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Publication Date
1984-11-01
Submitted for publication

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November 1984

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Prepared for the U.S. Department of Energy under Contract DE-AC03-76SF00098
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Models for inflation with a low supersymmetry-breaking scale

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ABSTRACT. We present models where the same scalar field is responsible for inflation and for the breaking of supersymmetry. The scale of supersymmetry breaking is related to the slope of the potential in the plateau region described by the scalar field during the slow rollover, and the gravitino mass can therefore be kept as small as $M_{W}$, the mass of the weak gauge boson. We show that such a result is stable under radiative corrections. We describe the inflationary scenario corresponding to the simplest of these models and show that no major problem arises, except for a violation of the thermal constraint (stabilisation of the field in the plateau region at high temperature). We discuss the possibility of introducing a second scalar field to satisfy this constraint.

The new inflationary universe scenario provides a simple and elegant way of solving a large number of cosmological problems of the standard big bang model [1]. But to be implemented, it needs the presence of a very weakly coupled scalar field. Locally supersymmetric theories with a hidden sector provide such a scalar field [2]. Whether
they just are a general framework or supersymmetry and inflation are more closely related is a matter of taste, but it seems desirable to require that the sector that drives inflation is also the one that breaks supersymmetry. Actually, in the case where this sector consists of one scalar field — called inflaton — the thermal constraint imposes such a breaking of supersymmetry \([3,4]\). The thermal constraint is the requirement that at high temperatures, a sufficient amount of energy is stored in the scalar field to give enough inflation — in other words, the inflaton field must start its evolution far away from its global minimum, slowly roll down (causing the universe to inflate) and eventually settle at its global minimum. The problem with this approach is that supersymmetry must be broken at a very large scale: typically \([4]\), the mass of the gravitino \(m_{3/2}\) must be greater than \(\mu^2/M = M_\rho/\sqrt{8\pi} = 2.4 \times 10^{18}\text{GeV}\) where \(\mu^4\) is the energy density of the false vacuum (a typical value for \(\frac{\mu}{M}\) of \(10^{-3}\) to \(10^{-4}\) is required to give rise to density fluctuations with the right amplitude; \(m_{3/2}\) is then greater than \(10^{10}\text{ GeV}\)). This has to be reconciled with models describing our low energy world where the breaking of \(SU(2) \times U(1)\) gauge invariance is driven by soft terms induced by supergravity — which scale like \(m_{3/2}\) \([5]\). Therefore in these models, the gravitino mass and the mass of the weak gauge boson \(M_W\) must be of the same order.

This problem has been addressed recently by Ovrut and Steinhardt who solve it by using two scalar fields in the inflationary sector \([6]\). They employ a mechanism \([7]\) which sets the symmetry-breaking scale to a much smaller value than the scale \(\mu\): typically, the gravitino mass is of order \(\frac{\mu^4}{M}\) which coincides therefore with the weak interaction scale \((\frac{\mu}{M} \sim 10^{-4})\). This can be worked out into a successful inflationary
universe scenario [6] at the price however of some fine-tuning (at least in the explicit example given in Ref. 6).

In this letter, we will take a different point of view and relate the smallness of the scale of supersymmetry breaking to the smallness of a parameter which is of basic importance in any inflationary universe scenario: the slope $\epsilon$ of the potential near the origin. Actually, since we want a scale of supersymmetry-breaking very small compared to the scales of relevance in the inflation sector (of the order of the Planck mass), it seems plausible that the ground state must be obtained by perturbing a supersymmetry conserving ground state. We will see in Sect. 1 that this imposes some constraints on the model that we are starting from. We do not know for the moment what is the nature of the perturbation but it has to be characterized by a parameter which must be very small. A natural (or possible) choice is precisely the slope $\epsilon$: if we want a slow roll-down along the plateau region of the potential, the slope has to be very small at the origin. Actually, in most models, it is taken to be zero. No symmetry argument supports such a choice (except simplicity, which is not thought to be a symmetry) hence we have no clue as to why $\epsilon$ is so small. But taking it for granted, we will show that the supersymmetry breaking scale can be related to it for a particular class of potentials. Moreover, even though $\epsilon$ is arbitrarily small, the scale of supersymmetry breaking that we obtain is stable under radiative corrections. In other words, in our approach, choosing the gravitino mass of the order of $M_W$ is natural in the technical sense. In Sect. 2, we describe the inflationary scenario that arises in a model which we consider as a typical example of our approach and we discuss what kind of constraints we obtain for
the parameters $\epsilon$ and $\mu$. It turns out that the thermal constraint mentioned earlier is violated. In Sect. 3, we show how to circumvent this by introducing a second scalar field in the inflation sector.

1. We first detail the procedure that we adopt to find a model that fulfills our requirement. The idea is to start with a potential for which $\epsilon = 0$ and the ground state is supersymmetry-conserving, then perturb this potential by taking $\epsilon \neq 0$ and see under which conditions the minimum becomes supersymmetry-breaking.

Let us first prove a result that applies to this situation in general, independently of the nature of the parameter $\epsilon$. Consider a scalar field $\phi$ in a locally supersymmetric theory. Its interactions are described by a superpotential $f(\phi)$ and the corresponding potential reads (assuming a flat Kähler potential) [8]:

$$V(\phi) = e^{\left|\phi\right|^2 / M^2} \left[ |D\phi f(\phi)|^2 - \frac{3}{M^2} |f(\phi)|^2 \right]$$

where

$$D\phi f(\phi) = \frac{\partial f(\phi)}{\partial \phi} + \frac{\phi^*}{M^2} f(\phi)$$

and $M$ is the reduced Planck mass $M = M_p / \sqrt{8\pi} \approx 2.4 \times 10^{18}$ GeV. The variable $\epsilon$ parametrizes a perturbation on the coefficients of the superpotential, which is left unspecified for the time being.

*If the minimum $- \sigma_0 M$ of the potential $V$ (with a zero cosmological constant) is supersymmetry-conserving when $\epsilon = 0$, then a necessary condition in order that the perturbed minimum (with zero cosmological constant) breaks supersymmetry is that:*

$$\left. \frac{\partial^2 f}{\partial \phi^2} (\sigma_0) \right|_{\epsilon=0} = 0$$

(3)
An equivalent formulation involving the potential is that its second derivative (and then automatically its third one) is zero at the minimum:

\[
\frac{d^2V}{d\phi^2}(\sigma_0) \bigg|_{\epsilon=0} = \frac{d^3V}{d\phi^3}(\sigma_0) \bigg|_{\epsilon=0} = 0
\]

(4)

The proof is straightforward. Since, when \(\epsilon = 0\), the minimum \(\sigma_0(V(\sigma_0) = V'(\sigma_0) = 0)\) conserves supersymmetry:

\[
f(\phi_0)\big|_{\epsilon=0} = \frac{\partial}{\partial \phi} f(\sigma_0) \bigg|_{\epsilon=0} = 0
\]

(5)

On the other hand, since we want a breaking of supersymmetry when we turn \(\epsilon\) on, we must require that:

\[
f(\sigma) \approx 0(\epsilon) \neq 0 \quad D_\phi f(\sigma) \approx 0(\epsilon) \neq 0
\]

(6)

The minimum \(\sigma\) is determined by the equations (\(\phi\) and \(f\) are taken to be real):

\[
V(\sigma) = 0 \iff |D_\phi f(\sigma)|^2 = 3 |f(\sigma)|^2
\]

(7)

\[
V'(\sigma) = 0 \iff [\frac{\partial}{\partial \phi} D_\phi f(\sigma)]D_\phi f(\sigma) = 3[\frac{\partial}{\partial \phi} f(\sigma)]f(\sigma)
\]

(8)

Therefore combining Eqs. (6) and (7), we obtain from (8):

\[
\frac{\partial}{\partial \phi} D_\phi f(\sigma) = \pm \sqrt{3} \frac{\partial}{\partial \phi} f(\sigma) = 0(\epsilon)
\]

(9)

which in turn gives Eq. (3). It is immediate to show, using the form of the potential [Eqs. (1) and (2)] that (3) and (4) are equivalent.
We now apply this result to a specific example which will prove to be generic. Let us consider the superpotential

\[ f(\phi) = \mu^2 M (a_0 + \frac{\phi}{M} + a_2 \frac{\phi^2}{M^2} + a_4 \frac{\phi^4}{M^4}) \]  \hspace{1cm} (10)

The corresponding potential reads, near the origin on the real axis,

\[ V(\phi) = \mu^4 e^{\phi^2/M^2} [(1 - 3a_0^2) + 4(a_2 - a_0) \frac{\phi}{M} + ...] \]  \hspace{1cm} (11)

As a first step, we require that the potential is flat near the origin and that the minimum \( u_0 M \) is supersymmetry-conserving. The first condition \( V'(0) = 0 \) gives:

\[ a_2 = a_0 \equiv \beta_0 \]  \hspace{1cm} (12)

and the second [Eq. (5)] yields

\[ \beta_0 = \frac{-3\sigma_0}{2(2 + \sigma_0^2)} \right., \quad \sigma_0^2 = \frac{-\beta_0 \pm \sqrt{\beta_0 (\beta_0 + 12a_4)}}{6a_4} \]  \hspace{1cm} (13)

It has been noticed already [3] that for such a family of superpotentials, the fact that \( \beta_0 \) and \( \sigma_0 \) are of opposite signs leads to a violation of the thermal constraint [3,4]. We will return to that question in Sect. 3.

We then relax condition (12) by allowing a small slope near the origin and we write instead:

\[ a_2 = \sigma_0 - \epsilon \equiv \beta \]  \hspace{1cm} (14)
The location of the minimum $\sigma$ and the parameter $\beta$ are now given by series in the parameter $\epsilon$:

$$
\beta = \beta_0 + \beta_1 \epsilon + \beta_2 \epsilon^2 + 0(\epsilon^3)
$$

$$
\sigma = \sigma_0 + \sigma_1 \epsilon + 0(\epsilon^2)
$$

(15)

The requirement that supersymmetry is broken at the minimum — expressed by the condition (3) — gives

$$
6a_4\sigma_0^2 + \beta_0 = 0
$$

(16)

in which case we obtain from (13)

$$
a_4 = -\frac{1}{12} \beta, \sigma_0 = \sqrt{2}, \beta_0 = -\frac{3}{8} \sigma_0
$$

(17)

We have actually two sets of solutions:

$$
\beta_1 = \pm \frac{\sqrt{6}}{8}, \sigma_1 = \mp \frac{4}{9}\sqrt{6}, \beta_2 = \frac{\sqrt{2}}{3} \pm \frac{2}{9} \sqrt{3}
$$

(18)

The amount of supersymmetry breaking at the minimum is given by

$$
f(\sigma) = M \mu^2 \epsilon (1 + \frac{8}{3} \beta_1)
$$

(19)

which yields for the gravitino a mass

$$
m_{3/2} = \frac{1}{M^2} e^{\sigma_{3/2}} |f(\sigma)| = \frac{\mu^2 \epsilon \epsilon}{M}(1 \pm \frac{\sqrt{6}}{3}).
$$

(20)
Typically, as we shall see in Sect. 2, $\mu/M \sim 10^{-3}$ which gives a gravitino mass in the range of $M_W$ for $\epsilon \sim 10^{-10}$. Thus the scale of supersymmetry breaking is small because it is of order $\epsilon$. The smaller $\epsilon$ is, the more inflation we have: we will actually see that the length of the inflation era goes as $\epsilon^{-1/2}$. We could therefore say that, in this model, the scale of supersymmetry breaking is small for the same reason that we get inflation.

This will allow us to put an upper limit on the mass of gravitino. In the same way, an upper limit on the amount of inflation (which we do not have at hand presently) would set a lower limit on the mass of the gravitino.

To recapitulate, we start with the superpotential

$$f_0(\phi) = \mu^2 M \sqrt{2} \left[ - \frac{3}{8} + \frac{\phi}{M \sqrt{2}} - \frac{3}{4} \left( \frac{\phi}{M \sqrt{2}} \right)^2 + \frac{1}{8} \left( \frac{\phi}{M \sqrt{2}} \right)^4 \right]$$

which has a supersymmetry conserving minimum at $\sigma_0 = \sqrt{2}$.

We then make a small perturbation by allowing for a non-zero difference between the coefficients of the constant and quadratic term $\epsilon = a_0 - a_2$ (or, in other words, a small linear term in the corresponding potential). The new minimum breaks supersymmetry and the corresponding scale is of order $\epsilon$ [Eq. (20)].

The superpotential that we are starting from is unique within the class described by Eq. (10). Of course, we could allow for more general potentials – in particular include a cubic term in Eq. (10) — but we think that this restricted class, and therefore $f_0(\phi)$, is very representative of the general behavior of the superpotentials which, by obeying Eq. (3), give a super-symmetry-breaking scale of order $\epsilon$ (defined as the slope of the potential at the origin). Therefore, from now on, we will restrict ourselves to the case
of $f_0(\phi)$ [Eq. (21)], although we will stress the relevance of some of its properties to the general case.

The corresponding potential $V_0(\phi)$ is very flat near its minimum since its first three derivatives are zero [Eq. (4)]:

$$V_0(\phi) = \mu^4 e^2 \frac{9}{16} \left( \frac{\phi}{M} - \sigma_0 \right)^4 + O\left( \left( \frac{\phi}{M} - \sigma_0 \right)^4 \right) \tag{22}$$

$V_0$ is shown in Fig. 1, together with its shape at temperature $T = M$, as computed from the results of Ref. [4]. As stressed earlier, the temperature corrections do not stabilize the field at the origin for high temperatures.¹ We will return to that point in Sect. (3).

Turning $\epsilon$ on displaces the absolute minimum from $\sigma_0$ to $\sigma$ given by Eqs. (15), (17), and (18). The superpotential now reads

$$f(\phi) = \mu^2 M \left[ \beta + \epsilon + \frac{\phi}{M} + \beta \left( \frac{\phi}{M} \right)^2 - \frac{1}{12} \beta \left( \frac{\phi}{M} \right)^4 \right]$$

$$\beta = -\frac{3}{8} \sqrt{2} \pm \epsilon \frac{\sqrt{6}}{8} + \epsilon^2 \left( \frac{\sqrt{2}}{3} \pm \frac{2}{9} \sqrt{3} \right) \tag{23}$$

The corresponding potential $V(\phi)$ is computed in the standard way [Eqs. (1) and (2)]. The first terms of its expansion around the origin are, to order $\epsilon$:

$$\frac{V(\phi)}{\mu^4} = \left[ \frac{5}{32} + \epsilon \frac{9}{4} (\sqrt{2} \pm \frac{\sqrt{3}}{4}) \right] - 4 \epsilon \frac{\phi}{M} + \epsilon \frac{9}{4} \sqrt{2} \frac{\phi^2}{M^2}$$

$$- \left[ \sqrt{2} \frac{\phi^3}{M^3} + 2 \epsilon (1 \mp \frac{\sqrt{6}}{12}) \frac{\phi^4}{M^4} \right] + 0(\epsilon^2, \frac{\phi^4}{M^4}) \tag{24}$$

¹One can show that this is so even if we include a cubic term in Eq. (10)
and, around the minimum $\sigma M$, to order $\epsilon$:

$$
\frac{V(\phi)}{\mu^4} = \epsilon e^2 \frac{3}{2} (\sqrt{2} \pm \sqrt{3}) \left( \frac{\phi}{M} - \sigma \right)^2 + \epsilon \frac{e^2}{2} \left( 15 \pm \frac{13}{2} \sqrt{6} \right) \left( \frac{\phi}{M} - \sigma \right)^3 \\
+ \frac{e^2}{16} \left[ 9 + \epsilon (187 \sqrt{2} \pm 98 \sqrt{3}) \right] \left( \frac{\phi}{M} - \sigma \right)^4 + 0(\epsilon^2, \left( \frac{\phi}{M} - \sigma \right)^5)
$$

It is undistinguishable from potential $V_0$ on the scale of Fig. (1).

The objection that one could raise to our linking the scale of supersymmetry breaking to the parameter $\epsilon$ is that we need to choose an arbitrarily small value for $\epsilon$. This seems unnatural (in the technical sense); the radiative corrections could induce large corrections to the scale of supersymmetry, thus putting an end to our hopes of bringing that scale down to $M_W$. But, as we will now see, one has to take into account the very special properties of renormalization in supersymmetric theories and the unique features of the superpotential that we consider [Eq. (3)]. The one-loop radiative corrections to the potential $V$ of Eq. (1) are given by [9]

$$
\delta V_A = \kappa \left( V + e^{i|\phi|^2/M^2} \left| f(\phi) \right|^2 M^2 \right)
$$

where

$$
\kappa = \frac{1}{16\pi^2} \frac{\Lambda^2}{M^2} (N - 1)
$$

In this formula, $\Lambda$ is the cut-off and $N$ is the total number of chiral fields in the theory.

We first note that, since $f(\sigma_0)|_{\epsilon=0} = f'(\sigma_0)|_{\epsilon=0} = 0$, the ground state remains unchanged at the zeroth order in $\epsilon$ when we include the radiative correction. Moreover
it is straightforward to prove that if $V$ satisfies Eq. (4) then $V + \delta V_A$ satisfies also Eq. (4) (using Eq. (3) for $f$). Therefore, using our initial result, we conclude that radiative corrections to the potential will at most induce corrections of order $\epsilon$ to the scale of supersymmetry breaking.

To be more explicit, let us show how this works on the specific example of Eq. (23). When we include the radiative corrections [Eqs. (26) and (27)], the following changes occur: the minimum of the potential is displaced to:

$$
\sigma = \sqrt{2} + \epsilon \left[ \pm \frac{4}{9} \sqrt{6} + \kappa \left( \frac{2}{9} \pm \frac{4}{27} \sqrt{6} \right) \right] + 0(\epsilon^2)
$$

and the parameter $\beta$ is now:

$$
\beta = -\frac{3}{8} \sqrt{2} + \epsilon \left( \pm \frac{\sqrt{6}}{8} \right) \left[ 1 - \kappa \left( \frac{5}{6} \pm \frac{\sqrt{6}}{3} \right) \right] + 0(\epsilon^2)
$$

Note that no correction appears to the zeroth order in $\epsilon$, which is exactly what we have proved in the general case. The value of the superpotential at the minimum is therefore still given by [compare with Eq. (19)].

$$
f(\sigma) = M \mu^2 \epsilon \left( 1 + \frac{8}{3} \beta'_1 \right)
$$

where $\beta'_1$ is the coefficient of order $\epsilon$ in the expansion of $\beta$ [Eq. (29)]. Thus the mass of the gravitino is

$$
m_{3/2} = \frac{\mu^2}{M} \epsilon \left| \left( 1 \pm \frac{\sqrt{6}}{3} \right) - \kappa \left( \frac{2}{3} \pm \frac{5\sqrt{6}}{18} \right) \right|
$$
It is precisely the fact that this scale is of order $\mu^2 \epsilon$ that justifies our approach a posteriori. Had corrections of order $\mu^2 \epsilon$ for example appeared in Eq. (31), the smallness of the scale of supersymmetry breaking would have been an unnatural feature of our model. Equation (31) shows that no such corrections appear.

2. We now review the set of constraints that the models that we consider must satisfy in order to give rise to a successful cosmological scenario [see for example Ref. (10)]. We will do that for the explicit example of Eq. (23) but, its salient features being a consequence of Eq. (3) and therefore shared by more general potentials, we believe that this analysis is applicable to any of them. We first have to follow the evolution of the inflaton field with time. This evolution is summarized in Table 1.

The inflation period starts when the energy density becomes dominated by the energy stored in the vacuum:

$$\rho_0 = V_0 = \frac{5}{32} \mu^4$$

(32)

We assume that the inflaton field is initially located near the origin; then its value when inflation starts is of the order of the Hubble parameter:

$$\phi_0 \simeq H_0 = (\frac{\rho_0}{3M^2})^{1/2} = \sqrt{\frac{5\mu^2}{96M}}$$

(33)

As long as radiation can be neglected, the classical evolution of the inflaton field is governed by the equations

$$\ddot{\phi} + 3H \dot{\phi} = -\frac{\partial V}{\partial \phi}$$

$$H^2 = \frac{1}{3M^2} \left[ \frac{1}{2} \dot{\phi}^2 + V(\phi) \right].$$

(34)
During the slow rollover — i.e. the inflation period — the motion of $\phi$ is friction dominated and the $\ddot{\phi}$ term is negligible. In terms of the potential, this can be expressed as [10]

\begin{align}
V''(\phi) &\leq \frac{3}{M^2} |V(\phi)| \\
V'(\phi) &\leq \frac{\sqrt{6}}{M} |V(\phi)| 
\end{align}

(35)

It turns out that for the potential that we consider, it is the second of these equations that breaks down first, at a value $\phi_e$ that is almost independent of $\epsilon$:

$$\phi_e \simeq 0.71M, \quad V(\phi_e) \simeq 8 \times 10^{-2} \mu^4.$$ 

(36)

The number of $e$-foldings that the scale factor undergoes during inflation is given by

$$N = - \int_{\phi_0}^{\phi_e} \frac{3H^2}{V'(\phi)} d\phi.$$ 

(37)

We can approximate the Hubble parameter in the numerator by its value of the origin [Eq. (32)] and the potential in the denominator by the first terms of its expansion [Eq. (24)]. It turns out that when $\epsilon \ll M^4$ (in particular $\epsilon = 0$), the main contribution comes from the lower bound $\phi_0$ (where the field spends most of the inflation epoch):

$$N = \sqrt{\frac{5}{48}} \frac{M^2}{\mu^2} + O\left(\frac{M}{\mu}\right)$$

(38a)
On the other hand, when $\epsilon >> \frac{\mu^4}{M^4}$ (non-negligible linear term in the potential), the upper bound $\phi_e$ gives the leading contribution which happens to be independent of $\mu$:

$$N = \frac{5\pi}{32} \frac{1}{(24\sqrt{2})^{1/2}} \epsilon^{-1/2}$$  \hspace{1cm} (38b)

We checked numerically that these approximate formulas are very accurate and computed $N$ in the intermediate region ($\epsilon \approx \mu^4/M^4$). A value of $N$ typically greater than 60 is required [1] if we want our present observable Universe to have emerged from a single causally-connected patch. Using Eqs. (38a,b) and our numerical computation, we can use the condition $N > 60$ to constrain our parameters $\epsilon$ and $\mu/M$. To be more accurate, we must take into account the fact that some time elapses between the end of inflation and reheating. We will see that, during this period, the cosmic scale factor $R$ grows by a factor

$$\frac{R(t_e)}{R(t_e)} \approx \left(\frac{\rho_\phi(t_e)}{M^4}\right)^{1/4} \left(\frac{\rho_\phi(t_c)}{M^4}\right)^{1/12} \left(\frac{\Gamma}{M}\right)^{-2/3}$$

(c.f. Eqs. (42) and (46); $t_c$ is defined in Eqs. (43), (44) and $\Gamma$ is the inflaton decay rate). Expressing all these quantities in terms of $\mu$ and $\epsilon$, we obtain the condition on the number of e-foldings [10]

$$N > 66.5 + \frac{5}{3} \ln \frac{\mu}{M} - \frac{1}{12} \ln \epsilon$$  \hspace{1cm} (39)

We draw the corresponding curve (labelled "$N = 60$") in the $\epsilon, \mu/M$ plane of Fig. 2.
The inflation field reaches the value \( \phi_e \) at time \( t_e \simeq \frac{N}{H_0} \). It turns out that this value corresponds also to the point where the curvature of the potential changes sign. The field therefore starts oscillating around the minimum \( \sigma \). In a first stage, it does not feel the details of order \( \epsilon \) of the potential near the minimum [Eq. (25)] and therefore oscillates in the \( \phi^4 \) potential of Eq. (22). We show in Fig. (3) the first few oscillations of the inflation field \( (t_e < t < 1.006 t_e) \). It is straightforward to compute the frequency of these oscillations:

\[
\frac{\omega}{M} \simeq 0.2 \frac{\mu^2}{M^2}
\]  

(40)

We have

\[
\frac{\omega}{M} \simeq H_e \equiv \left( \frac{V(\phi_e)}{3M^4} \right)^{1/2} \simeq 0.13 \frac{\mu^2}{M^2}
\]  

(41)

and, after a few oscillation, \( \omega >> H \). We can therefore apply the results of Turner [11]: after averaging over an oscillation period, the energy associated with the coherent field oscillations behaves like relativistic matter. Therefore the cosmic scalar factor \( R \) and the coherent energy density \( \rho_\phi \) scale with time as:

\[
\frac{R(t)}{R(t_e)} = [1 + 2H_e(t - t_e)]^{1/2} \quad , \quad \frac{\rho_\phi(t)}{\rho_\phi(t_e)} = \left( \frac{R(t)}{R(t_e)} \right)^{-4}
\]  

(42)

This will go on until time \( t_4 \) when the field oscillations take place only in the close vicinity of the minimum where the potential can be approximated by the first term of its expansion in Eq. (25). This will happen approximately for:

\[
\left| \frac{\phi_t}{M} - \sigma \right| \simeq 3 \epsilon^{1/2} \quad , \quad V(\phi_t) \simeq 6 \times 10^2 \epsilon^2 \mu^4
\]  

(43)
which gives, from (42)
\[
t_c \simeq 4 \times 10^{-2} \epsilon^{-1} \frac{M}{\mu^2}
\] (44)

From \( t_c \) onward, we can consider that the field oscillates in a \( \phi^2 \) potential, with a frequency equal to the mass of the inflaton field:
\[
m_\phi \simeq \frac{\mu^2}{M} \epsilon^{1/2}
\] (45)

and (since \( m_\phi > H(\phi_t) \equiv H_t \)), according to Ref. [11], the coherent energy density behaves like non-relativistic matter. Therefore, for \( t > t_c \),
\[
\frac{R(t)}{R(t_c)} = \left[ 1 + \frac{3}{2} H_t (t - t_c) \right]^{2/3}, \quad \frac{\rho_\phi(t)}{\rho_\phi(t_c)} = \left( \frac{R(t)}{R(t_c)} \right)^{-3}
\] (46)

This will last until \( t \sim \Gamma^{-1} \) when reheating takes place through the decay of the inflaton field. The decay rate is, following Eq. (45),
\[
\Gamma \sim \frac{m_\phi^3}{M^2} \sim \frac{\mu^6}{M^5} \epsilon^{3/2}
\] (47)

The photon density at \( t = \Gamma^{-1} \) reads (assuming that the inflaton decays mostly into photons)
\[
\rho_\gamma(\Gamma^{-1}) = \rho_\gamma(t_c) \left( \frac{R(\Gamma^{-1})}{R(t_c)} \right)^{-4} + \rho_\phi(t_c) \left( \frac{R(\Gamma^{-1})}{R(t_c)} \right)^{-3}
\] (48)

One can check that \( \frac{R(\Gamma^{-1})}{R(t_c)} \gg 1 \) and \( \rho_\phi(t_c) \gg \rho_\gamma(t_c) \), in which case the second terms is dominant. Using Eq. (46), we thus obtain
\[
\rho_\gamma(\Gamma^{-1}) \simeq \frac{4}{3} (\Gamma M)^2
\] (49)
and the universe is reheated to a temperature

\[ T_{RH} = \left( \frac{40}{\pi^2 g^*} \right)^{1/4} (\Gamma M)^{1/2} \]  

(50)

where \( g^* \) is the number of effective spin degrees of freedom \( (g^* \sim 10^2) \). We see that, although our potential has some peculiar features, the result for the reheating temperature agrees with the standard one (in the so-called poor reheating case)[10,11,12]. Using (47) we find that

\[ T_{RH} \sim \frac{\mu^3}{M^2} \epsilon^{3/4} \]  

(51)

We wish to emphasize at this point the complete generality of this result. Because all the potentials that we consider must satisfy Eq. (4), the mass of the inflaton field must of of order \( \epsilon \) and is therefore given by Eq. (45) [in the general case, \( \mu \) is an overall scale defined as in (10)]. This in turn gives Eq. (47) for \( \Gamma \) and Eq. (51) for \( T_{RH} \).

Before discussing the consequences of such a reheating temperature, we have to further constrain the parameter \( \mu/M \) by studying the amplitude of the density fluctuations. It is well known [13-15] that inflationary models yield a scale independent spectrum (the so-called Harrison-Zel'dovich spectrum [16]) with an amplitude at time \( t_f \) when the fluctuations reenter the horizon in the \( FRW \) phase given by:

\[ \frac{\delta \rho}{\rho} (t_f) = 0(1) \frac{\delta \phi(t_i)}{\dot{\phi}(t_i)} \]  

(52)

where \( t_i \) is the time when the perturbations leave the horizon in the de Sitter phase, and \( \delta \phi(t) \) is the space-averaged perturbation of the scalar field.
Taking into account the details of the reheating period described above, one can show that the number of e-foldings that take place between $t_i$ and the end of inflation $t_e$ is given, for a scale $\ell$, by [10]

$$N_\ell = \int_{t_i}^{t_e} H \, dt = \frac{1}{3} \ell n \left( \frac{M_e \, T_{RH} \, V(\phi_e)^{1/2}}{M_\odot \, M eV / MeV^2} \right)$$

(53)

where $M_\ell$ is the corresponding mass scale. Considering the typical scale of a large galaxy ($M_\ell \simeq 10^{15} M_\odot$) and using Eqs. (36) and (51), this gives:

$$N_\ell = 60 + \frac{5}{3} \ell n \frac{M}{\mu} + \frac{1}{4} \ell n \epsilon$$

(54)

It is easy to show that the corresponding value for the scalar field $\phi(t_i)$ ($\gg \epsilon^{1/2}$) is given by

$$\frac{\phi(t_i)}{M} \simeq \frac{5}{48 \sqrt{2} \, N_\ell} \simeq 1.7 \times 10^{-3}$$

(55)

Since we are in the slow-rollover period of the evolution of the $\phi$ field, its motion is friction-dominated and, linearizing the equation of motion for $\delta \phi$, we can write Eq. (52) as:

$$\frac{\delta \rho (t_f)}{\rho} (t_f) = 0(1) \left| \frac{V(\phi(t_i))}{V'(\phi(t_i))} \right| \delta \phi(t_i) \simeq 0(1) \frac{\delta \phi(t_i)}{\phi(t_i)}$$

(56)

Taking $\delta \phi(t_i) \simeq \frac{H_o}{2\pi} [13-15,17]$, we obtain from (33) and (55)

$$\frac{\delta \rho (t_f)}{\rho} \simeq 20 \frac{\mu^2}{M^2}$$

(57)
Let us note that the uncertainty on the numerical factor (20) is at least of one order of magnitude. If we consider that the amplification factor due to the evolution of the fluctuations subsequent to $t_f$ is not larger than $10^5$ [see e.g. Ref. (18)], galaxy formation ($\frac{\delta \rho}{\rho} \sim 0(1)$) requires

$$\frac{\delta \rho}{\rho}(t_f) > 10^{-5} \quad \text{and} \quad \frac{\mu}{M} > 7 \times 10^{-4}$$

(58)

On the other hand, the scales relevant to the cosmic microwave background reenter the horizon when the universe is matter-dominated, which decreases the amplitude of the density fluctuations [Eq. (54)] by a factor $\frac{1}{10}$ [14]. Following Sachs and Wolfe [19], this gives an anisotropy in the cosmic microwave background

$$\frac{\delta T}{T} \simeq \frac{1}{2} \frac{\delta \rho}{\rho}(t_f) \simeq \frac{\mu^2}{M^2}$$

(59)

Allowing for an observed temperature anisotropy on large angular scales smaller than $10^{-4}$ puts a limit

$$\frac{\mu}{M} < 10^{-2}$$

(60)

Therefore the study of the amplitude of the density fluctuations restrict the parameter $\mu/M$ to the region $10^{-4} < \mu/M < 10^{-2}$, as shown in Fig. 2. On the same figure, we have also drawn the curve $m_{3/2} = M_W$ where $m_{3/2}$ is given by Eq. (20) (we choose the lower sign in this equation). If we restrict ourselves to such values of the supersymmetry-breaking scale, then $\epsilon$ is typically of order $10^{-10\pm 2}$.

We now turn to the problem of baryon number generation. The limits on $\frac{\mu}{M}$ [Eq. (60)] and $\epsilon$ (see Fig. 2) put a bound on the reheating temperature:

$$T_{RH} < 10^{+8} \text{GeV}$$

(61)
Similarly, because the mass of the $\phi$ field is related to the gravitino mass according to [compare Eqs. (20) and (45)]

$$m_\phi \simeq m_{3/2} \epsilon^{-1/2},$$

(62)

the $\phi$ field is too light (taking $m_{3/2} = M_W$ gives $m_\phi < 10^6 M_W = 10^8$ GeV) to decay into the color triplet isosinglet superheavy Higgs bosons whose decays can lead to baryon number generation. Therefore, such superheavy Higgs bosons cannot be produced directly by the decay of the coherent inflaton field oscillations, as in the standard poor reheating scenarios [20]. We thus have to resort to models of cosmological baryon generation at low temperature [21]. We will also see in Sect. (3) that in trying to fulfill the thermal constraint, we gain another possible solution to baryogenesis.

We finally consider the so-called gravitino problem. Light gravitinos such as the ones that we consider have a very long lifetime:

$$\Gamma_{3/2} \sim \frac{m_{3/2}^3}{M^2} \sim \frac{\mu^6}{M^5 \epsilon^3}$$

(63)

It is therefore quite plausible that they will become non relativistic and dominate the energy density of the universe before they decay, which would dramatically perturb

\footnote{Let us note however that in the case of interest to us ($m_{3/2} = M_W$), the reheating temperature is in the 1 GeV - 1 TeV region ($T_{RH} = (m_{3/2}/M)^{3/2} M \epsilon^{-3/4} \simeq 10^{-6}$ GeV $\epsilon^{-3/4}$). We therefore need to adapt the models of Ref. [21] to such a low reheating temperature. This is possible because the mass of the fields responsible for baryon number generation is only limited by the mass of the inflaton field which lies in the $10^6$ to $10^8$ GeV region [see Eq. (62)].}
the successes of the standard big bang scenario. Gravitinos produced before inflation are diluted away [22] and we need not consider them. But they can be produced by thermal equilibrium processes after reheating [23,24] or directly through the decay of the inflaton field [25]. In the first case, the density of gravitinos produced after reheating has been shown to be proportional to $T_{RH}$ [24] and a low value for $T_{RH}$ can solve the problem. It turns out that the most stringent bound comes from the analysis of deuterium dissociation caused by the photons resulting from gravitino decays; this gives [24]

$$T_{RH} < \left( \frac{m_{3/2}}{100 GeV} \right)^{-1} \times 10^{10} GeV$$

(64)

The corresponding curve is shown on Fig. 2. It is clear that, at least for $m_{3/2} = M_W$, this does not give any further constraint.

The second source of gravitinos is the decay of the inflaton itself. Using an argument due to Ovrut and Steinhardt [25], one can show that, because the mass of the inflaton field is much bigger than the reheating temperature, the gravitinos that it produces will remain relativistic for a long period and will decay before they dominate the energy density of the universe.

3. We stressed earlier that the temperature corrections to the potential $V_0$ or $V$ do not have an absolute minimum at the origin [see Fig. 1]. Therefore the thermal constraint is not satisfied. In this section, we wish to study in detail a remedy to this problem which has been recently suggested [12]. The idea is to introduce a second chiral field in the inflaton sector of the theory. We will denote its scalar component by $\Psi$. The
superpotential is chosen to be:

\[ W(\phi, \Psi) = f(\phi) + \Psi^2 g(\phi) \quad \text{(65)} \]

where \( f(\phi) \) is given by Eq. (23) and \( g \) is a function of the \( \phi \) field only. Actually, we will only be interested here in the first terms of \( g(\phi) \) and write

\[ g(\phi) = \mu^2 M [b_0 + b_1 \frac{\phi}{M}] \quad \text{(66)} \]

The corresponding potential \( \hat{V}(\phi, \Psi) \) is given by the standard formula, generalizing Eq. (1) to the case of two fields:

\[ \hat{V}(\phi, \Psi) = e^{\frac{|\phi|^2}{M^2} + \frac{|\Psi|^2}{M^2}} \left[ \frac{\partial W}{\partial \phi} + \frac{\phi}{M^2} W| \phi |^2 + \frac{\phi}{M^2} W| \Psi |^2 - \frac{3}{M^2} |W|^2 \right] \quad \text{(67)} \]

From the results of Ref. [4], it is easy to compute the temperature corrections to that potential. For a moment, we will restrict ourselves to the \( \Psi = 0 \) direction, where the potential at temperature \( T \) reads:

\[ \hat{V}_T(\phi, \Psi = 0) = V_T(\phi) + \frac{T^2}{24M^2} 12\mu^4 e^{\frac{|\phi|^2}{M^2}} |b_0 + b_1 \frac{\phi}{M}|^2 \quad \text{(68)} \]

\( V_T(\phi) \) is the non-zero temperature version of the potential \( V(\phi) \) studied in the previous sections. Its first terms in a \( \phi \) expansion are:

\[ V_T(\phi) = V(\phi) + \frac{T^2}{24M^2} 12\mu^4 e^{\frac{|\phi|^2}{M^2}} \left\{ \left( \frac{7}{8}N + \frac{13}{16} \right) - \frac{3}{4} \sqrt{2}(N + 2) \left( \frac{\phi^*}{M} + \frac{\phi}{M} \right) + 0(\epsilon) + ... \right\} \quad \text{(69)} \]
where $N$ is the total number of chiral fields in the theory \[4,26\]. Typically, $N$ is of the order of $10^2$. It is clear from Eq. (69) that the potential $V$ alone does not satisfy the thermal constraint since already the linear term in $\phi$ tends to destabilize the inflaton field towards the minimum $\sigma$. But if we allow the parameters of $g(\phi) = b_0$ and $b_1$ — to satisfy the relation:

$$b_0b_1 > \frac{1}{16}\sqrt{2}(N + 2)$$

(70)

the extra terms in Eq. (68) will thwart this effect and stabilize the field $\phi$ near the origin (at least along $\Psi = 0$). Similarly, the coefficients of higher order terms in $g(\phi)$ can be arranged in order to cancel destabilizing effects of higher order terms in $V_T(\phi)$.

Of course, if we consider the superpotential $W(\phi, \Psi)$ as a whole, the constraint (70) which imposes that certain parameters $(b_0, b_1)$ are of order $N \approx 10^2$ compared with the others, is extremely artificial. This could be a reason sufficient to reject the solution of introducing a second field in the inflation sector, and advocate a yet-to-be-known mechanism to explain why the scalar field $\phi$ starts its evolution near the origin.\(^1\)

We will however pursue that solution to see what we can gain from it. In fact, we will take $b_0$ of order $N$ and $b_1$ of order 1 [satisfying (70)] and show that this is enough to obtain an absolute minimum at high temperature near the origin and a valley of the potential (at $T = 0$ and $T \neq 0$) in the $\Psi = 0$ direction.

It is easy to realize first that, because $W(\phi, \Psi)$ has only terms independent of $\Psi$ or quadratic in $\Psi$, $\frac{dW}{d\Psi}$ is of order $\Psi$. Keeping only terms of order $b_0^2$ because they are

\(^1\)The chaotic inflation scenario of Linde [27] could actually provide an answer.
leading in $N(0(N^2))$, we have in fact (we take $\phi$ and $\Psi$ to be real)

$$
\frac{d\tilde{V}}{d\Psi} = \Psi \mu^4 b_0^2 e^{\frac{\mu^2}{M^2}} + \frac{\mu^2}{M^2} \left[ \frac{\phi^2}{M^2} \frac{\Psi^2}{M^2} (\frac{\phi^2}{M^2} + 2) + 4 + 6 \frac{\Psi^2}{M^2} + 4 \frac{\Psi^4}{M^4} + \frac{\Psi^6}{M^6} \right]
$$

This shows that the only possible extrema of $\tilde{V}$ are in the $\Psi = 0$ direction. Moreover the coefficient of $\Psi$ in (71) is always strictly positive which means that the $\Psi = 0$ direction is a valley of the potential $\tilde{V}$. A similar analysis can be performed on the temperature corrections which shows that $\Psi = 0$ direction remains a valley even at non-zero temperature. Therefore, at high temperature, the $\phi$ field is stabilized around the origin and when the temperature decreases it starts evolving along $\Psi = 0$, in precisely the way studied in the previous section since $\tilde{V}(\phi, \Psi = 0) = V(\phi)$. The only apparent effect of the $\Psi$ field is to give the right behavior at high temperatures.

But what happens to the $\Psi$ field from then on? To answer this question, it is interesting to note two points. First, the $\Psi$ field is heavy: from Eqs. (65),(66), (67), we find a mass

$$
m_{\Psi} = 2\sqrt{2} e^{\frac{\mu^2}{M}} (b_0 + b_1 \sigma) \approx 0(\frac{\mu^2}{M})
$$

Therefore, in the region $\frac{\mu^2}{M} \sim 10^{-2}$, the $\Psi$ field is heavy enough to decay into the superheavy color triplet Higgs field of GUTS, leading therefore to the standard scenario of baryogenesis.

The second point is that this decay occurs after the end of the inflation period. More precisely, the $\Psi$ decay rate is, similarly to the case of the $\phi$ field [Eq. (47)] given by

$$
\Gamma_{\Psi} \sim \frac{m_\Psi^3}{M^2} \sim \frac{\mu^6}{M^5} 0(N^{3/2})
$$
The decay occurs at a time $\Gamma^{-1}_\varphi$ which satisfies, according to Eqs. (33), (38), and (47),

$$t_\epsilon(\sim \frac{N}{H_0}) < \Gamma^{-1}_\varphi < \Gamma^{-1}$$

Therefore the decay products of the $\Psi$ field — the precious color triplet Higgs — are not washed away by inflation and, when reheating occurs, are very far from equilibrium. Baryon number generation can then take place. One might worry about other decay products of the $\Psi$ field — the gravitinos. But the same argument as in the case of the $\phi$ field applies here because $m_{\Psi}$ is much bigger than the reheating temperature.

To conclude, we have studied inflationary models where the scale of supersymmetry breaking is proportional to a small parameter which we chose to relate to the slope of the potential at the origin. This scale can therefore be as low as the mass $M_W$ of the weak gauge boson. The study of the quantum corrections shows that this scale is stable under radiative corrections. These models share in common a low reheating temperature which helps in solving some of the problems (e.g., the gravitino problem) that inflationary models usually face. Indeed, the study of the simplest of these models showed that no particular problem arises except for a violation of the thermal constraint: temperature corrections do not stabilize the inflaton field away from its minimum. We showed however that one can deal with this problem by introducing a second field in the inflaton sector, whose sole effect is to modify the temperature corrections. This is a very artificial way of solving the thermal constraint but we gain another mechanism for baryon number generation. Anyway, whether or not we introduce this second field,
this only field which plays a dynamical role as far as inflation is concerned is the original inflaton (being responsible for the de Sitter phase and for reheating). We conclude therefore that there exist viable cosmological scenarios which allow a mass for the gravitino as low as $M_W$ (see also Refs. [6] and [25]).

ACKNOWLEDGEMENTS

We benefited from very useful conversations with R. Brandenberger, M.K. Gaillard, I. Hinchliffe and T. Yanagida. And we have a special debt to S. Dawson for her readiness to answer our endless questions about computing.

This research was supported in part by the National Science Foundation under Grant No. PHY77-27084, supplemented by funds from the National Aeronautics and Space Administration. This work was also supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.
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**FIGURE CAPTIONS**

Figure 1: Potential $V/\mu^4$ corresponding to the superpotential $f(\phi)$ given by Eq. (23) (or Eq. (21) since, on this scale, they are indistinguishable). The dashed curve gives the shape of the potential at $T = M$ (taking $N = 50$ chiral superfields in the theory [4]).

Figure 2: Cosmological constraints on the parameters $\epsilon$ and $\mu/M$. The curve $N = 60$ limits the region where enough inflation takes place [see the condition given by Eq. (39)]. The study of the amplitude of density fluctuations gives limits on $\mu/M$ only [Eqs. (58)-(60)] and the gravitino problem gives a bound on the reheating temperature [Eq. (64)]. Finally, we have drawn the line $m_{3/2} = M_W$ [Eq. (20) with the lower sign] which corresponds to the successful low energy models [5].

Figure 3: Oscillations of the $\phi$ field around the minimum $\sigma M$ of potential $V$ (Fig. 1) immediately after the end of inflation ($t_e < t < 1.006 t_e$).
FIGURE 2
Inflation $\rightarrow$ Oscillations in a $\phi^4$ potential $\rightarrow$ Oscillations in a $\phi^2$ potential $\rightarrow$

$R = e^{H_0 t}$

$R \sim t^4$

$R \sim t^{2/3}$

$t_0 \rightarrow$ Horizon crossing $\rightarrow$ $t_c \rightarrow$ Reheating $\rightarrow$

$\frac{\phi_0}{M} = \frac{H_0}{M} \approx \frac{\phi_2}{M}$

$\frac{\phi_1}{M} = 1.7 \times 10^{-3}$

$\frac{\phi_e}{M} = 0.71$

$\frac{\phi_t}{M} = 0.3 \times 4$

Table 1
This report was done with support from the Department of Energy. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the Department of Energy.

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