynamic factor model 1 and Markov-switching dynamic factor model 2 are shown as following:
MSDF-Model 1:

Measurement equation: \( \Delta Y_t = H \beta_t \)

\[
[\Delta Y_{1t}] \\
[\Delta Y_{2t}] \\
[\Delta Y_{3t}]
= [Y_1 \ 1 \ 0 \ 0] \\
[Y_2 \ 0 \ 1 \ 0] \\
[Y_3 \ 0 \ 0 \ 1]
\]

Transition equation: \( \beta_t = \mu_S t + F \beta_{t-1} + v_t \)

\[
[\Delta f_t] \\
[e_{1t}] \\
[e_{2t}] \\
[e_{3t}]
= [\mu_S t] \\
[0 \ \phi \ 0 \ 0 \ 0] \\
[0 \ \varphi_{11} \ 0 \ 0 \ 0] \\
[0 \ 0 \ \varphi_{21} \ 0 \ 0] \\
[0 \ 0 \ 0 \ \varphi_{31} \ 0] + \left[\begin{array}{c}
[\Delta f_{t-1}] \\
[e_{1,t-1}] \\
[e_{2,t-1}] \\
[e_{3,t-1}]
\end{array}\right] + \left[\begin{array}{c}
[w_t] \\
[e_{1t}] \\
[e_{2t}] \\
[e_{3t}]
\end{array}\right]
\]

\( v_t \sim i.i.d. N(0, Q) \)

\[
Q = \begin{bmatrix}
\sigma_{w, S_t}^2 & 0 & 0 & 0 \\
0 & \sigma_1^2 & 0 & 0 \\
0 & 0 & \sigma_2^2 & 0 \\
0 & 0 & 0 & \sigma_3^2
\end{bmatrix}
\]

The models are estimated by using a combination of the dynamic factor model in the state-space representation and Markov switching, as implemented by Kim (1994). In his work, he provided filtering and smoothing algorithms for the Markov-switching dynamic factor model, with a maximum likelihood estimation of unknown parameters and unobserved factors. Augmented Dickey-Fuller unit root tests (1979) are applied to each of index variable. The unit root test results show that each variable has a unit root. Johansen (1988) cointegration test is also conducted, which provides evidence that there is no cointegration relationship among these variables. According to Stock and Watson (1991), time series with unit root but without cointegration should enter the dynamic factor model in their first difference. All the log differenced variables are standardized by subtracting sample mean and dividing by sample standard deviation.
MSDF-Model 2:

Measurement equation: $\Delta Y_t = H\beta_t$

$$
\begin{bmatrix}
\Delta Y_{1t} \\
\Delta Y_{2t} \\
\Delta Y_{3t}
\end{bmatrix} = 
\begin{bmatrix}
\gamma_1 & 1 & 0 & 0 & 0 & 0 \\
\gamma_2 & 0 & 0 & 1 & 0 & 0 \\
\gamma_3 & 0 & 0 & 0 & 0 & 1
\end{bmatrix} 
\begin{bmatrix}
\Delta f_t \\
e_{1t} \\
e_{1t-1} \\
e_{2t} \\
e_{2t-1} \\
e_{3t} \\
e_{3t-1}
\end{bmatrix}
$$

Transition equation: $\beta_t = \mu_{S_t} + F\beta_{t-1} + v_t$

$$
\begin{bmatrix}
\Delta f_t \\
e_{1t} \\
e_{1t-1} \\
e_{2t} \\
e_{2t-1} \\
e_{3t} \\
e_{3t-1}
\end{bmatrix} = 
\begin{bmatrix}
\mu_{S_t} \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix} + 
\begin{bmatrix}
\phi & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \varphi_{11} & \varphi_{12} & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \varphi_{21} & \varphi_{22} & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \varphi_{31} & \varphi_{32} \\
0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix} 
\begin{bmatrix}
\Delta f_{t-1} \\
e_{1t-1} \\
e_{1t-2} \\
e_{2t-1} \\
e_{2t-2} \\
e_{3t-1} \\
e_{3t-2}
\end{bmatrix} + 
\begin{bmatrix}
W_t \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
$$

$v_t \sim i. i. d. N(0, Q)$

$$
Q = 
\begin{bmatrix}
\sigma_{\omega, S_t}^2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \sigma_1^2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \sigma_1^2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \sigma_2^2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \sigma_2^2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sigma_3^2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

For identification, it is necessary to standardize one of the factor loadings $\gamma_i$ or factor variance $\sigma_{\omega, S_t}^2$ to be one. In our model setting, the factor variance follows a Markov-switching process to capture the asymmetry between bull and bear market in terms of growth rate and volatility. Therefore, we set second factor loading $\gamma_2$ to one. The econometric estimation procedures are shown in the Appendix at the end of this chapter, which includes Kalman filter, Hamilton filter, smoothing, and approximations.
It is likely that the effects of monetary policy on stock performance can be different in bear market and bull market, which is the focus of this study. This chapter provides the dates of each bear market and bull market to assist the analysis of effects of monetary policy on stock performance. In order to define the turning point of bear market and bull market, we need to define the procedure for identify these turns. The above Markov-switching dynamic factor model provides probabilities that can be used as the rule. During periods classified as good stock performance, smoothed probability of bear market regime $pr(S_t = 0|I_T)$ is mostly close to 0. This probability spikes upward sharply and remains high when stock market enters into a bear market. Although visual inspection is helpful to measure the time periods of bear markets and bull markets, a formal definition is needed to precisely date the turning points using probabilities. The commonly accepted method used by Hamilton (1989) and Chauvet and Piger (2003), a turning point is defined to take place when smoothed probability of bear market regime $pr(S_t = 0|I_T)$ moves across the 50 percent line, which separates the time periods when bear market is more likely from the time periods when bull markets is more likely. Therefore, the beginning date of the bear market is defined as the time point when smoothed probability of bear market regime $pr(S_t = 0|I_T)$ changes from below 50 percent into above 50 percent. The ending date of the bear market is similarly defined as the time point when smoothed probability of bear market regime $pr(S_t = 0|I_T)$ changes from above 50 percent into below 50 percent.
1.5.2 Empirical Model for the Analysis of Monetary Policy’s Impact on Stock Market

The Markov-switching dynamic factor model also produces a composite index to represent the overall stock market price movements, and calculates the probability of bear market and bull market. Then this chapter applies this stock price movement index into four time-varying parameter models to study the predictive and contemporaneous effect of monetary policy on stock market performance. Time-varying parameter model (see Kim and Nelson 1989) is chosen to study the effect of monetary policy on stock market for the following three reasons. First, the changing coefficients statistically measure the dynamic relationship between monetary policy and stock market in different time periods, which is also the focus of this study. Second, stock price reflect market participants’ expectation of the future. Investors in the stock market revise their expectations when new information becomes available. The changing coefficients capture the expectation revision of investors and show how investors have been changing the view on stock market. Third, time-varying parameter model is undertaken within the environment of a state-space model, which is calculated through a Kalman filter and the maximum likelihood estimation. As Harrison and Stevens (1976) and Kim and Nelson (1999) argued, an investor’s uncertainty about the future arises not only because of the uncertainty about future random disturbance, but also from the uncertainty about the accuracy of estimated parameter values of the model. The equation in the Kalman filter for the
variance of forecast error fully captures this property. In equations, the specification of the time-varying parameter model is presented as following.

**Time-Varying Parameter Model:**

\[
\Delta f_t = \beta_{0t} + \beta_{1t} \Delta M_t + \beta_{2t} \Delta i_t + u_t
\]

\[
\beta_{it} = \beta_{it-1} + \epsilon_{it} \quad i = 0,1,2
\]

Measurement equation: \( \Delta f_t = x_t \beta + u_t \)

\[
\Delta f_t = \begin{bmatrix} 1 & \Delta M_t & \Delta i_t \end{bmatrix} \begin{bmatrix} \beta_{0t} \\ \beta_{1t} \\ \beta_{2t} \end{bmatrix} + u_t
\]

Transition equation: \( \beta_t = \beta_{t-1} + \epsilon_t \)

\[
\begin{bmatrix} \beta_{0t} \\ \beta_{1t} \\ \beta_{2t} \end{bmatrix} = \begin{bmatrix} \beta_{0,t-1} \\ \beta_{1,t-1} \\ \beta_{2,t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{0t} \\ \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix}
\]

\( u_t \sim i.i.d. N(0, \sigma_{u^2}) \)

\( \epsilon_t \sim i.i.d. N(0, Q) \)

\[
Q = \begin{bmatrix} \sigma_0^2 & 0 & 0 \\ 0 & \sigma_1^2 & 0 \\ 0 & 0 & \sigma_2^2 \end{bmatrix}
\]

where \( \Delta f_t \) is a unobserved common factor extracted from the three major stock indices in the previous dynamic factor model measuring the overall stock price movement; \( \beta_{it} \) is time-varying coefficient which measures the relationship between monetary policy and stock prices; \( \Delta M_t \) is the difference of log broad monetary aggregate, which is measured by Divisia M4 in the first and second time-varying parameter model and by M2 in the third and fourth ones; \( \Delta i_t \) is the difference of log federal funds rate; \( u_t \) is the error term of the regression.
The first time-varying parameter model explores the contemporary relationship among M4, federal funds rate and stock market. This study also investigates lead-lag relationship among M4, federal funds rate and stock market in the time-varying parameter Model 2. As shown by Friedman (1988), monetary aggregate has different contemporary relationship and leading relationship with stock prices. Considering the fact that this chapter uses monthly data and many studies documented that the effects of monetary policy action on stocks are immediate, the analysis on the relationship between monetary policy and stock return with one month lag is conducted. In the time-varying parameter model 3 and time-varying parameter model 4, this chapter uses a narrower money supply measurement M2 to replace M4 for robustness check.

1.6. Empirical Results

The Maximum likelihood estimation results for the parameters of Markov-switching dynamic factor models are shown in the Table 1.1, with standard errors in the parentheses. The estimation results of Markov-switching dynamic factor model 2 are more favorable than Markov-switching dynamic factor model 1. Markov-switching dynamic factor model 1 has an insignificant variance for the second idiosyncratic term \( \sigma_2 \), indicating that the common factor was dominated by the second variable S&P500 index and the contribution of the other two indices is trivial. Besides, model 2 has a higher log likelihood value than model 1. Therefore, this chapter adopts model 2 as the Markov-switching dynamic factor model.
Table 1.1: The Estimation Results of Markov-Switching Dynamic Factor Models

<table>
<thead>
<tr>
<th>Parameter</th>
<th>MSDF-Model 1</th>
<th>MSDF-Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi )</td>
<td>0.213 (0.044)</td>
<td>0.216 (0.043)</td>
</tr>
<tr>
<td>( \varphi_{11} )</td>
<td>0.269 (0.043)</td>
<td>0.303 (0.046)</td>
</tr>
<tr>
<td>( \varphi_{12} )</td>
<td></td>
<td>-0.023 (0.007)</td>
</tr>
<tr>
<td>( \varphi_{21} )</td>
<td>0.108 (0.000)</td>
<td>-0.091 (0.082)</td>
</tr>
<tr>
<td>( \varphi_{22} )</td>
<td></td>
<td>-0.922 (0.051)</td>
</tr>
<tr>
<td>( \varphi_{31} )</td>
<td>0.345 (0.042)</td>
<td>0.373 (0.045)</td>
</tr>
<tr>
<td>( \varphi_{32} )</td>
<td></td>
<td>-0.035 (0.009)</td>
</tr>
<tr>
<td>( \sigma_1 )</td>
<td>0.297 (0.009)</td>
<td>0.288 (0.010)</td>
</tr>
<tr>
<td>( \sigma_2 )</td>
<td>0.0002 (0.007)</td>
<td>0.025 (0.011)</td>
</tr>
<tr>
<td>( \sigma_3 )</td>
<td>0.453 (0.014)</td>
<td>0.452 (0.015)</td>
</tr>
<tr>
<td>( \sigma_{\omega,1} )</td>
<td>1.423 (0.106)</td>
<td>1.416 (0.106)</td>
</tr>
<tr>
<td>( \sigma_{\omega,2} )</td>
<td>0.622 (0.035)</td>
<td>0.616 (0.036)</td>
</tr>
<tr>
<td>( \gamma_1 )</td>
<td>0.955 (0.014)</td>
<td>0.964 (0.014)</td>
</tr>
<tr>
<td>( \gamma_3 )</td>
<td>0.855 (0.021)</td>
<td>0.859 (0.021)</td>
</tr>
<tr>
<td>( \mu_0 )</td>
<td>-0.376 (0.149)</td>
<td>-0.383 (0.149)</td>
</tr>
<tr>
<td>( \mu_1 )</td>
<td>0.140 (0.042)</td>
<td>0.143 (0.043)</td>
</tr>
<tr>
<td>( P_{00} )</td>
<td>0.829 (0.070)</td>
<td>0.822 (0.075)</td>
</tr>
<tr>
<td>( P_{11} )</td>
<td>0.927 (0.028)</td>
<td>0.924 (0.030)</td>
</tr>
</tbody>
</table>

The factor loading measures the contribution of each stock index to the dynamic common factor. The estimates of factor loadings \( \gamma_i \) in the MSDF-Model 2 are all significantly positive, which means all the indices have positive contributions to the underlying common factor. The model allows the intercept and the variance of the
common factor to follow Markov switching between two regimes, and they are all statistically significant and very different from its own counterpart. The intercept of bear market regime $\mu_0$ has expected negative sign while the intercept of bull market regime $\mu_1$ has expected positive sign, implying that the underlying common factor has downward movements in bear markets but upward movements in bull markets. It is also shown by the estimation results that stock market is more volatile in bear market than bull market, given that $\sigma_w,1$ is larger than $\sigma_w,2$. Moreover, the probability for the bear market to stay in the bear market is $P_{00} = p(S_t = 0|S_{t-1} = 0) = 82.96\%$. This shows that the expected duration of bear market is 5.6 months, which is calculated by using formula $1/(1 - P_{00})$. Similarly, the probability for the bull market to stay in the bull market is $P_{11} = p(S_t = 1|S_{t-1} = 1) = 92.4\%$. The expected duration of bull market is about 13.2 months, calculated by $1/(1 - P_{11})$.

Figure 1.1 plots the smoothed probability of the bear market in the Markov-switching dynamic factor model. The reason for presenting the smoothed probability rather than the filtered probability lies in the fact that the filtered probability is based on information available up to currently available time $t$, but the smoothing is based on all the information through all time periods $T$. Therefore, the smoothed probability has more information available than the filtered probability, and provides a more accurate inference on the unobserved state vector and its covariance matrix.
Figure 1.1: The Smoothed Probability of Bear Market for the U.S. Stock Market

Figure 1.1 successfully captures all the bear markets in the sample period, namely stock crash in 1973 mainly caused by the economy stagflation and oil price rise, 1980 Silver Thursday sharp stock price drop caused by the silver market crash, 1982 stock price huge decline impacted by Kuwait’s stock market losses, 1987 Black Monday stock crash, early 1990s’ stock crash caused by the burst of Japanese property price bubble, bear market in 1998 caused by Russian financial crisis, stock crash in late 2001 caused by September 11 terrorist attack, bear market in 2002 generated by the burst of internet technology bubble, stock market crash in 2007 affected by subprime mortgage crisis, and stock market downturn in 2010 and 2011 caused by European sovereign debt crisis. This provides the evidence showing that the two-state Markov switching model successfully captures the dynamics of regime changes between bear market and bull market of the U.S. stock market. This chapter
applies the 0.5 value cut off line to the smoothed probabilities of bear market as the rule to determine the dates of bear market.

The beginning and ending dates of each bear market is shown in Table 1.2 and the time periods of bear market is demonstrated by the green area in Figure 1.2. The areas between red lines in Figure 1.2 denote the periods of economic recession of the U.S., announced by National Bureau of Economic Research. Figure 1.2 shows that every economic recession is associated with a bear market, but a bear market is not necessarily associated with a domestic economic recession. It confirms that stock market is related to the domestic economy but more volatile, because the underlying domestic economic condition is just one of the driving factors of stock market fluctuation. Stock market is affected by many other factors besides the domestic economic condition. For instance, the fluctuations of global market influence the U.S. stock market to a large extent. What’s more, the U.S. stock market is also substantially affected by political issue, unexpected events, natural disaster, investors’ fears, and etc. Most of them do not give rise to turns in business cycle of economy. Another important phenomenon demonstrated by the plot is that the stock market occasionally falls into a bear market in advance of the economic recession, confirming that stock market is a leading indicator of the economy. For example, the stock market switches into a bear market four months before the arrival of 2007 economic recession. This coincides with existing studies showing that the stock index is a leading indicator of business cycle (see, for example, Chauvet 1998/1999, and
Chauvet and Potter (2000, 2001). Chauvet and Potter (2001) used a dynamic factor model with Markov switching to date turning points of bear and bull markets as well. The data series used by them and their dating results reflecting turning points are different from those of this chapter.

Table 1.2: The Dates of Turning Points of Bear Market

<table>
<thead>
<tr>
<th>Begin (Peak)</th>
<th>End (Trough)</th>
<th>Begin (Peak)</th>
<th>End (Trough)</th>
</tr>
</thead>
<tbody>
<tr>
<td>March 1980</td>
<td>April 1980</td>
<td>August 2007</td>
<td>March 2009</td>
</tr>
</tbody>
</table>

Figure 1.2: The Periods of Bear Market and Economic Recession
Having demonstrated the time periods of U.S. bear/bull market above, we now turn to the question of monetary policy’s effects on these stock market movements across the bull and bear market, as well as different regimes of monetary policy.

Table 1.3: The Estimation Results of Time-Varying Parameter Model 1

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Time-Varying Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_u$</td>
<td>0.875 (0.032)</td>
</tr>
<tr>
<td>$\sigma_0$</td>
<td>-0.038 (0.02)</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>0.126 (0.038)</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>0.007 (0.003)</td>
</tr>
</tbody>
</table>

Log likelihood value | 697.39

Table 1.4: The Estimation Results of Time-Varying Parameter Model 2

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Time-Varying Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_u$</td>
<td>0.937 (0.032)</td>
</tr>
<tr>
<td>$\sigma_0$</td>
<td>0.000 (0.010)</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>0.096 (0.040)</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>0.0013 (0.0007)</td>
</tr>
</tbody>
</table>

Log likelihood value | 709.88

Table 1.5: The Estimation Results of Time-Varying Parameter Model 3

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Time-Varying Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_u$</td>
<td>0.878 (0.034)</td>
</tr>
<tr>
<td>$\sigma_0$</td>
<td>0.041 (0.019)</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>0.084 (0.048)</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>0.012 (0.004)</td>
</tr>
</tbody>
</table>

Log likelihood value | 701.67
Table 1.6: The Estimation Results of Time-Varying Parameter Model 4

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Time-Varying Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_u$</td>
<td>0.978 (0.031)</td>
</tr>
<tr>
<td>$\sigma_0$</td>
<td>0.000 (0.016)</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>0.011 (0.019)</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>0.0015 (0.0008)</td>
</tr>
</tbody>
</table>

Log likelihood value | 717.48

Time-varying parameter model are chose to examine the potential asymmetry over time. The Maximum likelihood estimation results for time-varying parameter models are shown through Table 1.3 to Table 1.6. Figure 1.3 plots the time-varying coefficient $\beta_{1t}$ which measures the contemporary relationship between broad monetary aggregate Divisia M4 and stock movements. The time periods of bear market is still depicted by the green area in Figure 1.3. The areas between red lines indicate the periods of economic recession of the U.S., announced by National Bureau of Economic Research. It is shown that there is a sharp drop in the time-varying parameter in every bear market, indicating the expansionary monetary policy of increasing monetary aggregate is less influential during a bear market. The sign of time-varying parameter has switched from positive to negative since 1987. 1987 is the year when Alan Greenspan became the Federal Reserve chairman and abandoned the monetary aggregate as the monetary target. This leads the conclusion that the signaling effect of monetary policy action of changing monetary aggregate only functions during the periods when it is used as the monetary policy target. A further
interpretation of this result is that the Federal Reserve’s action of changing monetary aggregate has positive effects on stock return only if it is considered by the market participants as a meaningful indicator of monetary policy. If the monetary aggregate is not used as monetary target, the stock market may not respond to the changes in monetary aggregate in a regular manner, and the negative impacts of monetary aggregate increase on stock performance that explained in the theoretical background would dominate the positive effects. During a bear market, a drop in the correlation makes the negative relationship more negative, which arrives at the conclusion that an expansionary monetary policy action of increasing monetary aggregate can even deteriorate the stock performance during a bear market within the periods when monetary aggregate is not the policy target.

Figure 1.3: Monetary Aggregate Parameter $\beta_{1t}$ in Time-Varying Model 1
As is evident from Figure 1.4, the concurrent relationship between changes in federal funds rate and stock price movements is inconsistent, switching between positive and negative as expected. The positive coefficient means the positive effects shown in the previous theoretical framework section dominate the negative effects, and vice versa. During the periods that the federal funds rate was used as a monetary policy target (1974-1980, and 1990-2008), the sign of the relationship between federal funds rate and stock market is negative, indicating that the expansionary monetary policy of reducing federal funds rate is positively influential on stock performance. This parameter becomes positive during other periods (1980s and after 2008), which illustrates that monetary action of reducing federal funds rate is useless in improving stock performance. This dynamics reinforces the conclusion that the signaling effects of monetary policy influence investors’ sentiment successfully only when the market participants believe the Federal Reserve’s action is meaningful. Besides, the coefficient also has a sharp decrease during every bear market. These drops make a positive coefficient negative, and a negative coefficient even more negative. If the Federal Reserve wants to apply an expansionary policy to stimulate the stock market by reducing the federal funds rate in a bear market, it will have a substantial effect, given that it is during the periods when federal funds rate is used as an effective monetary target. This result is consistent with the findings of Jansen and Tsai (2010) and Kurov (2010).
Figure 1.5 plots the time-varying coefficient $\beta_{1t}$ which measures the predictive relationship between monetary aggregate Divisia M4 and stock price one month later. One result refers to the fact that there exists a sharp drop in the coefficient in every bear market, indicating that the leading effect of changing monetary aggregate is much weaker in a bear market. In most bear markets, the coefficient reduces even below zero, presenting a negative relationship between money supply and stock market. If the Federal Reserve uses expansionary monetary policy to improve stock market performance during a bear market by increasing money supply, it is futile and may even deteriorate the stock market. Money supply is positively associated with future stock performance during most bull markets, with the exception of time periods in early 1990s and 2000s. The most recent two economic recessions in 2000s were all
followed by a slow and sluggish economy recovery. The economic recession in early 1990s was followed by a four-year slow recovery, and the economy started to take off in the middle of 1990s. A positive predictive relationship between money supply and stock market occurs during the periods of robust economic growth, not during the periods of economic recession or slow recovery. The lead-lag relationship between monetary policy and stock market is more related to the business cycle than monetary policy regimes.

Figure 1.5: Monetary Aggregate Parameter $\beta_{1t}$ in Time-Varying Model 2

![Figure 1.5](image)

Figure 1.6 depicts the dynamic association between the changes in stock prices and changes in federal funds rate. It shows the predictive relationship between changes in federal funds rate and stock price movements is negative during all periods.
This finding provides the evidence that the expansionary monetary policy of reducing federal funds rate is very influential in all monetary policy regimes and all stock market regimes. This negative relationship becomes weaker since late 2008, where the coefficient of lagged federal funds rate is close to zero. This is due to the fact that the federal funds rate was reduced to the zero lower bound in late 2008, and can’t be used as an expansionary tool for further reduction.

**Figure 1.6: Interest Rate Parameter $\beta_{2t}$ in Time-Varying Model 2**

![Graph](image)

If we replace M4 with M2 in time-varying parameter model 3 and 4, the results are similar. The dynamic pattern of federal funds rate is the same as in model 1 and 2 (see Figure 1.8 and 1.10). Figure 1.7 shows that the concurrent relationship between M2 and stock market is similar to that between M4 and stock. However, the lead-lag relationship between M2 and stock market (see Figure 1.9) is strikingly different from
that between M4 and stock. The curve is very flat and the insignificant parameter of variance indicates that there is no too much volatility in the relationship. The relationship remains positive until 1987, where the parameter reduces fundamentally to zero. This is consistent with the previous finding that the monetary aggregate change’s signaling effect only works during periods when monetary aggregate is used as the monetary policy target. The relationship turns into negative during the 2007 financial crisis. The lead-lag relationship between M2 and stock performance does not demonstrate a distinguished feature in different regimes of stock market and different phases of business cycle, confirming the fact that M4 is a broader measure of monetary aggregate.

Figure 1.7: Monetary Aggregate Parameter $\beta_{1t}$ in Time-Varying Model 3
Figure 1.8: Interest Rate Parameter $\beta_{2t}$ in Time-Varying Model 3

Figure 1.9: Monetary Aggregate Parameter $\beta_{1t}$ in Time-Varying Model 4
1.7 Conclusion

As mentioned in the introduction, previous literatures found that the Federal Reserve’s monetary policy has played an important role in affecting stock returns, but the empirical literature on the asymmetric effects of monetary policy on stock returns over time is limited and, unfortunately, mixed. The purpose of this chapter is to improve on the earlier literature by conducting another empirical analysis of the time-varying effects of monetary policy on stock performance in different monetary policy regimes and stock market regimes during the last four decades. More specifically, how have the different views on applying monetary policy by Burns in the 1970s, Volcker in the 1980s, Greenspan in the 1990s and early 2000s, and
Bernanke from mid 2000s to 2013 affected the stock market? How has the nature of the dynamic relationship between monetary policy and stock return vary during the bull and bear market? The substantial stock market volatility under current expansionary monetary policy emphasizes the necessity and urgency of the study on this issue.

This chapter begins with the exploration of the dates of the turning points of bear and bull market by applying a Markov-switching dynamic factor model on major stock indices, and produces a new composite measure to represent the overall stock market movement more broadly and comprehensively. The Markov-switching dynamic factor model extracts the comovement among stocks across different sectors and stock exchanges with an unobserved underlying common factor. The Markov-switching feature catches the nonlinear asymmetry in bear and bull market in terms of growth rate and volatility because of its nonlinearity setting, and is capable of statistically identifying the turning points of stock market regimes by using its inherent transition probabilities. It estimates the probabilities of bear market and bull market of every time point in the sample periods. The results successfully capture all the bear markets in the sample history. The findings indicate bear markets are more volatile than bull markets, and the average durations of bear market is shorter than that of bull market. The chapter shows that bear markets frequently occur in advance of economic recessions, confirming that stock market is a leading indicator of business cycle of economy. It is also shown that every domestic economic recession is
associated with a bear market, but not vice versa. This coincides with the widely accepted notion that underlying domestic economic condition is the most essential driving force for stock market fluctuation, but the stock market fluctuation is also affected by many other factors as well. These findings help to understand in which state of stock market fluctuation cycle is and where it is moving towards.

Having illustrated the characteristics of U.S. stock market movements above, this chapter turns to the more difficult question of the dynamic relationship between these stock market movements and monetary policy. The newly extracted unobserved factor is then applied into a time-varying parameter model as a composite measure of stock market movements. The results provide the evidence that the relationship between monetary policy and stock returns varies over time, and the responses of stock returns to monetary policy are asymmetric during bull and bear markets, and across different monetary policy regimes. Specifically, the contemporary signaling effects of increases in monetary aggregates or reductions in federal funds rate are positive on stock returns only during periods when they are used as the monetary policy target by the Federal Reserve. In other words, the desired effects of Federal Reserve’s action through changes in monetary aggregates or federal funds rate is strong on stock market only if it is considered by the market participants as a meaningful indicator of monetary policy. Besides, a positive predictive relationship between monetary aggregate and stock returns one month later is detected during the periods of robust economic growth, but not during the periods of economic recession or slow recovery.
The observation of a sharp drop in the value of the correlation between monetary aggregate and stock return in every bear market indicates that the impacts of the monetary policy of increasing monetary aggregates are much weaker in a bear market, and can even deteriorate stock market. However, the expansionary monetary policy of reducing federal funds rate has strong positive effect on stock market performance during a bear market within the periods when federal funds rate is used as monetary policy target by the Federal Reserve.
Bibliography


Chapter 2

Nonlinear Relationship between Monetary Policy and Stock Returns: A Markov-Switching Dynamic Bi-Factor Approach\(^1\)

2.1 Introduction

Monetary policy is one of the macroeconomic variables that are closely followed by the stock market participants. The overall relationship between monetary policy and stock market return has been heatedly discussed in both academia and industry. Understanding this topic is very valuable for both stock market investors and policymakers. Obviously, it is important for stock market investors to understand the extent to which their stock investment is affected by the monetary policy. Determining the influence of monetary policy on stock market is also a key step for policymakers to formulate appropriate policy decisions.

Two strands of debate arise when people investigate the relationship between monetary policy and stock market. The first strand of debate is about whether the relationship runs from monetary policy to stock market or from stock market to monetary policy. The second one is about whether the relationship, if at all, is positive or negative.

\(^1\) This chapter is co-authored with Dr. Marcelle Chauvet
As documented in many economic research, monetary policy actions have a significant effect on stock market returns (see, for example, Jensen, Mercer, and Johnson, 1996; Patelis, 1997; Thorbecke, 1997; Rigobon and Sack, 2004; Bernanke and Kuttner, 2005; Chen, 2007). However, the evidence on the direction of the monetary policy’s effects on stock return is mixed. Thorbecke (1997), Rigobon and Sack (2004), and Bernanke and Kuttner (2005) all suggested a negative relationship between the Federal Reserve’s monetary policy interest and the stock market returns. In particular, rises in the monetary policy interest rate would reduce the stock market return, and reductions in monetary policy interest rate would generate the reverse effect. However, Cornell (1983) and Sellin (2001) demonstrated that the link between money supply and asset prices can be either positive or negative, depending on the assumption and hypothesis.

On the other hand, there exist some arguments stating that the performance of stock market also influences monetary policy maker's decision, meaning that the Federal Reserve’s monetary policy responds to stock market fluctuations. For example, whether it is correct or not, many people believe that the monetary contraction that began in late 1990s was a direct or indirect result of the "irrational exuberance" of stock market. It is well known that the Federal Reserve’s two ultimate objectives for its monetary policy are supporting maximum sustainable output and employment, and maintaining stable price level. It should be noticed upfront that stock price is not the one of the objectives of the Federal Reserve. However, many
still believe that stock prices may contain useful information about the current and future economic conditions useful to the central banks, because stock prices contain market participants’ expectation of economic variables such as inflation and output. As a result, stock price is a leading indicator that predicts the economy, and stock price movement is an important determinant of the monetary policy through their influence on the macroeconomic variables concerned by the Federal Reserve, such as price stability and economic growth. Stock market price can be a leading indicator that has implications on behavior of macroeconomic variables to some extent. In addition, Tobin’s $q$ theory (Brainard and Tobin 1968; Tobin 1969) suggests that increased stock prices would stimulate economic activities (Mishkin 2006). Tobin’s $q$ is defined as the market value of the company divided by the replacement cost of capital. A higher stock price will generate a higher $q$, leading to a higher market price of the company relative to the replacement cost of capital. The new plant and equipment capital will become relatively cheap, and the company will expand the investment by purchasing new capital investment products with a small issue of stock.

Another two strands of theory supporting the statement that monetary policy respond to stock prices are based on wealth effect channel and credit channel of monetary policy mechanism. The wealth effect channel theory argues that changes in monetary policy such as interest rate movement would affect financial assets’ value, such as the value of bonds and stocks, which in turn have an impact on the wealth of asset holders, their consumption and aggregate demand accordingly (Ludvigson,
Steindel, and Lettau 2002). The credit channel theory demonstrates that the stock prices changes would affect households and firms through their effects on the balance sheet conditions and financial positions of households who hold stocks and firms who issue and hold stocks, and hence their ability to access the credit. For example, households and firms may use stocks as their collateral for borrowing, and a drop in the stock price would reduce the value of their collateral for credit. A decline in the availability of credit deteriorates the consumption and investment, and negatively affects the aggregate demand and output as well. Stock returns have an impact on households’ consumption by altering their wealth condition, and also affect firms’ capability to finance future projects and hence their investment decisions. Monetary authorities should pay attention to stock prices movements, since they influence key economic variables such as consumption, investment, and output. Some studies also provide formal statistical evidence showing that stock market performance affects the changes of monetary policy (For example, see Rigobon and Sack, 2003, 2004).

The purpose of this chapter is to explore the above mentioned two questions. The chapter makes two main contributions to the literature. First, the chapter sheds new light on the forces driving the U.S. monetary policy regime switching and extracts a latent common factor from monetary policy variables. The second and most important contribution of the chapter is exploring an alternative perspective on the relationship between monetary policy and stock return, by establishing a Markov-switching dynamic bi-factor model.
In terms of measuring monetary policy, most studies used monetary aggregate and monetary policy interest rate (Patelis 1997, Thorbecke 1997, among others). This article extends the existing literature by including credit as one of monetary policy variable. The monetary policy actions affect the stock prices through credit in a direct way and an indirect way. The direct way argues that monetary policy actions such as changing interest rate cause a change in the price of credit. Most firms rely on the bank loans to finance their investment projects. The changes in the price of credit directly have an influence on firms’ borrowing cost, earnings, and stock prices. The indirect way is two-fold. First, the theoretical analysis for Federal Reserve’s monetary policy transmission mechanism indicates the impact of the monetary policy on the macroeconomy comes primarily through several channels, including the interest rate channel, the wealth effect channel and the credit channel. According to many influential literatures (see, for example, Bernanke and Blinder 1988, 1992, Bernanke and Gertler 1989, 1995, 2001, Gertler and Gilchrist 1994, Kiyotaki and Moore 1997), there are two sub-channels under the credit channel mechanism, a balance sheet channel, and a bank lending channel. Through balance sheet channel and bank lending channel, a contractionary monetary policy shock leads banks to have higher lending interest rates or less supply of new credit, leading to a decline in consumption, investment, and hence economic conditions. Second, it is commonly accepted that stock return is fundamentally affected by the economic condition. Therefore, even though Federal Reserve’s monetary policy is ultimately directed to macroeconomic
variables such as unemployment and price level, the Federal Reserve will only influence the macroeconomic variables and stock returns by changing interest rate, money supply and credit to alter people’s economic behavior.

This chapter employs a univariate Markov switching model on each of the three monetary policy variables. The estimation results of univariate Markov switching models show that they all display significant regime switching patterns, and the dynamics of all the three monetary policy variables are related to the economic cycles, which makes it reasonable to extract a common factor to represent the comovement among these monetary policy variables. The Markov-switching dynamic bi-factor model is used to simultaneously extract two latent common factors from monetary policy variables and stock indices to represent monetary policy changes and stock price movements separately. The Markov switching smoothed probabilities results demonstrate that low monetary policy regimes follow economic recessions, and that bear markets usually occur before economic recessions. The results also show that almost every economic recession is associated with a bear market, but not vice versa. The Maximum likelihood estimation results for the Markov-switching dynamic bi-factor model show that contractionary monetary policy has a significantly negative impact on stock returns, but stock returns don’t influence monetary policy decision.

The remainder of the chapter is organized as follows. The second section discusses the related literature. The third section describes the data used in the model. The univariate Markov-switching model and Markov-switching dynamic bi-factor
model are illustrated in the fourth section. The fifth section presents the empirical results. This chapter is concluded in the sixth section.

2.2 Literature Review

A large body of literature on monetary policy transmission present that changing monetary policy has important effects on aggregate demand, which influence consumption and output in real economy. According to Mishkin (2006), there are several monetary policy transmission channels in which monetary policy can be translated to the effects on real economy, such as interest rate channel, asset pricing channel and credit channel.

The interest rate channel is the most traditional monetary policy transmission mechanism. When the Federal Reserve, for example, loosens the monetary policy and conducts open market purchase, the increased reserve will drive down the federal funds rate. The lower federal funds rate will reduce other short-term interest rates in the loanable funds market. Given certain degree of price stickiness in the short run, lower short-term interest rates lead to lower real interest rates, which in turn generate a lower borrowing cost. Consumers are more likely to finance the big purchase on products, such as cars. Firms who borrow loans to finance their investment projects are also positively affected. Consequently, the increased level of aggregate demand can have a positive impact on real economic activities. This channel also works in the scenario of zero lower bound interest rate, where an
expansion in the money supply can raise the expected price level and hence expected inflation. Based on Fisher equation, real interest rate will be reduced even if the nominal interest rate is fixed at zero level, and stimulate the economy though spending and investment.

The asset price channel is related to the wealth effect (Lettau and Ludvigson, 2001). Changes in monetary policy such as interest rate movements result in changes in asset prices, such as the price of households’ houses and financial assets. These assets account for a large proportion of households’ total wealth. Changes in households’ wealth will affect their consumption and hence the real economy.

The third monetary policy transmission channel is the credit channel mechanism, which was demonstrated by a large body of literature (see Bernanke and Blinder 1988, 1992, Bernanke and Gertler 1989, 1995, Gertler and Gilchrist 1994, Kiyotaki and Moore 1997). As Bernanke and Gertler (1995) illustrated, there are two sub-channels under the credit channel, which are balance sheet channel and bank lending channel. The balance sheet channel is consistent with the effects of monetary policy changes on borrowers’ financial statements variables, such as borrowers’ net worth, cash flow, and assets. Through the balance sheet channel, the effects of monetary policy adjustments on short-term interest rate are amplified by changes in the external finance premium, which is defined as the difference in financing cost between funds raised internally such as retained earnings and funds raised externally such as issuing equity or debt. Bernanke and Gertler (1995) presented evidence showing that a
tightening monetary policy worsens borrower’s financial position, and a borrower’s external finance premium is negatively correlated with borrower’s financial position. Therefore, the size of the external finance premium changes in the same direction of the federal funds rate. A tightening monetary policy action reduces the value of the firms’ assets that function as the collateral for loans, and consequently raises the external finance premium, leading to a more difficult external finance. This results in a higher borrowing cost and a reduced access to bank loans, forcing the firm to lower its level of investment, and ultimately decreasing the real economic activities. Moreover, the bank lending channel suggests that the Federal Reserve’s monetary policy actions alter the amount of bank credit that firms and consumers can have access to, which in turn influences the investment and consumption. As Bernanke and Blinder (1988, 1992) pointed out, reservable deposit is an important source of funds for bank loans. They stated that Federal Reserve’s tightening monetary policy action such as open market sales would reduces reserves and deposits in the banking system. This can decrease the loanable funds available to banks, and accordingly lower the supply of bank loans. Since there is a big portion of firms and consumers rely on bank credit financing, a reduction in the availability of bank credit will depress consumers’ spending and firms’ investment, leading to a decrease in real activities.

It is well documented in the literature that stock market performance is fundamentally determined by the condition of real economic activities such as consumption and investment. Lettau and Ludvigson (2001) pointed out that the
change in aggregate consumption is a very useful predictor for stock return. Using U.S. quarterly stock market data, they find that consumption-wealth ratio plays an important role in predicting the stock market return. Ferson and Harvey (1991) also found evidence supporting the view that stock market returns fundamentally reflect the real economy conditions. By using an asset pricing model, they suggest that consumption and interest rate indicate most of the predictable variations in stock return, which can be explained by effects through stocks’ risk exposures such as betas and market risk premium. Yogo (2006) provided consumption-based explanation of variation in expected stock returns, and points out that stock returns are unexpectedly low at business cycle recessions, when durable consumption falls sharply. Hamilton and Lin (1996) investigated the joint time series behavior of monthly stock returns and growth in industrial production. They employed a bivariate model in which changes in stock return and industrial production are driven by related unobserved variables, and found that economic recessions are the most important factor that drives fluctuations in the volatility of stock returns, which strong supports the linkage between real economy and stock returns.

Therefore, based on the fact that monetary policy actions influence the real economy and the fact that real economy condition is the fundamental factor affecting stock market, it is reasonable to investigate the interrelationship between monetary policy actions and stock market, through the changes in interest rate, money supply and credit to alter people’s economic behavior.
Most studies show that monetary policy actions have an impact on stock market
(see, for example, Jensen, Mercer, and Johnson, 1996; Patelis, 1997; Thorbecke, 1997;
Rigobon and Sack, 2004; Bernanke and Kuttner, 2005; Chen, 2007). In a more recent
study, Sola, Spagnolo, and Spagnolo (2007) empirically showed the reaction of the
stock market returns to changes in monetary policy. They extended a multivariate
Markov switching model by allowing the transition probability matrix time-varying
and to be a function of monetary policy variables. Their chapter provided evidence
showing that monetary variables such as the interest rate play an important role in
predicting changes in stock returns.

However, some literature indicates that monetary policy also responds to the
changes in stock market. As illustrated by Rigobon and Sack (2003, 2004), short-term
interest rates significantly react to the movements in equity index, indicating the
endogenous response of monetary policy to stock market fluctuation. They
emphasized that the causality between interest rates and stock prices may run in both
directions, leading to a possible endogeneity problem. After dealing with this
endogeneity problem, they found empirical evidence showing that there exists a
significant monetary policy response to the stock market movements.

Even among the literatures that study the effect of monetary policy on stock
returns, the results are ambiguous and there is not yet a consensus on this conclusion.
Some studies showed the effect is positive and some studies argue that the effect is
negative. Thorbecke (1997) employed a monthly VAR model to analyze the link and
found a positive relationship between the expansionary monetary policy of reducing policy interest rate and the stock return. A positive relationship between the expansionary monetary policy and stock market return has also been found by Patelis (1997), Lastrapes (1998) and many others. However, several articles provided counter examples on the direction of monetary policy effects on stock return. Cornell (1983) and Sellin (2001) demonstrated that the link between money supply and asset prices can be either positive or negative, depending on the assumption and hypothesis.

2.3 Data

The overall price level of stock market is measured by the composite stock index. The most popular and influential stock indices in the U.S stock market, such as Dow Jones Industry Average, Standard & Poor’s 500, and NASDAQ Composite, are used to capture the movement in stock prices. For monetary policy, this chapter uses an interest rate variable, a monetary aggregate variable, and a credit variable to represent the changes in monetary policy. As well documented in the Federal Reserve’s history, both federal funds rate and different measures of monetary aggregates have been used as the monetary policy target. Federal funds rate and Divisia M4, which is modernized to be consistent with the current financial innovations, are used as the interest rate variable and the monetary aggregate variable in the model.

This chapter chooses total outstanding consumer credit owned and securitized as the credit variable to represent the monetary policy effect through credit channel, in
order to support the existence of a credit channel. Consumer credit is defined as the credit financed through banks for the purchase of consumer durables, such as TV and cars, on installment basis. A wide range of literature (see, for instance, Ludvigson 1998, Brady 2010, Gertler and Gilchrist 1994) showed empirical results that among three main private sector borrowing including consumer credit, commercial credit, and real estate credit, consumer credit is mainly affected by the Federal Reserve’s monetary policy. Since NASDAQ Composite was established in 1971, this chapter studies monthly data ranging from March 1971 to November 2012. The data is obtained from the websites of Federal Reserve Bank of St. Louis FRED database, Yahoo Finance, and Center for Financial Stability.

2.4 Empirical models

2.4.1 Univariate Markov Switching Model

It was well documented in the literature that many economic and financial variables follow the nonlinear pattern of regime switching (see, for example, Sims and Zha 2006, and Ang and Timmermann 2011). Regime switching models have become more and more popular in economic research and financial modeling. Andolfatto, and Gomme (2003), Ang and Bekaert (2002), Owyang and Ramey (2004), and Liu, Waggoner, and Zha (2009, 2011) particularly found that U.S. monetary policy variables such as policy interest rate showed regime-switching patterns. Hamilton (1989) first applied regime switching to business cycle recessions and expansions and
successfully showed the regimes naturally captured cycles of real economy, as Hamilton’s regimes periods closely match recession periods identified by the NBER business cycle dating committee.

This chapter specifies a variant of Hamilton’s (1989) work and estimates a univariate Markov switching model separately for each monetary policy variables, which are federal funds rate, broad money supply Divisia M4, and consumer credit. The important feature of such Markov switching model is that it can capture a particular form of nonlinear dynamics in the monetary policy movements. The assumptions of the model in this chapter are simple in order to keep the model parsimonious. This chapter allows the mean and variance of the monetary policy variables to be evolving according to a two-state Markov-switching process, depending on whether the monetary policy is in the expansionary regime or the contractionary regime. The variable switches between the two regimes controlled by the transition probabilities of an unobserved Markov chain. By doing this, the dynamics of contractionary monetary policy actions can be significantly differentiated from those of expansionary monetary policy actions. Following Chauvet (1998/1999), the changes of monetary policy variables are models as a AR(0) process each. Every monetary policy is decomposed into two integrated components: a Markov trend term, $\mu S_t$, and a Gaussian component $e_t$. Both $\mu S_t$ and the variance of $e_t$ are controlled by an unobserved state variable $S_t$. $S_t$ follows a two-state Markov processes with constant transition probabilities, which means that $S_t$ is allowed to assume one of the
two different values represented by the integers 0 and 1. When \( S_t = 0 \), the mean growth rate of the variable takes the value of \( \mu_0 \), while when \( S_t = 1 \), the mean growth rate is given by the parameter of \( \mu_1 \). The same rule is also applied to the variance of error term \( e_t \). A probability law is designed to govern the variable to switch between two regimes, in which the transition probability is assumed to be time-fixed. Taking one of the monetary policy variables federal funds rate (FFR) for example, the univariate Markov switching model is set as follows:

\[
\Delta FFR_t = \mu_{S_t} + e_t
\]

\[
e_t \sim i. i. d. N(0, \sigma_{S_t}^2)
\]

\[
\mu_{S_t} = \mu_0(1 - S_t) + \mu_1 S_t
\]

\[
\sigma_{S_t}^2 = \sigma_0^2 (1 - S_t) + \sigma_1^2 S_t
\]

\[
Pr[S_t = 0|S_{t-1} = 0] = P_{00}
\]

\[
Pr[S_t = 1|S_{t-1} = 1] = P_{11}
\]

This chapter also repeats the same procedures for other two monetary policy variables, broad money supply Divisia M4 (M4) and total consumer credit (CREDIT).

### 2.4.2 Markov Switching Dynamic Bi-factor Model

This chapter proposes a variant of Chauvet and Senyuz (2012)’s work and constructs a Markov switching dynamic bi-factor econometric model to investigate the joint dynamic interrelationship between the monetary policy of the Federal Reserve and the U.S. stock market performance. The regime-switching patterns of
monetary policy variables (federal funds rate, money supply M4 and consumer credit) and the comovement among those variables are demonstrated in the above univariate Markov-switching models. Besides, many studies (see Pagan and Sossounov 2003) indicates that U.S. stock market show significant regime-switching pattern between bull and bear market in growth and volatility, and documented that major stock indices presents similar movements. Some studies (Chauvet and Potter 2000, 2001) also used Markov switching dynamic factor model to date the bear and bull markets.

Therefore, this chapter employs a Markov switching dynamic bi-factor to extract a Markov-switching latent factor, $MF_t$, from these monetary variables to represent changes in the monetary policy, and also simultaneously extracts a second latent factor, $SF_t$, from major stock market indices to represent the stock market movements. One state space model is cast to include two unobserved factors, representing the common factor in the Federal Reserve’s monetary policy variables and major stock indices. These two unobserved factors as well as their lead-lag relationship with each other are estimated simultaneously in a joint nonlinear econometric model from the observable variables. The unobserved factors are set in the framework of a bivariate vector autoregression to examine the dynamic interrelationship between stock market phases and monetary policy regimes. This chapter allows the drift term and the variance of error term to switch between the two distinct Markov states. The switching is controlled by the transition probabilities of the first order two-state Markov process.
\[ p_{ij}^{MF} = Pr [S_{t}^{MF} = j | S_{t-1}^{MF} = i] \]
\[ p_{ij}^{SF} = Pr [S_{t}^{SF} = j | S_{t-1}^{SF} = i], i = 0,1 \]
\[ \sum_{j=0}^{1} p_{ij}^{MF} = 1, \text{ and } \sum_{j=0}^{1} p_{ij}^{SF} = 1 \]

The Markov state \( S_{t}^{MF} \) represents the expansionary or contractionary monetary policy regime, and \( S_{t}^{SF} \) measures the bull or bear stock market regime. Because the monetary policy and stock market are unlikely to shift at the same time, the latent factors representing monetary policy and stock market are not restricted by the model to switch between different Markov regimes simultaneously, which means they are allowed to follow different two-state Markov-switching process. The bivariate vector autoregression model has following form:

\[
\begin{bmatrix}
\Delta MF_t \\
\Delta SF_t
\end{bmatrix}
= \begin{bmatrix}
\mu_{t}^{MF} \\
\tau_{t}^{SF}
\end{bmatrix}
+ \begin{bmatrix}
\phi_1 & \phi_2
\end{bmatrix}
\begin{bmatrix}
\Delta MF_{t-1} \\
\Delta SF_{t-1}
\end{bmatrix}
+ \begin{bmatrix}
v_{1t} \\
v_{2t}
\end{bmatrix}
\]

\[ v_{1t} \sim N \left( 0, \sigma_{v1,S_{t}^{MF}}^2 \right) \text{ and } v_{2t} \sim N \left( 0, \sigma_{v2,S_{t}^{SF}}^2 \right) \]

The coefficients in the 2x2 matrix \( \begin{bmatrix} \phi_1 & \phi_2 \\ \varphi_1 & \varphi_2 \end{bmatrix} \) can measure the lead-lag relationship between monetary policy latent factor and stock market latent factor. As other dynamic factor models, the Markov switching dynamic bi-factor model is also made up of a measurement equation and a transition equation. The Markov switching joint dynamic bi-factor model takes the form as following:

Measurement equation: \( Y_t = H \beta_t \)
Transition equation: \( \beta_t = \Pi_{st} + F\beta_{t-1} + V_t \)

\[
\begin{bmatrix}
\Delta MF_t \\
\Delta SF_t \\
u_{1t} \\
u_{2t} \\
u_{3t} \\
u_{4t} \\
u_{5t} \\
u_{6t}
\end{bmatrix}
\begin{bmatrix}
\mu_{st}^{MF} \\
\tau_{st}^{SP} \\
\phi_1 \\
\phi_2 \\
\varphi_1 \\
\varphi_2 \\
k_1 \\
k_2
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
k_1 \\
k_2
\end{bmatrix}
+ \begin{bmatrix}
\varphi_1 \\
0 \\
k_3 \\
k_4
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
k_5 \\
k_6
\end{bmatrix}
= \begin{bmatrix}
\Delta MF_{t-1} \\
\Delta SF_{t-1} \\
u_{1t-1} \\
u_{2t-1} \\
u_{3t-1} \\
u_{4t-1} \\
u_{5t-1} \\
u_{6t-1}
\end{bmatrix}
\begin{bmatrix}
v_{1t} \\
v_{2t} \\
\epsilon_{1t} \\
\epsilon_{2t} \\
\epsilon_{3t} \\
\epsilon_{4t} \\
\epsilon_{5t} \\
\epsilon_{6t}
\end{bmatrix}
\]

\[V_t \sim i.i.d. N(0, Q)\]

\[
Q = \begin{bmatrix}
\sigma_{v1st}^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \sigma_{v2st}^2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \sigma_{u1}^2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \sigma_{u2}^2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \sigma_{u3}^2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sigma_{u4}^2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \sigma_{u5}^2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{u6}^2
\end{bmatrix}
\]

The dynamic factors are assumed to be uncorrelated with the idiosyncratic terms, \(u_{lt}\), at all lags and leads. It is also assumed that the variance covariance matrix \(Q\) is diagonal. The Markov switching dynamic bi-factor model estimate all latent factors and parameters simultaneously in one step.

This chapter constructs the Markov switching dynamic bi-factor model by combining the dynamic factor model in the state space representation and the Markov switching feature.

The model is estimated using the algorithm shown by Kim and Nelson (1999). The details are illustrated in Chapter one and Appendix. The unknown parameters and the latent factors are estimated through a nonlinear optimization procedure to maximize the likelihood function. His work also provides filtering and smoothing
algorithms for the Markov-switching dynamic factor model estimation, and suggests that the increasing amount of Markov cases is collapsed and truncated at each iteration step using an approximation. Since there are two latent factors, it is necessary to standardize two of the factor loadings or two factor variances to be one for identification purpose. In this model setting, the factor variance is designed to follow a Markov-switching process to capture the nonlinearity in monetary policy volatility and stock market volatility. Therefore, factor loadings $\lambda_1$ and $\gamma_1$ are standardized to be one.

Augmented Dickey-Fuller unit root tests are applied to each of monetary policy variables and stock index variables. The unit root test results show that each variable has a unit root. This chapter uses log difference data series, and trims the outlier observations outside the range of three standard deviations. All the modified variables are then standardized by subtracting the sample mean and dividing by sample standard deviation.

2.5 Empirical Results

2.5.1 Univariate Markov Switching Model Estimation Results

Table 2.1 presents the maximum likelihood estimates of the parameters of the univariate Markov switching models for each monetary policy variable, with standard errors in the parentheses. All the coefficients in these Markov switching models are statistically significant. In each model, different Markov states show very distinct
patterns in both mean and variance. The mean of growth rate is negative in the low federal funds rate regime and positive in the high federal funds rate regime, indicating that federal funds rate declines during the expansionary monetary policy regime, and increases during the contractionary regime. However, the means of growth rates are positive but smaller in the low money supply regime and low credit regime, meaning that the evolvements of money supply and consumer credit are always ascending, but they just increase at a lower rate during the low regimes.

Table 2.1: The Results of Markov-Switching Models for Monetary Variables

<table>
<thead>
<tr>
<th>Parameters</th>
<th>MS Model for FFR</th>
<th>MS Model for M4</th>
<th>MS Model for CREDIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{00}$</td>
<td>0.932 (0.021)</td>
<td>0.976 (0.012)</td>
<td>0.959 (0.016)</td>
</tr>
<tr>
<td>$P_{11}$</td>
<td>0.918 (0.027)</td>
<td>0.975 (0.013)</td>
<td>0.981 (0.008)</td>
</tr>
<tr>
<td>$\mu_0$</td>
<td>0.583 (0.226)</td>
<td>0.648 (0.022)</td>
<td>1.052 (0.024)</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>-2.174 (0.635)</td>
<td>0.291 (0.033)</td>
<td>0.348 (0.019)</td>
</tr>
<tr>
<td>$\sigma_0^2$</td>
<td>8.374 (1.024)</td>
<td>0.069 (0.007)</td>
<td>0.079 (0.009)</td>
</tr>
<tr>
<td>$\sigma_1^2$</td>
<td>81.746 (8.335)</td>
<td>0.152 (0.015)</td>
<td>0.116 (0.009)</td>
</tr>
</tbody>
</table>

Log likelihood value

1215.331
325.357
310.729

The smoothed probabilities of low regime of each monetary policy variables are shown by the black curves in Figure 2.1, Figure 2.2, and Figure 2.3. The green shaded areas represent the time periods of economic recession announced by the National
Bureau of Economic Research. It can be seen from Figure 2.1 that the low states of federal funds rate correspond well with economic recessions, which is consistent with Ang and Bekaert (2002). Every economic recession is associated with a high probability of low federal funds rate, leading to the conclusion that reducing federal funds rate is the main expansionary monetary policy tool that is used by the Federal Reserve to respond to the economic recession. In addition, high probabilities of low federal funds rate regime occur more frequently than economic recession, indicating that the Federal Reserve applies federal funds rate tool more often than the frequency of economic recessions. Figure 2.2 shows the patterns of low money supply regime, which is somewhat related to economic cycle. This is because the Federal Reserve mainly uses open market operation and federal funds rate as the tools to affect the economy, and the application of open market operation only directly leads to changes in reserve and monetary base, which indirectly affects money supply. Low money supply is associated with low aggregate demand and hence economic recession. Figure 2.3 demonstrates that low consumer credit regime is strongly linked to economic recession. During the economic recession, it is harder for consumers to get access to the bank credit. In particular, consumer credit maintains low after the economic recession in 2001. In short, the dynamics of all the three monetary policy variables are related to the economic conditions, and hence it is reasonable to generate a common factor to represent the comovement among these monetary policy variables including money supply, federal funds rate, and credit.
Figure 2.1: The Smoothed Probability of Low Federal Funds Rate Regime

Figure 2.2: The Smoothed Probability of Low Broad Money Supply M4 Regime
2.5.2 Markov Switching Dynamic Bi-factor Model Estimation Results

The Maximum likelihood estimation results for Markov-switching dynamic bi-factor models are shown in Table 2.2, with standard errors in the parentheses. The Markov-switching dynamic bi-factor model simultaneously extracts a latent common factor from monetary policy variables and a latent common factor from stock indices. The mean growth and variance of two factors are significantly different between two regimes, indicating that regime switching exists in both monetary policy and stock market. The mean growth rate is negative in the first regime of stock market, but positive in the second regime, indicating that first regime represents bear market and second regime represents bull market. The variance of latent factor in the first stock market regime is higher than that in the second regime, which is consistent with the fact that bear market is more volatile than bull market. The negative mean growth rate
in the first regime of monetary policy demonstrates that the first regime is expansionary monetary policy regime, as federal funds rate dominates other two monetary variables in the correlation with the factor. The higher variance in the first regime shows that expansionary monetary policy actions are more volatile.

### Table 2.2: The Results of Markov-Switching Dynamic Bi-factor Model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Estimation results</th>
<th>Parameters</th>
<th>Estimation results</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_1$</td>
<td>0.245 (0.089)</td>
<td>$\sigma_{v1,0}^2$</td>
<td>1.289 (0.086)</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>0.048 (0.034)</td>
<td>$\sigma_{v1,1}^2$</td>
<td>0.334 (0.037)</td>
</tr>
<tr>
<td>$\varphi_1$</td>
<td>-0.114 (0.043)</td>
<td>$\sigma_{v2,0}^2$</td>
<td>1.355 (0.101)</td>
</tr>
<tr>
<td>$\varphi_2$</td>
<td>0.200 (0.043)</td>
<td>$\sigma_{v2,1}^2$</td>
<td>0.590 (0.034)</td>
</tr>
<tr>
<td>$\mu_0$</td>
<td>-0.167 (0.132)</td>
<td>$\sigma_{u1}^2$</td>
<td>0.192 (0.053)</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>0.107 (0.074)</td>
<td>$\sigma_{u2}^2$</td>
<td>0.712 (0.023)</td>
</tr>
<tr>
<td>$\tau_0$</td>
<td>-0.397 (0.143)</td>
<td>$\sigma_{u3}^2$</td>
<td>0.905 (0.029)</td>
</tr>
<tr>
<td>$\tau_1$</td>
<td>0.150 (0.040)</td>
<td>$\sigma_{u4}^2$</td>
<td>0.297 (0.009)</td>
</tr>
<tr>
<td>$p_{00}^{MF}$</td>
<td>0.916 (0.033)</td>
<td>$\sigma_{u5}^2$</td>
<td>0.000 (0.024)</td>
</tr>
<tr>
<td>$p_{11}^{MF}$</td>
<td>0.950 (0.017)</td>
<td>$\sigma_{u6}^2$</td>
<td>0.453 (0.014)</td>
</tr>
<tr>
<td>$p_{00}^{SP}$</td>
<td>0.830 (0.062)</td>
<td>$k_1$</td>
<td>0.892 (0.042)</td>
</tr>
<tr>
<td>$p_{11}^{SP}$</td>
<td>0.932 (0.025)</td>
<td>$k_2$</td>
<td>0.702 (0.032)</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>-0.009 (0.036)</td>
<td>$k_3$</td>
<td>0.425 (0.041)</td>
</tr>
<tr>
<td>$\lambda_3$</td>
<td>0.023 (0.050)</td>
<td>$k_4$</td>
<td>0.269 (0.043)</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>1.047 (0.015)</td>
<td>$k_5$</td>
<td>0.306 (0.000)</td>
</tr>
<tr>
<td>$\gamma_3$</td>
<td>0.895 (0.026)</td>
<td>$k_6$</td>
<td>0.345 (0.042)</td>
</tr>
</tbody>
</table>

| Likelihood | 41.638 |

The Markov-switching dynamic bi-factor model also calculates the smoothed probabilities of monetary policy regimes and stock market regimes, which are
depicted by blue curves and red curve respectively in Figure 2.4. The green shaded areas denote the periods of economic recession defined by the National Bureau of Economic Research. The Markov switching results of monetary policy demonstrate that low monetary policy regimes follow economic recessions, confirming that monetary policy is the main tool used by the monetary authority to respond to the economic recession. The Markov switching results of stock market show that bear markets usually occur before economic recessions, indicating that stock price is a leading indicator of business cycles. The results also show that every economic recession is associated with a bear market, but not vice versa.

**Figure 2.4: The Smoothed Probability of Low Monetary Regime and Bear Market Regime**
The bi-variate VAR model estimation results reveal the dynamic lead-lag relationship between monetary policy and stock market performance. The statistically significant autoregression parameters $\phi_1$ and $\varphi_2$ show that both monetary policy factor and stock market factor are positively affected by their own information in the previous period. Parameter $\phi_2$ is statistically insignificant, indicating that the past information of stock market doesn’t have an effect on monetary policy decisions. This is consistent with the view that monetary policy should focus on price level and inflation in goods markets, and should not look at the financial asset prices unless they affect the inflationary forces in the good markets, such as through wealth effects on consumption and demand. However, parameter $\varphi_1$ is statistically significant, meaning that the past information of monetary policy plays an important role in affecting the future stock prices. Parameter $\varphi_1$ is significantly negative, which implies that the Federal Reserve’s expansionary monetary policy such as a decrease in the federal funds rate would raise stock return. The estimation result shows that the current stock market price is affected by the past information of monetary policy. This can be explained by discounted dividend theory of stock price. In this theory, stock prices are influenced by the expectations of future earnings and discount rates of future dividends, which in turn are highly associated with the expectations of future economic condition, inflation, and monetary policy. In particular, since the past monetary policy actions contain useful information for the future monetary policy and economic activities, the past monetary policy actions affect the stock prices as the
foundation for market participants to form the expectations of future monetary policy, economic condition, discount rates, and earnings.

2.6 Conclusion

The relationship between monetary policy and stock return has been a heated topic for decades. This study is designed to solve two questions in this subject. The first question is whether the relationship runs from monetary policy to stock market or from stock market to monetary policy. The second question is whether the relationship is positive or negative.

This chapter first constructs a univariate Markov switching model for each of three monetary policy variables, including federal funds rate, Divisia M4, and consumer credit. The estimation results show that the mean and variance of each monetary policy variable are significantly different in different regimes, and the high smoothed probabilities of low regime of each monetary policy variables are all related to economic cycles, making it reasonable to extract a latent common factor to represent the comovement among these monetary policy variables. Another major contribution of the chapter is establishing a Markov-switching dynamic bi-factor model to simultaneously extract a latent factor from monetary variables and a second latent factor from stock indices, and to investigate the dynamic relationship between monetary policy and stock return. The model also calculates the smoothed probabilities of different monetary regimes and stock market regimes. The Markov
switching results of monetary policy factor demonstrate that low monetary policy regimes follow economic recessions, showing that the Federal Reserve applies monetary policy as the main tool to achieve its economic objectives. The Markov switching results of stock market factor show that every economic recession is associated with a bear market, but not vice versa. It is also documented by the Markov switching results that bear markets usually occur before economic recessions, indicating that stock market is a leading indicator of business cycles.

The two unobserved factors as well as their lead-lag relationship with each other are estimated simultaneously in a joint nonlinear econometric model from the observable variables. The two latent factors are set in the framework of a bivariate vector autoregression to examine the dynamic interrelationship between stock market returns and monetary policy actions. The drift term and the variance of error term in the vector autoregression of the latent factors are allowed to switch between the two distinct Markov states. The switching between two regimes is governed by the transition probabilities of the first order two-state Markov process. The estimation results show that the mean growth rate in the stock market return is negative in the bear market, and positive in the bull market. The variance of latent factor in the bear stock market regime is higher than that in the bull stock market regime. This result is consistent with the fact that stock price in the bear market is more volatile than that in the bull market. It also can be seen from the Markov switching results of monetary policy that low monetary policy regimes follow economic recessions, confirming that
monetary policy is the main tool used by the Federal Reserve to respond to the economic recession.

The maximum likelihood estimation results of dynamic bi-factor model show that both monetary policy and stock market are positively affected by their own information in the previous period. It also indicates that monetary policy does not respond to the previous information of stock market, but monetary policy has an impact on stock return. In particular, an expansionary monetary policy action such as a decrease in the federal funds rate would generate a rise in stock return.
Bibliography


Chapter 3

Monetary Policy and Foreign Exchange Reserve Accumulation: A Vector Error Correction Model Approach

3.1 Introduction

For the past decade, central banks of Asian countries accumulate huge amount of foreign exchange reserves. Among these economies, Hong Kong’s foreign exchange reserve holding increased dramatically. Hong Kong’s foreign exchange reserve holdings were 94.3 billion U.S. dollars at the beginning of year 2000. It went up to 316.9 billion U.S. dollars by March 2014. Hong Kong becomes one of the largest foreign exchange reserve holder in the world. Moreover, the foreign exchange reserve to GDP ratio of Hong Kong is even more striking, which is almost the largest one in the world. In addition, Hong Kong maintains currency board monetary system and is under the monetary policy of exchange rate targeting. The management of its foreign exchange reserves becomes a major issue of Hong Kong’s monetary authority. There exists a growing debate on why the foreign exchange reserves accumulate on such a large scale have drawn plenty of attention during the past decade. This chapter is conducted to shed some light on this puzzle. It particularly examines the impact of monetary policy variables on the foreign exchange reserve accumulation and finds a reason for the large amount of foreign exchange reserve holdings.
Foreign exchange reserve is the foreign currency and foreign bonds held by a country’s monetary authority. According to the existing literature (Aizenman and Lee 2007, 2008; Edwards 1984; Ford and Huang 1994; Green and Torgerson 2007; Heller 1966; Kelly 1970), the purpose of holding foreign exchange reserve by central banks can be summarized as the following.

First, central banks hold foreign exchange reserves to self-insure against sudden shocks to the balance of payment. Hence it can stabilize its capital account and current account in bad scenarios. Maintaining adequate foreign exchange reserve is helpful to limit external vulnerability during crisis time, especially when the channels of borrowing from external or internal sources are obstructed. Second, central banks need foreign exchange reserves to act as a lender of last resort to commercial banks that have high foreign currency liability. Holding enough foreign exchange reserves can provide confidence to the market that the country is able to meet its external responsibilities, and hence, attract foreign capital and investments. Third, central banks need foreign exchange reserves for intervention, such as maintaining an exchange rate target or moderating deflation and inflation. This is also called mercantilist motive, because foreign exchange reserve is held as a tool to keep exchange rate constantly undervalued in order to promote international trade competitiveness.

Many questions were raised about the necessity of the huge amount of foreign exchange reserve accumulation by Asian central banks. This is because not only does
foreign exchange reserve accumulation has benefits in different aspects, but it generates a lot of cost as well (see Aizenman and Marion 2003; Green and Torgerson 2007; Landell-Mills 1989).

First, excessive foreign exchange reserve holding generates sterilization cost. In order to moderate inflation, central banks use the open market operations to control money supply. By using open market sale and issuing government bonds, the central banks decrease monetary supply and sterilize the inflationary impact of reserve accumulation. However, the excess of the interest rate paid to the domestic bond buyer over the return from foreign exchange reserves induces sterilization cost. Second, holding excessive foreign exchange reserves generates opportunity cost. The resources spent on buying foreign exchange reserve can be invested in many other ways. It is quite possible that the return from other investment can be much higher than the return from holding foreign exchange reserve. Third, foreign exchange reserves can lose value when domestic currency appreciates or foreign currency depreciates. Fourth, reserve accumulation may create a false sense of economic stability and security, which delays necessary adjustments and reforms.

Therefore, holding an optimal amount of foreign exchange reserve is very valuable for monetary authorities. This topic is especially important for Hong Kong’s monetary authority, as Hong Kong’s foreign exchange reserve holding has increased dramatically and almost has the highest foreign exchange reserve to GDP ratio in the world. However, few studies have been completed focusing on Hong Kong’s foreign
exchange reserves during the past decade by using time series approach. This chapter contributes to the literature by investigating the relationship between foreign exchange reserve in Hong Kong and factors that affect its accumulation such as exchange rate and broad money supply, and explaining why foreign exchange reserves held by Hong Kong’s monetary authority increase at a high pace during the past decade.

Empirical time series approaches such as cointegration test, Granger causality test, and vector error correction model are applied in the chapter. The cointegration test finds a negative long-run relationship between broad money supply and foreign exchange reserves, and finds no significant long-run relationship between exchange rate and foreign exchange reserves, indicating that changes in money supply and exchange rate don’t contribute to the large foreign exchange reserve accumulation. These results are also confirmed by the Granger causality tests. This chapter also established a vector error correction model, and its results show that the foreign exchange reserve has a low speed of adjustment of its departure from the long-run equilibrium, providing a reason for the large amount of foreign exchange reserves.

The remainder of this chapter is structured as follows. In the second section, related literatures are reviewed. This chapter provides the theoretical foundation for the empirical study in the third section. In the fourth section, the empirical framework is illustrated. In the fifth section, this chapter shows the empirical results and analysis. This chapter concludes in the sixth section.
3.2 Literature review

Holding an optimal amount of foreign exchange reserve has received a lot of attentions. There is a growing body of literature in the field of optimal amount of foreign exchange reserve accumulation and its determinants (see, for example, Green and Torgerson 2007). The literatures on the quantitative study of optimal foreign exchange reserve holding can be classified into two main categories: univariate ratio analysis and multivariate regression analysis.

For the univariate ratio analysis of optimal foreign exchange reserves accumulation, there are three conventional ratios: foreign exchange reserve to short-term external debt ratio, foreign exchange reserve to broad money supply ratio, and foreign exchange reserves to imports ratio. These benchmarks are all measured against a single statistics. These three ratios are discussed subsequently below.

The foreign exchange reserve to short-term external debt ratio is also called Greenspan-Guidotti rule. This rule suggests that developing countries, which have limited access to the international capital market, should maintain foreign exchange reserves equal to all external debt coming within the next year. It measures the country’s capacity to fulfill its external liabilities in the coming year. This ratio analysis about the optimality of foreign exchange reserve accumulation receives some empirical supports. For example, Jeanne and Ranciere (2011) supported the Greenspan-Guidotti rule as the optimal level of foreign exchange reserve holding for countries with lower interest rate on their external debt.
The foreign exchange reserve held as a portion of broad domestic money supply is also supported by empirical studies. Frenkel (1971) investigated 55 countries and found evidence for the ratio between foreign exchange reserves and broad money supply for less developed countries that can increase the creditworthiness in the value of domestic currency. Wijnholds and Kapteyn (2001) suggested that countries should hold foreign exchange reserves between 5% and 20% of M2, and countries with fixed exchange rate regime should have more foreign exchange reserves than those with flexible exchange rate regime.

Since one of the main purposes of foreign exchange rate holding is to protect the country from international trade shocks, the ratio of foreign exchange reserve to imports was used to measure the number of months of imports that a country can finance. Pineau, et al (2006) claimed that foreign exchange reserves holding should be equal to the value of 3 or 4 months’ import of the country.

However, these univariate ratio analyses have received a lot of criticism recently. For example, Green and Torgerson (2007) investigated each of the above mentioned ratio benchmarks by using the data of foreign exchange reserve holding of emerging markets in 2005. The results concluded that all the top foreign exchange reserve holders held much more reserves than those benchmarks suggested and stated that these ratio benchmarks against a single statistics were no longer appropriate criterion for central banks nowadays. The authors also examined the beneficial motivations and the cost of foreign exchange reserve accumulation among the emerging economies.
For current Hong Kong, the ratio of foreign exchange reserve to broad money supply is about 25%, and its foreign exchange reserve accumulation is high enough to cover more than 7 months’ imports, which largely exceed the benchmarks. Therefore, for the foreign exchange reserve analysis in current Hong Kong, these univariate ratios are not able to serve as accurate measurements.

Besides the above mentioned ratio benchmarks against a single statistic, there is another widely used approach estimating the demand for foreign exchange reserves, which is the regression analysis in a multivariate setting. The demand for foreign exchange reserves is determined by several variables, rather than a single variable.

Many studies on the demand for foreign exchange reserve have been conducted by using multivariate regression analysis (see, for instance, Lane and Burke 2001, Aizenman and Marion 2003). The explanatory variables used in these studies can be classified into several groups: current account vulnerability measured by imports or exports, monetary policy indicated by broad money supply and interest rate changes, economic size measured by GDP and population, and exchange rate.

In theory, the reason why these variables determine the level of foreign exchange reserve accumulation can be explained by the three reserve holding purposes mentioned above. Since one important purpose for reserve holding is to stabilize its capital account and current account in face of a sudden shock, measures of the vulnerability of capital account and current account should, therefore, be considered as explanatory variables. Factors in current account, such as international trade, highly
correlated with economic size and exchange rate, hence most empirical models also investigate them. A more flexible exchange rate regime should reduce the demand for foreign exchange reserves for cushion purpose.

Lane and Burke (2001) used the multivariate regression analysis to study the determinants of cross-country variation of foreign exchange reserve holding from 1981 to 1995. The empirical results indicate that financial openness is not significant in determining the optimal amount of foreign exchange reserves, but financial deepening is positively correlated with foreign exchange reserve accumulation. The results also demonstrate that smaller countries have higher foreign exchange reserves to GDP ratio, because they are more financially volatile and need more reserves.

Aizenman and Marion (2003) used a sample of 125 developing country and show that foreign exchange reserve holdings are predictable before 1997 financial crisis by a few key factors, such as size and volatility of international transactions, the exchange rate arrangement, and some political considerations. The results show that the foreign exchange reserve holdings are positively related with population, the size of the economy, and the size of export and import. Since a more flexible exchange rate regime should reduce the cushion purpose demand for foreign exchange reserves, the foreign exchange reserve holdings should be negatively associated with exchange rate volatility.

Aizenman, Lee, and Rhee (2007) empirically analyzed the demand for foreign exchange reserves for Korea by using the multivariate regression method. The results
confirmed the idea that self-insurance motivation became stronger after the Asian financial crisis and hence boosted the accumulation of foreign exchange reserves. The authors considered the Asian financial crisis in 1997 as a motivation for the increase in the foreign exchange accumulation of Asian emerging market countries. The results also indicated that after the Asian financial crisis, the explanatory variables measuring current account vulnerability such as imports and exports became less significant, and the explanatory variables measuring capital account vulnerability such as broad money supply tend to be more significant in explaining the foreign exchange reserve accumulation.

Aizenman and Lee (2007) empirically compared the financial precautionary motivation and the mercantilist motivation for a panel of emerging market economies. The results found little empirical support for the mercantilist motivation which indicates that exports and imports were less important in contributing in the large accumulation of foreign exchange reserves in current emerging market countries.

Ford and Huang (1994) employed the error correction model to investigate the foreign exchange reserve accumulation in China from 1950s to 1990s. They showed that reserve holdings in China have a stable relationship with its determinants.

Narayan and Smyth (2004) studied the relationship between China’s foreign exchange reserve holding and exchange rate. They used cointegration method and Granger causality testing to find that in the long run foreign exchange reserve holding Granger causes the exchange rate to change.
Gosselin and Parent (2005) used annual data ranging from 1980 to 2003 over eight Asian emerging market countries to analyze their foreign exchange reverse holdings. The chapter conducted unit root test and panel cointegration test. The results of panel error correction model suggested that the speed of foreign exchange reserve accumulation in these countries would slow down and this may generate negative risks for the U.S. dollar.

Prabheesh, Malathy, and Madhumati (2007) used cointegration and vector error correction approach to analyze India’s demand for foreign exchange reserve from 1983 to 2005. The model investigated the relationship among imports to GDP ratio, broad money to GDP ratio, interest rate difference, and exchange rate flexibility. The results implied that India’s reserve accumulation was highly related to capital account vulnerability and less related to its opportunity cost.

Kasman and Ayhan (2008) used monthly data from 1982 to 2005 to investigate the relationship between exchange rate and foreign exchange reserve holding in Turkey. They applied unit root test and cointegration test, and found that in the long run exchange rate Granger cause foreign exchange reserves.

3.3 Variables and Data

This chapter is designed to explain why Hong Kong foreign exchange reserve holdings increase at such a large scale, and to particularly investigate the role played by the monetary policy. Therefore, two monetary policy variables broad money
supply and exchange rate are included as the explanatory variables. Broad money supply M2 measures the magnitude of overall domestic banking system and the financial depth of economy of the country, which in turn affect capital account vulnerability, leading to precautionary self-insuring foreign exchange reserve accumulation. What’s more, Hong Kong’s large foreign exchange reserve holding increase associates with the rise of money supply. It is natural to take money supply into account. Moreover, Obstfeld, Shambaugh, and Taylor (2010) have empirically proved that it is broad money supply M2 rather than money base M0 that truly drive the foreign exchange reserve accumulation. The long-run relationship between M2 and foreign exchange reserve accumulation is expected to be either positive or negative as the theoretical model indicates.

Moreover, Hong Kong is under the currency board monetary system and applies exchange rate targeting monetary policy. Exchange rate targeting can be an essential purpose of foreign exchange reserve accumulation, exchange rate should be considered as another factor which affects the foreign exchange reserve in Hong Kong. Exchange rate changes also affect import and export, which have an impact on capital account vulnerability, resulting in a precautionary self-insuring foreign exchange reserve accumulation. Greater exchange rate flexibility may reduce the demand for foreign exchange reserves for cushion purpose.

This chapter eliminates explanatory variables such as interest rate, population, short-term external debt, GDP, imports and exports. Hong Kong is a small open
economy, which has a perfect access to the world capital market, and it can borrow and lend as much as it wants in the international capital market. The interest rate in the small open economy is determined by the world interest rate. Hong Kong’s monetary authority has no control on its domestic interest rate.

According to the empirical study results of Obstfeld, Shambaugh, and Taylor (2010), variables such as population and short-term external debt are insignificant in explaining reserve holdings. GDP is significant in the regression without M2 but becomes insignificant in the regression with M2. It is because GDP is positively correlated with broad money supply M2, and acts as the proxy for M2 when M2 is omitted. The inclusion of the true driver M2 will make GDP insignificant in explaining the demand for foreign exchange reserve.

Variables measuring current account such as imports and exports tend to be less significant with the existence of exchange rate. Theoretically, net export, measured by the difference between export and import, is a function of exchange rate for an open economy. This conclusion is consistent with the research result of Aizenman and Lee (2007). Green and Torgerson (2007) also confirmed that the import and export measurement is only useful in low-income countries which have limited access to international capital market. Rodrik (2006) also indicated that the foreign exchange reserve accumulation of emerging market countries is driven by the size of their domestic financial sector rather than the magnitude of international trade. Lane and Burke (2001) found that the financial openness is insignificant under their
cross-country empirical study. Therefore for my empirical study in Hong Kong, above mentioned variables are excluded since they are not the factors strongly affecting Hong Kong’s foreign exchange reserve accumulation.

Data used in this chapter is obtained from the website of the International Financial Statistics of IMF and the Hong Kong Monetary Authority, which is the central bank of Hong Kong. The monthly data range from January 2000 to November 2007, which make the sample size large enough to enable the time series econometric study. Many empirical studies (Aizenman, Lee, and Rhee 2007; Aizenman and Marion 2003; Mendoza 2004) demonstrated that the Asian financial crisis occurred in 1997 produced great effects on the demand for foreign exchange reserve accumulation in Asian countries. It strengthens the self-insurance incentive and hence increases the foreign exchange reserve holding. The effects of Asian financial crisis have faded away since 1999. Therefore, the period ranging from January 2000 isn’t strongly affected by the financial crises. Moreover, according the business cycle date announced by U.S. National Bureau of Economic Research, the currency economic recession started from December 2007. In order to investigate the direct effect of foreign exchange reserve’s determinants, this chapter chooses the period ranging from January 2000 to November 2007 to avoid the disturbance of economic crisis.

The foreign exchange reserve held by the Hong Kong Monetary Authority is measured in the unit of millions of U.S. dollar. It eliminates the gold reserve, since gold is not used by the country as an intervention asset. This chapter uses M2 to
measure the money supply, which includes cash, checking account deposits, saving account deposits, retail money market mutual fund balance, and small time deposit. The M2 data is originally measured in the unit of millions of Hong Kong dollar, and is transformed into million U.S. dollars. All the variables are studied in the form of logarithm. Hong Kong Monetary Authority follows the fixed exchange rate regime, which targets a fixed exchange rate with respect to U.S. dollar. It only allows for a limited fluctuation within a very small interval. Hence, this chapter uses the real effective exchange rate (REER) to measure the exchange rate of Hong Kong dollar. Real effective exchange rate is the weighted average of the currency’s exchange rate with respect to a basket of major foreign currencies adjusted for the effect of inflation. The weight is determined by the balance of international trade.

3.4 Theoretical Foundation

Since Hong Kong is a small open economy under fixed exchange rate regime, this chapter uses a variant of Mundell-Fleming model (Mundell 1960 and 1963; Fleming 1962; Mankiw 2010) and a variant of Obstfeld, Shambaugh, and Taylor analysis (2010) as the theoretical foundation to explain the relationship among foreign exchange reserve holding, money supply and exchange rate, which assumes that the economy is a small open economy with perfect capital mobility. “Small” means that the economy takes the world interest rate as given and it has negligible impacts on the world economy. For example, the domestic interest rate increases, international
investors or arbitrageurs will start to lend to this country to take advantage of the high interest rate. The capital inflow will drive the interest rate downward to the original level. By “open”, it means that the economy has perfect access to the world capital market, and capital is able to flow in and flow out freely. Both of them are exactly the case of Hong Kong.

The exchange rate is defined as the amount of foreign currency per unit of domestic currency, and a drop in the value of exchange rate indicate a depreciation of the value of domestic currency. Money supply is exogenously controlled by the central bank, and money demand is negatively related to world interest rate and positively related to income. This chapter allows the price to be flexible so that the model can explain the long run relationship.

A fixed exchange rate monetary policy system would force the central bank to buy or sell domestic currency in exchange of foreign currency at the predetermined price to maintain the exchange rate constant. If the central bank applies open market purchase to buy bonds, money supply would increase. In a small open economy, the interest rate is constant and the level is determined by the world interest rate. When the increased money supply generates downward pressure on the interest rate, investors or arbitrageurs seek higher returns in other countries and capital flows out of the economy. The outflow of the capital reduces the supply of capital, and prevents the interest rate from falling. Therefore, the interest rate maintain fixed. Moreover, the capital outflow raises the supply of domestic currency in the foreign currency
exchange market, generating a depreciating pressure on the exchange rate. From a different point of view, the demand for the domestic currency decreases with the capital outflow, leading to potential depreciation in the domestic currency. The theoretical model indicates that an increase in the money supply potentially lowers the exchange rate and increases income.

Arbitrageurs would quickly respond to take advantage of the falling exchange rate by selling the domestic currency to the central bank for foreign currency, which is equivalent to buying foreign currency from the central bank using domestic currency. This transaction is profitable for the arbitrageurs because market exchange rate can be lower than the fixed exchange rate committed by the central bank. Hence arbitrageurs can sell domestic currency to the central bank at a higher fixed exchange rate committed by the central bank. In other words, the essence is that arbitrageurs try to get rid of the domestic currency that is losing value, and give the domestic currency back to that country’s central bank. The central bank has to buy back the domestic currency from arbitrageurs using its foreign currency, which is a major component of its foreign exchange reserve holdings. By giving out foreign currency, the central bank’s foreign exchange reserve holding is reduced. Therefore, we can reach the conclusion that an increase in the money supply decreases the foreign exchange reserve holding by the central bank, and the relationship between money supply and foreign exchange reserve holding is expected to be negative. By buying back the domestic currency from arbitrageurs, the central bank reduces the money supply in
the market. A decrease in money supply leads to the return of the exchange rate back to the original level committed by the central bank. Hence, the long-run relationship between exchange rate and foreign exchange reserve holding is expected to be nil.

3.5 Empirical Models

This chapter empirically studies the relationship among exchange rate, broad money supply, and foreign exchange reserves in Hong Kong by using time series approach such as unit root test, cointegration test, Granger causality test and vector error correction model approach (Hamilton 1994). It strives to explain why foreign exchange reserve holdings increase considerably in Hong Kong.

This chapter first conducts the Augmented Dickey-Fuller unit root test (Dickey and Fuller 1979) and Phillips-Perron unit root test (Phillips and Perron 1988) for all the three variables. The unit root properties of the variables are very important for the estimation of relationship among them. If the variables are stationary in their level, we can easily carry out the standard regression under the VAR model, and obtain the reliable relationship among the level variables and their lags. If the variables are not stationary in their level, which means they have unit root, the ordinary regression will become a spurious regression and reach incorrect conclusion. Even if the two integrated variables are uncorrelated, the spurious regression could produce a significant coefficient parameter. What’s more, this problem will not disappear even if the sample size dramatically increases.
In this situation, we can take the differences of variables and transform them into the stationary variables. However, if the non-stationary variables are cointegrated (Granger and Newbold 1974), which means variables move together in the long run and have common unit root factor, the regression can make sense. Therefore we can explore the long-run relationship among foreign exchange reserve holdings, broad money supply and exchange rate through the cointegration analysis.

There are two methods for cointegration test. The first one is introduced by Engle and Granger (1987), and is based on unit root test. If the residual of the regression is stationary or the combination of the variables is stationary, the variables are cointegrated. The second method is introduced by Johansen (1988), which has advantages over the first method when testing cointegration with more than two variables. Johansen’ method is also a joint procedure which contains the estimation of the vector error correction model and long run equilibrium relations. Since this chapter studies the long-run relationship among three variables, it uses Johansen’s method to test cointegration among foreign exchange reserves, money supply, and exchange rate.

The Johansen’s cointegration involving vector error correction model is as follows:

$$\Delta X_t = C + A_0 X_{t-1} + A_1 \Delta X_{t-1} + A_2 \Delta X_{t-2} + A_3 \Delta X_{t-3} + \cdots + A_p \Delta X_{t-p} + \varepsilon_t$$

$X_t$ is a vector which contains all three non-stationary variables such as foreign exchange reserve holding, broad money supply, and exchange rate. The number of
lags $p$ can be determined by Akaike information criterion (AIC) (Akaike, 1973) and Bayesian information criterion (BIC) (Schwarz, 1978).

If the variables are cointegrated, the regressions among these non-stationary variables are valid. Therefore we can investigate the short-run dynamics through the vector error correction model by adding the error correcting term into the regression of difference of variables (see Pesaran, Shin and Smith 2000). The endogenous variables changes in response to the deviation from the long-run equilibrium of the previous period. The coefficient $A_0$ in the vector error correction model is the product of adjustment speed parameter $B$ and cointegrating vector $D$, i.e. $A_0 X_{t-1} = -BDX_{t-1} = -BZ_{t-1}$. The cointegration guarantees that the added error correction term is also I(0), and hence the stationary equation can hold. The error correction term represents the potential effect of departure from long-run equilibrium which is gradually corrected through short-run adjustments. In other words, the error correction term can measure the speed and tendency of each variable to restore from short-run dynamics back to its own long-run equilibrium.

3.6 Empirical Results

The Table 3.1 shows the results of Augmented Dickey-Fuller (ADF) unit root test and Phillips-Perron unit root test (PP) for all the three variables. $Ex$ denotes the logarithm of exchange rate, $M2$ denotes the logarithm of money supply, and $Reserve$ denotes the logarithm of foreign exchange reserve holding. The asterisk denotes
rejection of null hypothesis at 5% significant level. The p-values are shown in the parenthesis. The results of both unit root tests show that all three variables are I(1).

Table 3.1: ADF and PP Unit Root Test Results

<table>
<thead>
<tr>
<th>Variable</th>
<th>ADF Test statistics</th>
<th>PP Test statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ex</td>
<td></td>
<td></td>
</tr>
<tr>
<td>level</td>
<td>0.292 (0.977)</td>
<td>0.607 (0.989)</td>
</tr>
<tr>
<td>first difference</td>
<td>-6.880* (0.000)</td>
<td>-6.852* (0.000)</td>
</tr>
<tr>
<td>M2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>level</td>
<td>2.569 (1.000)</td>
<td>3.358 (1.000)</td>
</tr>
<tr>
<td>first difference</td>
<td>-9.382* (0.000)</td>
<td>-9.377* (0.000)</td>
</tr>
<tr>
<td>Reserve</td>
<td></td>
<td></td>
</tr>
<tr>
<td>level</td>
<td>-0.683 (0.845)</td>
<td>-0.583 (0.868)</td>
</tr>
<tr>
<td>first difference</td>
<td>-5.744* (0.000)</td>
<td>-5.714* (0.000)</td>
</tr>
</tbody>
</table>

Since there are three variables, this chapter uses Johansen cointegration test to explore the long-run relationship among these variables. Johansen cointegration Test also involves the estimation of vector error correction model. Therefore, before we carry out the Johansen cointegration test, we need to determine the number of lags of vector error correction model using Akaike information criterion (AIC) and Bayesian information criterion (BIC), and apply the number of lags to the cointegration test.

As shown in Table 3.2, lowest value of AIC indicates the lag number is three, but the lowest value of BIC indicates that the lag number is one. The asterisk indicates the lag number selected by that criterion. As documented by Bickel and Zhang, (1992), BIC is designed to identify the true model and consistent while in contrast AIC is not.
This chapter follows the BIC and chooses one lag for the vector error correction model setting.

### Table 3.2: Lag Selection Results

<table>
<thead>
<tr>
<th>Lag number</th>
<th>AIC</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-17.261</td>
<td>-16.921*</td>
</tr>
<tr>
<td>2</td>
<td>-17.466</td>
<td>-16.871</td>
</tr>
<tr>
<td>3</td>
<td>-17.623*</td>
<td>-16.773</td>
</tr>
<tr>
<td>4</td>
<td>-17.582</td>
<td>-16.477</td>
</tr>
<tr>
<td>5</td>
<td>-17.544</td>
<td>-16.184</td>
</tr>
<tr>
<td>6</td>
<td>-17.432</td>
<td>-15.816</td>
</tr>
<tr>
<td>7</td>
<td>-17.393</td>
<td>-15.522</td>
</tr>
<tr>
<td>8</td>
<td>-17.305</td>
<td>-15.180</td>
</tr>
</tbody>
</table>

### Table 3.3: Johansen Cointegration Test Results

<table>
<thead>
<tr>
<th>Cointegrating Vectors Number</th>
<th>Trace Statistics</th>
<th>Critical Value</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Cointegrating Vector*</td>
<td>36.433</td>
<td>29.797</td>
<td>0.007</td>
</tr>
<tr>
<td>At most one</td>
<td>9.704</td>
<td>15.494</td>
<td>0.304</td>
</tr>
<tr>
<td>At most two</td>
<td>0.005</td>
<td>3.841</td>
<td>0.945</td>
</tr>
</tbody>
</table>

From the result of Table 3.3, it can be seen that the null hypothesis of no cointegrating vector is rejected at the 5% significant level, and the alternative
hypothesis that there is at most one cointegrating vector is not rejected at 5% significant level. Therefore the results of Johansen’s cointegration test demonstrate that there is one cointegrating vector among three variables. The following cointegration equation clearly indicates the long-run relationship among variables.

\[
\text{Reserve} = 0.025Ex - 0.663M2 - 3.07
\]

\[
(0.111) \quad (0.114)
\]

From the above equation we can see that the correlation coefficient of exchange rate is insignificant, which confirms that the long-run relationship between exchange rate and foreign exchange rate holding is nil under the fixed exchange rate regime as theory indicates. Moreover, broad money supply M2 has significantly negative long-run relationship with foreign exchange reserve holding as expected. This is consistent with the model for the small open economy under fixed exchange rate. These results indicate that neither exchange rate change nor money supply increase can explain the large scale of foreign exchange reserve accumulation in Hong Kong.

This long-run relationship is also confirmed by the results of Granger causality test (Granger 1969, 1986), which is shown in Table 3.4. The Granger causality test demonstrates whether the past information of one variable is useful in predicting the future value of another variable. The asterisk denotes rejection of null hypothesis at 5% significant level in Table 3.4. The Granger causality test results imply that only broad money supply M2 Granger causes foreign exchange reserve holding, which means the past information of broad money supply is helpful in predicting the future value of
foreign exchange reserve holding. Therefore, it reinforces the idea that foreign exchange reserve holding is affected by broad money supply in the long run, but has no significant relationship with exchange rate in the long run.

Table 3.4: Granger Causality Test Results

<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>F-Statistic</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ex does not Granger Cause Reserve</td>
<td>1.638</td>
<td>0.204</td>
</tr>
<tr>
<td>Reserve does not Granger Cause Ex</td>
<td>1.403</td>
<td>0.239</td>
</tr>
<tr>
<td>M2 does not Granger Cause Reserve*</td>
<td>19.367</td>
<td>0.00003</td>
</tr>
<tr>
<td>Reserve does not Granger Cause M2</td>
<td>0.348</td>
<td>0.557</td>
</tr>
<tr>
<td>M2 does not Granger Cause Ex</td>
<td>0.008</td>
<td>0.927</td>
</tr>
<tr>
<td>Ex does not Granger Cause M2</td>
<td>0.647</td>
<td>0.423</td>
</tr>
</tbody>
</table>

In order to investigate the speed of adjustment of departure from long-run equilibrium to analyze the reason of large foreign exchange reserve accumulation in Hong Kong, the chapter also builds up the vector error correction model. Since this chapter follows the BIC as the information criterion to choose the optimal lag number of one, the vector error correction model with one lag is established as follows.

\[
\Delta X_t = C + A_0 X_{t-1} + A_1 \Delta X_{t-1} + \varepsilon_t
\]

where \( X_t \) is a vector containing three variables of \( Reserve_t, M2_t, \) and \( Ex_t. \)

From the vector error correction model results shown in Table 3.5, we can see that foreign exchange reserves change in response to both its past fluctuation and its
deviation from long-run equilibrium of the previous period. Previous changes in exchange rate and money supply don’t have a significant on foreign exchange reserve accumulation in the short-run. The coefficient of error correction term indicates the speed of adjustment of each variable to restore to its long-run equilibrium. From the results of Table 3.5, we can see that the coefficient of adjustment of error correction term of $\Delta Reserve$ is -0.10308, which implies that about 10.3% of the departure from long-run equilibrium is removed each month. The speed of adjustment is relatively low, which indicates that the past error in the foreign exchange reserve holdings could be corrected in a slow pace. The recovery of long-run equilibrium would take a relatively long time. According to Clark (1970), a low speed of adjustment of departure to long-run equilibrium generates a large foreign exchange reserve holding. This indicates that the monetary authority needs a huge amount of foreign exchange reserve in hand to actively intervene to maintain the fixed exchange rate and the balance of its current account and capital account. This explains why Hong Kong’s foreign exchange reserve holding is so huge. The sign of the coefficient of error correction term of foreign exchange reserve is negative, which means the foreign exchange reserve holdings of Hong Kong is excessive, and they have to be reduced to restore the long-run equilibrium. This is also consistent with the finding of Prabheesh, Malathy, and Madhumati (2007) in India foreign exchange reserve study. The low speed of adjustment of departure to long-run equilibrium also suggests that the Hong Kong Monetary Authority should hold considerable amount of foreign exchange
reserves in order to take active measures to intervene and manage the foreign exchange reserve in order to promote the recovery of long-run equilibrium.

Table 3.5: Vector Error Correction Model Results

<table>
<thead>
<tr>
<th></th>
<th>$\Delta \text{Reserve}_t$</th>
<th>$\Delta \text{Ex}_t$</th>
<th>$\Delta \text{M2}_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error correction</td>
<td>-0.103 (0.027)</td>
<td>-0.0397 (0.026)</td>
<td>-0.063 (0.045)</td>
</tr>
<tr>
<td>$\Delta \text{Reserve}_{t-1}$</td>
<td>0.323 (0.103)</td>
<td>-0.053 (0.099)</td>
<td>-0.307 (0.005)</td>
</tr>
<tr>
<td>$\Delta \text{Ex}_{t-1}$</td>
<td>0.011 (0.114)</td>
<td>0.212 (0.110)</td>
<td>0.199 (0.120)</td>
</tr>
<tr>
<td>$\Delta \text{M2}_{t-1}$</td>
<td>0.032 (0.435)</td>
<td>-0.155 (0.069)</td>
<td>-0.074 (0.120)</td>
</tr>
<tr>
<td>$C$</td>
<td>0.003 (0.001)</td>
<td>-0.002 (0.001)</td>
<td>0.007 (0.002)</td>
</tr>
</tbody>
</table>

3.7 Conclusion

During the past decade, Hong Kong’s foreign exchange reserve holding increases dramatically and becomes one of the largest foreign exchange reserve holder in the world. The debate regarding the optimal level of foreign exchange reserve holding in Hong Kong and other emerging market economies motivates this study. This chapter examines the reason why Hong Kong’s foreign exchange reserve holding increases at such a large scale, and particular investigates the contribution of monetary policy variables such as exchange rate and broad money supply.

This chapter reviews the existing literature, and states the benefits and costs of foreign exchange reserve accumulation. It uses the monthly data ranging from January 2000 to November 2007. The Augmented Dickey-Fuller test and Phillips-Perron unit
root test results indicate that all the three variables are I(1). The Johansen’s cointegration test results illustrate that three non-stationary variables are cointegrated and there exists only one cointegrating vector among them. The cointegration test results also demonstrate that there exists negative long-run relationship between broad money supply and foreign exchange reserve holding, and no significant relationship between exchange rate and foreign exchange reserve holding in the long run, which is consistent with the theoretical analysis. The Granger causality test results also confirm this relationship.

This chapter builds up the vector error correction model to measure the short-run dynamics and the tendency of each variable to restore to its long-run equilibrium. The coefficient of error correction term implies a relatively low speed of adjustment of foreign exchange reserve. About only 10.3% of the departure from long-run equilibrium is eliminated each month. The low speed of adjustment indicates that the monetary authority of Hong Kong has to hold large amount of foreign exchange reserves to be active in foreign exchange reserve intervention and management. The sign of the coefficient of error correction term of foreign exchange reserve is negative, indicating that the foreign exchange reserve holdings are excessive to its optimal level, and need to be reduced to restore the long-run equilibrium.
Bibliography


A. Appendix for Chapter 1

A.1 Estimation Procedure of Markov-switching Dynamic Factor Model

This study follows Kim and Nelson (1999) for estimation procedure of Markov-switching dynamic factor model. Let \( I_t \) denote the information set which contains the observations available up to time \( t \). In Markov-switching dynamic factor model, the forecast of unobserved state vector \( \beta_t \) is not only dependent on information set \( I_{t-1} \), but also based on state variable \( S_t \) that takes on the value of \( j \) and \( S_{t-1} \) that takes on the value of \( i \). The forecast of state variable \( \beta_t \) and its covariance matrix is as follows:

\[
\beta^{(i,j)}_{t|t-1} = E[\beta_t|I_{t-1}, S_{t-1} = i, S_t = j]
\]

\[
P^{(i,j)}_{t|t-1} = E[(\beta_t - \beta_{t|t-1})(\beta_t - \beta_{t|t-1})'|I_{t-1}, S_{t-1} = i, S_t = j]
\]

Based on Markov switching states \( S_{t-1} = i \) and \( S_t = j \), the Kalman filter is:

\[
\beta^{(i,j)}_{t|t-1} = \mu_j + F_j \beta^{(i)}_{t-1|t-1}
\]

\[
P^{(i,j)}_{t|t-1} = F_j P^{(i)}_{t-1|t-1} F_j' + Q_j
\]

\[
\theta^{(i,j)}_{t|t-1} = \Delta Y_t - \Delta Y^{(i,j)}_{t|t-1} = \Delta Y_t - H_j \beta^{(i,j)}_{t|t-1}
\]

\[
\tau^{(i,j)}_{t|t-1} = H_j P^{(i,j)}_{t|t-1} H_j'
\]

\[
\beta^{(i,j)}_{t|t} = \beta^{(i,j)}_{t|t-1} + P^{(i,j)}_{t|t-1} H_j' \theta^{(i,j)}_{t|t-1} = \beta^{(i,j)}_{t|t-1} + K_t\theta^{(i,j)}_{t|t-1}
\]

\[
P^{(i,j)}_{t|t} = (I - P^{(i,j)}_{t|t-1} H_j' [\tau^{(i,j)}_{t|t-1}]^{-1} H_j) P^{(i,j)}_{t|t-1}
\]

where \( \beta^{(i)}_{t|t-1} \) and \( P^{(i)}_{t|t-1} \) are inferences on \( \beta_{t-1} \) and \( P_{t-1} \) conditional on information up to time \( t-1 \) and \( S_{t-1} = i \); \( \theta^{(i,j)}_{t|t-1} \) is the prediction error of \( y_t \) conditional on information up to time \( t-1 \), given values of the two states \( S_{t-1} = i \) and
In order to make the loop of above Kalman filter operable, it is necessary to transfer $\beta^{(i,j)}_{t|t}$ and $P^{(i,j)}_{t|t}$ at the end of each iteration into $\beta^{(j)}_{t|t}$ and $P^{(j)}_{t|t}$, and use $\beta^{(j)}_{t|t}$ and $P^{(j)}_{t|t}$ to represent $\beta^{(i)}_{t-1|t-1}$ and $P^{(i)}_{t-1|t-1}$ for the next period. Kim (1994) showed an algorithm for transferring. The algorithm involves approximation:

$$
\beta^{(j)}_{t|t} = \left[ \sum_{S_{t-1} = i} pr(S_{t-1} = i, S_t = j|I_t)\beta^{(i,j)}_{t|t} \right] / pr(S_t = j|I_t)
$$

$$
P^{(j)}_{t|t} = \left[ \sum_{S_{t-1} = i} pr(S_{t-1} = i, S_t = j|I_t)P^{(i,j)}_{t|t} + \left( \beta^{(j)}_{t|t} - \beta^{(i)}_{t|t} \right) \left( \beta^{(j)}_{t|t} - \beta^{(i)}_{t|t} \right)^T \right] / pr(S_t = j|I_t)
$$

The probability terms $p(S_{t-1} = i, S_t = j|I_{t-1})$ and $pr(S_t = j|I_t)$ in the above equations have to be estimated to complete the Kalman filter involving approximation. By using Hamilton (1989) filter along with Markov switching, the inference on the above probability terms can be calculated and shown as follows:

$$
p(S_{t-1} = i, S_t = j|I_{t-1}) = pr(S_{t-1} = i|I_{t-1})pr(S_t = j|S_{t-1} = i)
$$

$$
f(y_t, S_{t-1} = i, S_t = j|I_{t-1}) = f(y_t|S_{t-1} = i, S_t = j, I_{t-1})pr(S_{t-1} = i, S_t = j|I_{t-1})
$$

$$
f(y_t|I_{t-1}) = \sum_{S_{t-1} = i} \sum_{S_t = j} f(y_t, S_{t-1} = i, S_t = j|I_{t-1})
$$

$$
pr(S_{t-1} = i, S_t = j|I_t) = pr(S_{t-1} = i, S_t = j|y_t, I_{t-1})
$$

$$
= f(y_t, S_{t-1} = i, S_t = j|I_{t-1})/f(y_t|I_{t-1}) = f(y_t|S_{t-1} = i, S_t
$$

$$
= f(y_t, I_{t-1})pr(S_{t-1} = i, S_t = j|I_{t-1})/ f(y_t|I_{t-1})
$$

$$
pr(S_t = j|I_t) = \sum_{t} pr(S_{t-1} = i, S_t = j|I_t)
$$

$S_t = j$; and $\tau^{(i,j)}_{t|t-1}$ is the conditional variance of the prediction error. The details of the derivation of the above Kalman filter can be referred to Hamilton (1994).
The transition probabilities capture the Markov switching between two states and are estimated by Maximum Likelihood estimation as one of the unknown parameters. For the inference of conditional density \( f(y_t|S_{t-1} = i, S_t = j, l_{t-1}) \), prediction error decomposition involving conditional forecast error and its variance obtained from the previous Kalman filter is used as follows.

\[
f(y_t|S_{t-1} = i, S_t = j, l_{t-1}) = \frac{1}{(2\pi)^{n/2}} \left| \theta_{t|t-1} \right|^{-1/2} \exp \left\{ -\frac{1}{2} \theta_{t|t-1} H_t^T \theta_{t|t-1} \right\}
\]

\[
f(y_t|l_{t-1}) = \sum_{S_t} \sum_{S_{t-1}} f(y_t|S_{t-1} = i, S_t = j, l_{t-1}) p(S_{t-1} = i, S_t = j|l_{t-1})
\]

\[
l(\theta) = \sum_{t=1}^{T} \ln (f(y_t|l_{t-1}))
\]

Initial values \( \beta_{0|0}^{(j)} \) and \( p_{0|0}^{(j)} \) for Kalman filter and \( pr(S_0 = j|l_0) \) for Hamilton filter are assigned to start the iteration. As soon as the Kalman filter and Hamilton filter are completed, smoothing procedures for \( \beta_t \), \( P_t \) and probability terms begin.

The smoothing algorithm iterates backwards and it has the following procedure:

\[
\beta_{t|T}^{(j,k)} = \beta_{t|t}^{(j)} + p_{t|t}^{(j)} F_k \left[ p_{t+1|t}^{(j,k)} \right]^{-1} (\beta_{t+1|T}^{(k)} - \beta_{t+1|t}^{(j,k)})
\]

\[
p_{t|T}^{(j,k)} = p_{t|t}^{(j)} + p_{t|t}^{(j)} F_k \left[ p_{t+1|t}^{(j,k)} \right]^{-1} (p_{t+1|T}^{(k)} - p_{t+1|t}^{(j,k)}) p_{t|t}^{(j)} F_k \left[ p_{t+1|t}^{(j,k)} \right]^{-1},
\]

\[
pr(S_t = j, S_{t+1} = k|\varphi_T) \approx pr(S_{t+1} = k|\varphi_T) pr(S_t = j|\varphi_t) pr(S_{t+1} = k|S_t = j)
\]

\[
pr(S_{t+1} = k|\varphi_\tau) = \sum_{k=0}^{1} pr(S_t = j, S_{t+1} = k|\varphi_T)r
\]

The initial values for the smoothing \( \beta_{T|T}^{(k)} \), \( p_{T|T}^{(k)} \) are obtained from the last iteration of Kalman filter and Hamilton filter. The smoothing algorithm also need to
transfer \( \beta^{(j,k)}_{t|T} \) and \( P^{(j,k)}_{t|T} \) into \( \beta^{(j)}_{t|T} \) and \( P^{(j)}_{t|T} \). The calculation method is similar to the one of filters.

### A.2 Estimation Procedure of Time-varying Parameter Model

This study follows Kim and Nelson (1999) for Estimation procedure of time-varying parameter model. In the simple state space model without Markov switching, the goal of Kalman filter is to use a recursive process to produce a forecast of unobserved state vector \( \beta_t \) and its covariance matrix with information available up to time \( t-1 \). They do not dependent on state information. The forecast of \( \beta_t \) and its covariance matrix of \( P_{t|t-1} \) are denoted as

\[
\beta_{t|t-1} = E[\beta_t|I_{t-1}]
\]

\[
P_{t|t-1} = E[(\beta_t - \beta_{t|t-1})(\beta_t - \beta_{t|t-1})'|I_{t-1}].
\]

The Kalman filter iteration process is as follows:

\[
\beta_{t|t-1} = \mu + F\beta_{t-1|t-1}
\]

\[
P_{t|t-1} = FP_{t-1|t-1}F' + Q
\]

\[
\theta_{t|t-1} = y_t - x_t\beta_{t|t-1}
\]

\[
\tau_{t|t-1} = x_tP_{t|t-1}x_t' + \sigma_u^2
\]

\[
\beta_{t|t} = \beta_{t|t-1} + P_{t|t-1}x_t'[\tau_{t|t-1}]^{-1}\theta_{t|t-1}
\]

\[
P_{t|t} = (I - P_{t|t-1}x_t'[\eta_{t|t-1}]^{-1}x_t)P_{t|t-1}
\]

where \( \theta_{t|t-1} \) is the prediction error of \( y_t \) conditional on information up to time \( t-1 \); and \( \tau_{t|t-1} \) is the conditional variance of the prediction error. Initial value of \( \beta_{0|0} \)
and $P_{0|0}$ are given to start the Kalman filter iteration. Maximum likelihood estimation is conducted for unknown parameters based on the prediction error decomposition. The forecasting error variance equation tells that an investor’s uncertainty about the future arises not only from the uncertainty about future random terms, but also from the uncertainty about the accuracy of parameter values of the model.