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MR-RELAXATION OF O17 IN AQUEOUS SOLUTIONS OF VANA-DYLPERCHLORATE AND THE RATE OF ELIMINATION OF WATER MOLECULES FROM THE FIRST COORDINATION SPHERE

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NMR-relaxation of $^{17}O$ in aqueous solutions of vanadyl-perchlorate and the rate of elimination of water molecules from the first coordination sphere.

K. Wüthrich and Robert E. Connick

September 1966
NMR-Relaxation of $^{17}O$ in Aqueous Solutions of Vanadyl-
Perchlorate and the Rate of Elimination of Water Molecules
from the First Coordination Sphere.

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Summary. An investigation was made of the temperature dependence
of the transverse nuclear relaxation time $T_2$ for $^{17}O$ in aqueous vanadyl
perchlorate solutions in the range 5°-170°C. In the major part of this
temperature region the relaxation due to the influence of the $VO^{2+}$ ions
was found to be controlled by the rate of exchange of water molecules
between the first coordination sphere of the metal ions and the bulk of
the solution. Values for the rate constant and the activation energy of
the water exchange, and for the constant which describes the scalar
coupling between $VO^{2+}$ and the $^{17}O$ nuclei of the coordinated water mole-
cules have been established. The rate of the water exchange was found
to be slow compared to the rates found in corresponding experiments with
other doubly charged metal ions. This indicates that the high positive charge of $V^{4+}$ is an important factor not only in the formation of the vanadyl group but also in the bonding of the water molecules in the first coordination sphere.

Additional relaxation effects are observed at low temperatures. It is shown that these effects are most likely explained by interactions of the vanadyl ion with $O^{17}$ in water molecules loosely coordinated outside the first coordination sphere.
In aqueous solutions of paramagnetic metal ions a certain number of water molecules are coordinated to each metal ion. Because of the magnetic interactions between the paramagnetic ions and the magnetic moments of the $^17O$ nuclei of these coordinated water molecules, the transverse n.m.r. relaxation time of these oxygens is greatly decreased. For dilute solutions of the metal ion only a single resonance can be observed which corresponds to that of the bulk water modified by the exchange of water molecules in and out of the coordination spheres of the metal ions.

The property observed is the line width of the resonance. Half the width at half height, $\delta\omega$, expressed in radians per second, is related to the apparent transverse relaxation time $T_2$ by Equation (1).

$$\delta\omega = \frac{1}{T_2}$$  \hspace{1cm} (1)

The line width arises from the relaxation of the bulk $^17O$ nuclei that would occur in the absence of paramagnetic ions, i.e. $1/T_{2H_2O}$, and the contribution from the paramagnetic ions, $1/T_{2p}$, as given by (2).

$$\frac{1}{T_2} = \frac{1}{T_{2H_2O}} + \frac{1}{T_{2p}}$$  \hspace{1cm} (2)
Analysis of $1/T_{2p}$ in terms of the properties of the dissolved paramagnetic ions has been used in earlier studies to evaluate the water exchange rate and various parameters which characterize the relaxation in aqueous solutions of a series of 3d metal ions. In the present paper the application of the n.m.r. technique to the study of aqueous vanadyl-perchlorate solutions is described.

I. Theory. The shape of the n.m.r.-spectrum of a chemically stable system is described by the Bloch equations. A modification of the Bloch equations by McConnell which includes the possibility of chemical exchange has been extended by Swift and Connick to the conditions present in dilute aqueous solutions containing paramagnetic metal ions. They found that the effect of the paramagnetic ions on the relaxation of the $^{17}$O nuclei in a solution where the water may exchange between the bulk of the solution and one kind of coordination site on the metal ions can be given by Equation (3). $\tau_{H_2O}$ and $\tau_M$ are the lifetimes with respect to chemical exchange for the water molecules in the bulk of the solution and on the coordination sites of the metal ions, $T_{2M}$ is the transverse relaxation time of the $^{17}$O nuclei in the first coordination sphere of the metal ions, and $\Delta\omega$ is the difference in the precessional frequencies of the $^{17}$O nuclei of the bulk water and those of the coordinated water molecules.
An analysis of Equation (3) reveals two relaxation mechanisms, one involving $\Delta \omega$ and the other involving $T_{2M}$. The "$\Delta \omega$-mechanism" can be of importance if $\Delta \omega$ is sufficiently large so that transverse relaxation can occur through the change in precessional frequency which arises when the nuclei exchange between the bulk of the solution and the coordination sites of the metal ion. The relaxation through $T_{2M}^5$ can be important if the nuclei of the coordinated water molecules undergo fast relaxation because of either scalar or dipolar coupling with the unpaired electrons of the metal ions. In certain cases the mechanism governing $T_{2p}$ can be established from the temperature dependence of $T_{2p}$. The following limiting cases A and B are important for the treatment of the $^{17}$ nuclear relaxation in VO(ClO$_4$)$_2$ solutions:

$$\frac{1}{T_{2M}^2} \gg \frac{1}{\tau_M^2}, \Delta \omega^2 : \frac{1}{T_{2p}} = \frac{P_M}{\tau_M^2} = \frac{1}{\tau_{H_2O}}$$ (4)

Relaxation by $T_{2M}$ is fast; $1/T_{2p}$ is controlled by the rate of the chemical exchange which is given by the lifetimes of the water molecules in the different environments. The probability factor $P_M$ is given closely by $n[M]/55.5$, where $[M]$ is the molar concentration of the paramagnetic ion and $n$ is the number of water molecules in the coordination sites characterized by the lifetime $\tau_M$. 
Chemical exchange is rapid; $1/T_{2p}$ is controlled by the $T_{2M}$ relaxation process. The possibility that the relaxation is controlled by a change in the precessional frequency, i.e. $\Delta \omega^2 \gg \frac{1}{T_{2M}}$, is no longer included in this discussion, since it can readily be eliminated from consideration: an impossibly high value of the constant which characterizes the scalar coupling between $\text{VO}^{2+}$ and the $\text{O}^{17}$ nuclei of the coordinated water molecules would be necessary to explain the experimental data in terms of a $\Delta \omega$-mechanism.

The variation of $\tau_M$ with temperature is given by the usual expression for the temperature dependence of the rate of chemical reactions (6),

$$
\tau_M = \frac{\hbar}{kT} \exp \left( \frac{\Delta H^+/RT - \Delta S^+/R} \right)
$$

where $\Delta H^+$ and $\Delta S^+$ are the enthalpy and entropy of activation for the water exchange between the first coordination sphere of the vanadyl ions and the bulk of the solution. The variation of $T_{2M}$ with temperature is determined by the mechanism which interrupts the interaction between $\text{VO}^{2+}$ and the $\text{O}^{17}$ nucleus. Both, dipolar coupling and scalar coupling, can be interrupted by the relaxation of the unpaired electron, and by the rate of elimination of the coordinated water molecule. The dipolar interactions can further be interrupted by the Brownian motion of the molecules.
The transverse relaxation time of the $^{17}$-nuclei in the first coordination sphere due to scalar coupling is expected to be given by (7), where $A/\hbar$ is the scalar coupling constant in radians per second,

$$\frac{1}{T_{2M}} = \frac{1}{3} S(S+1) \frac{A^2}{\hbar^2} \tau_e$$

and $S$ is the electron spin quantum number. The correlation time $\tau_e$ is given by $1/\tau_e = 1/T_{1\text{el}} + 1/\tau_M$, where $T_{1\text{el}}$ is the longitudinal relaxation time of the electronic system. Equation (7) corresponds to only the second term of Abragam's Equation (127), p. 311 (or the first term of Equation (122), p. 309) being of importance in our case. Since the temperature coefficient for $T_{1\text{el}}$ is usually much smaller than for $\tau_M$, the tendency will be for $T_{2M}$ to be controlled at high temperature by the interruption arising from chemical exchange and at low temperature by the electronic spin flip, i.e. at high temperature the observed line-broadening would be due to scalar relaxation of the first kind and at lower temperatures to scalar relaxation of the second kind. In our experiments with VO($\text{ClO}_4$)$_2$ the temperature never became high enough for chemical exchange to dominate.

For the dipole-dipole coupling induced relaxation in the first coordination sphere only the interruption of the coupling through rotational tumbling of the hydrated vanadyl-ions has to be taken into consideration, since the chemical exchange rate and the $T_{1\text{el}}$ process are much slower in aqueous VO($\text{ClO}_4$)$_2$-solutions. A quantitative treatment given by Solomon assumes that the Brownian motion of the molecules
can be described by Stoke's diffusion equation, i.e. the molecules are treated as rigid spheres of radius r moving in a medium of viscosity \( \eta \). The result is given by Equation (8), where \( \gamma_r \)

\[
\frac{1}{\tau_{2M}} = \gamma_r^2 \gamma_S^2 \hbar^2 S(S+1) \frac{1}{15d^6} \left\{ \frac{4\tau_c}{1+(\omega_1-\omega_S)^2 \tau_c^2} \right. \\
+ 3 \frac{\tau_c}{1+\omega_1^2 \tau_c^2} + 6 \frac{\tau_c}{1+\omega_S^2 \tau_c^2} + 6 \frac{\tau_c}{1+(\omega_1+\omega_S)^2 \tau_c^2} \right\} \tag{8}
\]

\[\tau_c = \frac{4\pi r^3}{3k} \frac{\eta}{T}\]

and \( \gamma_S \) are the gyromagnetic ratios for the \( ^{17}O \) nucleus and for the unpaired electron in VO\(^{2+} \), \( S \) is the electron spin quantum number, \( d \) is the distance between the interacting spins, \( \tau_c \) is the correlation time for the interruption of the dipole-dipole coupling through rotational motion, \( \omega_1 \) and \( \omega_S \) are the Larmor frequencies for the spins of the \( ^{17}O \) nucleus and the unpaired electron of VO\(^{2+} \) in the applied external magnetic field, and \( k \) is the Boltzmann constant.

While scalar coupling is expected to influence appreciably only nuclei of water molecules in the first coordination sphere we have to include the possibility of dipole-dipole coupling between VO\(^{2+} \) and the nuclei in outer coordination spheres. We distinguish between two possible cases: If the lifetime of the water molecules in a second coordination sphere is long compared to the correlation time arising from the rotational motion of the molecules VO(H\(_2\)O)\(_n\)(H\(_2\)O)\(_m\)\(^{2+} \), where \( n \) and \( m \) denote the number of water molecules in the first and in the
second coordination sphere, we have formally the same system as for the treatment of the dipolar coupling between $\text{VO}^{2+}$ and the $^{17}\text{O}$ nuclei in the first coordination sphere, and the relaxation effects are given by Equation (8). If the lifetime of the water molecules in the second coordination sphere of the complex is short compared to the rotational correlation time, the interruption of the dipole-dipole coupling is controlled by the relative translational motions of $\text{VO}(\text{H}_2\text{O})_n^{2+}$ and the $^{17}\text{O}$ nuclei in the bulk water. Assuming that the rate of the translational motions is diffusion controlled, a quantitative treatment of this case leads to expression (9) for the evaluation of the relaxation effects to be expected, where $N_{\text{Me}}$ is the concentration of paramagnetic ions expressed in cm$^{-3}$, $r_1$ is the radius assumed for the rigid sphere representing $\text{VO}(\text{H}_2\text{O})_n^{2+}$, and $r_2$ the radius of the sphere representing $\text{H}_2\text{O}$. Note that (9) gives directly the experimentally observable relaxation time $T_{2p}$ and not a quantity corresponding to $T_{2M}$ as in (7) and (8). Equation (9) is only valid in the above form in the case of "extreme motional narrowing," i.e. if $\frac{6\pi n}{kT} (r_1 + r_2)r_1 r_2 \omega \ll 1$ for all the spins involved, where $\omega$ is the Larmor frequency of the spins. When this condition does not hold the numerical factor in (9) can become as small as $\frac{28\pi^2}{75}$. 

\[
\frac{1}{T_{2p}} = \frac{16\pi^2}{15} N_{\text{Me}} \gamma_1^2 \gamma_2^2 \hbar^2 S(S+1) \frac{n}{kT} \frac{4r_1 r_2}{(r_1 + r_2)^2} \tag{9}
\]
A comparison of expressions (6)-(9) shows that a study of the temperature dependence of \( T_{2p} \) should enable us to distinguish between relaxation controlled by chemical exchange, by scalar relaxation of the first kind, by scalar relaxation of the second kind, and by relaxation through dipole-dipole coupling.

The first term in (2) which includes the relaxation time \( T_{2H_2O} \) for \( ^{17}O \)-nuclei in pure water has been studied by Meiboom. In acidic solution \( T_{2H_2O} \) is believed to be mainly due to interactions between the spin-moment and the quadrupolar moment of the \( ^{17}O \) nucleus interrupted by rotational tumbling of the water molecules. For the case of "extreme motional narrowing," which is a good approximation for the treatment of the \( ^{17}O \)-relaxation in water, the transverse relaxation time due to quadrupolar coupling is given by (10), where it is assumed that the Brownian motions of the molecules can be described by Stoke's diffusion equation. In Equation (10), \( I \) is the nuclear spin quantum number, \( \xi \) is the asymmetry parameter, \( Q \) is the nuclear quadrupole moment, \( q \) is the electric field gradient, \( r \) is the radius assumed for a sphere which represents the water molecule, and \( \eta \) is the viscosity of the solution under consideration.

\[
\frac{1}{T_{2H_2O}} = \frac{3}{40} \frac{2I+3}{I^2(2I-1)} \left( 1 + \frac{\xi^2}{3} \right) \left( \frac{eQq}{\pi} \right)^2 \frac{4\pi r^3 \eta}{3kT} \tag{10}
\]

II. Experimental. Employing the side-band detection technique we recorded the \( ^{17}O \) n.m.r. spectra with a modified Varian Associates Model V-4200 wide-line spectrometer, operated with a fixed frequency unit V-4311 at 8.134 Mc. A further modification, the use of a PAR lock-in amplifier
Model HR-8, made it possible to vary the modulation frequency continuously in the range 1-4000 c.p.s. Samples of ca. 0.5 cc. in volume were studied in tubes of 7-8 mm outer diameter.

For studies at variable temperature a n.m.r.-probe with an all-glass Dewar insert which contained the sample tube and the receiver coil was used. Heated or cooled nitrogen was then passed through the probe and the resulting temperature was measured simultaneously outside and at the center of the sample tube. The temperature range covered was 275-450°K.

The following chemicals were used: water enriched to 3.5% and 10% in $^{17}O$ obtained from the Weizmann Institute of Science, $\text{VOSO}_4 \cdot 5\text{H}_2\text{O}$ obtained from Fairmont Chem. Co., $\text{BaCO}_3$ reagent A.C.S. and $\text{HClO}_4$ 70% reagent A.C.S. obtained from Allied Chemicals. The $^{17}O$ n.m.r. spectrum of the water was measured in the temperature range of interest and the obtained $T_{2\text{H}_2\text{O}}$ values were in good agreement with earlier data. Vanadyl-perchlorate solutions were made from $\text{VOSO}_4 \cdot 5\text{H}_2\text{O}$, $\text{BaCO}_3$, and $\text{HClO}_4$. Through evaporation of the water in vacuum the $\text{VO(ClO}_4)_2$ solutions were concentrated to ca. 2.5-M. The vanadyl-perchlorate concentration was determined through titration with KMnO$_4$. The samples for the NMR experiments were prepared by adding known volumes of the concentrated $\text{VO(ClO}_4)_2$-solution and of 70% HClO$_4$ with micropipets to the $^{17}O$-enriched water. Since $T_{2p}$ decreases very much with increasing temperature in the major part of the temperature region examined, the vanadyl concentration was lowered in several steps from 0.83-M for the experiments at room temperature to 0.016-M for measurements above 390°K in order to have line-widths appropriate to the side-band technique. The samples contained 0.1-M or 0.3-M HClO$_4$ and ca. 0.01-M $\text{Ba(ClO}_4)_2$. 
In order to check relaxation effects which might arise from paramagnetic impurities the VO(ClO₄)₂ solutions used in our experiments were analyzed spectrochemically. The analyses were made by G. V. Shalimoff of the Lawrence Radiation Laboratory on a Baird 3 meter grating spectrograph. None of the elements known to form ions which influence the O¹⁷ nuclear relaxation in aqueous solutions could be detected and the following upper limits, given in weight % metal relative to Vanadium as metal, could be established: Mn < 0.001, Cu < 0.001, Ni < 0.005, Co < 0.005, Fe < 0.005. Using these limits and the data on relaxation effects arising from the presence of the paramagnetic ions formed by these metals¹ it is seen that none of them can contribute appreciably to the relaxation effects observed in VO(ClO₄)₂-solutions, except for Mn²⁺ which could cause at most one third of the "additional relaxation effects" (Fig. 1, curve c) observed at low temperatures.

III. Results. The O¹⁷-enriched aqueous solutions used for the n.m.r. studies contained various concentrations of VO(ClO₄)₂, HClO₄, and traces of Ba²⁺-ions. In Equation (2) we assumed that the relaxation of the O¹⁷-nuclei in pure water and the relaxation arising from the presence of the paramagnetic ions are two independent processes. On the basis of expressions (4), (5), or (9) we expect the line-widths \( \delta \omega = 1/T_{2H,O} + 1/T_{2p} \) to be linearly dependent on the metal-ion concentration. In Fig. 1 this was found to be the case for VO(ClO₄)₂ solutions. Further experimental checks showed that \( \delta \omega \) is independent of the perchloric acid concentration in the range 0.1-M to 0.5-M. As would be expected no change in \( \delta \omega \) was observable after addition of various amounts of Ba(ClO₄)₂.
The temperature dependence of that part of the overall transverse relaxation time for the $^{17}$O-nuclei arising from the presence of the VO$^{2+}$ ions is given in Fig. 2. The data indicate that $T_{2p}$ is controlled by the rate of exchange of water molecules between the bulk of the solution and the first coordination sphere of VO$^{2+}$ in the temperature range 330-390 K, since $T_{2p}$ decreases with increasing temperature. It has been shown by X-ray studies$^{13}$ that the VO$^{2+}$-ion in solid VO$\text{SO}_4\cdot5\text{H}_2\text{O}$ binds 4 water molecules and one sulfate-oxygen in the first coordination sphere. Presumably in aqueous solution each VO$^{2+}$ will have four equivalent oxygens of water in the equatorial plane and one water oxygen in the axial position opposite the vanadyl-oxygen.$^{14}$ One expects the bonding of the axial water molecule to be different from the bonding of the equatorial waters. Thus the number $n$ of the coordinated H$_2$O molecules which take part in the exchange reaction observed by n.m.r. is not known a priori. Therefore, we cannot yet characterize the exchange reaction numerically but rather by the stoichiometry (11) and the expressions in

$$\text{VO(O}_2\text{H}_2\text{O})_{5-n}(\text{H}_2\text{O})_n^{2+} + n\text{H}_2\text{O}^* \rightleftharpoons \text{VO(O}_2\text{H}_2\text{O})_{5-n}(\text{H}_2\text{O}^*)_n^{2+} + n\text{H}_2\text{O} \quad (11)$$

Table I which include $n$. The reaction rate constant $k$ is the first order rate constant for the loss from the first coordination sphere of a particular one of the exchanging waters. Its value at room temperature, $k(298)$, has been calculated from $k(333^\circ)$ and $\Delta H^\ddagger$. 
Table I
Parameters for Exchange of $^{17}O$ Between the First Coordination Sphere of $VO^{2+}$ and Bulk Water.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta H^\ddagger$</td>
<td>13.7 Kcal Mol$^{-1}$</td>
</tr>
<tr>
<td>$k(333^\circ K)$</td>
<td>$(1/n) \ 2.3 \cdot 10^4$ sec$^{-1}$</td>
</tr>
<tr>
<td>$k(298^\circ K)$</td>
<td>$(1/n) \ 2.0 \cdot 10^3$ sec$^{-1}$</td>
</tr>
<tr>
<td>$\Delta S^\ddagger$</td>
<td>$(2.2-4.6 \log n)$ e.u.</td>
</tr>
<tr>
<td>$A/h$</td>
<td>$(1/\sqrt{n}) \ 7.6 \cdot 10^6$ cps</td>
</tr>
</tbody>
</table>

Above 390°K the temperature dependence of $T_{2p}$ deviates from the slope found for the exchange controlled region. This implies that at high temperatures the relaxation is in part controlled by the $T_{2M}$ process, since, as mentioned earlier, the observed results cannot be explained by a $\Delta \omega$-mechanism. Using the data given in Table I it is found that $\tau_M \approx T_{2M}$ is of the order of magnitude of $n \cdot 10^{-6}$ to $n \cdot 10^{-7}$ sec. in the temperature region considered here. It is easy to show by inserting suitable numbers into Equations (8) and (9) that such a small value for $T_{2M}$ cannot be explained by dipole-dipole coupling and therefore most likely arises from scalar coupling. The interruption of the coupling will occur essentially entirely from the longitudinal electronic relaxation since the chemical exchange is much slower than the $T_{1 \text{el}}$ process in the temperature region examined. Only the transverse
relaxation time $T_{2\text{ el}}$ of vanadyl perchlorate in aqueous solution and its temperature dependence are known above 400°K: the average value for $T_{2\text{ el}}$ for the e.s.r. spectrum of a $10^{-2}$-M aqueous vanadyl perchlorate solution is ca. $1.0 \cdot 10^{-8}$ sec at 400°K and is to a good approximation independent of temperature between 400 and 450°K.\(^{15}\) From work done by Myers and McCain\(^{16}\) at room temperature and in a magnetic field of 1000 gauss $T_{1\text{ el}}$ for VO\(^{2+}\)-ions in aqueous solution appears to be only slightly longer than $T_{2\text{ el}}$. In going to 14,000 gauss, the magnetic field used for the 0\(^{17}\)n.m.r. experiments, and ca 430°K it seems still a good approximation to set $T_{1\text{ el}}$ equal to $T_{2\text{ el}}$, since the estimated value for $\omega_s \tau$, where $\omega_s$ is the electron spin resonance frequency and $\tau$ the correlation time in aqueous solution, probably does not vary greatly through this change of experimental conditions.

Following the above discussion we would expect $T_{2M}$ to be essentially independent of temperature in the region of interest. Neglecting the terms including $A$ in Equation (3) we find that $T_{2p}$ is given by (12).

$$T_{2p} = \frac{1}{P_M} (\tau_M + T_{2M}) \quad (12)$$

The data in Fig. 3 for $T_{2p}$ measured in a 0.016-M VO(ClO\(_4\))\(_2\)-solution are well approximated by adding to the contributions of the chemical exchange as calculated from Table I a temperature-independent value $T_{2M}/P_M = 6.10^{-4}$ sec which corresponds to $T_{2M} = 1.7 \cdot 10^{-7}$ sec. Inserting this value and $T_{1\text{ el}} \approx T_{2\text{ el}} = 1.0 \cdot 10^{-8}$ sec into Equation (7) we find for the scalar coupling constant $A/h$ the value given in Table I.
At temperatures below ca. 310°K the line-broadening of the n.m.r. spectrum of \( \text{O}^{17} \) from the presence of \( \text{VO(ClO}_4)_2 \) is small and varies only little with temperature. Fig. 4 shows the results of a study in 0.83-M \( \text{VO(ClO}_4)_2 \)-solution. In evaluating \( T_{2p} \) from \( T_2 \) through Equation (2) it is necessary to correct the pure water values of \( T_{2H_2O} \) for the increased viscosity of a 0.83-M \( \text{VO(ClO}_4)_2 \) solution (see Equation (10)). The data then show that an additional relaxation path is important at low temperature: after subtracting from the values of curve a the contributions arising from the high viscosity and from chemical exchange (calculated from Table I) there is left a small relaxation effect given by curve c which must be due to other interactions between \( \text{VO}^{2+} \) and the \( \text{O}^{17} \) nucleus.

In order to check whether interactions with the nuclei in water molecules outside the first coordination sphere could be responsible for this effect, we calculated the line-broadening to be expected from different possible mechanisms: (i) Inserting suitable numbers into Equation (9) we found that contributions to the relaxation arising from dipolar coupling between \( \text{VO(H}_2\text{O})_n^{2+} \) and the bulk water interrupted by the translational motion of the particles would be negligibly small. (ii) We assumed that a molecule \( \text{VO(H}_2\text{O})_5(H_2\text{O})_8^{2+} \) with a lifetime which is long compared to the correlation time characteristic for its rotational tumbling exists in the solutions at lower temperatures. The 8 water molecules bound in a second coordination sphere might be thought of as residing on the faces of the tetragonal bipyramid formed by \( \text{VO(H}_2\text{O})_5^{2+} \) (Fig. 5). Using the parameters \( r = 5 \) Å and \( d = 3.5 \) Å for this species, and the viscosities determined experimentally for 0.83-M \( \text{VO(ClO}_4)_2 \)-solutions at various temperatures, one calculates the curve d (Fig. 4)
for the relaxation effects to be expected from dipolar coupling with
the O$_{17}^-$ in the second coordination sphere.  (iii) We have further
calculated the relaxation effect which would arise if one of the 5
water-molecules in the first coordination sphere were exchanging
rapidly and subject to dipolar coupling only, assuming $r = 4.5$ Å and
d = 2.4 Å.  Fig. 4, curve e, shows that a result very similar to the
result for VO(H$_2$O)$_{13}^{2+}$ is obtained.

IV. Discussion.  As mentioned earlier it is believed that complexes
of the form VO(H$_2$O)$_5^{2+}$ exist in aqueous solutions of vanadyl ions.$^{14}$
The most striking feature of the VO(H$_2$O)$_5^{2+}$ molecule is the multiple
bond between V$^{4+}$ and one of the ligand oxygens which implies a strong
axial symmetry in the molecule (Fig. 5).  A detailed discussion of
the electronic structure of the hydrated vanadyl ion has been given by
Ballhausen and Gray.$^{14}$  It is seen from Fig. 6 that they give the
configuration [$(\text{Ia}_1)^2 (\text{IIa}_1)^2 (\text{b}_1)^2 (\text{e}_g)^4 (\text{IIIA}_1)^2 (\text{e}_\pi)^4 (\text{b}_2)^1$] for the
ground state of the molecule, whereby the unpaired electron is believed
to be in a mainly nonbonding orbital of $b_2$ symmetry.  From the symmetry
of the molecule it follows that the bonding of the four equatorial
water molecules should be equivalent, while different wave functions are
involved in the bonding of the axial water molecule.  Experimental
evidence for the equivalence of the 4 equatorial coordination sites
has been found from x-ray data$^{13}$ and from e.s.r. measurements.$^{17}$
From the present knowledge of the properties of the hydrated vanadyl ion it appears most likely that the number $n$ of exchanging water molecules in reaction (11) is 1 or 4, perhaps even 5. $n=1$ corresponds to the case where the four equatorial water molecules would exchange too slowly to influence the $^{17}O$ relaxation of the nuclei in the bulk water, and thus only the effect of the exchange of the axial water molecule would be observable. For the case $n=4$ only the effect of the water exchange in the equatorial positions would be observable, the scalar coupling of $^{17}O$ in the axial position being too small for the exchange of the axial water, which may occur at a high rate, to contribute appreciably to the relaxation in the bulk water. The case $n=5$ would correspond to all the coordinated water molecules having a uniform scalar coupling constant of $^{17}O$ and exchanging at the same rate despite the differences in the bonding. The rate constants and the scalar coupling constants calculated for these possible cases are given in Table II.

<table>
<thead>
<tr>
<th>$n$</th>
<th>Rate Constant $k(298^\circ)$</th>
<th>Scalar Coupling $A/n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$2 \cdot 10^3$ sec$^{-1}$</td>
<td>$7.6 \cdot 10^6$ cps</td>
</tr>
<tr>
<td>4</td>
<td>$5 \cdot 10^2$ sec$^{-1}$</td>
<td>$3.8 \cdot 10^6$ cps</td>
</tr>
<tr>
<td>5</td>
<td>$4 \cdot 10^2$ sec$^{-1}$</td>
<td>$3.4 \cdot 10^6$ cps</td>
</tr>
</tbody>
</table>
The influence of the exchange of the vanadyl-oxygen on the $^{17}$O relaxation in the bulk of the solution has been neglected in this discussion. From various observations it appears that the VO$^{2+}$ entity is very stable in aqueous solution and therefore one would expect an exchange of the oxygen to be rather slow. This view receives support from experiments with aqueous solutions of various VO$^{2+}$-compounds which showed that if there is an exchange of the vanadyl-oxygen in these complexes its rate is too slow to be observed by the $^{17}$O n.m.r. relaxation technique.

No other measurements exist for the rate of exchange of water molecules in the first coordination sphere of VO$^{2+}$. A comparison with the rates obtained for the water exchange in the first coordination sphere of a series of other doubly charged transition metal ions shows that the exchange in vanadyl perchlorate solutions is comparatively very slow for all the values chosen for n. This may be due to a high electrostatic contribution to the bonding of the ligands. The vanadyl ion can be considered as made up of V$^{4+}$ and $O^{2-}$ and it is likely that it acts as a cation with a charge of more than +2 not only towards the $O^{2-}$ but also towards the five other ligands. The influence of the high positive charge acting on the coordinated water molecules presumably would also be reflected in the value found for the activation enthalpy $\Delta H^\ddagger$ (Table I) which is high compared to $\Delta H^\ddagger$ found for the water exchange with other doubly charged metal ions. One would expect that crystal field effects play no important role for d$^1$-systems.
A comparison with earlier results\textsuperscript{1} shows that the scalar coupling constant of $^{17}O$ for the $\text{VO}^{2+}$-aquo complex is small compared to the values found in corresponding experiments with the hydrated $\text{Mn}^{2+}$, $\text{Fe}^{2+}$, $\text{Co}^{2+}$ and $\text{Ni}^{2+}$ ions. This indicates that scalar interactions of the unpaired electrons with the $^{17}O$ of the coordinated waters occur predominantly via $\sigma$-bonding rather than $\pi$-bonding when both possibilities exist: $\text{Mn}^{2+}$, $\text{Fe}^{2+}$, $\text{Co}^{2+}$, and $\text{Ni}^{2+}$ have 2 half-filled d-orbitals available for $\sigma$-bonding, while the unpaired electron of $\text{VO(H}_2\text{O)}_5^{2+}$ is believed to be mainly localized in a molecular orbital of $b_2$-symmetry (Fig. 6)\textsuperscript{14} which for symmetry reasons cannot contribute appreciably to $\sigma$-bonding of the ligands (Fig. 5). From the above mentioned theoretical considerations\textsuperscript{14} of the ground state of $\text{VO(H}_2\text{O)}_5^{2+}$ one might expect scalar interactions to occur via $\pi$-bonding (Fig. 5). This may be checked by considering the proton scalar coupling constant in $\text{VO(H}_2\text{O)}_n^{2+}$, since $\pi$-bonding has been shown to be primarily responsible for the scalar coupling of protons in hydrated metal ions.\textsuperscript{21} From a reinterpretation of Hauser and Laukien's\textsuperscript{22} data on the proton n.m.r. relaxation in aqueous $\text{VOSO}_4$ solution, $A/h$ of protons in $\text{VO(H}_2\text{O)}_n^{2+}$ was found to be given by $(1/\sqrt{p}) \times 3.3 \times 10^6$ cps, where $p$ is the number of coordinated protons. This value is indeed of the same magnitude as the proton scalar coupling constants for $\text{Mn}^{2+}$, $\text{Fe}^{2+}$, and $\text{Co}^{2+}$\textsuperscript{21,23} which also have half-filled d-orbitals available for $\pi$-bonding, and it is considerably larger than $A/h$ of protons in the hydrated $\text{Ni}^{2+}$ and $\text{Cu}^{2+}$ ions\textsuperscript{21} which have no half-filled $t_{2g}$-orbitals.
The variation of $T_{2p}$ with temperature between 275 and 310°K (Fig. 4) indicates that in this temperature region a large part of $T_{2p}$ (curve c) cannot be explained by the mechanism involving the exchange reaction (11), i.e. a new path for relaxation becomes important. In Part III it has been shown that dipole-dipole interactions between VO$^{2+}$ and the $^1_7$ nuclei of water molecules coordinated to VO$^{2+}$ with a much shorter chemical exchange lifetime than the n ligands characterized by reaction (11) could account for the observed additional line broadening. First, if n=4, the dipolar coupling with a loosely bound water molecule in the axial position of the first coordination sphere could be responsible for the measured relaxation effect (curve e). Secondly, if the whole effect of the first coordination sphere on the relaxation were accounted for through the mechanism which involves the water exchange (11), dipolar coupling with the nuclei of water molecules bound in a second coordination sphere could explain the line broadening at low temperatures (Fig. 4, curve d). The curves 4d and 4e agree as well with the experimental values as can be expected if one takes into consideration the approximation made in describing such systems by Stoke's diffusion equation, as in Equation (8).

As mentioned earlier (Part I) Equation (8) applies only if the loosely bound water molecules remain coordinated for a time long compared to the rotational correlation time of the hydrated vanadyl ion, i.e. ca. $10^{-10}$ sec. at 25°C. This seems not impossible, particularly in view of the high effective charge of the vanadium as implied by the slow water exchange from the first coordination sphere.
It should be pointed out, however, that quadrupolar coupling (10) is also a possible mechanism for the additional relaxation at low temperature. The contributions to $T_2p$ arising from quadrupolar coupling would be unimportant if the loosely bound waters were able to rotate as rapidly about their principle axes as unbound waters, since their quadrupolar contribution to the relaxation would then not be influenced by coordination to the vanadyl ions. On the other hand, if the loosely coordinated waters were bound so tightly that they could tumble only through the tumbling of the whole hydrated vanadyl ion, the correlation time $\tau_q$ for quadrupolar coupling would be given by (13), where $\tau_c$ is

$$\frac{1}{\tau_q} = \frac{1}{\tau_c} + \frac{1}{\tau_e} \tag{13}$$

the correlation time for rotational tumbling of the hydrated vanadyl ion, and $\tau_e$ is the lifetime with respect to chemical exchange of the loosely bound waters. $\tau_q$ is also effectively the correlation time for dipole-dipole coupling of Equation (8). Therefore the relative contributions from quadrupolar coupling and dipolar coupling will be determined by the ratio of the coupling constants in Equations (8) and (10). The quadrupolar effect turns out to be ca. 10 times greater. It follows that for this model, i.e. tumbling of the loosely coordinated water molecules only through the tumbling of the whole vanadyl-aquo complex, the observed additional relaxation at low temperature would occur primarily through quadrupolar coupling interrupted by chemical exchange with a lifetime of ca. $10^{-11}$ sec.
The interpretation of the $^{17}\text{O}$ relaxation in VO$^{2+}$ solutions in terms of interactions with water molecules in two different kinds of coordination sites receives support from a comparison with the temperature dependence of the proton n.m.r. relaxation in aqueous VOSO$_4$ solutions: The data by Hausser and Laukien$^{22}$ can be explained by an analogous interpretation which assumes that at higher temperature the relaxation is controlled by the exchange of protons from the first coordination sphere, while at lower temperature the contributions from protons of water molecules coordinated to VO$^{2+}$ with a much shorter lifetime than those in the first coordination sphere become important.

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References

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Figure Captions

Figure 1. Variation of the $^17$ n.m.r. line-widths with vanadyl-ion concentration. $T = 345^\circ K$.

Figure 2. Dependence on reciprocal of temperature of $\log T_{2p}$ in solutions of VO(ClO$_4$)$_2$.

Figure 3. Dependence on reciprocal of temperature of $\log T_{2p}$ for a 1.6・10$^{-2}$-M VO(ClO$_4$)$_2$-solution above 390$^\circ$K with the lines resulting from the curve-fitting. Experiment: ●

Figure 4. Dependence on reciprocal of temperature of $\log T_{2p}$ for a 0.83-M VO(ClO$_4$)$_2$-solution below 310$^\circ$K. a) Experimental data using $T_{2\text{H}_2\text{O}}$ of pure water in Equation (2). b) Contribution of the chemical exchange (11) calculated from Table I. c) Data given in curve a after subtraction of curve b and correction of the water blank for the high viscosity in 0.83-M VO(ClO$_4$)$_2$ (calculated using Equation (10) and the following data: at 280$^\circ$, $1/T_{2\text{H}_2\text{O}} = 251$ radians per second, $\eta_{\text{H}_2\text{O}} = 1.43\cdot10^{-2}$ poise, $\eta_{0.83\text{-M VO(ClO}_4\text{)}} = 1.93\cdot10^{-2}$ poise; at 290$^\circ$, $1/T_{2\text{H}_2\text{O}} = 188$, $\eta_{\text{H}_2\text{O}} = 1.08\cdot10^{-2}$, $\eta_{\text{VO(ClO}_4\text{)}} = 1.54\cdot10^{-2}$; at 300$^\circ$, $1/T_{2\text{H}_2\text{O}} = 151$, $\eta_{\text{H}_2\text{O}} = 0.85\cdot10^{-2}$, $\eta_{\text{VO(ClO}_4\text{)}} = 1.24\cdot10^{-2}$. This viscosity correction has been included in the values for $T_{2p}$ plotted in Fig. 2). d) Calculated relaxation effect from dipolar coupling with the nuclei of 8 water molecules bound with a lifetime longer than ca. $10^{-10}$ sec in a second-neighbor coordination.
sphere in VO(H₂O)₁₃²⁺. e) Calculated relaxation effects from dipolar coupling with one rapidly exchanging (lifetime ca. 10⁻¹⁰ sec) water molecule in the first coordination sphere of VO(H₂O)₅²⁺.

Figure 5. Structure of VO(H₂O)₅²⁺. |νₓᵧ|² outlines the areas of high unpaired electron density for the ground state of the molecule.

Figure 6. Energy level diagram for VO(H₂O)₅²⁺, according to Ballhausen and Gray.¹⁴
\[ \delta \omega = \frac{1}{T_{2H_2O}} + \frac{1}{T_2P} \] (radians/sec)

![Graph with data points and lines representing the equation \( \delta \omega = \frac{1}{T_{2H_2O}} + \frac{1}{T_2P} \).]

**Fig. 1**
Fig. 2
Fig. 3

\[ \frac{1}{P_M} \left( \tau_M + T_{2M} \right) \]

\[ \frac{1}{P_M} \tau_M \]

\[ \frac{1}{P_M} T_{2M} \]
Fig. 4
Fig. 5
Vanadium orbitals

Molecular orbitals

Ligand orbitals

\(-E\) cm\(^{-1}\) \(\cdot 10^3\)

Fig. 6
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