Title
STUDY OF THE BEAM BREAKUP MODE IN LINEAR INDUCTION ACCELERATORS FOR HEAVY IONS

Permalink
https://escholarship.org/uc/item/1ss5s132

Author
Chattopadhyay, S.

Publication Date
2010-03-24

Peer reviewed
STUDY OF THE BEAM-BREAKUP MODE IN LINEAR INDUCTION ACCELERATORS FOR HEAVY IONS

S. Chattopadhyay, A. Faltens, and L. Smith

March 1981
STUDY OF THE BEAM-BREAKUP MODE IN LINEAR INDUCTION ACCELERATORS FOR HEAVY IONS

S. Chattopadhyay, A. Faltens, and L. Smith

Accelerator and Fusion Research Division
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

March 1981

*This work was supported by the Director, Office of Energy Research, Office of Inertial Fusion, Research Division of the U.S. Department of Energy under Contract No. W-7405-ENG-48.
STUDY OF THE BEAM BREAKUP MODE IN LINEAR INDUCTION ACCELERATORS FOR HEAVY IONS*

Chattopadhyay, A. Faltens, and L. Smith
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

Abstract

A simple theoretical study and numerical estimate is presented for the transverse amplitude growth of a nonrelativistic heavy ion beam in an induction linac, as envisaged for use in commercial power plants, due to the nonregenerative coherent beam breakup mode. An equivalent electrical circuit has been used to represent the accelerating induction modules. Our calculation shows that for the parameters of interest, the beam breakup amplitude for a heavy ion beam grows extremely slowly in the time scales of interest, to magnitudes insignificant for transport purposes. It is concluded that the coherent beam breakup mode does not pose any serious threat to the stability of a high current (kA) heavy ion beam in an induction linac.

I. Introduction

High current heavy ion beams are being actively studied as potential drivers for inertial confinement fusion. Such high current nonneutral beams are subject to coherent and incoherent, transverse and longitudinal, collective instabilities arising from the beam space charge (self-force) and its interaction with the environment (external impedances, cavities etc.). In this paper, we study the growing coherent transverse motion of a high current (-kA) heavy ion beam due to an oscillatory transverse mode (analogous to TM_{10} mode of a pill-box cavity) excited by the beam in the accelerating modules. The subject has been studied extensively in connection with electron linacs by several authors(1-5), who computed the upper limit of transportable total charge set by the growth of beam breakup amplitude. However no such study has been reported for heavy ion beams transported by induction linacs.

II. Model for Transport

Our theoretical model of transport is a semi-infinite series of identical accelerating induction modules with identical focussing elements between them (see Fig. 1). If the beam centroid is off center (or if the beam is centered in an azimuthally asymmetric structure), it will excite a transversely deflecting mode (analogous to TM_{10} mode of a pill-box cavity) excited by the beam in the accelerating modules. The subject has been studied extensively in connection with electron linacs by several authors(1-5), who computed the upper limit of transportable total charge set by the growth of beam breakup amplitude. However no such study has been reported for heavy ion beams transported by induction linacs.

III. Induction Module Response

An induction linac module differs drastically from an r.f. cavity in its response to excitation by a particle beam. There is no accelerating mode as such(6); the longitudinal interaction of beam and module is best represented by an equivalent circuit involving the external drive, corresponding typically to a frequency of a few megacycles and strongly overdamped by the low drive-impedance. For the asymmetric modes of interest to the beam breakup phenomenon, the module looks like a pill-box with conducting end walls and a lossy outer wall traversed longitudinally by one or more conducting straps. Accordingly, we take as a model the excitation of the TM_{10} mode of a pill-box cavity with a radius of about half a meter and a Q of about 10.

The vector potential can be written as:

$$A_z = A(t) J_1 \left( \frac{\Omega}{c} r \right) \cos \theta$$

and A satisfies the differential equation:

$$\dddot{A} + m A + \Omega^2 A = \frac{\nu_0^2 I}{\pi \epsilon_0 \eta_0} \frac{\xi(t)}{j_1(j_1)} \xi(t)$$  \hspace{0.5cm} (1)$$

where I is the beam current, \( \xi(t) \) is the transverse beam displacement at time \( t \) following beam arrival at the module and the other symbols have their conventional meanings. In traversing the cavity, the beam experiences a change in slope (see Fig. 2) given by:

$$\Delta \theta = \left( \frac{Z_0}{2m_0 v_0} \right) \frac{\nu_0}{c} \left( \frac{Z_0}{\Delta m_0} \right) \Delta A$$

*This work was supported by the Director, Office of Energy Research, Office of Inertial Fusion, Research Division of the U.S. Department of Energy under Contract No. W-7405-ENG-48.
Using infinite and binomial series expansions for the cosine and \((\omega_b t + 16/s)^n\) respectively and making use of the Laplace inversion formula

\[
L^{-1} \left( \frac{1}{s^n} \right) = \frac{s^{n-1}}{n!}
\]

we get an expression for \(X(z,t)\) involving a double sum over integers, one of which can be summed in closed form to give spherical Bessel functions. We finally get:

\[
X(z,t) = \sum_{l=0}^{\infty} (-1)^l \frac{1}{(2l)!} \left( \frac{Gz}{w_b} \right)^l y_l(j_l(z))
\]

After a few betatron wavelengths down the accelerator, \(w_b z \gg 1\) and we use:

\[
j_{-1} (w_b z) \rightarrow \frac{1}{(w_b z)^{1/2}} \cos \left( \frac{w_b z}{Z \cdot A} \right) \]

Using (4), (5) and (2), we finally arrive at the expression for the transverse beam displacement \(X(z,t)\) at location \(z\) and time \(t\) following the arrival of the front of the beam, in closed form, as follows:

\[
X(z,t) = \frac{G}{2} e^{-\alpha T} \left[ \cos \left( w_b z - \alpha T \right) I_0 \left( \frac{Gz}{w_b} \right) + \cos \left( w_b z + \alpha T \right) J_0 \left( \frac{Gz}{w_b} \right) \right]
\]

where \(I_0\) and \(J_0\) are zero-order Bessel and modified Bessel functions respectively.

We note that in the limit of no focusing at all \((w_b = 0)\), we have:

\[
x(z,t) = \frac{G}{2} \sum_{l=0}^{\infty} (-1)^l \frac{1}{(2l)!} \left( \frac{Gz}{2a} \right)^l (2l-1) \]

so that the absolute square of the slowly varying amplitude grows as:

\[
|X(z,t)|^2 = \frac{G^2}{4} \sum_{l=0}^{\infty} \sum_{m=0}^{2m} \frac{(-1)^{m-n} (m+2m)!}{(2m)! (4n-2m)!}
\]

in agreement with Panofsky and Bander (2) and hence is expected to scale similarly as:

\[
|X(z,t)|^2 \sim e^{s} \text{ with } s = (Gz/2a)^{1/3}
\]

V. Numerical Estimates:

We observe from expression (6) that the beam displacement is damped on the whole if \(a > Gz/2a_b\); if \(a \ll (Gz/2a_b)\), the maximum in \(\alpha\) of the amplitude of displacement comes at \(\alpha = (Gz/2a - \alpha)\) and has a magnitude:

\[
x = \frac{d}{\sqrt{2s}} \left( \frac{w_b}{2s} \right)^{1/2} Gz/2a_b
\]

As a numerical example, we consider an induction linac that accelerates singly charged Uranium ions, with a 30° phase advance between modules. Example beam parameters (6) for two significant cases and parameters of equivalent induction module cavities are listed in Table I below.
For the square bucket, single harmonic \((l = 3, 4)\) rates are comparable to those of a transverse random load with a factor of 2. It should be noted that the longitudinal random load provides an effective random load of a uniform distribution in phase space.

Finally, the last entries in Table 1 are for longitudinal runs. Effective gcooling rates/step are given after 200 and 1000 correction steps are given. The phase space orbits are elliptical with amplitude variation of synchrotron frequency. Cooling rates degraded as mixing lessens with higher phase space density.

Acknowledgement

We thank L.O. Laslett for providing his coasting beam cooling code and for giving his invaluable advice in modifying it.

Conclusions

Synchrotron frequency spread provides the necessary mixing mechanism for bunched beam cooling. In addition, it appears that the natural non-linearities of a long RF bucket can provide mixing comparable to a single, long bucket, which can provide mismatches of higher frequency than those associated with the gross bunch structure. However, as the bunch length decreases degradation of cooling occurs as the mixing mechanism couples neighboring Schottky bands.6)

References

1. Tevatron Phase 1 Project, FNAL, 1980.