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Author
Walsh, Carl E.

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Optimal Economic Transparency*

Carl E. Walsh
Department of Economics, University of California, Santa Cruz

In this paper, I explore the optimal extent to which the central bank should disseminate information among private agents. Individual firms are assumed to have diverse private information, and the central bank provides public information either implicitly, by setting its policy instrument, or explicitly, by making announcements about its short-run targets. The optimal degree of economic transparency is affected differently by cost and demand shocks. More-accurate central bank forecasts of demand shocks reduce optimal transparency, while more-accurate forecasts of cost shocks increase optimal transparency. Increased persistence in demand (cost) disturbances increases (reduces) optimal transparency.

JEL Codes: E52, E58, E31.

1. Introduction

A major development in central banking in recent years has been the increase in monetary policy transparency. Inflation-targeting central banks in particular have gone the furthest in adopting mechanisms to ensure greater transparency.¹

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¹In recent years, even central banks that have not formally adopted inflation targeting have become more transparent. Eijffinger and Geraats (2006) provide an index of transparency for a set of developed economies that includes some inflation targeters (Australia, Canada, New Zealand, Sweden, and the United Kingdom) as well as nontargeters (Japan, Switzerland, and the United States).
Transparency has many dimensions. Geraats (2002) identifies five different forms of transparency: political, procedural, economic, policy, and operational. Briefly, these correspond to transparency about objectives, about the internal decision-making process, about the central bank’s forecasts and models, about the central bank’s communications of its policy actions, and about its instrument setting and control errors.

Most of the existing theoretical literature on central bank transparency has focused on political and operational transparency, employing models in which only policy surprises have real effects, the central bank’s preferences are stochastic and unknown, and the central bank’s policy instrument, taken to be the money supply, is observed with error. Private agents observe the current money growth rate but are unable to disentangle the effects of control errors from shifts in central bank preferences. Thus, there is opaqueness about political objectives and operational implementation. Transparency was typically modeled as a reduction in the noise in the signal on the policy instrument. Under a less transparent regime, disinflations are more costly, as it takes private agents longer to recognize that the central bank’s preferences have shifted away from greater output expansion. However, a more transparent regime allows private agents to assess better the shifting preferences of the central bank, and this reduces the ability of the central bank to create economic expansions when they are most desired. These two competing forces determine the optimal degree of transparency, and Cukierman and Meltzer (1986) show that the central bank may prefer to adopt a less efficient operating procedure than is technically feasible (i.e., not reduce the control-error variance to its minimum possible level).

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They find that between 1998 and 2002 transparency increased for virtually all the central banks they studied. Even the Federal Reserve, which has so far resisted calls to establish a formal inflation target, has moved to make its policy practices more transparent.

2See, for example, Cukierman and Meltzer (1986) and Faust and Svensson (2002).

3See also Faust and Svensson (2002), who show that when the choice of transparency is made under commitment, patient central banks with small inflation biases will prefer minimum transparency. They argue that this result might account for the (then) relatively low degree of transparency that characterized the U.S. Federal Reserve.
In contrast to these earlier models, standard policy models today imply that predictable policies are most effective, the preferences of inflation-targeting central banks are known, and the policy instrument is likely to be a nominal interest rate that is easily observable. Thus, results from models that emphasized unpredictable policies and money-supply control may not carry over. And while modern central banks may be operationally transparent, they may still be opaque with respect to internal forecasts about the economy; economic transparency may be incomplete.

In this paper, I examine the optimal degree of economic transparency. The model developed in the paper contrasts in several ways with previous work on monetary policy transparency. First, I employ a New Keynesian model of price setting rather than the type of Lucas supply curve commonly employed in the earlier literature on transparency. Second, I ignore the issue of the central bank’s intentions and focus on inflation-targeting central banks that have already developed a reputation for maintaining low and stable inflation. The public understands that the policymaker will maintain average inflation at zero; the public also understands the manner in which the bank will respond to shocks that lead to short-run fluctuations in inflation and the output gap. Private agents still face uncertainty about monetary policy, however, because they have only imperfect knowledge of the information on which the central bank bases its policy. A transparent central bank reveals its information about the economy to the public.\(^4\)

\(^4\)Jensen (2002) studies transparency using a two-period model in which inflation is forward looking in a manner consistent with recent monetary policy models. His focus, like that of Faust and Svensson (2002), is on political transparency. Greater transparency implies that policy has a larger impact on future expectations and therefore on current inflation. This leads to greater caution on the part of the central bank in its policy actions. Transparency improves welfare if the central bank is prone to an inflation bias, but it can limit stabilization policy if the central bank’s output objective is already consistent with the economy’s natural rate of output.

\(^5\)Walsh (1999, 2003) also investigates aspects of economic transparency. In Walsh (1999), the ability of the central bank to announce a state-contingent inflation target improves stabilization policy, while in Walsh (2003), transparency about the central bank’s information improves monitoring by the public and makes it optimal for the central bank to place greater weight on achieving its inflation objectives.
Third, I drop the standard assumption that private agents have common information, assuming instead that information is diverse, with individual firms receiving idiosyncratic signals about current aggregate cost and demand shocks. Since firms care about their price relative to other firms, individual firms must form expectations about what other firms are expecting. Thus, higher-order expectations (expectations of expectations of expectations . . . ) play a role, and this can influence the way public information about monetary policy affects inflation. Morris and Shin (2002) have argued that, when private agents have individual sources of information, there can be a cost to providing more-accurate public information. Agents may overreact to public information, making the economy more sensitive to any forecast errors in the public information.

Fourth, I model transparency, not in terms of a control-error variance but in terms of the extent to which the central bank disseminates information about its views on the state of the economy. At one extreme, the central bank may make no announcements. At the other extreme, it may undertake to publish detailed inflation reports that are widely read and discussed by the public. In between these extremes, the central bank may partially publicize information through speeches, less widely read press releases, or other means that reach a limited audience. The extent to which information on the central bank’s short-run targets is made available to private agents provides a measure of transparency.

In modeling transparency in this way, I follow Cornand and Heinemann (2004), who demonstrate that the partial release of information in the Morris-Shin model can be useful. Wide release of

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6Woodford (2003) has investigated the role of higher-order expectations in inducing persistent adjustments to monetary shocks in the Lucas-Phelps islands model. See also Hellwig (2002).

7The possibility that the private sector may overreact to central bank announcements does capture a concern expressed by some policymakers. For example, in discussing the release of Federal Open Market Committee (FOMC) minutes, Janet Yellen expressed the view that “financial markets could misinterpret and overreact to the minutes” (Yellen 2005). However, Svensson (2006) has argued that the Morris-Shin result is not a general one. He shows that welfare is increased by more-accurate public information in the Morris-Shin model for all but unreasonable parameter values. A similar result is found by Hellwig (2004).
information causes public information to coordinate expectations, and this can make the economy sensitive to any noise in the public information; this is the cost of announcements. The gain is that they provide information that leads the public to have more-accurate expectations. When it is costly for the central bank to provide information, it may still pay to engage in a limited release of information. If only a few agents receive the central bank’s information, private-sector expectations will, on average, be more accurate, but because only a few agents receive the information, it has little effect on the typical agent’s expectations of what others are expecting. The impact of the noise in the public information is limited.

Just as the earlier literature on transparency employed models at odds with current policy frameworks (only surprises mattered, money supply was the instrument), the analysis of Morris and Shin (2002) is conducted within a framework that fails to capture important aspects of actual monetary policy. For example, the public information in Morris and Shin (2002) is a signal on an exogenous disturbance, yet most of the monetary policy debate on transparency has focused on the endogenous signals a central bank might release. When private agents observe a change in the central bank’s instrument or receive announcements about the central bank’s inflation forecast, they are obtaining public signals that depend on both the central bank’s policy objectives and its assessment of economic conditions.

Amato and Shin (2003) have cast the Morris-Shin analysis in a more standard macro model. In their model, the central bank has perfect information about the underlying shocks. This ignores the uncertainty policymakers themselves face in assessing the state of the economy. Nor do Amato and Shin (2003) allow the private sector to use observations on the policy instrument to draw inferences about the central bank’s information. They also assume one-period price setting and represent monetary policy by a price-level targeting rule. In Hellwig (2004), prices are flexible and policy is given by an exogenous stochastic supply of money; private and public information consists of signals on the nominal quantity of money. In contrast, I employ a standard Calvo-type model of imperfect price flexibility, modifying it by assuming that those firms adjusting each period must do so before observing the actual aggregate price level. Thus, the need to infer what other firms are doing is present, as in
Amato and Shin (2003) and in Hellwig (2004), but the approach is more consistent with standard New Keynesian models.\footnote{Hellwig (2004) provides a more microfounded analysis that I pursue here, showing that this can be important for assessing the welfare effects of better information. Some comments on how results might differ if a welfare-based measure were used are discussed in the concluding section.}

Walsh (2006) examines how the degree of economic transparency affects the monetary transmission mechanism and shows that the impact of an interest rate change on inflation depends importantly on the information revealed by the central bank and on the quality of that information. In the model used in that paper, however, as in the related model of Baeriswyl and Cornand (2005), firms adjusting prices do so before observing any actual shocks. This means that inflation responds to expected cost shocks and not to the actual realizations of the shock. Transparency, by revealing information, can make expectations more volatile and increase the variability of inflation. Thus, a central bank concerned with stabilizing inflation may prefer to limit transparency. Walsh (2006) also assumed that firms received private information on the cost shock but not on an aggregate demand shock.\footnote{Baeriswyl and Cornand (2005) make a similar assumption.} In the present paper, I allow firm-specific cost shocks (and not just expectations of these shocks) to directly affect price-setting behavior, and I assume that firms receive private signals on both the cost shock and the shock to aggregate demand, and the underlying cost and demand shocks are allowed to display persistence. This last aspect is important when current inflation depends on expectations of future inflation.

Geraats (2005) also analyzed the role played by the release of central bank forecasts. However, she assumes that agents do not observe the bank’s policy instrument prior to forming expectations and she employs a traditional Lucas supply function. Her focus is on reputational equilibria in a two-period model with a stochastic inflation target. Thus, the model and the questions addressed are quite different than those pursued here.

Besides providing a new framework for analyzing transparency, several new insights into optimal transparency are obtained. First, improved central bank forecasting can have ambiguous effects on the optimal degree of transparency. If the central bank obtains
more-accurate signals on cost shocks, optimal transparency increases; if it obtains more-accurate signals on demand shocks, optimal transparency decreases. Optimal transparency is also affected differently by changes in the stochastic processes governing the cost and demand shocks. Thus, much as in the classic Poole analysis of instrument choice, the properties of exogenous shocks matter for determining the optimal degree of transparency.

The remainder of the paper is organized as follows. Section 2 sets out the basic model. Equilibrium with partial announcements is discussed in section 3. In section 4, numerical results are reported that examine the optimal degree of transparency and how it is affected by changes in various aspects of the model. Conclusions are summarized in section 5.

2. The Model

Assume that there are a continuum of firms of measure one, each producing a differentiated product using an identical technology. Firms face a Calvo-type fixed probability of adjusting their price each period. I assume that firms do not observe the current aggregate cost or demand shocks or the prices set by other firms until the period is over. Since any firm that is setting its price is concerned with its price relative to those of other firms, it will need to form expectations about the factors that determine its optimal relative price and about the behavior of other firms, since it must forecast the average price of other firms. Each period, private firms receive noisy signals on aggregate shocks. Each firm’s signal is private information to that firm, so individual firms will have different information. The central bank also has private, noisy information on aggregate shocks. The central bank may make an announcement about its output-gap target. It then sets its policy instrument. I assume that firms that adjust their price in period $t$ do so after observing the central bank’s instrument. Because the central bank receives information about the aggregate cost and demand shocks,

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10 In the model, this is equivalent to announcing an inflation target. Given the structure of the model, it is more convenient to view any announcement as an announcement about the output-gap target.
firms cannot infer perfectly the central bank’s information on each of the two shocks by only observing the instrument.

2.1 Price-Setting Behavior

Suppose firm $j$ is setting its price in period $t$. Let $p_{jt}^*$ denote the log price it chooses. It will be convenient to treat $\pi_{jt}^* \equiv p_{jt}^* - p_{t-1}$ as the choice variable, where $p_{t-1}$ is last period’s aggregate log price level. Let $\bar{\pi}_t^*$ be the average of $\pi_{jt}^*$ across the firms adjusting in period $t$, and let $\pi_t$ be the aggregate inflation rate.

The probability that a firm does not have the opportunity to adjust its price is $\omega$. Thus,

$$ p_t = (1 - \omega)\bar{p}_t^* + \omega p_{t-1}, $$

where $\bar{p}_t^* = \int_0^1 p_{jt}^* dj$. Equation (1) implies that $\bar{p}_t^* - p_t = \omega(\bar{p}_t^* - p_{t-1})$

and

$$ \pi_t = p_t - p_{t-1} = (1 - \omega)(\bar{p}_t^* - p_{t-1}) = \left(\frac{1 - \omega}{\omega}\right)(\bar{p}_t^* - p_t). $$

Let $\varphi$ denote log real marginal cost and assume a steady-state inflation rate of zero. If firm $j$ can adjust its price, it sets its current price equal to the expected discounted value of current and future nominal marginal cost $\varphi + p$. Future marginal cost is discounted by the probability that the firm has not received another opportunity to adjust, $\omega$, and by the discount factor, $\beta$. I assume the price of firm $j$ is also affected by a cost shock $s_{jt}$ that alters the firm’s desired price. Hence,

$$ p_{jt}^* = (1 - \omega\beta)\sum_{i=0}^{\infty}(\omega\beta)^i(E_t^j \varphi_{t+i} + E_t^j p_{t+i} + E_t^j s_{jt+i}), $$

where $E_t^j$ denotes the expectations based on the information available to firm $j$. Equation (3) can be rewritten as

$$ p_{jt}^* = (1 - \omega\beta)(E_t^j p_t + E_t^j \varphi_t + s_{jt}) + \omega\beta E_t^j p_{jt+1}^*. $$

Note that it has been assumed that the firm observes its own firm-specific cost shock, $s_{jt}$, prior to setting its price but that it does not
observe the current aggregate price level or current realized nominal marginal cost.

Individual firms may set different prices because they base expectations on different information sets. And, if information sets differ, each adjusting firm’s expectations about what it would do if it is again able to adjust in \( t + 1 \) may also differ. To simplify, I assume that any idiosyncratic information is i.i.d. and that all aggregate information is revealed at the end of each period. This will imply that \( E_t^j p^*_{jt+1} = E_t^j \tilde{p}^*_{t+1} \); each firm expects that, if it can adjust in \( t + 1 \), it will set the same price as other adjusting firms.

Using (2) and the definition of \( \pi^*_jt \), one obtains, after some manipulation,

\[
\pi^*_jt = (1 - \omega) E_t^j \tilde{\pi}^*_t + (1 - \omega \beta) E_t^j \phi_t + (1 - \omega \beta) s_{jt} \\
+ \left( \frac{\omega \beta}{1 - \omega} \right) E_t^j \pi_{t+1},
\]

where \( \tilde{\pi}^*_t = \tilde{p}^*_t - p_{t-1} \).

Assume that real marginal cost is linearly related to an output-gap measure \( x_t \): \( \phi_t = \kappa x_t \). Then

\[
\pi^*_jt = (1 - \omega) E_t^j \tilde{\pi}^*_t + (1 - \omega \beta) \kappa E_t^j x_t + (1 - \omega \beta) s_{jt} \\
+ \left( \frac{\omega \beta}{1 - \omega} \right) E_t^j \pi_{t+1}.
\]

Hence, firm \( j \) adjusts its price based on its signal on the cost shock, its expectations of what other adjusting firms are choosing \( (E_t^j \tilde{\pi}^*_t) \),

\[\text{Equation (4) has the form}\]

\[
\pi^*_jt = (1 - \omega) E_t^j \tilde{\pi}^*_t + \omega E_t^j \theta_t,
\]

where

\[
E_t^j \theta_t \equiv \left( \frac{1 - \omega \beta}{\omega} \right) (E_t^j \phi_t + s_{jt}) + \left( \frac{\beta}{1 - \omega} \right) E_t^j \pi_{t+1}.
\]

This is the basic form of the decision rule at the heart of the Morris-Shin analysis. The adjustment by firm \( j \) depends on the firm’s expectations about \( \theta_t \) and on what firm \( j \) expects other firms to do. In the present analysis, however, decisions depend on expectations of future inflation, not just on expectations concerning current variables.
its expectations about the output gap, and its forecast of next-period aggregate inflation.\textsuperscript{12}

2.2 Aggregate Demand

Monetary policy is represented by the central bank’s choice of an instrument $x_t^I$ and by any announcements the central bank might make. I assume $x_t^I$ is observed at the start of the period so that any firm that sets its price in period $t$ can condition its choice on $x_t^I$. The output gap differs from $x_t^I$ by a demand shock $v_t$:

$$x_t = x_t^I + v_t. \quad (6)$$

2.3 Information

There are two primitive, aggregate disturbances in the model: (i) $s_t$, representing cost factors that, for a given output gap and expectations of future inflation, generate inefficient inflation fluctuations and (ii) $v_t$, an aggregate demand disturbance. Each is assumed to follow independent $AR(1)$ processes given by

$$s_t = \rho_s s_{t-1} + \xi_t$$

and

$$v_t = \rho_v v_{t-1} + \varphi_t.$$
Firms (in setting prices) and the central bank (in setting its policy instrument) must act before learning the actual realizations of the aggregate shocks. Firm $j$’s idiosyncratic cost shock $s_{jt}$ is related to the aggregate shock according to

$$s_{jt} = s_t + \phi_{j,t}.$$  

In addition, the firm receives a noisy signal $v_{jt}$ about the aggregate demand shock, where

$$v_{jt} = v_t + \psi_{j,t}.$$  

For convenience, both $\phi_{j,t}$ and $\psi_{j,t}$ will be referred to as noise terms, but $\phi_{j,t}$ is actually the idiosyncratic component of the firm’s cost shock. The noise terms $\phi_j$ and $\psi_j$ are identically and independently distributed across firms. These signals are private in the sense that they are unobserved by other agents.

In a similar manner, the central bank receives private signals on the two aggregate disturbances:

$$s_{cb,t} = s_t + \phi_{cb,t}$$
$$v_{cb,t} = v_t + \psi_{cb,t}.$$  

The noise terms $\phi_{cb}$ and $\psi_{cb}$ are assumed to be independently distributed and to be independent of $\phi_j$ and $\psi_j$ for all $j$ and $t$. All stochastic variables are assumed to be normally distributed.

2.4 Monetary Policy

The central bank’s objective is to minimize a standard quadratic loss function that depends on inflation variability and output-gap variability. Specifically, loss is given by

$$L = \left( \frac{1}{1 - \beta} \right) \left( \sigma_\pi^2 + \lambda \sigma_x^2 \right),$$  \hspace{0.5cm} (7)

where $\sigma_\pi^2$ and $\sigma_x^2$ are the variances of inflation and the output gap. I consider linear policy rules of the form

$$x_t^I = \delta_1 x_{t-1} + \delta_2 E_t^{cb} s_t + \delta_3 E_t^{cb} v_t,$$  \hspace{0.5cm} (8)
where the $\delta_i$ coefficients are chosen to minimize (7) subject to the equilibrium process for inflation and the information structure faced by the central bank and firms. Rules of this form are consistent with optimal policy under both commitment and discretion in the standard New Keynesian model. Under optimal discretion, policy is a function of the state, and $\delta_1 = 0$, as $s_t$ and $v_t$ are the only state variables. Under optimal commitment, inertia is introduced by policy actions, making $x_{t-1}$ an additional state variable in the equilibrium solution of the model.

Since $x_t = x_t^I + v_t$, the central bank’s time $t$ implicit target for the output gap is

$$x_t^T \equiv x_t^I + E_t^{\text{cb}} v_t = \delta_1 x_{t-1} + \delta_2 E_t^{\text{cb}} s_t + (1 + \delta_3) E_t^{\text{cb}} v_t.$$  (9)

Equation (9) and the aggregate version of (5) also imply an implicit time $t$ target for inflation. These targets for the output gap and the inflation rate can be interpreted as short-run targets. Under a credible inflation-targeting regime, the long-run inflation target is zero.

From the distributional assumptions about the central bank’s information, $E_t^{\text{cb}} s_t = \rho_s s_{t-1} + \theta_s^{\text{cb}} (s_{\text{cb},t} - \rho_s s_{t-1})$, where $\theta_s^{\text{cb}} = \sigma_\xi^2 / (\sigma_\xi^2 + \sigma_{\phi,\text{cb}}^2)$, $\sigma_\xi^2$ is the variance of $\xi_t$, and $\sigma_{\phi,\text{cb}}^2$ is the variance of $\phi_{\text{cb},t}$. Similarly, $E_t^{\text{cb}} v_t = \rho_v v_{t-1} + \theta_v^{\text{cb}} (v_{t}^{\text{cb}} - \rho_v v_{t-1})$, where $\theta_v^{\text{cb}} = \sigma_\varphi^2 / (\sigma_\varphi^2 + \sigma_{\psi,\text{cb}}^2)$.

Firms that set prices must form expectations about what other firms are expecting, as in Amato and Shin (2003), but they must also form expectations about the central bank’s output-gap target, which implicitly involves forming expectations about the central bank’s expectation of shocks (and implicitly, therefore, about what other firms are expecting that the central bank is expecting). Because firm $j$ has private information on the aggregate shocks, its expectations of $s_t$ and $v_t$ may differ from what it thinks the central bank’s expectations are. For example, $E_{t}^{j} (E^{\text{cb}} s_{t}) \neq E_{t}^{j} s_{t}$. Because the private sector may have different information than the central bank has, private expectations of shocks can differ from the central bank’s expectations of those shocks. To predict the output gap, firms must guess what the central bank thinks the aggregate cost shock is, for example, and not simply guess what the cost shock is.
3. Equilibrium with Partial Announcements

Discussions of transparency generally focus on actions by the central bank that are designed explicitly to provide information. For example, the publication of the central bank’s forecasts for inflation or output or its announcement of short-run targets for inflation are among the forms of public information designed to increase policy transparency. Private agents will use the central bank’s announcements to infer something about the central bank’s assessment of the state of the economy. This means that errors in the central bank’s assessment of the economy will similarly infect private-sector forecasts and expectations. This may introduce undesirable volatility into private-sector expectations.

Even in the absence of announcements, the public can infer something about the central bank’s information by observing the short-term interest rate used as the policy instrument, and changes in the policy interest rate are typically widely publicized. However, observing the central bank’s instrument imperfectly reveals the central bank’s forecasts of demand and cost shocks (see (8)). A change in $x^I$ could reflect the central bank’s belief that a cost shock has occurred, or it could indicate that a demand shock has occurred. These have different implications for the expected output gap, and if they could be disentangled, they would affect firms’ price-setting decisions differently. Private agents will be uncertain whether an interest movement arises because the central bank is attempting to neutralize inflation and output in response to a demand shock or because it is actively adjusting the output gap to stabilize inflation in the face of a cost shock. For example, if $x^I$ is decreased to neutralize the effects of a positive demand shock, the fall in $x^I$ will be interpreted partially as the central bank’s reaction to a positive cost shock. Firms will revise their expectations about the cost shock and about the output gap, and, as a result, actual inflation ends up being affected by the demand shock. If the central bank announces its output-gap target $x^T$, the private sector has two public signals ($x^I$ and $x^T$) from which it will generally be able to disentangle the central bank’s forecasts of the aggregate cost shock $E_t^{cb} s_t$ from the central bank’s forecast of the aggregate demand shock $E_t^{cb} v_t$. 
Intuitively, one would expect that announcing the central bank’s output-gap target would improve economic outcomes.\footnote{As noted previously, this is equivalent to announcing an inflation target.} Since private firms are now able to distinguish between interest rate movements that are designed to offset demand disturbances and those reflecting the central bank’s estimate of the cost shock, the central bank could neutralize demand shocks without introducing any volatility into the inflation rate. At the same time, releasing information on $x_t^T$ in no way hampers the central bank’s ability to achieve its output-gap target. Thus, greater transparency should improve welfare. However, providing more public information may make private-sector expectations more sensitive to the announced target than they were to the instrument. Consequently, any errors the central bank makes in forecasting the cost shock will generate greater volatility in the inflation rate. If this channel dominates the reduction in volatility that occurs because demand shocks no longer affect inflation, loss can actually rise when targets are announced. Whether transparency reduces or increases loss will depend on the quantitative characteristics of the economy.

Rather than comparing the case of no announcement with the case in which all firms have information on the output-gap target, I consider the partial release of information along the lines of Cornand and Heinemann (2004). Suppose the central bank announces $x_t^T$ in a manner such that only a fraction $P$ of all firms receive the information.\footnote{One might interpret this partial release of information in terms of the notion of rational inattention emphasized by Mankiw and Reis (2002). Perhaps all firms observe the announcement but only a fraction $P$ actually incorporate the new information into their decisions.} Firms will be in one of three classes each period: (i) those that do not receive an opportunity to adjust their price, (ii) those that do adjust but do not receive the central bank’s announcement, and (iii) those that adjust and receive the announcement. Consider first those adjusting firms that receive information about $x_t^T$. There are a fraction $P$ of such firms. For these informed firms, their expectations of the current shocks will depend on their private information, on the central bank’s instrument setting, and on the announced output-gap target. For the $1 - P$ fraction of adjusting firms that do not observe $x_t^T$, expectations can be based only on private signals and the central
bank’s instrument. Firms that adjust prices in period $t$ must form expectations about what other firms are expecting, and this will now depend on the fraction of firms that receive information about the central bank’s output-gap target.

### 3.1 Expectations

The information problems faced by informed and uninformed firms differ. Consider first those firms that receive information about $x^T_t$. These firms observe $s_{jt}, v_{jt}, x^I_t$, and the central bank’s output-gap target $x^T_t$. Let $j$ index such a firm. The new information for informed firm $j$ is

$$\zeta_{jt} = \begin{bmatrix} s_{jt} - E_{t-1} s_{jt} \\ v_{jt} - E_{t-1} v_{jt} \\ x^I_t - E_{t-1} x^I_t \\ x^T_t - E_{t-1} x^T_t \end{bmatrix} = \begin{bmatrix} \xi_t + \phi_{jt} \\ \varphi_t + \psi_{jt} \\ \xi_t + \phi_{cb,t} \\ \varphi_t + \psi_{cb,t} \end{bmatrix}$$

where

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \delta_2 \theta_s & \delta_3 \theta_v \\ 0 & 0 & \delta_2 \theta_s (1 + \delta_3) \theta_v & \delta_1 \end{bmatrix}.$$

Define $Z'_t = [s_t \ v_t \ x_t]$ and $\Omega'_t = [\xi_t \ \varphi_{cb,t} \ \psi_{cb,t}]$. We can write the processes for the exogenous shocks and the output gap as

$$Z_t = CZ_{t-1} + D\Omega_t,$$

where

$$C = \begin{bmatrix} \rho_s & 0 & 0 \\ 0 & \rho_v & 0 \\ \delta_2 \rho_s & (1 + \delta_3) \rho_v & \delta_1 \end{bmatrix}.$$
and

\[
D = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
\delta_2 \theta_s^{cb} & (1 + \delta_3 \theta_v^{cb}) & \delta_2 \theta_s^{cb} & \delta_3 \theta_v^{cb}
\end{bmatrix}.
\]

Now let \( V_{\zeta \Omega} \) be the \( 4 \times 4 \) covariance matrix between \( \zeta_{jt} \) and the unobserved variables \( \Omega_t \), and let \( V_{\zeta \zeta} \) be the \( 4 \times 4 \) covariance matrix of \( \zeta_{jt} \). Then firm \( j \)'s expectation of \( \Omega_t \) is equal to

\[
E_j^{jt} \Omega_t = H \zeta_{jt},
\]

where \( H = V_{\Omega \zeta} V_{\zeta \zeta}^{-1} \).

Those firms that do not receive the announcement (the uninformed firms), denoted by \( h \), must base their expectations about current aggregate shocks on their private signals and the central bank's instrument. We can write the information of these firms as

\[
z_{ht} = W \zeta_{ht},
\]

where

\[
W = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}.
\]

Hence, for these firms,

\[
E_h^{ht} \Omega_t = GW \zeta_{ht},
\]

where \( G = V_{\Omega \zeta} W' (WV_{\zeta \zeta} W')^{-1} \).

In Morris and Shin (2002), Amato and Shin (2003), and Hellwig (2004), the weights placed on private and public information in the individual firm’s forecast are independent of any aspect of the central bank’s policy decisions. This is not true in the present case, because the public signals are the central bank’s instrument and, for a subset of firms, the central bank’s output-gap target. Thus, both \( H \) and \( G \) will depend on the policy parameters \( \delta_i \).
Finally, because the ideiosyncratic firm information averages to zero across firms, define the aggregate information (over all firms) as

$$\zeta_t \equiv M \begin{bmatrix} \xi_t \\ \varphi_t \\ \xi_t + \phi_{cb,t} \\ \varphi_t + \psi_{cb,t} \end{bmatrix} = L \Omega_t,$$

where

$$L = M \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}.$$ 

### 3.2 Inflation and the Output Gap

As detailed in the appendix, the equilibrium strategy of informed firms, those receiving the central bank’s announcement, is given by

$$\pi_{j,t}^* = a_{i,1}Z_{t-1} + a_{i,2}\zeta_{j,t}.$$ 

The equilibrium strategy for an uninformed firm is

$$\pi_{h,t}^* = a_{u,1}Z_{t-1} + a_{u,2}W\zeta_{h,t}.$$ 

Note that while $a_{i,2}$ in (13) is $1 \times 4$, $a_{u,2}$ in (15) is $1 \times 3$. Since $Z_{t-1}$ is common information to both types of firms, $a_{i,1} = a_{u,1} \equiv a_1$. The appendix shows that

$$a_1 = \left(1 - \frac{\omega \beta}{\omega}\right) [\kappa e_3 + e_1]C(I_3 - \beta C)^{-1}.$$  

(11)

Given $a_1$, the appendix shows how the equilibrium values of $a_{i,2}$ and $a_{u,2}W$ can be found.

Once $a_1$, $a_{i,2}$, and $a_{u,2}$ have been obtained, equilibrium inflation is given by

$$\pi_t = (1 - \omega) \left[ P \int \pi_{j,t}^* dj + (1 - P) \int \pi_{h,t}^* dh \right]$$

$$= (1 - \omega) [a_1Z_{t-1} + (Pa_{i,2} + (1 - P)a_{u,2}W)L\Omega_t],$$
while the equilibrium output gap is

\[ x_t = e_3(CZ_{t-1} + D\Omega_t), \]

where \( e_3 = [0 \ 0 \ 1] \).

4. Results

To explore the impact of transparency on the behavior of inflation and the output gap, the model is numerically solved. I set \( \omega = 0.5 \), \( \kappa = 1.8 \), and \( \beta = 0.99 \). A value of 0.5 for \( \omega \) is consistent with evidence on the frequency of price adjustment in the United States (Bils and Klenow 2004). In microfounded models, \( \kappa \) is the sum of the coefficient of relative risk aversion and the inverse of the wage elasticity of labor supply. Values of 1 for relative risk aversion and 0.8 for the inverse of the wage elasticity of labor supply are not uncommon in the literature, yielding \( \kappa = 1.8 \). The value chosen for the discount factor \( \beta \) is standard when dealing with quarterly data. I set the variances of the cost and demand shocks equal to each other and normalize so that \( \sigma_\xi^2 = \sigma_\phi^2 = 1 \). For the benchmark case, I assume that the private-sector noise variances \( \sigma_{\phi,j}^2 \) and \( \sigma_{\psi,j}^2 \) both equal 0.4. While Amato and Shin (2003) assume that the central bank has perfect information on the shocks, I assume that the noise variances in the central bank’s signals \( \sigma_{\phi,cb}^2 \) and \( \sigma_{\psi,cb}^2 \) also equal 0.4. For the baseline case, I set \( \rho_s = \rho_v = 0 \).

4.1 Policy Incentive Effects

In a standard New Keynesian model of optimal monetary policy with a loss function given by (7), the central bank would neutralize demand shocks to prevent them from affecting either the output gap or inflation. The central bank would partially stabilize inflation from the effects of cost shocks. Thus, both inflation and the output gap would fluctuate in the face of cost shocks, while neither would move in response to demand shocks. If the central bank faces a signal extraction problem, certainty equivalence still holds, and the central bank would offset expected demand shocks completely (i.e., \( \delta_3 = -1 \)) and stabilize in response to expected cost shocks (i.e., \( \delta_2 < 0 \)).
In the present model, a lack of transparency has what Geraats (2002) labels an incentive effect on policy. Suppose the central bank attempts to fully insulate the output gap from demand shocks. As it moves its instrument in response to forecasts of demand shocks, private agents will attribute some of the change in $x^I$ as due to cost shocks. This will result in firms altering their assessment of the aggregate cost shock, and inflation will be affected. Inflation is not fully insulated from demand shocks when $\delta_3 = -1$, and, as a consequence, the central bank will no longer find it optimal to set $\delta_3 = -1$.

Because firms partially attribute movements in $x^I$ to the central bank’s forecast of a cost shock, there are actually three effects of a change in $x^I$ on inflation. First, firms will use the information they extract from $x^I$ to reassess their expectations about the aggregate cost shock and therefore about what they expect other price-adjusting firms to do. Second, any reassessment of the aggregate cost shock will affect expectations of future inflation. Third, firms will alter their expectations about the aggregate output gap. This directly affects price-adjusting firms’ decisions about their own price and it alters such firms’ expectations about the prices other firms are setting.

Under a regime of complete transparency, the central bank announces its target to all firms. Private agents can now infer the central bank’s forecast of demand and cost shocks. By setting $\delta_3 = -1$, the central bank neutralizes the expected effect of demand shocks on both inflation and the output gap; the resulting movements in its instrument are no longer confused with responses to the cost shock. This should make inflation more stable, since expected demand shocks are completely neutralized. Thus, transparency can make both inflation and the output gap more stable.

However, once the central bank announces its output-gap target to all firms, inflation can become very sensitive to the central bank’s target. The increased volatility of expectations in the face of additional information is a standard cost of transparency (Geraats 2002). Any noise in the central bank’s cost-shock signal will now have a greater impact on inflation. If expectations and inflation react strongly to the central bank’s announced output-gap target, and therefore to any noise in the central bank’s estimate of the cost shock, inflation could become more volatile. In addition, because the
central bank reacts more strongly to its signal on demand shocks, any noise in that signal will have a bigger impact on the output gap. Walsh (2006) discussed the effects of transparency (as measured by $P$) on the optimal responses of policy to cost and demand shocks. In the present model, for example, $\delta_3 = -0.95$ when $P = 0$; the central bank does not fully offset expected demand shocks because the movements in $x^t$ needed to do so lead to excessive fluctuations in inflation. When $P = 1$, the optimal value of $\delta_3$ is $-1$, and expected demand shocks are fully offset. Thus, incentive effects are present but small.

Let $\delta^*(P)$ denote the policy coefficients optimized for a given $P$. For example, $\delta^*(1)$ would denote the policy rule optimized for complete transparency, and $\delta^*(0)$ is the policy rule optimized for the case of no announcements. The importance of accounting for changes in the optimal policy rule as the degree of transparency varies is illustrated in table 1. A switch from a regime with no announcements to one of full transparency increases loss as measured by (7) if the policy rule remains fixed at $\delta^*(0)$. Given the structure of the model, transparency has no effect on the variance of the output gap as long as the policy rule remains unchanged. With policy fixed at $\delta^*(0)$, however, transparency results in greater inflation rate volatility, and this accounts for the rise in loss. Inflation volatility rises because the additional information contained in $x^T_t$ makes firms’ expectations about $x_t$ and $\pi^*_t$ more volatile. The optimal policy rule, $\delta^*(1)$, involves a smaller (in absolute value) response to the central bank’s signal on cost shocks, $|\delta_2^*(1)| = 0.5205 < 0.5964 = |\delta_2^*(0)|$, and this tempers the volatility of private-sector expectations. Inflation volatility still rises with $P = 1$ and $\delta = \delta^*(1)$, but this is compensated by the fall in output-gap volatility as the central bank reacts less to cost shocks and fully stabilizes the output gap from expected demand shocks.

<table>
<thead>
<tr>
<th>Table 1. Effects of Policy Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P = 0$</td>
</tr>
<tr>
<td>$P = 1$</td>
</tr>
<tr>
<td>$P = 1$</td>
</tr>
</tbody>
</table>
Table 2. Effects of Policy Rule: $\rho_s = 0.8$

<table>
<thead>
<tr>
<th>$P$</th>
<th>$\delta^*(0)$</th>
<th>$\sigma^2_\pi$</th>
<th>$\sigma^2_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>11.04</td>
<td>0.66</td>
<td>1.10</td>
</tr>
<tr>
<td>1</td>
<td>11.53</td>
<td>0.74</td>
<td>1.10</td>
</tr>
<tr>
<td>1</td>
<td>11.13</td>
<td>0.72</td>
<td>1.06</td>
</tr>
</tbody>
</table>

As a consequence, loss declines with full transparency as long as the central bank correctly optimizes its policy rule to reflect the new level of transparency. Note, however, that even though loss is reduced under transparency (as long as policy also adjusts), inflation is more volatile than it was without any announcements.

When disturbances are serially correlated, information that alters agents’ expectations about current aggregate shocks will also affect their forecasts of future values of the disturbances and future inflation. This generates additional effects on inflation since current inflation depends on expected future inflation. Table 2 illustrates how persistence in the aggregate cost shock affects outcomes under the extreme cases of no announcements and complete announcements. In contrast to the baseline case with $\rho_s = 0$, loss is lower when the output-gap target is not announced.

In contrast, adding persistence to the demand shock makes transparency superior to opaqueness. In fact, when both aggregate shocks are persistent as in table 3, based on $\rho_s = \rho_v = 0.8$, loss is reduced when the central bank announces its output-gap target even if the policy rule is held fixed at $\delta^*(0)$. Transparency allows the central bank to completely insulate the output gap and inflation from demand shocks. Doing so is particularly important when demand shocks are serially correlated; otherwise, a demand shock affects the output gap and inflation directly as well as by altering expected future inflation.

The results reported in tables 1–3 illustrate the importance of allowing the policy rule to vary optimally when the degree of

---

15 From (11), the vector $a_1$ depends on the matrix $C$ giving the effects of $Z_{t-1}$ on $Z_t$, and $a_{i,2}$ and $a_{u,2}$ depend on $a_1$ (and so therefore on $C$). See the appendix for details.
transparency changes. They show, too, how the value of transparency can be affected by the persistence in the aggregate shocks. Finally, tables 2 and 3 reveal that demand and cost shocks can have asymmetric effects on the desirability of transparency. Persistence in the cost shock lowers the value of transparency; persistence in demand shocks raises it.

4.2 The Optimal Degree of Transparency

In this section, the optimal degree of partial transparency is investigated. Reported outcomes for different degrees of transparency are always evaluated using the policy rule coefficients that are optimal for the particular value of $P$.\(^\text{16}\)

The solid line in figure 1 shows the percentage change in loss relative to the case of no announcement (i.e., the case of $P = 0$) as a function of $P$ for the baseline parameter values. While loss is lower with complete transparency ($P = 1$) than it is in the absence of any announcements, the optimum occurs when $P = 0.725$. That is, it is optimal to be fairly transparent but not completely transparent.

Also shown in figure 1 is loss as a function of $P$ when the disturbances are serially correlated. The case of a serially correlated demand shock ($\rho_v = 0.8$) is shown by the dashed line with circles in the figure. The optimal degree of transparency increases (the optimal $P$ increases from 0.725 to 0.825) when demand shocks are persistent. In contrast, as shown by the dotted line with diamonds, introducing serial correlation in the cost shock ($\rho_s = 0.8$) decreases the optimal degree of transparency (the optimal $P$ decreases from 0.725 to 0.5).

The reason for these differing effects on optimal transparency can be see from figure 2, which plots the variances of inflation and

\(^{16}\)That is, outcomes for each $P$ are always evaluated using the policy $\delta^*(P)$. 

<table>
<thead>
<tr>
<th>$P$</th>
<th>Loss</th>
<th>$\sigma_\pi^2$</th>
<th>$\sigma_x^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\delta^*(0)$</td>
<td>11.28</td>
<td>0.74</td>
</tr>
<tr>
<td>1</td>
<td>$\delta^*(0)$</td>
<td>11.22</td>
<td>0.73</td>
</tr>
<tr>
<td>1</td>
<td>$\delta^*(1)$</td>
<td>11.13</td>
<td>0.73</td>
</tr>
</tbody>
</table>

Table 3. Effects of Policy Rule: $\rho_s = \rho_v = 0.8$
the output gap as a function of $P$ for the baseline parameters (no markers), $\rho_s = 0.8$ (indicated by diamonds), and $\rho_v = 0.8$ (indicated by circles). Consider first the case of a serially correlated cost shock. By increasing transparency, the central bank provides firms with information that can be useful in forecasting the current aggregate cost shock. When $\rho_s \neq 0$, this information is also useful for forecasting future $s_{t+i}$ and therefore future inflation. As expectations fluctuate in response to the greater information provided with announcements, current inflation becomes more volatile. As indicated by the figure, inflation becomes significantly more variable as $P \to 1$ when $\rho_s = 0.8$. This places a limit on how transparent the central bank wants to be.

Now consider the situation when the demand shock is serially correlated. Transparency allows the central bank to more fully neutralize the impacts of demand shocks. When these shocks are serially correlated, it becomes more important to offset them since the impact on current inflation depends on the present discounted value of any current and future demand shock that is not offset by policy.
As shown in figure 2, the variance of the output gap is reduced considerably relative to the $P = 0$ case, as $P \to 1$ when $\rho_v = 0.8$, while the variance of inflation is similar when $P = 0$ and $P = 1$.

4.3 The Effects of Central Bank Noise

Morris and Shin (2002) suggested that more-accurate central bank information could reduce welfare by making private expectations too sensitive to the noise in the information. In the present model (and consistent with Svensson 2006), reductions in the variances of the noise in the central bank’s signals about the aggregate shocks always reduce loss. However, more-accurate central bank signals can have ambiguous effects on the optimal degree of transparency.

Table 4 shows how the optimal degree of transparency varies with the noise in the central bank’s signals, holding constant the variance of the true aggregate shock. The upper half of the table shows that increased noise in the central bank’s signal on the cost shock decreases optimal transparency. As $\sigma_{\phi,cb}^2$ increases, the central
bank’s ability to engage in active stabilization is reduced. A less transparent regime limits the volatility of inflation expectations by reducing the public information provided by the central bank. This effect is stronger when the central bank has a more-accurate signal on demand disturbances in that the optimal $P$ is lower for any given $\sigma^2_{\phi,cb} > 0$. A lower $\sigma^2_{\psi,cb}$ implies that the central bank is less concerned with limiting the impact on expectations of its demand forecast errors since these errors are smaller. Being less transparent reduces the effects on inflation of noise in the central bank’s signal on cost disturbances.

The effects of altering the informational content of the central bank’s signal on the demand disturbances are quite different. The bottom half of table 4 shows that optimal transparency increases when the central bank’s signal on demand shocks contains more noise (i.e., when $\sigma^2_{\psi,cb}$ increases). Recall that in the absence of transparency, central bank errors in forecasting demand spill over to affect inflation. As these errors become larger, it is optimal to become more transparent to limit their impact on inflation. This effect is stronger when the noise in the central bank’s cost signal is reduced from the baseline case of $\sigma^2_{\psi,cb} = 0.4$ to a value of 0.2. With better information on cost shocks, the central bank engages in more-active stabilization. The gains to reducing private-sector confusion about the central bank’s information rise, leading to an increase in the optimal degree of transparency for any given value of $\sigma^2_{\psi,cb}$ until

Table 4. Optimal Transparency as Function of Noise Variances

<table>
<thead>
<tr>
<th>$\sigma^2_{\phi,cb}$</th>
<th>0</th>
<th>.2</th>
<th>.4</th>
<th>.6</th>
<th>.8</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma^2_{\psi,cb} = 0.2$</td>
<td>1.0</td>
<td>0.93</td>
<td>0.40</td>
<td>0.20</td>
<td>0.10</td>
<td>0.03</td>
</tr>
<tr>
<td>$\sigma^2_{\psi,cb} = 0.4$</td>
<td>1.0</td>
<td>1.00</td>
<td>0.73</td>
<td>0.48</td>
<td>0.33</td>
<td>0.23</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\sigma^2_{\psi,cb}$</th>
<th>0</th>
<th>.2</th>
<th>.4</th>
<th>.6</th>
<th>.8</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma^2_{\phi,cb} = 0.2$</td>
<td>0.58</td>
<td>0.93</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$\sigma^2_{\phi,cb} = 0.4$</td>
<td>0.15</td>
<td>0.40</td>
<td>0.73</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>
transparency is complete. Thus, consistent with the results on serial correlation, the impact of noise on optimal transparency differs depending on the source.

5. Summary

In this paper, I have investigated the role of economic transparency when private information is diverse and the central bank provides public information either implicitly, by setting its policy instrument, or explicitly, by making announcements about its short-run targets. In contrast to earlier work that interpreted transparency as a reduction in the central bank’s control error, I model transparency as the extent to which announcements are disseminated among the public. Being transparent is not an all-or-nothing proposition. Partial announcements provide one means of investigating how widely central banks should disseminate information about their targets. Under full transparency, the central bank’s announced target reaches all firms.

By announcing its short-run output-gap target (equivalently, its short-run inflation target), the central bank reveals information about its internal forecast of demand and cost shocks. This provides more-accurate public information to price-setting firms, but it also makes private-sector decisions more sensitive to the central bank’s forecast errors. As a result, inflation may become more volatile when the central bank announces its short-run target.

The degree of optimal transparency is affected differently by demand and cost disturbances. When the central bank’s forecasts of cost disturbances improve, or such disturbances become less persistent, optimal transparency increases. In contrast, when the central bank’s forecasts of demand disturbances improve, or such disturbances become less persistent, optimal transparency decreases.

To determine the optimal extent to which information should be made public, I employed a standard quadratic loss function. As Hellwig (2004) demonstrates, this can be misleading and will tend to undervalue the gains from transparency. The reason is based on the underlying distortion that makes inflation costly in New Keynesian models. These costs are due to the increase in price dispersion across firms that inflation generates. When firms have private information,
this introduces a new source of price dispersion and exacerbates the welfare costs of inflation. By providing information that is common to all firms, the central bank can reduce the extent of price dispersion. This represents a welfare gain. In terms of the model of partial announcements, employing an explicit welfare criterion is likely to increase the optimal degree of transparency.

Appendix

The pricing decision of an informed firm satisfies

\[
\pi_{j,t}^* = (1 - \omega)E_t^j \pi_t^* + (1 - \omega \beta)\kappa E_t^j x_t + (1 - \omega \beta) s_{jt} + \frac{\omega \beta}{1 - \omega} E_t^j \pi_{t+1},
\]

where expectations are with respect to the information set \( \{Z_{t-1}, \zeta_{j,t}\} \). Assume the equilibrium strategy for an informed firm is

\[
\pi_{j,t}^* = a_{i,1} Z_{t-1} + a_{i,2} \zeta_{j,t}.
\]

The pricing decision of an uninformed firm satisfies

\[
\pi_{h,t}^* = (1 - \omega)E_t^h \pi_t^* + (1 - \omega \beta)\kappa E_t^h x_t + (1 - \omega \beta) s_{ht} + \frac{\omega \beta}{1 - \omega} E_t^h \pi_{t+1},
\]

where expectations are with respect to the information set \( \{Z_{t-1}, W\zeta_{h,t}\} \). Assume the equilibrium strategy for an uninformed firm is

\[
\pi_{h,t}^* = a_{u,1} Z_{t-1} + a_{u,2} W\zeta_{h,t}.
\]

Note that while \( a_{i,2} \) in (13) is \( 1 \times 4 \), \( a_{u,2} \) in (15) is \( 1 \times 3 \).

The strategies (13) and (15) will be used by all adjusting firms in forming expectations about \( \bar{\pi}_t^* \), since

\[
\bar{\pi}_t^* = P \int \pi_{j,t}^* dj + (1 - P) \int \pi_{h,t}^* dh = \alpha_1 Z_{t-1} + \alpha_2 \zeta_t,
\]
where

\[ \alpha_1 = Pa_{i,1} + (1 - P)a_{u,1} \]
\[ \alpha_2 = Pa_{i,2} + (1 - P)a_{u,2}W. \]

Hence, for firms that observe \( x_t^T \),

\[ E_t^j \pi_t^* = \alpha_1 Z_{t-1} + \alpha_2 E_t^j \zeta_t = \alpha_1 Z_{t-1} + \alpha_2 LH\zeta_{j,t}, \]

while for firms that do not observe \( x_t^T \),

\[ E_t^h \pi_t^* = \alpha_1 Z_{t-1} + \alpha_2 E_t^h \zeta_t = \alpha_1 Z_{t-1} + \alpha_2 LGW\zeta_{h,t}. \]

Actual inflation will be

\[ \pi_t = (1 - \omega)\pi_t^* = (1 - \omega)(\alpha_1 Z_{t-1} + \alpha_2 \zeta_t). \tag{16} \]

Equation (16) implies that next-period inflation satisfies

\[ \pi_{t+1} = (1 - \omega)\pi_{t+1}^* = (1 - \omega)(\alpha_1 Z_t + \alpha_2 \zeta_{t+1}), \]

and so for informed firms,

\[ E_t^j \pi_{t+1} = (1 - \omega)\alpha_1 E_t^j Z_t = (1 - \omega)\alpha_1 (CZ_{t-1} + DH\zeta_{j,t}), \]

where (10) and \( E_t^j \Omega_t = H\zeta_{j,t} \) have been used. Similarly, for uninformed firms,

\[ E_t^h \pi_{t+1} = (1 - \omega)\alpha_1 E_t^h Z_t = (1 - \omega)\alpha_1 (CZ_{t-1} + DGW\zeta_{h,t}). \]

Firms must also forecast the output gap. Since \( x_t = e_3 Z_t \), where \( e_3 = [0 0 1] \), \( E_t^j x_t = e_3(CZ_{t-1} + DH\zeta_{j,t}) \) and \( E_t^h x_t = e_3(CZ_{t-1} + DGW\zeta_{h,t}). \)
Substituting the expressions for $E_j^t \bar{\pi}_t$, $E_j^t x_t$, and $E_j^t \pi_{t+1}$ into the price equation for informed firms (equation (12)),

\[
\pi_j^* = (1 - \omega)(\alpha_1 Z_{t-1} + \alpha_2 LH \zeta_{j,t}) + (1 - \omega \beta)\kappa e_3 (CZ_{t-1} + DH \zeta_{j,t}) \\
+ (1 - \omega \beta)(e_1 CZ_{t-1} + \bar{e}_1 \zeta_{j,t}) + \omega \beta \alpha_1 (CZ_{t-1} + DH \zeta_{j,t}),
\]

where $e_1 = [1 \ 0 \ 0]$ and $\bar{e}_1 = [1 \ 0 \ 0 \ 0]$.

Equating coefficients with those in (13),

\[
a_{i,1} = (1 - \omega)\alpha_1 + (1 - \omega \beta)[\kappa e_3 + e_1]C + \omega \beta \alpha_1 C,
\]

and

\[
a_{i,2}[I_4 - (1 - \omega)PLH] - (1 - P)(1 - \omega)a_{u,2}WLH \\
= (1 - \omega \beta)[\kappa e_3 DH + \bar{e}_1] + \omega \beta \alpha_1 DH.
\]

Turning to the uninformed firms, substituting the expressions for $E_h^t \bar{\pi}_t$, $E_h^t x_t$, and $E_h^t \pi_{t+1}$ into the price equation for informed firms (equation (14)) yields

\[
\pi_h^* = (1 - \omega)\alpha_1 Z_{t-1} + \alpha_2 LGW \zeta_{h,t} + (1 - \omega \beta)\kappa e_3 \\
\times (CZ_{t-1} + DGW \zeta_{h,t}) + (1 - \omega \beta)(e_1 CZ_{t-1} + e_1 W \zeta_{h,t}) \\
+ \omega \beta \alpha_1 (CZ_{t-1} + DGW \zeta_{h,t}).
\]

Equating coefficients with (15),

\[
a_{u,1} = (1 - \omega)\alpha_1 + (1 - \omega \beta)[\kappa e_3 + e_1]C + \omega \beta \alpha_1 C,
\]

and

\[
a_{u,2}W[I_4 - (1 - \omega)(1 - P)LGW] - (1 - \omega)Pa_{i,2}LGW \\
= (1 - \omega \beta)[\kappa e_3 DG + e_1]W + \omega \beta \alpha_1 DGW.
\]

Notice that the right-hand sides of (17) and (19) are the same. Therefore,

\[
a_1 \equiv a_{i,1} = a_{u,1} = \alpha_1.
\]
Taking the $P$-weighted average of (17) and (19) and solving for $a_1$, 

$$a_1 = \left( \frac{1 - \omega \beta}{\omega} \right) \left[ \kappa e_3 + e_1 \right] C(I_3 - \beta C)^{-1}. \quad (22)$$

Given $a_1$, (18) and (20) can be solved for $a_{i,2}$ and $a_{u,2}$ (recall that $a_{i,2}$ is $1 \times 4$, while $a_{u,2}$ is $1 \times 3$). Define 

$$\bar{a} = [a_{i,2}, a_{u,2}W]$$

as a $1 \times 8$ vector of the unknown coefficients whose last element is equal to zero. Then 

$$\bar{a} = (1 - \omega \beta) \left[ \kappa e_3 DH + e_1 \right] \left[ \kappa e_3 DG + e_1 \right] W \left[ \begin{array}{cc} A_{11} & A_{21} \\ A_{12} & A_{22} \end{array} \right]^{-1}$$

$$+ \omega \beta \alpha_1 \left[ DH \quad DGW \right] \left[ \begin{array}{cc} A_{11} & A_{21} \\ A_{12} & A_{22} \end{array} \right]^{-1},$$

where 

$$A_{11} = [I_4 - (1 - \omega)PHL],$$
$$A_{21} = -(1 - P)(1 - \omega)LH,$$
$$A_{12} = -(1 - \omega)PLGW,$$

and 

$$A_{22} = [I_4 - (1 - \omega)(1 - P)LGW].$$

References


