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Assessing the Usefulness of Distributed Measurements in the Smart Grid

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Assessing the Usefulness of Distributed Measurements in the Smart Grid

A thesis submitted in partial satisfaction of the requirements for the degree of

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In

COMPUTER ENGINEERING

by

Theodore Framhein

March 2013

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# Table of Contents

I) Introduction 1  

II) Power Distribution Systems 3  
   A. A History of Distribution Control and Monitoring 5  
   B. Distribution Automation 7  
   C. Grid Reliability – Data and Networking 8  
   D. Grid Reliability – the Smart Grid 9  

III) Related Research 11  
   A. Fault Location 11  
   B. Characterization of Transient Phenomena 13  

IV) Power System Modeling for Transient Analysis 15  
   A. Theory 15  
   B. Transient Analysis Example – Capacitor Bank Switching 24  
      i. Capacitor Bank Switching – Steady State 25  
      ii. Capacitor Bank Switching – Transient 27  
      iii. Capacitor Bank Switching – Solution 29  

V) Simulation Tools – ATP 30  

VI) Simple Simulation Model and Fault Location by Voltage Sag 31  

VII) Simulation Model – IEEE 34 Node Feeder 41  
   A. Simulation Results – Line Faults 44  
      i. A-to-Ground 44  
      ii. A,B and A,B,C-to-Ground 47
iii. A-to-B 48
iv. High-Z A-to-Ground 50

B. Summary 51

C. Automated Fault Location 54

VIII) Simulation Results – Transient Analysis 58

IX) Conclusion 62

Appendices 63

References 78

Figures

Figure 1 - Investments in transmission systems, 1975-2000 .................................................. 9
Figure 2 - Switching transients, closing (a) and opening (b) ................................................. 15
Figure 3 - Integral of x (a) and trapezoidal rule (b) ................................................................. 17
Figure 4 - Inductor companion equivalent ............................................................................. 18
Figure 5 - Capacitor companion equivalent ............................................................................ 19
Figure 6 - Distributed components ......................................................................................... 20
Figure 7 - Lossless line companion equivalent ....................................................................... 22
Figure 8 - RLC results ............................................................................................................ 23
Figure 9 - Example circuit .................................................................................................... 24
Figure 10 - Steady state circuit ............................................................................................. 25
Figure 11 - Transient circuit .................................................................................................. 27
Figure 12 - Capacitor switching transient ............................................................................... 29
Figure 36 – A-to-ground: three phase voltage at 4 over time (A, B, C) ................................ 45
Figure 37 – A-to-ground: three phase voltage at 7 over time (A, B, C) ................................. 45
Figure 38 – A-to-ground: phase A voltage from substation to fault over time (1, 2, 3, 4) .... 46
Figure 39 – A-to-ground: phase A voltage past fault to end of feeder over time (5, 6, 7) ...... 46
Figure 40 – A,B-to-ground: phase A voltage down feeder over time (1, 2, 3, 4, 7) ............. 47
Figure 41 – A,B-to-ground: phase B voltage down feeder over time (1, 2, 3, 4, 7) ............ 47
Figure 42 – A,B,C-to-ground: three phase voltage at 1 (A, B, C) and 7 (A, B, C) over time .... 47
Figure 43 – A-to-B: three phase voltage at 1 over time (A, B, C) ........................................ 48
Figure 44 – A-to-B: three phase voltage at 4 over time (A, B, C) ........................................ 48
Figure 45 – A-to-B: three phase voltage at 7 over time (A, B, C) ........................................ 49
Figure 46 – A-to-B: voltage at 1 (A, B) and 4 (A, B) over time ............................................. 49
Figure 47 – High-Z A-to-ground: three phase voltage at 1 over time (A, B, C) .................... 50
Figure 48 – High-Z A-to-ground: three phase voltage at 4 over time (A, B, C) .................... 50
Figure 49 – High-Z A-to-ground: phase A voltage from substation to fault over time (1, 2, 3, 4) .................................................................................................................. 50
Figure 50 – High-Z A-to-ground: phase A voltage from fault to end of feeder over time (4, 5, 6, 7) ................................................................................................. 50
Figure 51 - A-to-ground: RMS voltage at measurement locations ........................................ 52
Figure 52 - A,B-to-ground: RMS voltage at measurement locations ..................................... 52
Figure 53 - A,B,C-to-ground: RMS voltage at measurement locations ............................... 53
Figure 54 - A-to-B: RMS voltage at measurement locations .................................................. 53
Figure 55 - High-Z A-to-ground: RMS voltage at measurement locations ......................... 54
Figure 56 - Normal operation: RMS voltage at measurement locations .............................. 54
Figure 57 - A-to-ground: |Faulted Voltage – Normal Voltage| at measurement locations ... 56
Figure 58 - A,B-to-ground: |Faulted Voltage – Normal Voltage| at measurement locations 56
Figure 59 - A,B,C-to-ground: |Faulted Voltage – Normal Voltage| at measurement locations
........................................................................................................................................... 56
Figure 60 - A-to-B: |Faulted Voltage – Normal Voltage| at measurement locations........... 57
Figure 61 - High-Z A-to-ground: |Faulted Voltage – Normal Voltage| at measurement
locations........................................................................................................................................ 57
Figure 62 - Capacitor switching as seen at substation......................................................... 60
Figure 63 - Generator switching as seen at substation ....................................................... 60
Figure 64 - Line switching as seen at substation................................................................. 61
Figure 65 - Motor switching as seen at substation.............................................................. 61

Tables

Table 1 - Fault simulation results using IEEE 34-node model........................................ 51
Table 2 - Fault simulation results compared to normal operation (|Faulted Voltage – Normal
Voltage|)........................................................................................................................................ 58
Table 3 – Peak magnitude of transient activity at various locations (|V|)...................... 62

Appendices

Appendix A: RLC circuit Matlab code 63
Appendix B: Capacitor switching circuit Matlab code 64
Assessing the Usefulness of Distributed Measurements in the Smart Grid

Theodore Framhein

Abstract: The move to modernize power distribution, including the deployment of smart meters in the field, opens new possibilities for monitoring and control with the historically volatile distribution system. Given measurements from smart meters and other intelligent devices, location and characterization of the three basic fault types (line-to-ground, multi-line-to-ground, and line-to-line) is possible using voltage sag characteristics. In addition, distributed measurement locations result in greater fidelity in transient analysis, given waveforms captured closer to their source.
I) Introduction

The development of a “smart grid” offers many potential benefits to electric power systems. Reliability of the electrical grid, as seen by individual consumers, requires a reliable distribution system. Detection of incipient failures and outages, and rapid repair of failed (or failing) distribution infrastructure, could significantly contribute to a reduction of the number and duration of power outages. Today, most failures in the distribution systems are brought to the attention of the utility by a call from a customer without power, or with dim lights, etc. The typical response of the utility is to dispatch repair personnel to the locale, and to have them visually inspect the lines and search the subsystem (in a vehicle or on foot) to identify the probable cause of the outage or other failure. This approach to power service repair or restoration in the distribution system is not “modern”; rather, it has been the normal practice for utilities for many decades. With induction of “smart meters” and other intelligence in the distribution system, and the corresponding explosion of potentially available real-time measurements, it is our thesis that reliability of the power delivery system, as measured by the percentage of time when power is not available or is of degraded quality, can be significantly improved. Furthermore, this improvement in power availability can be obtained at very modest cost, and potentially even at less cost than that of today’s maintenance and repair practices.

Digital data systems are providing a starting point for true modernization in the way power is generated, transmitted, and distributed. Many of the ‘analog’ switches, relays, and measuring points commonly found in power systems are being supplemented by digital intelligence, enabling more timely and dynamic operation. Perhaps the most widely known
of these initiatives is the move to smart meters, in place of older analog meters, at residences and businesses. These devices include processing and communications modules that can allow them not only to monitor power usage, but also to sample incoming voltage and current waveforms, generating usable data. Given an overlying data network – not at all a rare thing in modern society – measurements from multiple meters can be gathered and used together.

A separate but no less significant area for innovation in power systems has been the drive to make power distribution – that is, the segment of the system beyond the substations that divide local distribution from long-range transmission – more reliable. Typical estimates suggest that 80% of faults occur within distribution systems, and this is not hard to imagine when one considers their relative volatility – power lines that are closer to the ground and various hazards, and the increasing complexity of the modern power grid. Many existing techniques suggested for achieving greater reliability in distribution systems involve measurements taken at the substation, not because these readings are more revealing, but rather because there is a lack of measurement equipment anywhere else on the feeder.

This paper proposes a melding of two ideas in the creation of a distributed feeder monitoring system. By utilizing the ability of smart meters to read and store voltage waveforms with high accuracy in both time and amplitude, a more complete picture of the sequence of events leading up to failure can be pieced together. Faults can be located to a single segment (defined as the connection between separate measurement locations), and transient waveforms can be further characterized to deduce source (type and location) and
severity. Such a system has the potential to make recovery from faults easier and faster, and even to allow for the prevention of outages before they occur.

II) Power Distribution Systems

Power distribution is conventionally recognized as the segment of the power system between the substation and the consumer. At the substation, the power flow is converted from high-voltage transmission to medium- and low-voltage distribution using high-capacity transformers.

Beyond the substation, power is transferred using lines and transformers, typically over shorter distances with conditioning and control handled by various other apparatus. Overhead transmission lines are generally non-transposed, and most systems are radial with one-, two-, and three-phase laterals [1]. Where reactive power – also called imaginary power, a result of inductive loading – is a problem, switchable capacitor banks are installed to provide local VAR (volt-amperes reactive) support, thereby improving voltage levels.

Reclosers break short-circuit current and attempt to re-close and restore power after a short-term fault (e.g. tree across lines), typically three times. In the event of a longer-term fault (e.g. line down), supporting sectionalizers, relays, fuses, and circuit breakers permanently isolate affected portions of the distribution grid. Variable taps on transformers, as well as voltage regulators, can be used to improve voltage levels down the feeder. Most of these devices act based on instructions from onboard controllers, and these actions often go unnoticed by power system workers.
It is difficult, due to general secrecy among utility companies, to gain a true understanding of the day-to-day operations of a typical distribution system. Utility workers are often unaware of major malfunctions and outages until they receive notification from customers of flickering lights or a total lack of power [2][15]. Customer notification is spotty at best: it is not at all uncommon for outages to last several minutes or more before any of the affected contacts the utility, particularly at night. By this time, various parts of the system have been stressed due to failure. At its base, this approach is reactionary and not particularly elegant.

Upon confirmation of an outage or other malfunction, and without any measurements down the line from the substation, utilities turn to workers in company vehicles to physically search for the source [2]. Based on location and customer notification, utility workers can narrow their search to specific areas, typically without any idea of what to look for. Short-term problems, including temporary line shorts and incipient device failure, can be exceedingly difficult to pin down; in addition, damage due to fault current and other irregularities often goes unnoticed.

Additionally, the tools used by power systems workers have a focus that is inconsistent with real-time analysis. According to [3], most tools used in distribution analysis are designed primarily to deal with measurements at a single point in time (e.g. phasors), as opposed to the waveforms generated in transient analysis. Additionally, the capacity for system dynamics studies is uncommon among the simulation tools typically used. This focus on long-term static measurements is a hindrance to many of the real-time measurement and control methods currently being researched.
A) A History of Distribution Control and Monitoring

Control and monitoring of power distribution has been a practical consideration for utilities in the United States since at least the 1940’s [4]. At that time, simple pilot wires were used for communication of commands. These utilized pairs of wires between sites, forming direct connections between devices. Multiplexing could be used to minimize the number of wires between devices. This expansion necessitated a select/check/operate scheme, whereby a device was first selected, its identity confirmed, and finally operated upon, generating a final acknowledgement.

Between 1950 and 1965, telephone relay systems were used as a communication medium. These used coded, timed, or sized pulses to relay information to remote devices, and utilized the aforementioned select/check/operate system for oversight. Westinghouse’s Visicode supervisory control is an example of this, using pulse count to encode information.

On the monitoring side, transducers have been used to generate a current scaled with the measured quantity, which was then communicated over pilot wires. Pulse rate systems similar to the above have also been used to communicate measurements over pilot wire, powerline carrier, and microwave media; Westinghouse’s Teledac system is an example of this. Other vendors included General Electric and Control Corporation.

Data aggregation architectures since the 1950’s have been multiplexed, again minimizing the number of direct connections required. Continuous scanning systems were made possible by solid-state digital devices, and automatic data loggers relieved dispatchers from having to log measurements manually.
Modern computing systems allowed for further system expansion, given their increased processing speed, memory capacity, and networking throughput. With the feasibility of computer-based master stations came SCADA – Supervisory Control And Data Acquisition – systems, in the mid 1960’s. Functions of a typical SCADA system include data aggregation, grid status monitoring, alarm triggering, and conveyance of all of this to utility workers through simple displays and user interfaces. The master station communicates with Remote Terminal Units (RTU’s), which are solid state devices distributed throughout the power system that can continuously record system conditions. RTU’s typically include backup power so that they can operate during outages, and use proprietary communication protocols (although there have been efforts to standardize message formats). Communications systems used range from microwave to powerline carrier.

A separate but related concept is that of Automated Generator Control (AGC). AGC systems estimate total system load and adjust generation accordingly – typically every few minutes – and execute Load Frequency Control (LFC) every few seconds. They have been in service since the 1950’s, and were converted to run on computer systems around the same time as the development of SCADA. At this point, AGC combined with SCADA and formed what became known as System Operation Computers (SOC) systems, which forecasted load and scheduled generation and power interchanges between utilities.

With increasing computational capability came more advanced system monitoring applications. In the 1970’s, Energy Management Systems (EMS) entered deployment. These execute continuous load-flow measurements using various mathematical techniques to improve processing time – first Gauss-Seidel, and later Newton-Raphson. Redundancy was
always a concern: a typical specification suggests a required 99.8% or better availability [4]. The introduction of powerful server computers in the 1980’s and 1990’s further improved efficiency and reliability.

B) Distribution Automation

SCADA, AGC, and EMS are all systems that aid in Distribution Automation (DA), the goal of which is to automate computationally intensive and otherwise time-consuming tasks within power systems, thereby increasing operational efficiency [5]. Functions include transformer load balancing, which provides near-real time data; voltage regulation using Load Tap Changers (LTC’s); remote interconnect, capacitor bank, and load switching; load management and Automated Meter Reading (AMR); and outage reporting. Importantly, there is also the possibility of automated fault isolation and sectionalizing using distributed voltage measurements, which will be discussed in detail later.

Many utilities prefer to avoid changes to their systems until those changes have been proven effective by other utilities. There is a tendency to rely on customer-initiated alarms – that is, phone calls from customers reporting outages and other operational irregularities – and where data is used, it is often on a month-to-month basis, typically to predict required load. Distribution automation systems, if found to be worth pursuing based on cost and potential benefits, will likely necessitate pilot projects before functional deployment.
C) Grid Reliability – Data and Networking

At present, the major push in distribution system monitoring is for measurements taken at the substation, given a lack of overarching data network. 'Disturbance recorders,' a category of measurement devices, include transient fault recorders, state-recording sequence of events recorders, and PMUs (phasor measurement units) [6]. Of these, PMUs and fault recorders have the greatest potential within the context of this paper. The former provide a GPS time-stamped phasor measurement of voltage (accurate to within 1µs), while the latter typically reside in the substation and record voltage and/or current at sampling rates between 64 and 385 samples per 60Hz cycle (3840-23100Hz effective). Some digital relays, which are distributed along the feeder, are also capable of sampling between 32 and 128 times per 60Hz cycle (1920-7680Hz effective). Some disturbance recorders and relays are able to make fault location estimations, as well, but due to complexities within the power system these are not always reliable. Taken together, devices like these may someday enable real-time power system state estimation and even measurement, which would enable full characterization of the entire system; this paper, however, focuses on transient (event) recording and analysis using distributed devices.

What to do with the data? Silver Spring, Cisco, and others are pushing separate data networks based on various technologies. In addition, wireless telephone carriers including AT&T, Verizon, and Sprint are looking to utilize their existing cellular networks to enhance the connectivity available to monitoring devices [7]. The preferred technology will depend on the architecture of the system, which is defined by the processes used to aggregate and analyze the gathered data, as well as required data rates and radio ranges. Wifi (802.11),
WiMAX (802.16), and cellular networks provide a direct tradeoff between range and data rate, with wifi supplying the highest data capacity and cellular networks providing the greatest range (and the added benefit of existing infrastructure). Palak P. Parikh et al. have summarized the usefulness of several wireless communication options [8].

Additionally, it should be noted that one of the primary concerns of utility workers is data overload – more data is worthless if the utility is already overwhelmed. Future work may involve the creation of an overarching data network with this and other tradeoffs in mind, but this aspect of distribution system monitoring is not covered to any greater depth here.

D) Grid Reliability – the Smart Grid

A major hurdle in increasing the reliability of modern power distribution is a lack of situational awareness. Historically, most distribution system analysis has been based on measurements from the substation alone. As a result, utility workers must react to problems with very little supporting information, causing more frequent and longer-lasting fault activity. The technology for an over-arching data network that would give workers the information they need to properly administer the power system exists, and the recent push for smart grid technology shows some promise; before this, however, the first hurdle to overcome has been cost. There is little incentive, upfront, for utilities to invest in grid modernization, as the cost of such an upgrade is
prohibitive and the benefits too slow in coming to fruition. Figure 1 gives a snapshot of this problem in power transmission, owing itself primarily to uncertainty in responsibility (government vs. companies at various levels of the power system)[9].

This trend has shown signs of reversing, however, as mentioned above – in particular, there has been significant government funding for smart grid initiatives. Under the American Recovery and Reinvestment Act of 2009, President Barack Obama announced 100 grants totaling $3.4 billion for use in grid modernization, including the deployment of 2.6 million smart meters [10]. Events including the 2003 blackout in the northeastern United States, a result of cascading failures due to uncoordinated protective device operation, have brought the lack of awareness and coordination in power grid operations under public scrutiny.

Reliability becomes increasingly important when one considers grid modernization as a whole. The advent of renewable generation and grid storage, in addition to new types of loads (e.g. electric vehicles), presents new sources of volatility that will require additional control given the relative age of existing infrastructure. There is a push to create a ‘flat’ load curve with this combination of variable sources and loads, and smart grid technologies allowing real-time measurement and control will be essential in realizing this goal.

Smart meters, themselves, provide a potential platform for data acquisition within the distribution system. The GE kV2c meter that PGE uses within Santa Cruz, for example, claims to digitize both current and voltage waveforms 1.68 million times per second per signal; the onboard converter provides an oversampling rate of 512 with an effective sampling rate of 3280 per signal (each has its own converter)[11]. Given a data network, this
information could be very useful indeed, not only in making distributed generation and other unpredictable sources and loads a reality, but also in making the system more reliable – as will be discussed later.

This, in conjunction with the more general goal of creating a more reliable distribution system, creates the impetus for this paper – the push to create a distributed system capable of increasing situational awareness and ease of failure location and diagnosis through the use of high-fidelity measurements from various locations along the feeder, most notably smart meters.

III) Related Research

A) Fault Location

Most fault location algorithms have been designed for use in comparatively simple transmission systems [2], where interconnections are generally single lines; the complex nature of distribution system architectures, with common lateral connections and more variety in equipment, makes the estimation of fault location in this way difficult. In the simplest case, where two identical parallel laterals are fed off of a single feeder line, faults on either of the laterals would be indistinguishable based on substation measurements alone without additional data. In [2], Senger et al. present a method that uses pre-existing knowledge of the properties of the system in addition to these measurements in an attempt to locate and categorize faults with greater accuracy. Stored data include system topology and electrical parameters of components such as cable types, spacing, and transformer properties. The algorithm estimates the distance of the fault from the point of intersection of the lateral with the primary feeder using known resistance values (depending on fault
type), and based on this distance determines if a fault down the lateral is possible. Out of all possible fault locations, the algorithm determines the most likely based on breaker/fuse operation and level of load rejection – overvoltage due to the disconnect of load – measured. Practical tests returned fault location accuracy to within 90 meters.

In [12] fault location using the discrete wavelet transform (DWT), which is similar to Fourier transform analysis but characterizes signals in both the time and frequency domain by comparing to a specified wavelet, is discussed. This is of particular use in underground distribution systems, which are more reliable but difficult to observe. The DWT of zero sequence current, in this case using the daubechies 4 wavelet, allows for accurate determination of fault start time. Travelling wave techniques are used to estimate distance from the substation; transient amplitude decreases and time delay increases with increasing distance. Testing returned accuracies of between 0.0416 and 1.0640km, which is acceptable, but given the velocity of wave propagation in electrical systems, estimations of this sort can be difficult without proper clock synchronization and very high sampling rates.

A summary of relevant challenges and current work in this field is presented in [13]. As suggested above, algorithms that use travelling waves or impedance measurements to estimate fault location and type operate on very few measuring points within the system. Given the nontrivial architecture of modern feeders, this is a decided disadvantage; impedance-based methods in particular sometimes return multiple location estimates. With the advent of digital devices (e.g. relays) and smart meters, here categorized as IEDs (intelligent electronic devices), measurements at high sampling rates from locations beyond the substation can be used to achieve greater accuracy while using relatively simple
algorithms. Sampling rates and accuracies are considered, as well as time synchronization methods and triggering options, with the aim of providing measurements that are accurate without being overwhelming.

Specific to distribution systems, [14] suggests the use of locations along the feeder – still short of the quantity presented by the installation of smart metering – to supplement measurements made at the substation. Their algorithm cycles through each node in the feeder, testing the likelihood of a fault by injecting calculated fault current at that location and estimating during-fault voltages using load flow techniques. This result is compared to the aforementioned measurements; the most likely node, as determined by the difference between simulated and actual measurements, is chosen as the most likely location of the fault. The algorithm triggers on high fault current measured at the substation as well as significant voltage drops down the line.

B) Characterization of Transient Phenomena

A separate but no less relevant area of research has been the characterization and analysis of power system transients, with the goal of not only sensing but also potentially preventing equipment failure. Researchers at Texas A&M, in conjunction with EPRI (Electric Power Research Institute) and local utilities, have created what they call a distribution fault anticipator (DFA)[15][16][17][18]. Their system uses voltage and current waveforms gathered at the substation to diagnose transient activity.

Using a database of thousands of recorded transients due to apparatus failure, the DFA monitors distribution feeders for similar activity. Instrumentation uses a sampling rate
of 256 samples per 60Hz cycle (15,360 samples per second), and is designed to trigger on even very small irregularities given the relatively low-magnitude activity under consideration. This results in an important tradeoff between amount and usability of data. Too much data can make results impossible to analyze. In one instance, device failure over the course of three weeks generated 2800 waveforms totaling 8.4GB. This necessitates the use of automated processes, as opposed to utility workers viewing data, to sort and categorize activity more efficiently.

As examples of DFA operation, the authors cite hot-line clamp failure, transformer bushing failure, tree limb- and animal-induced shorts, and recloser misoperation, among others. These are irregularities that are commonly missed by power system workers until disruptions in service occur, as existing measurement techniques do not trigger on such subtle activity. After utility workers are made aware of each situation, the debugging process begins, introducing yet another source of inefficiency: without knowledge of what has failed, corrective action often involves misdiagnosis, repeated replacement of devices and testing of the system.

Given a priori knowledge of failing devices, these complications can be avoided. In the case of the failing hot-line clamp, crews were dispatched four times in response to customer complaints about flickering lights and outages, and two transformers were replaced. Both transformers were healthy, with one suffering a blown fuse on its primary. Ultimately, the failing clamp was found and replaced, fully resolving the issue. Had utility workers known the source of the irregularities, as enabled by the DFA, these complications and further outages could have been avoided.
It is noteworthy that this research incorporates measurements from the substation alone. Transient activity will naturally decay as it propagates throughout the distribution system due to travelling wave effects (particularly attenuation), so it may be beneficial to use waveforms gathered further down the feeder – particularly at smart meters. Incorporation of this data should, at least, allow for less aggressive triggering and easier triage of data. This is later investigated through simulation.

**IV) Power System Modeling for Transient Analysis**

**A) Theory**

The simulation of transient activity in modern distribution systems requires modeling of the various components contained therein [19]. At their most basic, these can be categorized as sources, both internal (e.g. generation) and external (e.g. lightning strikes); passive elements, including transformers and transmission lines; and switches, which include in-line devices (e.g. circuit breakers) as well as temporary fault impedances.

Figure 2 - Switching transients, closing (a) and opening (b)

Switching transients are treated as compensating voltage sources, as shown in figure 2 [19]. The steady-state solution of the system, without any transient activity, is
simulated on its own. Superposition allows the addition of the transient waveform, simulated separately, to the steady-state solution, giving the final system reaction to the switching operation under analysis. The latter operation is accomplished by shorting all sources and treating the switch terminals as a voltage or current source – depending on whether the switch is closing or opening, respectively – that presents the voltage or current that would have existed across the terminals had the operation not occurred.

In 1968, Hermann Dommel introduced companion equivalents for common circuit components – inductors, capacitors, and transmission lines [20]. These take advantage of the trapezoidal integration rule applied to the well-known differential equations defining the behavior of inductances and capacitances, and have been the basis for most power system transient simulators (EMTP, or electromagnetic transient program) due to their relative efficiency, which will be discussed later. Given the differential equation:

$$\frac{dy(t)}{dt} = x(t)$$

$y$ is the integration of $x$ over a given period defined by $t$:

$$y(t) = y(0) + \int_0^t x(z)dz$$
The following estimation for integration can be made:

\[ y_n = y_{n-1} + \frac{x_{n-1} + x_n}{2} (t_n - t_{n-1}) \]

That is, the value of the function \( y \) at time \( n \) is equal to the value of \( y \) at time \( n-1 \) plus the average value of the function \( x \), over the same span of time, multiplied by the length of time elapsed (see figure 3 for a graphical representation). As \( y \) is defined as the integration of \( x \) over a period of time, the total area under the curve defined by \( x \) after each time step is equivalent to its value at the previous time step plus the area of the trapezoidal approximation of the function to the next time step. As \( x \) is assumed to vary slowly over each time step, this is only an approximation, but as the step size is decreased, accuracy is increased.

The above can be rewritten to be more simulation-friendly:

\[ y(t) = y(t - \Delta t) + \frac{\Delta t}{2} [x(t) + x(t - \Delta)] \]
Here, it can be seen that given a step size $\Delta t$, the value of $y(t)$ can be approximated over any length of time.

From the differential equation relating voltage and current through an inductor:

$$v_k(t) - v_m(t) = v_{km}(t) = L \frac{di_{km}(t)}{dt}$$

Where $k$ and $m$ are the terminals of the inductor, application of the trapezoidal rule gives

$$i_{km}(t) = i_{km}(t - \Delta t) + \frac{\Delta t}{2L} [v_{km}(t) + v_{km}(t - \Delta t)]$$

Solving for current results in:

$$i_{km(t)} = \frac{\Delta t}{2L} v_{km}(t) + I_{km(t)}$$

$$I_{km(t)} = \left[ \frac{\Delta t}{2L} v_{km}(t - \Delta t) + i_{km(t - \Delta t)} \right]$$

This, in circuit component form, is a resistance defined by inductance and time step, with a current source in parallel supplying the device’s dependence on previous conditions (figure 4).
The current and voltage through a capacitor are defined by:

\[ i_{km}(t) = C \frac{d}{dt} [v_k(t) - v_m(t)] = C \frac{dv_{km}(t)}{dt} \]

Using the trapezoidal rule in the same fashion as above:

\[ v_{km}(t) = v_{km}(t - \Delta t) + \frac{\Delta t}{2C} [i_{km}(t) + i_{km}(t - \Delta t)] \]

And solving for current:

\[ i_{km}(t) = \frac{2C}{\Delta t} v_{km}(t) + I_{km}(t) \]

\[ I_{km}(t) = -\left[ \frac{2C}{\Delta t} v_{km}(t - \Delta t) + i_{km}(t - \Delta t) \right] \]

Similarly to the companion circuit for the inductor, the capacitor is represented here by a resistance defined by capacitance and time step in parallel with a current source lending the dependence of the device on previous conditions (figure 5).
The single-phase lossless transmission line is, in this case, represented using distributed R, L, and C components (figure 6) as opposed to the commonly-used ‘pi’ model, for increased accuracy and ease of use.

Self-inductance and capacitance to ground are modeled as:

\[
\frac{\partial v(x,t)}{\partial x} = -L \frac{\partial i(x,t)}{\partial t}
\]

\[
\frac{\partial i(x,t)}{\partial x} = -C \frac{\partial v(x,t)}{\partial t}
\]

where L and C are expressed as per-unit-length values. These represent the series voltage drop due to inductance, and shunt current drop due to capacitance, as functions of distance from sending end \(x\) and time \(t\).

\[
\frac{\partial^2 v(x,t)}{\partial x^2} = LC \frac{\partial^2 i(x,t)}{\partial t^2}
\]

\[
\frac{\partial^2 i(x,t)}{\partial x^2} = LC \frac{\partial^2 v(x,t)}{\partial t^2}
\]

The general solution for voltage has the following form:

\[v(x,t) = f_1(x - vt) + f_2(x + v)\]
Where:

\[ v = \frac{1}{\sqrt{LC}} \]

is the propagation velocity of traveling waves within the line. The general solution for current is of the form:

\[ i(x, t) = \frac{f_1(x - vt) - f_2(x + vt)}{Z_c} \]

Where:

\[ Z_c = \sqrt{\frac{L}{C}} \]

is the surge, or characteristic, impedance of the line.

Each of these equations involves two functions, \( f_1 \) and \( f_2 \), that represent voltage traveling waves toward increasing and decreasing \( x \), respectively. Combination of voltage and current into a single function results in:

\[ v(x, t) + Z_c i(x, t) = 2f_1(x - vt) \]

Let the value \( \tau \) be equal to the travel time from one end, \( k \), to the other, \( m \), of the line. As long as \( x - vt \) remains constant, the value of \( v(x, t) + Z_c i(x, t) \) also remains constant. As a result, the function holds the same value at position \( m \), time \( \tau \) as it does at position \( k \), time 0; the wave represented by \( f_1 \) travels from \( k \) to \( m \) in this time:

\[ v_m(t) - Z_c i_{mk}(t) = v_k(t - \tau) + Z_c i_{km}(t - \tau) \]
For waves traveling from $m$ to $k$:

$$v_k(t) - Z_c i_{km}(t) = v_m(t - \tau) + Z_c i_{mk}(t - \tau)$$

Solving for currents:

$$i_{km}(t) = \frac{v_k(t)}{Z_c} + I_k(t)$$

$$I_k(t) = -\left[\frac{v_m(t - \tau)}{Z_c} + i_{mk}(t - \tau)\right]$$

![Diagram showing the circuit](image)

Figure 7 - Lossless line companion equivalent

$$i_{mk}(t) = \frac{v_m(t)}{Z_c} + I_m(t)$$

$$I_m(t) = -\left[\frac{v_k(t - \tau)}{Z_c} + i_{km}(t - \tau)\right]$$
These yield the familiar
resistance, defined by the
line’s surge impedance, in
parallel with a current
source providing coupling
between its end points
(figure 7). Notice the lack of
$\Delta t$ term; the single-phase
lossless line depends instead
on line transit time $\tau$. In running simulations, the time step $\Delta t$ can be no larger than $\tau$ for any
line in the system under consideration (to ensure accuracy).

Appendix A includes Matlab code representing a simple RLC circuit supplied by a
ramp voltage with unity magnitude. Results are shown (figure 8) for $R=1\Omega$, $L=160\mu H$, and
$C=16.6\mu F$. The waveforms shown are consistent with expectations for damping and ringing
frequency. Critical damping occurs when $\zeta = \frac{R}{2} \sqrt{\frac{C}{L}} = 1$, requiring $R=6.21\Omega$; in this case,
$\zeta = 0.16$, and the circuit exhibits ringing. Its natural frequency is $f = \frac{1}{2\pi \sqrt{LC}} = 3088Hz$,
consistent with the oscillations seen.

Taken together, these equivalents form the basis for an efficient, accurate
(depending on time step) simulation.
B) Transient Analysis Example – Capacitor Bank Switching

As an example, consider the circuit in figure 9, composed of a voltage source \( e(t) \), single-phase lossless line, and inductive load, with a switchable shunt capacitor bank supplying reactive power (i.e. compensating for reactive power consumed nearby). Voltage is supplied as a 120V RMS, 60Hz sine wave, with a 60Ω source impedance. The line is 6,000 feet long, and the load is represented as a series RL circuit with \( R=120\Omega \) and \( L=16\text{mH} \). The surge impedance of the line is 550Ω, based on a typical value of 300Ω per 1km\. A time step of \( \frac{1}{327840} \) s is used, which is half of the transit time of the line based on the speed of light, and the simulation completes after 21856 steps, or four 60Hz cycles.

The capacitor bank is, in this case, switched into the circuit 4.2ms after the start of the simulation, concurrent with a local maximum in the voltage source. This results in a transient of maximum amplitude.
i) **Capacitor Bank Switching – Steady State**

The simulation begins by producing results for the steady state system – that is, without the shunt capacitor bank in place (figure 10). The general method involves the solution of a matrix equation at each time step. As it is a known quantity – that is, the source – voltage at point 1 is not included in the set of equations solved; it is handled separately. Note that history terms are denoted with a capital ‘I’, whereas electrical current terms are denoted with a lowercase ‘i’.

The history terms for the lossless line and inductor (load) are:

\[
I_{t1}(t) = -\left[ \frac{v_2(t - \tau)}{Z_c} + i_{21}(t - \tau) \right]
\]

\[
I_{t2}(t) = -\left[ \frac{v_1(t - \tau)}{Z_c} + i_{12}(t - \tau) \right]
\]

\[
I_L(t) = \frac{\Delta t}{2L} [v_2(t - \Delta t) - v_3(t - \Delta t)] + i_{23}(t - \Delta t)
\]
By KCL and KVL, the circuit is defined by the equations:

\[ i_{21}(t) = -i_{23}(t) \]

\[ i_{23}(t) = i_3(t) \]

With the following defined given the aforementioned known source voltage \( e(t) \):

\[ i_{12}(t) = \frac{e(t) - i_{12}(t)R_s}{Z_c} + I_{l1}(t) \]

\[ i_{12}(t) = \frac{e(t) + I_{l1}(t)Z_c}{Z_c + R_s} \]

\[ v_1(t) = e(t) - \frac{i_{12}(t)}{R_s} \]

Expanding the first two current equations above:

\[ \frac{v_2(t)}{Z_c} + I_{l2}(t) = -\frac{\Delta t}{2L} [v_2(t) - v_3(t)] - I_L(t) \]

\[ \frac{\Delta t}{2L} [v_2(t) - v_3(t)] + I_L(t) = \frac{v_3(t)}{R} \]

Represented in matrix form:

\[
\begin{bmatrix}
\frac{1}{Z_c} + \frac{\Delta t}{2L} & -\frac{\Delta t}{2L} \\
-\frac{\Delta t}{2L} & \frac{1}{R} + \frac{\Delta t}{2L}
\end{bmatrix}
\begin{bmatrix}
v_2(t) \\
v_3(t)
\end{bmatrix}
= \begin{bmatrix}
-I_{l2}(t) - I_L(t) \\
I_L(t)
\end{bmatrix}
\]

The significance of this equation from a computational point of view is that the impedance matrix, which must be inverted to form a solution for voltage given each time
step’s history vector (right hand side), is constant. Therefore, only a single inversion operation is required for the entirety of the simulation, resulting in greatly increased efficiency. At each time step, currents at various points in the system can be derived using solved voltages and basic circuit theory.

![Figure 11 - Transient circuit](image)

**ii) Capacitor Bank Switching – Transient**

Results of the simulation of the transient that occurs as the capacitor bank is switched into the circuit can be added to the above steady state results (superposition) to give the final solution to voltage and current along the circuit.

The transient simulation differs from above in that the voltage source at the start of the lossless line is short-circuited, with a voltage source added across the terminals of the previously open switch representing the voltage that would have existed in that location had the switch never closed (figure 11). Notice also that the capacitor bank is now represented.
The solution to this circuit proceeds in the same way as above. Until the time at which the switch is closed, the single voltage source and as such every node in the system is at 0 voltage; no transient has yet been generated. At the appropriate time step, the voltage source is linked to the voltage at node 2 in the steady state simulation.

The history term for the capacitor is:

\[ I_C(t) = -\left( \frac{2C}{\Delta t} v_5(t - \Delta t) + i_5(t - \Delta t) \right) \]

Through the application of KCL, the system can be represented by the following equations:

\[ i_5(t) = -i_4(t) - i_6(t) \]
\[ i_6(t) = i_{60}(t) \]

Expanding the first:

\[ \frac{2C}{\Delta t} v_5(t) + I_C(t) = -\left[ \frac{\Delta t}{2L} [v_4(t) - v_6(t)] + I_L(t) \right] - \left[ \frac{1}{Z_c} v_4(t) + I_{12}(t) \right] \]

And noting that \( v_5(t) = v_4(t) + v_2(t) \),

\[ \left( \frac{2C}{\Delta t} + \frac{1}{Z_c} + \frac{\Delta t}{2L} \right) v_4(t) + \frac{\Delta t}{2L} v_6(t) = -I_L(t) - I_C(t) - I_{12}(t) - \frac{2C}{\Delta t} v_2(t) \]

Expanding the second current equation:

\[ \frac{\Delta t}{2L} v_4(t) - \left( \frac{\Delta t}{2L} + \frac{1}{R} \right) v_6(t) = -I_L(t) \]

28
These result in the matrix equation:

\[
\begin{bmatrix}
\frac{2C}{\Delta t} + \frac{1}{Z_c} + \frac{\Delta t}{2L} & \frac{\Delta t}{2L} \\
\frac{\Delta t}{2L} & \frac{\Delta t}{2L} + \frac{1}{R}
\end{bmatrix}
\begin{bmatrix}
v_4(t) \\
v_6(t)
\end{bmatrix}
= \begin{bmatrix}
-l_L(t) - I_C(t) - I_{L2}(t) - \frac{2C}{\Delta t} v_2(t) \\
-l_L(t)
\end{bmatrix}
\]

which, again, will be solved at each time step using the history terms computed based on the previous time step.

Matlab code based on the above set of data is included in appendix B. Figure 12 shows simulation results, displaying the expected behavior: a voltage waveform that dips to almost zero as the capacitor bank is charged, and that exhibits ringing due to reactions with inductances in the ideal line as well as reactive load. Current through the circuit reaches a peak shortly after the capacitor bank is switched and as it is charged.
V) Simulation Tools – ATP

Among the simulators that use the solution method discussed above, ATP-EMTP was chosen for use in the remainder of this paper (manual [21]). As opposed to such tools as PSCAD/EMTDC and EMTP-RV, ATP is somewhat less user friendly – it includes no GUI in its standard installation – but, importantly, free of charge to individuals not involved in the sale of other EMTP simulators.

Figure 13 - ATPDesigner

Third party GUIs are available for ATP, namely ATPDraw and ATPDesigner. Both convert proprietary file types to ATP input files; however, neither can convert in the other direction, nor read files made in the opposite GUI. Interestingly, ATPDesigner is not free, but is available without charge to those in academia. Its interface is shown in figure 13. Various graphing and signal analysis tools are also available; ATPDesigner happens to include its own,
with the ability to use discrete Fourier transform (DFT) analysis on the waveforms generated during the execution of ATP. Of the two, ATPDesigner was used, as the flow from ATP input files to signal analysis is more easily handled.

Other limitations of ATP include some missing models, including voltage regulators and surge arresters; however, it features an active user base and the ability to add custom models (although these wouldn’t necessarily be supported directly in any GUI).

**VI) Simple Simulation Model and Fault Location by Voltage Sag**

![Simple simulation model](image)

In order to test the efficacy of fault location and categorization based on distributed voltage magnitude measurements, the simple radial distribution model shown in figure 14 was used. It consists of 19 equally spaced 1MVA balanced loads, each with a 0.95 power factor, and uses overhead line configuration 301 from the IEEE 34 node test feeder[22] (discussed later) for all transmission lines, each of which is 2km long. There is a single lateral,
and the network infeed supplies 24.9kV service with a short-circuit power (defined by open-circuit voltage and source impedance) of 20MVA. Nominal load current is 226A per phase. Fault location 1 is used for all further testing, and a fault impedance of 0 is assumed. This results in fault currents of 89A in phase-to-ground, 122A in three-phase-to-ground, and 129A in phase-to-phase faults. Note that the data presented below is summarized in appendix C in table form, and that all measurements were obtained using ATP’s short circuit analysis, which returns phasor measurements for RMS voltage and current at all nodes by running the simulation to steady state.

Figure 15 – A-to-ground: RMS voltage at measurement locations

Figure 16 – A-to-ground: RMS voltage at measurement locations
Figure 17 – A,B-to-ground: RMS voltage at measurement locations

Figure 18 – A,B-to-ground: RMS voltage at measurement locations

Figure 19 – A,B,C-to-ground: RMS voltage at measurement locations
Results for one-, two-, and three-phase-to-ground faults are shown in figures 15-20, for each of the two paths down the feeder. In all cases, the location of fault can be predicted to a single segment (here defined as a line separating two loads). Where a phase or phases are shorted to ground, voltage at that point reaches 0, and remains close to that value further down the feeder. Phases on the unfaulted lateral do not approach 0.
Phase-to-phase faults yield similar results (figures 21 and 22). The difference in voltage between the phases involved in the fault reaches 0 at the point of fault, and remains there further down the feeder. The unfaulted lateral does not exhibit this behavior.

In summary, under ideal conditions, all four fault types are easily located and diagnosed. Generally, the voltage curve moving down the feeder reaches a ‘knee’ at the point of fault, where its downward slope decreases sharply. Voltage at this point is equal across faulted phases, and 0 when a phase is shorted to ground.

With lines shortened to 200m each, the magnitude of voltage at the base of the feeder decreases as it is physically (and electrically) closer to the fault, but for all four fault types, results are similar to those above (A-to-ground shown in figure 23). There is a clear
knee in faulted phase’s voltage sag curve; numerically, at this point and beyond, voltages are 0 for grounded phases and equal across faulted phases.

Figure 24 - Simple simulation model with transformer

Figure 24 shows the same model as above, with a three phase transformer added between nodes 3 and 4. It has grounded wye primary and secondary connections, and has a turns ratio of 1; that is, it doesn’t change voltage or current levels, but still provides coupling due to shared cores.

Figure 25 – Transformer, A-to-ground: RMS voltage at measurement locations
Figure 26 – Transformer, A-to-ground: RMS voltage at measurement locations

Figure 27 – Transformer, A,B-to-ground: RMS voltage at measurement locations

Figure 28 – Transformer, A,B,C-to-ground: RMS voltage at measurement locations
Figures 25-29 show results for the four phase types with this transformer in place. Due to shared cores within the transformer, fault conditions are more difficult to see between it and the network infeed. Past the transformer, however, the same behavior as above is apparent for each of the four fault types, albeit with less definition (that is, the knee in the curve is not as sharp as in either case above).
Figure 30 - High-Z A-to-ground: RMS voltage at measurement locations

Figure 31 - High-Z A-to-ground: RMS voltage at measurement locations
A particular area of interest in fault detection is the diagnosis and localization of high-impedance faults. These are loosely defined as those faults resulting in fault current that is insufficient to trip any protective devices, generally under 100A. ATP allows the simulation of nonzero-impedance faults, and results suggest that location can still be estimated to a single line. The results for A-to-ground and A-to-B faults with fault
impedances of 100Ω - resulting in fault currents of 24A and 26A, respectively – are shown in figures 30-33. The A-to-ground fault shows a noticeable change in voltage slope past node 7; the A-to-B fault exhibits this same condition on both phases involved. This is the same ‘knee’ as seen previously. Note also that the faulted lateral exhibits a greater downward slope than the unfaulted lateral in both cases. At the point of high impedance A-to-ground fault, for example, the downward slope decreases sharply from -269V/node to -105V/node.

It should be noted that all of the above represent simplifications of a typical distribution system. Given unequal line lengths, unbalanced and varying loads, voltage regulation, and other complications, results may not be so well defined, and increased accuracy in measurements may be required. The conditions at the point of fault remain the same, however, and comparisons of voltage magnitude measurements from different locations along the feeder and on different phases can reveal information about fault conditions that are difficult or impossible to deduce using substation measurements alone.

VII) Simulation Model – IEEE 34 Node Feeder

IEEE provides a set of test radial distribution feeders of various sizes and complexities with the stated purpose of enabling the reliable testing of solution methods and simulators [22]. Data specified includes load model type and power usage; distribution capacitor bank locations; overhead line spacing information and conductor data; underground spacing information and cable data; and voltage regulator and transformer specifications.
For purposes of transient testing, as well as further testing of fault location by voltage magnitude, the IEEE 34 node test feeder – which is based on an actual feeder in Arizona – has been implemented in ATPDesigner. Input data and topography is listed in appendix D. The feeder is very long and lightly loaded, with two regulators and a single (substation) transformer transferring power to various load configurations. As they are not modeled in ATP, voltage regulators are not included.

The input data first had to be converted to a form that ATPDesigner could use. In particular, IEEE lists line impedances, depending on configuration, in the phase domain: self and mutual impedances for phases A, B, and C. ATPDesigner accepts line impedances in the sequence domain (symmetrical component analysis), that is, the zero, positive, and negative sequences. Additionally, it is assumed that self and mutual impedances are equal among the phases, which is typical especially where they are transposed and have the same conductor properties, resulting in zero coupling between sequences. This conversion proceeds using the transformation equation:

\[ Z_{012} = T^{-1}Z_{ABC}T \]

Where \( Z_{012} \) and \( Z_{ABC} \) are the sequence and phase domain impedance matrices, respectively. The transformation matrix \( T \) is defined as:

\[
T = \begin{bmatrix}
1 & 1 & 1 \\
1 & \alpha & \alpha^2 \\
1 & \alpha^2 & \alpha
\end{bmatrix}
\]

Where \( \alpha = 1 \angle 120^\circ \).
Additionally, the transformer data given includes only rated power, primary and secondary voltages, and resistive and reactive losses in %. ATPDesigner requires resistive losses in W, and total impedance as a %, both of which are easily derived.

For more complete transient and fault testing, several changes were made to this model, the final topology of which is shown in figure 24.

![Figure 34 - Modified IEEE 34 node test feeder](image)

Additions include two 2MVA (represented as single balanced load) residential-style laterals, including step-down (120V) transformers and switchable capacitor banks;
switchable 500kW renewable generation, modeled as a current source with harmonic content; switchable three-phase 450kW motor, representing a typical industrial load; and a set of empirical function generators that yield input current consistent with a lightning strike, along with a metal oxide varistor acting as a surge arrester.

The network infeed is internally modeled as three single phase voltage sources, 120° phase shifted, in a grounded wye configuration, with a series impedance on each phase (a classic three phase generator model). The infeed provides 69kV service (stepped down to 24.9 kV for distribution, with two 120V residential laterals) and a short-circuit power of 15MVA.

Measurement locations are marked, and are distributed so as to allow maximal observation of the distribution system, and record both voltage and current. Fault locations are also noted, and fault impedance is assumed to be 0. The time step used is 10µs, resulting in an effective sampling rate of 10KHz, and each simulation proceeds for 10 60Hz cycles.

A) Simulation Results – Line Faults

i) A-to-Ground

Consider an A-to-ground fault at fault position 3 (which is at the end of the line labeled closest to the transformer), a residential-style lateral roughly midway down the feeder.

Figures 35-39 show simulation results at various locations. Figure 35 shows voltage waveforms from all three phases at measurement location 1 – the substation. At 0.1s, phase
A shorts to ground at fault position 3, resulting in a small transient and changed steady state operation. Due to coupling between phases at various points along the feeder, particularly three phase transformers (owing to their shared-core design), the faulted phase is not necessarily at 0 voltage at all locations.

Figure 35 – A-to-ground: three phase voltage at 1 over time (A, B, C)

Figure 36 – A-to-ground: three phase voltage at 4 over time (A, B, C)

Figure 37 – A-to-ground: three phase voltage at 7 over time (A, B, C)
Here, the important measure is during-fault voltage magnitude. Note the difference in phase A moving from measurement location 1 to measurement location 4 (figure 36), closest to the location of the fault, after 0.1s: phase voltages reach their minimum at points nearest this location. Indeed, assuming a zero impedance to ground, voltage on phase A is necessarily 0 at the point of fault.

Figure 37 shows waveforms for all three phases at measurement location 7; voltage magnitudes here are similar to those recorded at point 4.

Figure 38 – A-to-ground: phase A voltage from substation to fault over time (1, 2, 3, 4)

In summary, during an A-to-ground fault, the feeder exhibits a voltage drop on phase A, with voltage magnitude decreasing from the substation to the point of fault (figure 38). This measure remains nearly constant past the point of fault (figure 39).
ii) **A,B and A,B,C-to-Ground**

![Graph](image1.png)

**Figure 40 – A,B-to-ground: phase A voltage down feeder over time (1, 2, 3, 4, 7)**

![Graph](image2.png)

**Figure 41 – A,B-to-ground: phase B voltage down feeder over time (1, 2, 3, 4, 7)**

![Graph](image3.png)

**Figure 42 – A,B,C-to-ground: three phase voltage at 1 (A, B, C) and 7 (A, B, C) over time**

Two-phase-to-ground faults exhibit similar characteristics. Figures 40 and 41 show measurements taken of an A- and B-to-ground fault, again at position 3. Notice that voltage magnitudes on each phase vary in the same way as the faulted phase in a single-phase-to-
ground fault: there is a decrease in these measures from the substation to the fault location, and moving further down the feeder they are nearly constant.

Three-phase-to-ground faults extend this trend to all three phases, as shown in figure 42, which represents such a fault at position 3.

iii) A-to-B

![Figure 43](image-url-1)  
*Figure 43 – A-to-B: three phase voltage at 1 over time (A, B, C)*

![Figure 44](image-url-2)  
*Figure 44 – A-to-B: three phase voltage at 4 over time (A, B, C)*
Phase-to-phase faults behave somewhat differently in that their defining characteristic is equal voltage over two phases, as opposed to shorted voltage on a particular phase or phases. Figures 43-46 show measurements taken of an A-to-B fault at position 3. At the substation, phases A and B are still distinct and separated by close to 120° (figure 43). At measurement locations 4 and 7, however, the phase voltages are nearly equal (figures 44 and 45). Figure 46 shows a general trend: the difference in voltage magnitude between phases A and B decreases from the substation to the point of fault, beyond which it remains nearly constant.
iv) High-Z A-to-Ground

Figure 47 – High-Z A-to-ground: three phase voltage at 1 over time (A, B, C)

Figure 48 – High-Z A-to-ground: three phase voltage at 4 over time (A, B, C)

Figure 49 – High-Z A-to-ground: phase A voltage from substation to fault over time (1, 2, 3, 4)

Figure 50 – High-Z A-to-ground: phase A voltage from fault to end of feeder over time (4, 5, 6, 7)

Additional testing of high impedance faults yielded similar results. Here, an impedance value of 500Ω is used in an A-to-ground fault at position 3, resulting in fault
current of 18.52A seen at the substation on phase A. Figures 47 and 48 show results for three phase substation and point of fault voltages, respectively; it can be seen that the voltage on the faulted phase drops more quickly down the feeder. Figures 49 and 50 show phase A voltage waveforms between substation and fault and between fault and the end of the feeder, respectively; voltage magnitude decreases down the feeder to the location of the fault, past which it stays almost constant.

B) Summary

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Table 1 - Fault simulation results using IEEE 34-node model

These analyses suggest the possible use of distributed during-fault voltage magnitude measurements in localization and categorization, as there are clear distinctions between the four basic fault types. Data from this section is collected in table 1 above, and in figures 51-56 below. Importantly, faulted phases in each case exhibit more extreme voltage sag down the feeder than unfaulted phases and compared to normal operation,
consistent with results seen using the simple simulation model. According to [23], phase-to-ground faults result in an average of 8.35 60Hz cycles of fault current before being cleared by protective devices. This suggests that, given some alternate source of power to the data network and meter for a short duration, meters may be able to record and transmit these during-fault measurements before protective equipment operates.

**Figure 51 - A-to-ground: RMS voltage at measurement locations**

**Figure 52 - A,B-to-ground: RMS voltage at measurement locations**
Figure 53 - A,B,C-to-ground: RMS voltage at measurement locations

Figure 54 - A-to-B: RMS voltage at measurement locations
C) Automated Fault Location

Given the properties in voltage sag measurements highlighted above, an automated fault location algorithm can be developed. As previously mentioned, results shown here are specific to the systems simulated and can vary depending on a number of factors, including:
• Line model: transmission lines can be above- or underground, transposed or untransposed, and include one, two, or three phases. In addition, spacing has a direct effect on line characteristics including mutual inductance and capacitance.

• In-line devices: Between generator and load, there will be a number of transformers of various types and capacities, as well as other devices including voltage regulators and switchable capacitors. Each of these has a direct effect on voltage levels.

• Load characteristics: Most loads on distribution feeders are naturally not static, and can present varying levels of inductance to the system – resulting in varying voltage drop.

Perhaps the simplest way to account for these is to compare voltage measurements at a particular point in time to an accepted baseline – ‘normal’ operation or, more realistically, a moving average of recent measurements. This methodology is presented in figures 57-61 (summarized in table 2), which show the difference in voltage magnitude at each measurement location between unfaulted (normal) and faulted operation for each of the fault types analyzed thus far, using the IEEE 34 node feeder model. Fault activity is easier to isolate and diagnose, as faulted phases diverge noticeably from their unfaulted counterparts. Additional measurement locations result in greater accuracy in localization, but trend line techniques can be used to estimate fault location even when measurements are relatively sparse – as in the system under test.
Figure 57 - A-to-ground: |Faulted Voltage – Normal Voltage| at measurement locations

Figure 58 - A,B-to-ground: |Faulted Voltage – Normal Voltage| at measurement locations

Figure 59 - A,B,C-to-ground: |Faulted Voltage – Normal Voltage| at measurement locations
Figure 60 - A-to-B: |Faulted Voltage – Normal Voltage| at measurement locations

Figure 61 - High-Z A-to-ground: |Faulted Voltage – Normal Voltage| at measurement locations
In addition to the fault analysis presented thus far, the modified IEEE 34 node test feeder was also used to test the behavior of transient waveforms as they propagate through the distribution system. Switching transients analyzed include capacitor bank switching at each of the two residential laterals, line/lateral switching, generator switching, and three phase motor switching. Lightning transient data was added to the model in ATPDesigner; however, due to a software bug in the implementation of surge arresters in ATPDesigner, this could not be used in any realistic capacity and will not be included here. Each of the simulations below was executed separately.

Capacitor bank operation, as previously mentioned, involves the switching of capacitance into the distribution system, which helps to supply reactive power locally, but can result in a significant charging transient and subsequent oscillations due to interactions between inductances and capacitances. At t=0.08s, the capacitor bank within the residential
lateral at location 3, closer to the substation, is switched on, and at t=0.2s, the bank on a similar lateral at location 2, farther from the feeder, is switched on (figure 62).

The line switching operation in this case involves the charging of a transmission line as well as the addition to the distribution circuit of a (residential) load. The residential lateral at location 3 is switched into the system at t=0.1s, the residential lateral at location 2 at t=0.2s (figure 63).

The generator used in this model is a rough estimation of a 500kW wind generator included with the installation of ATPDesigner. It acts as a multi-frequency current source, introducing not only fundamental but also harmonic content to the system. It is switched on at t=0.1s and off at t=0.2s (figure 64).

A 450kW three phase motor is included in the system model, and is switched on at t=0.1s and off at t=0.2s (figure 65).

After processing through ATP, output waveforms were transferred to Matlab. The discrete wavelet transform (DWT) was used to decompose each signal, thereby isolating and characterizing, in both time and frequency domains, the transient phenomena. C. H. Kim provides a thorough introduction to wavelet transform analysis and its usefulness in power systems applications [24][25].

The daubeches 4 mother wavelet is a common choice for power system analysis, as it effectively isolates the oscillatory transient activity commonly seen. Six to eight levels of decomposition were used, depending on the frequency of the signal in question.
Results are also gathered in table 3. In every case, the magnitude of transient activity is at its lowest at the substation, and actually increases down the feeder – regardless of transient source.

Figure 62 - Capacitor switching as seen at substation

Figure 63 - Generator switching as seen at substation
Figure 64 - Line switching as seen at substation

Figure 65 - Motor switching as seen at substation
| Location | Capacitor | | Generator | | Line | | Motor |
|----------|----------|---|----------|---|---------|---|
| 1        | 0.08s: 3268 0.20s: 702 | | 4 | 0.08s: 3918 0.20s: 869.6 | | 7 | 0.08s: 4723 0.20s: 1146 |
| 4        | 0.10s: 485.1 0.20s: 461.3 | | 5 | 0.10s: 673.2 0.20s: 617.4 | | 7 | 0.10s: 749.4 0.20s: 704.6 |
| 7        | 0.10s: 1942 0.20s: 1627 | | 4 | 0.10s: 2402 0.20s: 1966 | | 7 | 0.10s: 2567 0.20s: 2141 |
| 3        | 0.10s: 78.13 0.20s: 24.39 | | 1 | 0.10s: 94.97 0.20s: 30.04 | | 4 | 0.10s: 98.78 0.20s: 30.83 |

Table 3 – Peak magnitude of transient activity at various locations (|V|)

IX) Conclusion

Through simulation, the usefulness of voltage measurements from various locations along distribution feeders, specifically from smart meters, has been shown. Measurements taken at the substation are useful; however, in creating a ‘true’ smart grid, other sources can and should be used in making power distribution more reliable and efficient. Given an overlying data network, voltage phasor measurements – specifically magnitude – can be used to characterize and locate faults, and complex waveform analysis can benefit from the increased fidelity offered by a distributed system.
Appendix A: RLC circuit Matlab code

clear all;
delta = 1; %relative to storage vectors
C = 16.6*10^-6;
L = 160*10^-6;
R = 1;
damping = R/2*sqrt(C/L) %damping
damping
freq = 1/(2*pi*sqrt(L*C)) %frequency
timestep = 1*10^-6; %in s
e(1) = 0; %start
V1(1) = 0;
V2(1) = 0;
I(1) = 0;
IC(1) = 0;
IL(1) = 0;
G = [1/R + timestep/(2*L), -timestep/(2*L);
    -timestep/(2*L), timestep/(2*L) +
    2*C/timestep]; %admittance matrix
for i=2:2500;
e(i) = 1; %unit step
IL(i) = timestep/(2*L)*(V1(i-delta) - V2(i-delta)) + I(i-delta);
IC(i) = -2*C/timestep*V2(i-delta) - I(i-delta);
Ihistory = [e(i)/R - IL(i); %history terms
    IL(i) - IC(i)];
X = G\Ihistory; %solve for voltages
V1(i) = X(1);
V2(i) = X(2);
I(i) = 1/R*(e(i) - V1(i)); %solve for current
end
time = (1:2500)*timestep*1000;
subplot(1, 2, 1); plot(time, V2); xlabel('Time (ms)');
ylabel('Voltage (V)'); title('Voltage across capacitor');
subplot(1, 2, 2); plot(time, I); xlabel('Time (ms)');
ylabel('Current (A)'); title('Current through circuit');
Appendix B: Capacitor switching circuit Matlab code

clear all;
Freq = 60;
Afreq = 2*pi*Freq;
Voltage = 120*sqrt(2);
Zc = 550;
delta = 1; %index to previous time step. Note tau=2*delta
Rs = 60;
C = 16.6*10^-6;
L = 16*10^-3;
R = 120;
Load = R + j*Afreq*L;
timestep = 1/327840;
Fs = 1/timestep;
Nsamps = 21856;
tsw = Nsamps/16;

V(1) = 0;
for i=1:Nsamps;
    V(i) = Voltage*sqrt(2)*sin(2*pi*(i-1)/5464);
end

V1(1) = 0; %steady-state start
V2(1) = 0;
V3(1) = 0;
Il1(1) = 0;
Il2(1) = 0;
IL(1) = 0;
Il2(1) = 0;
I2(1) = 0;
I23(1) = 0;

G = [1/Zc+timestep/(2*L), -timestep/(2*L);
    -timestep/(2*L), 1/R+timestep/(2*L)]; %steady-state admittance

for i=2:Nsamps;
    Il1(i) = -1/Zc*V2(max([i-2*delta,1])) - I21(max([i-2*delta,1]));
    Il2(i) = -1/Zc*V1(max([i-2*delta,1])) - I22(max([i-2*delta,1]));
    IL(i) = timestep/(2*L)*V2(i-delta) - timestep/(2*L)*V3(i-delta) + I23(i-delta);
    Il2(i) = (V(i)/Zc + Il1(i))/(1 + Rs/Zc);
    V1(i) = V(i) - Il2(i)/Rs;
    Ihistory = [-Il2(i) - IL(i); IL(i)]; %history terms
    X = G\Ihistory; %solve for voltages
V2(i) = X(1);
V3(i) = X(2);
I23(i) = V3(i)/R; %solve for currents
I21(i) = -I23(i);
end

G = [2*C/timestep + 1/Zc + timestep/(2*L), timestep/(2*L);
     timestep/(2*L), -timestep/(2*L) - 1/R]; %transient admittance

for i=1:tsw-1; %transient start
    V4(i) = 0;
    V6(i) = 0;
    I4(i) = 0;
    I5(i) = 0;
    I6(i) = 0;
    IS(i) = 0;
    VS(i) = 0;
    Il1(i) = 0;
    Il2(i) = 0;
    IL(i) = 0;
    IC(i) = 0;
    V5(i) = 0;
end

for i=tsw:Nsamps;
    IC(i) = -2*C/timestep*V5(i-delta) - I5(i-delta);
    I11(i) = -1/Zc*V4(max([i-2*delta,1])) - I4(max([i-2*delta,1]));
    I12(i) = -1/Zc*VS(max([i-2*delta,1])) - IS(max([i-2*delta,1])); %added source impedance
    IL(i) = timestep/(2*L)*V4(i-delta) - timestep/(2*L)*V6(i-delta) + I6(i-delta);
    Ihistory = [-IL(i) - I12(i) - 2*C/timestep*V2(i) - IC(i);
                 -IL(i)];
    X = G\Ihistory; %solve for voltages
    V4(i) = X(1);
    V6(i) = X(2);
    V5(i) = V4(i) + V2(i);
    I4(i) = 1/Zc*V4(i) + I12(i); %solve for currents
    I5(i) = 2*C/timestep*V5(i) + IC(i);
    I6(i) = 1/R*V6(i);
    VS(i) = (1/((1/Rs) + (1/Zc)))*I11(i);
    IS(i) = 1/Rs*VS(i);
end

time = (1:Nsamps)*timestep*1000;
subplot(2, 2, 1); plot(time, V4); xlabel('Time (ms)'); ylabel('Voltage (V)'); title('Voltage Transient (across capacitor)');

subplot(2, 2, 3); plot(time, V2+V4); xlabel('Time (ms)'); ylabel('Voltage (V)'); title('Voltage at Load');

subplot(2, 2, 2); plot(time, I5); xlabel('Time (ms)'); ylabel('Current (A)'); title('Current Transient (through capacitor)');

subplot(2, 2, 4); plot(time, I12+IS); xlabel('Time (ms)'); ylabel('Current (A)'); title('Feeder Current');

FT = abs(fft(V4));
f = Fs*(0:Nsamps)/Nsamps; %FFT
Appendix C: Summary of results using simple simulation model in ATP
All measurements are RMS phase-to-ground voltage at the indicated measurement location

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**High-Z A-to-ground**

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Appendix D: IEEE 34 node test feeder topology and data
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Shunt Capacitors

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**Line Impedances**

Configuration 300:

---------- Z & B Matrices Before Changes ----------

Z (R +jX) in ohms per mile

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B in micro Siemens per mile

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Configuration 301:

Z (R +jX) in ohms per mile

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B in micro Siemens per mile

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**Configuration 302:**

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**Configuration 303:**

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**Configuration 304:**

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References


