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Wetlands Mitigation Banks: A Developer’s Investment Problem

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Abstract

We study a land developer's decision to invest in a wetlands mitigation bank. The state at which it is optimal to "cash in" the investment in return for restoration credits increases with uncertainty. We calibrate and numerically solve a stochastic control model which describes the developer's investment problem. We study the effect of the parameters of the model on the investment trajectory and the optimal stopping state. A subsidy increases the option value of the investment and the stopping state. A small decrease in the variance of the state dynamics decreases the option value of investment and the stopping state.
Wetlands Mitigation Banks: 
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I. INTRODUCTION

Wetlands provide public goods as "kidneys of the landscape", by contributing to water quality purification, groundwater recharge and flood control. Since private users and owners do not capture all benefits from wetlands there is insufficient incentive to preserve them. Wetlands functions such as wildlife habitat and assimilation of pollutants are not efficiently allocated if the market does not reflect their functions' relative scarcity and value. Therefore, public resource agencies serve a role in establishing and enforcing regulations to protect the natural capital and mitigate wetland losses.

Wetlands Mitigation Banks (WMBs) are a component of recent national and state legislation to curtail loss of wetlands and regain acreage that supports wildlife habitat and other functions. The WMB program designates sites for the creation, restoration and/or enhancement of wetlands. These sites are used to offset the unavoidable losses from future development in the same watershed.

We study a land developer's investment in a WMB. The primary contribution of this paper is to develop a method that will help in answering two questions about restoration using WMBs. How much expenditure in restoration is optimal? At what state of wetlands quality is it optimal to stop the restoration? The answers to these questions offer insight for WMB policy by indicating how ecological factors affect economic behavior.

The number of credits that a developer obtains per dollar of investment depends on the success of the wetlands restoration, which is stochastic. The developer is able to "cash in"
credits in exchange for permission to develop other wetlands. Therefore, demand for credits in the WMB is derived from the demand for development projects. The developer minimizes the cost of obtaining credits which will be exchanged for permission to undertake future development projects (e.g., housing, shipping, port expansion).

The WMB program was introduced as a national policy in August 1993. There is insufficient data to estimate a complete model. We use the small amount of available data to calibrate a stochastic control model which describes the developer's investment problem. We solve the model numerically and perform sensitivity analysis. This procedure: (i) provides information on the optimal dynamics of investment; (ii) it enables us to study the qualitative effect of changes in various parameters; and (iii) it suggests the order of magnitude of changes in investment resulting from changes in parameter values. Given the paucity of data, the third type of information is particularly important, because it indicates the kind of data that is most urgently needed to improve policy prescriptions. We study changes in parameter values of the restoration costs, the (stochastic) biological growth equation, and the interest rate. Some of these parameter changes may be endogenous, if, for example, they are due to subsidies. We show how these parameters affect the trajectory of investment, the value of the investment program, and the optimal stopping state (i.e., the state at which it is optimal to cash in the investment). For some parameter values, it does not pay the developer to invest in the WMB. In these cases, subsidies are needed to support the WMB.

Section II describes WMB policy, and Section III discusses relevant literature. The analytical model is described in Section IV. The data for the empirical application is
discussed in Section V, and the model is solved in Section VI. Section VII provides concluding remarks.

II. INSTITUTIONAL BACKGROUND

WMBs are designed to assist in meeting the goal of the 1993 Federal Wetlands Policy and Clean Water Act regulations [Section 404 (b) (1)] for promoting "no overall net loss" of wetlands (Federal Register [10]). The broadly defined goal is to maintain a steady state of physical and biological functions and human use values of wetlands. Under the regulations, land developers must mitigate future unavoidable damage to wetlands by creating, restoring, or enhancing other wetlands before receiving a permit to develop. There has been net loss when development and restoration have been approved simultaneously. The WMB policy may stem this loss by requiring restoration before approving development projects. Urban and industrial land developers are subject to this national policy regulating discharges of dredge and fill material to wetlands through land development. Agricultural developers face different regulations. Section 404 of the Clean Water Act expressly exempts discharges to wetlands associated with "normal agricultural activities" such as plowing, seeding, cultivating, minor drainage and harvesting (Carriker, [5]).

The WMB program works in the following manner: In order to obtain permission to develop a different wetlands area, a developer is required to have credits obtained from investment in the completed rehabilitation of a WMB site. WMB credits are based on the value of restored wetlands functions. The program encourages protection and rehabilitation of some wetlands areas as a precondition for developing other areas.
WMB sites are determined through a process referred to as "advanced identification" (ADID). This process is an important feature of WMB's, and it provides a rationale for one of the policy experiments that we study later in this paper. We therefore discuss it in some detail here. ADID is a means of setting watershed conservation priorities by identifying wetlands functions and sites for protection and restoration (King and Bohlen [20]). It is widely used in planning WMB sites in California, Florida, and Louisiana. ADID produces maps of viable sites, it assesses functions and values of wetlands in a watershed, and it designs a restoration program. ADID identifies the most degraded, least valuable wetlands for future development, and selects other sites for restoration. In this manner, the process promotes the goal of maximizing public benefits from wetlands, while satisfying the constraint of no overall net loss of wetlands functions.

There are substantial costs involved in ADID. Some of these costs, such as the design of a restoration program for a particular area, are unavoidable. The fact that the public, rather than the WMB investor, bears these costs, means that ADID constitutes an implicit subsidy. That is, current WMB policy provides a built-in investment subsidy. In view of this, we use our model to investigate the effect of such a subsidy on the private investor's decision.

The subsidy promotes restoration efforts, which results in investors earning more credits. That, in turn, allows investors to undertake more development. Viewed in this light, ADID, or any other WMB investment subsidy, appears to be an indirect subsidy for development. Since WMBs are part of an effort to preserve wetlands, incorporating an implicit development subsidy into their design may appear perverse. Since, in addition, the policy requires public funds, it appears especially unattractive in a period of tight public
budgets. This observation may suggest that an investment subsidy is not a reasonable policy, and that perhaps it does not merit serious study. We have three responses to this objection. First (and most importantly), regardless of whether an investment subsidy is an intelligent policy option, it is implicitly included in the current design of WMBs. It is therefore worth learning about its effects on investor behavior. Second, an investment subsidy may be sensible, if we consider the political economy in which wetlands policy is determined. The goal of preserving wetlands functions resulted from a political process. The cost of meeting this goal has to be distributed in some manner, and an investment subsidy is one means of cushioning developers. Some such protection from loss may have made it easier (or possible) to push through the necessary legislation. Third, the particular type of subsidy implied by ADID may be efficient, because of the variety of the tasks that ADID comprises. Due to coordination problems, some of these tasks can be carried out more efficiently by a public agent, responsible for an entire watershed, rather than by an individual investor. If, as is likely, it is difficult to allocate these costs to individual projects, it is sensible to pay for them with public finance. In this case, there may be no intention of subsidizing investors. Nevertheless, the policy is effectively a subsidy. For example, the California Coastal Conservancy is a state agency that takes the lead in preparing restoration plans and pays for other ADID-type tasks. According to Hefelfinger [16] the agency believes that the likelihood of success of the restoration project is determined at the planning stage. The public thus benefits from state control. For all of these reasons, it is worth studying the effect of an investment subsidy.
The WMB policy addresses the lag time and uncertainty in any restoration effort. There may be interrupted flow of wetlands services and "net loss" due to the uncertainty of ecosystem replacement. Restoration means returning an ecosystem to an approximation of its condition prior to disturbance (National Research Council [25]). This requires ensuring that the ecosystem structure and functions are operating again. The multiple biological, chemical and physical factors that affect hydrologic, vegetative, and faunal recovery of a particular wetland ecosystem make it inappropriate to assume a static, deterministic relation between input costs and resulting quantities of recovered wetlands (Castelle [6]).

The diversity of species in ecosystems such as wetlands is correlated with the size of the habitat area; larger areas devoted to restoration in a watershed have greater potential to sustain ecosystems (National Research Council [25]). WMBs are intended to create large, high quality habitats which incorporate entire ecosystems, in contrast to previous mitigation efforts which tended to be fragmented and threatened by adjacent land uses (Anderson and Rockel [1]).

The larger site can be used by multiple developers, who are able to take advantage of economies of scale which do not occur on smaller, fragmented sites. There is evidence of a 3.1% decline in costs per acre for each 10% increase in project size (King and Bohlen [21]). The multiple investors in the WMB can pool financial resources, planning and scientific expertise (Reppert [28]). Often restoration tasks on one site involve joint efforts of public resource agencies, the private sector, and non-profit environmental groups. The momentum generated may lead to a more successful restoration project with all participants informed of mitigation activities throughout the regulatory process (Griswold [14]). This involvement
benefits the developer attempting to win support for a development project, provided restoration is successful.

Regulatory approval of a development project depends on the level of restoration credits in the developer’s WMB account. Credits are denominated in Habitat Units (HU) and are a measure of habitat value. The number of HU’s is the product of the number of species or functions per acre at a wetlands, times the number of acres. The number of species or functions per acre is a measure of natural processes present in wetlands (hydrology, chemical transformation, flora and fauna production) (USFWS [32]). This number is measured before and after a restoration project to indicate the level of wetland habitat recovery. The number of HU’s are used to determine the "compensation ratio", defined as the number of wetlands acres which must be replaced for each acre damaged by development (Cruickshank [7]).

The rules for transactions between the WMB manager, regulators and developers, such as those for issuing credits and assessing improvements resulting from investment, are spelled out in a contract called a Memorandum of Agreement. This legal basis diminishes any credibility problem that regulators might face (e.g., the fear that the regulator might expropriate a developer’s investment by changing the rules of the game after the investment has been sunk.) The WMB policy at this point does not provide a clear endorsement of tradable credits. The developer and the WMB manager are able to buy and sell credits, but the rules for exchange between developers have not been determined.

The restoration investment can be one asset in a diversified portfolio. For some transportation development projects affecting several wetlands along a route, investing in more than one WMB may be optimal.
III. LITERATURE REVIEW

There has been little previous work in economic modelling of wetlands restoration. Several papers study the conversion of pristine wetlands to agricultural use and the potential reversion through farmland abandonment (Stavins [30], Stavins and Jaffe [31], Van Kooten [33], Kramer and Shabman [22]). These examine impacts of federal farm support and other programs which promote the drainage of wetlands for farming. Studies by Parks and Kramer [26] and Heimlich [17] examine restoration of wetlands through the Farm Bill's Wetlands Reserve Program (WRP). Parks and Kramer [26] explain participation in the WRP using land attributes and owner attributes. Using national data on currently cropped wetlands acreage, they measure potential acreage enrolled in the program. Their measure of wetland quality is the proportion of acreage that is idle. The authors acknowledge the limitation of this measure: it incorrectly suggests that cropped acreage returns to functional wetlands when left idle from farming activity. Heimlich [17] estimates the foregone crop revenues and costs of restoring currently cropped wetlands acreage through the WRP. He emphasizes the need for studies of the value of WMB credits resulting from restoration. Restoration under the regulatory WMB program appears to have significantly higher costs and monitoring requirements than restoration under the voluntary WRP. The higher costs are due to more complex hydrology and ecosystems outside of the midwest farmlands (of the WRP) that require more complex tasks and inputs for restoration (King and Bohlen [21]). Monitoring costs are significant for restoration under the regulated WMB program, but they may be negligible under the voluntary WRP.
Fisher and Hanemann [12] use the concept of an option value to model the decision to preserve or to develop pristine wetlands. In their model, preservation is equivalent to not developing the site, and is therefore passive. A WMB involves restoration rather than merely preservation; the former requires an active investment program, which must be determined.

The demand for WMB credits and restored wetlands is a derived demand resulting from uncertain future development. Credits in a WMB enable a developer to meet future demand. An inventory of restored wetland functions make it possible to comply with environmental regulations. In any period, the current wetlands inventory is a capacity constraint on current development plans. An individual may want to invest in a WMB when a restoration opportunity arises, even in the absence of plans for development, because of the option value of cashing in at a later time.

Applications of stochastic control in the literature on capital theory provide guidelines for determining the level of investment and optimal stopping state in our problem. Pindyck [27] examines models where investment expenditure is a sunk cost and the future value of a project fluctuates stochastically. He studies the effect of irreversibility on the planner's choice of when to invest. McDonald and Siegel [24] include both the investment costs and future project value as two stochastic processes in a model which they use to derive an investment rule. The assumption that both processes follow geometric Brownian motion leads to a rule to invest when the project value is twice the project costs. Brennan and Schwartz [2] derive a rule for operating and closing a mine. The rule determines the value of the resource as a function of the current state of the mine (open or closed) and the stochastic resource price.
Brock et al. [3] show how to determine stopping times (states) in a model of an asset whose intrinsic value follows a diffusion process with instantaneous mean and variance. Using various boundary conditions, they derive comparative statics for the interest rate and instantaneous variance. Our model is similar, except that we are concerned with investment decisions as well as stopping times. In addition, due to data limitations we cannot use any of the boundary conditions that they consider, and therefore rely on a different method to solve the problem.

IV. THE MODEL

The economic-ecological model for the WMB is used to determine the optimal stopping rule and path of investment. The ecological component of the model is a stochastic differential equation which describes the evolution of the wetlands habitat as a function of the current state of the habitat, investment in conservation and restoration, and a stochastic term of uncertain exogenous factors that change the wetlands habitat. The economic component of the model is an optimization problem. The developer chooses the optimal level of investment activity (a control rule) as a function of the state of the wetlands quality, and decides when to "cash in" the investment. The latter decision involves exchanging investment credits for permission to proceed with a commercial development which damages some other wetlands areas. The action (cashing in) taken at the final time is called stopping. The state of wetlands quality at the time of optimal stopping (a "Markov time") is called the stopping state.
The model uses the following notation:

\[ z(t) = \text{Quality of wetlands habitat at time } t. \]

\[ m(t) = \text{Level of discretionary spending on maintenance and restoration services (engineering, revegetation activities).} \]

\[ T = \text{Time at which developer cashes in her investment and earns restoration credits.} \]

\[ K(z) = \text{Present expected discounted market value of restoration credits for } z, \text{ the "intrinsic value" or "cash-in value" of } z. \]

\[ C(m,z;\beta) = \text{Restoration cost function depending on level of discretionary spending, site quality, and } \beta, \text{ a vector of other parameters (property tax, etc.)} \]

\[ r = \text{Developer's discount rate.} \]

The state variable \( z(t) \), the index of restored wetlands quality, follows a continuous stochastic process. The level of \( z \) determines the number of credits that a developer obtains from cashing in her investment. The expected present discounted value of these credits is \( K(z) \). We assume that the function \( K \) (but not its argument) is fixed and deterministic. A more complete model would allow the function \( K \) to evolve stochastically, to reflect changing demand for development projects.\(^1\) Our assumption of nonstochastic \( K \) focuses the model on the uncertainty associated with restoration efforts.

The domain of the state variable is divided into two regions, a continuation region and a stopping region. If the state is in the continuation region it is optimal to hold the asset, and to invest in restoration. If the state is in the stopping region, it is optimal to cash in the

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\( ^1 \) For example, \( K(z) \) might depend on the demand for housing in the area which evolves stochastically. In this case, the decision of whether to cash in the credits (i.e., the optimal stopping state), and of how much to invest in discretionary expenses would depend on both \( z \) and the stochastic parameters of \( K \).
investment. The boundary separating these regions is the optimal stopping boundary, or "stopping state" (Malliaris and Brock [23]), which we denote as $z^*$. 

The definition of quality is described in more detail in the next section. The level and speed of wetlands recovery depends on the amount of restoration maintenance and existing wetlands quality. The stochastic term captures the uncertain exogenous factors (biological, chemical, physical) which contribute to a change in wetlands. The following stochastic differential equation for $z$ includes an ecological uncertainty component $W$, which evolves according to Brownian motion.

$$dz = g(z,m)dt + \sigma z dW$$

where: $g(z,m)dt =$ expected trend or drift in wetlands quality; $\sigma z =$ the instantaneous standard deviation in site quality change; $dW =$ increment of the stochastic Weiner process, which is normally distributed.

The value function $J$ is the expected present value, under optimal behavior, of restoration costs, minus the value of credits obtained when the investment is cashed in. The choice variables are the investment rule $m(z)$ and stopping state $z^*$. The value function is

$$J(z,t) = \min_m \left\{ E \int_T^T C(m,z;\beta)e^{-r(T-t)}d\tau - K[z(T)]e^{-rT} \right\}$$

s.t. (1), $z(0) = z_0$, $m(z) \geq 0$.

The Bellman dynamic programming equation (DPE) is:

$$0 = J_t + \min_m \left\{ C(z,m) e^{-rt} + J_z g(z,m) + \frac{1}{2} \sigma^2 z^2 J_{zz} \right\}.$$
In order to eliminate the time dependence, we use the fact that (2) is autonomous, so \( J(z,t) \) has the separable form \( J(z,t) = V(z)e^{-rt} \). This implies: \( J_t = -rV(z)e^{-rt}, \ J_z = V_z e^{-rt}, \ J_{zz} = V_{zz} e^{-rt} \).

Substituting these expressions into equation (3) gives

\[
0 = -rV(z) + \min_m \left( C(z,m) + V_z g(z,m) + \frac{1}{2} \sigma^2 z^2 V_{zz} \right).
\]

The functions for costs and wetlands quality, \( C \) and \( g \) are assumed to be twice continuously differentiable. The first and second derivatives of the functions satisfy the following inequalities:

\[
C_z > 0, \ C_m > 0, \ C_{zz} \geq 0, \ C_{mm} \geq 0, \ C_{zz} C_{mm} - C_{z m}^2 \geq 0
\]

\[
g_m > 0, \ g_{zz} \leq 0, \ g_{mm} \leq 0, \ g_{zz} g_{mm} - g_{zm}^2 \geq 0
\]

Costs are increasing and convex in \( z \) and \( m \), and the growth equation is increasing and concave in these arguments. The curvature assumptions insures that the first order condition to the DPE in \((3^*)\) gives a local minimum; the first order condition is also a necessary and sufficient condition for optimality in the deterministic version of our problem, obtained by setting \( \sigma^2 = 0 \). The signs of the first derivatives of \( C \) and \( g \) are easily motivated. By definition, an increase in discretionary expenditures, \( m \), increases instantaneous costs. It is reasonable to assume that there are also non-discretionary expenditures which increase with habitat quality, so \( C_z \geq 0 \). Discretionary expenditures would never be undertaken unless they led to an increase in quality, so \( g_m \geq 0 \) for optimal \( m \).

We assume that the optimal value of \( m \) is positive, i.e. we have an interior solution. This is true for our formulation in Section 5. In some cases it might be optimal to set \( m = 0 \),
that is, to allow natural regeneration for the ecological system to operate without intervention.

In this case the problem is simpler, since then the only question is when to cash in the investment; methods described by Brock et al. [3] can be used. Assuming an interior solution, the first order necessary condition from equation (3*) yields the optimal control rule for $m$:

\[
C_m + V g_m = 0.
\]

Equation (6) states that the marginal cost of discretionary investment equals the value of the marginal product of investment. The latter quantity equals the marginal product of discretionary investment, $g_m$, times the shadow price, $-V_z$.

Two boundary conditions for the DPE are:

\[
V(z^*) = -K(z^*) \tag{7}
\]

\[
V_z|_{z^*} = -K_z|_{z^*} \tag{8}
\]

In the continuation region, where $V(z) < -K(z)$, it pays to continue holding the asset. Since cashing in the asset is always an option, it must be the case that $V(z) \leq -K(z)$. The value matching condition in (7) states that if it is optimal to cash in, then the value of the program equals the intrinsic value of the state. Equation (8) is known as the smooth pasting condition. It states that the shadow value of the stock evaluated at the optimal stopping state equals the marginal salvage (intrinsic) value.

Substitution of $m^*$, the optimal control rule, and the boundary conditions into (3*) yields the following relationship at the optimal stopping state, $z^*$:
Equation (9) contains two unknowns, the values of $z^*$ and $V_{zz}(z^*)$, and therefore is not sufficient to determine the value of the stopping state. In the deterministic version of this problem ($\sigma^2 = 0$), equation (9) implicitly defines $z^*$. In that case, the condition for cashing in the investment is the familiar requirement that the opportunity cost of holding the investment for a unit of time, $rK$, must equal the increase in value of the investment, $Kg$, minus the flow cost, $C$, per unit of time.

When $\sigma^2 > 0$, we need one more piece of information to obtain $z^*$. Once $z^*$ is known, equations (7) and (8) provide two boundary conditions for the second order ordinary differential equation (ODE) that we obtain by substituting (6) into (3*). In other words, we need one more boundary condition in order to solve the problem. The determination of this missing boundary condition is discussed in Brock et al. [3]. They consider a simpler problem, in which the control $m$ is absent. Although their arguments can be generalized to apply to our problem, the resulting methods are not useful in the present context because of lack of data. In particular, we would need to know about the value of the program as $z$ approaches a lower bound (some finite value, or $-\infty$). For example, if we were told that the investment were worthless for $z \leq \tilde{z}$, and knew the value of $\tilde{z}$, we would have the additional boundary condition $J(\tilde{z}) = 0$. The investment opportunity may be worthless if the wetlands quality is so low that restoration is not feasible. Restoration feasibility depends on retention of a wetlands' pre-existing hydrologic conditions, a viable seed-source, and adjacent fauna and flora for recolonization (Holderman, [16]). These requirements may determine a value of
below which restoration is infeasible. However, we do not have the information needed to determine this value.

In the absence of information on a lower bound, we use the following two-step procedure to find the missing boundary condition for our empirical application. The first step is to choose a "trial value" for the stopping state, which we denote as \( z^* \). We then solve the ODE (3*) using the boundary conditions (7) and (8) evaluated at \( z^* \). We denote the solution to this ODE as \( V^{z*}(z) \). This function gives the value of the investment program under optimal discretionary expenditures, conditional upon cashing in at the (possibly suboptimal) level \( z^* \).

The superscript \( z^* \) indicates that the stopping state is fixed at \( z^* \); \( z \) is an argument of the function, since whatever is the stopping state, the value of the program still depends on the current level of \( z \). The second step is to vary the trial value \( z^* \), and for each value to obtain the function \( V^{z*}(z) \). We choose the optimal stopping state as the value of \( z^* \) that minimizes \( V^{z*}(z) \). That is, the optimal value function \( V(z) \) is given by

\[
(*) \quad V(z) = \min_{z^*} V^{z*}(z).
\]

In order for this procedure to work, the optimal stopping state must be independent of the current value of \( z \). This means that if \( z^j \) is the optimal stopping state, then the graph of \( V^{z_j}(z) \) must lie below the graph of \( V^{z_j}(z) \) for all \( z^j \neq z \) and for all values of the state \( z \). Figure 1 illustrates this "no crossing condition". Neither \( z^j \) nor \( z^j \) are candidates for the optimal stopping state because the graphs of \( V^{z_j}(z) \) and \( V^{z_j}(z) \) cross. If these two graphs cross, then whether it is better to cash in at \( z^j \) or \( z^j \) depends on the current value of \( z \) (whether it is above or below the point of intersection of the graphs). Since the optimal stopping state does
not depend on the initial condition, neither \( z^1 \) nor \( z^2 \) are candidates. Figure 1 also shows the graph of \( V^3(z) \) lying everywhere below the graphs of the other functions. In this case, \( z^3 \) is a candidate for the optimal stopping state. In our empirical application, there were no stopping values such that the graphs of \( V^3(z) \) crossed. This made it easy to select the lowest graph and the corresponding optimal stopping state.

Brock \textit{et al.} [3] show that the comparative statics of the stopping state, \( z^* \), with respect to exogenous parameters, do not depend on the type of missing boundary condition. With this encouraging result in mind, it seems reasonable to look for analytic results in our model.

Totally differentiating (9) with respect to \( z^* \) and \( \sigma^2 \), gives

\[
\frac{dz^*}{d\sigma^2} = \frac{\frac{1}{2} V_{zz} z^2}{-C_z + K_z(g_2 - r) + gK_{zz} - \frac{1}{2} \sigma^2 V_{zzzz} - \sigma^2 V_{zz}} .
\]
In (10) all functions are evaluated at \( z = z^* \). We cannot sign this expression in general.

However, we obtain some insight by considering the limiting case as \( \sigma^2 \to 0 \), so that the last two terms in the denominator of (10) vanish. In order to evaluate the resulting expression, it helps to consider the deterministic problem, using the Maximum Principle. The Hamiltonian for this problem is \( H = C(\cdot) + \lambda g(\cdot) \), where \( \lambda \) is the costate variable, and \( \dot{\lambda} = (r - g_z)\lambda - C_z \).

(A dot over a variable indicates differentiation with respect to time.) Using this relation, and the transversality condition \( \lambda = -K_z \) at the stopping state, we can rewrite (10) as

\[
(11) \quad \frac{dz^*}{d\sigma^2} = \frac{z^2 V_{zz}}{2 \left[ \dot{\lambda} + K_{zz} \frac{z}{z^*} \right]} = \frac{z^2}{2 \dot{z}} \left[ \frac{V_{zz}}{V_{zz} + K_{zz}} \right]
\]

For the second equality we have used the relation \( \dot{\lambda}/\dot{z} = d\lambda/dz = V_{zz}(z) \). Under the assumption that the value of \( z \) is initially small, the state approaches the stopping region from the left (if \( z > z^* \) it is optimal to stop immediately), so \( \dot{z} > 0 \) evaluated at \( z^* \).

For the special case where \( K \) is linear, so that \( K_{zz} = 0 \), (11) implies that a small increase in the variance increases the stopping state. For non-linear \( K \) we can show that 
\[
\text{sign} \left\{ \frac{dz^*}{d\sigma^2} \right\} = \text{sign} \left\{ -V_{zz}(z^*) \right\}
\]

This is done using the fact that \( V_{zz}(z^*) + K_{zz}(z^*) < 0 \). This inequality is established by noting that \( V(z^* - \varepsilon) + K(z^* - \varepsilon) < 0 \) for \( \varepsilon > 0 \). For small \( \varepsilon \), we can approximate both functions using a second order Taylor expansion. Using this approximation and (7) and (8) in our last inequality implies \( V_{zz}(z^*) + K_{zz}(z^*) < 0 \). We summarize these results in

**Proposition 1:** For values of \( \sigma^2 \) close to 0, a small increase in \( \sigma^2 \) leads to an increase in the value of the stopping state if either (i) the cash-in function \( K \) is linear, or (ii) the value function \( V \) is concave.
Brock et al. [3] obtain analogous results for their simpler model. Their results are valid even for large values of \( \sigma^2 \), whereas Proposition 1 is a local result, for \( \sigma^2 = 0 \). However, their result is obtained for the case where both \( g(\cdot) \) and \( K(\cdot) \) are linear. Brennan and Schwartz [2] also show that an increase in variance increases the value of the asset. Under condition (i), we see that an increase in uncertainty increases the value of holding the investment in the WMB. The developer delays cashing in her investment. (An increase in \( z^* \) means that it is profitable to hold the asset in more states of nature.) This is also true for non-linear \( K \), provided that the function \( V \) remains concave in the neighborhood of \( z^* \). This is very intuitive: we know that an increase in uncertainty decreases the expected value of a concave function. In our context, where we are minimizing a functional, this decrease in expected value is an improvement.

The developer’s restoration investment problem is similar to a decision of when to exercise a stock option. However, our problem contains the added feature that the level of investment must be chosen at each point in time before cashing in. The model’s value function incorporates the future consequences to wetlands from current restoration decisions. In the deterministic version of the model it is optimal to cash in when the opportunity cost of holding the investment for a unit of time equals the increase in value of the investment, minus the flow of cost per unit of time. Proposition 1 shows when the inclusion of a small amount of uncertainty increases the stopping state of wetlands quality.

We need to solve the problem numerically in order to obtain sensitivity results that are valid for non-infinitesimal changes in parameter values, and also to estimate the probable magnitude of effects of those changes. We now turn to the empirical application.
V. MODEL CALIBRATION

In this section we explain how the index of quality, \(z\), is determined. We then present the functional forms, and explain how we calibrated the model which is used in the following section.

Investment in restoration changes wetlands quality, which refers to specific wetlands functions. The Habitat Evaluation Procedure (HEP) is a method of quantifying the habitat (USFWS [32]). The procedure involves the estimation of the quantity of various wetlands attributes known to be important to one or more selected indicator species of flora or fauna. The species act as an indicator of overall ecosystem integrity. The species chosen might be based on their economic value (e.g. hunting, trapping). The HEP produces a Habitat Suitability Index (HSI) ranging from 0-1 for each indicator species. The number of indicator species times the HSI is \(z\). In our application the range of \(z\) is from 0-2, since the tidal marsh site for which we have data can support two rare and endangered indicator species. Non-integer values indicate the presence of contributing factors that support the indicator species. Habitat units are converted to restoration credits by multiplying the number of acres supporting the species by the HSI.

We chose the following functional forms: 

\[
C = \alpha z^2 + m; \quad dz = \beta \sqrt{m} + \rho z + \sigma z dW; \quad \text{and} \quad K = \gamma z^2/2 + \eta z.
\]

The natural expected growth rate, i.e. the growth rate when discretionary expenditures are 0 and \(dW = 0\), is \(\rho\); \(\beta\) determines the effect of discretionary expenditures on growth. Without more data, we cannot evaluate whether the functional forms are appropriate.
An important advantage is that they involve very few parameters. Since we rely on numerical methods to solve the model, more complicated functional forms would not be an obstacle.²

We obtain "estimates" of three parameters, \( \alpha, \beta \) and \( \rho \) by means of calibration. We have insufficient data to obtain even rudimentary estimates of the remaining three parameters, \( \gamma, \eta \) and \( \sigma \), so we assign them "reasonable" values and then do numerical sensitivity analysis.

Data of real estate value of tidal marsh provides our only means of estimating the parameters of the salvage function.

Restoration site data from Bracut Marsh, California, for a six year period, includes expenditures for the first, third, and sixth year (CCC [4]). We allocate the cost data into categories of discretionary and non-discretionary expenditures. We think that this is a useful distinction: whether an item is discretionary or non-discretionary may have an important effect on the optimal path for aggregate investment, and on the value of an investment opportunity. In practice it may be unclear to which category an item belongs. The determination is likely to be a matter of policy rather than of physical and biological laws.

For example, the design of a WMB may state that certain activities must be performed when the state reaches a given level, whereas other activities can be undertaken at the investor's discretion. In that case, the former activities entail non-discretionary expenditures from the

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² Note that if we define \( y = \sqrt{m} \), the model is equivalent to a linear-quadratic control problem. However, it makes more sense to think of instantaneous costs as being linear in discretionary expenditures, and the growth equation as being non-linear, then vice-versa. Despite this linear-quadratic structure, the value function \( V \) is not quadratic; this is because the investment program will be stopped at some state \( z^* \). To verify this, suppose to the contrary that \( V \) were quadratic. Using a quadratic form of \( V \) in (3*) and proceeding in the usual manner to "equate coefficients" of powers of \( z \), leads to algebraic equations for the parameters of \( V \). Given these parameter values, the boundary conditions (7) and (8) provide two equations in one unknown, \( z^* \). There is, in general, no value of \( z^* \) that solves both equations, since they are linearly independent.
standpoint of the investor, even though all activities are discretionary from the standpoint of society (or the designer of the WMB).

For our calibration exercise, we include as discretionary expenditures: labor and depreciation of capital equipment for planting, land excavation, and hydrological engineering. Non-discretionary expenditures consist of costs of physical inputs used to establish the ecosystem habitat. These include seeds, plants, and soil material, which contribute to the vegetation associated with the indicator species for wetlands quality. (Costs are in units of thousands of dollars per acre.) We refer to the parameter estimates obtained using this allocation of costs as the "base parameters", and we use these for sensitivity studies. In order to determine the effect of this allocation of costs on our results, we also consider the extreme case in which all costs are discretionary. That is, we allocate all the costs in our data to the discretionary category, and recalculate the model parameters. We refer to these as the "alternate parameters".3

An estimate of \( \alpha \) is obtained by solving \( \alpha \sum_{i=1}^{6} z_i^2 = N \), where \( N \) (total non-discretionary expenditures) equals the sum of total project expenses for plants, seeds, and soil inputs. The

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3 This experiment answers the following question: Suppose that we had misunderstood how the WMB works, and that in fact, all expenditures are discretionary; given our data, how would recognition of this mistake change our parameter estimates? The experiment does not answer the following, more difficult question: Suppose that the WMB is re-designed to allow the investor more flexibility, in the sense that previously non-discretionary expenditures are now discretionary; what effect does this have on the model parameters? It is important to bear in mind this distinction when interpreting the sensitivity analysis of the next section.

Non-discretionary expenditures, which by definition depend on \( z \), alter the drift term in the equation of motion. The parameter \( \rho \) incorporates both the biological/physical effect of \( z \) on the drift, and the effect of non-discretionary expenditures induced by \( z \). Therefore, a design change in the WMB that allows investors more discretion would lead to a decrease in \( \alpha \) and a decrease in the estimate of \( \rho \). The decrease in \( \alpha \) would benefit investors, but this would be partly offset by the decrease in \( \rho \). Note that the "alternate estimate" of \( \rho \) is higher than the base estimate: our experiment answers the first question, but it clearly does not answer the second.
The unit of measure for wetlands quality is the number of indicator species times their HSI. We have only three years of data on \( z_0, z_3 \) and \( z_6 \). We obtain estimates for the missing years by linear interpolation. For our sample, \( N = 1.68 \) ($1680 per acre) and \( \sum_{i=1}^{6} z_i^2 = 10.09 \), which results in an estimate of .16 for the parameter \( \alpha \). The value of \( N \) is comparable to some estimates of costs associated with tidal marsh restoration in the larger region of coastal California (King and Bohlen [20]).

Estimates for parameters \( p \) and \( \beta \) are obtained by using the data of \( z \) and \( m \) in the deterministic version of the state equation, obtained by setting \( \sigma^2 = 0 \). We solve this equation to obtain

\[
(12) \quad z_{t+3} = z_te^{3p} + (e^{3p} - 1)\frac{\beta \sqrt{m_t}}{p} \quad \text{for } t = 0, 3.
\]

Using the following observations on \( z \), \( z_0 = 0 \), \( z_3 = 1 \) and \( z_6 = 2 \) in (12) gives two equations which we solve for \( \beta \) and \( p \). We set the value of \( m_t \) for these two equations equal to the average discretionary expenditures, in the three year period beginning at time \( t \): for the first three years we have \( m_t = 4.96 \) and for the last three years \( m_t = .462 \). These imply estimates of \( \beta = .11 \) and \( p = .18 \). We have no degrees of freedom left to estimate \( \sigma^2 \). For our base case we arbitrarily set \( \sigma^2 \) equal to 1. Holderman [18] discusses some of the sources of randomness, such as methane in the soil substrate and poor water circulation.

In Section II we discussed the process of "advanced identification" (ADID) and we explained why this is an implicit subsidy for investment. For our Bracut Marsh data,
approximately 50% of the costs which we have designated nondiscretionary, are associated with the type of activities potentially covered by ADID. Therefore, we regard a 50% subsidy on nondiscretionary expenditures as a reasonable approximation of a relevant policy. In order to model the effects of this subsidy, we replace our point estimate of \( \alpha = 0.16 \) by \( \alpha = 0.08 \), leave the other parameter values unchanged, and resolve the model.

Another experiment is prompted by the recognition that we are not certain which expenditures are truly discretionary, from the standpoint of the investor. Therefore, in addition to considering the case described above, where we used our judgement to allocate expenditures between the two categories, we also consider the extreme case where all expenditures are discretionary. In that case, by definition \( \alpha = 0 \). To estimate the model with this maintained assumption, we reallocate the expenses we previously defined as non-discretionary, to the discretionary category, and recalculate \( \beta \) and \( \rho \), obtaining \( \beta = 0.09 \), \( \rho = 0.183 \). These are the "alternate parameters".

One of the proposed ADID tasks is to convert the physical measure of wetlands functions (Habitat Units) to a dollar value. If we had data on this conversion, we would be able to estimate the parameters of \( K(z) \), \( \gamma \) and \( \eta \). Unfortunately, we do not have this data, but we think that a value of \$6000 per acre of wetlands with \( z = 2 \) is reasonable; this figure is based on the sales price of coastal marsh in California (Eliot and Holdeman [8]). Our extrapolation from the real estate market is a rough gauge of marketed and non-marketed values of a tidal wetlands. We set \( \eta = 6 \) and \( \gamma = -3 \), values which are consistent with \( K(2) = 6 \). Rather than using only land price to estimate the parameters of \( K(z) \), we might also incorporate estimates of values for general categories of wetlands functions, such as the value for recreational use.
Wetlands offer other valuable functions such as flood control, water purification, and groundwater recharge. There are some unresolved issues in estimating these and other wetlands functions. Even if we had monetary estimates of values for our site, it is not clear how we would aggregate such estimates. Here, in the interests of simplicity, we use only the land price to approximate the dollar value of restored acreage. These values also imply that $K$ is maximized at $z = 2$. This means that $z^* \leq 2$, since it would never be optimal to incur a cost of holding an investment when the value of the investment cannot increase.

The remaining parameter is the interest rate, $r$, which we set equal to .1 for the base case. This corresponds to the 10% real interest rate in 1982, the year the restoration project began (Federal Reserve, [11]).

VI. RESULTS

Using the parameter values in the previous section, we solve the second order ordinary differential equation obtained by substituting (6) into (3*), using the boundary conditions (7) and (8). We search over the interval $(0, 2)$ to find the optimal $z^*$, as described in Section IV. This section reports results of the base case and sensitivity studies, which are illustrated in Figures 2 - 5 and summarized in Table I. We solve the model for the following six sets of parameter values: 1) the base parameter values described in Section V; 2) the cost parameter $\alpha$ decreases by 50%; 3) the variance of biological recovery increases by 10%; 4) the interest rate increases from 10% to 12%; 5) the market value of credits increases; 6) the "alternate parameters" described in Section V.
This choice of sensitivity studies has two motivations. The first is that we want to vary parameters that resource agencies involved in WMB policy consider important determinants of restoration efforts; except for the interest rate, we have very imprecise estimates of these parameters. The sensitivity studies give us an idea of how important is our lack of precise estimates. The second motivation is that we want to model the effect of actual or plausible policies. We discussed at length the role of AID as an implicit subsidy in general. In the previous section we explained why it is reasonable to approximate this as a 50% subsidy for our example. An alternative means of promoting wetlands restoration is to increase the market value of restoration credits. This increase may incorporate scarcity of wetlands functions. Policy changes may also affect investors' discount rate.

Figure 2 shows the graphs of $-K(z)$ and two functions of $V(z)$, for the base case parameters, and with $x$ reduced from .16 (the base case) to .08. We first discuss the base case. For these parameter values the stopping state is $z^* = 1$. For a given value of $z$, the difference between cashing in immediately and behaving optimally, $-K(z) - V(z)$, is defined as the option value of the investment. [This definition differs from another usage of the term, in which the option value is the amount an individual is willing to pay (a premium) to ensure future availability of an amenity.] For $z = .1$ the option value is 18.5% of the value of the investment. The option value is negligible at $z = .6$, where $K = 3.06$ (50% of its maximum value).

If we assume that $x$ represents the true social costs associated with non-discretionary restoration activities, then private decisions are socially optimal. However, in the illustration above, the investor has little incentive to restore the wetlands to a level close to the private
(and social) optimum; she looses a negligible amount by stopping restoration too soon. Therefore, if the investor is uncertain about the "true model", it is likely that adequate restoration would not occur. A subsidy on non-discretionary expenditure increases the investor's incentive to restore the wetlands.

We examine the effect of a 50% subsidy by reducing the parameter $\alpha$ from .16 to .08. The resulting value function is graphed in Figure 2. The stopping state, $z^*$, increases to 2, the level that maximizes the intrinsic value $K(z)$. The subsidy causes the privately optimal stopping state to be twice the socially optimal stopping state, and increases the value of the investment program. The option value, as a percentage of the value of the program, increases to approximately 75% at $z = .1$. The option value is 26% (instead of .5% with $\alpha = .16$) for $z = .6$, and does not fall to 1% until $z = 1.5$. This subsidy has a substantial effect in increasing the investor's incentives to restore the wetlands. Failure to invest at all (i.e. cashing in when $z$ is negligible), leads to a large loss. However, the subsidy can lead to excessive restoration (under the assumption that social costs are represented by $\alpha = .16$).

The subsidy also alters the (privately) optimal profile for discretionary investment, $m(z)$. This function, obtained using (6), is graphed for the two values of $\alpha$ in Figure 3. For low values of $z$, the subsidy increases discretionary expenditures. For example, at $z = .1$, the subsidy increases discretionary expenditures from $391$ to $563$ per acre per year. The reason for the increase is that the subsidy makes it less costly, and therefore more attractive, to have the state reach a high level. However, for values of $z > .2$, the subsidy decreases discretionary expenditures. This is because the subsidy decreases the cost of waiting to cash in, during which time non-discretionary expenditures are incurred. This increases the...
investor's incentive to allow the state to increase at its natural rate, rather than as a result of discretionary expenditures.

We obtain a certainty equivalent approximation of total undiscounted discretionary costs by taking the integral, from the initial state \( z_o \) to \( z^* \), of the function \( m(z)/g(z,m(z)) \). We denote this integral as \( D(z_o) \). To show that this equals the total undiscounted discretionary cost of driving the state from \( z = z_o \) to \( z = z^* \), when the decision rule \( m(z) \) is used, and \( \sigma^2 = 0 \), we use the following relation:

\[
D(z_o) = \int_0^T m(t) dt = \int_0^T \frac{m(z(t))}{dz/dt} = \int_{z_o}^{z^*} \frac{m(z)}{g(m,z)} dz.
\]

The 50% subsidy in non-discretionary expenditures may cause discretionary expenditures to increase or decrease at a point in time, but aggregate discretionary expenditures fall by approximately 30%. Since the wetlands are restored to a higher level, and discretionary expenditures fall, non-discretionary expenditures must increase. In this sense, discretionary and non-discretionary expenditures are "substitutes in production". Just as is the case in a static production model, where two inputs are substitutes and one is subsidized, the subsidy leads to a decrease in the use of the unsubsidized input.

To summarize, the subsidy has two effects. First, it increases the option value of investment. This can be socially beneficial if private investors would not undertake restoration activities which have only a small positive expected return. However, it can be

\[4\] The exact value of total expected discounted discretionary costs, denoted \( L(z) \) can be obtained by solving the second order ODE \( 0 = -rL(z) + m(z) + L'(z)g(z,m(z)) + \sigma^2 z^2 L''(z)/2 \), with boundary conditions \( L(z^*) = 0 = L'(z^*) \).
socially harmful if it leads to an excessive level of restoration. Second, the subsidy shifts
discretionary investment forward in time, decreases aggregate discretionary expenditures, and
increases aggregate non-discretionary expenditures (only half of which are paid by the private
investor). These changes tend to lower social welfare, since they represent an inefficient
allocation of inputs, both over time, and across categories of discretionary and non-
discretionary expenditures. When discretionary expenditures are lower, as is the case under
the subsidy for $z \in (.2, 1)$, the expected improvement in wetlands occurs more slowly.
Therefore, although the subsidy would probably eventually result in a higher quality of
wetlands, it is likely to cause a delay in the expected arrival time of reaching a moderate
level of quality (e.g., $z = 1$).\footnote{Using the same type of equation described in footnote 4, we could calculate exactly the expected arrival time under optimal behavior. We could also calculate other measures that might be of interest, such as the expected present value of the cost of the subsidy.}

Small changes in the variance lead to large changes in the optimal investment strategy.
Figure 4 shows the graph of the value functions with $\sigma^2 = 1.0$ and $\sigma^2 = 1.1$. Consistent with
Proposition 1 and results from previous literature, an increase in the variance increases the
value of the investment program. The magnitude of the change is surprising. This larger
variance causes an increase in the stopping state from $z^* = 1$ to $z^* = 2$. The option value, as
a percentage of the value of the program, is approximately 46% at $z = .1$, but it falls to 6.7%

Figure 5 graphs the control rules for the two values of $\sigma^2$. For very low values of the
state, discretionary investment is higher for $\sigma^2 = 1.1$, but for most values of $z$, it is lower with
the higher variance. The measure of aggregate discretionary expenditures increases slightly,
although the investment is held until a much higher level of $z$. With a lower variance, it does not pay the investor to wait around in the hope of getting lucky. Instead, when it is worth holding the asset she uses higher discretionary expenditures, but cashes in sooner.

Beginning with the base parameter values and increasing the interest rate from $r = .1$ to $r = .12$ decreases the stopping state from $z^* = 1$ to $z^* = .6$. The value of the program decreases considerably, as shown in Table I. The higher interest rate increases discretionary expenditures for all values of $z$ at which the investment is held. With a higher interest rate, the investor wants to cash in quickly, if she invests at all. The investor therefore undertakes discretionary expenditures rather than relying on the natural growth rate of the state. The measure for aggregate discretionary expenditures, $D(.1)$ is approximately 75% of the base case level.

We also experimented with changes in the parameters of $K(z)$. These led to changes in optimal behavior, in the direction expected. For example, an increase in $K(2)$ from $6,000$ per acre of coastal marsh, to $7,000$ per acre implies parameters values $\gamma = -3.5$ and $\eta = 7$. The increase in the credits makes it optimal to restore wetlands to a higher level. The stopping state is $z^* = 1.8$ instead of $z^* = 1$ as in the base case. The option value at $z = .1$ is 61% of the value of the program and does not decline to 3% until $z = 1.5$. The increased value of credits has a substantial increase in the incentive to engage in restoration.

Finally, we examined the importance of our assumption concerning which expenditures are discretionary, using the alternate parameter estimates described in the previous section. The last row of Table I presents the results. The most important parameter change is for $\alpha$, which becomes 0. The effect of a decrease in $\alpha$ was described above in the discussion of the
subsidy. The changes in optimal behavior are simply magnified here, although the interpretation is different. (In the present context, the change in $\alpha$ is due to correcting a "mistake" in our model, rather than to providing a subsidy.) The changes in the estimates of $\beta$ and $\rho$ work in the same direction as the change in $\alpha$.

VII. CONCLUSION

We formulated a stochastic control model of investment in a Wetlands Mitigation Bank. We calibrated a simple version of the model and solved it numerically. This approach to the problem makes efficient use of the data we have, and it also suggests where we would most benefit from better data.

By assumption, non-discretionary costs increase with the quality of the wetlands. The optimal level of discretionary expenditure, on the other hand, decreases with quality. Since we expect, on average, the quality to be increasing over time, this means that most of the discretionary investment comes early in the program.

The value of delaying cashing in the investment and continuing restoration (the option value) is largest when the quality of the wetland is low. It decreases monotonically as the quality improves. The incentive is negligible even when the quality is far below the (privately and socially) optimal stopping state in the base case. Since the investor is unlikely to know exactly what this state is (due to incomplete knowledge or bounded rationality, for example), this result suggests that wetlands may not be restored to their optimal level.

A subsidy on non-discretionary expenditures increases the option value and therefore encourages continued restoration, possibly to a level higher than is socially optimal. The
subsidy increases initial discretionary expenditures but then decreases these expenditures for a range of wetlands quality. Therefore, the subsidy accelerates improvements in wetlands at first (for low levels of \( z \)), but then delays them. The net effect of the subsidy is to decrease aggregate discretionary expenditures. The subsidy introduces a distortion; by decreasing the amount of non-discretionary costs that the developer must pay, it decreases her willingness to incur discretionary costs. The net effect on social welfare of the subsidy may therefore be negative. This particular subsidy is a blunt instrument. A more finely tuned policy, e.g. a subsidy which changes with the state of the wetlands, would result in a smaller distortion. Of course, such a policy requires more information and is harder to administer.

The quantitative results are sensitive to parameter values, which are based on inadequate data. For example, the magnitude of the variance was important in determining both the incentives for investment, and the optimal investment path. Less uncertainty decreases the option value of investment. However, conditional on investment occurring, discretionary investment is higher with less uncertainty. A decrease in uncertainty makes it less tempting to rely on good fortune.

In order to make our model more useful, it is especially important to improve our knowledge about the index of quality, \( z \). The dynamics of this variable need to be modelled carefully, a task which requires better data. In addition, the relation between the quality index and the value of credits, \( K(z) \), has to be understood better. This requires a clear definition of the relation between the quality index and the number of credits a developer receives. It also requires that we know more about the monetary value of restored wetlands. Since WMB's are a recent innovation, the current lack of data is not surprising.
The model could be expanded to include a stochastic salvage function if there were data to estimate demand for housing and other development projects. A combined Poisson-Weiner process may be appropriate to describe demand in a format similar to McDonald and Siegel [24].

While the paper focuses on the restoration of degraded wetlands it may offer insight for investment in restoring other natural resources. The stochastic optimal stopping problem is useful to examine investment with uncertain outcomes in such endeavors as restoring an aquifer and cleaning up hazardous waste in multi-media (air, water, soil). Since wetlands are ecosystems containing a multitude of natural resources, the model may also be used to examine strategies for inducing recovery of one or more endangered species in a wetlands or other ecosystem setting. A particular species of interest would be identified in the stochastic state equation by its growth function. There are resource policies, such as the Endangered Species Act, that could be examined with a species-specific view of this study’s model.
References


### Table I

**Sensitivity Analysis Results**

<table>
<thead>
<tr>
<th>Parameter value</th>
<th>$z^*$</th>
<th>option value, $z = .1$ percentage of investment value</th>
<th>dollar amount</th>
<th>option value, $z = .6$ percentage of investment value</th>
<th>dollar amount</th>
<th>Approximate total discretionary costs, $D(.1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>base case values</td>
<td>1</td>
<td>18.5%</td>
<td>$133</td>
<td>.5%</td>
<td>$7</td>
<td>$2,909</td>
</tr>
<tr>
<td>$\alpha = .08$</td>
<td>2</td>
<td>74.6%</td>
<td>$1719</td>
<td>26%</td>
<td>$1074</td>
<td>$2,043</td>
</tr>
<tr>
<td>$\sigma^2 = 1.1$</td>
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<td>46%</td>
<td>$505</td>
<td>6.7%</td>
<td>$222</td>
<td>$2,951</td>
</tr>
<tr>
<td>$r = .12$</td>
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<td>6%</td>
<td>$41</td>
<td>0%</td>
<td>$0</td>
<td>$2,267</td>
</tr>
<tr>
<td>$\alpha=0, \beta=.09, \rho=.183$</td>
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<td>86%</td>
<td>$3611</td>
<td>44%</td>
<td>$2412</td>
<td>$839</td>
</tr>
</tbody>
</table>

*Note: the base case values are: $\alpha=.16, \beta=.11, \rho=.18, \sigma^2=1, r=.1, \gamma=-3, \eta=6*
Figure 3. Control Rule $w$ for Base Case and Decrease in Non-Discretionary Cost Parameter.
Figure 4. Base case and effect of increase in variance from 1.0 to 1.1.