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A SIMPLE METHOD FOR INVESTIGATING THE PARENTAGE OF STATES USING TWO-NUCLEON TRANSFER REACTIONS

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April 16, 1969

A promising method is described for investigating the parentage of nuclear states. It involves the simultaneous observation of \( (p,t) \) and \( (p,^3\text{He}) \) reactions on \( 0^+ (T > 0) \) targets producing analogue final states with the same isospin as the target. Experimental results for \( 20 \leq A \leq 36 (T = 1) \) reveal a number of states with apparently very simple parentage for which \( j^2 \) "paired" pick-up predominates.

The simultaneous observation of \( (p,t) \) and \( (p,^3\text{He}) \) reactions on a target with isospin \( T_\| \) has been used for some time\(^1\text{–}^5\) to locate and identify analogue final states with isospin \( T_\| = T_\| + 1 \). The signature of a pair of such states is that the angular distributions of the corresponding tritons and \( ^3\text{He} \) particles have the same shape, and their magnitudes are related by a simple calculable factor. In this letter we shall first examine what is now a fairly large body of relevant experimental data, and use it as a means of establishing the validity of the approximations made in calculating the cross section ratio. We will then use the same approximations to calculate ratios for analogue final states with \( T_\| = T_\| \). In certain cases the same experimental techniques can be used not only to identify such states, but also to determine information regarding their structure. Experimental data presented on such states in sd-shell nuclei
indicate a striking simplicity in their parentage. Evidently the experimental method described provides a new and useful spectroscopic tool for investigating the parentage of nuclear states.

An expression for the differential cross section for the two-nucleon pick-up reaction \( A(a,b)B \) can be derived using the distorted wave Born approximation (DWBA). If spin-orbit forces are neglected, the result obtained is:

\[
\frac{d\sigma}{d\Omega} \propto \frac{k_b}{k_a} \sum_A b_{ST}^2 \left( T_{N_B N_T} N_{T_A N_A} \right)^2 \left| \sum_B^{N_{NLSJT}} b_{LN}^{\text{NLSJT}} (k_a k_b) \right|^2
\]

(1)

Here \( b_{ST}^2 \) is a spectroscopic factor for the light particles \( a \) and \( b \); it equals \( 1/2(\delta_{S0} \delta_{T1} + \delta_{S1} \delta_{T0}) \) for \((p,^3\text{He})\) and \( \delta_{S0} \delta_{T1} \) for \((p,t)\). The function \( D(S,T) \) depends upon the strength of the spin and isospin exchange terms in the interaction potential. The term \( b_{LN}^{\text{NLSJT}} \) contains distorted waves and the form factor of the transferred nucleons, while \( G_{\text{NLSJT}} \) contains all the information on nuclear structure. The latter is written:

\[
G_{\text{NLSJT}} = \sum \rho_1 \rho_2 (2-\delta_{\rho_1 \rho_2})^{1/2} \Omega_n \rho_{1/2}^{\text{AB}} (\rho_1 \rho_2; \text{LSJT}) \langle \text{nONL}:L|n_1^1, n_2^1 l_2^1 :L \rangle
\]

(2)

Here the summation is over all possible configurations \( \rho_1 \left[ \left[ n_1 \right], \rho_2 \right] \) for the transferred particles; \( \Omega_n \) is the overlap of the relative motion of these particles in the target with their relative motion in the triton (or \(^3\text{He}\) particle); and \( \langle | \) is a Moshinsky bracket. The spectroscopic amplitude, \( \rho_{1/2}^{\text{AB}} \), is an overlap integral which is a measure of the probability that the nucleons which are common to the initial and final nuclear states will have identical configurations in both and that the extra nucleons in \( A \) are in shell-model states \( \rho_1 \) and \( \rho_2 \).

For those cases in which \((p,t)\) and \((p,^3\text{He})\) reactions on the same target produce analogue final states with \( T_f = T_i + 1 \), the sum over \( S \) and \( T \)
in Eq. (1) is restricted to a single term \((S = 0; \, T = 1)\) and all factors in
Eq. (2) except \(\Omega_n\) are the same for both reactions. The ratio of their dif-
ferential cross sections may then be written (with \(G_{NLJ} \equiv \sum_n \Omega_n A_{NLJ}\)) as:

\[
R = \frac{\frac{d\sigma}{d\Omega}(p,t)}{\frac{d\sigma}{d\Omega}(p,^3\text{He})} = \frac{k_t}{k_{^3\text{He}}} \cdot \frac{2}{(2T_f-1)} \left\{ \frac{\sum_{\Lambda}^L J \left| \sum_{\Lambda}^N \rho_1 \rho_2 A_{NLJ} \left[ \Omega_n B_{\Lambda} \right]_{(p,t)} \right|^2}{\sum_{\Lambda}^L J \left| \sum_{\Lambda}^N \rho_1 \rho_2 A_{NLJ} \left[ \Omega_n B_{\Lambda} \right]_{(p,^3\text{He})} \right|^2} \right\}
\]

The numerator and denominator of the bracketed factor on the right hand side of Eq. (3) differ principally inasmuch as the mass-3 wave functions and the reaction Q-values are different. The effects of both can be estimated. If the target nucleus has \(A \leq 60\), then the difference in the mass-3 wave functions affects \(R\) by \(<2\%\); and by choosing a suitably high bombarding energy
\((E_p = 45\, \text{MeV})\) the effect of the different Q-values \((2.5 \leq \Delta Q \leq 8.5)\) on \(R\) is \(<5\%\). Thus, to a good approximation the bracketed factor may be assumed equal to one.

Experimental cross-section ratios \((15^\circ \leq \theta_{cm} \leq 45^\circ)\) to such analogue states for a number of nuclei with \(14 \leq A \leq 40\) (and \(1 \leq T_f \leq 3\)) are listed in Table 1 together with the values of \(R\) calculated from the first two factors in Eq. (3). The agreement is uniformly excellent and justifies neglecting the third factor in that equation.

What has not been recognized previously is that by making the same approximations for any \(0^+\) target a simple result can also be derived for the cross-section ratios corresponding to certain analogue final states with \(T_f = T_i\). Then, only transitions to natural-parity levels \((\pi = (-)^J)\) will be excited in both \((p,t)\) and \((p,^3\text{He})\) reactions, and each will be characterized by a single \(L\)-value, namely that for which \(L = J\). Two general cases will be considered:
(i) If both picked-up nucleons come from the same shell (i.e. referring to Eq. (2), \(p_1 = p_2\) for all terms in the sum) then, in order to produce a natural parity state, \(J\) must be even and the total isospin of the transferred nucleons is restricted to \(T = 1\) in order to preserve the anti-symmetry of their wave function. This is the same restriction, though for a different reason, as that which led to the derivation of Eq. (3). The subsequent approximation which was used to reduce that equation can again be used with the result:

\[
R = \frac{k_t}{k_3^{\text{He}}} \cdot \frac{2}{T_f^3} . \tag{4}
\]

(ii) If, on the other hand, the two picked-up nucleons come from different shells (i.e., \(p_1 \neq p_2\) for at least one term in the sum in Eq. (2)) then \(T = 0\) and \(T = 1\) are both permitted in the \((p, ^3\text{He})\) reaction. Thus, any mixture of both "paired" and "unpaired" particles or combinations of different "unpaired" particles involved in the transfer would in general result in a ratio less than that given by Eq. (4).

As a quantitative example, consider the transfer of specific unpaired particles characterized by \(p_1\) and \(p_2\). Using algebraic expressions for the spectroscopic amplitudes \(S_{AB}^{1/2}(p_1p_2; JT)\) and invoking a variant of the same approximation, the cross-section ratio can be expressed in the form:

\[
R = \frac{k_t}{k_3^{\text{He}}} \cdot \frac{2}{T_f^3} \left[ 1 + \frac{1}{3} \frac{\lambda^2}{T^2} \frac{(T_f + 1)^2}{L(L+1)} \right]^{-1} ,
\]

where \(\lambda = [l_1(l_1+1) - l_2(l_2+1)] - [j_1(j_1+1) - j_2(j_2+1)]\)

and \(\tau = [t_1(t_1+1) - t_2(t_2+1)] - [t_1'(t_1'+1) - t_2'(t_2'+1)]\) . \(\tag{5}\)
The factor 1/3 is the approximate value for \([D(1,0)/D(0,1)]^2\) determined in Refs. 8 and 10; also \(t_i\) and \(t'_i\) \((i = 1,2)\) are the initial and final isospins of the shells from which the nucleons are picked up. In the sd-shell the ratio calculated from Eq. (5) would typically be less than the "paired" value by \(\approx 30\%\).

To summarize, the angular distributions of \((p,t)\) and \((p,^3\text{He})\) reactions on \(0^+\) nuclei leading to analogue final states with \(T_f = T_i\) should be identical in shape and their relative magnitudes should acquire a maximum value when the wave function of the final state differs from that of the target ground state by the removal of paired nucleons. It should be emphasized that in principle any number of shells can be involved, and that there is no restriction on the complexity of the wave functions of the initial and final states. These transitions can be discerned since any unpaired particles involved in the transfer would reduce the magnitude ratio.

To provide a preliminary experimental investigation of this theory, we have examined the \((p,t)\) and \((p,^3\text{He})\) reactions on even-even \(T = 1\) target nuclei in the region \(22 \leq A \leq 38\). The experiments were carried out using the external 45.0 MeV proton beam of the Berkeley 88-inch cyclotron. Reaction products were detected and identified using a solid-state counter telescope; spectra of tritons and \(^3\text{He}\) particles were recorded simultaneously. A detailed description of the apparatus is given elsewhere.

Five targets were bombarded: \(^{22}\text{Ne}, \, ^{26}\text{Mg}, \, ^{30}\text{Si}, \, ^{34}\text{S}, \, \text{and} \, ^{38}\text{Ar}\). As an example of the experimental results, triton and \(^3\text{He}\) spectra from the \(^{22}\text{Ne}\) target are shown in Fig. 1. As with the other targets, angular distributions were extracted for all statistically significant peaks whose energies might correspond to pairs of \(T = 1\) final states; the angular range covered was
typically $15^\circ \leq \theta_{\text{cm}} \leq 45^\circ$. If the angular distributions for two of these states had the same shape, then they were positively identified as being $T = 1$ analogues. In mass 20, two pairs were observed and these have been marked in the figure. A list of such states covering all targets investigated is given in Table 2 together with cross-section ratios extracted from the data. The top half of the table includes only the lowest natural-parity $T = 1$ state in each nucleus, while in the bottom half a number of excited $T = 1$ states are shown. Those energies for which error bars are given were determined from this work; the others were taken from Ref. 11.

Calculated values for the cross-section ratios, assuming the picked-up nucleons are "paired", comprise the last column of Table 2. Those cases for which the experimental ratio is significantly less than the calculated one must involve some "unpaired" pick-up; then the calculated ratio in the table is bracketed. Two observations can be made immediately from the bottom half of the table. First, the two pairs of $0^+$ levels, which could only be produced by paired pick-up, do indeed show the maximum ratio; and second, the $(3^-)$ levels, which could not involve paired pick-up, give a reduced value as expected.

The most striking result, however, appears in the top half of the table. Here, three transitions are indicated as being dominated by the transfer of paired nucleons. These transitions are just the ones which would have been predicted from the simplest shell model interpretation as being $j^n \rightarrow j^{n-2}$; the remaining transitions would have been predicted as crossing subshells with unpaired transfer. There can be no doubt that the wave functions of the states involved in these reactions are vastly more complex than is indicated by such a simple model, so it is therefore all the more surprising that at least in the case of paired transfer the parentage remains simple. These
The conclusions are further confirmed by results of the reactions $^{22}_{\text{Ne}}(d,a)^{20}_{\text{F}}$ and $^{38}_{\text{Ar}}(d,a)^{36}_{\text{Cl}}$. Since the $(d,a)$ reaction must transfer $T = 0$, paired pick-up to natural parity states is forbidden. Three of the five transitions listed in Table 2 for these target nuclei were characterized as being "paired"; as expected, all were observed to be weak ($\leq 8\%$ of the strongest transition).

The usefulness of the experimental method as a general spectroscopic tool is certainly not limited to the mass region discussed here. The approximations are widely applicable and, although Eqs. (4) and (5) indicate that $R$ decreases with increasing $T$, the percentage difference between "paired" and "unpaired" transfer becomes greater. Thus with larger $A$ (and $T$) the sensitivity of the method will be increased. The striking results already obtained provide useful restrictions for future model calculations, and invite more general and detailed experimental investigations.

FOOTNOTES AND REFERENCES

* This work performed under the auspices of the U. S. Atomic Energy Commission.


5. H. Brunnader, J. C. Hardy, and J. Cerny, (to be published).


9. T. A. Brody and M. Moshinsky, Tables of Transformation Brackets,
   (Monograficis del Instituto de Fisica, Mexico, 1960).


11. For $^{20}$Ne: J. D. Pearson and R. H. Spear, Nucl. Phys. 54, 434 (1964); and
    R. D. Macfarlane and A. Siivola, Nucl. Phys. 59, 168 (1964); for all others

Table 1. Experimental and calculated relative cross-sections
d$\sigma$/d$\Omega$ (p,t)/d$\sigma$/d$\Omega$ (p,$^{3}$He) for states with $T_f = T_i + 1$

<table>
<thead>
<tr>
<th>Final States $J^m, T_f$</th>
<th>Allowed L-value(s)</th>
<th>Target nucleus</th>
<th>Cross-section ratio</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>R(exp)</td>
<td></td>
</tr>
<tr>
<td>$0^+, 1$</td>
<td>0</td>
<td>$^{16}$O</td>
<td>2.19±0.22</td>
<td>2</td>
</tr>
<tr>
<td>$0^+, 1$</td>
<td>0</td>
<td>$^{36}$Ar</td>
<td>1.92±0.19</td>
<td>5</td>
</tr>
<tr>
<td>$2^+, 1$</td>
<td>2</td>
<td>$^{36}$Ar</td>
<td>1.54±0.20</td>
<td>5</td>
</tr>
<tr>
<td>$3/2^+, 3/2$</td>
<td>0, 2</td>
<td>$^{21}$Ne</td>
<td>1.05±0.10</td>
<td>4</td>
</tr>
<tr>
<td>$5/2^+, 3/2$</td>
<td>0, 2, 4</td>
<td>$^{25}$Mg</td>
<td>0.85±0.09</td>
<td>4</td>
</tr>
<tr>
<td>$5/2^+, 3/2$</td>
<td>0, 2, 4</td>
<td>$^{27}$Al</td>
<td>0.89±0.09</td>
<td>3</td>
</tr>
<tr>
<td>$5/2^+, 3/2$</td>
<td>2</td>
<td>$^{31}$P</td>
<td>0.71±0.11</td>
<td>3</td>
</tr>
<tr>
<td>$0^+, 2$</td>
<td>0</td>
<td>$^{22}$Ne</td>
<td>0.70±0.09</td>
<td>1, 4</td>
</tr>
<tr>
<td>$0^+, 2$</td>
<td>0</td>
<td>$^{26}$Mg</td>
<td>0.61±0.06</td>
<td>4</td>
</tr>
<tr>
<td>$0^+, 2$</td>
<td>0</td>
<td>$^{30}$Si</td>
<td>0.54±0.10</td>
<td>5</td>
</tr>
<tr>
<td>$0^+, 2$</td>
<td>0</td>
<td>$^{34}$S</td>
<td>0.66±0.06</td>
<td>5</td>
</tr>
<tr>
<td>$0^+, 2$</td>
<td>0</td>
<td>$^{38}$Ar</td>
<td>0.62±0.07</td>
<td>5</td>
</tr>
<tr>
<td>$0^+, 2$</td>
<td>0</td>
<td>$^{42}$Ca</td>
<td>0.60±0.05</td>
<td>5</td>
</tr>
<tr>
<td>$0^+, 3$</td>
<td>0</td>
<td>$^{40}$Ar</td>
<td>0.36±0.04</td>
<td>5</td>
</tr>
</tbody>
</table>
Table 2. Experimental and calculated relative cross-sections
\( \frac{d \sigma}{d \Omega} \left( p, t \right) / \frac{d \sigma}{d \Omega} \left( p, \text{He} \right) \) for states with \( T_f = T_i \).

<table>
<thead>
<tr>
<th>Final states nucleus</th>
<th>( E_x )</th>
<th>( J^\pi, T )</th>
<th>Target nucleus</th>
<th>Cross-section ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{20}\text{Ne} )</td>
<td>10.275</td>
<td>( 2^+, 1 )</td>
<td>(^{22}\text{Ne} )</td>
<td>2.00±0.20</td>
</tr>
<tr>
<td>(^{20}\text{F} )</td>
<td>g.s.</td>
<td></td>
<td></td>
<td>1.88</td>
</tr>
<tr>
<td>(^{24}\text{Mg} )</td>
<td>9.517</td>
<td>( 4^+, 1 )</td>
<td>(^{26}\text{Mg} )</td>
<td>2.50±0.30</td>
</tr>
<tr>
<td>(^{24}\text{Na} )</td>
<td>g.s.</td>
<td></td>
<td></td>
<td>1.86</td>
</tr>
<tr>
<td>(^{28}\text{Si} )</td>
<td>9.379</td>
<td>( 2^+, 1 )</td>
<td>(^{30}\text{Si} )</td>
<td>1.15±0.10(^a)</td>
</tr>
<tr>
<td>(^{28}\text{Al} )</td>
<td>0.031</td>
<td></td>
<td></td>
<td>[1.84]</td>
</tr>
<tr>
<td>(^{32}\text{S} )</td>
<td>7.005</td>
<td>( 2^+, 1 )</td>
<td>(^{34}\text{S} )</td>
<td>1.20±0.30</td>
</tr>
<tr>
<td>(^{32}\text{P} )</td>
<td>0.078</td>
<td></td>
<td></td>
<td>[1.82]</td>
</tr>
<tr>
<td>(^{36}\text{Ar} )</td>
<td>6.612</td>
<td>( 2^+, 1 )</td>
<td>(^{38}\text{Ar} )</td>
<td>1.90±0.20</td>
</tr>
<tr>
<td>(^{36}\text{Cl} )</td>
<td>g.s.</td>
<td></td>
<td></td>
<td>1.81</td>
</tr>
</tbody>
</table>

\(^{20}\text{Ne} \)  | 12.25±0.03 | \( (3^-), 1^b \) | \(^{22}\text{Ne} \) | 1.40±0.15         |
| \(^{20}\text{F} \)  | 1.851     |             |                | c                  |
| \(^{28}\text{Si} \) | 10.70±0.03 | \( 0^+, 1^b, d \) | \(^{30}\text{Si} \) | 1.85±0.20         |
| \(^{28}\text{Al} \) | 1.35±0.03 |             |                | 1.84               |
| \(^{28}\text{Si} \) | 10.909   | \( (2^+), 1^b \) | \(^{30}\text{Si} \) | 1.80±0.20         |
| \(^{28}\text{Al} \) | 1.633    |             |                | 1.83               |
| \(^{36}\text{Ar} \) | 8.55±0.03 | \( 2^+, 1 \) | \(^{38}\text{Ar} \) | 1.45±0.20         |
| \(^{36}\text{Cl} \) | 1.949    |             |                | [1.81]            |
| \(^{36}\text{Ar} \) | 9.70±0.03 | \( 0^+, 1^c \) | \(^{38}\text{Ar} \) | 2.20±0.70         |
| \(^{36}\text{Cl} \) | 3.12±0.10 |             |                | 1.80               |

(continued)
The ground state of $^{28}$Al was not resolved from its $2^+$ state, so the quoted experimental ratio is a lower limit. For various reasons (e.g., pure $L = 2$ angular distribution) it seems unlikely that this ratio will approach the "paired" value.

These spin-parities were determined from this work by fitting experimental angular distributions with DWBA calculations.

As a simple example of an "unpaired" ratio for this transition, pick-up from the $p_{1/2}$ and $d_{5/2}$ shells yields a value for $R$ of 1.30.

A level is known in $^{28}$Al at 1.372 MeV which is certainly $1^+$; the strong $0^+$ observed by us presumably indicates the presence of a doublet.

---

**FIGURE CAPTION**

Fig. 1. Energy spectra from the reactions $^{22}$Ne($p$,t)$^{20}$Ne and $^{22}$Ne($p$,${}^3$He)$^{20}$F. The target was a 50:50 mixture of neon and methane, the neon being 92% enriched in $^{22}$Ne. The methane was used to provide an energy calibration.
$^{22}\text{Ne}(p, t)\ ^{20}\text{Ne}$
Target: $^{22}\text{Ne} + \text{CH}_4$
$E_p = 45.0$ MeV
$\theta_{\text{lab}} = 36.2^\circ$

Fig. 1

$^{22}\text{Ne}(p, ^3\text{He})\ ^{20}\text{F}$
Target: $^{22}\text{Ne} + \text{CH}_4$
$E_p = 45.0$ MeV
$\theta_{\text{lab}} = 36.2^\circ$
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