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Publication Date
1971-08-01
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AEC Contract No. W-7405-eng-48
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THE DEPENDENCE OF MULTIPLICITY ON RECOIL MOMENTUM IN HIGH-ENERGY COLLISIONS

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August 31, 1971

ABSTRACT

Determinations of the average multiplicity of secondaries as a function of the recoil momentum of the target particle will measure momentum-space correlation ranges in high-energy inelastic collisions.

Multiparticle production processes at very high energies are predicted by various models to exhibit the phenomenon of scaling [1-5], together with a logarithmic energy dependence of the average multiplicity of secondaries [1,4,5]. In order to distinguish between these models, it will be important to investigate the correlations between produced particles, and, in particular, to determine the ranges in momentum space over which such correlations can extend. It might appear that these investigations would inevitably require the difficult experimental task of measuring a large number of high-momentum secondary tracks in each event. However, it is the purpose of this article to point out that some two-particle correlation ranges can be determined by a measurement of one low-momentum track, namely that of the recoiling target particle, in conjunction with a simple count of the number of charged secondaries produced. We shall also discuss some related predictions concerning the results of this kind of measurement.

Consider first an inclusive process of the type
\[ a + b \rightarrow b' + c + X, \]
where 'b' is the target and 'b' is the recoiling target, 'c' is another observed secondary, and 'X' represents any combination of other particles. The particle 'b' will be easily distinguished from the other secondaries at the low momenta we wish to consider. Let \( f_{aab'c}(s; \mathbf{p}_b', \mathbf{p}_c) \) be the two-particle distribution function for this process at c.m. energy-squared 's',
\[
f_{aab'c}(s; \mathbf{p}_b', \mathbf{p}_c) = \frac{d^3p_b'}{E_b'} \frac{d^3p_c}{E_c} \frac{d\sigma_{ab}(s)}{d\sigma_{ab}} \, ,
\]
where \( \sigma_{ab}(s) \) is the ab total cross section. Let \( f_{aab'}(s; \mathbf{p}_b) \) be the single-particle distribution for the process \( a + b \rightarrow b' + X, \) in which 'c' is not observed. Then the average multiplicity of particle type 'c' in conjunction with a recoil of momentum \( p_b', \) is
\[
\bar{n}_{aab'}(s; \mathbf{p}_b') = \int f_{aab'c}(s; \mathbf{p}_b', \mathbf{p}_c) \, d^3p_c/E_c \, f_{aab'}(s; \mathbf{p}_b').
\]
for $p_c$ very different from $p_b$, where $f_{abc}$ is the single-particle distribution for the process $a + b \rightarrow c + X$.

In order to display the phase space of particle $c$, it is most convenient to introduce the longitudinal rapidity variable $y$ [3,5], in terms of which we may write the laboratory four-momentum of $c$ as $p_c = (\mu \cosh y, p_{T1}, p_{T2}, \mu \sinh y)$, where $\mu = (p_{T1}^2 + p_{T2}^2 + m_c^2)^{1/2}$. When the particle $b'$ is nonrelativistic its rapidity is simply $\beta_L$, the longitudinal component of its velocity. The corresponding rapidity of the system $cX$ for large $s$ is $\frac{1}{2} \ln(s/\mu^2 m_b)$, and the invariant mass-squared of this system is $s' = s \beta_L$. Consequently the longitudinal phase space of $c$ is given by

$$\ln(\mu/\beta_L m_b) \leq y \leq \ln(s/\mu m_b).$$

This region is shown in fig. 1, together with the form of single-particle distribution that is expected if two-particle correlations have finite rapidity ranges [5].

If we do not observe the recoil $b'$, the phase space of $c$ extends throughout the region

$$\ln(\mu^2 m_b) \leq y \leq \ln(s/\mu m_b),$$

and the overall multiplicity of particle type $c$ in $ab$ collisions has the form

$$\overline{n}_{abc}(s) = \int dp_{T1} \int dp_{T2} \int dy f_{abc}(s; p_c) \ln(\mu/\beta_L m_b)$$

$$= A^c \ln(s/m_c^2) + B_a^c + B_b^c + O(s^{-1/2} ln s),$$

where $A^c$ (independent of $a$ and $b$) is associated with the "pionization" region of the rapidity plot, $B_a^c$ (independent of $b$) is associated with the "fragmentation of $a$" at the high-rapidity end of the plot, and correspondingly for $B_b^c$.

If, on the other hand, we do observe $b'$, but at a recoil momentum sufficiently low that $\ln(\mu/\beta_L m_b)$ is greater than the range of the rapidity correlation between $b'$ and $c$, then we may use the factorization property (3) and eq. (2) to obtain

$$\overline{n}_{abc'}(s; p_b) = \int dp_{T1} \int dp_{T2} \int dy f_{abc}(s; p_c) \ln(\mu/\beta_L m_b)$$

which gives

$$\overline{n}_{abc'}(s; p_b) = \overline{n}_{abc}(s; p_b) = A^c \ln(s/m_c^2) + B_a^c \ln \beta_L + B_b^c + O(s^{-1/2} \ln s),$$

where the constants $A^c$ and $B_a^c$ are the same as those appearing in eq. (6). Equation (8) expresses the fact that in the kinematic region we have chosen, which is essentially the triple-Regge region [7], the multiplicity depends logarithmically on the missing mass $s'$ and does not receive any contribution from the fragmentation of $b$. If eqs. (6) and (8) are summed over all charged-particle types $c$ and the terms involving $\ln m_c$ are absorbed in the constant $B_a^c$, these equations become predictions of the average multiplicity of charged secondaries.

Equation (8), which is our main result, shows that at fixed, small longitudinal recoil velocities the multiplicity $\overline{n}_{abc'}$ is completely determined by the overall multiplicity $\overline{n}_{ab}$, and is independent
of the transverse recoil velocity†† and of the type of target particle \( c \). The value \( B_{\text{max}} \) of \( B \) at which the prediction (8) begins to break down is related to the rapidity range \( \Delta \) of the correlation between particles \( b' \) and \( c \):

\[
\Delta \approx 4 \ln(\mu_0/B_{\text{max}}).
\]

(9)

A common expectation, based on the multi-Regge model \([5,8]\), is that \( \Delta \) might be between 1 and 2 units of rapidity. If one assumes all the charged secondaries to be pions, the average value of \( \Delta \) can be computed from their transverse-momentum distribution to be

\( \langle \Delta \rangle \approx 1 \text{ GeV}. \) This implies that eq. (8) would apply up to longitudinal recoil momenta in the range 50-130 MeV/c. However, if almost all the secondary pions are in fact due to the decay of \( \rho \) mesons, then \( \Delta \) could be considerably less than 1, because pions with rapidities near that of \( b' \) would be coming from \( \rho \) mesons with higher, and therefore uncorrelated, rapidities. Such a mechanism would thus increase the range of recoil momenta for which eq. (8) is valid. At the other extreme, if there are very long correlation ranges eq. (8) will not be true at all. This in itself would be an experimental observation of great interest.

It is a pleasure to acknowledge conversations with M. B. Green.

†† In other terms, which may be more familiar, we may say that at fixed missing mass \( s' \) in this kinematic region the multiplicity is independent of \( t = (p_b - p_{b'})^2 \).

‡ This is essentially the case in the pion-exchange multiperipheral model, ref. [1].

REFERENCES


FIGURE CAPTION

Fig. 1. Longitudinal rapidity plot. The phase space of particle \( c \) lies between the dashed lines.
Fig. 1
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