The Additive Judge:
On the Abstraction of Explicit Knowledge of Cue-Criterion Relations

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Abstract
An experiment with a multiple-cue judgment task tested the hypothesis that humans can only abstract explicit representations of cue-criterion relations when the cues are related to the criterion by an additive function. It is proposed that the sequential and capacity-constrained nature of controlled, explicit thought can only induce and execute linear additive cue integration; non-additive environments require exemplar memory. The results showed that an additive task induced processes of cue abstraction and cue integration, while a multiplicative task induced exemplar processes. The results suggest flexible interplay between distinct representation-levels, a preference to abstract explicit “rules” whenever possible, although this capacity is constrained to additive cue-criterion relations.

Introduction
In this article we make three general claims about the cognitive processes involved in multiple-cue judgment: a) the cognitive system has multiple qualitatively distinct representations that “race” to control the judgments in a specific task (e.g., Ashby, Alfonso-Reese, Turken, & Waldron, 1998; Erickson & Kruschke, 1998; Juslin, H. Olsson, & A-C. Olsson, 2003); b) humans prefer to abstract explicit “rule-based” knowledge when the task feedback and the task structure allows for it (see also Ashby et al., 1998; Juslin et al., 2003); and c) because the sequential and capacity-constrained nature of explicit thought processes only allows induction and execution of additive cue integration, human’s are only capable of abstracting explicit cue-criterion representations when cues are related to the criterion by an additive function. These claims—and the last one, in particular—are tested in an experiment that relies on a task that allows us to identify whether cue abstraction has been successfully achieved (Juslin et al., 2003).

Linear, additive models often fit judgment data well in research on multiple-cue judgment (Brehmer, 1988; Cooksey, 1996). The issue of whether humans integrate information in an additive manner is, however, a core topic also in other fields of psychology, such as developmental psychology and perception. A main claim by Anderson’s (1981) Information Integration Theory is that humans integrate information with an additive rule. It is likewise suggested that perception of depth is enabled by adding the cues (Bruno & Cutting, 1988).

We propose that the results from multiple-cue judgment reflect a general architectural property of the controlled and explicit thought processes of the human mind. The idea is that the sequential, real-time consideration of multiple cues is a process of successive adjustment of the judgment, a process structurally compatible with a linear, additive cue-integration rule (Einhorn, Kleinnuntz, & Kleinnuntz, 1979). This hypothesis suggests that explicit and controlled thought processes are particularly apt at performing cue-integration in tasks where the cue-criterion relations are additive. By contrast, a task that involves non-linear or multiplicative cue-criterion relations requires the capitalization on some more implicit process, such as exemplar memory (Medin & Schaffer, 1978; Nosofsky & Johansen, 2000). Exemplar memory makes no computational commitments to a particular task structure (linear or otherwise).

In this study, we test this hypothesis in the context of two additional assumptions. First, because abstract and explicit knowledge is more general, useable in a more flexible and controlled manner, and more easily verbalized and communicated to others, as soon as the environment and task feedback allows for it humans have a preference to abstract explicit knowledge (Ashby et al., 1998; Juslin et al., 2003). Second: when people are unable to abstract explicit cue-criterion relations they retreat to exemplar memory (Juslin et al, 2003). Therefore, we predict that we can induce a shift between qualitatively distinct cognitive processes by manipulating the deep-lying structural properties of the task environment: additive cue-criterion relations should promote explicit cue abstraction and multiplicative cue-criterion relations should promote exemplar memory.

Judgment Task and Cognitive Models
The task requires participants to use four binary cues to infer a continuous criterion. (Juslin et al., 2003). The judgments involve the toxicity of subspecies of a fictitious bug. The different subspecies vary in concentration of poison from 50 ppm (harmless) to 60 ppm (le-
The toxicity can be inferred from four binary visual cues ($C_1$, $C_2$, $C_3$, and $C_4$) of the subspecies (e.g., the length of their legs, color of their back, the length of their nose and spots or no spots on the forehead). The cue structure is shown in Table 1. There are two conditions, one additive and one multiplicative.

Table 1: The 16 exemplars with their cues and criteria prior to addition of random error for both the additive (Add) and the multiplicative (Multi) condition. $E =$ Extrapolation exemplar, $T =$ Training exemplar, $O =$ Old comparison exemplar presented in training, matched on the criterion to one of the new exemplars, $N =$ New comparison exemplar presented at test.

<table>
<thead>
<tr>
<th>Exemplar</th>
<th>Cues</th>
<th>Criteria</th>
<th>Role</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 1 1 1</td>
<td>60</td>
<td>32.75</td>
</tr>
<tr>
<td>2</td>
<td>1 1 1 0</td>
<td>59</td>
<td>59.00</td>
</tr>
<tr>
<td>3</td>
<td>1 1 0 1</td>
<td>58</td>
<td>53.94</td>
</tr>
<tr>
<td>4</td>
<td>1 1 0 0</td>
<td>57</td>
<td>52.08</td>
</tr>
<tr>
<td>5</td>
<td>1 0 1 1</td>
<td>57</td>
<td>52.08</td>
</tr>
<tr>
<td>6</td>
<td>1 0 1 0</td>
<td>56</td>
<td>51.40</td>
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<tr>
<td>7</td>
<td>1 0 0 1</td>
<td>55</td>
<td>51.15</td>
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<td>8</td>
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</tr>
<tr>
<td>9</td>
<td>0 1 1 1</td>
<td>56</td>
<td>51.40</td>
</tr>
<tr>
<td>10</td>
<td>0 1 1 0</td>
<td>55</td>
<td>51.14</td>
</tr>
<tr>
<td>11</td>
<td>0 1 0 1</td>
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<tr>
<td>12</td>
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<td>53</td>
<td>51.02</td>
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<tr>
<td>13</td>
<td>0 0 1 1</td>
<td>53</td>
<td>51.02</td>
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<tr>
<td>14</td>
<td>0 0 1 0</td>
<td>52</td>
<td>51.00</td>
</tr>
<tr>
<td>15</td>
<td>0 0 0 1</td>
<td>51</td>
<td>51.00</td>
</tr>
<tr>
<td>16</td>
<td>0 0 0 0</td>
<td>50</td>
<td>51.00</td>
</tr>
</tbody>
</table>

In the additive condition the toxicity $c$ of a subspecies is a linear additive function of the cue values:

$$c = 50 + 4 \cdot C_1 + 3 \cdot C_2 + 2 \cdot C_3 + 1 \cdot C_4 \quad (1)$$

$C_1$ is the most important cue with coefficient 4 (i.e., a relative weight 4), $C_2$ is the second to most important with coefficient 3, and so forth. A subspecies with feature vector (0, 0, 0, 0) thus has no poison concentration 50 ppm; a subspecies with feature vector (1, 1, 1, 1) has 60 ppm. In the multiplicative condition the toxicity $c$ of a subspecies is a multiplicative function of the four cue values (the criterion values are shown in Table 1):

$$c = 51 + 0.0009875 \cdot e^{c_1 + c_2 + c_3 + c_4} \quad (2)$$

with the same coefficients as in the additive task (Eq. 1). The additive and multiplicative task environments are construed to produce equal training ranges for the two conditions (all training exemplars have a toxicity between 51 and 59). Moreover, the criteria in the multiplicative condition are a simple exponential function of the criteria presented in the additive condition. In both the additive and the multiplicative condition a random error is added to the criterion values, implying a probabilistic relation between cues and criteria ($R^2 = .9$).

We use two models to derive predictions for the two conditions, a cue-abstraction and an exemplar model. The cue-abstraction model assumes that the participants abstract explicit cue-criterion relations in training, which are mentally integrated at the time of judgment. When presented with a probe the participants retrieve rules connecting cues to the criterion from memory (e.g., “Green back goes with being poisonous”). The rules specify the sign of the relation and the importance of each cue with a cue weight. For example, after training the rule for cue $C_1$ may specify that $C_1 = 1$ goes with a large increase in the toxicity of a subspecies.

Cue abstraction with additive cue integration implies that the participants compute an estimate of the continuous criterion $c$. For each cue, the appropriate rule is retrieved and the estimate of $c$ is adjusted according to the cue weight $\omega_{i,j}$ ($i=1...4$). The final estimate $\hat{c}_{CA}$ of $c$ is a linear additive function of the cue values $c_i$:

$$\hat{c}_{CA} = k + \sum_{i=1}^{4} \omega_{i,j} \cdot C_i, \quad (3)$$

where $k = 50 + .5 \cdot (10 - \sum \omega_{i,j})$. If $\omega_{1,j} = 4$, $\omega_{2,j} = 3$, $\omega_{3,j} = 2$, and $\omega_{4,j} = 1$, Eq. 1 and 3 are identical and the model produce perfect judgments. The intercept $k$ constrains the function relating judgments to criteria to be regressive around the midpoint (55) of the interval [50, 60] specified by the task instructions.

As outlined in the introduction, our hypothesis is that explicit cue abstraction is essentially constrained to the linear additive form in Eq. 3. However, when the models are fitted to data below, we also consider the possibility that the participants have correctly abstracted the multiplicative cue-criterion relations by fitting a multiplicative cue-abstraction model to the data:

$$c = 51 + 0.0009875 \cdot e^{c_{1} + c_{2} + c_{3} + c_{4}} \quad (4)$$

where $\omega_{i,j}$ are the best fitting subjective cue weights in the multiplicative cue abstraction model.

The exemplar model implies that the participants make judgments by retrieving similar exemplars (subspecies) from long-term memory. When the exemplar model is applied to judgments of a continuous criterion variable, the estimate $\hat{c}_{x}$ of the criterion $c$ is a weighted average of the criteria $c_i$ stored for the $J$ exemplars, where the similarities $S(p,x_j)$ are the weights:

$$\hat{c}_{x} = \sum_{j=1}^{J} S(p,x_j) \cdot c_j. \quad (5)$$

$p$ is the probe to be judged, $x_j$ is exemplar $j$ ($j=1...J$), and $S(p,x_j)$ is the similarity between probe $p$ and exemplar $x_j$. Eq. 5 is the original context model (Medin & Schaffer, 1978) applied to a continuum (see DeLosh, Busemeyer & McDaniel., 1997; Justlin et al., 2003).

The similarity between probe $p$ and exemplar $x_j$ is computed according to the multiplicative similarity rule of the context model (Medin & Schaffer, 1978):

$$S(p,x_j) = \prod_{i=1}^{4} d_i, \quad (6)$$

where $d_i$ is an index that takes value 1 if the cue values on cue dimension $i$ coincide (i.e., both are 0 or both are 1), and 0 if they deviate (i.e., one is 0, the other is 1). $s_i$
are four parameters in the interval \([0, 1]\) that capture the impact of deviating cues values (features) on the overall perceived similarity \(S(p, x_j)\). A value of \(s_j\) close to 1 implies that a deviating feature on this cue dimension has no impact on the perceived similarity and is considered irrelevant. A value of \(s_j\) close to 0 means that the similarity \(S(p, x_j)\) is close to 0 if this feature is deviating, thus assigning crucial importance to the feature. For low \(s_j\), only identical exemplars have a profound effect on the judgments; with \(s_j\) close to 1 all exemplars receive the same weight, regardless of their features.

In the experiment five of the subspecies in the test phase are not included in the training phase (Exemplars 1, 5, 6, 7, & 16 in Table 1). This makes it possible to distinguish between the models as they provide different predictions (see Figure 1). In the training phase, all exemplars have toxicity between 51 and 59. If the participants have estimated the correct cue weight for each cue there should be no problem to compute the most extreme judgments for the extreme exemplars that are left out in the training phase (i.e., Exemplars 1 & 16). More specifically, whenever the participants have correctly identified the sign of the impact of each cue (i.e., whether it increases or decreases toxicity) they should always make the most extreme judgments for Exemplars 1 (all cues present) and 16 (all cues absent), as illustrated on the left-side of Figure 1. (This holds both for the additive and the multiplicative cue-abstraction models). By contrast, the exemplar model computes a weighted average of the stored exemplars with toxicity between 51 and 59 and this can never produce a value outside of the observed range (DeLosh et al., 1997; Erickson & Krusckhe, 1998). Moreover, because of the multiplicative similarity rule in Eq. 6, the most extreme judgments are made for the second to most extreme exemplars (Exemplars 2 & 15). For these exemplars the judgment is dominated by retrieval of identical stored exemplars and these identical exemplars are the most extreme that have been encountered in training. These predictions are illustrated on the right side of Figure 1.

When the new exemplars in the mid range of toxicity (Exemplars 5, 6, & 7) are judged, cue abstraction suggests no systematic difference between these three new exemplars and three old exemplars matched in toxicity (Exemplars 4, 9, & 10): the cognitive process is the same regardless of whether a specific exemplar has been encountered before or not. The exemplar model, however, predicts more precise judgments for the old exemplars because for these exemplars the participants can benefit from previous identical exemplars with the correct criterion \(c\). In addition, with most similarity parameters \(s_j\) these new exemplars have a high overall similarity to exemplars with a lower criterion than the correct value. Therefore, in general the exemplar model will predict that the new exemplars are underestimated.

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**The Experiment**

In the experiment we manipulated whether participants were confronted with a task that involved additive or multiplicative cue-criterion relations. For reasons outlined in the introduction, we predicted that the additive task (Eq. 1) should promote explicit cue abstraction with additive cue integration (Eq. 3). A multiplicative task (Eq. 2) should cause a shift to a qualitatively different process, that is, to exemplar memory (Eq. 5).

We also examined an alternative way to induce shifts between cognitive processes. Because working memory is more involved in the integration of explicitly ab-

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1 The optimal parameters are multiplied by 0.8 to yield functions that are more descriptive of the noise in data.

2 When the predictions by the exemplar-based model are illustrated in Figure 1 the similarity parameters \(s_j\) have been arbitrary set to 0.1. When the model is applied to data below, \(s_j\) are free parameters fitted to data.
stracted cues, we hypothesized that a distracter of the working memory in the test phase should affect cue abstraction more than exemplar processes. Therefore, introduction of a working memory distraction task should promote a shift towards more exemplar processes, in particular, in the additive task where cue abstraction is expected.

Method

Participants
Eighty undergraduate students volunteered, receiving a payment of 60-99 SKr, depending on their performance. Thirty-nine participants were men and 41 were women. They were all between 20 and 37 years old.

Materials and Procedure
The participant judged the toxicity of the subspecies in a training phase, followed by two test phases. The subspecies were presented to the participant on a computer screen as visual pictures, one at a time, and the participant controlled the time of exposure. The subspecies varied in terms of four binary cues; short, blue or long, green legs; short, darkblue or long, grey nose; spots or no spots on the fore back; and brown or geen buttock. In the training phase, 11 different subspecies were shown 20 times each, requiring a total of 220 judgments. Subspecies nr 1, 5, 6, 7 and 16 were left out, as imposed by the task design (see Table 1). The subspecies were shown in random order for every participant. The cue weights were also randomized, so that what was the most important cue differed across participants. With each subspecies, a question in written text on the screen was to be answered, asking for “how poisonous is this subspecies?” In the training phase, feedback was given on the correct criterion after each judgment (“This subspecies has toxicity 57 ppm”). There were two test phases. One had an additional working memory distracter task. This task was to listen to a recorded voice reading Swedish words at a rate of 1 s., and simultaneously perform the judgments, while remembering the number of words heard that denotes something alive (for example the word “dog”). It was outbalanced so that half of the participants had the undistracted test phase first and the distracted test phase last, and vice versa. Each test phase consisted of 16 different subspecies. The subspecies were shown twice each, requiring a total of 32 judgments for one test phase. Subspecies nr 1, 5, 6, 7 and 16 were introduced, as imposed by the task design. In the test phases, no feedback was given on the correct criterion.

Dependent Measures
The measure of performance is Root Mean Square Error (RMSE) of the judgments (i.e., between judgments and criteria). Measures of model fit are the coefficient of determination ($r^2$) and Root Mean Square Deviation (RMSD) between predictions and data computed on the basis of the data from the test phase. The exemplar index, $\Delta E$, is a measure of to what extent the judgments are dominated by an exemplar-based process. $\Delta E$ is the sum of two measures; old-new difference and extrapolation. The old-new difference is computed as the difference between the absolute deviation between judgments and criteria for the old exemplars and the absolute deviation between judgments and criteria for the matched new exemplars (denoted “O” and “E” in Table 1). Extrapolation is computed as the absolute deviation from the judgment predicted by linear regression of the judgments for training exemplars on the criterion. When this measure equals zero, the participants extrapolate appropriately for the extreme exemplars. The old-new difference and the extrapolation measure will be added for every participant, yielding a single measure of exemplar effects, the exemplar index $\Delta E$:

$$\Delta E = \sum_\tau \Delta ON + Extap. \quad (7)$$

A negative $\Delta E$ is predicted when the participant makes systematically poorer judgments for new exemplars compared with old exemplars. A cue-abstraction model predicts no systematic differences between judgments on old and new exemplars (see Juslin et al, 2003, for a further discussion of the $\Delta E$ measure).

Results
A two-way ANOVA with environment (additive or multiplicative) as between-subject factor and the two test phases (undistracted and distracted) as within-subject factor, shows two main effects on RMSE (Table 2).

First, there is significantly better performance (lower RMSE) for the additive condition ($F(1.78)=160.69; MSE=2.032; p=0.000$). Second, there is a significant effect on performance of the working memory distracter. The RMSE is better in the undistracted test phase ($F(1.78)=5.9112; MSE=0.5982; p=0.017$).

Table 2. Judgment performance in the experiment as measured by the Root Mean Square Error (RMSE) between judgment and criterion.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Test</th>
<th>Index</th>
<th>Add.</th>
<th>Mult.</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Undistracted RMSE</td>
<td>2.55</td>
<td>5.30</td>
<td>3.92</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distracted RMSE</td>
<td>2.74</td>
<td>5.70</td>
<td>4.22</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean RMSE</td>
<td>2.64</td>
<td>5.50</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A more negative exemplar index $\Delta E$ was hypothesized for the multiplicative condition. $\Delta E$ was moreover hypothesized to be more negative in the additive condition with the introduction of a working memory distracter. A two-way ANOVA with environment (additive or
multiplicative) as between-subjects factor and the two test phases (undistracted and distracted) as within-subjects factor showed one main effect and one significant interaction. First, $\Delta E$ is significantly lower in the multiplicative condition, suggesting more reliance on exemplar-memory ($F(1,782) = 43.12; \text{MSE} = 33.74; p<0.000$). Second, the significant interaction between environment and test ($F(1,782) = 9.19; \text{MSE} = 6.03; p<0.0025$, Figure 2) suggests that the effect of a working memory distracter was different in the two conditions; in the Additive condition less strong reliance on cue abstraction was induced, while in the multiplicative condition less strong reliance on exemplar memory was the result.

![Figure 2](image.png)

Figure 2. Exemplar index, $\Delta E$ in the Additive and Multiplicative conditions over the two test phases (undistracted and distracted).

Although significant, because the difference between the two test-phases was small and difficult to discern from visual inspection of data, and the aim of this paper is to investigate how the learning task affects cognitive processes, the mean judgments in the additive and multiplicative conditions were collapsed over the two test phases. The mean judgments are shown in Figure 3. In the additive condition, the judgments are a linear function of the criterion. Although there is some noise, there are no visible extra- or interpolation effects. In the multiplicative condition, the judgments do not follow the optimal line, nor the best fitting regression line. Although the judgments are a positive function of the criteria in the training range (51-59), the inability to extrapolate is striking. Notably, the judgments for $c=72$ is significantly lower than for $c=59$. In sum, these results show no signs of exemplar-processes in the additive condition, but clear signs of exemplar processes in the multiplicative condition.

Model fits were obtained by fitting the models described in the introduction with Mean Square Error between predictions and data as the error function. The models were further applied through a method of projective fit (Juslin et al., 2003). The models were thus fitted to data from the latter half of the training phase (i.e., based on 11 subspecies) and then applied with the parameters fitted to this data set to the data with all 16 subspecies in the test phase. This implies cross-validation for the 11 exemplars that were presented in training and genuine predictions for the new exemplars. Both the exemplar model and the cue-abstraction model were fitted to the data collapsed over the two test phases, as well as the multiplicative version of the cue abstraction model. Table 3 shows the fit for the models.

![Figure 3](image.png)

Figure 3. Mean judgments in the additive condition (Panel A) and multiplicative condition (Panel B), with the best-fitting regression line.

<table>
<thead>
<tr>
<th>Cond.</th>
<th>CAM(A)</th>
<th>CAM(M)</th>
<th>EBM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$r^2$</td>
<td>RMSD</td>
<td>$r^2$</td>
</tr>
<tr>
<td>Add.</td>
<td>0.95</td>
<td>0.32</td>
<td>-</td>
</tr>
<tr>
<td>Mult.</td>
<td>0.49</td>
<td>2.94</td>
<td>0.65</td>
</tr>
</tbody>
</table>

As predicted in the additive task condition the additive cue-abstraction model fits data better than the exemplar model, while the reverse is very clearly true in the multiplicative condition. In the multiplicative condition the multiplicative cue-abstraction model performs somewhat better than the additive cue abstraction model. This is probably an effect of its higher correlation with the predictions by the exemplar model.
3 provides no evidence for cue abstraction in the multiplicative condition. There were no clear differences between the test phase with and without working memory distraction in regard to the model fits and the same pattern as in Table 3 was observed in both conditions.

Discussion

The broad claim that essentially humans are only capable of extracting explicit cue-criterion relations from training with outcome feedback when they are related by an additive function is supported by the experiment. The cue-abstraction model provided both a qualitatively and quantitatively better explanation of the data in the Additive condition, while the exemplar model is a better explanation in the Multiplicative condition. That the exemplar-based model produces a relatively good fit to data in the additive condition as well could be interpreted in terms of quasi-rationality (e.g. Brehmer, 1994; Cooksey, 1996; Hammond, 1996; Justin et al, 2003); the additive task might have induced exemplar-memory for some participants.

What might be seen as a clear and simple mathematical manipulation of the task structure is sufficient to induce qualitatively different cognitive processes. These results suggests that: a) By manipulating the cue-criterion relations one can induce shifts between cognitive processes; b) The shift arises when the preference for explicit representations (i.e. abstract knowledge of cue-criterion relations) can not be met because of cognitive limitations, and people turn to the back-up process of exemplar memory; and c) The reason for this cognitive limitation is an architectural constraint on explicit and controlled thought processes only allowing for abstraction and integration in a sequential, additive manner. The results in regard to the manipulation of working memory load were less clear. Both visual inspection of data and the model fits indicate little difference between the conditions. The performance suggests that working memory distraction affected performance in a negative manner, but with no clear signs of a representational shift (i.e., the significant interaction is difficult to interpret).

It may be objected that the stimuli vary in few dimensions which may have promoted exemplar memory (see Smith & Minda, 2000, for a discussion). However, it remains the fact that qualitatively and quantitatively different results were obtained over the two conditions, with the same stimuli. The criteria in the multiplicative condition are concentrated around the low fifties. This could possibly have imposed a simple learning rule like “[always guess on 51].” However, the results show a significantly positive regression-line suggesting that the participants have learned the positive relation, yet both the inability to extrapolate and the model fits suggest that the knowledge is in form of exemplars. If some kind of additive rule was used the judgments on the extreme exemplar #1 (c=72) would not be expected to be lower than the judgments on exemplar #2 (c=59).

The aspects proposed in this paper needs to be further addressed. A comparision between how the cue-abstraction model and exemplar-models augmented with linear extra-polation (for example EXAM, DeLosh et al, 1997) describe data would be particularly valuable. Taken together, the results in the paper suggest that the type of cue-criterion relations has a powerful effect on cognitive processes.

Acknowledgments

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References


