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Abstract

Essays in Exchange Rate Dynamics

by

Jae Hoon Choi

This dissertation studies the dynamics of exchange rates and their effect on nominal and real macro variables and furthermore on policy choices.

The dissertation provides a theoretical framework where a policymaker can choose a path of international policy portfolio of capital controls and exchange rate regimes under financial frictions. The paper presents a novel theoretical approach to explain the coexistence of active use of capital controls and volatile exchange rates, which has become a robust feature in emerging market economies. Building upon the small open economy framework, I create an environment where the policymaker can decide the level of exchange rate regimes – instead of a binary choice of exchange rate regime, fixed or floating – in response to external shocks, where capital controls are introduced as a tax on international capital flows. I further assume that regime choice is subject to a financial friction; breaking the peg signals the country’s economic instability, which raises the country’s risk premium. Under this set-up, the floating exchange rate regime does not welfare-dominate the capital controls any longer because loosening/losing the controls over exchange rates may expose households to additional risk premium. The simulation results show that the coexistence of managed float and capital controls becomes optimal. Furthermore, this additional friction has a multiplying
effect, which makes exchange rate stabilization become important to prevent a bigger welfare loss. It also captures that optimal capital controls indirectly manage exchange rate depreciation, which allow policymakers to put less resource to stabilize the exchange rates.

Considering the fact that the countries actively intervene in the foreign exchange market, the dissertation re-investigates one of classical puzzles in international economics using a new estimation technique and a modern data categorization methodology. The Purchasing Power Parity puzzle states that even though real exchange rates may converge to parity in the long run, the consensus emerging from an extensive literature appears to be that the rate of mean-reversion is slow, where a half-life mean-reversion is between 3 - 5 years; however, this is much too long to be compatible with arbitrage. This paper first proposes that investigating the periods of de facto floating regime will explain seemingly unrealistic persistence in real exchange rates by presenting lower persistence in real exchange rates than the estimates of previous studies, which include the periods de facto fixed regime in their data set by using de jure regimes. Secondly, previous studies have included the periods when real exchange rates that are within “the regions of inaction”; because the trend of mean-reversion rates is non-linear, including the periods when real exchange rates are already converged to their means will bias the estimates toward zero, which are translated to the slow mean-reversion in real exchange rate estimates. Therefore, unbiased mean-reversion estimates can be estimated if I investigate the periods when the sample countries are under de facto float regimes and the periods when real exchange rates are statistically far from their
means. Studying the data of nineteen goods CPI for eleven countries confirms these propositions. The mean group estimation decreases the half-life by 28.26% (half-life of 23.14 months) compared to fixed-effects estimation. The exchange rates regime dummy decreases the half-life estimate to 19.81 months and the region of inaction dummy decreases the estimates to 15.22 - 20.54 months. Using both dummy variables elicits the results that make the puzzle less puzzling; the half-life estimates are 9.30 - 13.89 months.

The dissertation also explores the exchange rate regime-elastic risk premium quantitatively. This paper takes foreign investor’s perspective and studies how the trend of risk premium changes when the regime switches in ten emerging market economies through the event study framework. Using a daily data set, the events of de facto regime switching are identified following the comparable methodology used in Calvo and Reinhart (2002), and EMBI+ is used as a proxy for country risk. The results confirm that switching exchange rate regimes from fixed to floating incurs an abrupt increase in average risk premium. EMBI+ rises by 141.11 - 165.48 basis points (0.257 - 0.525 standard deviations) around the events and shows 205.72 - 340.30 basis points of 40-day average difference before and after the events. The abnormal return estimates during the events range from 0.0036 to 0.0075, which imply 8.31 - 225.85% increase in the returns of EMBI+ during the periods of breaking pegs.
To my parents, my wife,

and my son.
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Chapter 1

Capital Controls and Foreign Exchange

Market Intervention

1.1 Introduction

Since the financial crises, capital has been flowing back to emerging market economies. (Ostry et al., 2010) Increase in capital inflows can allow financially less-developed countries to allocate the resource more efficiently and raise growth rates by providing financing for high-return investment. It can also foster the diversification of investment risk and intertemporal trades. Excessive capital inflows, however, raise concerns over undue appreciation pressure on the currency, which can reduce the competitiveness of the emerging market countries’ export sector and raise the possibility of a sudden reversal in short-term inflows and concomitant risks to macroeconomic and financial stability. Such risks require appropriate policy responses. Capital controls are
the measures to restrict volatile movements of capital inflows and outflows, while employing capital controls is constrained by the economy’s monetary policies and foreign exchange regime choices.

Mundell’s trilemma states that a country cannot simultaneously have free capital flows, monetary sovereignty, and fixed exchange rates (Mundell, 1963); for example, if one intends to have a monetary policy that is independent from other countries and a freely moving capital over its borders, it is inevitable to allow exchange rates to freely float due to the potential arbitrage opportunities. This classic theory on international policy portfolio has been empirically validated; Obstfeld et al. (2005) study 130 years of policy portfolios in a trilemma framework and find that they are in general constrained by the trilemma. The countries’ international policy portfolios may be constrained by trilemma; however, they do not always reside in the corners of trilemma. As Popper et al. (2013) suggest, the choices of international policy portfolio are not final but can move along the lines of the triangle from corner to corner.

Especially, many developing countries’ policies tend to stay in the middle of corners. Figure 1.1 presents the levels of capital controls imposed in 72 countries (36 developed economies and 36 developing economies) from 1995 to 2013 – the capital control index from Fernández et al. (2015) represents the intensity of capital controls on a scale of 0 to 1. It is clear that developing countries have managed capital flows more actively than developed countries have. Figure 1.2 shows that while the average level of capital controls in developing countries is consistently higher than the one in advanced countries, the exchange rates in developing countries, however, are not steady,
and are in fact as volatile as in developed countries. This does not confirm that developing countries have freely floated their exchange rates as the developed countries may have; however, at least it implies that more countries in emerging markets have adopted managed float or managed peg regimes. The empirical studies also support this observation. Klein and Shambaugh (2008) study 3,253 exchange rate spells of 104 developing countries from 1973 to 2004 and find that 44.19% of the peg spells have been broken within 2 years, and that only 26.78% of the peg spells stay pegged for more than 5 years. Batini et al. (2006) also report that the share of developing countries adopting fixed exchange rate regimes fell from 75% in 1985 to 55% in 2005. Fiess and Shankar
Figure 1.2: Average level of capital controls (left) and exchange rate volatility (right) of developing, advanced, and Eurozone countries (1995 - 2013) (Data source: Fernández et al. (2015) and University of British Columbia – The Pacific Exchange Rate Service) (2009) study fundamental pressure on exchange rate regimes in 15 countries from 1985 to 2004 and identify that countries frequently release the pressure with low intervention on foreign exchange markets.

A large literature studies the mechanics of central banks’ foreign exchange market intervention process and furthermore suggest policy guidelines under managed peg/float regimes.\(^1\) There is also a growing literature that conducts model-based assessments of the welfare costs and benefits of capital controls.\(^2\) However, the attempts to study the portfolio of both international policies in a theoretical framework have been rare; one of the potential reasons for scarcity stems from a complexity of employing both capital flow management and foreign exchange market decision in a theoretical framework.

This paper contributes to the literature by introducing a novel and tractable

\(^1\)See Schmitt-Grohé and Uribe (2012), Farhi and Werning (2012), and Farhi and Werning (2014).

approach to allow us to assess the optimal portfolio of capital controls and foreign exchange market intervention. In the model, I assume that rational policymakers know the trajectories of shadow exchange rates under different regimes and decide how much to float or peg by simply making series of regime choices. Furthermore, incorporating the empirical findings that regime choice affects country’s risk premium, I introduce a financial friction – regime-elastic risk premium – in the model and avoid the uninteresting case, where currency price adjustment solves all problems, which is a common feature in standard small open economy models. This extension is made on the standard New Keynesian small open economy framework that is first constructed in Galí and Monacelli (2005) and Galí (2009) and that is later modified in Farhi and Werning (2014) by employing incomplete market to study the optimal level of capital controls. By minimizing the welfare loss function, the balanced path of optimal policy portfolio in response to the temporary regime-elastic risk premium shock is derived, and the results suggest that the coexistence of managed peg/float and capital controls are optimal to reduce the welfare loss of the economy. Furthermore, the model reflects “fear of floating” documented in Calvo and Reinhart (2002); the additional friction has a multiplying effect and it makes exchange rate stabilization become essential to prevent a further loss in welfare. It also captures that capital controls indirectly manage exchange rate depreciation, which allow policymakers put less resource to stabilize the exchange rates.

Section 1.2 reviews the literature. Section 1.3 sets up a model. Section 1.4 derives the equilibrium in a log-linearized form. Section 1.5 analyzes the optimal policy in the small open economy under a particular parameterization. Section 1.6 concludes.
1.2 Literature Review

This paper is related to a large literature on the volatility of international capital flows. The seminal study of Calvo (1998) sets a theoretical framework focusing on sudden stops, large and unexpected cutbacks in capital flows to a country. Galí and Monacelli (2005) and Galí (2009) set a standard New Keynesian small open economy framework to study monetary policies and their welfare implication in response to a temporary productivity shock, which creates volatile international capital movements. Later, the welfare implications of taxes has been explored in a context of open economies subject to volatile capital inflows. The research points to welfare-enhancing effects of taxes on capital inflows (Korinek, 2010) or on foreign debt (Bianchi, 2011). A related strand of literature emphasizes pecuniary externalities in the borrowing constraints. Jeanne and Korinek (2010) presents a simple model where a “financial accelerator” can be managed through the optimal Pigouvian tax. Increases in borrowing push up the price, and this raises the value of the collateral against which borrowing is secured. It then becomes easier to borrow to buy assets, which pushes prices even higher. This operates in reverse during a downturn. Falling prices erode the value of collateral, tightening credit and depressing demand. When they take on debt, borrowers fail to take into account the effect of their actions on the collateral constraints faced by others. To offset this, the authors propose a counter-cyclical tax on debt. Benigno et al. (2013), on the other hand, provides a different perspective that a commitment to a price support policy in the event of crisis welfare-dominates prudential capital controls. By
considering a broader set of policy instruments within the same theoretical framework (instead of assuming that a tax on debt is the only instrument), the authors show that capital are not necessary because they are the second-best instrument, while other tools can achieve the first-best allocation. The authors adopt a similar model economy as in Bianchi (2011) where a tax on borrowing is the only policy tool, and introduce two other distortionary policy instruments, a tax on nontradable consumption and a tax on tradable consumption.

These studies provide a rationale for controls on capital movements to prevent over-borrowing; however, because the models in these papers are in real terms, the optimal capital controls become independent of the choice of the exchange rate regimes. A few attempts have been made to assess capital flow management assuming nominal rigidity. Schmitt-Grohé and Uribe (2012) study second-best policies in economy with downward wage rigidity and a fixed exchange rate regime and finds that optimal capital controls are prudential and achieve large reduction in unemployment and increases in welfare. The authors characterize a wedge in the model with a innocuous and rather more realistic assumption on the labor market. However, the wedge is present only with fixed exchange rate, and switching to a float regime always achieves the first best. More recently, Farhi and Werning (2012, 2014) extend the framework designed by Galí and Monacelli (2005) to derive the optimal path of capital controls. The authors set an environment of imperfect risk sharing and introduce the international consumption wedge, which can be controlled by a tax on international capital flows. The results suggest that optimal level of capital controls is nonzero even under floating regime.
This paper also relates to the literature on sterilized foreign exchange intervention in the presence of market friction. The earlier literature includes Weber (1986) that considers the case in which the impossible trinity is not impossible, provided that bonds denominated in different currencies are not perfectly interchangeable. The model by Evans and Lyons (2002) employs the portfolio-balance channel under a microstructure approach. The authors estimate a partial equilibrium model in which the trading process reveals information contained in order flows. Maggiore and Gabaix (2015) build an analytically tractable 2-period general equilibrium model where constrained international financiers intermediate capital flows across countries. They provide a novel micro-foundation to the portfolio balance channel and analyze the welfare effects of heterodox policies such as foreign intervention. Cavallino (2016) employs the framework developed in Maggiore and Gabaix (2015) and studies the effects of exchange rate fluctuations driven by capital flows and characterize the optimal foreign exchange intervention.

1.3 Model

The model builds on the standard new Keynesian small open economy framework first introduced in Gali and Monacelli (2005) and Gali (2009) and further developed in Farhi and Werning (2014). The key difference between the two models is that Gali and Monacelli (2005) and Gali (2009) posit complete markets for securities traded internationally, while Farhi and Werning (2014) create an environment in which international
financial markets are incomplete and therefore risk-sharing between countries is limited. This imperfect risk-sharing between home and foreign households adds distortions to the Gali-Monacelli framework, and thus creates a room in the model for interventions. Using a similar methodology, this paper introduces an additional distortion: a signaling effect of breaking the peg on risk premium. This distortion creates an additional gap that the policy makers can close, and therefore allows both policies – capital controls and choice of exchange rate regime (potentially floating exchange rate regime) – to coexist. Given the equilibrium conditions developed in this section and the section 1.4, the social planner identifies the optimal path of time-varying Pareto weight, which provides the optimal levels of capital controls and foreign exchange market intervention over time.

1.3.1 Households

1.3.1.1 Exchange rate regime decision: Managed peg/float

Characterized as Mundell’s trilemma, capital controls become a unnecessary policy under floating exchange rate regimes. Therefore many models in the literature that assess the validity of capital controls implicitly assume fixed exchange rate regimes. In the model constructed in this paper, policymakers change regimes by controlling the levels of intervention on exchange rates, while controlling international capital flows. This modification is achieved through setting the dynamics of exchange rates as the geometric average of the change in floating exchange rates and the change in fixed
exchange rates, which can be expressed as the following:

\[ \frac{E_{t+1}}{E_t} = \left( \frac{E_{t+1}^f}{E_t^f} \right)^{\lambda_t} \left( \frac{\bar{E}_{t+1}}{E_t} \right)^{1-\lambda_t}, \]

(1.1)

where \( E_t \) is the effective nominal exchange rate at time \( t \), \( E_t^f \) is the nominal exchange rate under the floating exchange rate regime, \( \bar{E}_t \) is the nominal exchange rate under the fixed exchange rate regime, and \( \lambda_t \in [0, 1] \) denotes the level of intervention on the exchange rate. Therefore, when \( \lambda_t \) is high and close to 1, the level of intervention is low and the dynamic of the exchange rate is closer to the one under a floating regime. On the other hand, a lower \( \lambda_t \) implies that the economy requires a higher level of intervention on exchange rates.

For a tractability, I further assume that the country’s foreign reserve is sufficient enough to intervene in the exchange market without altering the money supply, so the exchange market intervention does not constrain its central bank’s other monetary policy decisions.

1.3.1.2 Exchange rate regime-elastic risk premium shock

Equation (1.1) is a convenient way to introduce exchange rate regime selection into the model, compared to the attempts made by previous studies to introduce foreign exchange market intervention in models. However, if a policymaker can control the exchange rate regime under a plain small open economy framework setting, the policymaker will always choose a freely-floating regime. This is mainly due to the assumption
made in the small open economy framework that the country is small enough that its policy or behavior do not alter world prices, interest rates, or incomes. Therefore, under this setting, the country’s decision to break the exchange rate peg shall induce only minimal costs within the system, and compared to the benefits from stabilization via adjustment in exchange rates, its cost is insignificant. Therefore I introduce an additional friction in the model by modifying the shock process based on the assumption that breaking exchange rate pegs signal the instability of the economy.

Empirical studies identify the effect of breaking the pegs and support the assumption made above. Alesina and Wagner (2006) find that switching exchange rate regimes from fixed to floating brings external costs into the economy. They study the countries whose de jure and de facto exchange rate regimes are different to argue that wide exchange rate fluctuations (especially devaluations) are taken by markets as an indication of poor economic management and that breaking pegs signal instability of the economy. An event study analysis on daily data also supports this assumption; the same results with more robust evidence were found.

The regime-elastic risk premium is introduced as a modified exogenous shock in the consumer’s budget constraint, and the section 1.3.1.4 discusses further how it is employed in the model.

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3Detailed setup and the full results of the study are documented in another chapter of my dissertation, “Cost of Floating Exchange Rates: Should we fear to float?”
1.3.1.3 Households problem

The world economy is modeled as a continuum of small open economies represented by the unit interval; \( i \in [0, 1] \). The focus is on the behavior of a single country called “home” and its interaction with the world economy. A representative household seeks to maximize

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_{t}^{1-\sigma} - N_t^{1+\phi}}{1-\sigma - N_t^{1+\phi}} \right]
\]  

(1.2)

where \( N_t \) is labor, and \( C_t \) is a composite consumption index defined by

\[
C_t \equiv \left(1 - \alpha \right)^\frac{\eta-1}{\eta} C_{H,t}^{\eta-1} + \alpha^\frac{1}{\eta} C_{F,t}^{\eta-1}
\]  

(1.3)

where \( \alpha \) corresponds to the share of domestic consumption allocated to imported goods, and \( C_{H,t} \) is an index of consumption of domestic goods given by the constant elasticity of substitution (CES) function

\[
C_{H,t} \equiv \left( \int_0^1 C_{H,t}(j) \left( \frac{1}{j} \right)^{\frac{\eta-1}{\gamma}} dj \right)^{\frac{\gamma}{\gamma-1}}
\]  

(1.4)

where \( j \in [0, 1] \) denotes the good variety. \( C_{F,t} \) is an index of imported goods given by

\[
C_{F,t} \equiv \left( \int_0^1 C_{F,t}(i) \left( \frac{1}{i} \right)^{\frac{\gamma-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}}
\]  

(1.5)
where $C_{i,t}$ is an index of the quantity of good imported from country $i$ and consumed by domestic households, and is given by a CES function

$$C_{i,t} \equiv \left( \int_0^1 C_{i,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}. \quad (1.6)$$

$\alpha \in [0,1]$ can be interpreted as a measure of openness of a country. $\epsilon > 1$ is the elasticity of substitution between varieties produced within any given country. $\eta > 0$ is the substitutability between domestic and world goods in home country, and $\gamma > 0$ measures the substitutability between domestic and world goods in different foreign countries.

### 1.3.1.4 Household budget constraints

Households maximizes (1.2) subject to a sequence of budget constraints of the form

$$\int_0^1 P_{H,t}(j)C_{H,t}(j) dj + \int_0^1 \int_0^1 P_{i,t}(j)C_{i,t}(j) dj di + D_{t+1} + \int_0^1 E_{i,t}D_{t+1}^i di \leq W_tN_t + \Pi_t + T_t + R_{t-1}D_t + \int_0^1 R_{t-1}^iE_{i,t}D_{t}^i di \quad (1.7)$$

where

$$R_t^i \equiv \tilde{R}_t^i \frac{1 + \frac{\tau_t}{\tau_t} \Psi_t^{1+\xi\lambda_t}}{\Psi_t^i} \quad (1.8)$$

for $t = 0, 1, 2, \ldots$, where all terms are expressed in domestic currency. $P_{H,t}(j)$ is the price
of domestic variety $j$, $P_{i,t}(j)$ is the price of variety $j$ imported from country $i$, $D_t$ is the home bond holdings of home agents, and $D^i_t$ is bond holdings of country $i$ of home agents. $W_t$ is the nominal wage, and $T_t$ denotes lump-sum transfers/taxes. Capital controls are introduced as a tax (or a subsidy) on foreign borrowing/lending. Risk premium shocks $\Psi_t$ and $\Psi^i_t$ create wedges between local and foreign investors, and $\psi_t \equiv \log \Psi_t$ and $\psi^i_t \equiv \log \Psi^i_t$ follow AR(1) processes $\psi_t = \rho_\psi \psi_{t-1} + \epsilon_{\psi,t}$ and $\psi^i_t = \rho_\psi \psi^i_{t-1} + \epsilon^i_{\psi,t}$. $\tau_t$ is a tax on net capital flows in the home country and $\tau^i_t$ is a tax on net capital flows in country $i$. The taxes collected here are rebated lump-sum to the households.

The prices of bonds are determined by the interest rate $R_t$ in the home country and the interest rate $R^i_t$ of country $i$. The interest $R^i_t$ that home agents pay for their loans from country $i$ is determined by the interest rate $\tilde{R}^i_t$ that the country $i$ charges to all agents including their local agents, the net level of risk premium shocks $\Psi_t^{1+\xi\lambda_t}/\Psi^i_t$, and the net capital controls $(1 + \tau_t)/(1 + \tau^i_t)$.

The exchange rate regime-elasticity of the risk premium shock is characterized by the exponential term $(1 + \xi\lambda_t)$ on the risk premium shock. $\lambda_t \in [0, 1]$ denotes the level of exchange market intervention as discussed in (1.1). $\xi > 0$ is the multiplier on the level of foreign exchange market intervention, and it reflects the investors’ exchange rate regime sensitivity. Suppose domestic investors are international net borrowers ($D^j_t < 0$). As the policy makers loosen or lose their controls over exchange rates ($\lambda_t > 0$), the foreign lenders perceive this policy change as an economic instability of home country and increase the price of their loans higher $(1 + \xi\lambda_t > 1)$. If home country’s risk premium is more sensitive to volatile exchange rates (high $\xi$), it is subject to face even higher...
risk premium shock \((1 + \xi \lambda_t)\) becomes higher given the same value of \(\lambda_t\). Introducing this friction creates an environment where floating regime may be a suboptimal exchange rate policy because it can magnify the shock to the economy and therefore can tighten consumers’ budget constraints, which lowers the level of total consumption and eventually lowers the level of welfare.

Price indices are defined as following. Home consumer price index (CPI) is defined as
\[
P_t \equiv \left( (1 - \alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta} \right)^{\frac{1}{1-\eta}}.
\] (1.9)

The terms of trade are defined by
\[
S_t = \frac{P_{F,t}}{P_{H,t}} = \frac{E_t P^*_t}{P_{H,t}}
\]
and the real exchange rate is
\[
Q_t = \frac{E_t P^*_t}{P_t}.
\]

Then log-linearized CPI around the steady state with \(P_{H,t} = P_{F,t}\) yields
\[
p_t \equiv (1 - \alpha)p_{H,t} + \alpha p_{F,t}
\]
\[
= p_{H,t} + \alpha s_t,
\] (1.10)
where the (log) terms of trade is

\[ s_t \equiv p_{F,t} - p_{H,t} \]
\[ = e_t + p^*_t - p_{H,t} \]  \hspace{1cm} (1.11)

and the real exchange rate is

\[ q_t \equiv p_{F,t} - p_t \]
\[ = e_t + p^*_t - p_{H,t} - \alpha s_t \]
\[ = s_t - \alpha s_t \]
\[ = (1 - \alpha)s_t. \]  \hspace{1cm} (1.12)

From (1.10), the following relationship between the domestic inflation and CPI inflation is derived

\[ \pi_t = \pi_{H,t} + \alpha \Delta s_t, \]  \hspace{1cm} (1.13)

where the domestic inflation is defined as \( \pi_{H,t} \equiv p_{H,t+1} - p_{H,t} \) and CPI inflation is defined as \( \pi_t \equiv p_{t+1} - p_t. \)

Home’s producer price index (PPI) is defined as

\[ P_{H,t} \equiv \left[ \int_0^1 P_{H,t}(j)^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}}. \]  \hspace{1cm} (1.14)
and the price index for imported goods is

\[ P_{F,t} \equiv \left[ \int_0^1 P_{t,1}^{1-\gamma} \, di \right]^{\frac{1}{1-\gamma}}, \quad (1.15) \]

where a country \( i \)'s PPI is defined as

\[ P_{t,i} \equiv \left[ \int_0^1 P_{t,1}^{1-\epsilon} \, dj \right]^{\frac{1}{1-\epsilon}}. \quad (1.16) \]

Then the optimal allocation of any given expenditure within each category of goods yields the demand functions\(^4\)

\[ C_{H,t}(j) = \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\epsilon} C_{H,t}; \quad C_{t,i}(j) = \left( \frac{P_{t,i}(j)}{P_{t,t}} \right)^{-\epsilon} C_{t,i} \quad (1.17) \]

and

\[ C_{t,i} = \left( \frac{P_{t,i}}{P_{F,t}} \right)^{-\gamma} C_{F,t}. \quad (1.18) \]

Assume further that the foreign countries are identical and also that there are no risk premium shocks in the foreign countries and foreign countries do not impose capital controls, then we can re-express the household budget constraints\(^5\) as

\[ P_tC_t + D_{t+1} + E_tD_t^∗ \leq W_t N_t + \Pi_t + T_t + R_{t-1} D_t + R_{t-1}^* E_t D_t^*, \quad (1.19) \]

\(^4\)See Appendix A.1 for derivation.
\(^5\)See Appendix A.2 for derivation.
where
\[ R_t^* = \tilde{R}_t^* (1 + \tau_t) \Psi_t^{1+\xi \lambda_t}. \tag{1.20} \]

The world variables are denoted with a star. The wedges that risk premium shocks and capital controls create are introduced to the model in a similar way. The key difference between two is that risk premium shocks affect both the interest rates that home agents perceive they can borrow and lend to the world and the interest rates that agents can borrow and lend to the world. On the other hand, capital controls affect only the interest rates that home agents perceive because the tax collected from capital controls is rebated to the home agents; the lump-sum rebate takes the following form

\[ T_t = -\tau_{t-1} \tilde{R}_{t-1}^* \Psi_{t-1}^{1+\xi \lambda_{t-1}} E_t^* D_t^* + \tau_L W_t N_t, \tag{1.21} \]

where \( \tau_L \) is a constant labor tax. In other words, capital controls divert home agent’s foreign borrowing/lending to domestic lenders/borrowers by changing the perceived world interest rates. Therefore it is clear that not only should the optimal level of capital controls offset the risk premium shocks but it should also account for home agent’s deviation from the optimal borrowing/lending allocation and corresponding nominal changes.\(^6\)

\(^6\)This provides the reason why capital controls simply offsetting risk premium shocks \( \tau_t = \Psi^{-1} - 1 \) are not optimal in this framework.
1.3.1.5 Optimality conditions of home and world households

The optimality conditions of households yield the standard Euler’s equation with respect to the domestic return

$$\frac{1}{R_t} = \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) \right\}$$  \hspace{1cm} (1.22)

and another Euler’s equation with respect to the world return

$$\frac{1}{R^*_t (1 + \tau_t) \Psi_t^{1+\xi \lambda_t}} = \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) \left( \frac{E_{t+1}}{E_t} \right) \right\}.$$  \hspace{1cm} (1.23)

Combining (1.22) and (1.23), we have the uncovered interest parity (UIP) condition,

$$R_t = \tilde{R}_t \frac{E_{t+1}}{E_t} (1 + \tau_t) \Psi_t^{1+\xi \lambda_t},$$  \hspace{1cm} (1.24)

where the risk premium shock $\Psi_t$ introduce a wedge, which may be magnified by loosening controls on exchange rates ($\lambda_t > 0$), in the UIP condition. The capital controls $\tau_t$ are expected to lean against the wind to close the gap.

The optimal labor supply policy is

$$\frac{W_t}{P_t} = C_t^{\sigma} N_t^{\phi}.$$  \hspace{1cm} (1.25)

Assuming all foreign countries are identical, when the world households solve their problems, the optimality conditions yield the symmetric Euler’s equation as (1.22),
\[
\frac{1}{R^*_t} = \beta E_t \left\{ \left( \frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \left( \frac{P_t^*}{P_{t+1}^*} \right) \right\}. \tag{1.26}
\]

Then, combining (1.23) and (1.26), together with the definition of the real exchange rate, yields the following expression of incomplete international risk sharing between home and world households

\[
\left( \frac{C_t}{C_t^*} \right)^{1/\sigma} Q_t = E_t \left\{ \left( \frac{C_{t+1}^*}{C_{t+1}^*} \right)^{1/\sigma} Q_{t+1} \right\} (1 + \tau_t) \Psi_t^{1+\xi \lambda_t}. \tag{1.27}
\]

After iteration, (1.27) yields the Backus-Smith condition

\[
C_t = \Theta_t C_t^* Q_t^{1/\sigma}, \tag{1.28}
\]

where \( \Theta_t \) is a relative Pareto weight\(^7\) whose evolution is given by

\[
\Theta_t = E_t \{ \Theta_{t+1} \} \left\{ (1 + \tau_t) \Psi_t^{1+\xi \lambda_t} \right\}^{-1/\sigma}. \tag{1.29}
\]

The log-linearized form of (1.28), using (1.12) is

\[
c_t = \theta_t + c_t^* + \frac{1}{\sigma} q_t
\]
\[
= \theta_t + c_t^* + \frac{1 - \alpha}{\sigma} s_t, \tag{1.30}
\]

\(^7\)In Gali and Monacelli (2005) and Gali (2009), by assuming the complete securities markets, \( \Theta_t \) is set as constant for all \( t \), while Farhi and Werning (2013) relaxes the assumption to introduce capital controls in the model.
where the movement of $\theta_t$ is found to be

$$\theta_t = E_t\{\theta_{t+1}\} - \frac{1}{\sigma} \{\tau_t + (1 + \xi \lambda_t)\psi_t\}. \quad (1.31)$$

### 1.3.2 Firms

A firm in the home country produces a differentiated good with a linear technology represented by the production function

$$Y_t(j) = A_t N_t(j), \quad (1.32)$$

The real marginal cost is common across domestic firms and given by

$$MC_t = (1 + \tau_L) \frac{W_t}{A_t P_{H,t}}, \quad (1.33)$$

where $\tau_L$ is an employment subsidy to have steady-state output equal to the efficient level,\(^8\) and the nominal marginal cost is

$$MC^n_t = (1 + \tau_L) \frac{W_t}{A_t}, \quad (1.34)$$

and its log-linearized form is given by

$$mc^n_t = -\nu + w_t - a_t, \quad (1.35)$$

\(^8\)The employment subsidy is the result of a balancing act between offsetting the monopoly distortion of individual producers and exerting some monopoly power as a country. Farhi and Werning (2012, 2013) find that the labor subsidy is constant. ($\tau_L = \frac{\gamma-1}{\alpha} \frac{(1-\alpha)(1-\gamma)}{(1-\alpha)\gamma-\alpha} - 1$).
where \( \nu \equiv -\log(1 + \tau_L) \).

Assuming the aggregate output is defined as \( Y_t \equiv \left[ \int_0^1 Y_t(j) \frac{e^j}{e^j - 1} \, dj \right]^{\frac{1}{\epsilon - 1}} \) as similarly assumed for the aggregate consumption, from

\[
N_t \equiv \int_0^1 N_t(j) \, dj = \frac{Y_t \int_0^1 Y_t(j) \, dj}{A_t},
\]

we have an aggregate relationship in a log-linearized form

\[
y_t = a_t + n_t. \quad (1.36)
\]

### 1.3.2.1 Price setting

This paper focuses on Calvo price setting; in every period, a randomly selected fraction \( 1 - \delta \) of firms resets their prices. The firms resetting their prices solve the following problem\(^9\) to choose the newly set domestic prices \( P_{H,t} \)

\[
\max_{P_{H,t}} \sum_{k=0}^{\infty} \delta^k \mathbb{E}_t \left\{ \left( \frac{1}{R_{t+k}} \right) \left( Y_{t+k} \left( P_{H,t} - MC^n_{t+k} \right) \right) \right\}
\]

subject to the demand constraints

\[
Y_{t+k} \leq \left( \frac{P_{H,t}}{P_{H,t+k}} \right)^{-\epsilon} (C_{H,t+k} + C^*_{H,t+k}) \equiv Y^d_{t+k}(P_{H,t}).
\]

Then, the log-linearized optimal price-setting strategy is given by

\[
\tilde{p}_{H,t} = \mu + (1 - \beta \delta) \sum_{k=0}^{\infty} (\beta \delta)^k E_t \{ mc_{i+k} \},
\]

(1.37)

where \( \mu \equiv \log \left( \frac{\epsilon}{\epsilon - 1} \right) \), the log of the markup in the steady state.

The world firms face the same price setting problem, and the optimal strategy is analogous to (1.37),

\[
\tilde{p}^*_t = \mu + (1 - \beta \delta^*) \sum_{k=0}^{\infty} (\beta \delta^*)^k E_t \{ mc_{i+k}^* \},
\]

(1.38)

and for simplicity, I assume the degree of price stickiness in the world economy \( \delta^* \) is identical to that in home country \( \delta \).

### 1.3.3 Market clearing

From (1.17), (1.18), and (A.1) we have

\[
C_{i,t}(j) = \alpha \left( \frac{P_{i,t}(j)}{P_{i,t}} \right)^{-\epsilon} \left( \frac{P_{i,t}}{P_{F,t}} \right)^{-\gamma} \left( \frac{P_{F,t}}{P_t} \right)^{-\eta} C_t
\]

(1.39)

and

\[
C_{H,t}(j) = (1 - \alpha) \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\epsilon} \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t.
\]

(1.40)

The goods market clearing condition in the home country requires

\[
Y_t(j) = C_{H,t}(j) + \int_0^1 C_{H,t}(j) dj,
\]

(1.41)

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where $C^i_H$ is the country $i$’s demand for home goods and takes the following form

$$C^i_{H,t}(j) = \alpha \left( \frac{P_{H,t}}{P_{H,t}} \right)^{-\epsilon} \left( \frac{P_{H,t}}{E_{i,t}P_{F,t}} \right)^{-\gamma} \left( \frac{P_{i}}{P_{F,t}} \right)^{-\eta} C^i_t$$

$$= \alpha \left( \frac{P_{H,t}}{P_{H,t}} \right)^{-\epsilon} \left( \frac{P_{H,t}}{E_{i,t}P_{F,t}} \right)^{-\gamma} \left( \frac{P_{i}}{P_{F,t}} \right)^{-\eta} C^i_t$$  \hspace{1cm} (1.42)

Combining (1.40), (1.41), and (1.42) and assuming $Y_t \equiv \left[ \int_0^1 Y_t(j) \frac{dj}{j} \right]^{\frac{1}{1-\epsilon}}$, I can rewrite the goods market clearing condition as

$$Y_t = (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \alpha \int_0^1 \left( \frac{P_{H,t}}{E_{i,t}P_{F,t}} \right)^{-\gamma} \left( \frac{P_{i}}{P_{F,t}} \right)^{-\eta} C^i_t dj. \hspace{1cm} (1.43)$$

Labor market clearing condition is

$$N_t = \frac{Y_t}{A_t} \int_0^1 \left( \frac{P_{H,t}}{P_{H,t}} \right)^{-\epsilon} dj$$

$$\hspace{1cm} \text{where } N_t = \int_0^1 N_t(j) dj.$$  \hspace{1cm} (1.44)

### 1.4 Equilibrium

#### 1.4.1 The demand side: Consumption and output determination

**1.4.1.1 World consumption and output**

The log-linearization of world Euler equation (1.26) is

$$c^*_t = E_t \{ c^*_{t+1} \} - \frac{1}{\sigma} (\bar{r}^*_t - E_t \{ \pi^*_t \} - \rho), \hspace{1cm} (1.45)$$
where \( \rho \equiv -\log \beta \), and combined with the market clearing condition \( y^*_t = c^*_t \), it yields

\[
y^*_t = E_t \{ y^*_{t+1} \} - \frac{1}{\sigma} (\bar{r}^*_t - E_t \{ \pi^*_{t+1} \} - \rho).
\]

(1.46)

### 1.4.1.2 Home consumption and output

Let \( C^*_{H,t}(i) \) denote the world demand for the domestic good \( i \). Then the aggregate demand for the domestic good \( i \) is

\[
Y_t(i) = C_{H,t}(i) + C^*_{H,t}(i)
\]

and then using the demand functions (1.17) and (1.28) and assuming the degrees of openness of home and world economies are the same \( \alpha = \alpha^* \),

\[
Y_t(i) = C_{H,t}(i) + C^*_{H,t}(i)
\]

\[
= \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{1-\epsilon} C_{H,t} + \left( \frac{P_{H,t}(i)}{P_{F,t}} \right)^{1-\epsilon} C^*_{H,t}
\]

\[
= \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{1-\epsilon} \left[ \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} (1-\alpha)C_t + \left( \frac{P_{H,t}}{E_tP_t} \right)^{-\eta} \alpha Y^*_t \right]
\]

\[
= \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{1-\epsilon} \left[ \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} (1-\alpha)\Theta_t C^*_t Q_t^{1/\sigma} + \left( \frac{P_{H,t}}{E_tP_t} \right)^{-\eta} \alpha Y^*_t \right]
\]

\[
= \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{1-\epsilon} \left[ \frac{P_{H,t}}{P_t} \right]^{-\eta} (1-\alpha)\Theta_t Q_t^{1/\sigma} + \left( \frac{P_{H,t}}{E_tP_t} \right)^{-\eta} \alpha \right]. \quad (1.47)
\]
Plugging (1.47) into the definition of aggregate output $Y_t \equiv \left[ \int_0^1 Y_t(i)^{1-\epsilon} di \right]^{\frac{1}{1-\epsilon}}$, we have

$$Y_t = \left[ \int_0^1 Y_t(i)^{1-\frac{1}{\epsilon}} di \right]^{\frac{1}{1-\epsilon}}$$

$$= \left[ \int_0^1 \left\{ \left( \frac{P_{H,t}(i)}{P_t} \right)^{-\epsilon} Y_t^* \left[ \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} (1 - \alpha) \Theta_t Q_t^{1/\sigma} + \left( \frac{P_{H,t}}{E_t P_t^*} \right)^{-\eta} \right] \right\}^{1-\frac{1}{\epsilon}} di \right]^{\frac{1}{1-\epsilon}}$$

$$= Y_t^* \left[ S_t^\eta Q_t^{-\eta} (1 - \alpha) \Theta_t Q_t^{1/\sigma} + S_t^\eta \right] \cdot \left[ \int_0^1 \left\{ \left( \frac{P_{H,t}(i)}{P_t} \right)^{-\epsilon} \right\} \frac{1-\epsilon}{\epsilon} di \right]^{\frac{1}{1-\epsilon}}$$

$$= Y_t^* S_t^\eta \left[ (1 - \alpha) \Theta_t Q_t^{1-\eta/\sigma} + \alpha \right]. \quad (1.48)$$

(1.48), up to a first order approximation, can be rewritten as

$$y_t = y_t^* + \frac{\omega_\alpha}{\sigma} s_t + (1 - \alpha) \theta_t, \quad (1.49)$$

where $\omega_\alpha \equiv 1 + \alpha(2 - \alpha)(\sigma_\eta - 1) > 0$. Combining (1.30) and (1.49) to substitute out for $s_t$, we have

$$c_t = \Phi_a y_t + (1 - \Phi_a) y_t^* + \{1 - (1 - \alpha) \Phi_a \} \theta_t, \quad (1.50)$$

where $\Phi_a = \frac{1 - \alpha}{\omega_\alpha} > 0$.

Then we can combine (1.50) with (1.13), (1.49), and home consumer’s Euler equation to derive a difference equation for domestic output in terms of domestic real
interest rates, world output, and relative Pareto weight:

\[ y_t = E_t \{ y_{t+1} \} - \frac{\omega}{\sigma} (r_t - E_t \{ \pi_{H,t+1} \} - \rho) + (\omega - 1)E_t \{ \Delta y^*_{t+1} \} + (\omega - 1 + \alpha)E_t \{ \Delta \theta_{t+1} \}, \]

(1.51)

where \( \Delta y^*_{t+1} \equiv y^*_{t+1} - y^*_t \) and \( \Delta \theta_{t+1} \equiv \theta_{t+1} - \theta_t \).

1.4.1.3 The trade balance

The net exports \( n x_t \) are denoted in terms of domestic output, expressed as a fraction of steady state output \( Y \),

\[ n x_t \equiv \frac{1}{Y} \left( Y_t - \frac{P_t}{P_{H,t}} C_t \right), \]

and its first-order approximation yields

\[ n x_t = y_t - c_t - p_t + p_{H,t}, \]

which combined with (1.49) and (1.50) implies

\[ n x_t = \alpha \left( 1 - \frac{\sigma}{\omega} \right) (y_t - y^*_t) - \left( 1 - \frac{1 - \alpha}{\omega} \right) \theta_t \]
\[ = \frac{\alpha \Lambda}{\omega} (y_t - y^*_t) - \left( 1 - \frac{1 - \alpha}{\omega} \right) \theta_t, \]

(1.52)

where \( \Lambda \equiv (2 - \alpha)(\sigma \eta - 1) + (1 - \sigma). \)
1.4.2 The supply side: Marginal cost and inflation dynamics

1.4.2.1 World marginal cost and inflation dynamics

By combining the optimal price setting equation (1.38) with the log-linearized equation of the evolution of aggregate price level, we have

\[ \pi_t^* = \beta \mathbb{E}_t \{ \pi_{t+1}^* \} + \frac{(1 - \delta)(1 - \beta \delta)}{\delta} \tilde{mc}_t, \]  

(1.53)

where \( \tilde{mc}_t \equiv mc_t^* + \varpi \) denotes the log real marginal cost, expressed as a deviation from its steady state value \( (-\varpi) \). Assuming the symmetric home and world economies, as implied in (1.35), the world log real marginal cost is given by

\[ mc_t^* = -\nu^* + w_t^* - p_t^* - a_t^*, \]

\[ = -\nu^* + \sigma c_t^* + \phi n_t^* - a_t^* \]

\[ = -\nu^* + (\sigma + \phi)y_t^* - (1 + \phi) a_t^* \]  

(1.54)

where \( \nu^* \equiv -\log(1 + \tau_L^*). \)

1.4.2.2 Home marginal cost and inflation dynamics

The dynamics of domestic inflation of home are described by an equation analogous to the world’s;

\[ \pi_{H,t} = \beta \mathbb{E}_t \{ \pi_{H,t+1} \} + \frac{(1 - \delta)(1 - \beta \delta)}{\delta} \tilde{mc}_t. \]  

(1.55)

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The home log real marginal cost is

\[ mc_t = -\nu + w_t - p_{H,t} - a_t \]

\[ = -\nu + (w_t - p_t) + (p_t - p_{H,t}) - a_t \]

\[ = -\nu + \sigma c_t + \phi n_t + \alpha s_t - a_t \]

\[ = -\nu + \sigma \left( \theta_t + c_t^* + \frac{1 - \alpha}{\sigma} s_t \right) + \phi (y_t - a_t) + \alpha s_t - a_t \]

\[ = -\nu + \phi y_t + \sigma y_t^* + s_t + \sigma \theta_t - (1 + \phi) a_t. \] (1.56)

Using (1.49) to substitute for \( s_t \), (1.56) can be rewritten as following

\[ mc_t = -\nu + \left( \frac{\sigma}{\omega_\alpha} + \phi \right) y_t + \sigma \left( 1 - \frac{1}{\omega_\alpha} \right) y_t^* + (1 - \sigma + \alpha) \theta_t - (1 + \phi) a_t. \] (1.57)

1.4.3 Equilibrium dynamics

I define the output gap \( x_t \) as the deviation of log output \( y_t \) from its natural level \( \overline{y}_t \), where the latter is in turn defined as the equilibrium level of output in the absence of nominal rigidities:

\[ x_t \equiv y_t - \overline{y}_t, \] (1.58)

and the counterpart for world is,

\[ x_t^* \equiv y_t^* - \overline{y}_t^*. \] (1.59)
1.4.3.1 World equilibrium dynamics

Under flexible prices, real marginal costs in the world economy will be constant over time, and given by \( mc^* \equiv -\varpi \), the level that would obtain under flexible prices. Then using (1.54), the natural level of world output can be expressed as

\[
\bar{y}_t^* = \frac{\nu^* + (-\varpi)}{\sigma + \phi} + \frac{1 + \phi}{\sigma + \phi} \delta_t^*.
\]

The relationship between real marginal cost and the output gap can be derived as the following equation

\[
y_t^* - \bar{y}_t^* = \frac{\nu^* + mc_t^*}{\sigma + \phi} + \frac{1 + \phi}{\sigma + \phi} a_t^* \left( \frac{\nu^* + (-\varpi)}{\sigma + \phi} + \frac{1 + \phi}{\sigma + \phi} a_t^* \right)
\]

\[
\Rightarrow \quad y_t^* - \bar{y}_t^* = \frac{mc_t^* - (-\varpi)}{\sigma + \phi}
\]

\[
\Rightarrow \quad x_t^* = \frac{\bar{mc}_t^*}{\sigma + \phi}
\]

\[
\Rightarrow \quad \bar{mc}_t^* = (\sigma + \phi)x_t^*,
\]

which, combined with (1.53), gives the New Keynesian Phillips curve:

\[
\pi_t^* = \beta E_t \{ \pi_{t+1}^* \} + \frac{(1 - \delta)(1 - \beta \delta)(\sigma + \phi)}{\delta} x_t^*.
\]

(1.46) can be also re-written in terms of the world output gap:

\[
x_t^* = E_t \{ x_{t+1}^* \} - \frac{1}{\sigma} (\tilde{r}_t^* - E_t \{ \pi_{t+1}^* \} - \rho).
\]
### 1.4.3.2 Home equilibrium dynamics

The steady state value of log real marginal cost is

$$-\mu = -\nu + \left( \frac{\sigma}{\omega_\alpha} + \phi \right) \bar{y}_t + \sigma \left( 1 - \frac{1}{\omega_\alpha} \right) y_t^* + (1 - \sigma + \alpha) \bar{\theta}_t - (1 + \phi) a_t, \quad (1.63)$$

which provides the natural level of output in home country

$$\bar{y}_t = \frac{\omega_\alpha (\nu - \mu)}{\sigma + \omega_\alpha \phi} + \frac{\sigma (1 - \omega_\alpha)}{\sigma + \omega_\alpha \phi} y_t^* - \frac{\omega_\alpha (1 - \sigma + \alpha)}{\sigma + \omega_\alpha \phi} \bar{\theta}_t + \frac{\omega_\alpha (1 + \phi)}{\sigma + \omega_\alpha \phi} a_t, \quad (1.64)$$

then the relationship between output gap and real marginal cost is derived as following

$$x_t = \frac{\omega_\alpha}{\sigma + \omega_\alpha \phi} \tilde{m}_c_t - \frac{\omega_\alpha (1 - \sigma + \alpha)}{\sigma + \omega_\alpha \phi} \tilde{\theta}_t. \quad (1.65)$$

The New Keynesian Phillips Curve can be derived using (1.55) and (1.65):

$$\pi_{H,t} = \beta E_t \{ \pi_{H,t+1} \} + \kappa \left( \frac{\sigma}{\omega_\alpha} + \phi \right) x_t + \alpha \kappa \tilde{\theta}_t, \quad (1.66)$$

where $\kappa \equiv \frac{(1-\delta)(1-\beta \delta)}{\delta}$.

Using (1.51), the IS equation can be expressed in terms of output gap:

$$x_t = E_t \{ x_{t+1} \} - \frac{\omega_\alpha}{\sigma} (r_t - E_t \{ \pi_{H,t+1} \} - \bar{\pi}_t) + (\omega_\alpha - 1 + \alpha) E_t \{ \Delta \tilde{\theta}_{t+1} \}, \quad (1.67)$$
where the home country’s Wicksellian interest rate $\overline{r}_t$ is defined as

$$
\overline{r}_t \equiv \rho + \frac{\sigma(1+\phi)}{\sigma + \omega_\alpha \phi} E_t \{\Delta a_{t+1}\} - \frac{\sigma \phi(1-\omega_\alpha)}{\sigma + \omega_\alpha \phi} E_t \{\Delta y^*_t\} \nonumber \\
+ \sigma \left(\frac{\omega_\alpha - 1 + \alpha}{\omega_\alpha} - \frac{1 - \sigma + \alpha}{\sigma + \omega_\alpha \phi}\right) E_t \{\Delta \hat{\theta}_{t+1}\}. \quad (1.68)
$$

1.5 Optimal Policy

The optimal policy path is derived in perspective of a social planner by minimizing the welfare loss with the resource constraints that are characterized in equilibrium conditions. Following the literature, I focus on the Cole-Obstfeld case $\sigma = \eta = \gamma = 1$ to derive a tractable second-order approximation of the welfare function.

1.5.1 Summarizing the economy

The demand block is summarized by the following equations:

$$
x_t = x_{t+1} - (r_t - E_t \{\pi_{H_t+1}\} - \overline{r}_t) + \alpha E_t \{\Delta \hat{\theta}_{t+1}\},
$$

$$
r_t = \tilde{r}^*_t + \lambda_t E_t \{\Delta e^t_{t+1}\} + E_t \{\Delta \hat{\theta}_{t+1}\} + (1 + \xi \lambda_t) \psi_t
$$

$$
\overline{r}_t = \rho - E_t \{\Delta a_{t+1}\} + \frac{\alpha \phi}{1 + \phi} (1 + \xi \lambda_t) \psi_t, \text{ and}
$$

$$
\sum_{i=0}^{\infty} \beta^i \hat{\theta}_t = 0,
$$

representing the Euler equation with goods market clearing condition and the Backus-Smith condition, the uncovered interest parity condition, Wicksellian interest rate, and
the budget constraint, respectively.

The New-Keynesian Philips Curve summarizes the supply block,

\[ \pi_{H,t} = \beta E_t(\pi_{H,t+1}) + \kappa (1 + \phi)x_t + \alpha \kappa \bar{\theta}_t. \]

### 1.5.2 The planning problem in gaps

The optimal allocation must maximize the utility of consumption and minimize the disutility of labor; that is, it must maximize the present value of the sum of future consumer utility \( \sum_{t=0}^{\infty} \beta^t \{ U(C_t) - V(N_t) \} \). This can be expressed in the form of welfare loss function, which is a second-order approximation of the welfare function, and a social planner solves the following planning problem (see the Appendix A.3 for the detailed derivation):

\[
\min_{\{\pi_{H,t}, x_t, \sigma_t, r_t, \lambda_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{2} \alpha \pi_{H,t}^2 + \frac{1}{2} x_t^2 + \frac{1}{2} \alpha_{\sigma} (\hat{\sigma}_t + \tilde{\theta}_t)^2 \right\},
\]
where $\alpha = \frac{\epsilon}{\kappa (1 + \phi)}$ and $\alpha_\theta = \frac{\alpha}{1 + \phi} \left( \frac{2 - \alpha}{1 - \alpha} + 1 - \alpha \right)$, subject to the following equilibrium conditions derived in the previous sections:

\[
\pi_{H,t} = \beta E_t \{ \pi_{H,t+1} \} + \kappa (1 + \phi) x_t + \alpha \kappa \hat{\theta}_t,
\]

\[
x_t = x_{t+1} - (r_t - E_t \{ \pi_{H,t+1} \} - \tau t) + \alpha E_t \{ \Delta \hat{\theta}_{t+1} \},
\]

\[
r_t = \hat{r}_t^* + \lambda_t E_t \{ \Delta e_{t+1}^f \} + E_t \{ \Delta \hat{\theta}_{t+1} \} + (1 + \xi \lambda_t) \psi_t,
\]

\[
\tau t = \rho - E_t \{ \Delta a_{t+1} \} + \frac{\alpha \phi}{1 + \phi} (1 + \xi \lambda_t) \psi_t,
\]

\[
E_t \{ \Delta \hat{\theta}_{t+1} \} = (1 + \xi \lambda_t) \psi_t,
\]

\[
\sum_{t=0}^\infty \beta^t \hat{\theta}_t = 0, \text{ and}
\]

\[
\lambda_t \in [0, 1]
\]

(1.31) shows that the wedge between home and foreign consumptions is widened by the risk premium shock and can be narrowed by the capital controls. That is, the policymaker controls the path of $\hat{\theta}_t$ and therefore the capital controls are characterized as $\tau_t = \Delta \hat{\theta}_{t+1}$.

### 1.5.3 Optimal policies

To gain an intuition of the effects of each policy, I hit the economy with AR(1) risk premium shock ($\psi_t = \rho \psi_{t-1} + \epsilon_{\psi,t}$) and compare how the system’s response to the shocks under four different cases of international policy portfolios: (1) the system under the home inflation-based Taylor rule with zero foreign exchange market intervention and
zero capital controls, (2) the system allowing foreign exchange market intervention only, 
(3) the system allowing capital controls only, and (4) the system under both policies.

Although it is possible to solve and present the optimal allocations of each case in closed forms, the expressions quickly become difficult to handle. Therefore I perform numerical simulations.\(^\text{10}\) Each case is simulated with two levels of the regime sensitivity, \(\xi = 1\) and \(\xi = 0.05\). Since the risk premium shock is characterized as \((1 + \xi \lambda_t) \psi_t\), under \(\xi = 1\), switching exchange rate regime from fixed to freely floating results in 100% increase in the risk premium shock, and it reflects the results of event study where the risk premium shows 85.7–108.5% abnormal returns in its trend when countries switch regimes. The setting of \(\xi = 0.05\) provides the benchmark case to illustrate how regime-elastic risk premium shock affects the economy. For other parameter values, I follow Galí and Monacelli (2005) by setting \(\alpha = 0.4\), \(\beta = 0.99\), \(\rho = 0.04\), \(\epsilon = 6\), and \(\phi = 3\) and Farhi and Werning (2014) by setting \(\delta = 0.68\) and the initial risk premium shock to be 5 percent, which diminishes at the rate of \(\rho \psi = 0.9\).

1.5.3.1 Under floating exchange rate regime and zero capital controls with home inflation-based Taylor rule

This case presents a benchmark case of home inflation-based Taylor rule. When the policymaker does not intervene in the foreign exchange market or levy taxes on

\(^{10}\)This simulation is meant to be an example and should not be thought of as a calibration exercise because the model is probably too stylized.
international capital flows, we have

\[ \lambda_t = 1 \quad \text{and} \quad \hat{\theta}_t = 0 \quad \forall t, \]

then the planning problem simplifies to

\[
\min_{(\pi_{H,t}, x_t)} \mathbb{E}_0 \sum_{t=0}^\infty \beta^t \left\{ \frac{1}{2} \alpha \pi_{H,t}^2 + \frac{1}{2} x_t^2 \right\},
\]
subject to

\[ \pi_{H,t} = \beta \mathbb{E}_t \{ \pi_{H,t+1} \} + \kappa (1 + \phi) x_t, \]

\[ x_t = x_{t+1} - (r_t - \mathbb{E}_t \{ \pi_{H,t+1} \} - \pi_t), \]

\[ r_t = \bar{r}_t^* + \mathbb{E}_t \{ \Delta e_{t+1} \} + (1 + \xi) \psi_t, \]

\[ \Delta r_t = \rho + \phi \pi_{H,t}, \quad \text{and} \]

\[ \bar{r}_t = \rho + \frac{\alpha \phi}{1 + \phi} (1 + \xi) \psi_t, \]

where Taylor rule coefficient \( \phi_\pi = 1.5 \) following Galí and Monacelli (2005).

Figure 1.3 displays the impulse responses of the system to a risk premium shock under the different levels of signaling effects. This case provides a benchmark to examine how the system works with different sets of policies. In addition, it is useful to consider this case to examine how signaling effect can affect the economy in general; the IRFs with a low \( \xi \) (blue line) represent the classical small open economy model with risk premium shock that is almost inelastic to the exchange rate regime change, while the IRFs with a high level of \( \xi \) (red line) depict how the regime-elastic shock changes the dynamics of classical models.

When the risk premium shock hits the economy under the floating exchange rate regime, the depreciation of the exchange rate is higher when the volatile exchange rates are considered to be a signal of economic instability (35.0 vs. 18.4 percent at impact); the regime-elastic premium shock is amplified under the floating regime, and without other policies, the price of local currency has to be rearranged to mitigate the
shock. The amplified risk premium shock and the floating exchange rates together bring in even higher impact on price levels and output. The home inflation under higher $\xi$ deviates from its steady state much more than the one under lower $\xi$ does (4.25 vs. 0.79 percent at peaks) and the output gap deviation is six times larger under high $\xi$ (42.9 vs. 7.02 percent at impact). As a result, a welfare loss is almost 30 times bigger (9.2 vs. 0.25 percent at impact), while the risk premium shock under the high $\xi$ is only about twice of that under the low $\xi$ (10 vs. 5.25 percent). The figures indicate that volatile exchange rates bring multiplying effects when $\xi$ is high, and therefore the optimal policies derived in the following sections are expected to stabilize the exchange rates.

1.5.3.2 Under managed exchange rates and zero capital controls

When policymakers are able to manage the exchange rates without capital controls, where

$$\hat{\theta}_t = 0 \quad \forall t,$$

then the problem becomes

$$\min_{\{\pi_{H,t}, x_t, r_t, \lambda_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{2} \alpha_\pi \pi_{H,t}^2 + \frac{1}{2} x_t^2 + \frac{1}{2} \alpha_\theta \theta_t^2 \right\},$$
Figure 1.4: Impulse response function of the system under managed exchange rates and zero capital controls: Low signaling effect (blue) and high signaling effect (red)

subject to

\[
\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \kappa (1 + \phi) x_t,
\]

\[
x_t = x_{t+1} - (r_t - E_t\{\pi_{H,t+1}\} - \overline{\pi}_t),
\]

\[
r_t = \tilde{r}^* + \lambda_t E_t\{\Delta e_{t+1}'\} + (1 + \xi \lambda_t) \psi_t,
\]

\[
\overline{\pi}_t = \rho + \frac{\alpha \phi}{1 + \phi} (1 + \xi \lambda_t) \psi_t,
\]

\[
E_t\{\Delta \hat{\theta}_{t+1}\} = (1 + \xi \lambda_t) \psi_t,
\]

\[
\sum_{t=0}^{\infty} \beta^t \hat{\theta}_t = 0, \quad \text{and}
\]

\[
\lambda_t \in [0, 1]
\]
Solving the planning problem, the optimal path of foreign exchange market intervention can be derived in terms of home inflation, shadow floating exchange rates, and the risk premium shock as following:

**Proposition 1.** *(Managed exchange rates under zero capital controls)* Suppose the policymaker intervenes in the foreign exchange market without capital controls. The optimal level of managed peg/float is given by

\[
\lambda_t = \left[ E_t \{ \Delta e_{t+1}^f \} + \frac{\xi(1 + \phi - \alpha \phi)}{1 + \phi} \psi_t \right]^{-1} \left[ \frac{1 + \phi - \alpha \phi}{1 + \phi} \psi_t - \{ \kappa(1 + \phi) \alpha \pi - 1 \} E_t \{ \pi_{H,t+1} \} \right]
\]

where \( \lambda_t \in [0, 1] \).

Proposition 1 implies that it is preferred to stabilize the exchange rates (\( \lambda_t \to 0 \)), when the floating exchange rates are volatile (\( \Delta e_{t+1}^f \uparrow \)) or the investors are more sensitive to broken pegs (\( \xi \uparrow \)) to avoid another wave of risk premium shock. The level of intervention does not totally lean against the wind, and it indicates the trade-off between the benefits of currency price-adjustment and the cost of impact on terms of trade and furthermore the local consumers’ purchasing power.

Figure 2.2 shows that when there are no capital controls, exchange rates shall be actively managed; \( \lambda_t \) does not deviate much from zero at impact and quickly converges back to zero under both levels of \( \xi \); especially under high level of \( \xi \), it is essential to stabilize the exchange rates to avoid the additional increase in risk premium, and managed peg/float lives only for two periods. As a result, price and output levels be-

\( ^{11} \)If the optimal value of \( \lambda_t \) is found to be less than 0 or greater than 1, the effective \( \lambda_t \) takes the value of 0 or 1 respectively.
Figure 1.5: Impulse response function of the system under floating exchange rates and capital controls: Low signaling effect (blue) and high signaling effect (red)

come stabilized, and the welfare loss has significantly decreased compared to the first case (9.2 percent under floating exchange rate regime to 0.35 percent under managed exchange rate regime at impact).

1.5.3.3 Under floating exchange rate regime and capital controls

When the policymaker can use capital controls under the floating exchange rate regime, where

\[ \lambda_t = 1 \quad \forall t, \]
the problem becomes

$$\min_{\{\pi_{H,t}, x_t, \theta_t, r_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{2} \alpha_{\pi} \pi_{H,t}^2 + \frac{1}{2} x_t^2 + \frac{1}{2} \alpha_{\theta} (\theta_t + \bar{\theta}_t)^2 \right\},$$

subject to

$$\pi_{H,t} = \beta E_t \{ \pi_{H,t+1} \} + \kappa (1 + \phi) x_t + \alpha \kappa \hat{\theta}_t,$$

$$x_t = x_{t+1} - (r_t - E_t \{ \pi_{H,t+1} \} - \bar{\pi}_t) + \alpha E_t \{ \Delta \hat{\theta}_{t+1} \},$$

$$r_t = \bar{r}_t^* + E_t \{ \Delta e_{t+1}^f \} + \alpha E_t \{ \Delta \hat{\theta}_{t+1} \} + (1 + \xi) \psi_t,$$

$$\bar{\pi}_t = \rho + \frac{\alpha \phi}{1 + \phi} (1 + \xi) \psi_t,$$

$$E_t \{ \Delta \bar{\theta}_{t+1} \} = (1 + \xi) \psi_t,$$

and

$$\sum_{t=0}^{\infty} \beta^t \hat{\theta}_t = 0.$$

Solving the planning problem brings to the following proposition:

**Proposition 2.** (Capital controls under floating exchange rate regime) Suppose the policymaker does not intervenes in the foreign exchange market. The optimal capital controls can be characterized by

$$\tau_t = -(1 + \xi) \psi_t - \frac{\alpha \kappa \alpha_{\pi}}{\alpha_{\theta}} E_t \{ \pi_{H,t+1} \}.$$

Optimal capital controls are proportional to the current risk premium shock. The tax $\tau_t$ has the opposite sign from the risk premium $\psi_t$ – the policy leans against the wind when
the home price level is rigid. High home inflation is expected due to volatile exchange rates, and therefore the optimal level of capital controls will increase accordingly.

Figure 2.3 shows that the capital controls are relatively more effective in mitigating the sudden stop; the welfare losses under capital controls are 0.06 and 0.22 percent, while the ones under managed exchange rates are 0.23 and 0.35 percent. The risk premium shock is transmitted to the local economy through the change in international capital flows; as it becomes more expensive to make loans internationally, the consumers face tighter budget constraints. By levying taxes on international capital flows, it directly counteracts the effect of the shock.

Furthermore, the model captures that the capital controls indirectly affect the foreign exchange market as well. Regulating the currency movements prevents a further depreciation of the local currency, and the example in Figure 2.3 shows that exchange rates depreciate 3 and 6 percent under capital controls, while in the first case (Figure 1.3) they depreciate 18.3 and 35 percent without capital controls. This implies that capital controls can be utilized as a partial substitute of foreign exchange market intervention to maximize consumer’s utility, and the next case validates this.

### 1.5.3.4 Under managed exchange rates and capital controls

The policymaker now solves the full problem laid out in the beginning of this section:

$$\min_{\{\pi_{H,t}, x_t, \theta_t, r_t, \lambda_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{2} \alpha \pi^2_{H,t} + \frac{1}{2} x^2_t + \frac{1}{2} \alpha_\theta (\hat{\theta}_t + \bar{\theta}_t)^2 \right\},$$

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subject to

\[ \pi_{H,t} = \beta E_t \{ \pi_{H,t+1} \} + \kappa (1 + \phi) x_t + \alpha \kappa \theta_t, \]

\[ x_t = x_{t+1} - (r_t - E_t \{ \pi_{H,t+1} \} - \pi_t) + \alpha E_t \{ \Delta \theta_{t+1} \}, \]

\[ r_t = r^*_t + \lambda_t E_t \{ \Delta e^f_{t+1} \} + E_t \{ \Delta \hat{\theta}_{t+1} \} + (1 + \xi \lambda_t) \psi_t, \]

\[ \pi_t = \rho + \frac{\alpha \phi}{1 + \phi} (1 + \xi \lambda_t) \psi_t, \]

\[ \sum_{t=0}^{\infty} \beta^t \hat{\theta}_t = 0, \text{ and} \]

\[ \lambda_t \in [0, 1] \]

The optimal policy portfolio can be characterized in the following proposition:

**Proposition 3.** *(Managed exchange rates and capital controls)* Suppose the policymaker is able to intervene in the foreign exchange market and levy taxes on international capital flows. The policy portfolio of \( \lambda_t \) and \( \tau_t \) is characterized by the following equations:

\[ \lambda_t = \left[ E_t \{ \Delta e^f_{t+1} \} + \frac{\xi (1 + \phi - \alpha \phi)}{1 + \phi} \psi_t \right]^{-1} \left[ \frac{1 + \phi - \alpha \phi}{1 + \phi} \psi_t - (1 - \alpha) \tau_t - \{ \kappa (1 + \phi) \alpha \pi - 1 \} E_t \{ \pi_{H,t+1} \} \right], \]

and

\[ \tau_t = -(1 + \xi \lambda_t) \psi_t - \frac{\alpha \kappa \alpha \pi}{\alpha \theta} E_t \{ \pi_{H,t+1} \}, \]

where \( \lambda_t \in [0, 1] \).

Proposition 3 illustrates the interaction between two policy tools. The first equation indicates that with a higher level of capital controls \( \tau_t \downarrow \); as seen in the previous
cases, \( \tau_t < 0 \) since we assume positive risk premium shock, capital flows are controlled with a subsidy on inflow or a tax on outflow), the optimal level of \( \lambda_t \) is higher; that is, the optimal exchange rate regime becomes closer to freely floating regime with a higher level of capital controls. This agrees with a theoretical evidence in the previous section that policymaker can partially mitigate the depreciation of exchange rates by controlling international capital flows. It suggests that capital controls can be utilized as a partial substitute of exchange rate policy, and this relationship further implies
that a policymaker may be able to achieve the same or a better welfare level with less
capital controls and less intervention in the foreign exchange markets than the previous
cases. According to the second equation, on the other hand, the lower foreign exchange
market intervention \((\lambda_t \rightarrow 1)\) shall bring in additional wave of risk premium shock that
capital controls are to handle, which is a tradeoff for implementing foreign exchange
market intervention policy. Therefore, solving two equations should provide us the
balanced path of the optimal policy portfolio against the exchange rate regime-elastic
risk premium shock.

Figure 1.6 depicts the optimal policy paths of managed exchange rates and
capital controls and the corresponding responses of the economy. When \(\xi\) is low, it
is optimal to float the exchange rates \((\lambda_t \text{ close to } 1)\) and instead to utilize the capital
controls, which can partially manage the exchange rate depreciation as well as mitigate
the shock by leaning against the wind. Similarly, even when the risk premium is more
sensitive to the exchange rate regime choice \((\xi \text{ is high})\), it is optimal to have a managed
floating exchange rate regime by letting exchange rates to depreciate 50 percent of
that under freely floating exchange rate regime \((\lambda_t = 0.5 \text{ at impact})\), while controlling
international capital flows at the same time. The length of non-fixed regime has been
extended from 2 periods to 9 periods, which may explain developing countries’ relatively
persistent managed peg/float and, as a result, their exchange rate volatility that is
comparable to the ones in advanced countries.

The effect of this policy implementation is evident when comparing price and
output levels under different levels of \(\xi\). The gaps between home inflations with different
values of $\xi$ have been brought down as well as the gaps between output gaps; the difference in home inflations is 0.16 percent at peaks and the difference in output gaps is 0.42 percent at impact. One can also notice a significant improvement in terms of trade. By allowing to float the exchange rates partially, the country can avoid a bigger risk premium shock, while alleviating the effect of the shock to its economy by taxing international capital flows. An improvement in terms of trade leads to a higher purchasing power of home consumers, and as a result, the difference in the welfare levels becomes minimal (0.02 percent at impact). That is, a proper use of these policies can neutralize the potential unfavorable effect of exchange rate regime choice.

This result is worth noting; as capital controls has become available as a policy tool to mitigate the shock, the policymakers in the countries that are sensitive to exchange rate regimes can now put less resources to stabilize exchange rates and still minimize the welfare loss from the shock by utilizing capital controls. That is, the coexistence of capital controls and managed peg/float regime can be explained using a theoretical framework.

1.6 Conclusion

I consider a New Keynesian small open economy model where international financial markets are not perfect. I introduce a novel and tractable approach to employ managed peg/float by assuming that a rational policymaker simply decides the level of intervention in the foreign exchange markets. Therefore, a policymaker in the model
makes a series of policy portfolio choices of capital controls and exchange rate regimes to minimize the loss in welfare in response to the exchange rate regime-elastic risk premium shocks, and the results suggest that the coexistence of active capital controls and managed peg/float regimes may be optimal that is one of robust features of developing countries’ policy portfolio decisions in the last decade. Despite the simple characterization of international policy choice – capital controls as a tax on international capital flows and foreign exchange market intervention as a choice of degree of exchange rate regimes, the model captures the major traits of the relationship between these international policies. A policymaker manages the perceived price of the international loans and therefore becomes able to indirectly manage exchange rate depreciation. It suggests that for an economy whose country risk is sensitive to its exchange rate regime choice, the role of capital controls becomes clear as it allows one to put less resource to stabilize the foreign exchange market.
Chapter 2

Revisiting the PPP Puzzle:
Nominal Exchange Rate Rigidity and Region of Inaction

2.1 Introduction

2.1.1 Motivation

Absolute purchasing power parity (PPP) posits that nominal exchange rate between two countries will be identical to the ratio of the price levels for those two countries. This concept is derived from a basic idea known as the law of one price, which states that the real price of a good must be the same across all countries. Even though real exchange rates may converge to parity in the long run, the consensus emerging from an extensive literature appears to be that the rate of mean-reversion is slow. Standard
metric of this rate is the half-life, i.e., the time it takes for half the effects of a given shock to dissipate. Rogoff (1996) talks of a consensus view of a half-life of 3 to 4 years; however, this is much too long to be compatible with arbitrage – hence, Rogoff argues, the “PPP puzzle.”

A number of studies have suggested potential solutions to this puzzle. Imbs et al. (2005) argue that the deviation from PPP seems more apparent due to a common econometric misinterpretation of panel analysis. Imbs and co-authors argue that existing estimates of real exchange rate persistence are based upon the implicit assumption that all relative prices that comprise the real exchange rate converge to parity at the same speed. However, there is little theoretical justification for this assumption, and it is hard to think of reasons why clothes and vegetables should revert to parity at the same speed. Allowing for heterogeneity in the price persistence of goods, as the variance of these persistences increase, the aggregate persistence will bias upward. They show that slow mean reversion in the aggregate real exchange rate is consistent with much faster adjustment of disaggregated relative prices. They provide the faster mean conversion rate estimate of 26 month-half-life using mean group estimator, compared to the faster mean conversion rate estimate of 36 month-half-life using the ordinary fixed effects estimator as documented in Rogoff (1996).

Nevertheless the half-life provided in Imbs et al. (2005) does not fully agree to the common beliefs on price levels yet. The 26 month half-life implies that “it takes 26 months for real exchange rates to convert back to 50% of its mean after a shock.” Considering the volume of international trade and global market integrations over the
last decades, it is difficult to believe that it takes more than two years on average to
observe approximately 50% of mean reversion in the relative real exchange rates.

This paper argues that previous literature has overlooked two aspects, the
government intervention on the foreign exchange market (de facto exchange rate regime)
and the non-linearity of real exchange rate conversion (the region of inaction), and that
therefore addressing these issues would make the puzzle less puzzling.

2.1.2 Hypotheses and Literature

2.1.2.1 De facto exchange rate regime

This paper first proposes that empirical study of de facto flexible nominal
exchange rates will estimate a lower half-life than ones in the previous studies using de
jure measures.

Froot and Rogoff (1995), Imbs et al. (2005), and Chen and Engel (2005) com-
monly estimate the following autoregressive process:

\[ q_{c,t} = \gamma + \rho q_{c,t-1} + \varepsilon_t \]  

(2.1)

where

\[ q_{c,t} = \ln \left( \frac{s_{c,t}P_{c,t}}{P_{b,t}} \right). \]  

(2.2)

\( q_c \) is country \( c \)'s real exchange rate deviation (from its mean), \( s_c \) is the nominal exchange
rate between country \( c \) and its base currency country \( b \) (typically the United States), and
\( P_c \) and \( P_b \) are the price levels of country \( c \) and country \( b \) respectively. The price levels
are generally assumed to be sticky. Then the persistence of real exchange rate becomes highly dependent on the persistence of nominal exchange rate. Engel and Rogers (2001) investigate European data from 1981 to 1997 and also confirm that the volatility of nominal exchange rates accounts for a large part of real exchange rate volatility in a short run. Therefore, if a country fixes its nominal exchange rate to another country, a gap between the countries’ real exchange rates would display persistence because prices are also assumed to be sticky in general. Therefore investigating data of the countries with flexible nominal exchange rates regimes naturally becomes essential for studying the motion of real exchange rates. For this reason, previous studies including Imbs et al. (2005) have investigated the countries that are known to have flexible exchange rate regimes. However, a problem may still remain.

Calvo and Reinhart (2002) provide the empirical evidence that countries that claimed to allow market forces to determine the exchange rate (de jure flexible exchange rate regime) in fact have intervened in markets to minimize fluctuations of other macro indicators (de facto fixed exchange rate regime) due to a “fear of floating.” For example, the UK is known as one of the countries which float their currencies against the US Dollar, and therefore the UK Pound data has been often studied in the PPP puzzle literature. According to the classification\(^1\) of de facto exchange rate regime in Klein and Shambaugh (2008), however, there are 11 years between 1971 and 2012 when the UK de-facto-fixed its exchange rate to the US Dollar. Moreover, the nominal exchange rate was fixed for 4 years between 1981 and 2000, which is the period that Imbs et al.\(^5\)  

\(^1\)The country is having a fixed exchange rate if the month-end official bilateral exchange rate stays within the same 2.5% band for the entire year.
Figure 2.1: Volatility of real exchange rates, nominal exchange rates, terms of trade during *de facto* fixed regime periods and *de facto* floating regime periods of 10 advanced countries; Belgium (BE), Germany (DE), Denmark (DK), Spain (ES), Finland (FI), France (FR), Greece (GR), Italy (IT), Netherlands (NL), Portugal (PT), and United Kingdom (UK): The volatility of real exchange rates (RER) in gray, the volatility of nominal exchange rates (NER) in red, and the volatility of terms of trade (TOT) in gold – Data source: Eurostat (Monthly, 1981–1995)

One of the assumptions\textsuperscript{2} under which PPP holds is that the price levels – both the prices of commodities and currencies – freely adjust. The evidence of rigidity in nominal exchange rates due to the central banks’ intervention, even in advanced countries, raises concerns that the mean-convergence rates estimated in previous studies are biased upwardly, and moreover, suggests a potential explanation of unrealistic persis-

\textsuperscript{2}Absolute PPP holds under the assumptions that all goods are tradable and preferences between countries are identical, and the basket of goods in the CPI is the same in all countries. Such rather stringent assumptions are useful in theoretical studies. On the other hand, empirical studies, including this paper, use relative PPP theory, which holds under less stringent and more practical assumptions; it suggests that prices in countries vary for the same product but that they differ by the same proportional rate over time, to account for taxes, shipping costs, differences in product quality and preferences, and etc.
tence in real exchange rates; previous studies have included data from periods of de facto fixed exchange rate regimes.

Figure 2.1 and Table 2.1 support this hypothesis. They provide a simple comparison of volatilities of real exchange rate and its components, nominal exchange rate and terms of trade, of 10 advanced countries that are studied in Imbs et al. (2005) in different periods. When we focus on the periods that countries’ exchange rate regimes were de facto fixed, we can observe that the volatility of real exchange rates is significantly lower in all countries during those periods as seen in Figure 2.1. Table 2.1 reports the standard deviations of real exchange rates, nominal exchange rates, and terms of trade of each country. While the total standard deviation of real exchange rates for all periods is 0.212, the standard deviation during de facto fixed periods is only 0.162, which indicates that the price adjustment was deterred by foreign exchange market intervention, and so was the mean reversion of real exchange rates during those periods. This implies that the half-life estimates from previous literature must be biased upwardly by including de facto fixed regime periods.

On the other hand, the real exchange rates during de facto floating regimes overall show higher volatility of 0.216. It may imply that the central banks’ intervention in foreign exchange market was minimal in these periods, and therefore the real exchange rates in fact must have moved more closely along the market force, Purchasing Power Parity, during these periods. Following this hypothesis and evidence, this paper

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3The specification of de facto fixed regimes in ? is used.
4The levels of volatility in terms of trade and nominal exchange rates of Greece are seemingly too high compared to other countries, and I address this issue in section 2.4.1 as a robustness check.
Table 2.1: Decomposition of volatility of real exchange rates (RER) into nominal exchange rates (NER) and terms of trade (TOT) – Data source: Eurostat (Monthly, 1981–1995)

estimates the mean-reversion rate of real exchange rates focusing on *de facto* floating periods.

### 2.1.2.2 Region of inaction

PPP holds under the assumption that the price levels adjust towards their long-run means when minimal deviations from the means exist. PPP bases its theory upon the arbitrage and consumers’ and investors’ incentives coming from its opportunity; if discrepancies in price levels between two countries exist, the investors can take advantage of the differences until the commodity prices and the currency prices adjust to the point where arbitrage opportunity becomes non-existent. However, previous studies, such as Dumas (1992), Michael et al. (1997), Imbs et al. (2003), have documented that the real exchange rates converge non-linearly and that there are “regions of inaction” around the long-run means of real exchange rates, where the price levels do not adjust. There are several potential explanations, such as transaction costs, learning costs, and

<table>
<thead>
<tr>
<th></th>
<th>BE</th>
<th>DE</th>
<th>DK</th>
<th>ES</th>
<th>FI</th>
<th>FR</th>
<th>GR</th>
<th>IT</th>
<th>NL</th>
<th>PT</th>
<th>UK</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>All periods</td>
<td>RER</td>
<td>0.21</td>
<td>0.19</td>
<td>0.22</td>
<td>0.24</td>
<td>0.23</td>
<td>0.20</td>
<td>0.22</td>
<td>0.21</td>
<td>0.18</td>
<td>0.28</td>
<td>0.17</td>
</tr>
<tr>
<td>NER</td>
<td>0.21</td>
<td>0.23</td>
<td>0.20</td>
<td>0.18</td>
<td>0.15</td>
<td>0.18</td>
<td>0.43</td>
<td>0.16</td>
<td>0.23</td>
<td>0.28</td>
<td>0.13</td>
<td></td>
</tr>
<tr>
<td>TOT</td>
<td>0.13</td>
<td>0.14</td>
<td>0.13</td>
<td>0.17</td>
<td>0.16</td>
<td>0.12</td>
<td>0.53</td>
<td>0.17</td>
<td>0.15</td>
<td>0.40</td>
<td>0.11</td>
<td></td>
</tr>
<tr>
<td>De facto</td>
<td>RER</td>
<td>0.14</td>
<td>0.11</td>
<td>0.13</td>
<td>0.20</td>
<td>0.24</td>
<td>0.13</td>
<td>0.19</td>
<td>0.15</td>
<td>0.12</td>
<td>0.23</td>
<td>0.13</td>
</tr>
<tr>
<td>fixed periods</td>
<td>NER</td>
<td>0.09</td>
<td>0.05</td>
<td>0.02</td>
<td>0.11</td>
<td>0.15</td>
<td>0.06</td>
<td>0.38</td>
<td>0.11</td>
<td>0.09</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>TOT</td>
<td>0.15</td>
<td>0.11</td>
<td>0.14</td>
<td>0.14</td>
<td>0.17</td>
<td>0.14</td>
<td>0.50</td>
<td>0.15</td>
<td>0.14</td>
<td>0.25</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td>De facto</td>
<td>RER</td>
<td>0.22</td>
<td>0.20</td>
<td>0.22</td>
<td>0.24</td>
<td>0.22</td>
<td>0.20</td>
<td>0.22</td>
<td>0.22</td>
<td>0.18</td>
<td>0.28</td>
<td>0.18</td>
</tr>
<tr>
<td>floating periods</td>
<td>NER</td>
<td>0.22</td>
<td>0.24</td>
<td>0.20</td>
<td>0.18</td>
<td>0.16</td>
<td>0.19</td>
<td>0.42</td>
<td>0.16</td>
<td>0.23</td>
<td>0.31</td>
<td>0.15</td>
</tr>
<tr>
<td>TOT</td>
<td>0.13</td>
<td>0.14</td>
<td>0.13</td>
<td>0.18</td>
<td>0.16</td>
<td>0.12</td>
<td>0.52</td>
<td>0.17</td>
<td>0.15</td>
<td>0.41</td>
<td>0.11</td>
<td></td>
</tr>
</tbody>
</table>
transportation costs. If we include more countries whose deviations from the long-run average are relatively small and are within the region of inaction, the persistence estimates will be biased upwardly. Figure 2.2 presents an example of the necessity of considering this issue. The graph presents the time series of real exchange rates of furniture sector between the United Kingdom and the United States for 48 months (from April 1990 to August 1994). From April 1990 to October 1992, the real exchange rates show large deviation from the long-run average real exchange rates. Due to the high incentives of arbitrage during these periods, it is expected to observe active adjustments in nominal exchange rates as well as price levels of furniture of the UK and the US, which is translated as a faster mean-reversion in real exchange rates. On the other hand, from November 1992 to August 1994, the real exchange rates do not deviate much from the long-run average but slightly fluctuate around it. If the mean reversion rate is estimated only for this period, the estimate of persistence must have been high, and it may even
indicate that the real exchange rates do not converge but rather follow a random-walk.

2.2 Empirical strategy

2.2.1 De facto exchange rate regime

This study provides the half-life estimate of real exchange rate mean-reversion by focusing on the periods where exchange rate regimes are de facto floating. De facto floating regime periods are defined as the periods when the countries do not intervene in the currency market to stabilize the nominal exchange rates. The foreign exchange markets are considered to be intervened, if the month-end official bilateral exchange rate stays within the same 2.5% band for 10 months. This definition is similar to the one introduced in Klein and Shambaugh (2008) where the authors find that pegs frequently break and also that there is a high degree of re-formation of pegs that have broken. One may argue that a country/year may be mis-classified as a peg simply due to a lack of shocks. According to Calvo and Reinhart (2002), the probability of having an exchange rate volatility less than 2.5% within a month while having flexible exchange rates is between 60% and 70%. Therefore, the probability of having 10 consecutive months of 2.5% below changes is estimated in between 0.6% and 3%, which is statistically insignificant.
2.2.2 Region of inaction

Using a similar strategy in the previous section, this paper provides the half-life estimate, which is estimated only with the data within the periods where deviations of real exchange rates are large enough for analysis of mean reversion to avoid region of inaction. However, previous studies do not provide a statistically rigorous standard of the region of inaction, and therefore, the statistical standard of “large” deviation remains indefinite.

The paper instead uses different sizes of the band around long-run means as regions of inaction. Since the statistical standard of region of inaction is indefinite, the paper uses rather conservative standards, and I investigate the observations, which are only certain standard deviations away from the long-run averages: ±0.05 standard deviation, ±0.1 standard deviation, and ±0.2 standard deviation around the long-run means. Figure 2.3 describes an example of setting region of inaction dummies with different levels of standard deviation from means, using the time series example presented in Figure 2.2. With a ±0.05 standard deviation range, there are 2 months of observations where real exchanges are in the region of inaction. As we increase the range of deviations to ±0.1 standard deviation and to ±0.2 standard deviation, the numbers of periods of stable real exchange rates increase to 3 months and 11 months. The wider range of standard deviations excludes the more already-stable periods. As the more periods when the real exchange rates are within the region of inaction are excluded, the shorter half-life estimates are expected.
Figure 2.3: 1990:04 - 1994:08 UK - US Real exchange rate of furniture (Shaded region: periods when real exchange rates are within regions of inaction) – Top panel: ±0.05 standard deviation / Middle panel: ±0.1 standard deviation band / Bottom panel: ±0.2 standard deviation band
2.2.3 Econometric methods

This paper studies panels of sectoral real exchange rates and therefore first uses a standard specification to investigate the speed of mean reversion in sectoral real exchange rates:

\[ q_{c,t} = \gamma_c + \sum_{k=1}^{K} \rho_{c,k} q_{c,t-k} + \varepsilon_{c,t}, \]  

(2.3)

where \( q_{c,t} \) is the real exchange rates in country \( c \) at time \( t \), \( K \) is the maximum number of lags to identify the long run estimates of mean reversion, and the fixed effects \( \gamma_c \) are allowed. However, as pointed out in Imbs et al. (2005), standard fixed effects estimates of real exchange rate persistence are based on the implicit assumption that all relative prices converge to parity at the same speed. Due to the nature of standard panel estimators, the fixed effects estimators take weighted average of sectoral coefficients and the sectoral coefficients that are more deviated from the means are weighted more. That is, the results with heterogeneous speeds of convergence, the slower convergence estimates (the higher coefficient estimates) get more weights than the faster convergence estimates. On the other hand, the standard mean group (MG) estimator introduced in Pesaran and Smith (1995) instead simply performs an arithmetic average of sector-specific slopes with equal weights. According to Imbs et al. (2005), the following specification is to be considered to allow for the cross-sectional heterogeneity of the panel:

\[ q_{i,c,t} = \gamma_{i,c} + \sum_{k=1}^{K} \rho_{i,c,k} q_{i,c,t-k} + \varepsilon_{i,c,t}, \]  

(2.4)
where \( i \) is an index of sectors, and slopes and intercepts are allowed to vary across the panel units. The standard mean group (MG) model estimates \( \{\rho_{i,c,k}\}_{k=1}^K \), the mean autoregressive coefficients.

Now, we consider the dummy variables, *de facto* fixed exchange rate regime dummy and region of inaction dummy that are discussed in the previous sections. Since the observations that are within \( K-1 \) distance from one invalid observation cannot be included in time-series estimation, dummy variables are employed as in the following specification:

\[
q_{i,c,t} = \gamma_{i,c} + \sum_{k=1}^{K} \rho_{i,c,k} q_{i,c,t-k} \cdot \prod_{k=1}^{2(K-1)} d_{i,c,t-k} + \varepsilon_{i,c,t}.
\] (2.5)

Using this regression specification, the paper reports following estimates: standard mean group estimate; mean group estimate with *de facto* regime dummy; and mean group estimate with region of inaction dummy. The estimates with region of inaction dummy will be reported with ±0.05, ±0.1, and ±0.2 standard deviation bands. These estimates are compared to the estimate of standard fixed effects model specified in (2.3). The fixed effects estimates with *de facto* regime dummy and region of inaction dummy are reported in section 2.4.2 as a robustness check.

2.3 Data

The paper studies sectoral real exchange rates and focuses on monthly price indices for different goods categories. We use the data set used in Imbs et al. (2005);
the data set consists of 12 countries’ monthly price indices for 19 goods categories from 1981 to 1995. The 12 countries include Belgium, Denmark, Finland, France, Germany, Greece, Italy, the Netherlands, Portugal, Spain, the United Kingdom, and the United States, and their de jure regimes are floating exchange rate regimes; that is, these countries are known to float their currencies against the US dollars. The goods categories are a mixture of low and high unit cost goods (e.g., bread and cereals versus vehicles), highly tradable and nontradable goods (e.g., clothing versus public transport and hotels), and goods for which there is wide variation in the degree of product differentiation (fuel versus sound and photographic equipment). The real exchange rates to be used in analysis are CPI-based and are defined against the US Dollar (a base or anchor currency). In particular, sectoral real exchange rates write

\[ q_{i,c,t} = \ln \left( \frac{S_{c,t}P_{i,c,t}}{P_{i,US,t}} \right), \tag{2.6} \]

where \( S_{c,t} \) is the nominal exchange rates between country \( c \) and the US at time \( t \), and \( P_{i,c,t} \) is the price level of the good \( i \) in country \( c \) at time \( t \).

11 sets of nominal exchange rates data are collected, and Table 2.2 provides a data description of nominal exchange rates between the currencies of 11 countries and the US Dollars and also the simple statistics of the numbers of months when the governments intervene the markets. There are 180 observations in each set and therefore there are 1,980 monthly observations in total. Applying the de facto fixed regime definition, 344 monthly observations, 17.37% of the total, are excluded when estimating the
<table>
<thead>
<tr>
<th>Country</th>
<th>Periods</th>
<th>Total number of months</th>
<th>Exchange Rates Mean</th>
<th>Exchange Rates Std. Dev</th>
<th>Number of months de facto fixed regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>1981:01 – 1995:12</td>
<td>180</td>
<td>0.0259</td>
<td>0.0052</td>
<td>16 (8.89%)</td>
</tr>
<tr>
<td>Germany</td>
<td>1981:01 – 1995:12</td>
<td>180</td>
<td>0.5236</td>
<td>0.1131</td>
<td>13 (7.22%)</td>
</tr>
<tr>
<td>Denmark</td>
<td>1981:01 – 1995:12</td>
<td>180</td>
<td>0.1396</td>
<td>0.0256</td>
<td>16 (8.89%)</td>
</tr>
<tr>
<td>Spain</td>
<td>1981:01 – 1995:12</td>
<td>180</td>
<td>0.0083</td>
<td>0.0014</td>
<td>26 (14.44%)</td>
</tr>
<tr>
<td>Italy</td>
<td>1981:01 – 1995:12</td>
<td>180</td>
<td>0.0007</td>
<td>0.0001</td>
<td>29 (16.11%)</td>
</tr>
<tr>
<td>France</td>
<td>1981:01 – 1995:12</td>
<td>180</td>
<td>0.1628</td>
<td>0.0274</td>
<td>23 (12.78%)</td>
</tr>
<tr>
<td>Greece</td>
<td>1981:01 – 1995:12</td>
<td>180</td>
<td>0.0079</td>
<td>0.0040</td>
<td>36 (20.00%)</td>
</tr>
<tr>
<td>Netherlands</td>
<td>1981:01 – 1995:12</td>
<td>180</td>
<td>0.4664</td>
<td>0.0998</td>
<td>16 (8.89%)</td>
</tr>
<tr>
<td>Portugal</td>
<td>1981:01 – 1995:12</td>
<td>180</td>
<td>0.0079</td>
<td>0.0029</td>
<td>48 (26.67%)</td>
</tr>
<tr>
<td>Finland</td>
<td>1981:01 – 1995:12</td>
<td>180</td>
<td>0.2119</td>
<td>0.0320</td>
<td>67 (37.22%)</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>1981:01 – 1995:12</td>
<td>180</td>
<td>1.6261</td>
<td>0.2098</td>
<td>40 (22.23%)</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>1981:01 – 1995:12</strong></td>
<td><strong>1,980</strong></td>
<td><strong>330 (16.67%)</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.2: Data description – Nominal exchange rates and de facto fixed exchange rates persistences. It shows that Finland, the UK, and Portugal are the countries which have intervened the currency markets the most frequently, and Germany, Belgium, Denmark, and the Netherlands are the countries that have floated their currencies more than other countries.

The numbers of observations that are within region of inaction are presented in Table 2.3. The numbers are derived at the range of ±0.1 standard deviations of each set. Among 11 countries, the price levels of the UK show the least deviations from the US price levels, and on the other hand, it shows that the price levels of Denmark show the largest gaps from the US price levels. We can also expect that tradable goods should show less deviations of prices than non-tradable goods. It is generally true because the prices of fruits, meat, and tobacco show the least deviation from the prices of the US. However, the prices of clothing, which is considered to be highly tradable, have unstable real exchange rates in most countries. This might be due to the seasonal fluctuations of clothing prices, but to understand fully, a closer investigation in pricing of clothing
Table 2.3: Data description – The number of months within region of inaction (±0.1 standard deviation band) by categories and countries

will be needed in the future.

2.4 Results

The estimation results are presented in Table 2.4, which reports the estimates of standard fixed effects model and mean group model. The mean group model has additional three variations; *de facto* regime dummy, region of inaction dummy (steady RER), and both *de facto* dummy and region of inaction dummy. The third column reports the sum of regression estimates, and the fourth column reports the half-life.
Table 2.4: Real exchange rates persistence estimates in half-life

<table>
<thead>
<tr>
<th>Model</th>
<th>Region of Inaction</th>
<th>( \sum_{k=1}^{K} \hat{\rho}_{i,c,k} )</th>
<th>Half-life (Months)</th>
<th>(95% C.I)</th>
<th>% change from Base</th>
<th>% change from FE</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Standard Panel</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Base</td>
<td>0.979 (0.0011)</td>
<td>32.26</td>
<td>(29.2, 36.1)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>Mean Group Panel</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Base</td>
<td>0.971 (0.0019)</td>
<td>23.14</td>
<td>(20.5, 26.6)</td>
<td>-</td>
<td>-28.26</td>
<td>-</td>
</tr>
<tr>
<td>De facto Regime</td>
<td>0.966 (0.0025)</td>
<td>19.81</td>
<td>(17.3, 23.2)</td>
<td>-14.39</td>
<td>-38.58</td>
<td>-</td>
</tr>
<tr>
<td>Steady RER</td>
<td>( \pm 0.05 \text{ s.d.} )</td>
<td>0.967 (0.0027)</td>
<td>20.54</td>
<td>(17.7, 24.4)</td>
<td>-11.25</td>
<td>-36.33</td>
</tr>
<tr>
<td></td>
<td>( \pm 0.1 \text{ s.d.} )</td>
<td>0.961 (0.0038)</td>
<td>17.55</td>
<td>(14.6, 21.9)</td>
<td>-24.18</td>
<td>-45.61</td>
</tr>
<tr>
<td></td>
<td>( \pm 0.2 \text{ s.d.} )</td>
<td>0.956 (0.0043)</td>
<td>15.22</td>
<td>(12.7, 18.9)</td>
<td>-34.24</td>
<td>-52.82</td>
</tr>
<tr>
<td>De facto &amp; Steady RER</td>
<td>( \pm 0.05 \text{ s.d.} )</td>
<td>0.951 (0.0068)</td>
<td>13.89</td>
<td>(10.8, 19.3)</td>
<td>-39.95</td>
<td>-56.93</td>
</tr>
<tr>
<td></td>
<td>( \pm 0.1 \text{ s.d.} )</td>
<td>0.948 (0.0074)</td>
<td>13.09</td>
<td>(10.2, 18.3)</td>
<td>-43.42</td>
<td>-59.41</td>
</tr>
<tr>
<td></td>
<td>( \pm 0.2 \text{ s.d.} )</td>
<td>0.928 (0.0155)</td>
<td>9.30</td>
<td>(6.4, 16.4)</td>
<td>-59.80</td>
<td>-71.17</td>
</tr>
</tbody>
</table>

* The number of lags are determined by Akaike information criterion (AIC) and Schwarz criterion (BIC).

The 95% confidence intervals are reported based on the standard errors in the third column.

The standard fixed effects and mean group estimators provide similar estimation results as in the previous literature. I find the fixed effects estimate of 32.26 months, which is similar to recent studies, such as Carvalho and Nechio (2011), which estimate 39 months of half-life of mean-reversion. The standard mean group estimate of 23.14 months also agrees to that of Imbs et al. (2005).

The estimates of implementing de facto regime dummy indicate that the real exchange rates are in fact less persistent than the estimates of previous literature; the
estimate shows a decrease to 19.81 month half-life, which is 14.39% lower than the standard mean group estimate and 38.58% lower than the standard fixed effects estimate. The estimates provide the evidence that the nominal exchange rate rigidity due central bank’s foreign exchange market intervention introduces biases in the estimate. It implies that previous literature has not controlled for this type of nominal rigidity, and that the foreign exchange market intervention may have accounted for a part of puzzle in the previously reported mean-reversion estimates.

The estimates with region of inaction dummy provide another potential explanation of the PPP puzzle. As the region of inaction band expands from ±0.05 to ±0.2 standard deviations, the mean-reversion estimate decreases: 20.54 months with ±0.05 standard deviation band, 17.55 months with ±0.1 standard deviation band, and 15.22 months with ±0.2 standard deviation band. The results imply that the mean-reversion is nonlinear; the further the real exchange rates are deviated from the long-run average, the faster the mean-reversion rate is estimated. In other words, the estimation results can also imply that if the deviation from the long-run average real exchange rate is minimal, the rate of mean-reversion is estimated to be low or the real exchange rates may even follow random walks. By excluding these observations that are close to long-run averages, estimated mean-reversion rates have decreased by 11.25 - 34.24% from the standard mean group estimate and by 38.58 - 52.82% from the standard fixed effects estimate. Due to the lack of rigorous study on the region of inaction, the study uses the arbitrary (but conservative) sizes of region of inaction band, and therefore, the estimation results may not represent the real exchange rate mean-reversion parameter. However,
the results still provide strong evidence that the mean-reversion estimates from previous studies are biased upwardly due to the non-linear and noncontinuous mean-reversion.

Combining both dummies provides even faster mean-reversion estimates, and it is consistent with the hypothesis that focusing on valid observations shall improve estimation and eventually lower the half-life estimates and eventually make the puzzle less puzzling. The half-life estimates are now 13.89 months with $\pm 0.05$ standard deviation band, 13.09 months with $\pm 0.1$ standard deviation band, and 9.30 months with $\pm 0.2$ standard deviation band, and these estimates are 56.93 - 71.17% lower than the standard fixed effect estimate and 39.95 - 59.80% lower than the standard mean group estimate.

One can argue that excluding invalid observations and decreasing the sample size may be a less favorable econometric method to avoid biases because it may induce an increase in the standard errors, indicating a loss of estimation accuracy. Especially, excluding the invalid observations in time series analysis should be executed with care because when regression estimation is over $K$ lags, excluding one invalid observation can result in excluding $2(K - 1)$ valid observations at most in estimation because $K - 1$ observations around the invalid observation can no longer be used in estimation.

However, despite the concern, excluding observations does not increase the standard errors to the extent that it changes the analysis of the results. Applying *de facto* regime dummy increases the standard errors from 0.0019 to 0.0025, which brings upper bound of the confidence interval close to the estimate of the standard mean group model. For the region of inaction dummy, although the standard errors increase as the
region of inaction band expands, the size of increase still remains minimal; 0.0027 for 
±0.05 standard deviation band, 0.0038 for ±0.1 standard deviation band, and 0.0043 
for ±0.2 standard deviation band. When both dummies are combined, the increase in 
standard errors is bigger because the number of invalid observations increases; 0.0068 for 
±0.05 standard deviation band, 0.0074 for ±0.1 standard deviation band, and 0.0155 for 
±0.2 standard deviation band. Especially, the standard error of the estimate with both 
de facto regime dummy and ±0.2 standard deviation band region of inaction dummy 
increases more than tenfold compared to the standard error of the standard mean group 
estimate; however, the estimate is far enough from the standard mean group estimate 
(and obviously the standard fixed effect estimate), so the confidence intervals of these 
estimates do not overlap. Therefore, it implies that even though excluding some valid 
observations may have lowered the accuracy of estimation, the new estimates are still 
statistically lower than the estimates of previous studies.

2.4.1 Robustness check: Greece

For a robustness check, I estimate the persistence of real exchange rates only 
for countries, which show similar price level volatilities. As we see Figure 2.1, Greece 
shows much higher volatilities of terms of trade and nominal exchange rates compared 
to other European countries, while maintaining a similar real exchange rate volatility. 
The volatilities are consistent even under de facto fixed exchange rate regimes (Figure 
2.1, right panel). This may imply that in Greece, price adjustments in both commodity 
and currency are more active than other countries and that the real exchange rate
<table>
<thead>
<tr>
<th>Model</th>
<th>Region of Inaction</th>
<th>$\sum_{k=1}^{K^*} \hat{p}_{c,k} \rho_i,c,k$</th>
<th>Half-life (Months)</th>
<th>(95% C.I)</th>
<th>% change from Base</th>
<th>% change from FE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Panel</td>
<td>Base</td>
<td>0.979 (0.0011)</td>
<td>33.29</td>
<td>(30.0, 37.4)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Mean Group Panel</td>
<td>Base</td>
<td>0.971 (0.0019)</td>
<td>23.25</td>
<td>(20.5, 26.8)</td>
<td>-</td>
<td>-30.16</td>
</tr>
<tr>
<td></td>
<td>De facto Regime</td>
<td>0.967 (0.0025)</td>
<td>20.89</td>
<td>(18.1, 24.6)</td>
<td>-10.15</td>
<td>-37.25</td>
</tr>
<tr>
<td></td>
<td>Steady RER ±0.05 s.d.</td>
<td>0.967 (0.0028)</td>
<td>20.50</td>
<td>(17.5, 24.7)</td>
<td>-11.83</td>
<td>-38.42</td>
</tr>
<tr>
<td></td>
<td>±0.1 s.d.</td>
<td>0.961 (0.0041)</td>
<td>17.57</td>
<td>(14.5, 22.3)</td>
<td>-24.43</td>
<td>-47.22</td>
</tr>
<tr>
<td></td>
<td>±0.2 s.d.</td>
<td>0.957 (0.0043)</td>
<td>15.71</td>
<td>(13.1, 19.7)</td>
<td>-32.43</td>
<td>-52.81</td>
</tr>
<tr>
<td></td>
<td>De facto Regime ±0.05 s.d.</td>
<td>0.952 (0.0076)</td>
<td>14.10</td>
<td>(10.7, 20.6)</td>
<td>-39.36</td>
<td>-57.65</td>
</tr>
<tr>
<td></td>
<td>&amp; Steady RER ±0.1 s.d.</td>
<td>0.954 (0.0059)</td>
<td>14.91</td>
<td>(11.8, 20.1)</td>
<td>-35.87</td>
<td>-55.21</td>
</tr>
<tr>
<td></td>
<td>±0.2 s.d.</td>
<td>0.934 (0.0165)</td>
<td>10.25</td>
<td>(6.7, 20.7)</td>
<td>-55.91</td>
<td>-69.21</td>
</tr>
</tbody>
</table>

* The number of lags are determined by Akaike information criterion (AIC) and Schwarz criterion (BIC).

Table 2.5: Real exchange rates persistence estimates in half-life (Excluding Greece)

mean-conversion is faster. Therefore, I estimate the persistence of real exchange rates excluding Greece data. The results are reported in Table 2.5.

The results are consistent with previous results, except that the overall persistence estimates are slightly higher than the previous ones. The standard fixed effects panel estimate and the standard mean group estimate are 33.29 and 23.25 months respectively; the estimates are higher, but they are statistically indistinguishable because large ranges of the confidence intervals overlap with the ones estimated including Greece.

The estimation excluding invalid observations provides the results showing a similar pattern. Including de facto regime dummy decreases the half-life estimate to 20.89 months that is slightly higher than the previous result of 19.81 months. Similarly to the previous results, as the region of inaction bands expands, the half-life estimate decreases. Employing both dummy variables provides the similar results. All esti-
\[
q_{c,t} = \gamma + \sum_{k=1}^{K^*} \rho_{c,k} q_{c,t-k} \cdot d_{c,k} + \varepsilon_{c,t},
\]
(2.7)

and the estimation results are reported in Table 2.6.

Including de facto regime dummy decreases the half-life estimate from 32.26
months to 29.05 months. Although the magnitude of decrease is only 9.96%, which is less than 14.39% of mean group estimate, the direction of change is consistent with the hypothesis and the standard error increases only by 0.0003, while the standard errors of mean group estimate increases by 0.0008.

As the region of inaction band expands, however, the half-life estimate increases: 34.42 months for \( \pm 0.05 \) standard deviation band, 37.98 months for \( \pm 0.1 \) standard deviation band, and 40.78 months for \( \pm 0.2 \) standard deviation band. This pattern is persistent when \textit{de facto} regime dummy is employed: 28.07 months for \( \pm 0.05 \) standard deviation band, 31.74 months for \( \pm 0.1 \) standard deviation band, and 32.99 months for \( \pm 0.2 \) standard deviation band. The same lags are used for both fixed effects model and mean group model, and therefore, it is assumed that excluding invalid observations aggravates the aggregation bias of the standard fixed effects panel estimation, and more studies on econometric methodology are required to understand the features of aggregation bias to explain such results.

\section*{2.5 Conclusion}

The paper contributes to the PPP puzzle literature by considering whether previous studies have used valid observations to study mean-reversion of real exchanges rates. Calvo and Reinhart (2002) and Klein and Shambaugh (2008) suggest that \textit{de jure} exchange rate regime accounts for seemingly slow reversion of real exchange rates. Therefore, by using \textit{de facto} regime dummy – categorization code used in Calvo and
Reinhart (2002) and Klein and Shambaugh (2008), lower half-lives of real exchange rate conversion rates are estimated, compared to those of previous studies. The paper also tests whether previous studies have investigated the real exchange rates data that are within the region of inaction. Since mean-reversion of real exchange rates is not linear in time due to different incentives for arbitrage, the estimate will suggest that the mean reversion rate is slow, if we study the time-series data of real exchange rates which are close to their means and therefore within the band of region of inaction. The standard for a “large” mean-deviation in real exchange rates has not been set or studied in the previous literature, and therefore I suggest three different conservative bands of region of inaction around means; real exchange rates are defined to be in the regions of inaction if they are within ±0.05, ±0.1, and ±0.2 standard deviations from the means. Studying the data of nineteen goods CPI for eleven countries provides empirical evidences to these propositions. Including the exchange rates regime dummy decreases the half-life estimate to 19.81 months and the region of inaction dummy decreases the estimates to 15.22 - 20.54 months. Using both dummy variables elicits the results that make the puzzle less puzzling; the half-life estimates are 9.30 - 13.89 months. Although the results are promising, excluding invalid observations may not be an effective methodology for standard fixed effects models. Rather, it seems aggravating the aggregation bias, and further study on econometric methodology is needed to fully comprehend such results.
Chapter 3

Cost of Floating Exchange Rates: Should we fear to float?

3.1 Introduction

The advocates of floating exchange rate regime argue that fixed exchange rate regimes result in sluggish growth economy, and more developing countries have switched their exchange rate regimes from fixed to floating in recent years. (Batini et al., 2006; Klein and Shambaugh, 2008) Since the choice of exchange rate regimes is a crucial part of international monetary policies that are bound by the “trilemma,” it is necessary to identify and characterize the costs of floating currency price levels to fully comprehend the dynamics of optimal monetary policy portfolio. Such numerical estimation of costs is useful for calibrating parameter values in theoretical studies.

A few empirical studies examine the effect of breaking exchange rate pegs and
switching to the floating regimes. Alesina and Wagner (2006) find that switching exchange rate regimes from fixed to floating brings external costs into the economy. They study the countries, whose de jure and de facto exchange rate regimes are different, to investigate why countries employ different exchange rate regimes from their de jure regimes. The authors argue that wide exchange rate fluctuations (especially devaluations) are taken by markets as an indication of poor economic management and that breaking pegs signal instability of the economy. The results are significant over 180 countries data, and they provide the empirical evidence that switching foreign exchange rate regimes from fixed to floating induces costs in exercising monetary policy, such as limitation in interest rate or capital control policies.

On the contrary, the findings in Jahjah et al. (2013) suggest the opposite results; the paper studies how exchange rate regime affects the level of risk premium, and finds that country spread is 88 basis points lower in the countries under floating regimes than under fixed regimes. However, the results are based on the annual data, which average out various effects of employing certain exchange rate regimes, and the implication on the shorter-run costs of breaking pegs is still illusive.

In this paper, I use the event study framework on daily data to study the short-term effect of switching exchange rate regimes on the country risk premium. Among various costs and benefits of switching regimes, I focus on the foreign investor’s perspective, and investigate how country spreads change when countries (especially developing countries) switch their exchange rate regimes from fixed to floating. Following the argument of Alesina and Wagner (2006), the study hypothesizes that foreign investors
prefer steadier exchange rates and less uncertainties in the currency price movements, and that they also consider breaking exchange rate pegs as central banks’ losing abilities to keep their monetary stability promises.

The results of this study suggest that switching exchange rate regime from fixed to floating results in increases in the risk premium in the short-run. The increase in the risk premium can be interpreted as the additional cost of switching exchange rate regimes because it implies that the price of foreign loans becomes more expensive for local entrepreneurs, and as the entrepreneurs are financially more constrained, the growth of economy eventually lingers.

Section 3.2 describes the empirical strategy; section 3.2.1 briefly describes event study framework performed in this study, section 3.2.2 illustrates a basis of coding of *de facto* exchange rate regime using the daily data, and section 3.2.3 provides a summary description of the data set. Section 3.3 provides the results, and Section 3.4 concludes.

### 3.2 Empirical Strategy

#### 3.2.1 Event study

Event studies are designed to estimate how asset prices react to economic events that include new information relevant for the value of the underlying assets. In this paper, I investigate the price index that represents the country risk premium around the identified events, when countries loosen or lose their controls of foreign exchange markets and switch their exchange rate regime from fixed to floating.
2 Empirical Strategy

2.1 Event study

Event studies are designed to estimate how asset prices react to economic events that include new information relevant for the value of the underlying assets. In this paper, I investigate the price index that represents the country risk premium around the identified events, when countries loosen or lose their controls of foreign exchange markets and switch their exchange rate regime from fixed to floating.

A timeline for a typical event study is shown in Figure 3.1. The interval from $T_0$ to $T_1$ is the estimation period (or the pre-event period), where the normal return of the price index (denoted as $R$) is estimated. The interval from $T_1$ to $T_2$ is the event period, and the abnormal return (denoted as $AR$) – the actual return compared to the estimated return from the pre-event period – and the cumulative abnormal return (denoted as $CAR$) are estimated in this period. Time 0 is the event date in calendar time, and the interval from $T_2$ to $T_3$ is the post-event period.

In the event study, the abnormal returns are crucial measure to assess the impact of an event. The abnormal return of event date $t$ is defined as the difference of the realized return and the expected return given the absence of the event:

$$AR_t = R_t - E[R_t|\Omega_t],$$

and it shows the effect of the event from other general market movements. It is considered that the event has a significant impact on the price index, if the estimated abnormal returns during the event window are statistically different from zero. Aggre-
gating abnormal returns across time (from $t_1$ to $t_2$) provides the cumulative abnormal return:

$$CAR_{(t_1, t_2)} = \sum_{t=t_1}^{t_2} AR_t,$$

and it provides the cumulated impact on the price index during the sub-period of the event window. I report both abnormal return estimates and cumulative abnormal return estimates to illustrate the impact on the risk premium in different perspectives.

In this study, various ranges of estimation and event periods are investigated because the nature of employing foreign exchange rate regimes; switching exchange rate regime can be executed on a single day, such as the case of the Swiss National Bank, which abandoned its peg on January 15th, 2015, but it can also take a few days for a policy to be implemented and to take an effect in the foreign exchange market. In addition, since we are interested in the times when *de facto* exchange rate regimes switch, the days of events are identified based on the movements of the exchange rates, and therefore, foreign investors may or may not recognize the events on the actual day of policy implementation. Thus, it is necessary to investigate different sizes of windows to observe the movements of country spreads around the series of events in this study.

### 3.2.2 Identification of regime-switching events

One of the crucial parts of this study is the identification of regime-switching events. To identify the events of switching foreign exchange rate regimes, I use a similar
methodology to that\(^1\) is used in Calvo and Reinhart (2002) and Klein and Shambaugh (2008); however, since their basis of coding is based on monthly data, I use a modified standard for \textit{de facto} exchange rate regimes. I define that the country keeps its exchange rate peg (and managed peg) to the US dollars on a certain day when the exchange rate of the day is within certain standard deviations (denoted as \(K\)) from the moving average exchange rates of the past 10 months. In other words, the date of switching exchange rate regimes is identified when the exchange rate of the day is outside of the band around the 10-month moving average.

The study tests two different sizes of band: 3 moving-standard-deviation band and 5-moving-standard deviation band. The sizes of band are selected to match the numbers of exchange rate regime switching identified in Klein and Shambaugh (2008). Since the data set provided by Klein and Shambaugh (2008) spans from 1973 only to 2004, the overlapping periods with the periods that this study investigates are limited.

\(^1\)The country is having a fixed exchange rate regime if the month-end exchange rates stay within \(\pm 2.5\%\) band for 10 months, and pegs that last less than 10 months are classified as floating.
Therefore, I use the data from 2000 to 2004 to calibrate the numbers of switching regimes, and use the standards to identify the events from 2004 to 2008 (2014 for Peru). Despite that the data set (the nominal exchange rate data and the daily country spread data\textsuperscript{2}) for most countries is available from 1998, due to the instability of the foreign exchange markets during the global financial crisis in emerging market economies in 1998 and 1999, I limit the periods of study to 2000 – 2008 (2014 for Peru).

Figure 3.2 provides an example of identification of \textit{de facto} regime-switching using nominal exchange rates of the South Africa rand against the US dollar. Between 2000 and 2014, 6 events under $K = 3$ band and 3 events under $K = 5$ band are identified, and one can find that the dates are identified as events when there are spikes after relatively flat movements of exchange rates for 10 months. The events under a higher threshold of $K = 5$ are within the set of the events under a lower threshold of $K = 3$; however, the dates of events under $K = 5$ may not perfectly align with the event dates under $K = 3$ but they are lagged for a few days. Because it takes a few days for a currency price to fully depreciate, under a lower threshold, the event can be recognized earlier than under a higher threshold.\textsuperscript{3}

3.2.3 Data

As a measure of sovereign credit risk, this study uses the J.P. Morgan Emerging Market Bond Index (EMBI+), the major index of foreign currency emerging market economy sovereign bond spreads. The EMBI+ is created as a benchmark that would

\textsuperscript{2}Detailed description is provided in the next section 3.2.3.

\textsuperscript{3}This is another reason why it is necessary to investigate various sizes of estimation and event window.
reflect returns from price gains and interest income on a “passive” portfolio of traded emerging markets debt, and it is constructed as a composite of its four markets: Brady bonds, Eurobonds, US dollar local markets, and loans. The index reflects the country risk but is independent from the exchange risk because the index presents the spread between the dollar-denominated bonds and the US Treasury bill. For this reason, EMBI+ is often used in the sovereign risk literature\textsuperscript{4}, and it fits well with this study as I intend to extract investor’s sentiment from country’s exchange rate policy change on the sovereign risk, which requires a measure that is highly correlated with sovereign risk and independent from (or less correlated with) the exchange risk. In other words, when the central banks break exchange rate pegs by loosening (or losing) controls over local currency prices, EMBI+ is expected to show zero abnormal returns, unless investors consider the volatile exchange rates as a signal of instable economy.

This study examines the data of 10 developing countries: Argentina (ARG), Brazil (BRA), Bulgaria (BGR), Colombia (COL), Indonesia (IDN), Mexico (MEX), Peru (PER), Turkey (TUR), Ukraine (UKR), and South Africa (ZAF). The summary description of EMBI+ of these countries is provided in Table 3.1. The levels of EMBI+ vary across countries; for example, the average EMBI+ of Argentina is 2201.49, while the one of South Africa is 158.12. However, since I examine the daily return of EMBI+, the estimation results are independent from the different average levels\textsuperscript{5}; the results in section 3.3.3, where I present estimation results without Argentina observations as a

\textsuperscript{4}See Kaminsky et al. (2002), Remolona et al. (2008), Gapen et al. (2008), Hilscher and Nosbusch (2010).

\textsuperscript{5}For a visualization purpose, I use the standardized measure to present the time series around the events.
Table 3.1: Summary description of EMBI+ of 10 developing countries from January 2nd, 1998 to December 23rd, 2008

<table>
<thead>
<tr>
<th>Country</th>
<th># of Obs</th>
<th>Average</th>
<th>Std Dev</th>
<th>Min</th>
<th>Max</th>
<th>Range of available data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>2752</td>
<td>2201.49</td>
<td>2315.06</td>
<td>185</td>
<td>7220</td>
<td>Jan 02, 1998 : Dec 23, 2008</td>
</tr>
<tr>
<td>Brazil</td>
<td>2752</td>
<td>664.28</td>
<td>418.67</td>
<td>138</td>
<td>2436</td>
<td>Jan 02, 1998 : Dec 23, 2008</td>
</tr>
<tr>
<td>Colombia</td>
<td>2401</td>
<td>441.71</td>
<td>211.96</td>
<td>95</td>
<td>1094</td>
<td>May 28, 1999 : Dec 23, 2008</td>
</tr>
<tr>
<td>Indonesia</td>
<td>538</td>
<td>309.71</td>
<td>195.73</td>
<td>141</td>
<td>1152</td>
<td>Nov 01, 2006 : Dec 23, 2008</td>
</tr>
<tr>
<td>Mexico</td>
<td>2752</td>
<td>298.04</td>
<td>183.95</td>
<td>71</td>
<td>1160</td>
<td>Jan 02, 1998 : Dec 23, 2008</td>
</tr>
<tr>
<td>Peru</td>
<td>4222</td>
<td>335.56</td>
<td>206.87</td>
<td>91</td>
<td>1061</td>
<td>Jan 02, 1998 : Sep 04, 2014</td>
</tr>
<tr>
<td>Turkey</td>
<td>2358</td>
<td>468.50</td>
<td>249.33</td>
<td>164</td>
<td>1194</td>
<td>Jul 30, 1999 : Dec 23, 2008</td>
</tr>
<tr>
<td>Total</td>
<td>24048</td>
<td>605.80</td>
<td>1012.28</td>
<td>42</td>
<td>7220</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.2: Number of identified events of exchange rate regime switching from fixed to floating in 10 developing countries from January 3rd, 2000 to September 4th, 2014

<table>
<thead>
<tr>
<th>ARG</th>
<th>BRA</th>
<th>BGR</th>
<th>COL</th>
<th>IDN</th>
<th>MEX</th>
<th>PER</th>
<th>TUR</th>
<th>UKR</th>
<th>ZAF</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>K = 3</td>
<td>6</td>
<td>8</td>
<td>3</td>
<td>7</td>
<td>6</td>
<td>5</td>
<td>6</td>
<td>5</td>
<td>6</td>
<td>58</td>
</tr>
<tr>
<td>K = 5</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

robustness check, show that the estimated abnormal returns are not sensitive to the differences in the overall average EMBI+ across countries.

I follow the basis of coding illustrated in section 3.2.2 to identify the dates of events, and the numbers of events identified are reported in Table 3.2. Figure 3.3 uses the example in Figure 3.2 to provide an example of how EMBI+ responds around the events. Since the daily South African EMBI+ data is only available from April 30th, 2002 to December 23rd, 2008, the index is evaluated around three events (one event with the higher threshold). The South African example shows that a depreciation of exchange rates raises a concern over the country credit, which is reflected in the upward movements of EMBI+ around the events. The same exercises are performed
on 9 other countries, and I find the average returns of EMBI+ around the events to evaluate whether the spreads “jump” or show abnormal returns that are statistically different from zero, when exchange rate regimes switch.

3.3 Results

In this section, I present the results in two ways; section 3.3.1 provides plots of EMBI+ and standardized EMBI+ around the events of regime-switching, and section 3.3.2 provides the abnormal returns and cumulative abnormal returns. As a robustness check, I present the results without the observations of Argentina in section 3.3.3, and I present the plots around the events with longer-term windows in section 3.3.4.
3.3.1 EMBI+ around the events

The plots of EMBI+ around the events provide the visual presentation that breaking pegs increases the country risk premium and eventually induces additional costs that monetary authorities should consider.

In Figure 3.4, I present the plots of the country spread discontinuity around the events under $K = 3$. The left panel presents the linear, quadratic, and cubic regression estimations before and after the events, which illustrate the discontinuities in expected values of EMBI+ around the events. The discontinuity is clear; the size of discontinuity ranges from 35.97 basis points to 81.34 basis points. The right panel
Figure 3.5: Discontinuity (linear, quadratic, and cubic) of EMBI+ around the events identified under $K = 5$

provides the linearly fitted line within the event period ($\pm 5$ days around the events) to highlight the difference among the rates of change in EMBI+ before the event, during the event, and after the event, and it provides another visual evidence showing that EMBI+ increases in a much faster rate during the event period compared to pre-event and post-event periods.

During the events identified under a higher threshold ($K = 5$), the discontinuities are more evident as shown in Figure 3.5; the size of discontinuity ranges 141.11 - 165.49 basis points. We can also find that the trend of EMBI+ movement changes after the event, and the results align well with the argument of Fiess and Shankar (2009),
which finds that the countries often break their exchange rate pegs to lessen the pressure of maintaining fixed regimes on other macro variables. Figure 3.5 shows that the country spread increases, when the pressure on the foreign exchange market becomes higher, and when countries switch their regimes and exchange rates start floating, the country risk premium spikes because of the uncertainty in exchange rates and its potential impact on sovereign risk; however, it becomes stabilized afterwards. Such findings are consistent during the events under $K = 3$ and $K = 5$ thresholds.

Due to the potential bias towards countries with high levels of EMBI+, such as Argentina, I also provide the plots of standardized EMBI+ around the events in Figure 3.6.
3.6 and Figure 3.7, where the standardized EMBI+ is computed as following:

\[
\text{Std.EMBI+}_{c,t} = \frac{\text{EMBI+}_{c,t} - \hat{\mu}(\text{EMBI+})_c}{\hat{\sigma}(\text{EMBI+})_c}.
\]

\(\hat{\mu}(\text{EMBI+})_c\) is the sample average of EMBI+ of country \(c\), and \(\hat{\sigma}(\text{EMBI+})_c\) is the sample standard deviation of EMBI+ of country \(c\). Unifying the units to the standardized deviation from its mean allows to compare EMBI+ of different groups.\(^6\) Similarly to the previous graphs, the standardized EMBI+ shows discontinuity around the events identified under \(K = 3\). Although the discontinuity of estimates is not as apparent (0.038 - 0.236 standard deviations) as seen in Figure 3.4, especially the one with cubic functions, the change in trend of standardized EMBI+ is clear as we can see in the right panel of Figure 3.6.

Figure 3.7 presents the discontinuity of standardized EMBI+ around the events identified under \(K = 5\) threshold, and the discontinuity is clearer; the discontinuity estimates range from 0.257 to 0.525 standard deviations and the slope estimate of EMBI+ trend increases from 0.017 (0.0039) to 0.074 (0.0452). As the more distinct events are filtered, the clearer feedbacks from investors can be identified.

Overall, the plots in this section provide visual presentations of positive discontinuities of EMBI+, when countries switch their exchange rate regimes from fixed to floating. In the next section, I provide the abnormal return estimates with various sizes of estimation and event windows.

\(^6\)Standardizing is not necessary for abnormal return and cumulative abnormal return estimation in section 3.3.2 because the estimates are computed on the daily returns of EMBI+, not on the absolute value of EMBI+.
Figure 3.7: Discontinuity (linear, quadratic, and cubic) of standardized EMBI+ around the events identified under $K = 5$

### 3.3.2 Abnormal returns and cumulative abnormal returns

In Table 3.3, I provide the abnormal return estimates ($AR$) and the cumulative abnormal return estimates ($CAR$) with different estimation (pre-event) and event window sizes. The time periods, $T_0$, $T_1$, and $T_2$ denote the days from events as illustrated in Figure 3.1. The table reports the estimates under $K = 3$ and $K = 5$ thresholds and the corresponding test statistics of the estimates, $t_{K=3}$ and $t_{K=5}$. The test statistics are computed based on the null hypotheses that the abnormal returns and the cumulative
Table 3.3: Abnormal return and cumulative abnormal return estimates with different pre-event and event window sizes; $T_0 - T_1$: Pre-event window and $T_1 - T_2$: Event window

<table>
<thead>
<tr>
<th>Time periods</th>
<th>$K = 3$</th>
<th>$K = 5$</th>
<th>$t_{K=3}$</th>
<th>$t_{K=5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_0$ - 5</td>
<td>4.096</td>
<td>2.989</td>
<td>0.115</td>
<td>0.090</td>
</tr>
<tr>
<td>$T_0$ - 3</td>
<td>5.102</td>
<td>4.237</td>
<td>0.115</td>
<td>0.120</td>
</tr>
<tr>
<td>$T_0$ - 2</td>
<td>4.112</td>
<td>3.763</td>
<td>0.115</td>
<td>0.120</td>
</tr>
<tr>
<td>$T_0$ - 1</td>
<td>4.756</td>
<td>4.237</td>
<td>0.115</td>
<td>0.120</td>
</tr>
<tr>
<td>$T_0$ 0</td>
<td>4.967</td>
<td>4.237</td>
<td>0.115</td>
<td>0.120</td>
</tr>
<tr>
<td>$T_0$ - 5</td>
<td>5.102</td>
<td>4.237</td>
<td>0.115</td>
<td>0.120</td>
</tr>
<tr>
<td>$T_0$ - 3</td>
<td>5.102</td>
<td>4.237</td>
<td>0.115</td>
<td>0.120</td>
</tr>
<tr>
<td>$T_0$ - 2</td>
<td>5.102</td>
<td>4.237</td>
<td>0.115</td>
<td>0.120</td>
</tr>
<tr>
<td>$T_0$ - 1</td>
<td>5.102</td>
<td>4.237</td>
<td>0.115</td>
<td>0.120</td>
</tr>
<tr>
<td>$T_0$ 0</td>
<td>5.102</td>
<td>4.237</td>
<td>0.115</td>
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<tr>
<td>$T_0$ - 5</td>
<td>5.102</td>
<td>4.237</td>
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<td>0.120</td>
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<td>$T_0$ - 3</td>
<td>5.102</td>
<td>4.237</td>
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<td>5.102</td>
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</tr>
<tr>
<td>$T_0$ - 1</td>
<td>5.102</td>
<td>4.237</td>
<td>0.115</td>
<td>0.120</td>
</tr>
<tr>
<td>$T_0$ 0</td>
<td>5.102</td>
<td>4.237</td>
<td>0.115</td>
<td>0.120</td>
</tr>
</tbody>
</table>

†: test-statistics less than 2.58 (Not statistically significant at 99% confidence level)
‡: test-statistics less than 1.96 (Not statistically significant at 95% confidence level)

The results in Table 3.3 show that the abnormal return estimates are statistically significant for most of the time periods. The test statistic for CAR is computed as following:

$$ t = \frac{CAR/N}{\hat{\sigma}(AR)/\sqrt{N}} $$

where $\hat{\sigma}(AR)$ is the sample standard deviation of abnormal returns, and $N$ is the number of events.
cally significant with few exceptions, which are marked as † and ‡. The size of abnormal return estimates under $K = 3$ thresholds ranges from 0.009 to 0.019, and these estimates indicate that the returns of EMBI+ are 17.99 - 166.82% higher during the periods of switching exchange rate regimes, compared to the expected returns, which range from 0.0036 to 0.0075. The abnormal return estimates are higher around the events identified under higher thresholds ($K = 5$); we expect more distinct feedbacks from the investors, when the exchange rates depreciate higher. The size of abnormal return estimates under $K = 5$ thresholds ranges from 0.011 to 0.029; the returns of EMBI+ are 8.31 - 225.85% higher during the periods of switching exchange rate regimes.

As the abnormal return estimates are statistically significant, the cumulative abnormal return estimates, which are the sum of the abnormal returns during the event periods, are statistically different from zero, with one case of exception ($T_0 = -20$, $T_1 = -2$, $T_2 = 2$). Therefore, the results imply that there are short-term cumulative impacts on investor’s sentiment on sovereign risks, when countries loosen or lose controls over the exchange rates, and that such impacts are expressed as over 100% increases in the daily rate of returns in EMBI+. In other words, when a country breaks its exchange rate peg, the price of foreign loans becomes more expensive for local investors, which incurs additional short-term costs to the economy because local entrepreneurs become financially constrained.
Table 3.4: Abnormal return and cumulative abnormal return estimates with different pre-event and event window sizes; \( T_0 - T_1 \): Pre-event window and \( T_1 - T_2 \): Event window

<table>
<thead>
<tr>
<th>Time periods</th>
<th>AR</th>
<th>CAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_0 )</td>
<td>( T_1 )</td>
<td>( T_2 )</td>
</tr>
<tr>
<td>-40 -5 5</td>
<td>0.010</td>
<td>3.763</td>
</tr>
<tr>
<td>-40 -3 3</td>
<td>0.008†</td>
<td>2.493</td>
</tr>
<tr>
<td>-40 -2 2</td>
<td>0.006‡</td>
<td>1.625</td>
</tr>
<tr>
<td>-40 -1 1</td>
<td>0.013†</td>
<td>2.881</td>
</tr>
<tr>
<td>-40 0 3</td>
<td>0.013†</td>
<td>3.156</td>
</tr>
<tr>
<td>-40 0 5</td>
<td>0.014†</td>
<td>4.089</td>
</tr>
<tr>
<td>-40 -3 0</td>
<td>0.007‡</td>
<td>1.723</td>
</tr>
<tr>
<td>-40 -5 0</td>
<td>0.008†</td>
<td>2.412</td>
</tr>
<tr>
<td>-20 -5 5</td>
<td>0.010</td>
<td>2.751</td>
</tr>
<tr>
<td>-20 -3 3</td>
<td>0.007‡</td>
<td>1.738</td>
</tr>
<tr>
<td>-20 -2 2</td>
<td>0.005‡</td>
<td>1.069</td>
</tr>
<tr>
<td>-20 -1 1</td>
<td>0.013‡</td>
<td>2.348</td>
</tr>
<tr>
<td>-20 0 3</td>
<td>0.012‡</td>
<td>2.552</td>
</tr>
<tr>
<td>-20 0 5</td>
<td>0.014‡</td>
<td>3.361</td>
</tr>
<tr>
<td>-20 -3 0</td>
<td>0.005‡</td>
<td>1.077</td>
</tr>
<tr>
<td>-20 -5 0</td>
<td>0.007‡</td>
<td>1.614</td>
</tr>
<tr>
<td>-60 -5 5</td>
<td>0.012</td>
<td>4.846</td>
</tr>
<tr>
<td>-60 -3 3</td>
<td>0.010</td>
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</tr>
<tr>
<td>-60 -2 2</td>
<td>0.008†</td>
<td>2.325</td>
</tr>
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<td>3.443</td>
</tr>
<tr>
<td>-60 0 3</td>
<td>0.014</td>
<td>3.837</td>
</tr>
<tr>
<td>-60 0 5</td>
<td>0.016</td>
<td>4.962</td>
</tr>
<tr>
<td>-60 -3 0</td>
<td>0.009‡</td>
<td>2.364</td>
</tr>
<tr>
<td>-60 -5 0</td>
<td>0.010</td>
<td>3.165</td>
</tr>
</tbody>
</table>

\( \dagger \): test-statistics less than 2.58 (Not statistically significant at 99% confidence level)

\( \ddagger \): test-statistics less than 1.96 (Not statistically significant at 95% confidence level)

3.3.3 Robustness check: Argentina

The average level and the standard deviation of EMBI+ of Argentina are much higher than the ones of other countries in the sample, as seen in Table 3.1, and one can argue that the returns of Argentina EMBI+ may bias the estimates upwardly. Therefore, I find the abnormal return and the cumulative abnormal return estimates, excluding the observations of Argentina as a robustness check.

The estimation results are reported in Table 3.4. More test-statistics of ab-
normal returns fall under the 1.96 or 2.58 thresholds compared to Table 3.3; however, for $K = 3$, 18 cases (out of 24 cases) still reject the null hypothesis at 95% confidence level, and for $K = 5$, 20 cases reject the null hypothesis at 95% confidence level. In addition, the cumulative abnormal return estimates still indicate the statistical significance of cumulative impacts of breaking the peg; 21 cases in $K = 3$ and 22 cases in $K = 5$ present statistically significant estimates of cumulative abnormal returns at 95% confidence level.

### 3.3.4 Robustness check: Long-term effect

The results in the previous sections suggest an argument contrary to the results of Jahjah et al. (2013), which use the same measure of country spread to gauge the impact of switching regimes. One of the main differences between their study and this study is that the authors construct econometric models to find the longer-term effect of implementing different exchange rate regime policies using the annual data, while this study presents the shorter-term effect by focusing on the daily change of country spreads, when the country switch its exchange rate regime.

Figure 3.8 presents the movement of standardized EMBI+ around the events in longer horizons of 80 days of pre-event and post-event windows and 120 days of pre-event and post-event windows. Examining the trends over longer periods provides crude evidence that switching the exchange rate regime indeed brings down the country spread in the long-run, as the linear, quadratic, and cubic trends within post-event periods in all four plots in Figure 3.8 are downwardly sloped. The event study framework is no
Figure 3.8: Discontinuity (linear, quadratic, and cubic) of standardized EMBI+ around the events with 80 days of pre-event and post-event windows (left panels) and 120 days of pre-event and post-event windows (right panels) (top panels are for $K = 3$ and bottom panels are for $K = 5$)

longer a best-fitting econometric tool with this time-horizon because it is designed to capture the short-term impact of policy change or announcement and it is not able to control for other policy instruments that might have been effective in longer terms. However, these plots hint the idea that the optimal monetary policy portfolio including loosening controls over the foreign exchange market may have been implemented and eventually induced the improvement in EMBI+.
3.4 Conclusion

This paper studies the short-term cost of switching the exchange rate regime from fixed to floating by investigating the country spread in the event study framework. I use EMBI+ as a measure of the country spread that is independent from exchange risk to extract the investor’s sentiment on the sovereign risk during the events of breaking pegs.

While the findings of Jahjah et al. (2013) suggest switching exchange rate regime from fixed to floating lowers the price of foreign loans and relaxes financial constraints of local investors in the long-run, the results of this paper indicate that the effect of switching regime is unfavorable in the shorter-run, and it rather increases the price of foreign loans, which is reflected as the increase in the rate of returns of EMBI+. The abnormal return estimate ranges from 0.0036 to 0.0075 and the cumulative abnormal return estimate ranges from 0.036 to 0.240, which imply 8.31 - 225.85% increase in the returns of EMBI+ during the periods of breaking the peg.
Appendix A

Appendix for Chapter 1

A.1 Derivations of demand functions

Suppose \( C_{H,t}(j) = \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\epsilon} C_{H,t} \), then using (1.4), we have

\[
C_{H,t} = \left( \int_0^1 \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\epsilon} C_{H,t} \right)^{\frac{1-\epsilon}{\epsilon}}
\]

\[
= \left( C_{H,t} \frac{\epsilon-1}{\epsilon} \int_0^1 P_{H,t}(j)^{1-\epsilon} dj \right)^{\frac{1}{\epsilon}}
\]

\[
= C_{H,t} P_{H,t} \left( \int_0^1 P_{H,t}(j)^{1-\epsilon} dj \right)^{\frac{1}{\epsilon-1}}
\]
Since I previously defined in (1.14) that

\[
P_{H,t} = \left( \int_0^1 P_{H,t}(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}
\]

\[
\Rightarrow P_{H,t}^\epsilon = \left( \int_0^1 P_{H,t}(j)^{1-\epsilon} dj \right)^{\frac{\epsilon}{1-\epsilon}}
\]

\[
\Rightarrow P_{H,t}^\epsilon \left( \int_0^1 P_{H,t}(j)^{1-\epsilon} dj \right)^{\frac{\epsilon}{\gamma-1}} = 1
\]

Therefore

\[
C_{H,t}(j) = \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\epsilon} C_{H,t}
\]

\[
\square
\]

With a similar proof,

\[
C_{i,t}(j) = \left( \frac{P_{i,t}(j)}{P_{i,t}} \right)^{-\epsilon} C_{i,t}
\]

\[
\square
\]

Again, suppose \( C_{i,t} = \left( \frac{P_{i,t}}{P_{F,t}} \right)^{-\gamma} C_{F,t} \), then using (1.5), we have

\[
C_{F,t} = \left( \int_0^1 \left( \frac{P_{i,t}}{P_{F,t}} \right)^{-\gamma} C_{F,t} \right)^{\frac{\gamma}{\gamma-1}} \left( \frac{\gamma}{\gamma-1} \right) di
\]

\[
= \left( \frac{C_{F,t}}{\gamma} P_{F,t}^{\gamma-1} \right) \left( \int_0^1 P_{i,t}^{\gamma-1} di \right)^{\frac{\gamma}{\gamma-1}}
\]

\[
= C_{F,t} P_{F,t}^{\gamma} \left( \int_0^1 P_{i,t}^{\gamma-1} di \right)^{\frac{\gamma}{\gamma-1}}
\]
Since I previously defined in (1.15) that

\[ P_{F,t} = \left( \int_0^1 P_{i,t}^{1-\gamma} dj \right)^{\frac{1}{1-\gamma}} \]

\[ \Rightarrow P_{F,t}^\gamma = \left( \int_0^1 P_{i,t}^{1-\gamma} dj \right)^{\frac{\gamma}{1-\gamma}} \]

\[ \Rightarrow P_{F,t}^\gamma \left( \int_0^1 P_{i,t}^{1-\gamma} dj \right)^{\frac{\gamma}{1-\gamma}} = 1 \]

Therefore

\[ C_{i,t} = \left( \frac{P_{i,t}}{P_{F,t}} \right)^{-\gamma} C_{F,t} \]

A.2 Derivation of budget constraint

First term of (1.7) is

\[ \int_0^1 P_{H,t}(j) C_{H,t}(j) dj = \int_0^1 P_{H,t} \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\epsilon} C_{H,t} dj \]

\[ = C_{H,t} P_{H,t}^\epsilon \int_0^1 P_{H,t}(j)^{1-\epsilon} dj \]

and (1.14) provides

\[ P_{H,t}^\epsilon \int_0^1 P_{H,t}(j)^{1-\epsilon} dj = P_{H,t} \]

therefore

\[ \int_0^1 P_{H,t}(j) C_{H,t}(j) dj = P_{H,t} C_{H,t} \]
Similarly,

\[ \int_0^1 P_{i,t}(j)C_{i,t}(j) dj = P_{i,t}C_{i,t} \]

Suppose that \( \int_0^1 P_{i,t}C_{i,t} di = P_{F,t}C_{F,t} \), then using (1.18), we have

\[ \int_0^1 P_{i,t} \left( \frac{P_{i,t}}{P_{F,t}} \right)^{-\gamma} C_{F,t} di = P_{F,t}C_{F,t} \]

\[ \Rightarrow P_{F,t} = P_{F,t}^{\gamma} \int_0^1 P_{i,t}^{1-\gamma} di \]

\[ \Rightarrow P_{F,t}^{1-\gamma} = \int_0^1 P_{i,t}^{1-\gamma} di \]

\[ \Rightarrow P_{F,t} = \left( \int_0^1 P_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} \]

Since we defined in (1.15), we verify that \( \int_0^1 P_{i,t}C_{i,t} di = P_{F,t}C_{F,t} \), and therefore the second term of (1.7) is

\[ \int_0^1 \int_0^1 P_{i,t}(j)C_{i,t}(j) dj di = P_{F,t}C_{F,t} \]

The optimal allocation of expenditures between domestic and imported goods is given by

\[ C_{H,t} = (1-\alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t; \quad C_{F,t} = \alpha \left( \frac{P_{F,t}}{P_t} \right)^{-\eta} C_t. \quad (A.1) \]

Combining (1.9) and (A.1), then we have

\[ P_{H,t}C_{H,t} + P_{F,t}C_{F,t} = P_tC_t, \quad (A.2) \]
and the proof is given as following: Suppose $P_{H,t}C_{H,t} + P_{F,t}C_{F,t} = P_tC_t$, then using (A.1), we have

$$P_{H,t}(1 - \alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}C_t + P_{F,t}\alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta}C_t = P_tC_t$$

$$\Rightarrow (1 - \alpha)P_{H,t}^{1-\eta}P_t^\eta + \alpha P_{F,t}^{1-\eta}P_t^\eta = P_t$$

$$\Rightarrow (1 - \alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta} = P_t^{1-\eta}$$

$$\Rightarrow P_t = \left[(1 - \alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}},$$

and therefore we verify that $P_{H,t}C_{H,t} + P_{F,t}C_{F,t} = P_tC_t$.

Since we treat the rest of the world as symmetric countries,

$$\int_0^1 E_{i,t}D_{i+1}^{*}di = E_t^{*}D_t^{*}; \quad \int_0^1 E_{i,t}\tilde{R}_{i-1}^{*}(1+\tau)(1+\xi\lambda)\Psi_tD_{i}^{*}di = E_t^{*}\tilde{R}_{i-1}^{*}(1+\tau)(1+\xi\lambda)\Psi_tD_t^{*}$$

With the assumptions of $\tau_i = 0$ (\forall t) and $\Psi_t = 0$ (\forall t) we have the following budget constraints,

$$P_tC_t + D_{t+1}^{*}E_tD_{t+1}^{*} \leq W_tN_t + \Pi_t + T_t + R_{t-1}D_t + R_{t-1}^{*}E_tD_t^{*},$$

where

$$R_t^{*} = \tilde{R}_t^{*}(1 + \tau)(1 + \xi\lambda)\Psi_t.$$
A.3 Derivation of the Loss Function

A.3.1 Utility of consumption

Backus-Smith condition is log-linearized as following

\[ C_t = \Theta_t C_t^* Q_t^{\frac{1}{\sigma}} \]

\[ \Rightarrow \log C_t = \log \Theta_t + \log C_t^* + \sigma \log Q_t \]

\[ \Rightarrow c_t = \theta_t + c_t^* + \sigma q_t \]

\[ \Rightarrow c_t = \theta_t + c_t^* + q_t \quad (\because \sigma = 1) \quad \square \quad (A.3) \]

Goods market condition is

\[ Y_t = (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \alpha \int_0^1 \left( \frac{P_{H,t}}{E_{i,t} P_{F,t}^i} \right)^{-\gamma} \left( \frac{P_{F,t}^i}{P_t^i} \right)^{-\eta} C_t^{di} dt \]

\[ \Rightarrow Y_t = (1 - \alpha) \left( \frac{P_t}{P_{H,t}} \right) C_t + \alpha \left( \frac{P_t^*}{P_{H,t}} \right) E_t C_t^* \quad (\because \sigma = \eta = \gamma = 1) \]
and since \( C_t = \Theta_t C_t^* Q_t^* \),

\[
Y_t = (1 - \alpha) \left( \frac{P_t}{P_{H,t}} \right) \Theta_t C_t^* Q_t + \alpha \left( \frac{P_t^*}{P_{H,t}} \right) E_t C_t^* 
\]

\[
\Rightarrow Y_t = (1 - \alpha) S_t \Theta_t C_t^* + \alpha S_t C_t^* 
\]

\[
\Rightarrow Y_t = S_t C_t^* [(1 - \alpha) \Theta_t + \alpha] 
\]

\[
\Rightarrow \log Y_t = \log S_t + \log C_t^* + \log [(1 - \alpha) \Theta_t + \alpha] 
\]

\[
\Rightarrow s_t = y_t - c_t^* - (1 - \alpha) \theta_t - \frac{1}{2} \alpha (1 - \alpha)^2 \theta_t^2 
\] \hspace{1cm} \square \hspace{1cm} (A.4)

Combining (1.12), (A.3), and (A.4), we have

\[
c_t = \alpha c_t^* + (1 - \alpha) y_t + \alpha (2 - \alpha) \theta_t - \frac{1}{2} \alpha (1 - \alpha)^2 \theta_t^2 
\] \hspace{1cm} (A.5)

and its natural allocation is

\[
\tilde{c}_t = \alpha c_t^* + (1 - \alpha) \bar{y}_t + \alpha (2 - \alpha) \bar{\theta}_t - \frac{1}{2} \alpha (1 - \alpha)^2 \bar{\theta}_t^2 
\] \hspace{1cm} (A.6)

then expression of home consumption in gap is

\[
\hat{c}_t = c_t - \tilde{c}_t = (1 - \alpha) x_t + \alpha (2 - \alpha) \hat{\theta}_t - \frac{1}{2} \alpha (1 - \alpha)^2 (\theta_t^2 - \bar{\theta}_t^2) 
\]

\[
= (1 - \alpha) x_t + \alpha (2 - \alpha) \hat{\theta}_t - \frac{1}{2} \alpha (1 - \alpha)^2 (\theta_t - \bar{\theta}_t)(\theta_t + \bar{\theta}_t) 
\]

\[
= (1 - \alpha) x_t + \alpha (2 - \alpha) \hat{\theta}_t - \frac{1}{2} \alpha (1 - \alpha)^2 \hat{\theta}_t(\hat{\theta}_t + 2 \bar{\theta}_t) \quad (: \theta_t = \bar{\theta}_t + \hat{\theta}_t) \hspace{1cm} (A.7)
\]
As $\sigma = 1$, the utility of consumption becomes

$$U(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma} \approx \log(C_t) = c_t = \bar{c}_t + \hat{c}_t$$

$$= \bar{c}_t + (1-\alpha)x_t + \alpha(2-\alpha)\hat{\theta}_t - \frac{1}{2}\alpha(1-\alpha)^2\hat{\theta}_t^2(\hat{\theta}_t + 2\hat{\theta}_t) \quad (A.8)$$

A.3.2 Disutility of labor

The disutility of labor can be expressed by the second-order approximation

$$V(N_t) = \frac{N_t^{1+\phi}}{1+\phi} \approx \tilde{V}_t + \tilde{V}'_t \tilde{N}_t \left\{ \hat{n}_t + \frac{1}{2}(1+\phi)\hat{n}_t^2 \right\}$$

Labor market clearing condition (1.44) is now

$$N_t = \frac{Y_t}{A} \int_{0}^{1} \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\epsilon} dj$$

$$\Rightarrow \quad \log N_t = \log Y_t - \log A + \log \left[ \int_{0}^{1} \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\epsilon} dj \right]$$

$$\Rightarrow \quad n_t = y_t - a_t + u_t, \quad \text{where} \quad u_t \equiv \log \left[ \int_{0}^{1} \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\epsilon} dj \right]$$

Since the productivity is assumed to be stable over time and the following holds at steady state

$$\log \left[ \int_{0}^{1} \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\epsilon} dj \right] = 0,$$

we have

$$\hat{n}_t = x_t + u_t. \quad (A.9)$$
Then I can rewrite the second-order approximation to the disutility of labor as

\[ V(N_t) = \frac{N_t^{1+\phi}}{1 + \phi} \approx \bar{V}_t + \bar{V}'_t \bar{N}_t \left\{ x_t + u_t + \frac{1}{2} (1 + \phi) x_t^2 \right\}. \]  

(A.10)

### A.3.3 Loss function

Under the optimal subsidy scheme assumed, the optimality condition \( \bar{V}'_t \bar{N}_t = 1 - \alpha \) holds for all \( t \). Then, the period utility is

\[
U(C_t) - V(N_t) = \bar{c}_t + (1 - \alpha)x_t + \alpha(2 - \alpha)\hat{\theta}_t - \frac{1}{2} \alpha(1 - \alpha)^2 \hat{\theta}_t(\hat{\theta}_t + 2\bar{\theta}_t) \\
- \bar{V}_t - (1 - \alpha) \left\{ x_t + u_t + \frac{1}{2} (1 + \phi) x_t^2 \right\} \\
= - (1 - \alpha) \left\{ u_t + \frac{1}{2} (1 + \phi) x_t^2 + \frac{1}{2} \alpha(1 - \alpha)^2 \hat{\theta}_t(\hat{\theta}_t + 2\bar{\theta}_t) - \frac{\alpha(2 - \alpha)}{1 - \alpha} \right\} + t.i.p.,
\]

where \( t.i.p. \) denotes terms independent of policy, which include constant terms.

Woodford (2001) shows

\[
u_t = 1 + \frac{\epsilon}{2} \text{var}_i \{ p_{H,t}(i) \} + o(||a||^3),
\]

where \( o(||a||^3) \) refers to terms of third or higher order. Under Calvo pricing assumption, we have

\[
\sum_{t=0}^{\infty} \beta^t \text{var}_i \{ p_{H,t}(i) \} = \frac{1}{\kappa} \sum_{t=0}^{\infty} \beta^t \pi^2_{H,t},
\]
where $\kappa \equiv \frac{(1-\delta)(1-\beta\delta)}{\delta}$. Then I get the following expression for the objective function:

$$L = -\frac{1-\alpha}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\epsilon}{\kappa} \pi_{H,t}^2 + (1 + \phi)x_t^2 + \alpha(1 - \alpha)\hat{\theta}_t(\hat{\theta}_t + 2\bar{\theta}_t) - \frac{\alpha(2 - \alpha)}{1 - \alpha} \hat{\theta}_t \right\}.$$  

Now I use a second order approximation of the budget constraint to replace the linear term $\hat{\theta}_t$ in the objective function. A second order approximation for $nx_t$ is

$$nx_t = -\alpha \left( \theta_t + \frac{1}{2} \theta_t^2 \right)$$

then, a second order approximation for the budget constraint can be expressed as following:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \hat{\theta}_t + \frac{1}{2} \hat{\theta}_t(\hat{\theta}_t + 2\bar{\theta}_t) \right\} = 0,$$

so we can replace the linear term in the objective function as following:

$$L = -\frac{1-\alpha}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\epsilon}{\kappa} \pi_{H,t}^2 + (1 + \phi)x_t^2 + \alpha \left( \frac{2 - \alpha}{1 - \alpha} + 1 - \alpha \right) \hat{\theta}_t(\hat{\theta}_t + 2\bar{\theta}_t) \right\}$$

or up to a constant

$$L = -\frac{(1-\alpha)(1 + \phi)}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \alpha_\pi \pi_{H,t}^2 + x_t^2 + \alpha_\theta (\hat{\theta}_t + \bar{\theta}_t)^2 \right\},$$

where $\alpha_\pi = \frac{\epsilon}{\kappa(1+\phi)}$ and $\alpha_\theta = \frac{\alpha}{1+\phi} \left( \frac{2-\alpha}{1-\alpha} + 1 - \alpha \right)$, and therefore the objective function
that the social planner minimizes is expressed as following:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{2} \alpha_n \pi_{H,t}^2 + \frac{1}{2} x_t^2 + \frac{1}{2} \alpha_\theta (\hat{\theta}_t + \bar{\theta}_t)^2 \right\}.$$
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