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What determines the number of parties in a national assembly? Previous work has emphasized either sociopolitical heterogeneity or electoral system permissiveness, or brought them together on an empirical basis. Here an equation is developed that satisfies two theoretical boundary conditions and expresses the effective number of assembly parties \(N\) in terms of both the number of politicized issue dimensions \(I\) and effective magnitude \(M\) of electoral system: \(N = I^{0.6} M^{0.15} + 1\). Actually, depending on circumstances, any of the three variables could become the dependent one, affected by the two others. Empirical evidence is presented, based on Lijphart’s (1984) data on 22 stable regimes.

THE NUMBER OF PARTIES AS A FUNCTION OF HETEROGENEITY AND ELECTORAL SYSTEM

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What determines the number of parties in a national assembly? This is one of the most important and long-standing questions in the study of democratic systems because, more than any other single factor, the number of parties affects the nature of politics in the national assembly. Two types of explanations have been offered.

On one hand, the number of parties depends on sociopolitical heterogeneity, that is, the number of social cleavages that are politicized. On the other, it also depends on the electoral system permissiveness toward small-party representation. This is not an either/or question, though, because the number of parties visibly depends on both. Moreover, sociopolitical heterogeneity and electoral system are interrelated, as pictured in Figure 1.

The triangle in Figure 1 has at its apexes three variables that reflect, respectively, the number of parties, heterogeneity, and electoral system permissiveness. The “number of issue dimensions” \(I\), as defined and tabulated

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in Lijphart (1984, p. 130), is taken as an admittedly imperfect measure of sociopolitical heterogeneity. As an also imperfect measure of an electoral system’s permissiveness, the “effective magnitude” \((M)\) is used, as defined and tabulated in Taagepera and Shugart (1989, pp. 138-139). For the number of parties the “effective number of assembly parties” \((N)\) is used (Laakso & Taagepera, 1979), as tabulated for the systems in question in Lijphart (1984, p. 122). The degree of validity of these choices of variables is discussed in Appendix A.

The causality relationships, symbolized by arrows in this \(N-I-M\) triangle, are multidirectional. Sociopolitical heterogeneity affects the number of parties but also influences the choice of electoral system. The number of parties may induce changes in electoral system and also affects the number of issue dimensions that are seen as politically distinct. (Thus, the single-member districts in the United States have induced an uneasy rapprochement of the issues of economic freedom and of moral coercion of the individual.) Electoral system affects the number of parties that can survive and thereby indirectly also the number of issue dimensions seen as politically distinct (see, e.g., Taagepera & Shugart 1989, pp. 98, 153-154).
THE GOAL: A SINGLE INTERDEPENDENCE EQUATION

Within this triangle of interdependence some pressures act more rapidly and directly, whereas others operate slowly and indirectly but maybe no less forcefully in the long run. If an extraneous force or event produces a change in any corner of the triangle, adjustments are eventually made at all corners, until equilibrium is restored. Such mutual interdependence can arise in many systems. One of the most familiar ones is the law of ideal gases, \( PVT = \text{constant} \), which joins pressure \( (P) \), volume \( (V) \), and temperature \( (T) \) of a closed system. Any two of these three factors could be modified from the outside, but then the third one must adjust so as to keep the total expression constant.

The purpose of the present study is to develop an equation to interconnect \( I, M, \) and \( N \) in an analogous way, so that, once two of them are given, the third one can be calculated. Beyond this notion (that any two determine the third) there is, of course, no connection to the ideal gas. It is also recognized that the context is more fluid. \( I, M, \) and \( N \) are imperfect measures of heterogeneity, impact of electoral rules, and number of parties, respectively. Moreover, other factors can enter. The effect would show up as error in results based on the simple \( N-I-M \) equation, something that can and will be tested.

The payoff in case of success is ability to estimate the number of parties whenever sociopolitical heterogeneity and electoral system permissiveness are known, within the error limits established. In mathematical terms, we are looking for a constant function of \( N, I, \) and \( M \): \( f(N, I, M) = \text{constant} \). With a question mark, this function is placed at the center of the triangle in Figure 1.

The starting point consists of two previously established equations, shown left and right in Figure 1, that connect the variables two at a time: \( N \) and \( I \) at an average value of \( M \) (Taagepera & Grofman, 1985), and \( N \) and \( M \) at an average value of \( I \) (Taagepera & Shugart, 1993). Eliminating \( N \) between these two equations results in a third equation, shown at the bottom of the triangle, that formally connects \( I \) and \( M \). These equations will be discussed in due time. Before doing so, previous work will be reviewed and data will be graphed to show visually that \( N \) does vary with varying \( I \) and \( M \).

PREVIOUS WORK

Considerable attention has been paid to the determinants of the number of parties, with varying emphasis on issues/cleavages and institutions (mainly electoral system), as reviewed recently by Amorim Neto and Cox (1997). In their opinion, institutions are stressed by scholars such as Duverger (1954),

Thus, among the “institutionalists,” Lijphart (1984, pp. 127-149) discusses in length the number of issue dimensions, assigning numerical values and establishing a correlation between $I$ and $N$. This correlation leads to the functional relationship $N = I + 1$ (Taagepera & Grofman, 1985), which is highlighted in Taagepera and Shugart (1989), another presumably institution-oriented work. Yet, this equation has no institutional ingredients. True, it is out of step with most culturalists in trying to establish an explicit equational tie between sociopolitical heterogeneity and the number of parties. In this light the diffuse groups of scholars delineated by Amorim Neto and Cox (1997) may have as much to do with differences of emphasis between philosophizing and measuring as between institutionalism and culturalism—and many of these scholars (notably Rokkan) have been involved in all four.

Statistical analyses have been carried out to determine the joint effect of various aspects of social heterogeneity and electoral system on the number of parties, notably by Powell (1982), Ordeshook and Shvetsova (1994), and Amorim Neto and Cox (1997). They all found that both factors have an effect.

All three studies expressed output in terms of the effective number of assembly parties (or its equivalent in terms of fractionalization), to which Ordeshook and Shvetsova (1994) and Amorim Neto and Cox (1997) added the effective number of elective parties (using votes rather than seats) and various other variables. Social heterogeneity was measured in terms of effective number of ethnic groups (or its equivalent in terms of fractionalization), to which Powell (1982) and Ordeshook and Shvetsova added other social characteristics. The relevant characteristics of the electoral system were expressed in terms of district magnitude (seats allocated in the district) and related measures such as the aforementioned effective magnitude that tries to include the impact of legal thresholds and multitiered elections.

A thorough comparison of all three studies is given in Amorim Neto and Cox (1997). It will not be repeated here, except for one issue that impinges directly on the core of the present study: Are the effects of heterogeneity and electoral system additive or multiplicative?

Powell (1982) used an addition of the two factors, but Ordeshook and Shvetsova (1994) found a multiplicative interaction superior. So did Amorim Neto and Cox (1997), who offered reasons why this should be the case on
conceptional grounds. Low heterogeneity puts a lid on the number of parties even in the presence of a very permissive electoral system, because there will be no demand for many parties. Similarly, a low magnitude (single-member districts) tends to put a lid on the number of parties even in the presence of strong heterogeneity, because few parties can gain representation. This mutual limitation can be obtained by multiplying the two effects, but not by adding them.

The issue is somewhat analogous to whether a person’s need for food and air is additive or multiplicative. Rare is the situation in which a surfeit of something can compensate for utter lack of something else, as suggested by addition. In contrast, multiplication leads to zero when either component is zero. See Appendix B for further discussion.

In determining relationships between variables one can follow two paths. One is to enter a number of plausible variables, assume simple relationships among them (addition, multiplication, possibly in conjunction with logarithmic or exponential transformations), and run empirical regressions. The other approach is to build first a rational model of how the variables should be interconnected and then test the predictions of this theoretical model. The three aforementioned studies used the former approach. In the present study the second one also enters.

THE DATA

Consider the 22 stable democratic systems in Lijphart’s (1984) classic study. Figure 2 shows the observed values of $N$ mapped on a field of $I$ versus $M$. A logarithmic scale is used for $M$, because $M$ varies over several orders of magnitude. The upward curve shown corresponds to the equation in $I$ and $M$ introduced at the bottom of the $N$-$I$-$M$ triangle in Figure 1; it will be discussed in the next section.

Visibly, the values of $N$ are lowest in the lower left corner, where both $I$ and $M$ are low. The number of parties tends to increase when either sociopolitical heterogeneity or electoral system permissiveness increases.

Some inconsistencies occur. Most glaringly, at the same effective magnitude, Switzerland has fewer issue dimensions yet many more parties than Norway. Also, with practically the same $I$ and $M$, Israel has many more parties than Iceland. Such inconsistencies may be due to faulty estimates of $I$ or $M$, both of which involve an element of judgment, or to the impact of some other factor not accounted for by $I$ or $M$.

When considering the effect of $I$ and $M$ on $N$, such inconsistencies are to be treated as random noise. They can be smoothed out by taking the mean $N$
for groups of systems with similar $M$ and $I$—the groups circled in Figure 2. More sophisticated smoothing techniques exist, but I believe not to have distorted the outcome by using the present simple approach. These smoothed values are shown in the first three columns of Table 1. (The last three columns will be discussed later.)

For Austria, Iceland, and France IV the effective magnitude is uncertain in the original tabulation (Taagepera & Shugart, 1989, p. 138), as indicated by question marks in Table 1. France IV is a doubly uncertain case because even the questionable value of $M$ is based only on the short period of 1945-1946.

THE N-I-M EQUATION

The starting point consists of two previously established equations based on rational models and in fair agreement with data. The first one ties the
It expresses a simple model: A newly politicized issue dimension does not split all or most existing parties (in which case the model would be \( N = 2^I \)). Rather, it generates only one new party that champions the new issue in opposition to all the established parties (Taagepera & Shugart, 1989, pp. 95-97). In the case of the 22 political systems investigated by Lijphart (1984), the actual values of \( N \) fall evenly between the lines \( N = I \) and \( N = I + 2 \).
Single-member districts tend to lead to values of $N$ on the low side of $I + 1$ (cf. graph in Taagepera & Grofman, 1985; Taagepera & Shugart, 1989, p. 93), suggesting that an effect of $M$ is superimposed to that of $I$. This means that $N = I + 1$ applies best at median levels of $M$, maybe around $M = 4$. Given that the number of parties also exerts an effect on which issues are considered politically distinct, it might be better to use a form that does not suggest unidirectional causality:

$$N - I = 1.$$  \hspace{1cm} (1')

Whatever the pressures on issue dimensions and the number of parties, the difference between $N$ and $I$ tends to be preserved.

The second equation ties the effective number of assembly parties to magnitude (Taagepera & Shugart, 1993):

$$N = 2.15 M^{3/16}. \hspace{1cm} (2)$$

It is based on the notion that in the absence of any further information, one is best off by guessing at the middle of the logically possible range, by taking the geometric mean of the lower and higher boundary values. For seat-winning parties in one single district these boundaries are 1 and $M$, and hence the expectation value is $M^{5/3}$. Through a succession of such estimates the nationwide effective number of assembly parties is obtained in terms of the product $MS$, $S$ being the number of seats in the assembly. Using the worldwide median value for $S$ results in the equation above. It is confirmed empirically, albeit subject to wide random variation, part of which may be due to the varying number of issue dimensions. Here, too, a nondirectional format might be preferable:

$$N/M^{3/16} = 2.15. \hspace{1cm} (2')$$

This leaves open the possibility that a preexisting number of parties may induce a change in electoral rules so as to make it safer for the existing parties or to exclude new competitors.

Eliminating $N$ between the two equations leads to an equation that joins $I$ and $M$—either

$$I = 2.15 M^{3/16} - 1 \hspace{1cm} (3)$$

or, more symmetrically,

$$(I + 1)/M^{3/16} = 2.15. \hspace{1cm} (3')$$
The corresponding curve is shown in Figure 2. Instead of the average trend, though, it looks more like the lower envelope of the data scatter. This outcome could be interpreted to mean that highly permissive electoral systems (high $M$) are adopted only when the number of issue dimensions is high, but the reverse is not always true: Even heterogeneous societies sometimes adopt fairly low-$M$ electoral systems (France V, in particular). When this is done, the electoral system, in turn, may exert pressure to reduce the number of politically distinct issue dimensions.

Whereas Equations 1 and 2 are based on rational models, Equation 3 is an indirect derivative and need not reflect the average relationship. But it is aesthetically pleasing to have an equation for each side of the $N$-$I$-$M$ triangle in Figure 1.

It is now a matter of fusing these equations that deal with variables $I$, $M$, and $N$ two at a time into one equation that includes all three variables. Start with Equation 1: $N = I + 1$. The “1” represents the minimal baseline: There must be at least one party, even if there are no issues to divide the polity ($I = 0$). Above this baseline, the first term ($I$) implicitly includes the effect of an average effective magnitude. A simple but still flexible way to introduce it explicitly is to replace $I$ with $kI^A M^B$, in which $k$, $A$, and $B$ are constants. The resulting equation, $N = kI^A M^B + 1$, implies that $N = 1$ for $I = 0$, regardless of $M$. This is indeed an essential theoretical requirement: In the absence of any issues, a mere increase in electoral system permissiveness should not create new parties. Note that no additive format, such as $N = kI + hM^B$ (where $h$ is another constant), could satisfy this requirement.

The following restricts the choice of $k$. A single issue dimension combined with single-member districts ($I = 1$, $M = 1$) must lead to a pure two-party nationwide constellation ($N = 2$). This is the classical Duverger Law. If so, then one must have $k = 1$. Hence, the following general format emerges:

$$N = I^A M^B + 1. \quad (4)$$

Simple ways should be tried unless evidence imposes the need for more complexity, and this format is the simplest way to satisfy two rational conditions: When $I = 0$, then $N = 1$ regardless of $M$; and when $I = M = 1$, then $N = 2$. Such extreme conditions might materialize exceedingly rarely, but a general equation must apply under all conditions. See Appendix B for further discussion of such “boundary conditions.”

The values of Constants $A$ and $B$ must be determined. Here rational considerations give only the broadest of guidance. Note that Equation 1 follows the format of Equation 4, with $A = 1$ and $B = 0$, giving full weight to $I$ and none to $M$. If both $I$ and $M$ actually affect the number of parties to an equal extent,
the impact of $I$ is reduced by one half, and consequently $A$ might be reduced by one half. Thus, the expectation would be

$$1 > A \times 0.5.$$ 

Now bring in Equation 2, which gives full weight to $M$ and none to $I$, so that $A = 0$, whereas $B$ is around $3/16 = 0.1875$, although the absence of the additive constant 1 in Equation 2 complicates comparison. If both $M$ and $I$ actually affect the number of parties to an equal extent, $B$ too might be reduced by one half. Thus, the expectation would be

$$0.1875 \times B \approx 0.094.$$ 

Beyond this point, rational model building gives way to empirical determination of the values of constants $A$ and $B$ so as to minimize disagreement with observed values of $N$ in Table 1. The best data fit is obtained with $A$ around 0.60 and $B$ around 0.15, both values being in the expected ranges. Thus, the final result is

$$N = I^6 M^{15} + 1. \quad (5)$$

In a format that avoids suggestion of causal directionality, it could be written as

$$N - I^6 M^{15} = 1. \quad (5')$$

On this empirical basis, $A = 4B$. I don’t know whether this simple ratio has theoretical implications. The disparity in values of constants is not to be construed as $I$ having more influence on the number of parties than $M$. Because $I$ varies from 1 to 4.5 by Lijphart’s (1984) estimates, $I^6$ varies from 1 to 400. This is of the same order as possible variation in $M$.

**EMPIRICAL DATA FIT**

Let us return to Table 1, which lists the mean values of $I$, $M$, and $N$ for groups of systems identified in Figure 2 as having similar levels of $I$ and $M$. Recall that the purpose of such groupings is to smooth out random variation in $N$ that could not possibly be expressed by any simple function $N = f(I,M)$. Only after such smoothing is done, we can decide whether the relationship proposed in Equation 5 can lead to reasonable predictions.

The last three columns in Table 1 show the values of $N$ calculated, respectively, from Equations 1 ($N$ determined from $I$ alone), 2 ($N$ determined from $M$ alone), and 5. The latter represents the best data fit possible with an equation
with the general format $N = I^4M^6 + 1$ (Equation 4). As such, it is bound to fit the average data points, but agreement with all points represents a test of the model.

Figure 3 shows the actual values of $N$ for such groupings in a field of $I$ versus $M$. The format is the same as for Figure 2. Added are isometric curves—curves along which $N$ has a constant value, according to Equation 5. Overall, there is fair consistency between the observed values and the isometric curves. Only France IV deviates from the expected value of $N$ by more than 0.3—and this is the case with a highly uncertain estimate of effective magnitude. No simple modification of Equation 5 seems able to improve the fit with data significantly. This conclusion holds when isometric curves are superimposed to Figure 2, in which all countries are shown separately—the picture just becomes more blurred.

The isometric curves indicate that when the number of issue dimensions is low, the electoral system permissiveness has little effect on the number of
parties, which remains low: $N$ is 2 at $M = 1$ and rises to 3 only by $M = 100$. There are no actual cases at a large $M$, because in reality $M$ and $I$ interact: Once high system permissiveness allows for more parties (possibly based on regional or personal differences), some previously collapsed issue dimensions are likely to become distinct issues, so that $I$, too, increases.

When the number of issue dimensions is median, the system permissiveness has a major impact. At $I = 3$, single-member districts can depress the number of parties down to $N = 3$ (cf. Australia and France IV), whereas an $M$ close to 100 can result in nearly five effective parties (cf. Iceland-Israel-Netherlands). At an even higher $I$, the degree of system permissiveness should have even more impact, but the actual cases are few among stable democracies.

A large number of parties is generally assumed to be destabilizing, but it might depend on how much of it is due to a large number of issue dimensions and how much to electoral system permissiveness. High-$N$ systems ($N > 4.5$) may be quite stable when they are by-products of high permissiveness at moderate $I$ (Israel, the Netherlands). Stability may be reduced when an equally high $N$ is rooted in a high number of issue dimensions (France IV).

Does Equation 5 offer any improvement over the fit with $I$ or $M$ separately (Equations 1 and 2)? Figure 4 shows the actual values of $N$ for country groupings with similar $I$ and $M$ graphed against the values calculated from Equations 1, 2, and 5, respectively.

The predictions based on $I$ alone ($r^2 = .58$) and $M$ alone ($r^2 = .30$) correlate with actual $N$ to about the same degree when one overlooks the questionable case of France IV. It may mean that sociopolitical heterogeneity and the electoral system affect the number of parties to about the same degree, but one must be cautious, given that $I$ and $M$ are imperfect measures for these broader notions.

Correlation improves markedly when both $I$ and $M$ enter ($r^2 = .90$). The observed values are within $+0.3$ of those calculated from Equation 5 (except for France IV). When individual countries are considered separately, the systematic aspects are blurred, and correlation coefficients ($r^2 = .51$ for Equation 1, .39 for Equation 2, .44 for Equation 5) depend heavily on a few outliers.

Equation 5 basically multiplies $I$ and $M$ (rather than adding them), but the detailed functional form is more complex than could be found by simply trying various standard combinations of arithmetic operations and functional forms. The addition of 1 is based on a rational condition. Statistical studies are most fruitful when guided by rational models—and rational models, often developed on the basis of limited data, profit from further testing with data. The present study is limited to developing a model based on theoretical considerations, plus empirical determination of the values of some constants.
Figure 4. Actual values of effective number of assembly parties versus those calculated from (a) the number of issue dimensions, (b) effective magnitude, and (c) both combined.
CONCLUSIONS

As political science has developed, discussion has increasingly shifted away from arguing for a single major determinant for the number of parties. It is by now widely accepted that both sociopolitical heterogeneity and electoral systems have an effect and that this effect is interactive in a roughly multiplicative sense. However, the explicit equations enabling one to estimate the effective number of parties remained segregated: \( N \) could be calculated either as a function of \( I \) or of \( M \) but not both.

The main contribution of the present study is to supply a single equation so as to calculate the number of parties on the basis of both a measure of sociopolitical heterogeneity and a measure of the electoral system permissiveness. This combined equation is partly theoretical and partly empirical, making use of a classical data set. Its basically multiplicative nature is in line with Ordeshook and Shvetsova (1994) and Amorim Neto and Cox (1997).

Although emphasis has been on explaining the number of parties in terms of heterogeneity and permissiveness, all three factors actually affect each other. A triangle of interdependence overshadows purely unilateral cause-effect linkages.

APPENDIX A

The Meaning of Variables Used

This study operationalizes the number of parties as the “effective number of assembly parties” \( N \), the electoral system permissiveness to small parties as “effective magnitude” \( M \), and sociopolitical heterogeneity as the “number of issue dimensions” \( I \). The question is to what degree these variables express the underlying broader notions.

The Laakso and Taagepera (1979) effective number of assembly parties is defined as \( N = (\Sigma S_i^2 / \Sigma S_i)^{1/2} \), where \( S_i \) is the number of seats of the \( i \)th party. It has gained wide acceptance, although other measures also exist.

Effective magnitude takes off from district magnitude, that is, the number of seats allocated within an electoral district. A low district magnitude certainly reduces small-party access to representation. When widely disparate district magnitudes, multtiered elections, nationwide adjustment seats, and legal thresholds are involved, estimating the nationwide effective \( M \) (or the corresponding effective threshold) can become quite problematic (Lijphart, 1994, pp. 25-30; Taagepera & Shugart, 1989, pp. 135-139, 266-269). Missing an apparently secondary and little-reported stipulation in the electoral rules may cause a large alteration of effective \( M \).
In face of complex electoral rules the estimation of effective $M$ may involve a subjective element. Still, effective $M$ (or the corresponding effective threshold) remains the best available overall measure of the permissiveness of an electoral system in allowing a large number of parties to gain representation. This study uses values of $M$ published nearly a decade ago, without any reevaluation that might be influenced by the current topic.

The number of issue dimensions is the most controversial of the three variables. The aforementioned three statistical studies (Amorim Neto & Cox, 1997; Ordeshook & Shvetsova, 1994; Powell, 1982) used a different variable for social heterogeneity—the effective number of ethnic groups. The latter can be determined precisely, provided that one can agree on which ethnic groups are distinct groups. However, most issue dimensions or cleavage lines are not ethnic, the socioeconomic being by far the most ubiquitous. Lijphart’s (1984) number of issue dimensions includes them but depends on qualitative judgment of whether an issue dimension is salient or only semisalient in political debate. Thus, $I$ comes only in integer and half-integer amounts, and educated observers may disagree somewhat (although in practice large disagreements might be rare).

The underlying difficulty is that social heterogeneity is not the same as political heterogeneity. The former deals with potential cleavages, the latter with the actually politicized ones. Race, language, ethnicity, and religion may play a political role in some countries and none in others. If a feature is not politicized, it may mean that people do not pay attention to it or, to the contrary, that it is such a potential hot potato that the elites agree to keep it out of politics; such was the case of the ever-smoldering language issue in Belgium up to the 1970s. In the reverse direction, some political issues do not reflect preexisting social heterogeneity. Foreign policy issues, such as whether to join the European Union, may divide the public in a Nordic country along a cleavage line lacking social basis.

In terms of correlating with the number of parties, $I$ appears much stronger than the ethnic indicator. Amorim Neto and Cox (1997) obtain no correlation at all between $N$ and the effective number of ethnic groups ($r^2 = .01$); $r^2$ goes up to .61 when only electoral variables are used, and to .69 when they are joined to the effective number of ethnic groups. In contrast, for the 22 systems used here, the correlation between $N$ and $I$ alone is also quite high: $r^2 = .56$ (Lijphart, 1984, p. 148) for the best fit line and $r^2 = .51$ for the theory-based line $N = I + 1$. But here another difficulty arises.

Politiciized issues mean issues on which some parties disagree. With a larger effective number of parties, more of such separate issues may be identified, so that the connection may be tautological. Rather than politicized issues determining the number of parties, the observed number of parties may determine how many separate issues can be seen. Note, however, that the correlation between $I$ and $N$ is far from perfect. Thus, something else than the number of parties must enter the estimate of $I$. And whatever this is, it yields, along with $M$ and through Equation 5, a much improved estimate of $N$ in Figure 4.
The highly subjective method of determining \( I \) is, of course, less than satisfactory. Much more operationalization is needed. But as of now, facing the choice between a precise measure (effective number of ethnic groups) that misses most of the politicized issue dimensions and a fuzzy one (Lijphart’s \( I \)) that includes them, this author is resigned to use the latter.

**APPENDIX B**

**Dimensionality and Boundary Conditions**

This appendix deals with two aspects inspired by model building in physics that could be useful in social model building. Of course, no analogies with specific physical quantities are involved.

**Dimensionality.** Regarding the advantages of multiplicative over purely additive models, an even broader argument can be added to those advanced by Amorim Neto and Cox (1997) in this particular case. It is based on the notion of dimensionality.

It is conceptually impossible to add kilometers and hours, because they do not have the same dimensionality—but one can multiply or divide them. Division produces velocity (in kilometers per hour), and kilometers-times-hours also can be given a meaning and is sometimes used. Addition of kilometers and hours becomes possible only when preceded by a constant of proper dimensionality. One can add kilometers and \( k \) times hours, provided that \( k \) itself is in kilometers per hour (meaning that \( k \) is a velocity), because \( \text{km/hr times hours} \) is also dimensionally in kilometers.

Although quantities like the number of issue dimensions or of parties are pure numbers lacking physical dimensionality, some elements of dimensional consistency still enter. Whenever one wants to add quantities \( A \) and \( B \), one must ask whether they are dimensionally compatible, or whether it should be \( A + kB \), where \( k \) has the same dimension as \( A/B \) (analogous to the kilometers/hour above). However, a multiplication, \( AB \), is dimensionally always possible. Therefore, it makes sense to expect a multiplicative relationship, unless an additive one imposes itself.

**Boundary conditions.** Establishing relationships between quantities by considering what values they must have in some extreme situations is a frequent practice in physics. These “boundary conditions” plus the requirement that the intervening change be continuous restrict the options. Indeed, at times the boundary conditions determine uniquely what happens in between. Thus, when it is stipulated which walls of a container conduct electricity and which are insulators, the entire electric field configuration within the container is determined.

The reasoning by conceptual extremes follows a similar pattern. The boundary conditions involved may be difficult or even impossible to reach in practice. The gas law mentioned in the main text applies only to a nonexistent ideal gas, the molecules of which have zero volume. Yet, the model built on this unrealistic basis is an indispensable starting point for the study of the more complex real gases.
In this study two such boundary conditions are invoked: What would happen if the number of issue dimensions went to zero, and what would happen if the effective magnitude went to one under specified conditions? Note that $I = 0$, $M = 1$, and $N = 1$ are the lowest values these variables could possibly take.

1. The least possible value of $I$ is zero. When there are no issue dimensions to create even a single political cleavage, there must be only one party: $I = 0 \rightarrow N = 1$, regardless of $M$. The moment we envisage a second party, we must give a reason for its appearance, which inherently means raising an issue. A different scenario for $I = 0$ involves a single catchall coalition party (somewhat on the Malaysian lines) that excludes all potential issues from public political debate. (I owe this example to Gary Cox.) Whatever the cause for $I = 0$, such a political nirvana is not to be expected in real democracies. But rational models must satisfy even unreachable ideal-boundary values. It is a restriction but also a powerful guide in devising appropriate models.

2. The least possible value of $M$ is one. If the number of issue dimensions is also one, then the nationwide effective number of assembly parties should be two, if the following conditions prevail. In each of the single-member districts, of course, the effective number of winners is one. Nationwide, $N = 2$ emerges if the single-issue dimension divides the public into two equal parts and if each party wins in one half of the districts. These conditions rarely prevail exactly, but if they did, then we would have $N = 2$ exactly. Once more, any general model must be able to fit this situation, too.

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