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Publication Date
1971-03-01
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AEC Contract No. W-7405-eng-48
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LIFETIMES OF GROUND-BAND STATES IN $^{154}$Sm

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March 1971

Abstract: The lifetimes of the $8 \rightarrow 6$, $6 \rightarrow 4$, and $4 \rightarrow 2$ ground-band transitions in $^{154}$Sm have been measured by a recoil-distance Doppler-shift technique following Coulomb excitation by back-scattered $^{40}$Ar projectiles. Within experimental error, the measured $B(E2)$ values are those of a rigid rotor. These results and those of a previous experiment on $^{152}$Sm are compared with current models.

NUCLEAR MOMENTS

$^{154}$Sm, measured $T_{1/2}$ of $8^+$, $6^+$, $4^+$ levels, deduced $B(E2)$'s.

* Work supported under the auspices of the U. S. Atomic Energy Commission.
1. Introduction

It is well-known that the spacings of the first few ground-band levels of doubly-even deformed nuclei are to a first approximation in agreement with the rigid-rotor expression\(^1\),

\[ E(I) = AI(I + 1), \]  

but in detail diverge from this simple formula to a greater or lesser degree, requiring an infinite series expansion in \(I(I + 1)\)

\[ E(I) = AI(I + 1) + BI^2(I + 1)^2 + CI^3(I + 1)^3 + \ldots \] \((2)\)

There seem to be a number of causes for this behavior, the most prominent being Coriolis anti-pairing, centrifugal stretching, and fourth-order cranking model corrections\(^2-17\). All of these effects apparently contribute deviations from eq. (1) with the same (negative) sign\(^12\). These effects should also cause deviations in the transition probabilities from the expectations of the simple rigid-rotor formula\(^18\),

\[ B(E2; I \rightarrow I - 2) = B_o(E2; 2 \rightarrow 0) \frac{(I020|I-20\rangle)^2}{(2020|00\rangle^2} \] \((3)\)

For centrifugal stretching we can relate the deviations in the transition probabilities to those in the energy-level spacings. But for the other two effects the relationships are not known.

With poor rotors, the deviations from rigid-rotor transition probabilities in the first few transitions may be 10-15\%, and so determinable. Recently, several groups of experimenters have reported on transition probabilities in
the ground-band of $^{152}$Sm. The multiple-Coulomb-excitation results$^{19-22}$, when corrected for the effects of an E4 moment$^{23}$, are in rough agreement with the recoil-distance Doppler-shift measurement$^{24}$, and all indicate transition probability deviations, in percent, of the same order of magnitude as those of the energy-level spacings. For better rotors the deviations should be smaller, and so more difficult to observe. We have chosen to study the ground-band transition probabilities in $^{154}$Sm by the recoil-distance Doppler-shift method$^{24-27}$ because we believe it currently gives the most accurate $\text{B(E2)}$ values, since multiple-Coulomb-excitation determinations must take into account additional effects including those of higher electric moments$^{23}$). In fact, by combining the present result with such multiple-Coulomb-excitation studies we can obtain the value of the E4 moment in $^{154}$Sm (Ref. 23a).
2. Experimental

A schematic drawing of the experimental arrangement is shown in fig. 1; the set-up is essentially the same as in previous studies\cite{24,28}. A collimated beam of 146 MeV $^{40}$Ar ions from the Berkeley Hilac passes through the aperture in a Si ring counter and strikes a 1 mg/cm$^2$ target of $^{154}$Sm metal which has been stretched flat. The recoiling (Coulomb-) excited nuclei gamma-cascade to ground either while in flight or after stopping in the lead-covered plunger. The gamma-rays are observed in a Ge(Li) detector placed behind the plunger at 0° to the original beam direction and operated in coincidence with backscattered (148-160°) $^{40}$Ar projectiles detected in the ring counter. Since the velocity of the recoiling Sm nuclei is ~3.4% that of light, the Doppler-shifted transition is moved to a sufficiently high energy to be clearly resolvable from the unshifted line. Changes in the distance between the target and the plunger can be measured to 0.003 mm. Such changes vary the relative intensities of the shifted and unshifted transitions, allowing determination of the mean lifetime of the (upper) state involved, if the average recoil velocity is also known. This velocity is evaluated from,

$$\frac{\Delta E}{E_o} = \frac{(1 - \beta^2)^{1/2}}{\beta(1 - \cos \theta_c)} \ln \left( \frac{(1 + \beta)(1 - \cos \theta_o)}{\beta \cos \theta_c - \cos \theta_o + [(\cos \theta_o - \beta \cos \theta_c)^2 + (1 - \beta^2)\sin^2 \theta_o]^{1/2}} \right)^{-1}$$

(4)

where $\Delta E$ is the observed shift in transition energy, $\theta_c$ is the half-angle subtended by the Ge counter, $\theta_o$ is the angle between the axis of the counter and the direction of the recoiling nucleus, $E_o$ is the unshifted transition energy, and $\beta = v/c$. The effective velocity determined was $v \cos \theta_o = (0.0340 \pm 0.0005)c$. 

3. Results

Some typical spectra observed at several target-plunger distances are shown in fig. 2. For each spectrum taken, the small number of accidental coincidence events have been subtracted, and then a background curve (third-order polynomial) was determined and subtracted. The remaining areas under the shifted and unshifted peaks have been integrated, and corrected for the small changes in solid angle at the Ge counter due to the change in position of the lead plunger.

Two further, partially compensating, corrections were made. One was for the variation in the Ge counter efficiency between the shifted and unshifted peaks (-2.3 to +1.7%). The other was a correction of the shifted transition intensity due to the motion of the recoiling nucleus. To first order this correction at 0° is

$$\Delta I_s(0) = 2 \beta \left[ (1 - \frac{1}{5} A_2) Q_1 + \left( \frac{6}{5} A_2 - \frac{2}{3} A_4 \right) Q_3 + \frac{5}{3} A_4 Q_5 \right]$$  \hspace{1cm} (5)

where $A_j$ and $Q_k$ are the usual angular correlation coefficients and finite geometry correction factors, respectively. This correction is usually larger and of opposite sign (-7%) to the previous one, so that they partially compensate. The total of these two corrections to the shifted intensities varies from -9.3 to -2.3%.

A semi-logarithmic plot of the fraction of unshifted intensity for the $\delta^+ + 6^+$, $\delta^+ + 4^+$, and $4^+ + 2^+$ transitions vs. the target-plunger distance is shown in fig. 3. To a first approximation, the slopes of these lines are

$$\lambda_1 / v \cos \theta_0$$

where $\lambda_1$ is the desired decay constant from the $i^{th}$ level and $v \cos \theta_0$ is the average recoil velocity determined from eq. (4).
Actually, two further corrections must be applied in the data analysis, and these were incorporated into a computer code to obtain an error-weighted least-square-fit to the data. One correction is for the feeding into the level of interest by transitions from higher-lying states. Feeding from only one higher state in the ground-band was considered in the computer program, but in the present case this is the dominant source of feeding. The amount of this feeding was determined experimentally from the intensity of the corresponding transition in the spectra, and also from the Winther-de Boer Coulomb excitation program assuming the ground-band B(E2) values to be those of a rigid-rotor (see below for justification). These two sets of feeding intensities agreed within 15% for the three transitions studied, and the lifetimes are not so sensitive (< 1% for a 50% increase in feeding) to the values taken. The other correction is for the attenuation of the angular distribution of the gamma rays because of the hyperfine field exerted on the nucleus by the unpaired electrons of the excited, recoiling ion. That is, the angular distribution of gamma rays from the recoiling nuclei must be modified by the addition of attenuation coefficients, $\tau_k$ such that

$$W(\theta, t) = 1 + e^{\frac{t}{\tau_2}} A_2 Q_2 P_2(\cos \theta) + e^{\frac{t}{\tau_4}} A_4 Q_4 P_4(\cos \theta)$$

(6)

From the work of other investigators, it is known that in the rare earth ions this attenuation is due to a time-dependent magnetic interaction, so that $\tau_2 = \frac{10}{3} \tau_4$. For Sm nuclei we have determined $\tau_2$ experimentally for the 551 keV $2^+ \rightarrow 0^+$ transition in $^{148}$Sm: $\tau_2 = 3 \times 10^{-11}$ sec. We have also extrapolated from the Rehovot group's data at lower recoil velocities by means...
of an empirical relationship, \( \tau_2 = k v^{-1.2} \), and obtained a similar value, 
\( \tau_2 = 4 \times 10^{-11} \) sec. Even though it probably becomes larger for the higher spin 
states\(^{35}\), we have used the former value for all three transitions as the 
change in lifetime caused by a two-fold increase in \( \tau_k \) is < 1 1/2% for the 
\( 6^+ \rightarrow 4^+ \) and \( 8^+ \rightarrow 6^+ \) transitions.

In Table 1, we have listed the transitions studied, their energies in 
keV, the measured half-lives, values for the total conversion coefficients\(^{36}\), 
and the values of \( B(E2; I \rightarrow I-2) \) derived therefrom. The errors assigned to 
the half-life determinations were obtained by combining the 1 1/2% uncertainty 
in the recoil velocity with the small errors from the uncertainties in the 
lifetime and feeding intensity of the preceding level, from the uncertainties in 
the angular distribution and attenuation coefficients, and most importantly, 
from the statistical uncertainties in the integrations. For the \( B(E2) \) values, 
a 2% error in the total conversion coefficients has been considered in addition; 
this makes its greatest impact on the \( 2^+ \rightarrow 0^+ \) value, and has least effect on 
the \( 8^+ \rightarrow 6^+ \) value.

Finally, it should be pointed out that the \( 2^+ \rightarrow 0^+ \) lifetime was not 
measured in this work, but is the mean value from two groups of investigators\(^{37,38}\) 
who determined this lifetime by direct electronic techniques. The two values 
are in good agreement, combining to an error of 1.3%, but in going to the 
\( B(E2) \) value the 2% uncertainty in the large conversion coefficient is a major 
factor in increasing the error in the transition probability to 2.5%. 

4. Discussion

The B(E2) values determined in this study are shown in the fifth column of Table 1. The sixth column lists calculated rigid-rotor values based on the experimental $2 + 0$ transition probability. It can be seen that within the experimental errors, the measured reduced transition probabilities agree with the rigid-rotor ones, although there appears to be a slight trend towards larger values at higher spins. A convenient way to express the deviations in the ground-band transition probabilities from those calculated for a rigid-rotor by eq. (3) is by means of the parameter $\alpha$, using the expression

$$B(E2; I \rightarrow I-2) = B_o(E2; 2 \rightarrow 0) \frac{\langle 1020|I-20\rangle^2}{\langle 2020|00 \rangle^2} \times \left\{1 + \frac{\alpha}{2} [I(I + 1) + (I - 2)(I - 1)]\right\}^2$$

(7)

where $B_o(E2; 2 \rightarrow 0)$ is the rigid-rotor or unperturbed value. Since experimentally one determines $B(E2; 2 \rightarrow 0)$, and not $B_o(E2; 2 \rightarrow 0)$, an alternative formulation\(^{39}\) is

$$B(E2; I \rightarrow I-2) = B(E2; 2 \rightarrow 0) \frac{\langle 1020|I-20\rangle^2}{\langle 2020|00 \rangle^2} \times \left\{1 + \frac{\alpha}{2} [I(I + 1) + (I - 2)(I - 1)]\right\}^2$$

(8)

These expressions come from a first-order band-mixing calculation, or alternatively, from an approximate treatment of a centrifugal-stretching model. Since we do not know what form the deviations caused by Coriolis anti-pairing or 4th order cranking corrections may take, we shall use eqs. (7) or (8) empirically to fit the present data, without meaning to imply that the cause of the deviations is stretching.
If there is any validity to this phenomenological approach, a plot of the square-root of the quantity $B(E2; I \rightarrow I-2) \left(\frac{2020|00\rangle}{|1020|I-20\rangle}\right)^2$ vs. $\frac{I(I+1) + (I-2)(I-1)}{2}$ should yield a straight line whose slope is $\alpha$ and whose intercept is $\sqrt{B_0(E2; 2 \rightarrow 0)}$. Such a plot for $^{154}$Sm is shown in fig. 4, yielding a least-square-fit, $\alpha = (0.6\pm0.6) \times 10^{-3}$, confirming the earlier observation that within the experimental accuracy of these measurements, the $B(E2)$ values determined are in agreement with those of a rigid rotor. On the other hand, this value of $\alpha$ includes the $2^+ \rightarrow 0^+$ transition which was not measured in the present work. There should be a greater internal self-consistency using only the three presently-measured transitions, as the error in the velocity determination drops out. The value so obtained is $\alpha = (1.1\pm0.8) \times 10^{-3}$, and the corresponding line is shown (dashed) in fig. 4. Because of the relatively large error, this answer still gives a reasonable probability of $\alpha = 0$, but both values suggest that $\alpha$ has a small positive value for the transitions in $^{154}$Sm. The difference in the two values of $\alpha$ and the difference in the extrapolated and measured $B(E2; 2 \rightarrow 0)$ might indicate that either the value of the half-life of the $2^+$ state is $1-1/2\%$ smaller than the value we have used or the mean velocity of the recoiling $^{154}$Sm nuclei is $1-1/2\%$ larger than we have determined, or some combination of these.

In fig. 4 are also plotted the corresponding data for the previously studied $^{152}$Sm. The value of $\alpha$ derived from all four points (including the electronically measured $2^+$ lifetime) is $(1.9\pm0.6) \times 10^{-3}$, while that from the three points of the recoil-distance method is $(2.1\pm0.9) \times 10^{-3}$. These values are in good agreement, as are also the extrapolated and the measured $B(E2; 2 \rightarrow 0)$ values.
The values of $\chi^2$ for both plots for $^{152,154}$Sm are less than 0.5. It would appear that a straight line can be passed through the data, and therefore that, to the present accuracy, eqs. (7) and (8) are appropriate.

From the expression for the intrinsic quadrupole moment of a rotor to first order in the quadrupole deformation parameter, $\beta_2$

$$Q(I) = \frac{3}{\sqrt{5\pi}} Z R_o^2 \beta_2(I) = \frac{3}{\sqrt{5\pi}} Z R_o^2 \beta_2(0) \left(1 + \frac{\Delta \beta_2(I)}{\beta_2(0)}\right)$$

(9)

we can derive an alternate expression for the $B(E2)$:

$$B(E2; I \rightarrow I-2) = B_o(E2; 2 \rightarrow 0) \frac{\langle I020|I-20 \rangle^2}{\langle 2020|00 \rangle^2}$$

$$\times \left\{1 + \frac{1}{2} \left(\frac{\Delta \beta_2(I)}{\beta_2(0)} + \frac{\Delta \beta_2(I-2)}{\beta_2(0)}\right)\right\}^2$$

(10)

By comparing eqs. (7) and (10), we obtain the relationship (to first order in $\Delta \beta_2(I)/\beta_2(0)$)

$$\alpha I(I + 1) = \frac{\Delta \beta_2(I)}{\beta_2(0)}$$

(11)

Equation (10), and therefore also (11), involve the assumption that these collective $B(E2)$ values are determined only by the nuclear shape. Equation (11) allows connection of the present results with other experimental measurements of change in nuclear shape. For example, the change in the mean-square-radius, $\Delta \langle r^2 \rangle$, of a nucleus between two states (usually the $0^+$ ground state and the first excited $2^+$ state) can be determined from a change in the nuclear transition
energy between those states in the presence of: two different electronic environments (isomer shifts in Mössbauer experiments); or with and without a muon in a K-orbit (muonic atom studies). Both types of experiments require a number of assumptions and involve a number of uncertainties in their evaluation, but the two methods seem in good agreement for the ground and 2\(^+\) levels of \(^{152}\)Sm (Refs. 40-43) and the similar 90-neutron nucleus \(^{154}\)Gd (Refs. 40,44), as shown in table 2.

With the further assumptions that the nuclei are axially symmetric spheroids with a sharp boundary given by \(R = R_0 (1 + \beta Y_2)\) and are of uniform density and incompressible*, the relationship, to first order in \(\Delta \beta_2 (I)/\beta_2 (0)\),

\[
\left( \frac{\Delta \langle r^2 \rangle}{\langle r^2 \rangle} \right)_{2^+} \approx \frac{\Delta \beta_2 (2)}{\beta_2 (0)^2} \frac{2\pi}{5} \text{ (12)}
\]

can be derived. Then through eq. (11),

\[
\alpha \approx \frac{\pi}{15 \beta_2 (0)^2} \left( \frac{\Delta \langle r^2 \rangle}{\langle r^2 \rangle} \right)_{2^+} \text{ (13)}
\]

As shown in table 2, the values of \(\alpha\) determined by means of the isomer-shift and muonic-atom studies are in agreement with, though perhaps somewhat smaller

* These assumptions are not correct in detail. Small amounts of higher-order deformations, namely \(\beta_4 Y_4\) and \(\beta_6 Y_6\), have been found in nuclei in this region of the periodic table by inelastic \(\alpha\)-scattering\(^{48}\), and the assumption of incompressibility of the nucleus has been questioned\(^{49}\).
than*, the values from the lifetime measurements on $^{152,154}$Sm. Within the (rather large) experimental errors, this suggests that a large proportion of the increase in B(E2) with spin for $^{152}$Sm comes from the increase in mean-square-radius indicated by the isomer-shift and muonic-atom studies. Only further experimentation will tell whether this is also true of other "soft" rotors.

It seems reasonable to connect this increase in $\langle r^2 \rangle$ or $\beta$ with centrifugal stretching or mixing of the $\beta$-vibrational band into the ground band. The amount of this mixing can be determined from the ratio of a pair of interband E2 transition probabilities from a given $\beta$-band level, where, to first order\textsuperscript{50,51}

$$B(E2; I_\beta \rightarrow I_g) = \langle I_{\beta} 0 |I_{I_0} | I_g \rangle^2 \{M_1 + M_2[I_g(I_g + 1) - I_\beta(I_\beta + 1))]^2 \}$$

or

$$B(E2; I_\beta \rightarrow I_g) = B_0(E2; I_\beta \rightarrow I_g) \{1 + Z_0[I_g(I_g + 1) - I_\beta(I_\beta + 1))]^2 \}$$

But the value depends rather sensitively on the choice of $\beta_2(0)$. We have employed the values determined by Elbek\textsuperscript{49a}) and these lead to magnitudes for $\alpha$ somewhat smaller than those of the lifetime measurements. However, if the effects of $\beta_4$ are included in the calculations of $\beta_2$ from B(E2) measurements, somewhat smaller values of $\beta_2$ result for $^{152,154}$Sm (Ref. 23a), and thus somewhat larger values of $\alpha$. These latter are closer to the values derived from the lifetime experiments, but consideration of higher powers of $\Delta \beta/\beta$ for both $\beta_2$ and $\beta_4$ in eqs. (12) and (13) partially cancels this increase. It seems best to continue to use the simple expressions, if one notes these problems and that the agreement may be somewhat better than appears in table 2.
with

\[ Z_0 = \frac{M_2}{M_1} = \varepsilon_o \left[ \frac{B_0(E2; 0 \rightarrow 2)}{B_0(E2; 0 \rightarrow 2_\beta)} \right]^{1/2} \] (16)

As before, \( B_0(E2) \) means the unperturbed value, and \( \varepsilon_o \) is the first-order, spin-independent mixing amplitude for the \( \beta \)-band ground-band coupling. Then

\[ \alpha_o = 2Z_0 \frac{B_0(E2; 0 \rightarrow 2_\beta)}{B_0(E2; 0 \rightarrow 2)} \] (17)

where \( \alpha_o \) means that part of \( \alpha \) coming specifically from \( \beta \)-band mixing.

Unfortunately, a severe difficulty with this first-order band-mixing picture is that the branching ratios to the ground band from the \( 2^+ \) and \( 4^+ \) levels in the \( \beta \)-bands of \( ^{152}\text{Sm} \) and \( ^{154}\text{Gd} \) do not yield unique values of \( Z_0 \) for either nuclei. This has been demonstrated by a number of workers\(^{19,52-56} \). Recently, though, there have been several approaches to this problem which have met with some success. Two groups have included higher-order terms in the interaction, although in different ways, and have used a mixing coefficient obtained by exact diagonalization\(^{57,58} \). A third group considers not just the \( \beta \)-band ground-band coupling, but simultaneously mixes the \( \beta \)-, \( \gamma \)-, and ground-bands to first order\(^{59} \). All three calculations give better agreement with the experimental ratios of the interband \((\beta + g)\) E2 transition matrix elements vs. the spin function \( I \frac{g(g + 1)}{2} - I_\beta \frac{\beta(\beta + 1)}{2} \) than does the simple first-order band-mixing theory, reproducing roughly the deviation of the data from the straight-line graph of the latter. Although none of these treatments give complete agreement with the data, they indicate that it may be possible to explain these
deviations in $^{152}$Sm and $^{154}$Gd within the rotational model. Surely a better, though more complicated, procedure would be diagonalization of the $\beta$, $\gamma$, and ground-band mixing using higher than first-order interaction terms. However, we can use the results of the $\beta$, $\gamma$, and ground-band mixing study already performed on $^{152}$Sm and $^{154}$, $^{156}$Gd by Rud, Nielsen, and Wilsky$^{59}$), as they give perhaps the best available fit to the branching ratios from the $2^+$ and $4^+$ levels of the $\beta$-band. The required amount of $\beta$- and ground-band mixing is reflected in an $\alpha_0$ of $2.2 \times 10^{-3}$ and $2.7 \times 10^{-3}$ for $^{152}$Sm and $^{154}$Gd, respectively (column 6, table 2). This is in general agreement with the value of $\alpha$ obtained from the lifetime measurement on $^{152}$Sm, since the contribution of $\gamma$-band mixing to $\alpha$ is small. These values are possibly larger, however, than those derived from the isomer-shift and muonic-atom studies. It will be important to see, when further work has solved this band-mixing problem, if this difference remains.

Still another determination of $\frac{\Delta \langle r^2 \rangle}{\langle r^2 \rangle}$,

$$\frac{\Delta \langle r^2 \rangle}{\langle r^2 \rangle} = \frac{10}{3} \frac{\varepsilon \ I(I + 1) \ \rho}{Z}$$

(18)

can be obtained from the reduced monopole transition amplitude, $\rho(I)$, between $\beta$- and ground-band states of spin $I$ and from the admixed amplitude of the one state in the other$^{61}$). The latter quantity usually is represented by the first-order term, $\varepsilon \ I(I + 1)$, and is evaluated by analysis of $\beta$-band branching ratios to the ground band as mentioned in the two previous paragraphs. Thus, although it suffers from the difficulties mentioned above, it still may provide an independent estimate for $\alpha_0$, as shown in column 7 of table 2. The results are in good agreement with the largest values in column 4, thus indicating that, at
least for these nuclei, β-band mixing, contrary to earlier thought, can explain the values of \( \frac{\Delta \langle r^2 \rangle}{\langle r^2 \rangle} \) from the isomer-shift and muonic-atom studies.

Finally, it is also possible to test whether \( \frac{\Delta \beta_2(I)}{\beta_2(0)} \), and thus \( \alpha \), is consistent with the deviations in the ground-band energy spacings. To do so, a particular form of the energy expression must be chosen. We shall consider the centrifugal stretching expression with the assumption that the moment-of-inertia \( \mathcal{G} \) goes as \( \beta^2 \). Then

\[
E(I) = AI(I + 1) \left[ 1 - \frac{\Delta \beta_2(I)}{\beta_2(0)} + 2 \left( \frac{\Delta \beta_2(I)}{\beta_2(0)} \right)^3 + ... \right] \tag{19}
\]

with \( A = \hbar^2/2\mathcal{G} \). If this expression is taken only to first order in \( \Delta \beta_2(I)/\beta_2(0) \), then from eq. (11)

\[
E(I) = AI(I + 1) \left[ 1 - \alpha I(I + 1) \right] \tag{20}
\]

which is equivalent to the first two terms of eq. (2), with \( \alpha = -B/A \) \tag{21}

However, since eq. (20) cannot fit the measured energy levels in \(^{152,154}\text{Sm}\), we adopt the point of view that the energies indicate values of \( \alpha \) that vary with \( I \). Thus for \(^{152}\text{Sm}\) the energies of the \( 0^+, 2^+, \) and \( 4^+ \) levels lead to \( \alpha = 6.7 \times 10^{-3} \), from the \( 2^+, 4^+, \) and \( 6^+ \) levels to \( \alpha = 2.9 \times 10^{-3} \), and from

\( \Delta \beta_2(I) \) is missing, so that this approximation is better than would otherwise be expected.

If initially the choice \( \mathcal{G} \) goes as \( \beta \), rather than \( \beta^2 \), is made, eq. (19) is different, leading to \( \alpha = -2B/A \).
the 4⁺, 6⁺, and 8⁺ levels to α = 1.7 × 10⁻³; for ¹⁵⁴Sm the corresponding values are 1.6 × 10⁻³, 1.2 × 10⁻³, and 0.9 × 10⁻³, respectively. It is apparent that these calculated values of α in both cases are initially larger than the values derived from the lifetime measurements. But they decrease with increasing spin of the levels involved, and appear to approach the values of the lifetime measurements, or possibly, the still smaller isomeric-shift quantities. The large values at the bottom of the band indicate rather clearly that the energies are influenced by effects in addition to (or other than) centrifugal stretching. However, the agreement at higher spins suggests that these additional factors may affect only the lowest 3 or 4 states of the band. One must be very cautious about this last conclusion, however, since more cases are needed and also energies and B(E2) values for higher spins.

Part of the above empirical conclusions have also been suggested by recent microscopic calculations leading to theoretical estimates of \( \frac{\Delta \langle r^2 \rangle}{\langle r^2 \rangle} \) or of the B coefficient in the energy expression. Other bands besides the β-(and γ-) band mix into the ground band; two prominent ones should be the proton and neutron pairing vibrational bands, whose admixture constitutes the Coriolis anti-pairing effect. These play an important role in the deviations of the energy-level spacings because of their effect on increasing the moment-of-inertia, \( \mathcal{S} \), but may have little effect on the E2 transition rates and hence on branching ratios. Such behavior might explain why the low-spin energy spacings give the largest values of α found in table 2. Of course, many other excited \( K = 0 \) bands can and do mix with the ground band and must be considered; they produce the effects treated by the cranking model to 4th order.
Thus, a number of calculations treat, to varying degree, the effects of rotation on the quadrupole deformation, on the pairing correlations, and on the motion of certain quasi-particle states. For example, the work of Bes, Landowne, and Mariscotti\(^{10}\) is essentially a microscopic calculation of the centrifugal stretching model but including, in addition, the effects of rotation on pairing. Values of \( \frac{\Delta \langle r^2 \rangle}{\langle r^2 \rangle} \) were calculated for \(^{152,154}\)Sm and \(^{158}\)Gd, among others, which by eq. (13) we can relate to \(10^3 \alpha\), obtaining 3, 0.7, and 0.3, respectively. They have also calculated the expectation values of the E2 operator in the \(^{2+}\) state from which values of \(10^3 \alpha\) of 2, 0.6, and 0.3, respectively, can be directly obtained. It is not clear to us why these two sets differ, but presumably this reflects the first-order approximations we have made in obtaining \(\alpha\).

Pavlichenkov\(^{13}\) considered \(\beta\) and \(\gamma\) vibration-rotation interactions on the basis of the cranking model. His calculations suggest that the effect of vibration-rotation interaction is important only at the beginning and end of a deformed region, and that in-between the major source of the \(\beta\) coefficient (of the \( r^2(I + 1)^2 \) term) is the effect of rotation on the quasi-particle motion. Agreement of calculation with experiment is moderately good.
Mikoshiba et al.\textsuperscript{16} calculated the nature of the first 10 \( K = 0 \) states in deformed nuclei across the rare earth region, and their effect on the B coefficient. They employ both pairing and quadrupole fields and consider non-adiabatic coupling between the bands using the cranking model. They find that usually the \( \beta \)-band is the first excited \( K = 0 \) state, but not always; there are exceptions, mainly in the 2\textsuperscript{nd} half of the rare earth region. In these cases the neutron-pairing vibration is calculated to be the lowest band, but more usually the pairing vibration is spread over several higher states. The effect of the mixing of all these states into the ground band gives approximately the correct energy-level spacing deviations. The Coriolis anti-pairing contributes a moderately constant amount to the B coefficient across the rare-earth region, and dominates the smaller coefficients in the middle of the region. Centrifugal stretching is calculated to dominate at the beginning of the rare earths, and by being coherent with the Coriolis anti-pairing effect causes very large energy deviations.

Marshalek\textsuperscript{6} considered the effects of stretching, Coriolis anti-pairing, and cranking, and calculated values for \( \frac{\Delta \langle r^2 \rangle}{\langle r^2 \rangle} \) and the B coefficient. Generally, both sets of values are a factor 2-3 too large compared to experiment. For example, from his values of \( \frac{\Delta \langle r^2 \rangle}{\langle r^2 \rangle} \) we can obtain values of \( 10^{-3} \alpha \) for \( ^{152,154}_{\text{Sm}} \), and \( ^{154,156,158}_{\text{Gd}} \) of 5, 0.4, 2.2, 0.3, 0.2, respectively. As expected, stretching makes up the largest part of these, although the cranking correction is not negligible after the beginning of the rare earths. The values of the B coefficients mostly come from the cranking corrections and the Coriolis anti-pairing, but at the beginning of the rare earths, the stretching term dominates.

Ma and Rasmussen\textsuperscript{12} consider the same corrections explicitly (they take particle-number conservation into account in treating pairing), but as the modes
of a generalized vibration. Thus, they show that the seemingly quite different empirical energy-level expressions from 4th order cranking, from stretching, and from the VMI model all have the same form and are related. This explains why they all give such good fits, as their empirically-derived parameters include all three major effects. The calculated B coefficients of Ma and Rasumssen are of the right order of magnitude in the rare-earth region except at the beginning, where they are too small and appear to give too small a stretching component. In general, they observe that the cranking correction is comparable to that due to change in pairing, and that these effects are larger than that of stretching on the B coefficient.

All of the above-mentioned microscopic calculations agree among themselves to within factors of two or so. As expected, calculations of \( \frac{\Delta \langle r^2 \rangle}{\langle r^2 \rangle} \) indicate stretching as the most important component. For the B coefficients, 4th order cranking corrections and Corolis anti-pairing are usually of the same order, and they appear to dominate the coefficient. Neither of them change dramatically with Z or A (by less than a factor of 5 or so) around the rare-earth region. On the other hand, quadrupole stretching appears to be most important at the beginning and end of the region, but the resulting B coefficient component decreases by two orders of magnitude in the middle of the region.
5. Summary

We have shown by recoil-distance Doppler-shift lifetime measurements that the E2 reduced transition probabilities of the $8^{+} \rightarrow 6$, $6^{+} \rightarrow 4$, $4^{+} \rightarrow 2$, and $2^{+} \rightarrow 0$ transitions in $^{154}\text{Sm}$ follow those of a rigid rotor to an accuracy of a few percent. However, the corresponding values for $^{152}\text{Sm}$ show a definite increase above those of the rigid rotor. A simple band-mixing or centrifugal-stretching model to first order gives an expression for the increase in $B(E2)$, and the data, though not as accurate as one would like, agree with the predicted linear dependence on $I(I+1)$. If possible, it would be important to determine the results more accurately, in order to see how well a first-order term in the interaction does explain the $B(E2)$ values. It is well known that the expression for the energy levels as a function of $I(I+1)$ requires many higher terms for "soft" nuclei, and so this difference in behavior would be an indication that different effects are involved. This may well be related to the difference in values of $\alpha$ obtained from the lowest energy-level spacings on the one hand, and from the other types of measurements shown in table 2 on the other. We take this to indicate that the other four measured quantities probably have a common cuase, most likely some kind of centrifugal stretching, but that additional factors affect the lowest ground-band energies. An example would be the pairing vibration, which upon mixing into the ground band would increase $\delta$ and so decrease the energy level spacing, but might not appreciably affect the beta- and ground-band interband or intraband E2 transition strengths. In this regard it may be important to note that the value of $\alpha$ obtained from the energy levels appears to drop sharply with increasing $I$, and already for the $4^{+}$, $6^{+}$, and $8^{+}$ states in both $^{152}\text{Sm}$ and $^{154}\text{Sm}$ one obtains $\alpha$ values consistent with those from the other methods.
It is important to know if all the measured quantities in table 2 except the ground-band energies (col. 8) are mutually consistent. Within the present error limits, this appears to be reasonably probable. More accuracy, as well as more measured cases, are badly needed to settle this question. However, it does seem clear that the results of the present experiments and of the other studies summarized in table 2, indicate that for the "soft" nuclei, $^{150}$Nd, $^{152}$Sm, and $^{154}$Gd, there is a marked increase in $(r^2)$ or $\beta$, with spin, of an order sufficient to explain a significant part, or most, of the changes in ground-band $B(E2)$ values and the deviations in $E2$ branching ratios from the $\beta$ to the ground band. For more rigid nuclei, such as $^{154}$Sm and $^{156}$Gd, the changes in $(r^2)$ or $\beta$ are an order of magnitude smaller than for the "soft" nuclei, and this result shows up in all the different kinds of measurements as well as in the microscopic calculations.

Acknowledgments

The authors would like to thank Dr. K. Nakai for help during some of the experiments, Mr. Tom Gee for his careful preparation of the targets, and the crew of the Hilac for providing the $^{40}$Ar beams.
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Table 1. $B(E2)$ Values for $^{154}$Sm.

<table>
<thead>
<tr>
<th>Transition</th>
<th>Energy (keV)</th>
<th>$T_{1/2}$ (psec)</th>
<th>$\alpha_T$</th>
<th>$B(E2; I \rightarrow I-2)$ ($e^2 b^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$a$</td>
<td>$b$</td>
<td>exp.</td>
</tr>
<tr>
<td>2 → 0</td>
<td>81.99</td>
<td>3017 ±38$c$</td>
<td>5.003</td>
<td>0.843±0.021</td>
</tr>
<tr>
<td>4 → 2</td>
<td>184.9</td>
<td>172.7 ± 5.0</td>
<td>0.277</td>
<td>1.186±0.039</td>
</tr>
<tr>
<td>6 → 4</td>
<td>277.4</td>
<td>23.34±0.69</td>
<td>0.074</td>
<td>1.374±0.047</td>
</tr>
<tr>
<td>8 → 6</td>
<td>359.1</td>
<td>6.17±0.62</td>
<td>0.032</td>
<td>1.49±0.15</td>
</tr>
</tbody>
</table>

$a$ The value of $v \cos \theta_0$ for the recoiling nuclei was (0.0340±0.0005)c.

$b$ From calculations by R. S. Hager and E. C. Seltzer, ref. 36).

$c$ Average value from refs. 37, 38).
Table 2. Derived Values of $\frac{\Delta \langle r^2 \rangle}{\langle r^2 \rangle}$ and $\alpha$.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$\frac{\Delta \langle r^2 \rangle}{\langle r^2 \rangle} \times 10^4$</th>
<th>$\alpha \times 10^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{150}\text{Nd}$</td>
<td>5.8±0.8$^a$</td>
<td>1.4±0.2</td>
</tr>
<tr>
<td>$^{152}\text{Sm}$</td>
<td>3.7±1.0$^b$ 5.5±0.6$^a$</td>
<td>1.0±1.3±0.3 1.9±0.6$^h$</td>
</tr>
<tr>
<td>$^{154}\text{Gd}$</td>
<td>7.5±2.3$^e$ 5.9±0.8$^a$</td>
<td>1.5±1.8±0.2</td>
</tr>
<tr>
<td>$^{152}\text{Sm}$</td>
<td>0.2±0.2$^r$</td>
<td>0.04±0.04 0.6±0.6$^i$</td>
</tr>
<tr>
<td>$^{156}\text{Gd}$</td>
<td>0.6±0.2$^g$</td>
<td>0.1±0.04</td>
</tr>
<tr>
<td>$^{174}\text{Hf}$</td>
<td></td>
<td>0.5$^k$ ~ 0.4$^k$</td>
</tr>
</tbody>
</table>

$^a$Ref. 40).
$^b$Ref. 42), but corrected for 5f electron shielding on solids as mentioned in Ref. 47).
$^c$Ref. 41), but corrected for 5f electron shielding on solids as mentioned in Ref. 47).
$^d$Ref. 43).
$^e$Ref. 44).
$^f$Ref. 45), but corrected for 5f electron shielding on solids as mentioned in Ref. 47).
$^g$Ref. 46).

(continued)
Table 2 (continued)

<table>
<thead>
<tr>
<th>h</th>
<th>i</th>
<th>j</th>
<th>k</th>
<th>l</th>
</tr>
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<tbody>
<tr>
<td>Ref. 24</td>
<td>Present work</td>
<td>Ref. 59</td>
<td>Ref. 60</td>
<td>Derived from Ref. 59 by use of eq. (11)</td>
</tr>
</tbody>
</table>
Figure Captions

Fig. 1. Schematic of the experiment. The Si ring counter detects the back-scattered $^{40}$Ar projectiles in coincidence with the gamma cascade from the Sm nuclei that have been Coulomb excited. Those of the latter nuclei that decay in flight yield Doppler-shifted transitions in the Ge(Li) detector at 0° to the beam; those that stop first in the lead-covered plunger give normal peaks. By varying the distance between the target and plunger, one can vary the ratio of unshifted to shifted intensities, and so obtain essentially a decay curve of the excited state (see text).

Fig. 2. Spectra from Coulomb excitation of $^{154}$Sm by 146 MeV $^{40}$Ar. The target-plunger distance is given in thousandths of an inch for each spectrum. The positions of the unshifted (shifted) lines are given at the top (bottom) of the figure.

Fig. 3. The fraction of each transition which is unshifted in energy vs. the separation distance of the target and plunger. The solid lines are the computer-calculated best-fit curves, allowing for one stage of feeding into the particular state.

Fig. 4. Plot of $\sqrt{B(E2; I \rightarrow I-2)} \frac{(2020|00\rangle}{(1020|I-2\rangle} \sqrt{\frac{I(I+1) + (I-2)(I-1)}{2}}$ for $^{154}$Sm, upper curves, and $^{152}$Sm, lower curves. The solid lines are least-square fits to the four points including the electronically-measured $2^+ \rightarrow 0^+$ transition, with the indicated errors. The dashed lines are least-square fits to the three upper points from the present measurements only.
Fig. 1
Fig. 2
Fig. 3
\[ \sqrt{\langle B_{E2; I \rightarrow I-2} \rangle^2} \langle I|2020|00\rangle \]

\[ \frac{I(I+1)+(I-2)(I-1)}{2} \]

Fig. 4
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