Negative Transfer in Matchstick Arithmetic Insight Problems

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Abstract

The current experiment examined whether successful solution on one type of problem, indicating the relaxation of a constraint, had a negative impact on subsequent problems that did not involve the same constraints. One hundred and forty-five participants solved a series of matchstick arithmetic problems. In one group, participants were given three relatively simple “chunk decomposition” problems (CD). A second group solved one “operator decomposition” (OD) problem, involving more constraints, between the baseline CD problem and two later problems. The third group solved three OD problems, similarly placed. Results indicated that successful solution of an OD problem produced negative transfer to subsequent CD problems in the form of longer solution times. Participants who did not successfully solve OD problems did not slow down on subsequent problems; they displayed evidence of positive transfer. The findings were interpreted with reference to theories of constraint relaxation and its relationship to problem solving performance.

Keywords: mental set; insight problem solving; negative transfer

Procedures in Problem Solving

Whenever new problems are encountered in everyday life, our general approach is to apply procedures or solutions that produced successful outcomes in the past. For example, in the event that you are moving a couch to a new apartment, it is possible that the couch will not fit through a particular doorway or up a particular flight of stairs. Solutions that worked in the past were to unscrew the feet on the couch, or to try the other stairway/doorway into the apartment. If we try one of these solutions and it works again, we will likely bring them to bear when similar situations arise in the future.

Similarly, procedures or solutions that have worked in one context are often evoked and applied to another context. In the first author’s most recent move, her bookcase was too large to fit up the front staircase, but could be brought up the back staircase with little trouble. Because the try-the-other-staircase procedure worked in a different situation, it may be that the procedure becomes generalized, thus making it more likely to be employed in a variety of situations.

However, known procedures do not always apply to new situations, and, in fact, may lead to situations of impasse. To use one final example, again from the first author’s most recent move, her box spring would not fit up the front staircase. The back staircase was next attempted with no success. She and her movers tried rotating the box spring in multiple orientations on each staircase to no avail. Over an hour was spent attempting to apply a known procedure that was not leading to any progress. Eventually, a neighbor suggested using a chainsaw to split the box spring and fold it in half. The chainsaw procedure was used and the box spring entered the apartment. Thus, a successful procedure was applied, but only after lengthy misapplications of known, and previously useful, procedures.

The preceding everyday example of misapplying previously successful procedures to the moving of furniture is analogous to the sequence that occurs when solving insight problems. An individual’s initial representation of an insight problem is often faulty because unhelpful prior knowledge and experiences are activated by the problem (Kershaw & Ohlsson, 2004; Knoblich, Ohlsson, Haider, & Rhenius, 1999; Ohlsson, 1992). The individual’s initial problem-solving attempts are guided by this unsuitable knowledge. These initial attempts are usually unsuccessful and the individual then enters a period of impasse, in which no overt problem-solving behavior occurs. In order to exit the impasse, the individual must relax constraints (Knoblich et al., 1999; Ohlsson, 1992) or overcome mental sets caused by incorrect application of procedures (Luchins, 1942). The likelihood of relaxing constraints or breaking mental set depends on the number and strength of the constraints or procedures.

Constraint Relaxation and Breaking Mental Set

The difficulty of a particular insight problem is dependent upon several factors. For many insight problems, including famous examples such as the nine-dot problem, the necklace...
problem, and the four tree problem, multiple types of constraints interact to make the achievement of solution difficult (Kershaw & Ohlsson, 2004). For example, Kershaw and Ohlsson identified perceptual (figural integrity and other Gestalt laws), knowledge (prior experiences and knowledge), and process (size and variability of search space) constraints that prevent solution of the nine-dot problem. Likewise, Flynn, Gordon, and Kershaw (2010) identified perceptual and knowledge constraints in the four tree problem.

Several researchers state that the difficulty of a particular problem can be found in the strength of constraints present in a particular problem. For example, Knoblich et al. (1999) identified three types of constraints in matchstick arithmetic problems: value, operator, and tautology. The value constraint, the weakest of the three, suggests that numerical values on one side of an equation cannot be changed without compensatory changes on the other. The operator constraint, which is described as having a moderate level of strength, signifies that arithmetic functions (operators) cannot be arbitrarily changed. The tautology constraint, which is the strongest of the three, signifies that arithmetic equations should follow a particular format in which a calculation is specified. That is, an arithmetic operation on one side of the equation should indicate a value on the other side of an equation, such as $V + I = VI$. While statements like $II = II = II$ are valid, they are not common in arithmetic and therefore violate the tautology constraint.

Knoblich et al. (1999) also classified the difficulty of matchstick arithmetic insight problems by the strength of the chunks that had to be decomposed in order to solve the problem. People tend to view Roman numerals as perceptual chunks, but the strength of these particular chunks depends on the numeral or other element of the equation. Tight chunks, such as $V$ and $I$, are composed of single units. Loose chunks, such as $VII$ and $III$, are composed of other chunks. For example, $VII$ is composed of three tight chunks, $V$, $I$, and $I$. Knoblich et al. also suggest there are intermediate chunks, such as operators like the plus sign (+) and the equal sign (=). Although these symbols are composed of other chunks, people are unlikely to have experience decomposing a $+$ into its horizontal and vertical components, for example.

A different explanation of the difficulty of a particular problem is the success of the procedures applied to the problems that preceded it. In a classic demonstration of mental set, Luchins (1942) gave participants a series of water jug problems. The first five problems could all be solved successfully using a particular procedure, but the last five problems either could not be solved using the known procedure or could be solved using a simpler procedure. Luchins (1942) found that participants continued to apply the known procedure to the last five problems, and that over half of the participants were unable to solve problems for which the known procedure could not be applied. That is, participants experienced impasse on some problems and were unable to break impasse to reach solution.

Thus, the difficulty of particular insight problems may be due to the number and strength of constraints or procedures present. Likewise, the likelihood of relaxing these constraints or procedures should also be affected by number and strength. Knoblich et al.’s (1999) theory presupposes that relaxing one weak constraint will be much easier than relaxing multiple strong constraints. Researchers have implemented experimental interventions to increase the likelihood of constraint and procedure relaxation. For example, Kershaw and Ohlsson (2004) and Flynn et al. (2010) developed training procedures that targeted particular constraints, such as practicing non-dot turns for the nine-dot problem (Kershaw & Ohlsson) or comparing solved analogs of the four tree problem (Flynn et al.). Luchins and Luchins (1950) tried to prevent mental set by limiting the amount of liquid available, adding a fourth jar to the problems, and giving participants physical objects (actual jars and water) instead of using paper-and-pencil forms of the problems. Of these three manipulations, only adding a fourth jar was successful, because participants needed to figure out the amount that each jar could hold for each problem. Luchins and Luchins’ other two manipulations did not work because participants were poor at keeping track of how much liquid they had used or they persisted in doing paper-and-pencil calculations prior to using the physical materials.

**The Current Experiment**

In the current experiment, we examine the connection between the strength of constraints and the effect of mental set by using matchstick arithmetic insight problems of two types. One type of problem we used required the decomposition of loose chunks. For example, to solve $VI = VII + I$, a participant needs to move a single matchstick (I) from VII to VI, thus making the solution of the problem $VI = VI + I$. Knoblich et al. (1999) states that these types of problems require the relaxation of the value constraint and the decomposition of loose chunks. For simplicity sake, we refer to these problems as chunk decomposition (CD) problems.

The second type of problem we used required the decomposition of the operator in the problem, in this case the plus sign (+). For example, to solve $VII = VII + I$, a participant needs to move the vertical matchstick from the $+$ to the second VII, thus making the solution of the problem $VII = VIII − I$. Knoblich et al. (1999) note that these type of problems require the relaxation of the value and operator constraints as well as the decomposition of loose and intermediate chunks. We refer to these problems as operator decomposition (OD) problems. Although Knoblich et al. make a conceptual distinction between constraint relaxation and chunk decomposition mechanisms, we group both into the general category of constraints in the current work. Thus, CD problems contain two constraints and OD problems contain four. Because OD problems contain a greater number of constraints, as well as stronger constraints, they should be harder to solve than CD
problems as well as require longer solution times, predictions that are supported by Knoblich et al.’s findings.

In this experiment, all participants solve three CD problems. Participants differed in the number of OD problems that they received. A baseline group of participants did not receive any OD problems, a second group received one OD problem, and a third group received three OD problems. The groups that received the OD problem(s) solved one CD problem, the OD problem(s), followed by two additional CD problems, which functioned as transfer problems.

Our first research goal was to examine how the sequencing of constraint relaxation types affected solution time. In the group that did not receive any OD problems, we expected a general decrease in solution time across the CD problems because, as stated by Knoblich et al. (1999), once constraints are relaxed they will remain relaxed. In the groups that receive the OD problem(s), we explored the possibility that solving the OD problem(s) would make it more difficult to solve the subsequent CD problems. Knoblich et al. (1999, cf. Ohlsson, 1992) posit that constraint relaxation occurs through the natural spreading of activation after persistent failure is experienced via impasse. Because activation to memory nodes decays, it is possible that the CD solution space might become reconstrained after participants spend some time exploring the OD solution space. Therefore, successful solution of OD problems may make solving subsequent CD problems difficult because the constraint would need to be re-relaxed, thus leading to longer solution times for the CD problems received after the OD problem(s) relative to the CD problem received prior to the OD problem(s).

Our second research goal involved the amount of time that participants spent using the procedure needed to solve the OD problems. Thus, we manipulated the mental set that participants experienced due to the OD problems. Some participants only received one OD problem, while others received three. We expected that participants who received three OD problems would show longer solution times on subsequent CD problems than the participants who only received one OD problem relative to the CD problem solved prior to the OD problem(s). We will refer to these post-OD problems as return-to-chunk-decomposition problems and therefore they will be labeled RCD1 and RCD2.

Öllinger, Jones, and Knoblich (2008) explored similar questions using matchstick arithmetic problems. In Experiment 2 they found that solving a series of CD problems did not affect the solution rate for one OD problem (type CR1 in their experiment), although they did affect the solution rate for other constraint relaxation problem types. In Experiment 3 they found that solving a series of constraint relaxation problems negatively impacted CD problems, but the constraint relaxation problems were of a different type than the OD problems used in the current experiment. Thus, while Öllinger et al. (2008) explored similar questions to the current experiment, the current work builds on these findings in terms of providing solvers with different problem types and in varying the number of OD problems between participants.

Overall, we made the following predictions for the experiment:
1) Participants who receive OD problems will have slower solution times than participants who do not receive OD problems on RCD1 compared to the baseline CD problem (B).
2) Participants who receive three OD problems will have slower solution times than participants who receive one OD problem on RCD1 compared to B.
3) Participants who do not receive OD problems will show faster solution times from B to RCD1 and from RCD1 to RCD2. Participants who receive OD problems will not show this pattern.

Method

Participants
Participants were 145 introductory psychology students who received research credit for their participation. Sixty of the participants were from the University of Illinois at Chicago and 85 of the participants were from the University of Massachusetts Dartmouth. No demographic data were collected about the participants.

Materials
A series of matchstick arithmetic insight problems were developed for the study. The problems were of two types, chunk decomposition (CD) and operator decomposition (OD). Following the terminology of Knoblich et al. (1999), the CD problems required the decomposition of loose chunks, which are composite Roman numerals (such as IV, VII, etc.). In each problem, one matchstick is moved from one numeral to another. For example, the problem \( V = VI + I \) is solved by moving one matchstick from VI to V, thus making the answer \( VI = V + I \) (an acceptable alternate solution is \( V = IV + I \)). The CD problems and their solutions are in Table 1.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Solution(s)</th>
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<tbody>
<tr>
<td>( XI = XII + I )</td>
<td>( XII = XI + I )</td>
</tr>
<tr>
<td>( V = VI + I )</td>
<td>( VI = V + I, V = IV + I )</td>
</tr>
<tr>
<td>( VI = VII + I )</td>
<td>( VII = VI + I )</td>
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<tr>
<td>( VII = VIII + I )</td>
<td>( VIII = VI + I )</td>
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The OD problems were akin to Knoblich et al.’s (1999) constraint relaxation (Type B) problems, and specifically required the relaxation of the operator constraint by decomposing the plus sign (+) into two matches and moving the vertical match elsewhere in the location, thus turning the operator into a minus sign (-). For example, the problem \( II = VIII + V \) is solved by moving the vertical matchstick from the + to the II, thus making the answer \( III = VIII - V \) (an
acceptable alternate solution is $II = VIII - VI$). The OD problems and their solutions are in Table 2.

Table 2: OD problems and solutions.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Solution(s)</th>
</tr>
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<tbody>
<tr>
<td>$VII = VII + I$</td>
<td>$VII = VIII - I$</td>
</tr>
<tr>
<td>$II = VIII + V$</td>
<td>$III = VIII - V, II = VIII - VI$</td>
</tr>
<tr>
<td>$V = VII + I$</td>
<td>$VI = VII - I, V = VII - II$</td>
</tr>
<tr>
<td>$I = V + III$</td>
<td>$II = V - III, I = IV - III$</td>
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Procedure

Participants were run individually. After completing the consent process, participants were given a packet containing the experimental materials. Rules for solving matchstick arithmetic problems were provided on each problem page in the packet. The rules were:

A) Only one matchstick is to be moved.
B) A matchstick cannot be discarded; that is, it can only be moved from one position in the equation to another.
C) A slanted stick cannot be interpreted as a vertical matchstick.
D) The result must be a correct arithmetic equation.

In addition to these rules, participants were given a list of Roman numerals and their Arabic numeral equivalents (e.g., $X = 10$).

The first problem for all participants was $XI = XII + I$, which served as a practice problem. Participants were given five minutes to work on the problem and were instructed to alert the experimenter when they came up with a solution. If the participant correctly solved the problem, the experimenter summarized the participant’s actions and stated that the solution was correct. The experimenter emphasized that one matchstick had been moved to create a correct equation. If the participant came up with an incorrect solution, the experimenter referred back to the rules to explain why the solution was incorrect. For example, the participant might be reminded that only one matchstick could be moved. If the participant did not solve the practice problem correctly within the time limit, the experimenter first checked to see if he/she had any questions and then gave him/her two more minutes to work on the problem. If, after this additional time, no solution was offered, then the experimenter explained how to move one matchstick to achieve the correct solution, $XII = XI + I$.

The second problem in the packet was the baseline chunk decomposition problem (B), $V = VI + I$. Participants had four minutes to work on this problem (and all subsequent problems). Participants wrote down their start time, worked on the problem, and wrote down the end time if they came up with a solution. The accuracy of the solution was checked by the experimenter. If the solution was incorrect, the experimenter used the rules to point out the inaccuracies of the solution.

The penultimate and final problems in the packet were also CD problems. As stated previously, the penultimate problem, $VI = VII + I$, will be referred to as the first return-to-chunk-decomposition problem (RCD1), and the last problem, $VII = VIII + I$, will be referred to as the second return-to-chunk-decomposition problem (RCD2). Participants received the same instructions and same amount of time to solve the B, RCD1, and RCD2 problems.

The problems in between B and the RCD1, RCD2 sequence differed by condition. One group of participants did not receive any OD problems. A second group of participants received one OD problem between B and the RCDs. A third group of participants received three OD problems between B and RCDs. On all OD problems, participants followed the same procedure as used for the CD problems by writing down their start and end times and checking their solutions with the experimenter.

After completing RCD2, participants filled out a problem familiarity survey, which asked participants if they had seen and solved any of the matchstick arithmetic problems prior to the experimental session. No participants had any familiarity with the matchstick arithmetic insight problems. At the end of the session, participants were debriefed and thanked for their participation.

Analysis

Participants were originally grouped by the number of OD problems they received. There were 46 participants who received no OD problems, 50 who received one OD problem, and 49 who received three OD problems. However, initial examination of the data revealed that not all participants in the OD conditions solved the OD problems. Therefore, participants were regrouped by the number of OD problems they solved. If participants did not solve the OD problems, we could not expect that they also relaxed this constraints associated with these problems. Thus, in the final analyses, there were 64 participants who solved no OD problems, 35 participants who solved one OD problem, and 46 participants who solved three OD problems.\(^1\)

Time to solve the B, RCD1, and RCD2 problems was calculated by subtracting the end time from the start time for each problem. If a participant did not solve one of these problems, then his/her time data were not included. The number of non-solvers was low for each problem: one participant did not solve B, five participants did not solve RCD1, and three participants did not solve RCD2. The time-to-solve data were then screened for outliers, which were defined as time to solve values that were greater than three standard deviations above the mean. Rather than deleting data list-wise, data points were removed case-wise. Three time-to-solve values were removed from the B and RCD1 values, and four time-to-solve values were removed from the RCD2 values.

Three variables were computed for the planned comparisons between the solution times. First, a value was removed participants who did not conform to their groups rather than regrouping participants led to the same pattern of results.
calculated for RCD1 – B, that is, the difference between the time needed to solve the baseline and first return to chunk decomposition problems. Next, a value was calculated for RCD1 – RCD2, that is, the difference between the time needed to solve the first and second return to chunk decomposition problems. Third, a value was calculated for B – RCD2, that is, the difference between the time needed to solve the baseline and second return to chunk decomposition problems.

**Results**

A one-way analysis of variance (ANOVA) compared participants on the difference between the time needed to solve the baseline chunk decomposition problem (B) and the time needed to solve the first return-to-chunk-decomposition problem (RCD1). The ANOVA was significant, $F(2, 135) = 4.44, p < .05, \eta^2 = .06$. Tukey post-hoc tests indicated that participants who solved three OD problems showed a significant increase in solution time from the B to the RCD1 problems ($M = 19.26$ seconds, $SD = 43.44$) compared to participants who did not solve any OD problems ($M = -4.18$ seconds, $SD = 40.99$), $p < .05$. Participants who did not solve any OD problems showed a decrease in time-to-solve between B and RCD1. There was also a marginal difference between participants who did not solve any OD problems and those who solved one OD problem ($M = 15.41$ seconds, $SD = 45.74$), $p = .09$. Importantly, there was no difference between participants who solved one OD problem and those who solved three.

A second analysis compared participants on the difference between the time needed to solve RCD1 and RCD2 (RCD1 – RCD2). A one-way ANOVA did not show any inter-group differences, $F(2, 129) = .86, p > .05, \eta^2 = .01$.

A third analysis compared participants on the difference between the time needed to solve B and RCD2 (B – RCD2). A one-way ANOVA showed an overall difference between the conditions, $F(2, 131) = 4.64, p < .05, \eta^2 = .07$. Tukey post-hoc tests indicated that participants who did not solve any OD problems needed significantly less time to solve RCD2 than to solve B ($M = 15.14$ seconds, $SD = 23.08$) compared to participants who solved three OD problems ($M = 2.35$ seconds, $SD = 22.25$), $p < .05$. There was also a marginal difference between participants who did not solve any OD problems and those who solved one OD problem ($M = 3.69$ seconds, $SD = 24.30$), $p = .07$. There was no difference between participants who solved one OD problem and those who solved three.

**Discussion**

The current work produced four main important findings. First, in the absence of successful OD performance, participants got progressively faster when solving CD problems. This finding suggests that relaxation of the value constraint made it easier to solve subsequent value constraint problems. In this sense, we found some evidence of positive transfer on problems that presumably required relaxation of the same constraint. This finding supported our third prediction, that participants who did not receive OD problems would show faster solution times from B to RCD1, while participants who received OD problems would not show this pattern. This finding also supports Knoblich et al.’s (1999) theory – once a constraint is relaxed, it will remain relaxed and affect subsequent performance on similar problems.

Second, the current results provide greater confidence that constraint relaxation can also negatively impact subsequent problem solving performance, particularly in the event that a different, more complex constraint was relaxed. Generally, successful solution of OD problems resulted in longer subsequent solution of CD problems compared to those who did not solve or were not presented with OD problems. Thus, the longer solution times indicate that there was at least some negative transfer associated with the relaxation of the operator constraint. Solution of an OD problem appeared to make it more difficult to solve the simple CD problems; this difficulty was absent for those who did not solve OD problems. This finding supports our first prediction, that participants who received OD problems would have slower solution times than participants who did not receive OD problems on RCD1 compared to B. Additionally, this finding replicates and extends the findings of Öllinger et al. (2008), who also found successful solution of constraint relaxation problems (of a different type) affected later problem solving performance.

Third, as stated previously, we varied the number of OD problems that were presented to participants and were solved in between the CD problems. Participants who solved one and three OD problems displayed similar indications of negative transfer on subsequent RCDs, as evidenced by longer solution times. This finding did not support our second prediction, in which we predicted relatively slower solution times for participants who received three OD problems than participants who received one OD problem. Our finding suggests that, indeed, after one successful solution of an OD problem, the operator constraint was relaxed. Moreover, there did not seem to be an additional slowing associated with solving multiple OD trials.

The final important point is that the negative transfer effects associated with the relaxation of the operator constraint were relatively lasting. That is to say that successful solution not only affected the immediate CD problem, but also the problem that followed. Although the current data does not provide an indication of how long-lasting this kind of negative transfer would be, there did seem to be a “downstreaming” effect into subsequent problem solving performance, beyond the problem situation that immediately followed the constraint relaxation.

Overall, the findings of this experiment point to the manner in which previously appropriate procedures can be persistently misapplied to a new situation. As demonstrated by Luchins (1942; Luchins & Luchins, 1950), mental set can hinder future problem solving. Mental set and other interference effects fit within the larger concept of negative
transfer, in which prior knowledge and experiences hinder learning in new situations that are similar to known situations. The type of negative transfer effects shown in this experiment are similar to those proposed by Singley and Anderson (1989): participants show behavioral slowing due to the misapplication of a procedure. However, the misapplication is an incorrect method, not a non-optimal method, and this misapplication of procedure lasts for more than one trial, thus lending some support to Woltz, Gardner, and Bell’s (2000) theory of negative transfer.

Further work is needed to address the direction and duration of negative transfer effects. It is possible that completing a series of CD problems could lead to negative transfer on the OD problems. Likewise, it would be interesting to determine if increases in solution time on the RCD problems lasts more than two iterations. The negative transfer literature is divided on whether negative transfer effects are fleeting (e.g., Singley & Anderson, 1989) or lingering (e.g., Woltz et al., 2000). Additional studies extending the number of to-be-solved CD problems may inform on this issue.

Another future direction for this research would be to examine the processes that underlie the interaction between constraint relaxation mechanisms. Our findings, as well as the findings of Öllinger et al. (2008), show that relaxing some constraints hinders the relaxing of other constraints. One explanation for these findings is that successful solution of OD problems may make solving subsequent CD problems difficult because the constraint would need to be re-relaxed. Thus, there would be longer solution times for the CD problems received after the OD problems relative to the CD problem received prior to the OD problems. Alternatively, relaxing a stronger constraint, such as the operator constraint present in the OD problems, may cancel out a weaker constraint, such as the value constraint present in the CD problems. Although this a different explanation the same effect would be expected, in which solution times are longer for the CD problems received after the OD problem(s) than for the CD problem received prior to the OD problem(s). A third possibility is that the relaxing of multiple constraints opens up the problem space too much, thus leading to a difficulty in finding a correct solution path (c.f. Ohlsson, 1996; Ormerod, MacGregor, & Chronicle, 2002). This third explanation would also lead to the same pattern of results. Future research should address the mechanisms that underlie constraint relaxation interactions and, if possible, attempt to tease apart which of these three possibilities best explains negative transfer in problem solving performance.

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