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Two-phase Flow in High Aspect-ratio Microchannels

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Two-phase Flow in High Aspect-ratio Microchannels    
DISSERTATION

submitted in partial satisfaction of the requirements    
for the degree of

DOCTOR OF PHILOSOPHY

in Mechanical and Aerospace Engineering

by

James M. Lewis

Dissertation Committee:
Professor Yun Wang, Chair
Professor John C. LaRue
Professor Derek Dunn-Rankin
Professor Russell Detwiler

2018
Dedication

To my family and friends, whose love and unwavering support made this pursuit possible.
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Acronyms

GDL Gas Diffusion Layer
PEM Polymer-Electrolyte Membrane

Greek Symbols

\( \alpha \) Void fraction
\( \alpha^* \) Aspect ratio (smallest dimension/largest dimension)
\( \alpha_E \) Mean volumetric liquid entrainment for Schmidt & Friedel (1996)
\( \beta \) Homogeneous void fraction
\( \beta_n \) Ratio of velocity profile coefficients obtained by Steinbrenner (2011)
\( \chi \) Gas quality
\( \Delta \) Change between two points
\( \delta^*P \) Percent error
\( \delta P \) Error
\( \epsilon \) Small perturbation
\( \eta \) Virtual displacement
\( \gamma \) Contact angle
\( \Gamma_e \) Downstream pressure correction term of Schmidt & Friedel (1996)
\( \hat{\mu} \) Viscosity ratio (\( \mu_L/\mu_G \))
\( \kappa \) \( \rho gz/\sigma \)
\( \Lambda \) Inverse liquid-only Suratman number
\( \lambda \) Lagrange multiplier
\( \mu \) Dynamic viscosity
\( \nu \) Kinematic viscosity
\( \Omega \) Area Domain
\( \omega \) Differential area of \( \Omega \)
\( \bar{\mu} \) Viscosity ratio \((\mu_G/\mu_L)\)
\( \Phi \) Modified capillary number
\( \phi^2 \) Two-phase flow multiplier
\( \rho \) Density
\( \sigma \) Surface tension
\( \sigma_A \) Area ratio \((A_1/A_2)\)
\( \sigma_e \) Root-mean-squared error
\( \sigma_{\%} \) Root-mean-squared percent error
\( \theta \) Corner half angle
\( \Upsilon \) Correlation equation of Attou & Bolle (1997)
\( \varepsilon \) Relative adhesion coefficient
\( \varphi \) Gravitational potential function
\( \rho \) Syringe pump screw pitch
\( \varsigma \) Function defining a surface
\( \psi \) Correlation equation of Attou & Bolle (1997)
\( \xi \) Air stoichiometric ratio
\( X \) Lockhart-Martinelli parameter

**Roman Symbols**

\( \dot{m} \) Mass flow rate per unit area
\( n \) Unit normal vector
\( \overline{\varepsilon} \) Mean error
\( \overline{\varepsilon}_{\%} \) Mean percent error
\( A_c \) Cross-sectional area
\( A_n \)  First coefficient for the velocity profiles obtained by Steinbrenner (2011)

\( AR \)  Aspect-ratio \((w/h)\)

\( b \)  \( w - c \)

\( B_n \)  Second coefficient for the velocity profiles obtained by Steinbrenner (2011)

\( Bo \)  Bond number

\( C \)  Chisholm parameter

\( c \)  Water film thickness

\( d \)  Diameter

\( D_H \)  Hydraulic diameter

\( D_s \)  Syringe diameter

\( E \)  Energy

\( f \)  Darcy Friction factor

\( f_f \)  Fanning friction factor

\( f_m \)  Mechanical frequency of the syringe pump

\( f_{f,app} \)  Apparent Fanning friction factor

\( G \)  Total mass flux

\( g \)  Acceleration of gravity

\( h \)  Channel height (gap spacing)

\( h_{ratio} \)  Water film thickness ratio

\( j \)  Slug velocity

\( K \)  Absolute permeability

\( k \)  Axial wavenumber

\( k_r \)  Relative permeability

\( L \)  Length

\( l \)  Solid boundary

\( n \)  Index

\( n_k \)  Relative permeability exponent
$N_{\mu_p}$ Two-phase viscosity number

$N_{con}$ Confinement number

$P$ Pressure

$Q$ Volumetric flow rate

$R$ Radius of curvature

$r$ Radius

$Re$ Reynolds number

$S$ Slip ratio

$s$ Saturation

$s_{L,e}$ Effective liquid saturation

$s_{L,r}$ Residual liquid saturation

$Su$ Suratman number

$U$ Superficial velocity

$u$ Axial component of velocity

$V$ Velocity

$w$ Channel width

$We$ Weber number

$x$ Coordinate along channel height

$y$ Coordinate along channel width

$z$ Downstream coordinate

HP Hydrophilic

HY Mixed-wettability

**Superscripts**

$\vec{\cdot}$ Vector

$r$ Correlation exponent of Attou & Bolle (1997)

**Subscripts**

$A$ Acceleration component
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<tr>
<td>$B$</td>
<td>Bubble</td>
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<tr>
<td>$cap$</td>
<td>Capillary</td>
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<tr>
<td>$e$</td>
<td>Effective condition for Schmidt &amp; Friedel (1996)</td>
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<td>$exit$</td>
<td>Across the channel exit</td>
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<td>$exp$</td>
<td>Experimental</td>
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<td>Frictional component</td>
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<td>$g$</td>
<td>Gravitational component</td>
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<td>$h$</td>
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**Other Symbols**

<table>
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<tr>
<td>$\hat{a}$</td>
<td>Scaling factor of Ma et al. (2010)</td>
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</table>
\( \hat{b} \) Correlation exponent of Ma et al. (2010)
\( \hat{c} \) Correlation parameter of Corey (1954)
\( \hat{n} \) Correlation exponent of Lin et al. (1991)
\( \hat{x} \) Dimensionless x-coordinate
\( \hat{y} \) Dimensionless y-coordinate
\( \hat{z} \) Integration variable
\( Z \) Shear force function
\( \mathcal{D}_1^{\pm} \), Concus-Finn region of nonexistence
\( \mathcal{D}_2^{\pm} \), Concus-Finn region of noncontinuous surface unit normal
\( \mathcal{L} \) Surface Area
\( \mathcal{R} \) Concus-Finn region of existence
\( \mathcal{I} \) Wetted surface area
\( \chi \) Lockhart-Martinelli’s specific correlation parameter
\( \overline{C} \) Friction factor correlation parameter
\( \overline{z} \) Dimensionless streamwise coordinate
\( z^* \) Dimensionless water injection location
\( \rho_1 \) Density function of Abdelall et al. (2005)
\( \rho_{11} \) Second density function of Abdelall et al. (2005)
\( V \) Volume
\( C^* \) Correlation parameter of Shah (1978)
\( c^* \) Second correlation parameter of Corey (1954)
\( k(\infty) \) Correlation parameter of Shah (1978)
\( K^* \) Correction factor of Wadle (1989)
\( s_m \) Model parameter of Burdine (1953)
\( W_{rel} \) Relation in the void fraction of Rouhani (1969)
\( x^+ \) Dimensionless length scale \((L/ReD_H)\)

E Cell potential
Operators

\( \langle \rangle \) Area-averaged quantity

\( \nabla \) Differential operator

\( \sum \) Summation
Acknowledgments

Over the course of my time at UCI, several individuals have impacted my research and me as an individual. First and foremost, I would like to thank Louise Yeager and Jean Bennett for always taking the time to answer questions about the requirements and paperwork a PhD entails—and ensuring I was aware of them. I owe a debt of gratitude to Allen Kine and Dr. Djmal Khelif for their guidance and helpfulness. I am also grateful for the advice Professor William A. Sirignano provided on different aspects of my research and his overall contribution to my education. I would like to thank Professor Derek Dunn-Rankin for his keen questions about my work, his guidance, and his time on my dissertation committee. I would also like to thank Professor Russell Detwiler for the interest he showed in my work, his time on my dissertation committee, and the feedback he provided. I would like to thank Professor John C. LaRue for being on my dissertation committee and the time he took to discuss different aspects of research & my overall graduate experience in general. I would also like to thank my advisor, Professor Yun Wang, for what he has taught me and for helping me to reach this milestone. I appreciate the suggestions of Ted Ediss, Steve Heck, and Tucker Parris on machining the experimental apparatus. Dr. Alejandro Puga sacrificed his time to help me in a multitude of ways and for this I thank him. Additionally, I thank my fellow graduate students Rayomand Gundevia, Tuan Nguyen, Timothy Koster, Hao Yuan, Rosa Padilla, Jesse Tinajero, Zahra Sheykhangafsheh, and Jingtian Wu for their helpful discussions and assistance. I am a firm believer that everyone I have encountered has taught me something and helped shape who I am today. To those individuals, I thank you as well.
Curriculum Vitae

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Abstract

Two-phase Flow in High Aspect-ratio Microchannels

by

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Doctor of Philosophy in Mechanical and Aerospace Engineering

University of California, Irvine, 2018

Professor Yun Wang, Chair

Two-phase flow in a microscale channel with a characteristic pressure drop, flow pattern, and liquid saturation occurs in a variety of applications. Understanding the behavior of two-phase flows and predicting the two-phase pressure drop can aid in the development and assessment of engineering designs; PEM fuel cells serve as a test case for this work. The experimental investigation of this work consists of measuring the two-phase pressure drop in a high aspect-ratio microchannel of dimensions 3.23mm wide by 0.304mm high combined with visualization of the two-phase flow. The tested superficial velocities represent common fuel cell operating conditions. In the first case, the microchannel consists of hydrophilic surfaces and the flow forms a stratified pattern. The two-phase pressure measurements led to the determination of a new relative permeability exponent (Wang, 2009) for stratified flow equal to 1.159, which predicts the two-phase pressure with a mean absolute percent error of 3.25%. The X-model and the correlations proposed by Lee & Lee (2001) and Kim & Mudawar (2012) well predict the experimental data with mean absolute percent errors of 3.32%, 4.1%, and 4.2%, respectively. Measurements of the water film thickness extracted from images of the flow follow the analytical result of Steinbrenner (2011). The new relative permeability exponent reasonably predicts the liquid saturation (dimensionless water film thickness).
In the second case, the base of the channel becomes hydrophobic while the other walls remain hydrophilic. This configuration forms a mixed-wettability microchannel, similar to the gas-supply channels found in PEM fuel cells. The change of the base wettability results in primarily rivulet flow along the hydrophobic surface and an increase in the two-phase pressure drop compared to the hydrophilic case. Researchers have presented inconsistent trends of the two-phase pressure drop with changing contact angle. This work proposes a critical liquid Capillary number in the range of $1.38 \times 10^{-4}$ to $9.63 \times 10^{-4}$ to clarify the trend for adiabatic flow in a single mixed-wettability microchannel. Furthermore, the work demonstrates the variability of the flow in the mixed-wettability channel. This suggests an instability of the rivulet flow, which prevents existing two-phase pressure models from collapsing the data. Optimizing the relative permeability exponent in the two-fluid model for rivulet flow led to a value of 1.747, which predicts the two-phase pressure measurements with a mean absolute percent error of 14.9%. The existing correlations of English & Kandlikar (2006) and Sun & Mishima (2009) predict the entire experimental data set with mean absolute percent errors of 22.7% and 23.7%, respectively.
Chapter 1

Introduction to the Thesis

1.1 Motivation

Two-phase flow in micro-scale channels has become ubiquitous in modern engineering and industrial applications. For example, high flux micro-heat exchangers/condensers exploit the latent heat of vaporization/condensation to enhance heat transfer for small scale electrons (Lee & Mudawar, 2005; Kim & Mudawar, 2012). Fluctuations of the phase distribution leads to pressure oscillations that can degrade the heat exchangers performance (Li & Hibiki, 2017). Geophysical processes ranging from oil extraction to geothermal energy production operate with two-phase flows, in which the pressure will determine the rate of extraction (Fourar & Bories, 1995). Water management in polymer-electrolyte membrane (PEM) fuel cells relies on removing water through the gas-supply channels, generating a two-phase flow (Wang, 2009). Water accumulation in the channel increases the pressure resulting in the maldistribution of the flow in parallel channels (Wang et al., 2008b). Thus, determining the two-phase pressure allows for the determination of its influence on the overall system performance.
With a wide variety of applications and operating conditions in which two-phase flow occurs, modeling the two-phase pressure remains an active area of research. Specifically, numerous relations exist to determine the two-phase pressure without a universally applicable relation. This necessitates the continued refinement of existing models under a variety of conditions to expand their applicability. This work utilizes the two-phase flow in a simulated PEM fuel cell gas-supply channel as a test case. While operational PEM fuel cell parameters set the test conditions, the findings of this work focus on a broader understanding of the two-phase pressure and flow behavior. This allows the work to extend to other applications that generate two-phase flow.

1.2 Water Management in PEM Fuel Cells

Water plays a crucial role in the function and performance of PEM fuel cells. The PEM fuel cell assembly consists of several different components: gas-supply channels, gas diffusion layers (GDL), catalyst layers, and the membrane (figure 1.1). The gas-supply channels have a cross-section dimension at the micro/millimeter scale and a length scale of one to tens of centimeters (Wang et al., 2013). The membrane, typically formed by perfluorosulfonic acid ionomers (Nafion), acts as both a separator between the electrodes and as an electrolyte. Thus the membrane forces electrons to traverse the external circuit while allowing the conduction of hydrogen ions to the cathode catalyst layer. However, the proton conductivity of the Nafion membrane decreases with a decrease in the membrane’s water content Zawodzinski Jr. et al. (1993), a condition called dry-out. The water required to hydrate the membrane can come from either the water formed by the fuel cell reaction or from the humidification of the reactant gases. The fuel cell reaction consists of two parts: the oxidation of hydrogen gas at the anode catalyst layer (equation 1.1) and the oxygen reduction reaction (equation 1.2) at the cathode catalyst later. The produced hydrogen ions (equation 1.1) traverse the
membrane while the electrons flow through the external load. The electrons and hydrogen ions combine with oxygen (equation 1.2), forming water. The production of the water on the cathode side generates a water concentration gradient, with a high concentration of water on the cathode side and low on the anode. As a result, water will diffuse across the membrane to equilibrate the water concentration. This mechanism, termed back diffusion, hydrates the membrane.

![PEM fuel cell diagram](image)

**Figure 1.1: PEM fuel cell diagram.**

\[
H_2 \rightarrow 2H^+ + 2e^- \quad (1.1)
\]

\[
\frac{1}{2}O_2 + 2H^+ + 2e^- \rightarrow H_2O \quad (1.2)
\]

Three other mechanisms impact the membrane’s hydration: electro-osmotic drag, thermal-osmotic drag, and pressure-driven hydraulic permeation (Dai et al., 2006). Pressure-driven hydraulic permeation results from a pressure gradient between the anode and cathode side. The pressure gradient will drive water from the high pressure side of the membrane to the low pressure side. A temperature gradient across the membrane induces water transport, termed
thermal-osmotic drag. The direction of the flow depends on the material properties of the membrane and becomes significant at high current densities (Kim & Mench, 2009). On the other hand, electro-osmotic drag results from the electromotive force between protons and polar water molecules; the positively charged proton attracts the negative pole of the water molecules and propagate together across the membrane. This results in water traversing from the anode side to cathode side (Piovar, 2006).

Quantifying the impact each mechanisms has on the total water transport proves difficult. Instead the net drag coefficient determines the combined effect of the different mechanisms. The value of the net drag coefficient depends on the dominant mechanism and will vary with operating conditions (Janssen & Overvelde, 2001). When the water moves from the anode to the cathode, the net drag coefficient will have a positive value; the net drag coefficient will have a negative value when water moves from the cathode to the anode. Back diffusion, electro-osmotic drag, thermal-osmotic drag, and pressure-driven hydraulic permeation not only influence the membrane hydration but represent the mechanisms by which water traverses from one side of the fuel cell to the other. The work focuses on the cathode side of the PEM fuel cell.

While water plays a critical role in maintaining the membranes polar conductivity, the production of water at the cathode catalyst layer can exceed the rate of water transport way from catalyst layer. The excess water leads to catalyst layer flooding, in which liquid water blocks catalyst sites and the pathways for oxygen to reach the catalyst (Das et al., 2010). The water can also block the pores of the GDL depriving the catalyst layer of reactants (Li et al., 2008). In both cases, the cell performance drops due to oxygen starvation. To prevent flooding, the water must traverse the catalyst layer and the gas diffusion layer, entering the gas-supply channel for removal by the gas stream. Although the porous nature of the GDL allows for the flow of oxygen to the catalyst layer from anywhere along the channel, water breaking through into the gas channel behaves differently. The water will form preferential
pathways through the gas diffusion layer and will break through at a few preferred regions in the GDL (Lu et al., 2010; Lister et al., 2006; Yang et al., 2004; Zenyuk et al., 2015). The addition of a micro-porous layer between the catalyst layer and the GDL reduces the number of break through locations (Lu et al., 2010) and also aids in water management (Owejan et al., 2010). The mechanisms behind the water movement in the porous media forming the catalyst layer and the GDL are beyond the scope of this work (cf. Liu et al., 2015). Due to the limited number of break through locations, a simplified model of the PEM fuel cell channel can consist of single-point water injection.

With the gas-supply channels serving to remove excess water, the channels themselves can become flooded. Channel level flooding occurs to varying degrees depending on how the two-phase flow forms in the channels at the given operating conditions (Liu et al., 2007). The two-phase pressure impacts the cell performance (Liu et al., 2007; He et al., 2003; Barbir et al., 2005) and serves as an indicator to flooding conditions. Figure 1.2 shows the time evolution of the cathode pressure and cell voltage for a fuel cell operating at a current density of 0.2 A/cm². As the air stoichiometry ratio ($\xi$)—the ratio of supplied oxygen rate divided the rate of oxygen consumption—decreases, the cell voltage drops but the two-phase pressure remains nearly constant. The decrease in air stoichiometry ratio corresponds to a decrease in the mass flow rate of oxygen, which should lead to a decrease in pressure. The pressure remaining nearly constant, indicating an increase in the channel’s water content (flooded). Conversely, a decrease in the pressure of the cathode side, combined with an increase in the cell resistance corresponds to membrane dry-out (Barbir et al., 2005). Therefore, measuring the two-phase pressure drop in the gas channels, together with measuring the cell voltage and resistance, can diagnose fuel cell flooding and dry-out.
While using the pressure drop as a diagnostic tool helps understand the complex water management in the fuel cell, predicting the two-phase pressure drop would allow engineers to optimize the fuel cell design for specific design requirements. At the system level, predicting the two-phase pressure would allow designers to select the required pumping power to supply the proper reactant flow rates. At the component level, designers can combine prediction methods for the two-phase pressure with reaction kinetics and transport phenomena throughout the cell to form a computation model (cf. Goshtasbi et al., 2016) to investigate water management during fuel cell operation. The prediction relies on semi-empirical models.

Researchers have proposed many different semi-empirical models to predict the two-phase pressure drop which typically fall into one of three categories: homogeneous flow, separated flow, and relative permeability models. The homogeneous flow model treats the two-phase flow as an equivalent single-phase flow by averaging the two fluid properties together, particularly the viscosity. The separated flow model follows the work of Lockhart & Martinelli (1949) and Chisholm (1967), accounting for the interaction of the two phases through the Chisholm parameter, $C$. The functional form of the $C$-value depends on experimental correlations. Finally, the relative permeability models use Dary’s equations to determine the two-phase pressure by modeling the form of the relative permeability in terms of the liquid saturation. In particular, Wang (2009) proposed a relative permeability model, termed the
two-fluid model, to investigate the flow in PEM fuel cells, treating the gas-supply channels as porous media that depends on the relative permeability exponent $n_k$.

The direct application of the models to operating fuel cells fails to match the experimentally measured pressure drop (Anderson et al., 2015). At a given operating condition, multiple flow regimes along the length of the channel can occur, as noted by Anderson et al. (2015) and Fontana et al. (2013), causing the models to break down. To address this, Kandlikar et al. (2015) proposed a 1-D down-the-channel model for the two-phase pressure drop. The algorithm breaks the channel into segments where reaction kinetics determine the local flow conditions, while two-phase pressure models calculate the local two-phase pressure drop. Using the $C$-value correlation proposed by Grimm et al. (2012), based on experiments with water injection through a GDL at 23°C, the algorithm of Kandlikar et al. (2015) predicted the two-phase pressure with a mean error of 5.2% compared to the pressure measured in the cathode channel of a fuel cell operating at 40°C with fully humidified air. The model predicted the two-phase pressure with a mean error of 40.2% for the anode side. The $C$-value correlation of English & Kandlikar (2006) based on flow originating from a single water location at room temperature agreed within 5.4% of the pressure drop compared to the same operating fuel cell. However, using other correlations for the Chisholm parameter resulted in 13%-18% error in the cathode channel under the same conditions. Therefore, determining and validating two-phase pressure models, even under simplified conditions, can provide useful engineering tools to design fuel cells.

The geometry of the gas-supply channels also plays a role in the fuel cell performance. A decrease in the channel size results in an increase in the cell voltage at a given current density as a result of better fuel utilization (Cha et al., 2004a). As the devices scale down, the surface to volume ratio of the channel increases, improving transport processes (McLean et al., 2000). However, as the channel dimensions continue to decrease, the channels become more susceptible to flooding (Cha et al., 2004b). Typically, fuel cell gas-supply channels
have square cross-sections. Researchers continue to investigate the influence of changing the channel’s aspect ratio ($AR = w/h$)—where $w$ defines the channel width and $h$ the channel height. Depending on the channel configuration in the fuel cell (parallel, interdigitated, serpentine, multiple-serpentine) increasing the aspect ratio can affect the performance of the fuel cell significantly (Wang et al., 2008a, 2009; Choi et al., 2011b; Manso et al., 2011). Thus, designers can optimize the channel geometry to achieve optimal fuel cell performance. This would require understanding how the channel geometry influences the two-phase pressure, as it plays a role in the performance as well. Consequently, this work focuses on a thin (high aspect-ratio) microchannel.

Furthermore, designers can modify the surface wettability of the GDL and gas channels to improve cell performance by improving water management. The contact angle ($\gamma$) defines the surface wettability (table 1.1). Specifically, surface wettability falls into two categories: hydrophilic when $\gamma < 90^\circ$ and hydrophobic when $\gamma > 90^\circ$.

Table 1.1: Definition of wettability.

<table>
<thead>
<tr>
<th>Contact Angle [°]</th>
<th>Wettability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Wetting</td>
</tr>
<tr>
<td>$0 &lt; \gamma &lt; 90$</td>
<td>Partially wetting</td>
</tr>
<tr>
<td>$90 \leq \gamma &lt; 180$</td>
<td>Partially non-wetting</td>
</tr>
<tr>
<td>180</td>
<td>Non-wetting</td>
</tr>
</tbody>
</table>

To prevent water retention, commercial GDLs undergo a hydrophobic treatment (cf. Park & Popov, 2009; Kumbur et al., 2008). This forms one side of the gas channels. The bipolar plate forms the remaining sides, which can have a different wettability than the GDL (cf. Owejan et al., 2007; Cai et al., 2006; Lu et al., 2011). Thus, PEM fuel cell gas channels typically form a mixed-wettability channel—one surface has a distinctly different wettability than the remaining surfaces that form the channel. Unfortunately, studies of the two-phase pressure drop as a function of channel wettability have produced inconsistent trends in how the two-phase pressure drop changes with contact angle, which this work investigates.
1.3 Overview of the Work

This work focuses on the two-phase flow in a high aspect ratio (thin) microchannel through experimental investigation. Air-water two-phase experiments were performed in an independent test bed of dimensions 3.23mm wide by 0.304mm high by 164mm long. The geometry results in an aspect-ratio of 10.6. The work falls into two parts (chapters 2 and 3). Chapter 2 demonstrates the applicability of the two-fluid model in the common flow range of an operational PEM fuel cell for high aspect ratio microchannels and provides a basis of comparison for chapter 3. In addition to selected existing two-phase pressure models, §2.2 introduces the two-fluid model. The analytical solution to stratified flow proposed in dimensionless form by Steinbrenner (2011) used for comparison to the optically measured water film thickness follows in §2.2.4. Section 2.3 details the experimental apparatus used to produce air-water flow and outlines the testing method. Finally, §2.4 discusses the validation of the experimental setup and an assessment of the two-phase pressure models. §2.4.4 discusses the new relative permeability exponent \( n_k \) and its validation in predicting the experimental two-phase pressure and water film thickness measurements.

Chapter 3 uses an identical test bed as chapter 2, except the channel base material changes from aluminum to PTFE. Changing the base creates a mixed-wettability microchannel. Chapter 3 demonstrates how the two-phase pressure drop changes with surface wettability and assesses the predictive accuracy of existing two-phase pressure drop models for mixed-wettability microchannels. §3.2 details the conflicting results for the two-phase pressure drop presented in literature (§3.2.1) and a discussion of the stability of rivulets (§3.2.2), the primary flow pattern in this study. The existing homogeneous, separated, and relative permeability models selected for comparison follow in §3.3. Section 3.4 details the experimental method to produce air-water flow in a mixed-wettability microchannel. The validation of the experimental set-up, the two-phase pressure drop results, and the observed flow patterns follow in §3.5. Placing the existing experimental data into context using dimensionless
numbers organizes the pressure trends in literature for adiabatic two-phase flow in a single mixed-wettability microchannel (§3.5.4). The comparison to the existing two-phase pressure models follows in §3.5.5 with the determination of an optimized relative permeability exponent ($n_k$) for rivulet flow (§3.5.6). The chapter concludes with a discussion of the stability of rivulet flow in relation to the predictive ability of the two-phase pressure models (§3.5.7). The work concludes with chapter 4 summarizing the conclusions and future work.
Chapter 2

Two-phase Frictional Pressure Drop and Water Film Thickness in a Thin Hydrophilic Microchannel

2.1 Abstract

This chapter focuses on the investigation of the two-phase pressure drop and water film thickness in a thin microchannel through experimental investigation and modeling. Air-water flow in the thin channel of dimensions 3.23mm wide by 0.304mm high produces a stratified flow over a range of test conditions. The experimental data allows for the assessment of several models including homogeneous, separated, and relative permeability models. A comparison of the two-phase pressure measurements to the recently developed two-fluid model (Wang, 2009) determines a new relative permeability exponent ($n_k$) of 1.159, producing a mean absolute percent error of 3.25%. Unlike previous models, the two-fluid model requires only a single correlation parameter, $n_k$, to determine the two-phase pressure drop. Imaging of
the air-water flow provides measurements of the water film thickness, which agrees with the analytical solution of Steinbrenner (2011) and the two-fluid model with a mean error of -0.035. The study demonstrates the applicability for the two-fluid model to predict both the two-phase pressure and the water film thickness in thin microchannels.

2.2 Background

Multiple mechanisms contribute to the overall two-phase pressure drop ($\Delta P_{tp}$) (Kim & Mudawar, 2012):

$$\Delta P_{tp} = \Delta P_F + \Delta P_g + \Delta P_A + \Delta P_{loss}$$  \hspace{1cm} (2.1)

where $\Delta P$ refers to pressure drop and the subscripts $F$, $g$, $A$, and $loss$ refer to frictional, gravitational, acceleration, and loss, respectively. The models in this section refer only to the frictional pressure drop. The current study focuses on the adiabatic flow of air and water in a horizontal microchannel. Under adiabatic conditions, the flow will not accelerate and $\Delta P_A$ equals zero. With the channel aligned horizontally, the gravitational pressure term also equals zero. The experimental technique seeks to minimize losses caused by entrance/exit effects and thus the loss term equals zero. Thus, the two-phase pressure equals the two-phase frictional pressure loss.

2.2.1 Homogeneous Flow Model

The homogeneous flow model treats the two-phase flow as an equivalent single-phase flow with aggregate properties under the condition that the two phases move with the same velocity. The two-phase pressure drop would follow:

$$\left( \frac{dP}{dz} \right)_{tp} = f_{tp} \frac{G^2}{2DH\rho tp}$$  \hspace{1cm} (2.2)
where $P$ stands for the pressure, $z$ the downstream coordinate, $f$ the Darcy friction factor, $G$ the total mass flux, $D_H$ the hydraulic diameter, and $\rho$ the density. The subscript $tp$ stands for two-phase. The total mass flux ($G$) equals:

$$G = \frac{\rho_G Q_G + \rho_L Q_L}{A_c} \quad (2.3)$$

where $A_c$ stands for the cross-sectional area and $Q$ the volumetric flow rate. The subscripts $G$ and $L$ stand for gas-phase and liquid-phase, respectively. The friction factor follows the standard definition for laminar flow:

$$f_{tp} = \frac{\overline{C}}{Re_{tp}} \quad (2.4)$$

where the correlation constant ($\overline{C}$) equals its single-phase equivalent but the Reynolds number becomes the two-phase Reynolds number ($Re_{tp}$) defined as:

$$Re_{tp} = \frac{GD_H}{\mu_{tp}} \quad (2.5)$$

Therefore, determination of the two-phase pressure drop only requires knowledge of the two-phase density ($\rho_{tp}$) and the two-phase dynamic viscosity ($\mu_{tp}$). Researchers often agree that the two-phase density has the form:

$$\rho_{tp} = \left(\frac{\chi \rho_G + 1 - \chi \rho_L}{\rho_G Q_G + \rho_L Q_L}\right)^{-1} \quad (2.6)$$

where $\chi$ represents the gas quality defined as:

$$\chi = \frac{\rho_G Q_G}{\rho_G Q_G + \rho_L Q_L} \quad (2.7)$$
However, researchers have proposed several forms for the two-phase viscosity. McAdams et al. (1942) proposed:

\[
\frac{1}{\mu_{tp}} = \frac{\chi}{\mu_G} + \frac{1 - \chi}{\mu_L} \quad (2.8)
\]
as the two-phase viscosity to follow the form of the two-phase density based on flow boiling experiments of benzene-oil in 2.69 cm tubes. Cicchitti et al. (1960) proposed:

\[
\mu_{tp} = \mu_G \chi + (1 - \chi) \mu_L \quad (2.9)
\]
for simplicity and justified the form by showing ±12% average difference between the experimentally measured and calculated friction factor for steam-water flowing vertically in 0.51 cm tubes of various lengths.

Flow may not meet the primary assumption of the homogeneous flow model that the two phases move with the same velocity. The slip ratio, defined as the ratio of the local gas velocity to the local liquid velocity, would equal one for a homogeneous flow. To account for a slip ratio deviating from one, Lin et al. (1991) modified the form of McAdams to:

\[
\mu_{tp} = \frac{\mu_L \mu_G}{\mu_G + \chi \hat{n}(\mu_L - \mu_G)} \quad (2.10)
\]
where \(\hat{n}\) takes into account the slip ratio. Fitting experimental data for R-12 refrigerant undergoing vaporization in 1 mm tubes, Lin et al. (1991) determined \(\hat{n} = 1.4\) for a relative error of 15.3%.

Other researchers took a fundamental approach to define the two-phase viscosity. Through the use of dynamic similarity, Dukler et al. (1964) determined:

\[
\mu_{tp} = \mu_L(1 - \beta) + \mu_G \beta \quad (2.11)
\]
where $\beta$ represents the homogeneous void fraction defined as:

$$\beta = \frac{Q_G}{Q_G + Q_L} \quad (2.12)$$

Dukler et al. (1964) tested two-component flows in tubes of diameter 2.54–12.7 cm (1-5 inch) over a range of viscosities. The measured pressure drop compared to the pressure drop calculated by equations 2.2, 2.6, 2.11 showed an agreement within $-19\%$ to $16\%$.

Fourar & Bories (1995) studied air-water flows in horizontal fractures composed of glass and baked clay. The glass fractures had a gap spacing ($h$) of 1 mm and a width of 0.5 m, while baked clay formed fractures 14 cm wide by 0.54, 0.4, and 0.18 mm high. The small heights compared to the width allowed Fourar and Bories to treat the flow as plane-poiseuille flow and defined the flow properties as:

$$U_{tp} = U_G + U_L \quad (2.13)$$
$$\rho_{tp} = s_L \rho_L + s_G \rho_G \quad (2.14)$$

where $U$ represents the superficial velocity ($Q/A_c$) and $s$ the saturation—the volume occupied by the phase divided by the volume of the channel. This results in a two-phase viscosity of the form:

$$\mu_{tp} = (1 - \beta) \mu_L + \beta \mu_G + \sqrt{\beta(1 - \beta) \mu_g \mu_L} \quad (2.15)$$

Equation 2.15 has the same form as equation 2.11 (Dukler et al., 1964) with an additional term. Measurements in the clay channel ($Re_{tp} < 4000$) agreed well with the friction factor calculated from equation 2.4 with $C = 96$. To calculate $Re_{tp}$ to use in equation 2.4, $2h$ replaces $D_H$ and equation 2.13 multiplied by $\rho_{tp}$ replaces $G$ in equation 2.5. The flow pattern did not influence the flow except for annular flow. Annular flow occurring in the glass channel at $Re_{tp} > 4000$ resulted in a significant deviation between the measured friction factor and the calculated values.
Beattie & Whalley (1982) looked to create a general two-phase viscosity by analyzing flow pattern specific $\mu_{tp}$. For bubble flow up to a voidage of 20% $\mu_{tp} = \mu_L (1 + 2.5\beta)$. For annular flows $\mu_{tp} = \mu_L (1 - \beta) + \mu_G \beta$. The two models represent the extremes of the flow pattern range and thus Beattie and Whalley proposed:

$$\mu_{tp} = \mu_L (1 - \beta) (1 + 2.5\beta) \mu_G \beta$$

(2.16)

The model showed good agreement for horizontal flows of fluid pairs other than steam-water but underpredicted steam-water experiments.

Awad & Muzychka (2008) determined the two-phase viscosity as:

$$\mu_{tp} = \mu_G \frac{2\mu_G + \mu_L - 2(\mu_G - \mu_L)(1 - \chi)}{2\mu_G + \mu_L + (\mu_G - \mu_L)(1 - \chi)}$$

(2.17)

through an analogy of the Maxwell-Eucken equation for the effective thermal conductivity of a porous media. For mini/microchannels, the model predicted the friction factor on average 5–6% better than McAdams’ and Cicchitti’s models. The authors based the comparison on experiments of R134a, R410A, and propane flow in horizontal tubes of diameters 2.58 and 2.46 mm, and channels of hydraulic diameter 1.4 mm and 0.148 mm.

### 2.2.2 Separated Flow Model

While the homogeneous flow model treats a two-phase flow as an equivalent single phase flow, the separated flow model seeks to account for how the interaction between the two phases influences the pressure drop. Martinelli et al. (1944) determined that the two-phase pressure drop varies depending on the flow mechanism—laminar or turbulent—of each phase. Since the phases have different flow rates, four possible combinations exist: laminar-laminar, turbulent-turbulent, laminar-turbulent, and turbulent-laminar liquid-gas flow, respectively.
Testing various liquids in air flow, Martinelli et al. (1944) determined the two-phase pressure over a change of length \( L \) should equal:

\[
\left( \frac{\Delta P}{\Delta L} \right)_{tp} = \phi_G^2 \left( \frac{\Delta P}{\Delta L} \right)_G
\]

where \( \phi_G^2 \), the gas two-phase flow multiplier, depends on a parameter \( \chi \). In a later paper, Martinelli et al. (1946) showed the relation held true for the laminar-laminar regime. The term \( \chi \) takes on a different form for each flow mechanism—an expected result considering the friction factor varies between laminar and turbulent flow. In further work, the authors found a similarity of shape and magnitude for plots of \( \phi^2 \) versus \( \chi \). Lockhart & Martinelli (1949) sought to generalize the correlation factor for the flow multiplier in terms of a new parameter \( X \), independent of the flow mechanism. Lockhart and Martinelli considered the two fluids separately, formulating the Fanning friction factor for each phase, as if each phase occupied a distinct fraction of the cross-sectional area. Under the constraints that both phases have equal static pressure drop and the volume occupied by each phase equals the volume of the pipe, the new correlation term—termed the Lockart-Martinelli parameter—becomes:

\[
X = \sqrt{\frac{\Delta P_L}{\Delta P_G}}
\]

in which \( \Delta P_L \) and \( \Delta P_G \) equal the pressure drop experienced along the channel if the respective phase flowed alone in the pipe. Comparisons between experimental air flow with oil, benzene, and other liquids in 1.49 cm to 2.58 cm diameter pipes showed \( \phi^2 = f(X) \).

The Lockhart-Martinelli analysis does not take into account the shear between the phases. Chisholm (1967), through a force balance on each phase, determined the relations for \( \phi_L \) and \( X \) in terms of geometric properties and a shear force function \( Z \). The complexity of the
relation for $Z$ and $\phi_L$ resulted in a recommendation of a simplified relation:

$$\phi^2_L = 1 + \frac{C}{X} + \frac{1}{X^2}$$  \hspace{1cm} (2.20)

The constant termed Chisholm’s parameter, $C$, takes into account the interaction between two phases. For air-water flows, Chisholm recommended the values shown in table 2.1 for each of the four flow mechanisms. In a previous work, Chisholm & Laird (1958) proposed

<table>
<thead>
<tr>
<th>Liquid Flow</th>
<th>Gas flow</th>
<th>$C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turbulent</td>
<td>Turbulent</td>
<td>20</td>
</tr>
<tr>
<td>Laminar</td>
<td>Turbulent</td>
<td>12</td>
</tr>
<tr>
<td>Turbulent</td>
<td>Laminar</td>
<td>10</td>
</tr>
<tr>
<td>Laminar</td>
<td>Laminar</td>
<td>5</td>
</tr>
</tbody>
</table>

that the turbulent-turbulent case should have $C = 21$, a value commonly referenced to by researchers. Equation 2.20 holds for the gas as well since:

$$\phi^2_G = X^2\phi^2_L$$  \hspace{1cm} (2.21)

Chisholm determined the values in table 2.1 for conventional tubes. Investigation of mini and microchannels reveals a need for further refinement. Mishima et al. (1993) investigated the influence of scale and geometry on the pressure drop of vertical-upward air-water flow in narrow gaps 40 mm wide by 1.07, 2.45, and 5.00 mm high by 200cm long. Later, Mishima & Hibiki (1996) investigated the same conditions for tubes of inside diameter 1–4 mm. The data from both experiments, combined with a database of other vertical and horizontal experiments, led the authors to propose:

$$C = 21 \left(1 - e^{-0.319DH}\right)$$  \hspace{1cm} (2.22)

The correlation predicted the pressure drop data to within 12%, except ammonia-ammonia
vapor flow data fell within 25%.

As the scale decreases, the importance of surface tension should increase. Based on the application of a neural network, Zhang et al. (2010) found that surface tension (\(\sigma\)) influences correlations for the two-phase pressure drop through the confinement number (\(N_{con}\)) more than \(D_H\) or the Weber number. As a result, Zhang et al. (2010) modified the result of Mishima & Hibiki (1996) to introduce the confinement number as:

\[
C = 21 \left(1 - e^{-\frac{0.674}{N_{con}}}\right)
\]

\[
N_{con} = \sqrt{\frac{\sigma}{g(\rho_L - \rho_G)D_H^2}}
\]

for adiabatic two-phase flows for \(Re_L\) and \(Re_G\) less than two-thousand. The variable \(g\) equals the acceleration due to gravity.

In the limiting case, as \(D_H\) increases, equation 2.22 produces Chisholm’s turbulent-turbulent \(C\)-value of 21. In small geometries the flow likely exhibits laminar characteristics. To investigate this, English & Kandlikar (2006) tested air-water flow in a horizontal rectangular microchannel of dimensions 1.124 mm wide by 0.93 mm high by 321 mm long. From the mixing section to the outlet, the channel had a length of 7 cm. The flow rates produced liquid Reynolds numbers between 0.56-24.6 and gas Reynolds numbers of 211-654. Based on the experimental data, English and Kandlikar proposed modifying Mishima and Hibiki’s correlation to:

\[
C = 5 \left(1 - e^{-0.319D_H}\right)
\]

where Chisholm’s laminar-laminar value of 5 replaces 21. The correlation predicted the experimental data with an average deviation of 3.3%.

Li & Wu (2010) compiled a database of adiabatic flow boiling experiments for multiple refrigerants with various qualities. The database contained a range of hydraulic diameters
between 0.148–3.25 mm. Of the data, 59% produced a Bond Number \((Bo = 1/N_{conf}^2)\) less than one. Consequently, Li and Wu sought to modify \(C\) to become a function of \(Bo\). For the data with \(Bo \leq 1.5\), a regression analysis showed:

\[
C = 11.9Bo^{0.45}
\]

(2.26)

fit 80% of the data within 30% with a mean absolute error of 20.8%.

For a given geometry and fluid pair, Chisholm (1967), Mishima & Hibiki (1996), Zhang \textit{et al.} (2010), English & Kandlikar (2006), and Li & Wu (2010) produce a constant \(C\) value regardless of the flow conditions. Several authors investigated the influence of test conditions on the Chisholm parameter. Sun & Mishima (2009) compiled a database of 2092 two-phase pressure drop measurements for various refrigerants and air-water experiments in channels of hydraulic diameters between 0.506–12 mm. Based on statistical analysis, Sun and Mishima determined the liquid Reynolds number as well as the confinement number influence the \(C\) value for laminar-laminar flow. Thus, Sun and Mishima proposed a modification to equation 2.23 such that:

\[
C = 26\left(1 + \frac{Re_L}{1000}\right)\left[1 - e^{-0.153N_{conf}^{0.8}}\right]
\]

(2.27)

Ma \textit{et al.} (2010) investigated the influence of aspect ratio for rectangular microchannels on the two-phase pressure drop. The authors created microchannels of dimensions (height \(\times\) width) 100\(\mu\)m \(\times\) 200\(\mu\)m, 100\(\mu\)m \(\times\) 400\(\mu\)m, 100\(\mu\)m \(\times\) 800\(\mu\)m, and 100\(\mu\)m \(\times\) 2000\(\mu\)m oriented horizontally. Based on least-squares regression Ma \textit{et al.} (2010) found \(C\) depends on the
aspect ratio and the capillary number as:

\[
C = \hat{a}Ca_L^{\hat{b}} \tag{2.28}
\]

\[
\hat{a} = 7.59 - 0.4237AR^{0.9485} + 0.0023Re_L
\]

\[
\hat{b} = 0.223 + 0.2AR^{-0.9778}
\]

\[
Ca_L = \frac{\mu_L U_L}{\sigma} \tag{2.29}
\]

\(Ca_L\) stands for the liquid capillary number. For air-water flows, equation 2.28 correlated the experimental measurements within a mean deviation of 8.5%.

Kim & Mudawar (2012) sought to improve two-phase pressure drop predictions for condensers. While condensers generate a two-phase flow by inducing a phase change via heat transfer, the flow structure of condensing flow closely resembles that of adiabatic flows. Kim and Mudawar compiled a database of condensing and adiabatic flows for various refrigerants and air-water. Of the data, 1919 points fell in the laminar-laminar regime, none of which fell in the range \(Re_L < 10\) and \(Re_G < 200\). The authors sought to determine the \(C\)-value as a function of dimensionless parameters. At small scales, the body forces should play a minimal role, leading Kim and Mudawar to eliminate Froude, Bond, and confinement numbers. This leaves a possibility of Reynolds, Weber, Capillary, and Suratman (\(Su\)) numbers. Additionally, to account for the combination of different fluids, the authors consider the density ratio. For each flow mechanism, Kim and Mudawar tried various combinations of the parameters and arrived at:

\[
C = 3.5 \times 10^{-5}Re_{lo}^{0.44}Su_{go}^{0.5} \left(\frac{\rho_L}{\rho_G}\right)^{0.48} \tag{2.30}
\]

which correlated the laminar-laminar data to a mean absolute percent error of 26.3%. In equation 2.30, \(Re_{lo}\) stands for the liquid-only Reynolds number defined as:

\[
Re_{lo} = \frac{GD_H}{\mu_L} \tag{2.31}
\]
and $Su_{go}$ stands for the gas-only Suratman number defined as:

$$Su_{go} = \frac{\rho_G \sigma D_H}{\mu_G^2}$$  \hspace{1cm} (2.32)

which relates the combined effects of surface tension and inertia to the influences of viscosity.

While Kim & Mudawar (2012) investigated flow condensing, Li & Hibiki (2017) investigated flow boiling. Li and Hibiki collected 1521 data points for adiabatic and diadiabatic flow for various refrigerants, water, nitrogen, and carbon dioxide. The circular and rectangular microchannels had hydraulic diameters between 0.1 and 3 mm. Eighty-five points fell in the laminar-laminar regime. The authors plotted the common logarithm of various dimensionless parameters versus the common logarithm of the experimentally measured $C$ value. Based on the analysis, the authors concluded that the gas quality, the two-phase Reynolds number, and the viscosity number ($N_{\mu_{tp}}$) determine the $C$-value as:

$$C = 41.7 N_{\mu_{tp}}^{0.66} R_{\epsilon_{tp}}^{0.42} \chi^{0.21}$$  \hspace{1cm} (2.33)

where the two-phase viscosity follows McAdams et al. (equation 2.8) to calculate the two-phase Reynolds number. The two-phase density equals:

$$\rho_{tp} = \chi \rho_G + (1 - \chi) \rho_L$$ \hspace{1cm} (2.34)

to determine the two-phase viscosity number as:

$$N_{\mu_{tp}} = \left( \frac{\mu_{tp}}{\rho_{tp} \sqrt{\frac{\sigma}{g(\rho_L - \rho_G)}}} \right)^{0.5}$$ \hspace{1cm} (2.35)

The correlation predicted the laminar-laminar database to a mean absolute error of 9.07%.

Suo & Griffith (1694) investigated slug flow in horizontal capillary tubes with radii of 0.5
and 0.8 mm for water-nitrogen/air, heptane-helium, and heptane-nitrogen flows. Through a Buckingham-Pi analysis, Suo and Griffith found two non-dimensional parameters:

\[
\Phi = \frac{\mu_L U_B}{\sigma} \quad (2.36) \\
\Lambda = \frac{\mu_L^2}{r \rho_L \sigma} \quad (2.37)
\]

where \( U_B \) equals the bubble velocity and \( r \) the tube radius. The dimensionless variables \( \Phi \) and \( \Lambda \) equate to a capillary number based on bubble velocity, and an inverse liquid-only Suratman number, respectively. These terms serve as the basis for the correlations of Lee & Lee (2001) and Saisorn & Wongwises (2010).

Lee & Lee (2001) conducted experiments in horizontal channels of 20mm width and 0.4, 1, 2, and 4mm heights. The experimental apparatus consisted of a porous plate of 40\( \mu \)m pore size to introduce air bubbles uniformly into a water stream. The test conditions result in \( Re_G \) between 8–3176 and \( Re_L \) between 133–13,615. Lee and Lee modified Suo and Griffith’s \( \Phi \) and \( \Lambda \) by using a liquid slug velocity \( (j) \) in \( \Phi \) and the hydraulic diameter in \( \Lambda \). Additionally, Lee and Lee utilized the liquid-only Reynolds number to account for the influence of mass velocity. The authors proposed that the \( C \)-value had the form \( C = f(\Lambda, \Phi, Re_{lo}) \) and through data regression found:

\[
C = 6.833 \times 10^{-8} \Lambda^{-1.317} \Phi^{0.719} Re_{lo}^{0.557} \quad (2.38)
\]

The correlation predicted the experimental data and a database of other narrow-gap/large-width experiments to within \( \pm10\% \).

Saisorn & Wongwises (2010) took a similar approach to Lee & Lee (2001) to determine the \( C \)-value for fused silica micro-tubes. The authors conducted air/nitrogen-water experiments in micro-tubes of 0.15, 0.22, and 0.53 mm diameter. The experimental flow rates produced \( 3.5 \leq Re_G \leq 1447 \) and \( 0.75 \leq Re_L \leq 1605 \). Under these conditions, the flow formed several
different flow patterns that resulted in different flow multipliers. To account for the variation in $C$ with different flow patterns—particularly bubble, transition, and liquid-ring flow—Choi & Kim (2011) proposed flow pattern specific correlations. However, Saisorn & Wongwisess (2010) proposed a single correlation as:

$$C = 7.599 \times 10^{-3} \Lambda^{-0.631} \Phi^{0.005} Re_{lo}^{-0.008}$$

(2.39)

where the authors modified $\Phi$ to use the sum of superficial velocities instead of the slug velocity used equation 2.38. Compared the experimental data, 52% of the data fell within ±40% of the two-phase pressure drop predicted by in equation 2.39.

### 2.2.3 Relative Permeability Based Two-Fluid Model

The final approach to understand the two-phase pressure drop treats the flow as a flow in a porous media. The channel represents a single pore with an absolute permeability of $K$. Under two-phase flow, the liquid in the channel forms a pore structure that the gas must flow through and vice versa. The presence of the second phase will reduce the available space for the first phase, decreasing the permeability of the first phase. Thus, the effective permeability of a phase should equal $K$ multiplied by a term (the relative permeability, $k_r$) to account for the influence of the second phase. This allows for a generalization of Darcy’s equations for steady-laminar flow in porous media as:

$$\rho_G \vec{U}_G = -\frac{k_{r,G}K}{\nu_G} \nabla P_G$$

(2.40)

$$\rho_L \vec{U}_L = -\frac{k_{r,L}K}{\nu_L} \nabla P_L$$

(2.41)

where $k_{r,G}$ takes into account the influence of the water on the gas and $k_{r,L}$ takes into account the influence the gas has on the water.
With the goal of understanding the pressure drop between the inlet and outlet of a channel, and making the assumption that the flow properties change only in the streamwise ($z$-direction) simplifies equations 2.40 and 2.41 to one-dimension as (Wang, 2009):

\[
\rho_G U_G = -\frac{k_{r,G} K dP_G}{\nu_G} d\frac{z}{dz} \tag{2.42}
\]

\[
\rho_L U_L = -\frac{k_{r,L} K dP_G}{\nu_L} d\frac{z}{dz} \tag{2.43}
\]

The pressure gradient of the gas replaces the pressure drop of the liquid phase for equation 2.43 by assuming negligible capillary pressure.

The flow need not start out as a two-phase flow, depending on the injection conditions. Let $\bar{z} = z/L$ equal a dimensionless streamwise coordinate and $\bar{z}^*$ equal the dimensionless starting location of the two-phase flow. The governing equations for each phase become:

\[
\rho_G U_G = -\frac{k_{r,g} K dP_G}{L \nu_G} d\frac{\bar{z}}{d\bar{z}} \tag{2.44}
\]

\[
\rho_L U_L = -\frac{k_{r,L} K dP_G}{L \nu_L} d\frac{\bar{z}}{d\bar{z}} \tag{2.45}
\]

Consider for now single-phase gas flow between the inlet and $\bar{z}^*$. For single-phase flow $k_{r,G} = 1$ and the integration of equation 2.44 gives:

\[
\Delta P_G = P(\bar{z}^*) - P(0) = -L \int_0^{\bar{z}^*} \frac{U_G \mu_G}{K} d\bar{z} \tag{2.46}
\]

Similarly, for the two-phase flow between $\bar{z}^*$ and the channel exit ($z = L, \bar{z} = 1$), integrating equation 2.42 gives:

\[
\Delta P_{tp} = P(1) - P(\bar{z}^*) = -L \int_{\bar{z}^*}^{1} \frac{U_G \mu_G}{K k_{r,G}} d\bar{z} \tag{2.47}
\]

The total pressure drop ($\Delta P = P(0) - P(1)$) in the channel would equal the sum of equa-
\[ \Delta P = -(\Delta P_{tp} + \Delta P_G) = L \left[ \int_0^{z^*} \frac{U_G \mu_G}{K} d\tilde{z} + \int_{z^*}^{1} \frac{U_G \mu_G}{K k_{r,G}} d\tilde{z} \right] \] (2.48)

The channel length and the absolute permeability only depend on the selected channel geometry, while the viscosity remains constant under adiabatic conditions. Thus \( L, K, \) and \( \mu_G \) remain constant. Assuming that the superficial velocity remains constant, factoring out constant terms from the integrals gives:

\[ \Delta P = \frac{\mu_G U_G L}{K} \left[ \int_0^{z^*} d\tilde{z} + \int_{z^*}^{1} \frac{1}{k_{r,G}} d\tilde{z} \right] \] (2.49)

Looking at equation 2.46, the factored term in equation 2.49 equals the single-phase pressure drop of the gas for the entire channel length. By equation 2.18,

\[ \frac{\Delta P_G}{\mu_G U_G L} = \phi_G^2 \]

\[ \phi_G^2 = \pi^* + \int_{\pi^*}^{1} \frac{1}{K_{r,G}} d\tilde{z} \] (2.50)

where the first term accounts for any single-phase gas flow at the entrance of the channel and the second term accounts for the interaction of the two phases. A similar expression arises for the liquid two-phase flow multiplier starting with 2.43, assuming single-phase liquid flow at the entrance. Note that the first term in equation 2.50 would equal zero if the channel began with only single-phase liquid flow and equals zero in the \( \phi_L^2 \) equation for single-phase gas flow at the entrance. Similar to solving for Chisholm’s \( C \)-value for the two-phase pressure drop, determination of the two-phase pressure drop using equation 2.50 requires knowing \( k_{r,G} \).

The determination of the relative permeability requires knowledge of the saturation. Saturation (s) equals the volume of the fluid divided by the volume of the pore/channel. Burdine
(1953) reduced the form of the relative permeability to saturation and capillary pressure ($P_{cap}$) as:

$$k_{r,L} = \left( \frac{s_L - s_{L,r}}{1 - s_{L,r}} \right) \int_0^{s_L} \frac{d s_L}{P_{cap}^2}$$

$$k_{r,G} = \left( 1 - \frac{s_L - s_{L,r}}{s_m - s_{L,r}} \right) \int_1^{s_L} \frac{d s_L}{P_{cap}^2}$$

(2.51)

(2.52)

Corey (1954) analyzed oil-gas capillary pressure curves and showed:

$$\frac{1}{P_{cap}^2} = \hat{c} s_{L,e}$$

(2.53)

where $s_{L,e}$, the effective liquid (oil) saturation equals:

$$s_{L,e} = \frac{s_L - s_{L,r}}{1 - s_{L,r}}$$

(2.54)

and $s_{L,r}$ accounts for residual oil remaining in the pore that the flow could not carry out. The constant $\hat{c}$ equals $c^*(1 - s_{L,r})$, with $c^*$ a correlation constant. Utilizing equation 2.53 in equation 2.51, Corey arrived at:

$$k_{r,L} = s_{L,e}^4$$

(2.55)

and making an assumption to first order $s_m$ equals one in equation 2.52, the relative permeability of the gas phase becomes:

$$k_{r,G} = (1 - s_{L,e})^2(1 - s_{L,e}^2)$$

(2.56)

Based on Corey’s analysis, $s_m$ may only result from the method of derivation and does not have physical significance. The author only utilized equation 2.56 to determine $s_{L,r}$ to use in equation 2.55. Two-thirds of oil-gas experimental experiments conducted by Corey agreed well with the liquid relative permeability calculated from equation 2.55.
The X-model provides the simplest form of the relative permeability. Based on the experimental work of Romm (1972), the relative permeability varies linearly with the saturation such that the sum of gas and liquid relative permeabilities equal 1. Thus:

$$k_{r,G} = (1 - s_{L,e})$$  \hspace{3cm} (2.57)

Nowamooz et al. (2009) investigated two-phase flow at high flow rates in a replica of a Vosges sandstone sample 26cm long and 15cm wide, approximately 429µm in height. The experiments started as single-phase water flow in the range of $Re_L$ from 0.07 to 0.45. Air injection followed at $0 \leq Q_G \leq 1.2 \times 10^{-3}$ m³/s to generate two-phase flow. The authors could not directly measure saturation. Instead, Nowamooz et al. (2009) utilized the solution to plane-poiseuille flow of Fourar & Bories (1995) to calculate the relative permeability from the measured pressure drop and utilized equation 2.19 to relate the relative permeability to measured quantities. Based on minimizing the mean absolute relative error between the experiments and the model for the relative permeability, the authors determined:

$$k_{r,L} = s_L^{1.15}$$  \hspace{3cm} (2.58)

$$k_{r,G} = (1 - s_L)^{3.05}$$  \hspace{3cm} (2.59)

The formation of the flow plays a critical role on the measured pressure drop and thus should influence the relative permeability. Chen et al. (2004) investigated nitrogen-deionized water flow in a fracture 30.48cm long by 10.16cm wide by 130µm high. Glass formed the top of a stainless steel-aluminum channel allowing visualization of the flow. From images of the flow Chen et al. (2004) determined the saturation and extracted the tortuosity ($\tau$)—the ratio of the smallest area bounding a phase divided by the channel area. Curve fits of $\tau$ versus $s_L$ combined with the proposal that the relative permeability equals the saturation divided by
\[ k_{r,L} = 0.2677 s_L^3 + 0.331 s_L^2 + 0.3835 s_L \]
\[ k_{r,G} = 0.502 s_G^3 + 0.1129 s_G^2 + 0.3483 s_G \]

which predicted the relative permeability within a mean error of 0.0485 relative to the experimental \( k_r \).

The relative permeability from Corey (1954), Nowamooz et al. (2009), and Chen et al. (2004) take into account the impediment of one phase on the other but not the shear interaction between the phases. Fourar & Lenormand (1998) modeled the 2-D co-current flow of immiscible fluids between parallel plates. The wetting fluid contacts both plates with the non-wetting fluid flowing in between. A planar interface defines the boundary between phases. The application of Stokes equation for each phase in combination with Darcy’s equations gave:

\[ k_{r,L} = \frac{s_L^2}{2} (3 - s_L) \]  
\[ k_{r,G} = (1 - s_L)^3 + \frac{3}{2} \mu s_L (1 - s_L) (2 - s_L) \]

where \( \mu = \mu_G / \mu_L \)—the ratio of the non-wetting to wetting fluid viscosities. Fourar & Lenormand (1998) validated the model using the air-water data of Fourar & Bories (1995). Utilizing a Lattice-Boltzmann method to numerically solve the same setup as Fourar & Lenormand, Huang et al. (2009) found the same liquid relative permeability (equation 2.62) but the gas relative permeability equaled:

\[ k_{r,G} = (1 - s_L) \left[ \frac{3}{2} \mu + (1 - s_L)^2 \left( 1 - \frac{3}{2} \mu \right) \right] \]

For fluid pairs where the wetting fluid has a significantly larger viscosity than the non-wetting fluid, \( \mu \) approaches zero and equations 2.63 and 2.64 become identical.
The difficulty in experimentally measuring the saturation led other researchers to approach the problem in a different manner. In addition to the homogeneous flow model, Fourar & Bories (1995) determined a saturation model and a relative permeability model through liquid saturation measurements in the glass fracture. The small heights compared to the width allowed Fourar & Bories to treat the flow as plane-poiseuille flow and relating the pressure equation to the Lockhart-Martinelli parameter \((X)\), the authors arrived at:

\[
s_L = \left( \frac{X}{1 + X} \right)^2 \quad (2.65)
\]

\[
k_{r,G} = (1 - \sqrt{s_L})^2 \quad (2.66)
\]

Equations 2.65 and 2.66 showed good agreement with the experimental two-phase pressure for both the glass fracture and baked clay fractures.

Wang (2009) arrived at a simplified form for the saturation through the manipulation of Darcy’s equations. Equations 2.42 and 2.43 have the gas pressure gradient in common. By combining the two equations through the gas pressure gradient and re-organizing,

\[
U_G = \frac{\mu_L}{\mu_G} \frac{k_{r,G}}{k_{r,L}} U_L \quad (2.67)
\]

results. By utilizing the effective saturation (equation 2.54), the relative permeabilities take the form:

\[
k_{r,L} = s_{L,c}^{n_k} \quad (2.68)
\]

\[
k_{r,G} = (1 - s_{L,c})^{n_k} \quad (2.69)
\]
with a general exponent \( n_k \). Combining equations 2.67, 2.68, and 2.69 gives:

\[
S_L = \frac{\left( \frac{U_{L\mu_L}}{U_{G\mu_G}} \right)^{\frac{1}{n_k}} + S_{L,r}}{\left( \frac{U_{L\mu_L}}{U_{G\mu_G}} \right)^{\frac{1}{n_k}} + 1}
\] (2.70)

Thus, the saturation only depends on fluid parameters and the set test conditions for superficial velocities. Similar to Chisholm’s analysis, the determination of an unknown correlation parameter \( n_k \) remains. In this work, the two-fluid model refers to the use of equation 2.50 combined with equations 2.69 and 2.70. Adroher & Wang (2011) and Cho & Wang (2014b) showed the applicability of the two-fluid model to standard gas flow channels in PEM fuel cells. In a channel of 1.6mm wide by 1mm high, Adroher & Wang (2011) found an \( n_k \) value of 2 only resulted in a qualitative agreement between the experimentally measured two-phase pressure drop for superficial gas velocities in the range of 5 – 10m/s. Cho & Wang (2014b) applied the two-fluid model to both hydrophilic and hydrophobic channels of dimensions 1.68mm wide by 1.00mm high by 150mm long. The authors found that \( n_k \) values varied with flow pattern but remain a constant value of 2.49, 2.15, and 1.96 for slug, wavy, and annulus flow patterns, respectively.

2.2.4 Theoretical Solution of the Water Film Thickness

Steinbrenner (2011) noted that in thin microchannels, the stratified flow regime persisted over a wide range of test conditions. The current study predominately generated stratified flow under the test conditions, which simplifies the determination of the saturation. Stratified flow forms when a liquid film in contact with a side wall occupies the entire height of the channel but only part of the channel width (figure 2.1). In figure 2.1, \( h \) equals the channel height, \( c \) the film thickness, and \( c + b \) the channel width \( (w) \); \( b \) indicates the width occupied by the gas phase. The liquid saturation equals the volume of liquid \( (h \times c \times L) \) divided by the
total volume of the channel \((h \times w \times L)\), which reduces to the film thickness ratio \(h_{ratio} = c/w\). Thus, determining the liquid saturation requires only measuring the film thickness ratio.

![Channel cross-section with regions defining stratified flow](image)

Figure 2.1: Channel cross-section with regions defining stratified flow. The flow moves in the \(z\)-direction, out of the page.

Through a Fourier sine transformation, Tang & Himmelblau (1963) determined a series solution to the velocity distribution in each phase under the assumptions of laminar flow with a planar interface (neglecting capillary and body forces) in an infinitely long channel. The boundary conditions impose zero velocity at the walls and equal shear at the interface for both fluids. Steinbrenner (2011) reorganized Tang & Himmelblau’s equations in terms of dimensionless variables (appendix A) such that the velocity equals a dimensional scaling factor multiplied by a dimensionless function of only \(\mu, h_{ratio}\), and \(AR\) where \(\mu = \mu_L/\mu_G\). Integrating the velocity in each phase over the cross-sectional area gives the volumetric flow rate of each phase. Taking the ratio of the volumetric flow rates will eliminate the scaling factor, leaving \(f(\mu, h_{ratio}, AR)\). Setting the fluid pair and the channel geometry fixes the \(AR\) and \(\mu\). Therefore, the ratio of volumetric flow rates only depends on \(h_{ratio}\) (saturation). Experimenters set the volumetric flow rates, fluid pairs, and channel geometry, allowing for the calculation of \(h_{ratio}\) for a given experiment without further measurement.
2.3 Experimental Method

The experimental work consists of air-water tests in a rectangular microchannel. The calculation of different properties relies on standard fluid properties of humid air and water at 20°C shown in table 2.2. Four different liquid flow rates of 177 µL/hr, 1.77 mL/hr, 59.07 µL/min, and 590.7 µL/min produce superficial liquid velocities of $5.0 \times 10^{-5}$, $5.0 \times 10^{-4}$, $1.0 \times 10^{-3}$, and $1.0 \times 10^{-2}$ m/s, respectively. The gas flow rates vary from 30 and 50–325 mL/min in 25 mL/min increments producing superficial gas velocities between 0.51 and 5.50 m/s. Characterizing the flow in terms of Reynolds numbers gives a $Re_L$ of 0.0108, 0.0277, 0.55, and 5.55 with gas Reynolds numbers varying between 18.2 to 197 for each liquid Reynolds number. The combination of Reynolds numbers produce a liquid-only Reynolds number between 0.35 and 9.19.

Table 2.2: Fluid properties.

<table>
<thead>
<tr>
<th>Property</th>
<th>Air</th>
<th>Water</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (kg/m³)</td>
<td>1.19</td>
<td>998.3</td>
</tr>
<tr>
<td>Viscosity (kg/m·s)</td>
<td>$1.846 \times 10^{-5}$</td>
<td>$1.002 \times 10^{-3}$</td>
</tr>
<tr>
<td>Surface Tension (N/m)</td>
<td>$72.86 \times 10^{-3}$</td>
<td></td>
</tr>
</tbody>
</table>

2.3.1 Experimental Assembly

Figure 2.2 depicts the testing apparatus for this work. The microchannel assembly (figure 2.2A) forms a 3.23 mm by 0.304 mm by 164 mm ($w \times h \times L$) rectangular microchannel, aligned horizontally. The geometry results in a hydraulic diameter of 557 µm. Three materials form the walls of the microchannel: 6061 aluminum forms the base, 304 full-hard stainless steel forms the side walls, and polycarbonate forms the top of the channel. The material properties for air-water flow result in a hydrophilic microchannel with the contact angles shown in table 2.3. In this work, the stated uncertainties are at a 95% confidence interval.
Table 2.3: Contact angle of the different components forming the microchannel.

<table>
<thead>
<tr>
<th>Component</th>
<th>Contact angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum Base</td>
<td>$76^\circ \pm 8^\circ$</td>
</tr>
<tr>
<td>Stainless Steel Sidewall</td>
<td>$82^\circ \pm 7^\circ$</td>
</tr>
<tr>
<td>Polycarbonate Top</td>
<td>$81^\circ \pm 7^\circ$</td>
</tr>
</tbody>
</table>

Through manual milling techniques, an end-mill cut a width of $3.23 \text{mm} \pm 10 \mu \text{m}$ into the stainless steel sheet, forming the channel side walls. The stainless steel has a manufacture stated thickness tolerance of $\pm 15 \mu \text{m}$ about the nominal $0.304 \text{mm}$ thickness. The polycarbonate allows optical assess for visualization of the flow. The microchannel assembly builds off the designs of Cho & Wang (2014b) and Pfund et al. (2000) in which layers form the channel. Forming the microchannel in this manner allows for changing the surface characteristics of the baseplate for different experiments without changing any other parameters of the microchannel. Additionally, the construction could allow the use of different channel plates of different channel dimensions without constructing a new apparatus. For this apparatus, the seal of the channel comes solely from compression generated by twenty-two bolts torqued to $5.65 \text{ N} \cdot \text{m}$ (50 in·lb).
jection occurs 10 mm downstream of the air inlet through a 365µm diameter hole in the aluminum base (figure 2.3a). Therefore, the first 10mm of the channel experiences only single-phase air flow. A syringe pump (New Era Pump System NE-300, figure 2.2B) loaded with 1, 10, or 30mL syringes depending on the flow rate, supplies room temperature (20°C ±2°C) deionized water to the system.

![Diagram of asymmetric water injection](image1)

(a) Diagram of asymmetric water injection.  

![Pressure tap locations](image2)

(b) Pressure tap locations. Dimensions are in millimeters.

Figure 2.3: Detailed diagram of the microchannel assembly.

MKS 100B mass flow controllers inside a Scribner and Associates 850e Fuel-cell Test Station (figure 2.2C) control the air flow from the main air supply within ±20mL/min. The air passes through a bubble humidifier containing 1500mL of DI-water to achieve 100% relative humidity (figure 2.2D) before entering the microchannel. A 1cm diameter hole acts as the inlet manifold while a 1.4cm diameter hole acts as an outlet manifold. The minimum straight distance between the edge of the manifolds defines the channel length.

### 2.3.2 Pressure Measurement

The measurement of the two-phase pressure drop, the primary experimental objective, requires the selection of pressure taps and pressure transducers. A Setra 230 differential pressure transducer (figure 2.2E) with a range of ±0.5psi (3.447kPa) takes the difference between two pressure taps in the microchannel with an accuracy of ±0.0025psi (17.2 Pascals). The pressure taps align with the channel centerline and consist of 365µm holes drilled...
through the optical plate to a depth of 1.27mm before expanding to a connecting line of 1.58mm diameter. The design of the taps follow the recommendations of Shaw (1960) to minimize static pressure error for the tap diameter and for the tap depth before expansion to a connecting line. The measured pressure difference occurs over a 154mm length of the channel, with one tap located at the entrance ($z = 0$mm) and another one located 12mm before the exit ($z = 152$mm) as shown in figure 2.3b.

### 2.3.3 Data Acquisition

The Setra 230 has a manufacturers stated response time of 2ms. This corresponds to 500 Hz. A digital filter (Alligator USBPGF-S1) filters the pressure transducer’s output at 700 Hz before a data acquisition card (DATAQ DI-245) logs the signal at 2000 samples per second. To maximize the dynamic range of the data acquisition card, a precision buck and gain amplifier subtracts out the mean voltage before amplifying the signal by a factor of 100.

### 2.3.4 Visualization and Saturation Measurements

In this work, the flow forms as a stratified flow and visualizing the flow formation allows for the determination of the liquid saturation. Several techniques exist to visualize the flow: laser-induced fluorescence (Steinbrenner, 2011), Schlieren (Chinnov et al., 2016), and shadowography based on back-lighting. This work applies a simplified approach relying on the shadow generated by the interface of the two phases. Unlike shadowography, this method does not utilize back-lighting allowing the use of any material for the base of the channel. Additionally, the method does not require seeding the fluid nor the expensive equipment required to perform LIF. The clear polycarbonate sheet forming the top of the channel allows optical access for a DSLR camera (Canon Rebel T3) to capture images of the entire channel length in 5 second increments. The surrounding lighting and the color of the base
need to allow for sufficient contrast between the shadow of the interface and its surroundings. Fortunately, standard room lighting provides enough contrast between the interface and the aluminum base.

Extracting the saturation from the images relies on an image processing algorithm in MATLAB. The process starts with the raw image (figure 2.4a). Cropping the image and converting the raw image to gray scale (figure 2.4b) limits the number of pixels to analyze and the single pixel values make the image easier to work with. The assigned pixel number inverts the image in figure 2.4b—note the compressed aspect ratio of the channel. The application of a Laplacian of Gaussian filter with a 5 × 5 kernel and standard deviation of 0.5 determines the location of intensity gradients—specifically the intensity gradient at the air-water interface. Along the image boundaries, the kernel will reference values outside the image boundaries. At the boundaries, the filter assumes the pixels outside the boundary have an intensity value equal to the intensity of the closest boundary pixel. The Laplacian part of the filter acts to reduce noise. Applying a Wiener filter with a region of 2 × 20 pixels further reduces the noise. Plotting the contour matrix of the filtered pixel intensities allows for the determination of the contour level corresponding to the interface. Extracting the contour values gives the location of the pixels in terms of the pixel number. By defining a coordinate system, the pixel numbers are converted to physical units (figure 2.4c). The lens introduces a curvature to straight lines (fisheye effect) clearly visible at the bottom of figure 2.4b and thus requires correction. A quadratic function well approximates the curved wall, allowing for the correction of the film location. The application of R-Loess regression converts the individual points into a continuous line (figure 2.4d). While the film appears to contain small oscillations, this results from the compression of the image aspect ratio and partially due to the selection of points by the R-Loess regression.

As with any experimental technique, the imaging method has strengths and limitations. While the method’s simplicity provides an advantage over other methods, the ability to
image the entire length and width of the channel proves the greatest strength. By providing a global behavior, one can identify regions of interest to study in further detail and quickly identify differences between flow behavior in different experiments. However, by imaging the entire channel, the pixel count becomes an issue. When converting the pixels to physical units, the number of pixels determines the smallest scale the method can resolve, anything smaller gets lost in a given pixel. Therefore, the more pixels, the more precise the scales. Using a higher resolution camera would improve the methods accuracy. Unlike LIF or Schlieren which generate sharp boundaries, the current method relies on lighting to generate the shadow/contrast. Therefore, one must take proper care to ensure proper lighting to generate a uniform shadow. Finally, the method only allows for two-dimensional, top-down imaging of the film. Particularly, the method cannot account for the three-dimensional curvature of the interface between the two fluids. Instead, the three-dimensional features get smeared into a two-dimensional shadow. As a result, the film will appear larger than its actual size—a limitation shared by both LIF and Schlieren.

The film thickness, shown in figure 2.5, where the water lies between the x-axis and the black line, varies over the length of the channel. Therefore, for comparison to Steinbrenner (2011) and for use in the relative gas permeability models, an equivalent film thickness is defined. Trapezoidal numerical integration of the film thickness data gives the area of water in the channel. By dividing the area of water by the length of the film gives an equivalent film thickness—essentially a film that has the same area as the experimental measurement but produces a flat interface between the fluids. Eighty pixels compose the width of the channel (3.23mm), meaning each pixel represents 0.04mm. The Wiener filter smooths the data over two pixels and the location of the wall can fall within two pixels as well. The film thickness ratio \( h_{ratio} \) equals the location of the film minus the location of the wall, all divided by the channel width. Through the Kline-McClintock method, the uncertainty of the film thickness equals 0.11mm or equivalently 0.035 for the \( h_{ratio} \). The propagation of the error through the numerical integration to define the equivalent film thickness gives the same uncertainty,
Figure 2.4: Image analysis process to extract the water film thickness shown for $U_G = 1.69$ m/s and $U_L = 1.0 \times 10^{-2}$ m/s.
neglecting the uncertainty of the film length.

Figure 2.5: Water film for experimental conditions of $U_L = 1.0 \times 10^{-3}$ m/s and $U_G = 3.81$ m/s.

2.4 Results and Discussion

2.4.1 Single-phase Validation

The variability in two-phase pressure correlations shown in §2.2.2 prevents validation of the experimental set-up with two-phase flow. Therefore, single-phase gas flow experiments were conducted for validation. Instead single-phase gas experiments validate the set-up. For single-phase flow, equation 2.2 still holds with $G$ replaced by $\rho_G U_G$ and $\rho_{tp}$ replaced by $\rho_G$. Equation 2.4 still defines the friction factor with $Re_{tp}$ replaced by $Re_G$. For single-phase flow in a rectangular duct, the Darcy friction factor depends on the aspect ratio of the channel as:

$$C = 96(1 - 1.35532\alpha^* + 1.9467\alpha^{*2} - 1.7012\alpha^{*3} + 0.9564\alpha^{*4} - 0.2537\alpha^{*5}) \quad (2.71)$$
given by Kakac et al. (1987) from fitting the exact solutions of Shah & London (1971) for different aspect ratios ($\alpha^*$). In this case, the aspect ratio ($\alpha^*$) equals the smallest dimension divided by the largest dimension. Figure 2.6 shows the comparison between the experimentally measured pressure drop and the theoretical value. The experiments fall within ±4% for all experiments except for the two lowest. At 0.51m/s and 0.85m/s, the measurements fall below the predicted value by 17% and 7%, respectively. The error bars for pressure in figure 2.6 accounts for the ±17.2Pa accuracy of the pressure transducer. The superficial gas velocity equals the volumetric flow rate of gas divided by the cross-sectional area. Utilizing the Kline-McClintock method for the equation $U_G = Q_G/A_c$, gives a velocity uncertainty of ±0.34m/s at $U_G = 0.51$m/s to ±0.43m/s at $U_G = 5.5$m/s.

![Figure 2.6: Comparison of theoretical and experimental single-phase gas pressure drop versus superficial gas velocity.](image)

The location of the first tap ($z = 0$) means that the pressure drop will include entrance effects. To account for the entrance effects, Shah defines an apparent Fanning friction factor...
(Shah, 1978) to replace equation 2.4 as:

\[
f_{f,\text{app}} Re = \frac{3.44}{\sqrt{x^+}} + \frac{(f_f Re) + \frac{k(\infty)}{4x^+} - \frac{3.44}{\sqrt{x^+}}}{1 + C^*(x^+)^{-2}}
\]  

(2.72)

where \(x^+\) defines a dimensionless length scale equal to \(L/(ReD_H)\). The variables \(f_f Re\), \(k(\infty)\), and \(C^*\) depend on the aspect ratio \((\alpha^*)\) with the tabulated values shown in table 2.4. To use equation 2.72 in equation 2.2, requires multiplying equation 2.72 by four to convert to a Darcy friction factor. Since the experimental aspect ratio \((\alpha^* = 0.09)\) falls between the values in table 2.4, the application of equation 2.72 requires interpolation of the values. Comparison of the calculated pressure drop with interpolation to ones calculated at the extremes of \(\alpha^* = 0 \& 0.20\) showed little difference.

Table 2.4: Shah (1978) variables for equation 2.72

<table>
<thead>
<tr>
<th>(\alpha^*)</th>
<th>(k(\infty))</th>
<th>(f_f Re)</th>
<th>(C^*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>1.43</td>
<td>14.227</td>
<td>0.00029</td>
</tr>
<tr>
<td>0.50</td>
<td>1.28</td>
<td>15.548</td>
<td>0.00021</td>
</tr>
<tr>
<td>0.20</td>
<td>0.931</td>
<td>19.071</td>
<td>0.000076</td>
</tr>
<tr>
<td>0.00</td>
<td>0.674</td>
<td>24.000</td>
<td>0.000029</td>
</tr>
</tbody>
</table>

Figure 2.6 also shows a comparison of the experimental data to equation 2.72. The experiments fall within \(\pm 3\%\) of the prediction defined by equation 2.72 for all experiments except for the two lowest. For 0.51m/s and 0.85m/s, equation 2.72 produces negligible change. Therefore, the placement of the tap at the inlet of the channel has minimal influence on the measured pressure drop.

### 2.4.2 Two-phase Pressure Results

Characterizing the two-phase pressure drop relies on the gas two-phase flow multiplier \((\phi_G^2)\). Experimentally determining \(\phi_G^2\) consists of measuring the single-phase pressure drop for gas flow and then measuring the two-phase pressure drop after water injection begins—the two-
phase pressure drop divided by the single-phase gas pressure drop gives $\phi_G^2$. Figure 2.7 shows $\phi_G^2$ versus the superficial gas velocity for the four different liquid velocities. Each experimental data point represents a 30 minute average of the measured two-phase pressure, while the average data points represent the average of all the experimental data points for a given case. The exception being the $U_L = 1.0 \times 10^{-2}$ m/s data set, which consists of 5 minute averages of the measurement. The application of the Kline-McClintock method on the equation $\phi_G^2 = \frac{P_{tp}}{P_G}$ determines the magnitude of the error bars. As the superficial gas velocity increases, the gas two-phase flow multiplier decreases. Increasing the superficial liquid velocity results in an increase in $\phi_G^2$. During the experiments for $U_L = 1 \times 10^{-2}$ m/s for $U_G = 0.51 - 1.27$ m/s the flow forms as plugs, which caused water to enter the pressure taps. Due to the influence of the exit (appendix B), placing the taps outside the channel does not provide a solution. As a result, the analysis excludes data in that range. Of particular note, the majority of the data fall within the measurement uncertainty. Several points however, fall outside the uncertainty range. Under the same experimental conditions, the stratified flow showed differences in film thickness between experiments and thus produced differences in the measured pressure (figure B.6). Additionally, figures 2.7a and 2.7b at $U_G = 0.51 \& 5.08$ m/s show $\phi_G^2$ dropping below 1. These measurements do not make physical sense and the uncertainty in the measurement accounts for this behavior.

The pressure signals themselves provide insight into the behavior of the flow. Figure 2.8 shows a one minute sample of the pressure time trace for $U_G = 5.50$ m/s at $U_L = 1.0 \times 10^{-2}$ m/s. The signal remains steady about the mean, a characteristic shared by all of the experiments. Therefore, after the development of the stratified flow, the mean pressure signal remains unchanged throughout the experiment.

The pressure signal, however, does not remain a constant value; figure 2.8 shows both short and long period oscillations. The relatively high frequency oscillation results from the bubble humidifier based on the frequency of bubble formation. The humidifier introduces a primary
Figure 2.7: Experimental gas two-phase flow multiplier versus superficial gas velocity.

(a) $U_L = 5.0 \times 10^{-5}$ m/s.

(b) $U_L = 5.0 \times 10^{-4}$ m/s.

(c) $U_L = 1.0 \times 10^{-3}$ m/s.

(d) $U_L = 1.0 \times 10^{-2}$ m/s.
frequency of 2 Hz at $U_G = 0.51\text{m/s}$ to 10 Hz at $U_G = 5.50\text{m/s}$. The longer period oscillation results from the mechanical oscillation from the syringe pump. Zeng et al. (2015) showed that pressure oscillations seen in experiments correspond to the mechanical frequency of the syringe pump ($f_m$) defined as:

$$f_m = \frac{4Q_L}{\pi D_s^2 \rho}$$

(2.73)

where $D_s$ equals the syringe diameter and $\rho$ equals the screw pitch of the syringe pump.

For the 10 mL syringe used for the experiment in figure 2.8, at $Q_L = 0.5907 \text{mL/min}$ ($U_L = 1.0 \times 10^{-2} \text{m/s}$), equation 2.73 gives a period of 17.5 seconds. Based on a power-spectral density calculation, the long wavelength in figure 2.8 has a period of 21 seconds. Therefore, equation 2.73 differs by 3.5 seconds but does indicate that the oscillation results from the syringe pump.

![Figure 2.8: Representative pressure time trace shown for $U_G = 5.50 \text{m/s}$ at $U_L = 1.0 \times 10^{-2} \text{m/s}$.](image)

Figure 2.8: Representative pressure time trace shown for $U_G = 5.50 \text{m/s}$ at $U_L = 1.0 \times 10^{-2} \text{m/s}$. 
2.4.3 Evaluation of Existing Two-phase Pressure Models

Section 2.2 presented three approaches to predicting the two-phase pressure drop accompanied by several different correlations derived for a variety of conditions. This section will look at how the experimental data compares to the existing models to reveal which models best predict the experimental data.

Statistical Method for Model Comparison

Determining how well models predict the experimental data relies on statistics of the error. As defined by Li & Hibiki (2017), the error ($\delta P_i$) equals the two-phase pressure drop calculated from the model ($\Delta P_{pre,i}$) minus the experimentally measured pressure drop ($\Delta P_{exp,i}$) for the $i$th experimental datum. Defining the percent error ($\delta^* P_i$) as the error ($\delta P_i$) divided by $\Delta P_{exp,i}$ gives a scale independent comparison. The mean absolute percent error defined as:

$$|\bar{\epsilon}_\%| = \frac{1}{n} \sum_{i=1}^{n} |\delta^* P_i|$$  \hspace{1cm} (2.74)

serves as the primary basis of comparison in this work and the cited works. The mean absolute error prevents individual errors from canceling out and thus gives the best indication of the error. Researchers typically quantify the percentage of data points that fall within a given range (Li & Wu, 2010; Sun & Mishima, 2009; Kim & Mudawar, 2012; Lee & Lee, 2001). Defining the root-mean-square percent error ($\sigma_\%$) as:

$$\sigma_\% = \left( \frac{1}{n} \sum_{i=1}^{n} (\delta^* P_i)^2 \right)^{0.5}$$  \hspace{1cm} (2.75)

will indicate the range of the data. For example, a $\sigma_\% = 15\%$ indicates 68% of the data falls within $\pm 15\%$ or 95% of the data falls within $\pm 30\%$, for normally distributed errors. While equations 2.74 and 2.75 represent scale independent quantities, scale dependent quantities
provide further insight. The mean error ($\bar{e}$) defined as:

$$\bar{e} = \frac{1}{n} \sum_{i=1}^{n} \delta P_i$$  \hspace{1cm} (2.76)$$

where $n$ equals the number of data points, demonstrates the trend in the data. A positive mean error shows the model over-predicts the experimental data, while a negative mean error shows an under-prediction. Taking the mean percent error ($\bar{e}_\%$) defined as:

$$\bar{e}_\% = \frac{1}{n} \sum_{i=1}^{n} \delta^* P_i$$ \hspace{1cm} (2.77)

represents the mean error independent of the scale. Finally, utilizing the root-mean-square error ($\sigma_e$) defined as:

$$\sigma_e = \left( \frac{1}{n} \sum_{i=1}^{n} (\delta P_i)^2 \right)^{0.5}$$ \hspace{1cm} (2.78)

indicates the variation of the error, which helps to determine how well the models collapse the experimental data to the prediction. Equations 2.74, 2.75, 2.76, 2.77, and 2.78 will serve as the basis of comparison between different predictions to understand the differences between models.

Figures plotting the experimentally measured two-phase pressure ($\Delta P_{exp}$) versus the value predicted by the two-phase pressure models ($\Delta P_{pre}$) help reinforce the statistics. The solid line in figures 2.9-2.13 represent a one to one prediction—i.e. the model perfectly predicts the two-phase pressure measurements. Each figure also shows the individual data points for the four superficial liquid velocities. Data above the solid line indicates the model under-predicts the experimental data whereas data below the solid line indicates an over-prediction. Data sets significantly far from the solid line result due to the values predicted by the models under the test conditions of the data set.
Homogeneous Flow Model Comparison

Table 2.5 summarizes the overall error statistics for the homogeneous flow models for all of the data. The models compared poorly to the experimental data by every metric, except for the viscosity models of Dukler et al. (1964) and Beattie & Whalley (1982). The model of Dukler et al. (equation 2.11) under-predicted the two-phase pressure drop (figure 2.9a) with the smallest $\bar{\varepsilon} = -32\,\text{Pa}$ at $U_L = 5.0 \times 10^{-5}\,\text{m/s}$ and increasing to $\bar{\varepsilon} = -102\,\text{Pa}$ at $U_L = 1.0 \times 10^{-2}\,\text{m/s}$. Thus, the predictive accuracy for the model of Dukler et al. decreases as the superficial liquid velocity increases for this experiment even though the mean absolute percent error equals 6.6%. Beattie & Whalley’s model (equation 2.16) initially outperforms equation 2.11 having a $\bar{\varepsilon} = -30\,\text{Pa}$ at $U_L = 5.0 \times 10^{-5}\,\text{m/s}$ to a $\bar{\varepsilon} = -11\,\text{Pa}$ at $U_L = 1.0 \times 10^{-3}\,\text{m/s}$ with a smaller scatter in this range (a $\sigma_{\varepsilon}$ of 64Pa versus 77Pa on average). At $U_L = 1.0 \times 10^{-2}\,\text{m/s}$, Beattie & Whalley’s model over-predicts the two-phase pressure (figure 2.9b) with $\bar{\varepsilon} = 420\,\text{Pa}$ ($|\varepsilon_{\%}| = 30\%$), leading to the overall root-mean-square error of 230 Pa. Based on the analysis, the viscosity model of Dukler et al. produces too low of a two-phase viscosity while Beattie & Whalley’s two-phase viscosity increases too rapidly at higher liquid superficial velocities.

Table 2.5: Overall error statistics of the homogeneous flow models.

| Model                          | $\bar{\varepsilon}$ (Pa) | $\sigma_{\varepsilon}$ (Pa) | $\varepsilon_{\%}$ | $\sigma_{\%}$ | $|\varepsilon_{\%}|$ |
|--------------------------------|--------------------------|-----------------------------|---------------------|--------------|-------------------|
| Dukler et al. (1964)           | -62.7                    | 89.07                       | -2.2%               | 8.2%         | 6.6%              |
| Beattie & Whalley (1982)       | 102.2                    | 230                         | 11.1%               | 20.5%        | 14.3%             |
| Fourar & Bories (1995)         | 350                      | 512                         | 32.6%               | 42%          | 33%               |
| McAdams et al. (1942)          | 834                      | 1447                        | 70%                 | 110%         | 71%               |
| Lin et al. (1991)              | 1616                     | 2824                        | 133%                | 217%         | 134%              |
| Awad & Muzychka (2008)         | 2445                     | 4117                        | 201%                | 305%         | 202%              |
| Cicchitti et al. (1960)        | 18016                    | 27044                       | 1531%               | 1956%        | 1531%             |
Figure 2.9: Comparison between the experimental and predicted two-phase pressure drop for homogeneous flow models.

**Separated Flow Model Comparison**

The separated flow models fall into two categories: \( C \)-values that remain constant and correlations where \( C \) varies with flow conditions. Of the correlations that produce a constant \( C \)-value for a given fluid pair and geometry, the correlation of English & Kandlikar (2006) predicted the data the best (table 2.6). The model of English and Kandlikar over-predicted the two-phase pressure with the mean error increasing from an \( \bar{e} = 2.09 \text{Pa} \) at \( U_L = 5.0 \times 10^{-5} \text{m/s} \) to \( \bar{e} = 364 \text{Pa} \) at \( U_L = 1.0 \times 10^{-2} \text{m/s} \) (figure 2.10d). The other correlations shown in table 2.6 also over-predict the two-phase pressure for all superficial liquid velocities with the error increasing as the superficial liquid velocity increases (figure 2.10). Statistically speaking, the homogeneous flow model of Dukler *et al.* (1964) out performed the separated flow model of English & Kandlikar (2006) for this experiment as equation 2.25 could not collapse the data over the range of test conditions (\( \sigma_e = 202 \text{Pa} \)). Therefore, developing a correlation that does not account for the flow conditions will not accurately predict the experimental data.
Figure 2.10: Comparison between the experimental and predicted two-phase pressure drop for Chisholm correlations producing constant $C$-value.
By allowing the $C$-value to vary with flow conditions, the correlations statistically performed far better than the homogeneous model or correlations with constant $C$-values (table 2.7). Lee & Lee (2001) derived equation 2.38 accounting for a slug velocity. Since the flow in this work formed as a stratified flow, the slug velocity does not apply and instead was replaced by the superficial gas velocity. Saisorn & Wongwisess (2010) replaced the slug velocity with the total superficial velocity. For this experiment, using the total superficial velocity negligibly changed the $C$-value, due to the low superficial liquid velocities. The correlations of Kim & Mudawar (2012) and Lee & Lee (2001) behave nearly identically (table 2.7), deviating at $U_L = 1.0 \times 10^{-2} m/s$ with $\bar{e} = -57 Pa$ and $\bar{e} = -50 Pa$, respectively. Visually the difference becomes indistinguishable (figures 2.11d and 2.11e). Although undiscernible in the figures, the models show an increasing mean error as the superficial velocity increases with $\bar{e} = -22 Pa$ to $\bar{e} = -44 Pa$ from $U_L = 5.0 \times 10^{-5} m/s$ to $U_L = 1.0 \times 10^{-3} m/s$.

The correlations of Ma et al. (2010) and Li & Hibiki (2017) also well predict the experimental data. Unlike Lee & Lee and Kim & Mudawar, Ma et al. (2010) initially under-predicts the data ($\bar{e} = -19 Pa$ to $-15 Pa$) before over-predicting the data at $U_L = 1.0 \times 10^{-2} m/s$ with $\bar{e} = 136 Pa$ (figure 2.11b). This explains the low mean percent error ($\bar{e}_\% = 0.9\%$) but a large mean absolute percent error ($|\bar{e}_\%| = 5.0\%$). Li & Hibiki (2017) generally over-predicts the data. Initially, the correlation only differs by $\bar{e} = 10 Pa$ at the lowest $U_L$ but increases to $\bar{e} = 219 Pa$ at $U_L = 1 \times 10^{-2} m/s$. The variation with $U_L$ leads to a $\sigma_e$ of 125.5Pa, with the data at $U_L = 1.0 \times 10^{-2} m/s$ consistently different than the prediction (figure 2.11c).
The work of Li & Hibiki (2017) requires further discussion. Figure 2.7 shows that the gas two-phase flow multiplier decreases with increasing superficial gas velocity. The correlation of Li & Hibiki (2017) produces varying trends for $\phi_G^2$ depending on the superficial liquid velocity. For $U_L = 1.0 \times 10^{-3}$ and $1.0 \times 10^{-2}\text{m/s}$, the trend of $\phi_G^2$ agrees with the experimental trend. However, equation 2.33 results in an increasing $\phi_G^2$ with increasing $U_G$ at $U_L = 5.0 \times 10^{-5}\text{m/s}$. Conversely, at $U_L = 5.0 \times 10^{-4}\text{m/s}$, the correlation predicts $\phi_G^2$ decreasing until $U_G = 3.81\text{m/s}$ and increasing thereafter. Li & Hibiki (2017) arrived at the correlation for flow boiling. Kim & Mudawar (2012) noted that boiling flows behave differently than condensing and adiabatic flows, meaning flow boiling should have its own specific correlations. Even though the model of Li & Hibiki (2017) reasonably predicted the two-phase pressure, the trend did not align with the experimentally observed behavior likely due to the correlation being based on experimental data for flow boiling.

Table 2.7: Overall error statistics of separated flow models using correlations producing variable $C$-values.

| Model                        | $\bar{e}$ (Pa) | $\sigma_e$ (Pa) | $\overline{\bar{e}}\%$ | $\sigma_\%$ | $|\bar{e}|\%$ |
|------------------------------|-----------------|-----------------|------------------------|-------------|--------------|
| Kim & Mudawar (2012)         | -40.1           | 55.0            | -3.9%                  | 5.6%        | 4.2%         |
| Lee & Lee (2001)             | -37.7           | 52.3            | -3.8%                  | 5.6%        | 4.1%         |
| Ma et al. (2010)             | 25.1            | 81.31           | 0.9%                   | 6.5%        | 5.0%         |
| Li & Hibiki (2017)           | 73.4            | 125.5           | 4.2%                   | 8.5%        | 6.7%         |
| Sun & Mishima (2009)         | 337.4           | 497.1           | 26.9%                  | 34.7%       | 27.0%        |
| Saisorn & Wongwises (2010)   | 1203.5          | 1718.6          | 97.6%                  | 120.8%      | 97.6%        |

Comparing the models in table 2.7 can give guidelines to correlating the two-phase pressure drop based on separated flow. Equations 2.28, 2.30, 2.33, and 2.38 all contain a form of the Reynolds numbers. For the correlations of Lee & Lee (2001) and Kim & Mudawar (2012), only the liquid only Reynolds number varies for different $U_L$. For Ma et al. (2010), both the liquid Reynolds number and liquid Capillary number vary. All terms in equation 2.33 vary for Li & Hibiki (2017) but the two-phase Reynolds number significantly influences the resulting $C$-value. The correlation of Sun & Mishima (2009) contains a liquid Reynolds number but does not well predict the experimental data (figure 2.11a). Consequently, other terms
Figure 2.11: Comparison between the experimental and predicted two-phase pressure drop for Chisholm correlations that vary based on flow conditions.
must influence the correlation but the correlation starts with the Reynolds number. Each of the above correlations have a term accounting for surface tension—Capillary number, Suratman number, a combination of both, or the viscosity number. While equation 2.27 (Sun & Mishima, 2009) contains the Confinement number, this term acts to modify the exponent, not the $C$-value directly. The current experiment only tested air-water flow, fixing the Suratman number and the density ratio (in equation 2.30). To clarify the best term to use to account for surface tension requires testing multiple fluid pairs to vary the surface tension while maintaining the same Reynolds number range. Based on how well it compared to the experimental data, equation 2.38 (Lee & Lee, 2001) serves as a good starting point for developing correlations for low Reynolds number flows but requires further investigation. Specifically, the exponents in equation 2.38 did not change from the original paper. Although done for tube flows, Saisorn & Wongwises (2010) changed the exponents and constant in equation 2.38. The resulting correlation (equation 2.39) poorly predicted the experimental data (table 2.7, figure 2.11f). The determination of the universality of the exponents at low Reynolds numbers requires comparisons to experimental data taken over a wide range of fluid pairs and channel geometries.

Relative Permeability Model Comparison

Using equation 2.50 with the X-Model outperformed the other relative permeability models based on the experimentally measured saturation (table 2.8). The X-Model over-predicted the two-phase pressure drop (figure 2.12f) for the three lowest liquid superficial velocities with a mean error of 6.4, 10.9, and 9.23Pa, respectively. At $U_L = 1.0 \times 10^{-2}$ m/s, the X-model under-predicts the data by -26.9Pa. The model proposed by Corey (1954) performs the second best of the relative permeability models. However, the model of Corey (1954) shows an increasing trend in over-predicting the data ($\tau = 38.5$ to 338.7Pa) with increasing superficial liquid velocity (figure 2.12a). The other models also over-predict the two-phase
pressure drop and show $\sigma_e$ increasing as the superficial liquid velocity increases (figure 2.12), including the model of Fourar & Bories (1995) using the accompanying saturation model (equation 2.65). Additionally, the models of Fourar & Lenormand (1998) and Huang et al. (2009) behave identically due to the small viscosity ratio (table 2.8).

Physically, the relative permeability exponent of 1 for the X-Model means the two fluids minimally impede one another (Chen et al., 2004). In stratified flow the gas essentially flows along a channel equal to the channel width minus the water film thickness; the water flows in a channel equal to the water film thickness. Thus, the fluids only interact at the air-water interface. The other relative permeability models with an exponent greater than 1 thus over-predict the interaction of the phases resulting in the over-prediction of the two-phase pressure drop. Since the X-model slightly under-predicts the experimental data while the model of Corey (1954) over-predicts the data, the $n_k$ value for the two-fluid model should fall between a value of 1 used by the X-Model and 2 used to leading order by the model of Corey (1954).

Table 2.8: Overall error statistics of the relative permeability models.

|                | $\bar{\varepsilon}$ (Pa) | $\sigma_e$ (Pa) | $\bar{\varepsilon}_\%$ | $\sigma_\%$ | $|\bar{\varepsilon}_\%|$ |
|----------------|--------------------------|----------------|-------------------------|-------------|---------------------------|
| X-Model        | -1.1                     | 36.2           | 0.64%                   | 4.79%       | 3.32%                     |
| Corey (1954)   | 145.8                    | 195.1          | 14.2%                   | 17.8%       | 14.3%                     |
| Chen et al. (2004) | 187.5              | 229.6          | 17.6%                   | 20.2%       | 17.6%                     |
| Fourar & Lenormand (1998) | 266.2            | 356.3          | 25.3%                   | 31.0%       | 25.3%                     |
| Huang et al. (2009) | 266.2            | 356.3          | 25.3%                   | 31.0%       | 25.3%                     |
| Nowamooz et al. (2009) | 285.1            | 383.0          | 27.1%                   | 33.2%       | 27.1%                     |
| Fourar & Bories (1995) | 344.3            | 505.1          | 27.5%                   | 35.3%       | 27.6%                     |
Figure 2.12: Comparison between experimental and predicted two-phase pressure drop for relative permeability models using experimentally measured saturation.
2.4.4 Determination of $n_k$ for Stratified Flow

The previous section detailed a comparison between existing models and the experimental data. The $C$-value correlations of Kim & Mudawar (2012) and Lee & Lee (2001) predicted the experimental data the best out of the homogeneous and separated flow models with a mean absolute percent error of 4.2% and 4.1%, respectively. Overall, the X-Model produced the lowest mean absolute percent error (3.32%) of the compared models. A new model cannot improve the result too much and will only prove beneficial if the model reduces the complexity of determining the two-phase pressure drop. Looking at the $C$-value equations of Lee & Lee (2001) and Kim & Mudawar (2012), one could optimize the equation by modifying the leading constant or one of the three exponents. The two-fluid model on the other hand only requires a single correlation parameter. Although straightforward to use, the X-model relies on experimentally measured saturation, which proves difficult to measure. As the two-fluid model depends only on a single correlation constant and models the saturation, the two-fluid model simplifies the prediction of the two-phase pressure drop.

Determining $n_k$ from Two-phase Pressure

Cho & Wang (2014b) previously determined $n_k$ values for slug, wavy, and annulus flow patterns in a regular fuel cell channel. The determination of an $n_k$ value for stratified flow follows. In the experiments water did not remain in the channel after water injection stopped; thus, the residual water ($s_{L,r}$) equals zero. This simplifies equation 2.70 and reduces the effective saturation to the liquid saturation. This leaves only the determination of the correlation constant, $n_k$.

Table 2.9: Overall error statistics for the optimized $n_k$ value.

| $n_k$ = 1.159 (Current Study) | $\tau$ (Pa) | $\sigma_\tau$ (Pa) | $\tau_\%$ | $\sigma_\%$ | $|\tau_\%|$ |
|------------------------------|-------------|---------------------|----------|----------|----------|
| -11.91                       | 41.63       | -1.58%              | 4.72%    | 3.25%    |
Optimizing the entire data set to minimize the variance between the experimental data and the prediction led to an $n_k$ value of 1.159. Table 2.9 shows the error statistics for the optimization. The $n_k$ value resulted in an under-prediction of the two-phase pressure by a mean error of $-11.91\text{Pa}$ and produces a mean absolute percent error of 3.25%. Since the optimization minimizes the variance, the standard deviation of the error represents the smallest possible value for this optimization and thus well collapses the data (figure 2.13). Tables 2.10 and 2.11 show the error statistics for each superficial liquid velocity correlated by $n_k = 1.159$. Based on the mean absolute percent error, the selection of the $n_k = 1.159$ agrees best at the lowest and highest superficial liquid velocity. The optimized $n_k$ value under-predicts the majority of the data, but over-predicts the data at $U_L = 1.0 \times 10^{-2}\text{m/s}$ with a mean error of 21.92Pa.

Figure 2.13: Comparison between experimental and predicted two-phase pressure drop for the optimized $n_k$ value using the two-fluid model.

The change from under-predicting to over-predicting suggests the $n_k$ value could vary with test parameters. To understand the variation requires investigating the optimization of the $n_k$ value for each superficial velocity data set. Tables 2.10 and 2.11 show the error statistics of the optimized $n_k$ value for each data set compared to $n_k = 1.159$. The comparison to
Table 2.10: Comparison between different $n_k$ values for the two lowest $U_L$.

<table>
<thead>
<tr>
<th>$n_k$</th>
<th>$U_L = 5.0 \times 10^{-5}$ m/s</th>
<th>$U_L = 5.0 \times 10^{-4}$ m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{e}$ (Pa)</td>
<td>1.159</td>
<td>1.675</td>
</tr>
<tr>
<td>$\sigma_e$ (Pa)</td>
<td>-20.31</td>
<td>0.95</td>
</tr>
<tr>
<td>$\bar{e}^%$</td>
<td>-2.84%</td>
<td>0.20%</td>
</tr>
<tr>
<td>$\sigma^%$</td>
<td>4.29%</td>
<td>2.85%</td>
</tr>
<tr>
<td>$</td>
<td>\bar{e}^%</td>
<td>$</td>
</tr>
</tbody>
</table>

Table 2.11: Comparison between different $n_k$ values for the two highest $U_L$.

<table>
<thead>
<tr>
<th>$n_k$</th>
<th>$U_L = 1.0 \times 10^{-4}$ m/s</th>
<th>$U_L = 1.0 \times 10^{-2}$ m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{e}$ (Pa)</td>
<td>1.159</td>
<td>1.363</td>
</tr>
<tr>
<td>$\sigma_e$ (Pa)</td>
<td>-28.45</td>
<td>7.35</td>
</tr>
<tr>
<td>$\bar{e}^%$</td>
<td>-3.48%</td>
<td>0.55%</td>
</tr>
<tr>
<td>$\sigma^%$</td>
<td>5.77%</td>
<td>4.35%</td>
</tr>
<tr>
<td>$</td>
<td>\bar{e}^%</td>
<td>$</td>
</tr>
</tbody>
</table>

$n_k = 1.159$ shows the opposite trend in the mean error, in which the model over-predicts the data but at $U_L = 1.0 \times 10^{-2}$ m/s the model under-predicts the data. Comparison of the other error statistics show a modest improvement.

The optimization at $U_L = 5.0 \times 10^{-4}$ m/s stands as an outlier. Optimization about the mean $\phi^2_G$ for each superficial gas velocity, instead of the individual data points, did not change the $n_k$ value. Conversely, by removing the data for $U_G = 0.51$ to 1.27 m/s, the $n_k$ value changed by $-0.065$, $0.135$, and $-0.037$, for the $U_L$ between $5.0 \times 10^{-5}$ and $1.0 \times 10^{-3}$ m/s, respectively. The uncertainty at low superficial gas velocity at $U_L = 5.0 \times 10^{-4}$ m/s, particularly at $U_G = 0.51$ m/s that shows several points at the low end of the uncertainty (figure 2.7b), causes the $n_k$ value to not follow the overall trend of $n_k$ decreasing with increasing $U_L$. For $U_L = 5.0 \times 10^{-4}$ m/s, the $n_k$ equals 1.436 when neglecting the data at $U_G = 0.51$ m/s.

Additionally, the theoretical velocity profiles determined by Tang & Himmelblau (1963) and analyzed by Steinbrenner (2011) provide further insight (appendix A). Steinbrenner showed that the thick water films have a parabolic profile characteristic of pressure driven
flows whereas thin water films have a linear profile characteristic to a shear driven flow. Therefore, the influence of water on the air changes, which could account for the variation of $n_k$ between the data sets (tables 2.10 and 2.11) and the prediction trend of the X-model. Nonetheless, attempts to correlate the change in $n_k$ with dimensionless parameters showed no clear correlation. Therefore, the $n_k$ value should equal 1.159 for stratified flow and remain constant. The universality of $n_k = 1.159$ can only be known as researchers add experimental work for various geometries and fluid pairs that produce stratified flow.

**Comparison of determined $n_k$ to Measured Water Film Thickness**

The saturation model defined in equation 2.70 has not previously been validated against saturation measurements. Figure 2.14 compares the experimental saturation and the saturation calculated using equation 2.70 with $n_k = 1.159$. The behavior of the saturation follows the mean error trend of the two-phase pressure shown in tables 2.10 and 2.11. From $U_L = 5.0 \times 10^{-5}$m/s to $U_L = 1.0 \times 10^{-3}$m/s, equation 2.70 under-predicts the saturation but over-predicts the saturation at $U_L = 1.0 \times 10^{-2}$m/s. This trend explains the differences between the optimized $n_k$ value and the X-model. To account for the under-prediction of the experimentally measured saturation used in the X-model, the two-fluid model would have to predict a greater interaction of the fluids to compensate for the difference. Conversely, the experimental method over approximates the thickness of the film due to the inability to account for the curvature of the interface, which would allow the X-model to have a lower relative permeability exponent than the two-fluid model. Overall, the predicted saturation of the two-fluid model produces a mean error of $-0.035$ with a root-mean-square error of 0.045 compared to the experimental data.

Figure 2.14 also shows the comparison between the experimental data, the analytical solution presented by Steinbrenner (2011), and the saturation model of Fourar & Bories (1995). The experimental data follow the same trend as the analytical solution, the $h_{ratio}$ decreases with
increasing ratio of volumetric flow rates. Additionally, the data consistently fall above the analytical solution, as expected due to the inability to account for the curvature of the interface. Steinbrenner (2011) showed similar characteristics when comparing experimental measurements to the analytical solution. The two-fluid model produces a higher saturation at low volumetric flow rate ratios and a lower saturation at higher volumetric flow rate ratios compared to Steinbrenner (2011) but follows the overall trend. The saturation model of Fourar & Bories (1995) defined by equation 2.65 under-predicts the experimental data, the analytical solution, and the two-fluid model. Fourar & Bories (1995) derived equation 2.65 based on plane-poiseuille flow, which does not well represent stratified flow when the flow forms between the narrow height of the channel instead of across the channel width.

Figure 2.14: Comparison between the experimental and expected solutions of the dimensionless water film thickness.
2.5 Concluding Remarks

This chapter discussed an experimental and modeling study of air-water two-phase frictional pressure drop in a microchannel of dimensions 3.23mm wide by 0.304mm high by 164mm long. The test conditions produced liquid Reynolds numbers between 0.0108 and 5.55 with a corresponding gas Reynolds number ranging between 18.2 and 197. Characterizing the measured two-phase pressure drop in terms of the gas two-phase flow multiplier ($\phi^G$) resulted in values between 1 and 1.44, where $\phi^G$ increased with increasing superficial liquid velocity and decreased with increasing superficial gas velocity. Comparing the two-phase pressure drop predicted by several models showed the correlations for the separated flow model proposed by Lee & Lee (2001) and Kim & Mudawar (2012) predicted the experimental two-phase pressure with mean absolute percent errors ($|\epsilon|\%\$) of 4.1% and 4.2%, respectively. Of the selected relative permeability models, the X-Model predicted the experimental data with a $|\epsilon|\%\$ of 3.32%. Other homogeneous, separated, and relative permeability models produced a $|\epsilon|\%\$ greater than 5%.

While existing models accurately predicted the two-phase pressure drop, the models rely on multiple correlation parameters and experimental measurements of saturation. The two-fluid model (Wang, 2009) can reduce the complexity in determining the two-phase pressure drop as the model relies solely on a relative permeability exponent ($n_k$) and models the liquid saturation to predict the two-phase pressure drop. The two-fluid model predicted the experimental two-phase pressure drop with a $|\epsilon|\%\$ of 3.25% when using the new $n_k$ value of 1.159 in the relative permeability.

This chapter also detailed an experimental and modeling study of the water film thickness. Imaging of the flow showed that the air-water experiments produced stratified flow for the range of test conditions. Measurements of the water film thickness show a decreasing trend with an increase in the ratio of air to liquid volumetric flow rate. The two-fluid model using
\( n_k = 1.159 \), predicted the experimentally measured water film thickness with a mean error of -0.035, following the trend of the analytical solution of Steinbrenner (2011). Based on the analysis, the two-fluid model applies to thin microchannels for predicting both the two-phase pressure and the water film thickness.
Chapter 3

Two-phase Frictional Pressure Drop in a Thin Mixed-wettability Microchannel

3.1 Abstract

This study focuses on the experimental investigation of the two-phase pressure drop in a thin mixed-wettability microchannel. Air-water flow in the thin channel of dimensions 3.23mm wide by 0.304mm high primarily produces rivulet flow. The two-phase pressure drop increases when the base contact angle changed from 76° to 99°, with the other walls remaining the same. Placing the result in the context of existing works demonstrates the trend of the two-phase pressure drop with contact angle depends on the liquid capillary number for adiabatic flow in a single mixed-wettability microchannel. The critical capillary number falls in the range of $1.38 \times 10^{-4}$ to $9.63 \times 10^{-4}$. Limiting the data to the rivulet flow regime leads to the determination of a new relative permeability exponent of 1.747 in the
two-fluid model, which produces a mean absolute percent error of 14.9%. The instability of the rivulet prevents the two-fluid model and other existing two-phase pressure models from collapsing the data.

### 3.2 Background

#### 3.2.1 Two-phase Pressure and Flow Pattern

Investigations of two-phase flow in microchannels consisting of at least one hydrophobic surface have produced inconsistent findings in terms of the influence surface wettability has on the two-phase pressure drop. Stevens et al. (2017) conducted air-water experiments in a microchannel 9.92mm wide by 360-380\(\mu\)m high. The microchannel consisted of three hydrophilic acrylic surfaces with a contact angle (\(\gamma\)) of 64° and one interchangeable surface. The interchangeable surface consisted of a hydrophilic silicon surface of \(\gamma = 60^\circ\) for the control tests and a superhydrophobic surface for the remaining tests. The superhydrophobic surface consists of parallels ribs 15-20\(\mu\)m in height with differing cavity ratios (ratio of rib surface area divided by the total plate surface area). The superhydrophobic surface had contact angles of 146°, 157°, & 155° in the streamwise direction and 132°, 149°, & 146° in the traverse direction, depending on cavity fraction. The pressure measurements by Stevens et al. (2017) showed little influence of the cavity fractions on the two-phase flow multiplier (\(\phi^2\)) but saw a reduction of 10% in \(\phi^2\)—beyond the 5-15% reduction in the single-phase measurement—relative to the prediction of Kim & Mudawar (2012). The control experiments agreed within a mean absolute percent error within 20% of the prediction of Kim & Mudawar (2012). The gas Reynolds number (\(Re_G\)) varied between 22-215 and the liquid Reynolds number (\(Re_L\)) varied between 55-220, which generated slug flow.
Wang et al. (2014b) also studied the influence of superhydrophobic surfaces on the two-phase pressure, finding inconsistent results. The microchannel had a 4mm square cross-section with a 150mm length consisting of a plexiglass top with the remaining walls formed by graphite with different surface treatments. It remains unclear as to the contact angle of the plexiglass, although typically plexiglass has hydrophilic contact angles. The surface treatment of the graphite produced a contact angle of 35° with silica particles, 145° when treated with PTFE, or 155° when treated with silica combined with PDMS-2. At a superficial liquid velocity ($U_L$) of 0.015m/s with superficial gas velocities ($U_G$) between 2–9m/s, the PTFE treatment resulted in a higher two-phase pressure drop than the silica treatment. The silica-PDMS-2 treatment resulted in the lowest two-phase pressure drop.

Cho & Wang (2014a) investigated two-phase air-water flow in a microchannel of dimensions 1.68mm×1.00mm×150mm with 0.55 ≤ $U_G$ ≤ 9.36m/s and 5.0 × 10^{-5} ≤ $U_L$ ≤ 1.0 × 10^{-3}m/s. The hydrophilic surface has a contact angle of 80° and the smooth hydrophobic PTFE surface had a contact angle of 104°. Identical hydrophilic surfaces formed the remainder of the microchannel in both cases. Contrary to Stevens et al. (2017) and Wang et al. (2014b), the two-phase pressure drop increased as the contact angle increased. Comparison to existing two-phase pressure models showed good agreement between the prediction and the experimental data, improving with increasing $U_L$. When optimizing the relative permeability exponent ($n_k$), Cho & Wang (2014a) found a slight increase from 1.96, 2.15, & 2.49 in the hydrophilic case to 2.47, 2.58, and 2.89 in the hydrophobic case for annular, mixed flow, & slug flow, respectively. In both the hydrophilic and hydrophobic cases, similar flow patterns exist, with a slight redistribution of fluid to the hydrophilic corners in the hydrophobic case. A rough carbon paper with a contact angle 128° also showed a pressure increase but existing two-phase pressure models did not compare well to the experimental data.

Lu et al. (2011) investigated the influence of surface wettability in 8 parallel rectangular channels, 0.4mm deep by 0.7mm wide. Water injection occurs through a gas-diffusion layer.
(GDL) with a contact angle of 138-145°. Different surface treatments on the remaining three walls produce contact angles of 11°, 85°, and 116°. At $U_L = 3.0 \times 10^{-4}$ m/s, the two-phase pressure increased with the contact angle in a range of superficial gas velocities between 0.98 m/s to 15 m/s and became similar for $U_G$ between 15–29.5 m/s. Conversely, at $U_L = 7.5 \times 10^{-4}$ m/s, the two-phase pressure generally decreased as the contact angle increased between $U_G = 0.98$ and 29.5 m/s. The authors note the hydrophilic channel meets the Concus-Finn condition for water to wick into the corners (appendix C), resulting in water movement in the channel as a continuous film instead of being sheared by the air flow, resulting in the slightly higher two-phase pressure.

Unlike the four previous works in which the authors conducted experiments under adiabatic conditions, Phan et al. (2011) conducted flow boiling experiments with different surface wettabilities. Different surface treatments resulted in contact angles of 26°, 49°, 63°, and 103° for the base, with a hydrophilic Pyrex glass top. The microchannel had dimensions 0.5 mm high by 5 mm wide by 180 mm long. Under total mass fluxes of water of 100 and 120 kg/m²·s, the two-phase pressure increased with increasing contact angle but existing two-phase pressure models did not well predict the behavior.

The five aforementioned studies focused on mixed-wettability channels, in which at least one surface had a differing wettability than the remaining three. Several authors have studied homogeneous rectangular channels, where all four walls have the same wetting properties. Wang et al. (2014a) studied 200 µm wide by 100 µm deep microchannels of glass, modified glass, and PDMS that had contact angles of 37°, 94°, and 135°, respectively. Testing at $U_L = 0.08$ and 0.12 m/s produced slug flow for $U_G = 0.05$–0.4 m/s. Tests at $U_L = 0.05$ and 0.18 m/s for $U_G = 1.0$–4.0 m/s produced continuous gas flows, i.e. the gas forms a continuous path from one end of the channel to the other. In both cases, the two-phase pressure decreased as the contact angle increased.

Choi et al. (2011a) also studied homogeneous microchannels with different contact angles for
nitrogen-water flows at $0.066 \leq U_G \leq 34.1\text{m/s}$ and $0.19 \leq U_L \leq 0.46\text{m/s}$ . The rectangular microchannels had cross-sections of 608.6$\mu$m wide by 410$\mu$m high for the hydrophilic channel and 617.2$\mu$m wide by 430.6$\mu$m high for the hydrophobic channel. Bare photosensitive glass of contact angle $25^\circ$ formed the hydrophilic microchannel while a treated photosensitive glass with contact angle $105^\circ$ formed the hydrophobic microchannel. While the hydrophilic case produced bubbly and liquid-ring flows, the hydrophobic case produced stratified flow with and without the entrainment of nitrogen. Consequently, the two-phase pressure decreased as the contact angle increased. Using existing correlations of the Chisholm parameter ($C$), the authors could not predict the two-phase pressure in the hydrophobic case.

Finally, Rapolu & Son (2011) investigated the influence of contact angle on air-water slug flow in square microchannels of dimensions $700\mu\text{m}^2$. The microchannels had contact angles of $25^\circ$, $60^\circ$, $105^\circ$, and $105^\circ$. In the range of liquid volumetric flow rates ($Q_L$) of $1.2 \times 10^{-8}$ to $4.6 \times 10^{-8}\text{m}^3/\text{s}$ and gas volumetric flow rates ($Q_G$) of $1.4 \times 10^{-6}$ to $1.7 \times 10^{-6}\text{m}^3/\text{s}$, the two-phase pressure increased as the contact angle increased. Combined with the other works discussed, the trend of changing contact angle on the two-phase pressure remains unclear.

By changing the contact angle, the two-phase flow patterns typically change as well. Barajas & Panton (1993) studied air-water flow in mini-tubes of 1.6mm diameter with contact angles of $34^\circ$, $61^\circ$, $74^\circ$, and $106^\circ$. The contact angle did not significantly alter the conditions at which slug, bubble, or dispersed flow formed. As the contact angle increased, rivulet flows replaced wavy flows for contact angles above $61^\circ$ and multiple rivulets replaced annular flow above $74^\circ$. Lee & Lee (2008a) found general agreement with Barajas & Panton (1993) but noted that flows could appear similar but have differing interactions with the wall. For example, the authors defined a wet-plug flow in which a thin water film lubricated the gas plugs whereas dry-plug flow does not have the lubricating film and the gas directly contacts the wall. Huh et al. (2009) studied air-water flow in microchannels $300\mu\text{m}$ wide by $100\mu\text{m}$ high. Under a range of gas and liquid superficial velocities, the authors identified 7
distinct flow patterns in the hydrophobic channel with a contact angle of 111°. In contrast, hydrophilic channels of contact angles 35° and 75° produced only two flow patterns; only one flow pattern, the annular-droplet flow, overlapped between the hydrophilic and hydrophobic cases. While the flow can behave similarly, the flow patterns often change with changing contact angle.

3.2.2 Rivulet Stability

Rivulet flows seen by Barajas & Panton (1993) and Lee & Lee (2008a) can become unstable due to perturbations of the flow. When a rivulet becomes unstable, its leading edge breaks apart into periodic droplets, where the wavelength of the instability nearly equals the average drop spacing (Diez et al., 2009), changing the behavior of the flow. Davis (1980) investigated the linear stability of infinite static rivulets on an inclined plane. With fixed contact lines, the rivulet has unconditional stability for all axial wavenumbers \( k \) if the surface has a contact angle \( \gamma \) less than \( \pi/2 \). If the surface contact angle falls in the range \( \pi/2 < \gamma < \pi \), the rivulet will remain stable for \( (kR)^2 > 1 \), where \( R \) is the radius of curvature of the rivulet. If the contact lines can move but the rivulet maintain a constant contact angle, a region of stability exits that does not encompass a complete range of wavenumbers (i.e. for a given wavenumber, instabilities will grow for a surface of contact angle \( \pi/4 \) but become stable for a contact angle of \( \pi/2 \).

Koplik et al. (2006) investigated the linear pearling instability of rivulets confined to a chemical channel, in which a gravitational force drives the liquid rivulet. A chemical channel consists of a wetting surface surrounded by non-wetting surfaces. The authors noted in the static case, instabilities grow for \( \theta > \pi/2 \). The driving force enhances the instability, if the instability also occurred in the static case. The driving force could not destabilize a stable state that exists in the static case.
Herrada et al. (2015) simulated horizontal air-ethanol flow in a microchannel of constant pressure gradient, where rivulets flow along a chemical channel. The surface had contact angles of 80°, 89°, and 120°. The case of $\theta = 80^\circ$ remains stable. However, the 89° and 120° cases demonstrated that the ethanol rivulets surrounded by a gas flow became unstable above a critical Reynolds number. The Weber number shifts the peak growth factor. Both the Reynolds number and Weber number are calculated from the properties of the rivulet and based on an average velocity inside the rivulet. The authors did not note the influence of the velocity or Reynolds number of the air.

Cheverda et al. (2013) conducted experiments of FC-72/nitrogen flows in a 30mm wide by 1.4mm by 67mm long microchannel specifically designed to generate rivulet flow. Under a wide range of liquid and gas Reynolds numbers the authors showed as the liquid Reynolds number increased for a fixed gas Reynolds number the rivulet width increases. Conversely as the gas Reynolds number increased for a fixed liquid Reynolds number the rivulet width decreases. In either case, the surface becomes wavier (less stable).

### 3.3 Two-phase Pressure Drop Models

Prediction methods of the two-phase pressure in hydrophobic channels often rely on models determined for flows in hydrophilic channels. As noted in §2.2, the models typically fall into three categories: homogeneous (table 3.1), separated (tables 3.2 and 3.3), and relative permeability based models (table 3.4). In this work, only friction contributes to the two-phase pressure drop in both the measurements and the models.
3.3.1 Homogeneous Flow Model

The homogeneous model treats the two-phase flow as an equivalent single-phase flow with averaged properties—the two-phase viscosity ($\mu_{tp}$) and the two-phase density ($\rho_{tp}$) shown in table 3.1—that determine the two-phase Reynolds number ($Re_{tp}$) used to determine the friction factor, such that the two-phase pressure equals:

$$\left(\frac{dP}{dz}\right)_{tp} = f_{tp} \frac{G^2}{2D_H\rho_{tp}}$$

(2.2)

where $P$ equals the pressure, $z$ the downstream coordinate, $f$ the Darcy friction factor, $G$ the total mass flux, $D_H$ the hydraulic diameter, and $\rho$ the density. The subscript $tp$ stands for two-phase. The two-phase properties depend on the gas quality ($\chi$), the homogenous void fraction ($\beta$), the fluid viscosities ($\mu$), and the volumetric flow rate ($Q$). The subscripts $G & L$ stand for gas-phase and liquid-phase, respectively.
Table 3.1: Viscosity models for the homogeneous flow model.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Viscosity Model</th>
<th>Test Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>McAdams et al. (1942)</td>
<td>( \frac{1}{\mu_{tp}} = \frac{x}{\mu_G} + \frac{1-x}{\mu_L} )</td>
<td>Benzene-oil, 2.69cm tubes</td>
</tr>
<tr>
<td>Cicchitti et al. (1960)</td>
<td>( \mu_{tp} = \mu_G x + (1-x)\mu_L )</td>
<td>Steam-water, 0.51cm tubes</td>
</tr>
<tr>
<td>Lin et al. (1991)</td>
<td>( \mu_{tp} = \frac{\mu_L \mu_G}{\mu_G + \chi^{1/4} (\mu_L - \mu_G)} )</td>
<td>R-12, 1mm tubes</td>
</tr>
<tr>
<td>Dukler et al. (1964)</td>
<td>( \mu_{tp} = \mu_L (1 - \beta) + \mu_G \beta )</td>
<td>Two-component, 2.54-12.7cm tubes</td>
</tr>
<tr>
<td>Fourar &amp; Bories (1995)</td>
<td>( \mu_{tp} = \frac{(1 - \beta)\mu_L + \beta \mu_G + \sqrt{\beta(1 - \beta) \mu_G \mu_L}}{\mu_G + \chi^{1/4} (\mu_L - \mu_G)} )</td>
<td>Fractures, 0.18-1mm gaps</td>
</tr>
<tr>
<td>Beattie &amp; Whalley (1982)</td>
<td>( \mu_{tp} = \mu_L (1 - \beta)(1 + 2.5\beta)\mu_G \beta )</td>
<td>Multiple fluids</td>
</tr>
<tr>
<td>Awad &amp; Muzychka (2008)</td>
<td>( \mu_{tp} = \mu_G \frac{2\mu_G + \mu_L - 2(\mu_G - \mu_L)(1-x)}{2\mu_G + \mu_L + (\mu_G - \mu_L)(1-x)} )</td>
<td>Refrigerants; 2.46, 2.58mm tubes; 0.148, 1.44mm channels</td>
</tr>
</tbody>
</table>

where

\[ \beta = \frac{\rho_L x}{\rho_L x + \rho_G (1-x)} \]
\[ \rho_{tp} = \left( \frac{x}{\rho_G} + \frac{1-x}{\rho_L} \right)^{-1} \]
\[ \chi = \frac{\rho_G Q_G}{\rho_G Q_G + \rho_L Q_L} \]
### 3.3.2 Separated Flow Model

Instead of averaging the properties of the two phases like the homogeneous flow models, the separated flow model follows the work of Lockhart & Martinelli (1949) and Chisholm (1967) to account for the interaction of the phases. Lockhart & Martinelli (1949) proposed the two-phase pressure equals:

\[
\frac{\Delta P}{\Delta L}_{tp} = \phi^2_G \left( \frac{\Delta P}{\Delta L} \right)_G
\]  

(2.18)

where the gas two-phase flow multiplier \( \phi^2 \) depends on the Lockhart-Martinelli parameter \( X \):

\[
X = \sqrt{\frac{\Delta P_L}{\Delta P_G}}
\]  

(2.19)

in which \( \Delta P_L \) and \( \Delta P_G \) equal the pressure drop experienced along the channel if the respective phase flowed alone in the pipe. Chisholm (1967) proposed:

\[
\phi^2_G = 1 + CX + X^2
\]  

(3.1)

where \( C \), the Chisholm parameter, takes on may forms that remain constant (table 3.2) or vary with flow condition (table 3.3). Chisholm (1967) proposed that \( C = 5 \) for laminar flows of both phases. The \( C \)-value correlations depend on different flow parameters such as the confinement number \( (N_{conf}) \), the Bond number \( (Bo) \) defined as \( 1/N^2_{conf} \), the Capillary number \( (Ca) \), the aspect ratio \( (AR) \), the Suratman number \( (Su) \), the liquid-only Reynolds number \( (Re_{lo}) \), and the two-phase viscosity number as:

\[
N_{\mu_{tp}} = \frac{\mu_{tp}}{\left( \rho_{tp} \sigma \sqrt{\frac{\sigma}{g(\rho_L - \rho_G)}} \right)^{0.5}}
\]  

(2.35)

where \( g \) equals the acceleration due to gravity and \( \sigma \) the surface tension. Equation 2.35 uses the viscosity model of McAdams et al. (1942) and the two-phase density shown in table 3.1.
Table 3.2: Correlations producing a constant $C$-value for the separated flow model.

<table>
<thead>
<tr>
<th>Reference</th>
<th>$C$ Correlation</th>
<th>Test Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mishima &amp; Hibiki (1996)</td>
<td>$C = 21 \left(1 - e^{-0.319D_h}\right)$</td>
<td>Air-water, 1.07–5.00mm gaps</td>
</tr>
<tr>
<td>Zhang et al. (2010)</td>
<td>$C = 21 \left(1 - e^{-0.674/N_{conf}}\right)$</td>
<td>$Re_G, Re_L &lt; 2000$</td>
</tr>
<tr>
<td>English &amp; Kandlikar (2006)</td>
<td>$C = 5 \left(1 - e^{-0.319D_h}\right)$</td>
<td>Air-water, 1.124 × 0.93mm²</td>
</tr>
<tr>
<td>Li &amp; Wu (2010)</td>
<td>$C = 11.9Bo^{0.45}$</td>
<td>$Bo &lt; 1.5$</td>
</tr>
</tbody>
</table>

For Lee & Lee (2001) and Saisorn & Wongwises (2010), the dimensionless variables $\Phi$ and $\Lambda$ equate to a capillary number and an inverse liquid-only Suratman number, respectively. The subscripts $go$ & $lo$ refer to gas-only and liquid-only.

Two authors proposed $C$-value correlations specific to hydrophobic channels. Lee & Lee (2008b) modified a correlation derived for hydrophilic channels (Lee & Lee, 2001) to account for the moving contact lines that result in dry flows such that:

$$C = 2.161 \times 10^{-21} \Lambda^{-3.703} \Phi^{-0.995} Re_{lo}^{0.486} \quad (3.2)$$

Wang et al. (2014a) proposed:

$$C = 18.1 \left(1 + \cos \gamma\right)^{0.200} We_L^{0.248} \quad (3.3)$$

which takes into account the influence of the contact angle ($\theta$) and the relative importance between the liquid inertia and surface tension through the liquid Weber number defined as:

$$We_L = \frac{\rho_L U_L^2 D_h}{\sigma} \quad (3.4)$$
Table 3.3: Correlations producing a variable $C$-value for the separated flow model.

<table>
<thead>
<tr>
<th>Reference</th>
<th>$C$ Correlation</th>
<th>Test Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun &amp; Mishima (2009)</td>
<td>$C = 26 \left( 1 + \frac{Re_L}{1000} \right) \left[ 1 - e^{0.27N_{conf} + 0.8} \right]$</td>
<td>$D_H = 0.506$–12mm</td>
</tr>
<tr>
<td>Ma et al. (2010)</td>
<td>$C = \hat{a}Ca_L^{\hat{b}}$</td>
<td>Channels 100µm by 200–2000µm</td>
</tr>
<tr>
<td></td>
<td>$\hat{a} = 7.59 - 0.4237AR^{0.9485} + 0.0023Re_L$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\hat{b} = 0.223 + 0.2AR^{-0.9778}$</td>
<td></td>
</tr>
<tr>
<td>Kim &amp; Mudawar (2012)</td>
<td>$C = 3.5 \times 10^{-5}Re_{lo}^{0.44}Su_{go}^{0.5} \left( \frac{\rho_L}{\rho_G} \right)^{0.48}$</td>
<td>Condensing, Adiabatic flows</td>
</tr>
<tr>
<td>Li &amp; Hibiki (2017)</td>
<td>$C = 41.7N_{\mu_p}^{0.66}Re_{tp}^{0.42} \lambda^{0.21}$</td>
<td>Flow boiling</td>
</tr>
<tr>
<td>Lee &amp; Lee (2001)</td>
<td>$C = 6.833 \times 10^{-8}\Lambda^{-1.317\Phi^{0.719}}Re_{lo}^{0.557}$</td>
<td>20mm wide by 0.4–4mm high Channels</td>
</tr>
<tr>
<td>Saisorn &amp; Wongwises (2010)</td>
<td>$C = 7.599 \times 10^{-3}\Lambda^{-0.631\Phi^{0.005}}Re_{lo}^{-0.008}$</td>
<td>Round tubes</td>
</tr>
</tbody>
</table>

### 3.3.3 Relative Permeability Model

Although typically applied for porous media, relative permeability models work well in predicting the two-phase pressure in microchannels (chapter 2 and Cho & Wang 2014b). Wang (2009) showed that:

$$\phi_G^2 = \bar{z}^* + \int_{\bar{z}^*}^{1} \frac{1}{k_{r,G}} d\bar{z}$$  \hspace{1cm} (2.50)

where the relative permeability can have different forms shown in table 3.4. The term $\bar{z}^*$ equals the location of water injection divided by the length of the channel and accounts for the flow initially starting as single-phase gas flow. In general the gas-relative permeability equals the term $1 - s_L$ raised to an exponent, $n_k$. Thus the relative permeability depends on the liquid saturation, $s_L$.

Typically, researchers measure the liquid saturation. However, Fourar & Bories (1995) pro-
<table>
<thead>
<tr>
<th>Reference</th>
<th>$k_{r,G}$ Model</th>
<th>Test Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>X-Model</td>
<td>$k_{r,G} = (1 - s_{L,e})$</td>
<td></td>
</tr>
<tr>
<td>Corey (1954)</td>
<td>$k_{r,G} = (1 - s_{L,e})^2(1 - s_{L,e}^2)$</td>
<td>Oil-gas</td>
</tr>
<tr>
<td>Nowamooz et al. (2009)</td>
<td>$k_{r,G} = (1 - s_{L})^{3.05}$</td>
<td>Air-water, fractures</td>
</tr>
<tr>
<td>Chen et al. (2004)</td>
<td>$k_{r,G} = 0.502s_G^3 + 0.1129s_G^2 + 0.3483s_G$</td>
<td>Nitrogen-water, fractures</td>
</tr>
<tr>
<td>Fourar &amp; Lenormand (1998)</td>
<td>$k_{r,G} = (1 - s_{L})^3 + \frac{3}{2}\mu s_L (1 - s_{L})(2 - s_{L})$</td>
<td>Air-water, fractures</td>
</tr>
<tr>
<td>Huang et al. (2009)</td>
<td>$k_{r,G} = (1 - s_{L}) \left[ \frac{3}{2} \mu + (1 - s_{L})^2 (1 - \frac{3}{2} \mu) \right]$</td>
<td>LBM Simulation</td>
</tr>
<tr>
<td>Fourar &amp; Bories (1995)</td>
<td>$k_{r,G} = (1 - \sqrt{s_L})^2$</td>
<td>Fractures</td>
</tr>
<tr>
<td>Wang (2009)</td>
<td>$k_{r,G} = (1 - s_{L,e})^{n_h}$</td>
<td></td>
</tr>
</tbody>
</table>

where

$$s_{L,e} = \frac{s_L - s_{L,r}}{1 - s_{L,r}}$$

$$s_G = 1 - s_L$$

$$\mu = \mu_G/\mu_L$$
posed a saturation model to accompany the relative permeability model as:

\[ s_L = \left( \frac{X}{1 + X} \right)^2 \]  

(2.65)

Wang (2009) arrived at a model for the saturation dependent on the relative permeability exponent \( n_k \) as:

\[ s_L = \left( \frac{U_L \mu_L}{U_G \mu_G} \right)^{\frac{1}{n_k}} + s_{L,r} \]

\[ \left( \frac{U_L \mu_L}{U_G \mu_G} \right)^{\frac{1}{n_k}} + 1 \]  

(2.70)

which accounts for any residual liquid saturation \( (s_{L,r}) \) that the gas stream cannot remove from the channel. In this work, the two-fluid model refers to the relative permeability model proposed by Wang (2009) combined with equations 2.50 and 2.70.

### 3.4 Experimental Method

The experimental work consists of air-water tests in a mixed-wettability rectangular microchannel where a hydrophobic surface forms the base and hydrophilic surfaces form the remaining channel walls. Chapter 2 provided the hydrophilic results used for comparison in this work. To accurately access the influence of changing the base from hydrophilic to hydrophobic, this work replicates the same test conditions as the hydrophilic case. Specifically, calculations use the fluid properties of humid air and water at 20°C shown in table 3.5. Water volumetric flow rates of 177μL/hr, 1.77mL/hr, 59.07μL/min, and 590.7μL/min produce superficial liquid velocities of \( 5.0 \times 10^{-5} \), \( 5.0 \times 10^{-4} \), \( 1.0 \times 10^{-3} \), and \( 1.0 \times 10^{-2} \) m/s, respectively—forming the four different data sets. For each data set, the gas volumetric flow rates vary from 30, 50–325 mL/min in 25 mL/min increments producing superficial gas velocities between 0.51 and 5.50 m/s. Characterizing the flow in terms of Reynolds numbers gives a \( Re_L \) of 0.0108, 0.0277, 0.55, and 5.55 with gas Reynolds numbers varying between
18.2 to 197 for each liquid Reynolds number. The combination of Reynolds numbers produce a liquid-only Reynolds number between 0.35 and 9.19.

Table 3.5: Fluid properties.

<table>
<thead>
<tr>
<th>Property</th>
<th>Air</th>
<th>Water</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (kg/m$^3$)</td>
<td>1.19</td>
<td>998.3</td>
</tr>
<tr>
<td>Viscosity (kg/m·s)</td>
<td>$1.846 \times 10^{-5}$</td>
<td>$1.002 \times 10^{-3}$</td>
</tr>
<tr>
<td>Surface Tension (N/m)</td>
<td>$72.86 \times 10^{-3}$</td>
<td></td>
</tr>
</tbody>
</table>

### 3.4.1 Experimental Assembly

The design of the microchannel assembly (figure 3.1A), in which individual layers form the channel, allows for one layer to change while the other layers remain the same. Thus, the mixed-wettability experiments use the same microchannel assembly as the hydrophilic experiments, except the base material changed from aluminum to PTFE. The change from a 6061 aluminum base to a PTFE base changed the base contact angle from $76^\circ \pm 8^\circ$ to $99^\circ \pm 10^\circ$ for air-water. The 304 full-hard stainless steel forming the side walls and polycarbonate forming the top of the channel have contact angles of $82^\circ \pm 7^\circ$ and $81^\circ \pm 7^\circ$, respectively. In this work, the stated uncertainties are at a 95% confidence level.

Manual milling techniques formed the channel width of 3.23mm $\pm 10\mu$m into the stainless steel sheet. The stainless steel has a manufacture stated thickness tolerance of $\pm 15\mu$m about the nominal 0.304mm thickness. A 1cm diameter hole in the PTFE base acts as the inlet manifold while a 1.4cm diameter hole in the base acts as an outlet manifold. The minimum straight distance between the edge of the manifolds defines the channel length and equals 164mm. The apparatus forms a 3.23 mm by 0.304 mm by 164 mm ($w \times h \times L$) rectangular microchannel, aligned horizontally. The microchannel assembly builds off the designs of Cho & Wang (2014b) and Pfund et al. (2000) in which layers form the channel. For this apparatus, the seal of the channel comes solely from compression generated by twenty-two
The softness and flexibility of the PTFE sheet caused several design challenges. A three-eighths inch (0.9525 cm) thick 6061 aluminum plate provides a base for the PTFE sheet. 3M Scotch-Weld 2216 B/A Grey epoxy bonds the etched side of the PTFE sheet to the aluminum. This forms a single piece of material that can undergo the same machining process as the original aluminum base to form the fluid inlets and the flow outlets without requiring further sealing. When compressing the channel components together to seal the channel revealed that the PTFE tended to intrude into the channel (i.e. reduce the channel height). The thickness of the PTFE played a role in this as 0.005 inch (127 µm) and one-sixteenth inch (1.5875mm) showed a greater tendency to intrude than the 0.015 inch (381 µm) thick sheet ultimately used in this work. The 0.005 inch PTFE sheet came with an adhesive backing softer than the PTFE itself, leading to its high intrusion. Adjusting the bolt torque from 5.65 N·m used in the hydrophilic case to 1.13 N·m further minimized the PTFE intrusion. Leak-testing the set up, using a Druck DPI-605, with the bolts torqued to 1.13 N·m showed a negligible leak rate (2.75 Pa/min) over 2 hours when pressurized to 4500 Pa.

The air and water flow enter the channel independently. Single-phase gas flows alone in
the channel for the first 10mm of the channel length, at which point water injection occurs through a 365µm hole in the PTFE base (figure 3.2a). Figure 3.2a also notes the contact angles of the surfaces. A syringe pump (New Era Pump System NE-300, figure 3.1B) loaded with 1, 10, or 30mL syringes depending on the flow rate, supplies room temperature (20°C ±2°C) deionized water to the system. MKS 100B mass flow controllers inside a Scribner and Associates 850e Fuel-cell Test Station (figure 3.1C) control the air flow from the main air supply within ±20mL/min. The air passes through a bubble humidifier (figure 3.1D) containing 1500mL of DI-water to achieve 100% relative humidity before entering the microchannel.

![Diagram](image1.png)  
(a) Diagram of asymmetric water injection and wettability.

![Diagram](image2.png)  
(b) Pressure tap locations. Dimensions are in millimeters.

Figure 3.2: Detailed diagram of the mixed-wettability microchannel assembly.

### 3.4.2 Pressure Measurement

A Setra 230 differential pressure transducer (figure 3.1E) with a range of ±0.5psi (±3.447kPa) provides the differential pressure between two pressure taps with an accuracy of ±0.0025psi (±17.2 Pascals). 365µm diameter holes drilled through the optical plate to a depth of 1.27mm before expanding to a connecting line of 1.58mm diameter form the pressure taps. This design follows the recommendations of Shaw (1960) to minimize static pressure error. The measured pressure difference occurs over a 154mm length of the channel, with one tap located at the entrance (z = 0mm) and another one located 12mm before the exit (z = 152mm) as shown in figure 3.2b.
3.4.3 Data Acquisition

To eliminate noise and improve the dynamic range of the voltage signal output from the Setra 230, the signal undergoes signal conditioning. A precision buck and gain amplifier subtracts out the mean voltage before amplifying the signal by a factor of 10. A digital filter (Alligator USBPGF-S1) filters the pressure signal at 700Hz—selected to put the 3dB point beyond the 500 Hz frequency response of the pressure transducer—before a data acquisition card (DATAQ DI-245) logs the signal at 2000 samples per second.

3.4.4 Flow Visualization

The clear polycarbonate sheet forming the top of the channel gives optical access for a DSLR camera (Canon Rebel T3) to capture images of the entire channel length in 5 second intervals (figure 3.1F) to determine the flow characteristics. The length of the channel compared to the channel width makes it difficult to demonstrate the flow behavior in this text using raw images. Instead, this work presents traces of the raw images with a compressed aspect ratio to illustrate the flow behavior.

3.5 Results and Discussion

3.5.1 Single-phase Validation

Single-phase experiments of gas flow validated the experimental apparatus through a comparison to the theoretical result and single-phase measurements for the hydrophilic case. For single-phase flow in a rectangular duct, the Darcy friction factor \( f \) depends on the aspect
ratio of the channel as:

$$\overline{C} = 96(1 - 1.35532\alpha^* + 1.9467\alpha^{*2} - 1.7012\alpha^{*3} + 0.9564\alpha^{*4} - 0.2537\alpha^{*5})$$ \hspace{1cm} (2.71)

given by Kakac et al. (1987) from fitting the exact solutions of Shah & London (1971) for different aspect ratios ($\alpha^*$). In this case, the aspect ratio ($\alpha^*$) equals the smallest dimension divided by the largest dimension. The Darcy friction factor depends on equation 2.71 and the gas Reynolds number ($Re_G$) as:

$$f = \frac{C}{Re_G}$$ \hspace{1cm} (3.5)

The theoretical pressure then follows the relation:

$$\left(\frac{dP}{dz}\right) = f \frac{\rho U_G^2}{2D_H}$$ \hspace{1cm} (3.6)

where $D_H$ equals the hydraulic diameter. The superficial gas velocity ($U_G$) equals the gas volumetric flow rate ($Q_G$) divided by the cross-sectional area ($A_c$).

Figure 3.3 shows the comparison between the experimentally measured pressure drop and the theoretical value. The data fall within $\pm 4.5\%$ for all experiments except for the two lowest. At 0.51m/s and 0.85m/s, the measurements fall below the theoretical value by 19% and 8.7%, respectively. The error bars for pressure in figure 3.3 account for the $\pm 17.2$Pa accuracy of the pressure transducer. Utilizing the Kline-McClintock method for the equation $U_G = Q_G/A_c$, gives a velocity uncertainty of $\pm 0.34$ m/s at $U_G = 0.51$m/s to $\pm 0.43$m/s at $U_G = 5.5$m/s.

Conversely, the data for the hydrophilic case fell within $\pm 4\%$ for all experiments except for the two lowest; at 0.51m/s and 0.85m/s, the measurements fall below the theoretical value by 17% and 7%, respectively (figure 3.3). Comparing the single-phase measurements between the hydrophilic and mixed-wettability cases, the measurements differed by less than $\pm 1.5\%$ relative to the hydrophilic case except differ by 2.6% and 1.75% for 0.51m/s and 0.85m/s. Therefore, the two cases agree well.
The location of the first tap ($z = 0$) means that pressure measurements will include entrance effects. The data falling above the theoretical line supports this. To account for the entrance effects, Shah defines an apparent Fanning friction factor (Shah, 1978) to replace equation 3.5. Comparing the experimental data to the correlation proposed by Shah (1978), the data fall within $\pm 3.7\%$ for the mixed-wettability case and within $\pm 3\%$ for the hydrophilic case for all experiments except the two lowest that show negligible change (figure 3.3). Therefore, the inclusion of the entrance region has minimal influence on the measured pressure drop for either case.

### 3.5.2 Two-phase Pressure Results

The gas two-phase flow multiplier ($\phi_{2G}^2$) equals the ratio of the two-phase pressure drop to the single-phase pressure drop, representing the influence the liquid-phase has on the flow. In this study, the two-phase multiplier shows a decreasing trend as the superficial gas velocity increases (figure 3.4). $\phi_{2G}^2$ increases with increasing superficial liquid velocity at low superficial gas velocities but after $U_G = 2.96$ m/s, little change occurs in $\phi_{2G}^2$ at different $U_L$. 

![Figure 3.3: Comparison of theoretical and experimental single-phase gas pressure drop versus superficial gas velocity for the hydrophillic (HP) and mixed-wettability (HY) cases.](image-url)
based on the average value. Each experimental data point represents a 30 minute average of the pressure signal, after allowing 30 minutes for the flow to develop. The exception being the $U_L = 1.0 \times 10^{-2}$ m/s data set, which consists of 5 minute averages of the measurement after allowing 5 minutes for the flow to develop. Comparing the measured two-phase flow multiplier between the mixed-wettability and hydrophilic cases show a significant increase in the two-phase flow multiplier. During the hydrophilic test at $U_L = 1 \times 10^{-2}$ m/s for $U_G = 0.51 - 1.27$ m/s water entered the downstream pressure tap and the results were excluded from the data set (figure 3.4d). While water entered the downstream tap only under the noted conditions in the hydrophilic case, the flow behavior in the mixed-wettability case caused water to enter the downstream tap in nearly all experiments. Based on measuring the single-phase pressure before the experiment, after the experiment, and after clearing the tap of water, the blocked tap resulted in an increase of the pressure from 0% up to 10%. However, a few experiments showed no water in the downstream tap and the calculated $\phi^2_G$ followed the same trend as the other measurements, producing a two-phase flow multiplier higher than the hydrophilic case. In figure 3.4, error bars show the experimental uncertainty referenced to the average of the measured $\phi^2_G$. The experimental uncertainty accounts for the accuracy of the pressure transducer of $\pm 17.2$ Pascals for both the single-phase and two-phase measurements determined by a Kline-McClintock uncertainty analysis. Additionally, the measured $\phi^2_G$ values show a variability at a given test condition that falls outside the uncertainty of the measurement. While the varying influence of blocked pressure taps could account for differences in $\phi^2_G$ up to 10%, the variation in $\phi^2_G$ often exceeds 10%. Investigating the flow behavior will provide insight into the pressure variation.
Figure 3.4: Comparison between the mixed-wettability (HY) and hydrophilic (HP) microchannels for the pressure ratio versus superficial gas velocity.
3.5.3 Flow Behavior

The interaction of air and water in a microchannel results in the formation of identifiable flow patterns. In the hydrophilic case, the flow formed as a stratified flow in which the water formed a film in contact with a side wall that occupied the entire height of the channel but only part of the channel width. Stratified flow formed for the range of test conditions. In the mixed-wettability case, flow visualization revealed multiple flow patterns (figure 3.5) over the range of test conditions, fundamentally different than the stratified flow visualized in the hydrophilic case.

Figure 3.5 demonstrates the observed flow patterns in this study presented as a top-down view. In the figure, for a given pair of $U_L$ and $U_G$, a single frame denotes a stationary behavior relative to the frame rate, while multiple frames show the periodic behavior of the flow occurring at the stated test conditions. The stationary behavior refers to the fact that the flow pattern remains unchanged between frames but water still flows through the channel. The left side of each frame corresponds to the water injection location and the frame ends just before the location of the downstream pressure tap.

Under the conditions of superficial liquid velocity ($U_L$) equal to $5.0 \times 10^{-5}$ m/s and superficial gas velocity ($U_G$) equal to 0.51 m/s, elongated droplets form and eventually contact the channel walls. The formation of a second droplet forms before the previous water clears. The water propagates downstream, leaving residual water on the channel walls, with another droplet forming behind (figure 3.5a). The residual water formed during the case of $U_L = 5.0 \times 10^{-4}$ m/s and $U_G = 0.51$ m/s (figure 3.5d) remains relatively unchanged as the droplets form. At the two highest superficial liquid velocities ($U_L = 1.0 \times 10^{-3}$ & $U_L = 1.0 \times 10^{-2}$ m/s) at $U_G = 0.51$ m/s, the flow forms as a stratified flow with entrained air bubbles (figures 3.5g,j), similar to the flow pattern seen by Choi et al. (2011a).

As the superficial gas velocity increases, the flow takes on the characteristics of a rivulet flow.
A rivulet consists of a thin film of water moving along the bottom surface of the channel but not contacting the side walls nor the top of the channel. The dashed lines in figure 3.5 represent the rivulet. Although drawn as straight lines, the rivulet tends to wind along the channel. Barajas & Panton (1993) noted a similar behavior in a 1.6mm diameter tube. At the end of the rivulet, drops periodically form and break off, leading to a series of confined droplets (figures 3.5h,k,l). The tests of $U_L = 1.0 \times 10^{-2}$ at $U_G = 2.96 \& 5.08$ m/s show the droplet length decreases with increasing superficial gas velocity (figures 3.5k,l). Jose & Cubaud (2014) identified a similar behavior for water-silicon oil flows (liquid-liquid flow) in a 250$\mu$m square microchannel; the water droplets formed spherical or bullet shapes depending on the capillary number with nearly uniform spacing. Conversely, in this work the confined water droplets have irregular shape and spacing. Additionally, the location of the rivulet end can change location and upon break-up of the drop leave residual water in the channel (figures 3.5b,c).

The rivulet in several experiments (figures 3.5e,f,i) extended nearly the entire observed length of the channel, terminating in a stationary drop. Repetition of the experiments shown in figures 3.5e,f,i produced the same flow behavior as shown. However, at $U_L = 5.0 \times 10^{-3}$ m/s with $U_G = 5.50$ m/s, the flow behaved similar to the flow shown in figure 3.5h. Between the mixed-wettability and the hydrophilic microchannel, only the wetting properties of the base changed. Thus, changing the wetting properties of the materials changes the flow behavior.
<table>
<thead>
<tr>
<th>$U_L$</th>
<th>$U_C$</th>
<th>0.51 m/s</th>
<th>2.96 m/s</th>
<th>5.08 m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5.0 \times 10^{-5}$ m/s</td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
<td></td>
</tr>
<tr>
<td>$5.0 \times 10^{-4}$ m/s</td>
<td>(d)</td>
<td>(e)</td>
<td>(f)</td>
<td></td>
</tr>
<tr>
<td>$1.0 \times 10^{-3}$ m/s</td>
<td>(g)</td>
<td>(h)</td>
<td>(i)</td>
<td></td>
</tr>
<tr>
<td>$1.0 \times 10^{-2}$ m/s</td>
<td>(j)</td>
<td>(k)</td>
<td>(l)</td>
<td></td>
</tr>
</tbody>
</table>

Figure 3.5: Observed flow patterns in the mixed-wettability microchannel (Flow from left to right; water indicated in blue).
Variability of Flow Behavior

Flipo et al. (2016) noted multiple flow patterns resulted for the same test conditions and could even change during a given test in a hydrophobic channel. In this work, measurements of the gas two-phase flow multiplier show several cases in which the measurements fell outside the range of uncertainty (figure 3.4). Visualization of the flow behavior demonstrated the pressure change corresponded to a change in the flow behavior.

The measurement of $\phi^2_G$ at $U_L = 5 \times 10^{-5}$ m/s for $U_G = 0.85$ m/s particularly stands out in terms of the variation of the pressure measurement (figure 3.4a). Under this condition, the water forms as an elongated confined droplet emanating from the water inlet, propagating downstream, and leaving residual water in the channel as another drop forms (figure 3.6). In the case of the gas two-phase flow multiplier, the residual water contacts the wall (figure 3.6a) while in the low case the residual water remains as a confined droplet (figure 3.6b). The two cases produce $\phi^2_G$ values of 2.51 and 1.94, respectively, based on averaging a 1 hour sample. The images of figure 3.6 correspond to the peaks in pressure near the 60 minute mark in figure 3.7. However, based on the slow period of oscillation the average value may not truly represent $\phi^2_G$. For example in Test 1, $\phi^2_G$ varies from 4 to 2.2 for the peak-to-trough at the 60 minute mark, while $\phi^2_G$ varies from 3.7 to 1.2 for the peak-to-trough at the 73 minute mark. Interestingly, at the 75 minute mark, the pressure signal in Test 1 drops below Test 2 (figure 3.7). At this time, residual water cleared the channel, leaving only the next forming droplet in the channel. The growth of the next drop produces a pressure signal lower than Test 2, which still has residual water in the channel. The propagation of the next droplet leaves residual water in the channel at elapsed time of 84 minutes, bringing the pressure back to the level of the trough of the oscillation at an elapsed time of 60 minutes. Test 2 remained consistent throughout. Therefore, the residual water remaining in the channel varies and influences the level of the measured pressure.
Figure 3.6: Variable flow behavior at $U_G = 0.85$ m/s and $U_L = 5.0 \times 10^{-5}$ m/s.

(a) $\phi_G^2 = 2.51$ (Test 1).

(b) $\phi_G^2 = 1.94$ (Test 2).

Figure 3.7: Pressure signals at $U_G = 0.85$ m/s and $U_L = 5.0 \times 10^{-5}$ m/s for the two experiments shown in figure 3.6.

In the case of $U_L = 5 \times 10^{-5}$ m/s for $U_G = 0.85$ m/s, the flow changed subtly but at different test conditions the flow can change fundamentally (figure 3.8). At $U_L = 5 \times 10^{-4}$ m/s for $U_G = 2.12$ m/s, a long rivulet formed terminating in a stationary drop in one case (figure 3.8a), while a shorter rivulet formed producing a series of confined droplets in another case (figure 3.8b). This produced $\phi_G^2$ values of 1.86 and 1.67, respectively. The time trace of the pressure signal demonstrates the nature of the two flows (figure 3.9). In the case of Test 2, the pressure signal shows an oscillating pressure corresponding to the movement of the droplets while the Test 1 shows little pressure variation in time. The pressure oscillation for Test 1 corresponds to the characteristic frequency of the syringe pump (equation 2.73). Additionally, the number and length of droplets changes in Test 2 (figure 3.8b) resulting in varying peak-to-peak pressure values. Therefore, under a given set of test conditions, the
flow can take on different characteristics.

(a) $\phi^2_G = 1.86$ (Test 1).

(b) $\phi^2_G = 1.67$ (Test 2).

Figure 3.8: Variable flow behavior at $U_G = 2.12$ m/s and $U_L = 5.0 \times 10^{-4}$ m/s.

Tests at $U_L = 1 \times 10^{-3}$ m/s for $U_G = 3.39$ m/s behaved similarly (figure 3.10) to the tests of $U_L = 5 \times 10^{-4}$ m/s for $U_G = 2.12$ m/s. In one case, the flow remained stationary while the other produced a series of confined droplets. The flows produced $\phi^2_G$ values of 1.50 and 1.25, respectively. The pressure signals behaved similarly to that of figure 3.9 but Test 2 shows a high pressure spike around 41 minutes elapsed time (figure 3.11). At this time, the droplet near the end of the top frame of figure 3.10b appeared longer, with a larger residual water droplet on the wall. As figure 3.10b shows, the number of droplets in a given sequence varies as well influencing the signal. Therefore, the behavior of the droplets and the flow formation significantly influence the pressure measurement.
\[
\phi_2^2 = 1.50 \quad \text{(Test 1)}.
\]

\[
\phi_2^2 = 1.25 \quad \text{(Test 2)}.
\]

Figure 3.10: Variable flow behavior at \( U_G = 3.39 \) m/s and \( U_L = 1.0 \times 10^{-3} \) m/s.

(a) \( \phi_2^2 = 1.50 \) (Test 1).

(b) \( \phi_2^2 = 1.25 \) (Test 2).

Figure 3.11: Pressure signals at \( U_G = 2.12 \) m/s and \( U_L = 1.0 \times 10^{-3} \) m/s for the two experiments shown in figure 3.10.

3.5.4 Comparison of Existing Experimental Results

As figure 3.4 illustrates, the experiments in the mixed-wettability case resulted in a consistently higher gas two-phase flow multiplier compared to the hydrophilic experiments. In other words, as the contact angle increases, the two-phase frictional pressure drop increases. This trend agrees with the results of Cho & Wang (2014a), Phan et al. (2011), and Rapolu & Son (2011). However, Stevens et al. (2017), Wang et al. (2014a), and Choi et al. (2011a) found that as the contact angle increases, the two-phase pressure drop decreases. Additionally, Lu et al. (2011) found that the trend changed for the same configuration with an increase in superficial liquid velocity while Wang et al. (2014b) found the two-phase pressure in the hydrophilic case fell between the pressure measurements of the two hydrophobic cases.
The differences between the reported trends of the two-phase pressure with contact angle could result from the variable test conditions, scales, and levels of hydrophobicity between the different works. Understanding the similarity and differences between experiments will rely on representing the flow conditions in terms of the liquid-only Reynolds number ($Re_{lo}$), the liquid Capillary number ($Ca_L$), and the gas-only Suratman number ($Su_{go}$). To fairly assess the experimental results requires comparing similar configurations and analyzing the different groups separately. Wang et al. (2014a), Choi et al. (2011a), and Rapolu & Son (2011) had homogenous surface properties whereas the current work, Stevens et al. (2017), Cho & Wang (2014a), Wang et al. (2014b), and Phan et al. (2011) had mixed-wettability channels composed of materials with differing contact angles. Furthermore, the apparatus of Lu et al. (2011) consisted of eight parallel channels and Phan et al. (2011) conducted flow boiling experiments—both of which differ from the other experiments conducted in a single channel under adiabatic conditions. While the analysis will note the results of Lu et al. (2011) and Phan et al. (2011), the conclusions of the analysis apply to adiabatic flow in a single channel differentiated as mixed-wettability and homogeneous experiments.

The liquid-only Reynolds number indicates the relative importance between the inertia of the total mass flux to the viscous force of the liquid. Figure 3.12 shows the range of $Re_{lo}$ indicating the contact angle tested and the direction of the pressure change. For the mixed-wettability case (figure 3.12a), the pressure trend of Wang et al. (2014b) and Phan et al. (2011) conflict with that of Stevens et al. (2017), while in the homogeneous case (figure 3.12b) the experiments of Wang et al. (2014a) conflict with Rapolu & Son (2011). The results of Lu et al. (2011) both agree and disagree with the current work. Thus, the liquid-only Reynolds number cannot explain the differences in the pressure trend.
Figure 3.12: Contact angle versus liquid-only Reynolds number (arrows indicate the direction of the pressure change, double lines indicate inconsistent trends).
The liquid Capillary number relates the viscous forces to the surface tension forces of the liquid. Figure 3.13 shows the range of $Ca_L$ versus a pressure indicator. A pressure indicator of 0.2 indicates that the pressure increases as the contact angle increases, a value of 0.1 indicates inconsistent trends, and a value of 0 indicates that the pressure increases as the contact angle decreases. The homogeneous experiments (figure 3.13b) show no consistent trend, with Rapolu & Son (2011) and Wang et al. (2014a) showing different trends for similar $Ca_L$. On the other hand, the $Ca_L$ re-organized the mixed-wettability data into a consistent trend (figure 3.13a). The pressure increases with increasing contact angle for this work and the work of Cho & Wang (2014a), falling in a $Ca_L$ range of $6.88 \times 10^{-7}$ to $1.38 \times 10^{-4}$. For Stevens et al. (2017), the pressure increases with a decrease in contact angle in a $Ca_L$ range of $9.63 \times 10^{-4}$ to $3.99 \times 10^{-3}$. The varying trend of Wang et al. (2014b) fell in between with a $Ca_L$ of $2.06 \times 10^{-4}$. This suggests that a critical $Ca_L$ exists for adiabatic flow in a single mixed-wettability rectangular microchannel in the range of $1.38 \times 10^{-4}$ to $9.63 \times 10^{-4}$, in which the trend of the two-phase pressure with the contact angle changes. As the analysis only contained four experiments for mixed-wettability channels with adiabatic single channel flow, future work will need to add to the ranges of $Ca_L$ of $1.38 \times 10^{-4}$ to $9.63 \times 10^{-4}$ to refine the range of $Ca_L$. Specifically, it remains unclear if the work of Wang et al. (2014b) finds inconsistent trends due to the $Ca_L$ or results from how the contact angles influence the flow (appendix C §C.1).

Attempting to extend the analysis beyond adiabatic flow in a single channel to have a greater number of experiments for comparison resulted in no correlation of the pressure-contact angle trend. For example, the results of Lu et al. (2011) would fall in the range of $Ca_L$ between $4.13 \times 10^{-6}$ to $1.03 \times 10^{-5}$, where the variable trend would counter the current results and the work of Cho & Wang (2014a). Based on the the data for the homogeneous case the critical $Ca_L$ could fall in a range of $6.74 \times 10^{-4}$ to $1.29 \times 10^{-3}$. This would lead to the conclusion that the critical liquid Capillary number falls below $1.29 \times 10^{-3}$. Conversely, the $Ca_L$ values of Phan et al. (2011) fall between $9.21 \times 10^{-2}$ to 0.12, negating any correlation of
the pressure-contact angle trend, with an opposing trend of both Stevens et al. (2017) and Choi et al. (2011a). This would lead to no correlation of the pressure-contact angle trend with the $Ca_L$. Consequently, the stated critical liquid Capillary number falling between $1.38 \times 10^{-4}$ to $9.63 \times 10^{-4}$ does not extend beyond a single, mixed-wettability microchannel with adiabatic flow.

![Graph](a) Mixed-wettability experiments. (b) Homogeneous experiments.

Figure 3.13: Pressure indicator versus liquid Capillary number (Pressure indicator: 0.2 indicates pressure increasing as the contact angle increases, 0.1 inconsistent trend, and 0 pressure increases with decreasing contact angle).

The gas-only Suratman number in this case only represents a scaled hydraulic diameter because all of the experiments in figure 3.14 use air as the gas phase. The pressure trend did not become consistent with $Su_{go}$ for the mixed-wettability case (figure 3.14a), indicating that the channel scale does not influence the trend. The aspect ratio behaved similarly. On the other hand, the pressure trend for the homogeneous cases became consistent with $Su_{go}$ (figure 3.14b). The aspect ratio showed a similar trend. However, as only one experiment differs in the homogeneous case and the remaining two cases have a smaller $Su_{go}$ than the mixed-wettability case, the pressure trend may become inconsistent as the $Su_{go}$ increases. This prevents a definitive conclusion on the influence of scale for the homogeneous case.
Figure 3.14: Pressure indicator versus gas-only Suratman number (Pressure indicator: 0.2 indicates pressure increasing as the contact angle increases, 0.1 inconsistent trend, and 0 pressure increases with decreasing contact angle).

3.5.5 Assessment of Existing Two-phase Pressure Models

The previous sections discussed the trend in the two-phase pressure drop and the behavior of the flow. This section will analyze how the existing two-phase pressure models discussed in section 3.2 predict the experimental data in a mixed-wettability microchannel.

Statistical Method for Model Comparison

The statistical method for assessing the two-phase pressure models follows the method of §2.4.3, relying on the mean absolute percent error (|$\bar{e}$|%), the root-mean-square percent error $\sigma_e$%, the mean error (|$\bar{e}$|), the mean percent error ($\bar{e}$%), and the root-mean-square error ($\sigma_e$). The scale dependent statistical quantities rely on the error ($\delta P_i$) defined as the two-phase pressure drop calculated from the model ($\Delta P_{pre,i}$) minus the experimentally measured pressure drop ($\Delta P_{exp,i}$) for the $ith$ experimental datum. Similarly, the scale independent statistical quantities rely on the percent error ($\delta^eP_i$) defined as the error ($\delta P_i$) divided by
Thus the statistical quantities have the definitions:

\[ |\bar{\varepsilon}| = \frac{1}{n} \sum_{i=1}^{n} |\delta^* P_i| \]  

(2.74)

\[ \sigma_%e = \left( \frac{1}{n} \sum_{i=1}^{n} (\delta^* P_i)^2 \right)^{0.5} \]  

(2.75)

\[ \bar{\varepsilon} = \frac{1}{n} \sum_{i=1}^{n} \delta P_i \]  

(2.76)

\[ \bar{\varepsilon}_% = \frac{1}{n} \sum_{i=1}^{n} \delta^* P_i \]  

(2.77)

\[ \sigma_e = \left( \frac{1}{n} \sum_{i=1}^{n} (\delta P_i)^2 \right)^{0.5} \]  

(2.78)

where \( n \) equals the number of data points.

**Comparison to Homogeneous Flow Models**

Of the selected homogeneous flow models, the model of Fourar & Bories (1995) outperformed the other models (table 3.6). Initially, the model of Fourar & Bories under-predicts the experimental data (figure 3.15a) with a \( \bar{\varepsilon} = -378 \text{Pa} \) at \( U_L = 5.0 \times 10^{-5} \text{m/s} \) and decreasing as \( U_L \) increases to a minimum mean error of \(-157.5 \text{Pa} \) at \( U_L = 1.0 \times 10^{-3} \text{m/s} \). At \( U_L = 1.0 \times 10^{-2} \text{m/s} \), the model of Fourar & Bories over-predicts the experimental data with a \( \bar{\varepsilon} = 402.5 \text{Pa} \). Conversely, the model of Beattie & Whalley (1982) better predicts the two-phase pressure at \( U_L = 1.0 \times 10^{-2} \text{m/s} \) with a mean error of 46.5Pa while under-predicting the remaining data sets (figure 3.15b). The under-prediction does not have a decreasing trend in the mean error, with values of \(-438.5, -485, \) and \(-404.6 \) for superficial liquid velocities...
from $5.0 \times 10^{-5}$ to $1.0 \times 10^{-3}$ m/s, respectively. This leaves the majority of the data clustered away from the prediction (figure 3.15b). The model of Dukler et al. (1964) showed the same trend for all the superficial liquid velocities (figure 3.15c) with means errors of $-441$, $-511$, $-457.6$, and $-474.5$ Pa for superficial liquid velocities from $5.0 \times 10^{-5}$ to $1.0 \times 10^{-2}$ m/s, respectively. The remaining models in table 3.6 follow the trend of McAdams et al. (1942) in which the model significantly over-predicts one or more of the $U_L$ data sets (figure 3.15d).

Based on the trends, the model of Dukler et al. (1964) produces too low of a two-phase viscosity, the viscosity model of Beattie & Whalley (1982) does not change rapidly enough at low $U_L$, while the model of Fourar & Bories (1995) changes too quickly at higher $U_L$.

Table 3.6: Overall error statistics of the homogeneous flow models for the mixed-wettability case.

| Model                        | $\bar{e}$ (Pa) | $\sigma_e$ (Pa) | $\bar{e}_\%$ | $\sigma_\%$ | $|\bar{e}_\%|$ |
|------------------------------|----------------|-----------------|--------------|-------------|------------|
| Fourar & Bories (1995)       | -29.9          | 391.7           | -5.1%        | 26.9%       | 23.1%      |
| Beattie & Whalley (1982)     | -262.5         | 389.4           | -17.8%       | 28.9%       | 23.8%      |
| Dukler et al. (1964)         | -474.1         | 500             | -32.8%       | 35.2%       | 32.8%      |
| McAdams et al. (1942)        | 637.4          | 1414            | 44.4%        | 106%        | 73.7%      |
| Lin et al. (1991)            | 1709           | 3069            | 124%         | 243%        | 147%       |
| Awad & Muzychka (2008)       | 2469           | 4179            | 167%         | 287%        | 182%       |
| Cicchitti et al. (1960)      | 18090          | 26681           | 1075%        | 1417%       | 1075%      |
Figure 3.15: Comparison between the mixed-wettability experimental data and the predicted two-phase pressure drop for the homogeneous flow models.
Comparison to Separated Flow Models

The separated flow model comparison consists of three categories: models producing constant $C$-values, models that produce $C$-values varying with test conditions, and models that researchers have proposed for hydrophobic microchannels. The model of English & Kandlikar (2006) predicted the experimental data better than the other constant $C$-value correlations (table 3.7). The correlation of English & Kandlikar under-predicts the experimental data with mean errors of $-393.6$, $-402.6$, $-304.6$, and $-96.8$ Pa for $U_L$ from $5.0 \times 10^{-5}$ to $1.0 \times 10^{-2}$ m/s, respectively. Statistically, the correlations of Mishima & Hibiki (1996), Zhang et al. (2011), and Li & Wu (2010) deviate less than that of English & Kandlikar in the range of $U_L = 5.0 \times 10^{-5}$ to $1.0 \times 10^{-3}$ m/s. The mean errors showed a decreasing trend before over-predicting the experimental data at $U_L = 1.0 \times 10^{-2}$ m/s (figure 3.16). The correlation of Chisholm (1967) produced smaller mean errors for the two lowest superficial liquid velocities but produced the largest mean error for the $U_L = 1.0 \times 10^{-2}$ m/s data set compared to the other correlations.

Table 3.7: Overall error statistics of separated flow models producing constant $C$-values for the mixed-wettability case.

| Model                        | $\tau$ (Pa) | $\sigma_e$ (Pa) | $\tau_\%$ | $\sigma_\%$ | $|\tau_\%|$ |
|------------------------------|-------------|-----------------|-----------|-------------|------------|
| English & Kandlikar (2006)   | -296.6      | 340.2           | -21.7%    | 29.0%       | 22.7%      |
| Zhang et al. (2010)          | 152.7       | 570.4           | 4.1%      | 35.0%       | 28.9%      |
| Li & Wu (2010)               | 186         | 607.7           | 6.1%      | 36.7%       | 30.0%      |
| Mishima & Hibiki (1996)      | 314         | 759.3           | 13.9%     | 43.9%       | 34.5%      |
| Chisholm (1967)              | 668         | 1208            | 35.4%     | 67.4%       | 50.2%      |

The analysis of the correlations where the $C$-value varies with test conditions demonstrates no significant improvement over the constant $C$-value relations. Of the selected relations, the correlation of Sun & Mishima (2009) produced the lowest mean absolute percent error (table 3.8) of 23.7%. The correlation initially under-predicts the experimental data with a decreasing mean error of $-362.1$, $-299.1$, $-153.6$ Pa as the superficial liquid velocity increases from $U_L = 5.0 \times 10^{-5}$ to $1.0 \times 10^{-3}$ m/s. For the highest superficial liquid velocity, the correlation
Table 3.8: Overall error statistics of separated flow models using correlations producing variable \( C \)-values for the mixed-wettability case.

| Authors                | \( \bar{e} \) (Pa) | \( \sigma_e \) (Pa) | \( \bar{e}_\% \) | \( \sigma_\% \) | \( |\bar{e}_\%| \) |
|------------------------|---------------------|---------------------|------------------|----------------|------------------|
| Sun & Mishima (2009)   | -45.2               | 383.4               | -8.0\%           | 28.0\%         | 23.7\%           |
| Li & Hibiki (2017)     | -329                | 370                 | -25.7\%          | 31.6\%         | 25.8\%           |
| Ma et al. (2010)       | -378                | 411                 | -28.3\%          | 33.1\%         | 28.3\%           |
| Lee & Lee (2001)       | -447                | 470                 | -32.5\%          | 36.2\%         | 32.5\%           |
| Kim & Mudawar (2012)   | 449                 | 472                 | -32.6\%          | 36.3\%         | 32.6\%           |
| Saisorn & Wongwises (2010) | 880              | 1482                | 48.3\%           | 82.2\%         | 60.2\%           |

over-predicts the data with a mean error of 347Pa (figure 3.17a). While deviating further in magnitude than the correlation of English & Kandlikar (2006) at \( U_L = 1.0 \times 10^{-2}\) m/s, the relation of Sun & Mishima (2009) produces smaller mean errors for the other superficial liquid velocities. Consequently, the correlation of Sun & Mishima has a slightly lower root-mean-square percent error than the relation of English & Kandlikar (28.0\% to 29.0\%). Although the two correlations produce different trends in correlating the experimental data, the similar statistics mean the two correlations have equal validity in correlating the data.

The remaining \( C \)-value correlations generally under-predict the experimental data (figure 3.17). The relations of Lee & Lee (2001) and Kim & Mudawar (2012), produce similar mean errors of \(-419\), \(-486\), \(-426\), and \(-450\)Pa for the range of superficial liquid velocities of \(5.0 \times 10^{-5}\) to \(1.0 \times 10^{-2}\) m/s (figures 3.17d and 3.17e). The correlations of Li & Hibiki (2017) and Ma et al. (2010) produce similar results, except achieve minimum mean errors of \(-224\)Pa and \(-287\)Pa, respectively, at \( U_L = 1.0 \times 10^{-2}\) m/s (figures 3.17b and 3.17c). The correlation proposed by Saisorn & Wongwises (2010) only under predicts the \( U_L = 5.0 \times 10^{-5}\) m/s data with a \( \bar{e} = -229.5\)Pa. As the superficial liquid velocity increases the mean error increases from 135Pa to 2163Pa (figure 3.17f).
Figure 3.16: Comparison between the mixed-wettability experimental data and the predicted two-phase pressure drop for separated flow correlations with constant $C$-values.
Figure 3.17: Comparison between the mixed-wettability experimental data and the predicted two-phase pressure drop for separated flow correlations with flow dependent $C$-values.
Analyzing the relations derived for hydrophobic channels provides interesting results (table 3.9). The correlation of Lee & Lee (2008b) under-predicts the experimental data (figure 3.18a) with mean errors of $-419$, $-486$, $-427$, and $-447$ as the $U_L$ increases from $5.0 \times 10^{-5}$ to $1.0 \times 10^{-2}$ m/s, respectively. This produces nearly the same statistics as the original relation for hydrophilic channels (Lee & Lee, 2001) shown in figure 3.17d. Additionally, the relation of Saisorn & Wongwis (2010), which modified the exponents of Lee & Lee (2001), did not accurately predict the two-phase pressure in this experiment. Therefore, adjusting the exponents in the relation of Lee & Lee (2001) tends to have specific applicability.

Table 3.9: Overall error statistics of separated flow models modified for hydrophobic channels.

|                | $\bar{\sigma}$ (Pa) | $\sigma_e$ (Pa) | $\bar{\tau}_%$ | $\sigma_\tau$% | $|\tau_\%|$ |
|----------------|----------------------|------------------|-----------------|-----------------|-------------|
| Lee & Lee (2008b) | -488.1               | 471.2            | -32.5%          | 36.1%           | 32.5%       |
| Wang et al. (2014a) | 80.3                 | 677.6            | -0.8%           | 42.1%           | 36.7%       |

Figure 3.18: Comparison between the mixed-wettability experimental data and the predicted two-phase pressure drop for separated flow correlations derived for hydrophobic channels.

As figure 3.18b illustrates, the relation of Wang et al. (2014a) produces varying trends dependent on the superficial liquid velocity. From $U_L = 5.0 \times 10^{-5}$ to $1.0 \times 10^{-3}$ m/s, the correlation under-predicts the data with mean errors of $-419$, $-486$, and $-426.8$ Pa,
respectively. At $U_L = 1.0 \times 10^{-2}$m/s, the correlation over-predicts the experimental data with a mean error of 847Pa. As a result, modifying the $C$-value with a function of the contact angle does not improve predictive accuracy for the experimental work, compared to other relations.

**Comparison to Relative Permeability Models**

While the imaging technique provided qualitative behavior of the flow, the method did not provide saturation measurements. As the relative permeability models require saturation values, the assessment of the models relied on a saturation model (equation 2.70). Calculating the saturation for the X-model, the model of Corey (1954), and Nowamooz *et al.* (2009) used $n_k$ values of 1, 2, and 3.05, respectively. The model of Fourar & Bories (1995) has an accompanying saturation model. The assessment does not include the models of Chen *et al.* (2004), Fourar & Lenormand (1998), or Huang *et al.* (2009), as a single $n_k$ value cannot simplify the relations, which prevents using the saturation model.

The combination of the saturation and relative permeability models of Fourar & Bories (1995) performed statistically (table 3.10) similar to the relation of Sun & Mishima (2009) and the homogeneous model also proposed by Fourar & Bories (1995). Initially, the model under-predicts the experimental data with mean errors of $-360.3$, $-293.5$, and $-145.8$Pa for $U_L = 5.0 \times 10^{-5}$ to $1.0 \times 10^{-3}$m/s, respectively. Subsequently at $U_L = 1.0 \times 10^{-2}$m/s the model over-predicts the experimental data with a mean error of 365.7Pa (figure 3.19d). The model of Corey (1954) follows a similar trend. The model first under-predicts the experimental data with mean errors of $-360$, $-283.7$, and $-126$Pa for $U_L = 5.0 \times 10^{-5}$ to $1.0 \times 10^{-3}$m/s, respectively. The model then over-predicts the data set of $U_L = 1.0 \times 10^{-2}$m/s with a mean error of 365.7Pa (figure 3.19b).

The model of Nowamooz *et al.* (2009) and the X-model provide bounds for the optimized
$n_k$ value of the data. The model of Nowamooz et al. (2009) primarily over-predicts the measurements with mean errors of 327, 685.5, and 2236Pa respective to the superficial liquid velocities of $5.0 \times 10^{-4}$ to $1.0 \times 10^{-2}$. The model under-predicts the measurements at $U_L = 5.0 \times 10^{-5}$ m/s with a mean error of -87Pa (figure 3.19c). Conversely, the X-model under-predicts all the experimental results, with nearly consistent mean errors of $-419.5$, $-488$, $-429$, and $-470$Pa from low to high superficial liquid velocity (figure 3.19a). Based on the analysis, the determination of an optimal $n_k$ value for the entire data set will fall between 1 and 3.05. Since the model of Corey (1954) predicted the results the closest out of the three models, the optimized $n_k$ value will likely fall near 2 for the entire data set.

Table 3.10: Overall error statistics of the relative permeability models based on modeled saturation.

| Model                      | $\bar{e}$ (Pa) | $\sigma_e$ (Pa) | $\bar{e}_\%$ | $\sigma_\%$ | $|\bar{e}_\%|$ |
|---------------------------|----------------|-----------------|--------------|-------------|---------------|
| Fourar & Bories (1995)    | -38.8          | 386.6           | -7.7%        | 28.1%       | 23.7%         |
| Corey (1954)              | 50.1           | 473.7           | -1.3%        | 32.2%       | 27.2%         |
| X-Model                   | -457.6         | 480.3           | -33.2%       | 36.8%       | 33.2%         |
| Nowamooz et al. (2009)    | 1019           | 1568.5          | 55.8%        | 85%         | 63.6%         |
Figure 3.19: Comparison between the mixed-wettability experimental data and the predicted two-phase pressure drop for the relative permeability models with modeled saturation.
3.5.6 Determining an optimized $n_k$ value

The previous section analyzed the capabilities of selected two-phase pressure models in predicting the experimental results. The homogeneous and relative permeability model of Fourar & Bories (1995) predicted the data with a mean absolute percent error of 23.1% and 23.7%, respectively. The separated flow models of Sun & Mishima (2009) and English & Kandlikar (2006) produced similar mean errors of 23.7% and 22.7%, respectively. By optimizing the relative permeability exponent ($n_k$) in the two-fluid model, the predictive accuracy can improve.

Figure 3.20a shows a comparison of the optimized $n_k$ value equal to 2.14 to the experimental measurements. The optimization minimized the variance between the entire experimental data set and the prediction. The optimized value produced similar statistics (table 3.11) to the statistics of Corey (1954) shown in table 3.10. The similarity of the results indicate the assessment of the data requires further refinement.

As shown in figure 3.5, the flow in this work takes on variable patterns: elongated drop with residual water at low pairs of superficial velocities, entrained stratified flow for low superficial gas velocities with the two highest superficial liquid velocities, and the remaining test conditions produce rivulet type flows. Chapter 2 and Cho & Wang (2014b) demonstrated that the $n_k$ value should depend on the flow pattern to account for the changing influence of the liquid on the gas. For $U_L = 5.0 \times 10^{-5}$m/s the rivulet flow begins at $U_G = 2.54$m/s.

Table 3.11: Overall error statistics for the optimized $n_k$ values in the mixed-wettability experiment.

| $n_k$ Value                  | $\bar{e}$ (Pa) | $\sigma_e$ (Pa) | $\bar{e}_\%$ | $\sigma_\%$ | $|\bar{e}_\%|$ |
|-----------------------------|----------------|----------------|--------------|-------------|----------------|
| $n_k = 2.14$ (all data)     | 58.8           | 462.5          | -1.8%        | 30.4%       | 25.68%         |
| $n_k = 2.25$ (Entrained-stratified) | 86.2       | 345            | 8.1%         | 39.4%       | 31.5%          |
| $n_k = 3.387$ (Elongated droplet) | -26.9      | 149            | -3.7%        | 20.7%       | 16.5%          |
| $n_k = 1.747$ (Rivulet)     | -148.6         | 330.7          | -8.42%       | 17.7%       | 14.9%          |
| Combined                    | -93            | 321            | -5.07%       | 23%         | 17.8%          |
while rivulet flow begins at $U_G = 1.69\text{m/s}$ for the three remaining $U_L$ data sets.

The optimization of the $n_k$ value produced varying results depending on the flow regime. The stratified flow with entrained bubbles occurred for $U_L = 1.0 \times 10^{-3}$ and $U_L = 1.0 \times 10^{-2}\text{m/s}$. The optimization produced an $n_k = 2.25$ with a mean absolute percent error of 31.5% (table 3.11). The relation under-predicts the experimental data for $U_L = 1.0 \times 10^{-3}\text{m/s}$ with a mean absolute percent error of 34% and over-predicts the pressure measurements $U_L = 1.0 \times 10^{-2}\text{m/s}$ with a mean absolute percent error of 30%, leading to the poor agreement.
of the prediction (figure 3.20b). The optimization indicates that, although visually similar (figure 3.5g,j) the water does not have the same influence in both cases. Additionally, the case of \( U_L = 1.0 \times 10^{-3} \text{m/s} \) did not correlate well with the elongated droplet flows.

The elongated droplet flow had an optimized \( n_k \) value of 3.387 and occurs for the two lowest superficial liquid velocities. The resulting prediction under-predicts the \( U_L = 5.0 \times 10^{-5} \text{m/s} \) data set with a mean error of \(-116\text{Pa}\) while over-predicting the \( U_L = 5.0 \times 10^{-4} \text{m/s} \) data with a mean error of 106Pa (figure 3.20c). This results in a mean absolute percent error of 16.5\% (table 3.11).

The correlation of the remaining data, which corresponds to rivulet type flows, produces an optimized \( n_k \) value of 1.747. The optimization yielded mean errors of \(-409\), \(-382\), \(-215\), and 167Pa for \( U_L = 5.0 \times 10^{-5} \) to \( 1.0 \times 10^{-2} \text{m/s} \). Thus the optimization under-predicts the three lowest superficial velocities while over-predicting the highest \( U_L \) (figure 3.20d). Overall, the mean absolute percent error equals 15\% (table 3.11).

To accurately compare the optimization to the other models requires combining the error statistics of the three optimized \( n_k \) values. Table 3.11 shows the combination, which results in a mean absolute percent error of 18\% and a \( \sigma_{\%} = 23\% \). Therefore, the optimization produced a better prediction than the other selected models. However, the optimized \( n_k \) values resulted from the experimental measurements of this work alone, requiring other researchers to add additional measurements in the specific flow regimes to determine the universality of the optimized relative permeability exponents.
3.5.7 Discussion of the Predictive Accuracy of the Selected Models

In the comparison between the experimental data and existing two-phase pressure models, the prediction equations did not collapse the experimental data, leading to the relatively high mean absolute percent errors. Furthermore, determining new correlation parameters (relative permeability exponents, $n_k$, in the two-fluid model) also failed to collapse the experimental data. As noted in §3.5.2, water entered the downstream pressure tap resulting in an increase in the measured two-phase pressure. Employing an ad hoc method of correcting the two-phase pressure measurements and repeating the analysis showed $\sigma$% and $|\varepsilon|$% decreased by 3–4% in most cases—particularly increasing for the models that over-predict the $U_L = 1.0 \times 10^{-2} \text{m/s}$ data. The method reduced the two-phase pressure by the percentage increase in the single-phase pressure measurements. The difference between the single-phase measurements before and after the experiment, divided by the initial single-phase pressure determined the percent difference. As the two-phase pressure will always have a greater magnitude than the single-phase pressure, this method will produce the greatest reduction of the two-phase pressure. Overall, the change does not produce an appreciable change in the variability of the data.

With the influence of the water in the pressure taps minimally contributing to the behavior of the measurements, instabilities of the rivulet could cause the inability of the two-phase pressure models to collapse the experimental data. In this work, the channel has a base contact angle of 99°. Based on the analysis of Davis (1980), Koplik et al. (2006), and Herrada et al. (2015), this contact angle could result in an unstable rivulet. The rivulet breaking into droplets under multiple test conditions indicates an instability. The syringe pump introduces a characteristic frequency to the system resulting from the steps of the stepper motor. The bubble humidifier also introduces characteristic frequencies based on the frequency of bubble formation (§2.4.2). These two mechanisms can provide perturbations to the system that can
destabilize the rivulet.

The four superficial liquid velocities produce different Reynolds and Weber numbers, which could lead to different levels of instability between the four data sets based on the analysis of Herrada et al. (2015). In turn, the instability will induce differing levels of interaction between the gas and liquid. Specifically figure 3.5 shows the length of the rivulet before breaking into droplets, the number, and the size of the droplets change between experiments. In the two-fluid model, the $n_k$ value indicates the influence of the interaction between the two-phases. Optimizing the $n_k$ value for each $U_L$ data set gave $n_k$ values of 3.09, 2.60, 2.22, and 1.63 for $U_L = 5.0 \times 10^{-5}$ to $1.0 \times 10^{-2}$m/s. The optimization produced mean absolute percent errors of 6.6%, 10.5%, 10.6% and 5.2%, respectively. The range of $n_k$ values indicates the interactions between the phases changed between the data sets. The inability of the selected models to collapse the data does not result from a fault in the models but a variable condition induced by the flow conditions. While rivulet stability remains an open area of research, further conclusions and investigations of the instability in this experiment would require more advanced imaging techniques such as Laser-induced Fluorescence (LIF) and/or schlieren combined with high speed photography to resolve the surface waves/droplet spacing. Particularly, the assessment of the stability condition of Davis (1980) would require topological information of the rivulet.

### 3.6 Concluding Remarks

This chapter discussed an experimental and modeling study of air-water two-phase frictional pressure drop in a mixed-wettability microchannel of dimensions 3.23mm wide by 0.304mm high by 164mm long. The test conditions produced liquid Reynolds numbers between 0.0108 and 5.55 with a corresponding gas Reynolds number ranging between 18.2 and 197. Compared to the measured two-phase pressure drop in a hydrophilic microchannel of
the same dimensions but differing only by the base contact angle, the two-phase pressure drop increased when the base contact angle changed from 76° to 99°. The flow in the mixed-wettability channel formed primarily as a rivulet type flow compared to the stratified flow in the hydrophilic channel. The flow behavior varied between experiments for fixed test conditions, resulting in different two-phase pressure drops.

Researchers have found inconsistent trends in how the two-phase pressure drop changes as the contact angle changes. Based on the current study and the selected works, the trend becomes consistent with the liquid Capillary number. A critical liquid Capillary number between $1.38 \times 10^{-4}$ to $9.63 \times 10^{-4}$ exists for adiabatic two-phase flow in a single mixed-wettability microchannel. Below the critical liquid Capillary number the two-phase pressure will increase with increasing contact angle and above the critical value the two-phase pressure will increase with decreasing contact angle.

Comparing the two-phase pressure drop predicted by several models showed the correlations for the separated flow model proposed by Sun & Mishima (2009) and English & Kandlikar (2006) predicted the experimental two-phase pressure with mean absolute percent errors of 23.7% and 22.7%, respectively. By optimizing the relative permeability exponent ($n_k$) for each of the three visualized flow patterns improved the prediction to a mean absolute percent error of 17.8%. An optimized $n_k$ value for rivulet flows alone equals 1.747 and predicted the two-phase pressure drop of rivulet flows with a mean absolute percent error of 14.9%. The predictions of the two-phase pressure do not collapse the experimental data due to the instability of the rivulet flow.
Chapter 4

Conclusion and Future Considerations

This work investigated the two-phase flow behavior and the ability of two-phase pressure models to predict the experimentally measured two-phase pressure in a high aspect ratio microchannel. Air and water flowed adiabatically at \(0.51 \leq U_G \leq 5.50\text{m/s}\) and \(5.0 \times 10^{-5} \leq U_L \leq 1.0 \times 10^{-2}\text{m/s}\). These superficial velocities represent conditions during PEM fuel cell operation. A 3.23mm wide by 0.304mm high by 164mm long microchannel represented a cathode gas-supply channel of a PEM fuel cell. In one set of experiments, hydrophilic materials composed all four walls of the rectangular microchannel; the second set of experiments replaced the base of the channel with PTFE, leading to a mixed-wettability microchannel.

4.1 Two-phase Frictional Pressure Drop and Water Film Thickness in a Thin Hydrophilic Microchannel

The high aspect ratio microchannel predominately formed stratified flow over the range of test conditions. The thickness of the water film decreased as the superficial gas velocity
increased and increased as the superficial liquid velocity increased. The gas two-phase flow multiplier had values between 1 and 1.44, showing a similar trend to the water film thickness. Comparing the two-phase pressure drop predicted by several models showed the correlations for the separated flow model proposed by Lee & Lee (2001) and Kim & Mudawar (2012) predicted the experimental two-phase pressure with mean absolute percent errors of 4.1% and 4.2%, respectively. Therefore, correlations for the Chisholm parameter should depend on the liquid-only Reynolds number and the liquid capillary number. The two-fluid model predicted the experimental two-phase pressure drop with a mean absolute error of 3.25% when using the newly determined $n_k$ value of 1.159 in the relative permeability model. Comparing measurements of the water film thickness to the value predicted by the two-fluid model showed the same trend as the prediction for the two-phase pressure.

4.2 Two-phase Frictional Pressure Drop in a Thin Mixed-wettability Microchannel

The mixed-wettability microchannel produced fundamentally different flow patterns than the hydrophilic channel. The flow patterns fell into three categories: elongated confined droplets with residual water, stratified flow with entrained air, and rivulet type flows. Between two experiments conducted at the same test conditions, the flow behavior changed, with a corresponding change in the two-phase pressure.

Researchers have presented inconsistent trends in how the two-phase pressure drop changed as the contact angle changed in microchannels. In this case, the two-phase pressure increased from the hydrophilic case to the mixed-wettability case. Between the two experiments, only the base material changed, going from a contact angle of 76° to 99°. The analysis of the current work and existing experiments found a critical liquid Capillary number between
1.38 \times 10^{-4} \text{ to } 9.63 \times 10^{-4} \text{ for adiabatic two-phase flow in a single mixed-wettability microchannel. Below the critical liquid Capillary number the two-phase pressure will increase with increasing contact angle and above the critical value the two-phase pressure will increase with decreasing contact angle.}

Comparing the two-phase pressure drop predicted by several models showed the correlations for the separated flow model proposed by Sun & Mishima (2009) and English & Kandlikar (2006) predicted the experimental two-phase pressure with mean absolute percent errors of 23.7% and 22.7%, respectively. By optimizing the relative permeability exponent ($n_k$) for each of the three flow patterns improved the prediction to a mean absolute percent error of 17.8%. An optimized $n_k$ value for rivulet flows alone equals 1.747 and predicted the two-phase pressure drop of rivulet flows with a mean absolute percent error of 14.9%. The predictions of the two-phase pressure did not collapse the experimental data due to the instability of the rivulet flow, leading to the relatively large mean absolute percent errors.

4.3 Recommendations for Future Work

Several aspects of this work could serve as research questions for future work. The hydrophilic case presented measurements of the water film thickness using a simplified optical technique. Using more advance imaging techniques could improve the comparison between the measured film thickness and the predicted values. Optimizing the current data determined the $n_k$ value for stratified flow of 1.159. Researchers can generate stratified flow to investigate if the value will hold for different aspect ratios and test conditions.

From the analysis in the mixed-wettability case, a critical liquid Capillary range arose. To further refine the critical liquid Capillary number requires measuring the two-phase pressure for at least two contact angles, in a flow that produces liquid Capillary numbers in the
range $1.38 \times 10^{-4}$ to $9.63 \times 10^{-4}$. Additionally, the stability of the rivulet requires further investigation, not solely for this case. It remains as open question as to the stability conditions for a rivulet on a hydrophobic surface. For this work, stability conditions would have helped indicate the differences between the flow at different test conditions that prevented the two-phase pressure correlations for collapsing the data.
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Appendices
A Dimensionless Velocity Equations for Stratified Flow

In chapter 2, the flow formed as a stratified flow, which simplifying the determination of the saturation. Stratified flow forms when a liquid film in contact with a side wall occupies the entire height of the channel but only part of the channel width. Let \( h \) equal the channel height, \( c \) the film thickness, and \( c + b \) the channel width (\( w \)).

Steinbrenner (2011) reorganized the equations of Tang & Himmelblau (1963) in terms of dimensionless variables as:

\[
\beta_n = \frac{A_n}{B_n} \quad (A.1)
\]

\[
A_n = \left( 1 - \frac{1}{\cosh \left[ (2n + 1)\pi h_{ratio}AR \right] } \right) \left( 1 - \hat{\mu} \left( 1 - \frac{1}{\cosh \left[ (2n + 1)\pi (1 - h_{ratio})AR \right] } \right) \right)
\]

\[
B_n = \sinh \left[ (2n + 1)\pi h_{ratio}AR \right] \cosh \left[ (2n + 1)\pi (1 - h_{ratio})AR \right] \\
+ \hat{\mu} \cosh \left[ (2n + 1)\pi h_{ratio}AR \right] \sinh \left[ (2n + 1)\pi (1 - h_{ratio})AR \right]
\]

\[
V_G = \frac{4h^2}{\mu_L} \left( \frac{\Delta P}{\Delta L} \right) \sum_{n=0}^{\infty} \hat{\mu} \frac{\sin \left[ (2n + 1)\pi x \right]}{(2n + 1)^3\pi^3} \\
\left\{ \beta_n \cosh \left[ (2n + 1)\pi h_{ratio}AR \right] \sinh \left[ (2n + 1)\pi \left( (1 - h_{ratio})AR - \hat{y} \right) \right] \right. \\
+ \left. \left( 1 - \frac{\cosh \left[ (2n + 1)\pi \hat{y} \right]}{\cosh \left[ (2n + 1)\pi (1 - h_{ratio})AR \right]} \right) \right\} 
\]
\[ V_L = \frac{4h^2 \left( \frac{\Delta P}{\Delta L} \right)}{\mu_L} \sum_{n=0}^{\infty} \frac{\sin \left[ (2n + 1)\pi \hat{x} \right]}{(2n + 1)^3 \pi^3} \]

\[
\left\{ -\beta_n \cosh \left[ (2n + 1)\pi (1 - h_{ratio}) AR \right] \sinh \left[ (2n + 1)\pi (h_{ratio} AR + \hat{y}) \right] \right. \\
+ \left. \left( 1 - \frac{\cosh \left[ (2n + 1)\pi \hat{y} \right]}{\cosh \left[ (2n + 1)\pi h_{ratio} AR \right]} \right) \right\}
\]  

(A.3)

where \( \hat{\mu} = \mu_L / \mu_G \); the aspect ratio \( AR = w/h \), \( \hat{y} = y/h \), and \( \hat{x} = x/h \).
B Investigating the Pressure Loss Associated with Two-Phase Flow in a Rectangular Microchannel Suddenly Expanding into a Manifold

B.1 Abstract

This study focuses on the experimental investigation of two-phase pressure loss occurring at the exit of a microchannel of dimensions 3.23mm wide by 0.304mm high by 164mm long. Air-water flow exits the microchannel into a circular manifold of 1.4 cm diameter oriented 90° relative to the flow direction. The majority of the data for the additional pressure loss across the channel exit fall between 150Pa to 400Pa with a slight dependence on superficial gas velocity but independent of the superficial liquid velocities. Visualizing the water accumulating at the exit shows varying behavior that corresponds to the variation of the pressure loss. Comparing selected sudden expansion pressure loss models to the data shows the model of Abdelall et al. (2005) came the closest to the data with a mean absolute percent error of 92%. However, the comparison reveals that the experimental pressure drop likely contains other loss mechanisms. Therefore, an important pressure loss occurs in the experiment but the exact mechanisms remain unclear and require further investigation.

B.2 Introduction

Pipe systems inevitably include bends, area contractions/expansions, or other geometric features that will produce a minor pressure loss. Polymer-electrolyte membrane (PEM) fuel cells in particular can consists of several parallel channels in which pressure loss can occur at the sudden expansion from the channels to the manifold. Additionally, the geometric change from a small channel to a large channel can result in the local accumulation of water.
However, understanding two-phase flow itself introduces enough complexity that researchers focus on characterizing the two-phase flow independent of any exit influences. For example, English & Kandlikar (2006) designed the exit of the microchannel specifically to mitigate its influence on the results and Grimm et al. (2012) used rolled up paper at the exit to prevent water build up. Beyond Abdelall et al. (2005), literature contains little discussion of the sudden expansion pressure loss for microscale channels, especially rectangular microchannels. This work discusses the pressure loss associated with the exit of a 3.23mm wide by 0.304mm rectangular channel into a 1.4cm circular manifold orientated 90° to the flow. Section B.3 outlines the experimental method and section B.4 demonstrates that a pressure loss at the channel exit occurs during the experiment, which does not represent the flow physics inside of the channel. The comparison to the sudden expansion models discussed in section B.8 reveal the loss in this experiment results from other mechanisms beyond the sudden expansion pressure loss.

**Sudden Expansion Pressure Loss**

Minor pressure losses rely on empirical measurements with the exception of the pressure loss due to a sudden area change. Defining a control volume (figure B.1) across an area expansion

![Figure B.1: Control volume for the sudden expansion analysis.](image-url)
and applying the conservation of mass and momentum, the relations (Delhaye, 1981):

\[
\langle u_1 \rangle A_1 = \langle u_2 \rangle A_2 \quad \text{(Mass)} \quad (B.4)
\]
\[
\langle P_2 \rangle - \langle P_1 \rangle = \rho \langle u_1^2 \rangle \sigma_A - \rho \langle u_2^2 \rangle \quad \text{(Momentum)} \quad (B.5)
\]

arise under the assumptions of steady, incompressible single-phase flow where no viscous losses occur in the control volume and the pressure at the flange (location 0) equals the upstream pressure \( P_1 \). The terms \( u, \rho, A, \) and \( \sigma_A \) represent the axial velocity, density, cross-sectional area, and area ratio \( (\sigma_A = A_1/A_2) \), respectively. Subscript 1 refers to the upstream location and subscript 2 the downstream location. To combine equations B.4 and B.5 requires relating \( \langle u \rangle^2 \) to \( \langle u^2 \rangle \) where:

\[
\langle u \rangle = \frac{1}{A} \int_A u dA \quad (B.6)
\]

defines an area-averaged axial component of velocity. For turbulent flows \( \langle u^2 \rangle / \langle u \rangle^2 \approx 1 \).

Combining equations B.4 and B.5 gives the general relation for single-phase flow as:

\[
P_2 - P_1 = \frac{\sigma_A(1 - \sigma_A)}{\rho} \dot{m}^2
\]

where \( \dot{m} = \rho \langle u_1 \rangle \) and the representation of the area averaging of the pressure has been dropped for simplicity.

The mechanical energy equation defined as:

\[
\int \frac{1}{2} \rho u^2 u \cdot n dA = - \int P u \cdot n dA
\]

provides a second approach to relate the pressure change across the exit to the area ratio. The vectors \( u \) and \( n \) represent the velocity vector and the unit normal to the area of the control volume, respectively. Under the same assumptions as equations B.4 and B.5 while
also neglecting body forces, equation B.8 gives:

\[ \frac{1}{2} \rho \langle u_3^2 \rangle A_2 - \frac{1}{2} \rho \langle u_1^3 \rangle A_1 + \langle P_2 u_2 \rangle A_2 - \langle P_1 u_1 \rangle A_1 = 0. \]  \hspace{1cm} (B.9)

For turbulent flow \( \langle u^3 \rangle/\langle u \rangle^3 \approx 1 \), allowing for the combination of equations B.4 and B.8 to give:

\[ P_2 - P_1 = \frac{(1 - \sigma_A^2)}{2 \rho} \dot{m}^2. \]  \hspace{1cm} (B.10)

As \( \sigma_A(1 - \sigma_A) < (1 - \sigma_A^2) \) for \( 0 < \sigma_A < 1 \), equation B.7 will predict a lower pressure change than equation B.10. Experimental measurements agree with equation B.7, while equation B.10 represents the possible reversible pressure change (Delhaye, 1981).

The simplest extension of the single-phase relations to two-phase flow follow the standard homogeneous model approach of defining an equivalent single-phase flow of weighted properties such that equation B.7 becomes:

\[ \Delta P = \frac{\sigma_A (1 - \sigma_A)}{\rho_{tp}} G^2 \]  \hspace{1cm} (B.11)

where \( \Delta P = P_2 - P_1 \) and \( \rho_{tp} \) equals the homogeneous density defined as:

\[ \rho_{tp} = \left( \frac{\chi}{\rho_G} + \frac{1 - \chi}{\rho_L} \right)^{-1}. \]  \hspace{1cm} (2.6)

The subscript \( tp \) stands for two-phase. The gas quality, \( \chi \), equals:

\[ \chi = \frac{\rho_G Q_G}{\rho_G Q_G + \rho_L Q_L} \]  \hspace{1cm} (2.7)

with \( G \), the total mass flux, defined as:

\[ G = \frac{\rho_G Q_G + \rho_L Q_L}{A_c} \]  \hspace{1cm} (2.3)
Equation B.11 uses $\chi$ and $G$ determined by the conditions in the smaller channel (location 1). $Q$ and $A_C$ stand for the volumetric flow rate and cross-sectional area, respectively, determined in the smaller channel. The subscripts $G$ and $L$ stand for the gas and liquid phase, respectively. Similarly, equation B.10 becomes:

$$
\Delta P = \frac{(1 - \sigma_A^2)}{2\rho_{tp}} \dot{m}^2 \tag{B.12}
$$

Following a similar analysis as used to define equations B.4 and B.5, Romie (1958) arrived at the two phase relation:

$$
\Delta P = \frac{\sigma_A G^2}{\rho_L} \left[ \chi^2 \rho_L \left( \frac{1}{\alpha_1} - \frac{\sigma_A}{\alpha_2} \right) + (1 - \chi)^2 \left( \frac{1}{1 - \alpha_1} - \frac{\sigma_A}{1 - \alpha_2} \right) \right] \tag{B.13}
$$

where $\alpha$, the void fraction, equals the area occupied by the gas divided by the total cross-sectional area of the channel. Romie (1958) allowed the upstream void fraction ($\alpha_1$) to vary from the downstream ($\alpha_2$).

Lottes (1961) simplified the analysis by neglecting the gas phase ($\chi \ll 1$) such that the pressure loss only occurs in the liquid phase and arrived at:

$$
\Delta P = \frac{\sigma_A (1 - \sigma_A) G^2}{\rho_L (1 - \alpha)^2} \tag{B.14}
$$

where the void fraction remains constant across the sudden expansion.

Collier & Thome (1996) followed a similar analysis used to determine equations B.4 and B.9, taking into account two phases to determine:

$$
\Delta P = \frac{\rho_{tp} (1 - \sigma_A^2) G^2}{2} \left[ \frac{(1 - \chi)^3}{(1 - \alpha)^2 \rho^3_L} + \frac{\chi^3}{\alpha^3 \rho^3_G} \right] \tag{B.15}
$$

Richardson (1958) simplified equation B.15 by considering only the liquid velocity such that:
\[ \Delta P = \frac{(1 - \sigma_A^2)\sigma_A G^2}{2\rho_L} \left[ \frac{(1 - \chi)^2}{1 - \alpha} \right] \]  
(B.16)

While the previously discussed models result from a generalization of the flow, other authors have applied the analysis to specific flows. Attou & Bolle (1997) treated the sudden expansion as a conical jet originating from a small circular cross-section. Applying a momentum balance, the authors arrive at:

\[ \Delta P = \sigma_A(1 - \sigma_A)\vartheta^r Y G^2 + (1 - \vartheta^r)\sigma_A(1 - \sigma_A)\frac{G^2}{\rho_L} \]  
(B.17)

\[ \vartheta = \frac{3}{1 + \sqrt{\sigma_A + \sigma_A}} \]  
(B.18)

\[ Y = \frac{\chi^2}{\alpha\rho_G} + \frac{(1 - \chi)^2}{(1 - \alpha)\rho_G} \]  
(B.19)

Comparing the model to experimental data using the void fraction relation of Rouhani (1969), the authors found \( r = 1.4 \) for air-water flows at small gas quality.

By restricting the flow to an annular-mist flow, Schmidt & Friedel (1996) arrived at:

\[ \Delta P = \frac{G^2 \left[ \frac{\sigma_A}{\rho_e} - \frac{\sigma_A^2}{\rho_e} - f_e\rho_e \left( \frac{\chi}{\rho_G\alpha_e} - \frac{(1 - \chi)}{\rho_L(1 - \alpha_e)} \right) \right] (1 - \sqrt{\sigma_A})^2}{1 - \Gamma_e(1 - \sigma_A)} \]  
(B.20)

that depends on the relations:

\[ \frac{1}{\rho_e} = \frac{\chi^2}{\rho_G\alpha_e} + \frac{(1 - \chi)^2}{\rho_L(1 - \alpha_e)} + \frac{\rho_L\alpha E(1 - \alpha_e)}{1 - \alpha E} \left[ \frac{\chi}{\rho_G\alpha_e} - \frac{1 - \chi}{\rho_L(1 - \alpha_e)} \right]^2 \]  
(B.21)

\[ \alpha_e = 1 - \frac{2(1 - \chi)^2}{1 - 2\chi + \sqrt{1 + 4\chi(1 - \chi)\left( \frac{\rho_L}{\rho_G} - 1 \right)}} \]  
(B.22)

\[ \alpha_E = \frac{1}{S_e} \left[ 1 - \chi \frac{1 - \chi}{1 - \chi(1 - 0.05 W e_0^{0.27} R e_0^{0.05})} \right] \]  
(B.23)
\[ S_e = \frac{\chi(1 - \alpha_e)\rho_L}{(1 - \chi)\alpha_e \rho_G} \]  
(B.24)

\[ We_e = G^2 \chi^2 \frac{d}{\rho_G \sigma} \frac{\rho_L - \rho_G}{\rho_G} \]  
(B.25)

\[ Re_e = \frac{G(1 - \chi)d}{\mu_L} \]  
(B.26)

\[ \Gamma_e = 1 - \sigma_{0.25}^2 \]  
(B.27)

\[ f_e = 4.9 \times 10^{-3} \chi^2 (1 - \chi)^2 \left( \frac{\mu_L}{\mu_G} \right)^{0.7} \]  
(B.28)

where \( \sigma \), \( d \), and \( \mu_L \) stand for surface tension, smaller pipe diameter, and liquid dynamic viscosity, respectively. Comparing data for \( 0 < \chi < 100\% \) at mass fluxes of \( 50 - 16000 \) kg/m\(^2\)-s for multiple fluid pairs showed a scatter of 61\% about the prediction.

On the other hand, several authors proposed correlations not directly derived from the conservation equations. Chisholm & Sutherland (1969) applied the separated flow model approach of Lockhart & Martinelli (1949) and Chisholm (1967) to the two-phase sudden expansion problem, such that the two-phase pressure across the sudden expansion equals its single-phase equivalent multiplied by a scaling factor. The relation thus equals:

\[ \Delta P = \frac{G^2}{\rho_L} \sigma_A (1 - \sigma_A)(1 - \chi)^2 \left[ 1 + \frac{C_h}{X_h} + \frac{1}{X_h^2} \right] \]  
(B.29)

\[ X_h = \left( \frac{\rho_g}{\rho_L} \right)^{0.5} \frac{(1 - \chi)}{\chi} \]  
(B.30)

\[ C_h = \left\{ 1 + 0.5 \left( 1 - \left( \frac{\rho_G}{\rho_L} \right)^{0.5} \right) \right\} \left\{ \left( \frac{\rho_G}{\rho_L} \right)^{0.5} + \left( \frac{\rho_L}{\rho_G} \right)^{0.5} \right\} \]  
(B.31)

which applies only for turbulent flow in rough tubes.

Wadle (1989) considered the pressure loss as being proportional to the difference in the dynamic pressure head such that:

\[ \Delta P = (1 - \sigma_A^2) \frac{\dot{m}^2}{2} K^* \left[ \frac{\chi^2}{\rho_G} + \frac{(1 - \chi)^2}{\rho_L} \right] \]  
(B.32)
For experimental data at $0 < \chi < 7\%$ in an area expansion of a 16mm diameter tube into a 80mm diameter tube with mass fluxes of 4500-11000 kg/m$^2$·s, the authors found $K^*$ equals 0.83 for air-water flows.

The equations for a sudden area expansion presented above apply to conventional scale channels. In the case of microscale tubes/channels, Abdelall et al. (2005) investigated the pressure loss for air-water flow through a 0.16mm tube into a 0.84mm tube. The authors used the relations:

\[
\Delta P = \Delta P_R + \Delta P_I \tag{B.33}
\]

\[
\Delta P_R = \frac{G^2}{2} \left( \frac{1 - \sigma^2}{\langle \rho_r \rangle^2} \right) \tag{B.34}
\]

\[
\Delta P_I = \frac{G^2}{2 \rho_L} \left[ \frac{2 \rho_L}{\rho_r} \sigma_A (\sigma_A - 1) - \rho_p \frac{\rho_L}{\rho_r^2} (\sigma_A - 1) \right] \tag{B.35}
\]

where equation B.34 accounts for reversible pressure loss while equation B.35 accounts for irreversible pressure losses. Equations B.34 and B.35 depend on:

\[
\rho_r = \left[ \frac{(1 - \chi^2)}{\rho_L (1 - \alpha)} + \frac{\chi^2}{\rho_G \alpha} \right]^{-1} \tag{B.36}
\]

\[
\rho_{\text{..}} = \left[ \frac{(1 - \chi)^3}{\rho_L^2 (1 - \alpha)^2} + \frac{\chi^3}{\rho_G^2 \alpha^2} \right]^{-0.5} \tag{B.37}
\]

When using the homogeneous void fraction ($\beta$) defined as:

\[
\beta = \frac{Q_G}{Q_G + Q_L} \tag{2.12}
\]

equation B.33 overestimated the data. However, using the ideal annular flow slip ratio of $S = (\rho_L/\rho_G)^{1/3}$ to define the void fraction as:

\[
\alpha = \frac{Q_G}{Q_G + SQ_L} \tag{B.38}
\]
in equation B.33, the prediction agreed with the experimental data for liquid-only Reynolds numbers (\(Re_{lo}\)) between 2500-3530.

With the exception of the models of Chisholm & Sutherland (1969), Wadle (1989), and the homogeneous model, the sudden expansion models rely on a correlation for the void fraction, which acts as a closure model for the sudden expansion relations. However, several correlations exist. In addition to equations 2.12 and B.38, Rouhani (1969) proposed:

\[
\alpha = \frac{\chi \rho_L}{\rho_G} \left(1+0.12(1-\chi)\right) + \frac{W_{rel}}{m} \quad (B.39)
\]

\[
W_{rel} = 1.18 \sqrt{\rho_L \left[g\sigma(\rho_L - \rho_G)\right]^{0.25}} \quad (B.40)
\]

where \(g\) equals the acceleration due to gravity. Both Wadle (1989) and Attou & Bolle (1997) used equation B.39 when comparing to experimental data. In evaluating the sudden expansion models for air-water flow in a rectangular duct of dimensions 3mm by 6mm expanding into a rectangular duct of 3mm by 9mm, Chen et al. (2010) used the relation defined by Kawahara et al. (2002) as:

\[
\alpha = \frac{0.03\beta^{0.5}}{1 - 0.97\beta^{0.5}} \quad (B.41)
\]

The authors found that the model of Wadle (1989) predicted the experimental data with a mean deviation of 200% at gas qualities between 0.001 and 0.8 at mass fluxes between 100-700 kg/m\(^2\)-s.

**B.3 Experimental Method**

The experimental method follows that of §2.3 in chapter 2. This section discusses deviations in the set up to measure the pressure across the exit. Figure B.2a illustrates the exit geometry for this experiment. The two-phase flow expands from a rectangular duct into a circular manifold oriented 90° relative to the flow direction. The top of the channel extends over the
exit manifold, while the side walls extend approximately 2mm over the manifold. Therefore, the flow first sees the bottom of the channel expand before the sides expand. Using the diameter of the manifold as the expansion area gives a value of 0.0064 for $\sigma_A$.

(a) Diagram of exit geometry of the microchannel. (b) Pressure tap locations. Dimensions are in millimeters.

Figure B.2: Detailed diagram of the microchannel assembly.

Pressure Measurement

The measured pressure difference occurs between two sets of taps as shown in figure B.2b. The first measurement measures the difference over a 154mm length of the channel, between the Tap 1 located at $z = 0$mm and Tap 2 located 12mm before the exit at $z = 152$mm. This measurement provides the two-phase pressure drop representative of the flow dynamics. The second measurement takes the pressure difference between the entrance (Tap 1 at $z = 0$mm) and a pressure tap (Tap 3) located in the exit manifold ($z = 171$mm). This measurement provides a pressure drop that will include any exit effects. A valve allows switching between the two measurements. The measurement between Taps 1 and 2 occurs first followed by a measurement between Taps 1 and 3.
B.4 Results and Discussion

Single-phase Validation

Single-phase gas flow experiments were conducted for validation of the experimental apparatus. For single phase flow, the pressure follows

$$\frac{dP}{dz} = f \frac{pU_G^2}{2D_H}$$

(3.6)

The superficial gas velocity ($U_G$) equals the gas volumetric flow rate ($Q_G$) divided by the cross-sectional area ($A_c$). The Darcy friction factor ($f$) depends on the gas Reynolds number and equals

$$f = \frac{\overline{C}}{Re_G}$$

(3.5)

where the correlation constant ($\overline{C}$) depends on the aspect ratio of the channel as:

$$\overline{C} = 96(1 - 1.35532\alpha^* + 1.9467\alpha^*^2 - 1.7012\alpha^*^3 + 0.9564\alpha^*^4 - 0.2537\alpha^*^5)$$

(2.71)

given by Kakac et al. (1987) from fitting the exact solutions of Shah & London (1971) for different aspect ratios ($\alpha^*$). In this case, the aspect ratio ($\alpha^*$) equals the smallest dimension divided by the largest dimension. Figure B.3a shows the comparison between the experimentally measured pressure drop and the theoretical value measured between Taps 1 and 2. The data fall within ±4% for all experiments except for the two lowest. At 0.51 m/s and 0.85 m/s, the measurements fall below the predicted value by 17% and 7%, respectively. Figure B.3b shows the comparison between the experimentally measured pressure drop and the theoretical value measured between Taps 1 and 3. The experiments fall within ±4.5% for all experiments except for the two lowest. At 0.51 m/s and 0.85 m/s, the measurements fall below the predicted value by 17% and 6%, respectively. The similarity in the deviation from theory for both measurements indicates that for single-phase flow, the exit accounts for less
Figure B.3: Single-phase pressure drop versus superficial gas velocity.

than 1% of the deviation. The error bars for pressure in figure B.3 accounts for the ±17.2Pa accuracy of the pressure transducer. The superficial gas velocity equals the volumetric flow rate of gas divided by the cross-sectional area. Utilizing the Kline-McClintock method for the equation $U_G = Q_G / A_c$, gives a velocity uncertainty of ±0.34 m/s at $U_G = 0.51$ m/s to ±0.43 m/s at $U_G = 5.5$ m/s.

The location of the first tap ($z = 0$) means that the pressure drop will include entrance effects. The data falling above the theoretical line supports this. To account for the entrance effects, Shah defines an apparent Fanning friction factor (Shah, 1978) to replace equation 3.5. Figures B.3a and B.3b also show a comparison of the experimental data to the entrance effects predicted by Shah (1978). The entrance effects only account for 1% of the difference between the experimental measurements and the theoretical prediction using equation 3.6.
B.5 Two-phase Pressure Results

The two-phase pressure results indicate a difference between measurements before the channel exit (Taps 1 & 2) versus after the exit (Taps 1 & 3). Characterizing the two-phase pressure drop relies on the gas two-phase flow multiplier ($\phi^2_{G}$) defined as the ratio of the experimentally measured two-phase pressure to the single-phase gas pressure. Figure B.4 shows a comparison between $\phi^2_{G}$ versus superficial gas velocity ($U_G$) for measurements taken between Taps 1 & 2 and between Taps 1 & 3. The experimental data points for Tap 2 represent a 30 minute average of the measured two-phase pressure, while the measurements shown for Tap 3 represent the average of all the experimental data points for a given superficial gas velocity. Both measurements show the same trend of $\phi^2_{G}$ decreasing with increasing superficial gas velocity and $\phi^2_{G}$ increasing with increasing superficial liquid velocity. However, the two measurements differ in terms of the magnitude of $\phi^2_{G}$. The measurement between Taps 1 & 3 fall significantly higher than the corresponding measurements between Taps 1 & 2.

The data for Taps 1 & 3 in figure B.4 only represent the average measurement of $\phi^2_{G}$ for clarity. Looking at the individual experimental data points for $\phi^2_{G}$ reveals an interesting behavior (figure B.5). Particularly at the lower superficial gas velocities, the value of $\phi^2_{G}$ varies well outside the uncertainty of the measurement determined from the Kline-McClintock method for the equation $\phi^2_{G} = P_{tp}/P_G$. Consequently, some physical mechanism—such as a change in flow behavior—must cause the difference between individual measurements and the difference between the two test cases (Taps 1-2 and Taps 1-3).
(a) $U_L = 5.0 \times 10^{-5}$ m/s.

(b) $U_L = 5.0 \times 10^{-4}$ m/s.

(c) $U_L = 1.0 \times 10^{-3}$ m/s.

(d) $U_L = 1.0 \times 10^{-2}$ m/s.

Figure B.4: Experimental gas two-phase flow multiplier versus superficial gas velocity comparing data at Tap 2 and Tap 3.
Figure B.5: Experimental gas two-phase flow multiplier versus superficial gas velocity measured between Taps 1 & 3.

(a) $U_L = 5.0 \times 10^{-5}$ m/s.

(b) $U_L = 5.0 \times 10^{-4}$ m/s.

(c) $U_L = 1.0 \times 10^{-3}$ m/s.

(d) $U_L = 1.0 \times 10^{-2}$ m/s.
B.6 Flow Behavior Along the Channel

With the test conditions remain the same between the two test cases, a change in the flow behavior likely results in the difference between individual measurements for Taps 1 & 3. In the channel, the air-water flow forms as a stratified flow, in which a water film moves along a side wall, filling the entire height of the channel but not the entire channel width. Visualization of the water film showed the film did not change during the duration of an experiment in the region between Taps 1 and 2. This allowed for the measurement of the two-phase pressure between Taps 1 & 2 first before subsequently measuring the two-phase pressure between Taps 1 & 3 by switching the valve. While ideally the measurements would occur simultaneously, the steady dynamics of the system allowed for the sequential measurements.

While the water film did not change during an experiment, the water film did change between different experiments conducted at the same test conditions. Figure B.6 shows a comparison between water films across several test conditions. The compressed aspect ratio of figure B.6 causes the wavy appearance of the films. Typically, the water film changes the most around the water inlet. For example, Test 2 showed a water droplet near the water inlet separate from the water film whereas Test 1 had a smooth connection to the inlet for \( U_G = 4.23 \) m/s at \( U_L = 1.0 \times 10^{-2} \) m/s (figure B.6b). The films, however, do remain similar. As the measurements between Taps 1 & 2 did not show such extreme variation outside of the uncertainty (figure B.4) when compared to the variation of measurements between Taps 1 & 3 (figure B.5), the change in the film behavior between experiments can neither account for the variation nor the differences between the two test cases (Taps 1-2 and Taps 1-3).
(a) $U_G = 0.85 \text{ m/s and } U_L = 5.0 \times 10^{-4} \text{ m/s.}$

(b) $U_G = 4.23 \text{ m/s and } U_L = 1.0 \times 10^{-2} \text{ m/s.}$

(c) $U_G = 5.08 \text{ m/s and } U_L = 1.0 \times 10^{-3} \text{ m/s.}$

Figure B.6: Comparison of the water film thickness for different tests.
B.7 Isolating the Influence of the Expansion to the Exit Manifold

With the behavior of the stratified flow before the exit unable to account for the variations of the measurements, determining whether the two-phase pressure measurements between Taps 1 & 2 or between Taps 1 & 3 better represent the pressure loss associated with stratified flow will narrow the focus to the water accumulation at the channel exit. In a stratified flow, a distinct boundary exists between the air and the water. Neglecting the capillary forces would mean each phase experiences the same streamwise pressure drop. To first approximation, one could assume that the velocity at the interface between the fluids equals zero—i.e. a separating wall. To calculate the two-phase pressure drop would then only require calculating the single-phase pressure drop in a channel of reduced width. Visualization of the water film allowed for the determination of the water film thickness. By subtracting the water film thickness from the channel width gives a single-phase gas flow in a smaller channel. Equations 3.6, 3.5, 2.71 determine the pressure drop of the channel of reduced width.

Figure B.7 compares the reduced width calculation to the two-phase measurement between Taps 1 & 2. The reduced width method shows that the two-phase pressure measurements between Taps 1 & 2 well represent the stratified flow behavior. The error bars for the pressure measurement account for the ±17.2Pa accuracy of the pressure transducer. The error bars for the reduced width calculation represent the uncertainty in the film thickness of ±0.11mm. Even with the uncertainty in the measurement, the water film does not block off enough of the channel width to produce the pressure drop seen in the measurement between Taps 1 & 3. Therefore, a loss must occur at the exit of the channel as the flow behavior agrees with the two-phase pressure measurement between Taps 1 & 2.

Investigating the water accumulation at the channel exit gives insight into the variation of the measured pressure between Taps 1 & 3 for a given test condition. The flow behaved two ways at the exit: periodic oscillations or stationary. In the periodic case, water at
Figure B.7: Comparison of the gas two-phase flow multiplier versus superficial gas velocity determined by experimental measurement and the reduced width method between Taps 1 & 2.
Figure B.8: Observed periodic behavior of the water at the channel exit viewed top-down (flow left to right; water indicated as blue).

the exit could periodically block the channel (figures B.8a and B.8b) and then break apart (figures B.8c and B.8d). Under the same test conditions, the water film would thicken near channel/manifold edge (figures B.9a and B.9b) and remain stationary for the duration of the experiment. Although the water appears to block the entire channel exit, the camera resolution prevents determining how close to the far wall the water extends. Unfortunately, top down images do not provide information on how the water blocks the channel height as a function of the channel width. For the test velocities of $U_L = 5.0 \times 10^{-5}$ to $1.0 \times 10^{-3}$m/s between $U_G = 0.51$ & 1.27m/s, the periodic case corresponds to the low $\phi_G^2$ while the stationary case corresponds to the higher $\phi_G^2$ in figure B.5. In the intermediate range of the gas two-phase flow multiplier both the periodic and stationary behavior can produce similar pressure measurements. Additionally, as the gas velocity increases, it becomes difficult to discern the flow behavior at the exit and appears to approach the stationary behavior.
Figure B.9: Observed stationary behavior of the water at the channel exit viewed top-down (flow left to right; water indicated as blue).

B.8 Pressure Increase at the Exit

Figure B.10 shows the pressure change across the exit ($\Delta P_{exit}$) defined as:

$$\Delta P_{exit} = P_{1,3} - P_{expected}$$  \hspace{1cm} (B.42)

where $P_{1,3}$ equals the experimentally measured pressure drop between Taps 1 & 3 whereas the expected pressure with no loss ($P_{expected}$) equals:

$$P_{expected} = \phi^2 G P_{sp,3}$$  \hspace{1cm} (B.43)

where $P_{sp,3}$ equals the single phase pressure measurement between Taps 1 & 3 while $\phi^2 G$ equals the gas-two phase flow multiplier determined from the two-phase pressure measurements between Taps 1 & 2. Defining $\Delta P_{exit}$ in this manner removes the contribution of the frictional two-phase pressure drop, isolating the pressure increase due to the exit.

Interestingly, the data does not show a clear dependence on the superficial liquid velocity as the data showed similar results for the four different $U_L$ (figure B.10). Additionally, the exit pressure data for $U_L = 1.0 \times 10^{-3}$ and $1.0 \times 10^{-2}$ m/s show a clear dependence on the superficial gas velocity, showing an increase with increasing $U_G$ but appearing to reach a...
Pa plateau after $U_G = 3.0 \text{ m/s}$. The behavior suggests that set parameters determine the exit pressure. For this experiment, the exit geometry and the surface tension remain constant. With only a single fluid pair (air-water), a comment cannot be made on the influence of surface tension, and instead will focus on the geometric effects.

**Comparison of Sudden Expansion Models to the Experimental Data**

As shown in figure B.2a, the microchannel ends at a circular manifold located 90° relative to the flow path, which equates to a sudden expansion. Section B.2 introduced several
models for the pressure change resulting from a sudden expansion. Of the selected models, the model of Abdelall et al. (2005) came closest to the experimental data (figure B.11) with a mean absolute percent error of 92% defined as the sum of the absolute difference between the prediction and the experimental measurement, both divided by the experimental measurement. The other selected correlations predicted pressures less than those of Abdelall et al. (2005). Thus, all the models under-predict the experimental data.

Several possible explanations could account for the difference between the experimental data and the prediction of the models. Of the selected models, only Abdelall et al. (2005) tested geometries with a hydraulic diameter less than 1 mm. However, all of the models assume a turbulent flow in circular pipe. The difference between a laminar or turbulent flow for a sudden expansion comes from the treatment of the area averaged velocity terms. For single-phase turbulent flow $\langle u^2 \rangle / \langle u \rangle^2 \approx 1$, allowing for the direct substitution of the continuity equation into the conservation of momentum equation. Numerically integrating the analytical solution for the single-phase velocity profile in a rectangular duct for both $\langle u^2 \rangle$ and $\langle u \rangle^2$ showed that $\langle u^2 \rangle / \langle u \rangle^2 \approx 1.2$. Therefore a correction of 20% to the turbulent prediction will account for laminar flow in a rectangular duct. However, a 20% increase of the prediction will not account for the difference between the measurement and the prediction. Abdelall et al. (2005) also took into account the reversible pressure change derived from the energy equation that would scale 2 to 1 for laminar to turbulent flow, which also cannot increase the prediction to the experimental measurement. Therefore, neither the flow type nor the geometry can account for the difference.

This leaves the correlations for the void fraction ($\alpha$) as a possible reason for the deviation between the measured exit pressure and the predictions. As detailed in section B.2, several void fraction correlations exist. For this experiment, the flow formed a stratified pattern
Figure B.11: Comparison of the measured $\Delta P_{\text{exit}}$ to the prediction of Abdelall et al. (2005).

which allows for the determination of the void fraction in the channel as

$$\alpha = 1 - s_L$$  \hspace{1cm} (B.44)

where $s_L$ equals the liquid saturation defined as the ratio of the volume of fluid to the volume of the channel. For stratified flow, this equals the ratio of the water film thickness to the width of the channel. However, using equation B.44 in the sudden expansion model of Abdelall et al. (2005) changed the exit pressure by only 0.1-20 Pa. Thus, the void fraction does not account for the difference between the prediction and the measurement.

Based on the comparison of existing models of the sudden expansion to the measured pressure loss at the exit of the microchannel, the experiment does not compare well to the sudden expansion models. However, the comparison itself breaks down. Sudden expansion correlations assume the flow remains horizontal and has achieved fully-developed flow at both locations used to determine the pressure difference. As figure B.2 shows these assumptions break down. In particular, the exit geometry has the flow turning $90^\circ$, which itself carries a loss. Additionally, the flow likely has not become fully developed before reaching Tap 3
due to the flow changing direction and the water accumulated at the edge of the manifold (figures B.8 and B.9a). Thus, the sudden expansion may account for a fraction of the pressure loss but the experimental measurement will measure a total loss including components the sudden expansion models cannot account for. Therefore, measurements of the two-phase frictional pressure drop across a geometric change is not recommended, as it remains unclear how to predict the pressure loss resulting from the channel expanding to a manifold, preventing the isolation of the two-phase frictional pressure drop.

B.9 Conclusion

This work conducted an experimental study of the pressure loss associated with air-water two-phase flow across an exit of a microchannel to a larger exit manifold. The microchannel has dimensions 3.23mm wide by 0.304mm high by 164mm long and exits into a circular manifold of 1.4 cm diameter oriented 90° relative to the flow direction. The major results include:

1. The majority of the exit pressure data fall between 150Pa to 400Pa with a slight dependence on superficial gas velocity but independent of the superficial liquid velocities.

2. Treating the exit as a sudden expansion and comparing the results to models for the pressure loss associated with a sudden expansion reveals the measured loss includes more than the loss associated with the sudden expansion as the best sudden expansion model (Abdelall et al., 2005) produced a mean absolute percent error of 92%.

3. Measuring the two-phase pressure drop across an exit is not recommended.
C The Concus-Finn Condition

The corners of non-circular channels greatly influence the behavior of the two-fluid interface. Consider, for example, two immiscible fluids (A and B) bounded by a solid surface (l) where the two fluids form an equilibrium interface of surface area \( \mathcal{L} \) shown in figure C.12. The total energy of the interface will consist of the free-surface energy (equation C.45), the wetting energy (equation C.46), the potential energy (equation C.47), and a general energy term to account for a change in volume (equation C.48) (Finn, 1986).

\[
\begin{align*}
E_s &= \sigma \mathcal{L} \quad \text{(C.45)} \\
E_W &= -\sigma \varepsilon \mathcal{J} \quad \text{(C.46)} \\
E_g &= \int \varphi \rho dx \quad \text{(C.47)} \\
E_v &= \sigma \lambda V \quad \text{(C.48)} \\
\end{align*}
\]

where \( \sigma \) is the surface tension, \( \varepsilon \) the relative adhesion coefficient equal to \( \cos \gamma \), \( \gamma \) the contact angle, \( \mathcal{J} \) the wetted surface area near the contact line, \( \varphi \) a potential function of energy per unit mass, \( \rho \) the density specific to the limits of integration, \( x \) a position within the medium, and \( \lambda \) a Lagrange multiple that allows the system to satisfy the volume (\( V \)) constraints. Let the equation \( z = \zeta(x, y) \) represent the interface over a domain \( \Omega \). The energy equation becomes:

\[
E = \sigma \left( \int_{\Omega} \sqrt{1 + \zeta_x^2 + \zeta_y^2} d\omega + \frac{1}{\sigma} \int \varphi \rho dx + \lambda \int_{\Omega} \zeta d\omega \right) \quad \text{(C.50)}
\]

where \( \mathcal{L} \) and \( V \) have been replaced with their integral forms, the subscripts \( x \) or \( y \) correspond to derivatives with respect to \( x \) or \( y \), and \( d\omega \) represents a differential area of \( \Omega \). In this case, the wetting energy (equation C.46) has been neglected as \( E_W \) will not change with a displacement of the surface as long as the surface properties remain the same. Substituting
\[ \tilde{\varsigma} = \varsigma(x, y) + \epsilon \eta(x, y), \]  
where \( \eta(x, y) \) represents a virtual displacement to the surface and neglecting terms of \( O(\epsilon^2) \), the application of the virtual work theorem results in:

\[ \nabla \cdot \left( \nabla \tilde{\varsigma} \frac{1}{\sqrt{1 + \tilde{\varsigma}_x^2 + \tilde{\varsigma}_y^2}} \right) = \frac{\rho \varphi}{\sigma} + \lambda \quad (C.51) \]

for the equilibrium surface \( \varsigma(x, y) \) since the energy should not change with a virtual displacement. If the only potential energy results from gravity, \( \varphi = gz \) with \( g \) the gravitational constant. The first term on the right hand side of equation C.51 becomes \( \rho gz/\sigma \). Defining \( \rho g/\sigma \) as \( \kappa \) with \( z = \varsigma(x, y) \), the term becomes \( \kappa \varsigma \). The surface must meet the solid boundary \( (l) \) at a constant angle \( \gamma \). Thus equation C.51 has the boundary condition:

\[ \left. \frac{\nabla \varsigma}{\sqrt{1 + \tilde{\varsigma}_x^2 + \tilde{\varsigma}_y^2}} \right|_l \cdot \mathbf{n} = \cos \gamma \quad (C.52) \]

where \( \mathbf{n} \) is the outward unit normal of the surface and \( \gamma \) the experimentally determined contact angle between the two fluids and the surface.
Concus & Finn (1969) studied the solution to equation C.51 in a wedge of interior angle $2\theta$ under the condition of equation C.52. The wedge forms from two vertical walls ($l_1$ & $l_2$) where gravity acts toward the base (figure C.13). When $\kappa = 0$, for micro-gravity conditions or $\kappa > 0$ for a gravitational field, a solution cannot exist if:

$$\gamma + \theta < \frac{\pi}{2}$$  \hfill (C.53) \]

Under the condition of equation C.53, the fluid seeks the corners of the domain and will rise to unpredictable heights. Later, Concus and Finn found that a bounded solution exists for a capillary surface in a gravitational field (Concus & Finn, 1974a) or in the absence of gravity (Concus & Finn, 1974b) if:

$$\frac{\pi}{2} - \theta \leq \gamma \leq \frac{\pi}{2} + \theta$$  \hfill (C.54) \]

when the right-hand side of equation C.51 does not equal zero. In the case of $\rho \varphi / \sigma + \lambda$ equal to zero, equation C.54 becomes a strict inequality.

To this point, the surfaces comprising the wedge have the same contact angle ($\gamma$). In practical situations, the materials forming the wedge need not have the same contact angle. Concus
\& Finn (1996) extended their previous work to a wedge with walls of different contact angle. Equation C.51 remains the same and the boundary conditions have the same form as equation C.52 except $\gamma$ changes to $\gamma_1$ on $l_1$ and $\gamma_2$ on $l_2$ (figure C.13). In this case, there exist five regions of existence and non-existence of a solution. The possible combinations of $\gamma_1$ and $\gamma_2$ that result in a solution to equation C.51 with continuous unit normal lie in the region $\mathcal{R}$ in figure C.14. In regions $\mathcal{D}_1^-$ and $\mathcal{D}_1^+$ a solution cannot exist. Solutions will exist in regions $\mathcal{D}_2^-$ and $\mathcal{D}_2^+$ but will not have continuous unit normal to the vertex of the wedge (Concus \& Finn, 1994).

Consider the domain shown in figure C.14 when the wedge forms a corner of $2\theta = \pi/2 \ (90^\circ)$, where both walls have the same contact angle ($\gamma_1 = \gamma_2$). The line of $\gamma_1 = \gamma_2$ will intersect $\mathcal{R}$ at $\gamma = (\pi - 2\theta)/2$ (figure C.15) which corresponds to equation C.53 and the lower bound of equation C.54. Therefore, as long as $(\gamma_1, \gamma_2)$ falls in $\mathcal{D}_1^+$, the fluid will rise to unpredictable heights. The line $\gamma_1 = \gamma_2$ will exit $\mathcal{R}$ at $\gamma = (\pi + 2\theta)/2$ which corresponds to the upper bound of equation C.54. In this condition a surface cannot exist that will include the corner. This becomes clearer when applying equation C.54 to a horizontal microchannel. Rath \& Kandlikar (2011) tested the behavior of a drop emerging from a gas diffusion layer into a corner for various corner angles. The authors found that in region $\mathcal{R}$, the drop would fill the corner. In region $\mathcal{D}_1^-$, the drop would not fill the corner, with the drop pinned to the two
The preceding discussion considered a capillary surface exposed to a stagnant fluid. In practical situations, the second fluid will have a nonzero velocity. Gopalan & Kandlikar (2011) investigated the influence of air velocity on emerging water drops in parallelogram shaped channels of different corner angles. The authors tested a range of superficial air velocities of 0.2-2 m/s. At a corner angle of $\pi/2$, the air velocity did not influence the Concus-Finn condition with water filling the corner under all velocities. Similarly, for corners of angle $\pi/6$, the water did not fill the corners, as expected from the Concus-Finn condition. Conversely, when the corner angles equaled $5\pi/18$ ($50^\circ$), the water fills the corners at low air velocities but not high air velocities. The test conditions thus influence a fluid filling a corner. However, the Concus-Finn conditions provides useful insight into the behavior of a capillary surface in a corner for both vertical and horizontal wedges.
C.1 Concus-Finn Condition for the Compared Mixed-wettability Experiments

In chapter 3 §3.5.4, a comparison of existing adiabatic two-phase pressure experiments in single mixed-wettability channels revealed that the trend of the two-phase pressure drop with contact angle became consistent with the liquid capillary number. The range of the critical capillary number—$1.38 \times 10^{-4}$ to $9.63 \times 10^{-4}$—contains the results of Wang et al. (2014b), which showed inconsistent trends. The current study and the works of Cho & Wang (2014a) and Stevens et al. (2017) fall in the domain $\mathcal{R}$ for both the hydrophilic and mixed wettability experiments (figure C.16). Conversely, the results of Wang et al. (2014b) do not meet the Concus-Finn condition, falling in regions $\mathcal{D}_1^-$ and $\mathcal{D}_1^+$ where water will wick into the corner and not fill the corner, respectively (figure C.16. This assumes the contact angle of plexiglass equals $60^\circ$. This could lead to the inconsistent trends seen by Wang et al. (2014b) instead of resulting from the dependence of the $Ca_L$.

![Figure C.16](image)

Figure C.16: The contact angles of different works plotted in the Concus-Finn domain. Filled symbols represent the corners in the mixed-wettability experiments and the larger-unfilled symbols represent the corners in the hydrophilic tests.