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SLOW CRACK GROWTH: MACROSCOPIC AND MICROSCOPIC ASPECTS

by

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ABSTRACT

Modern engineering design against fracture in "safety-critical" structures generally is based on the concept of defect- or damage-tolerance, where projected life is estimated in terms of the time for an assumed initial defect to propagate to some critical size. Accordingly, from a materials standpoint, increased resistance to failure can be achieved by retarding the sub-critical growth of cracks prior to final failure. In the current paper, an overview is presented of several recent advances in the understanding of the salient mechanisms of such slow crack growth, involving fracture under both monotonic and cyclic loading at ambient and elevated temperatures.
INTRODUCTION

In the understanding of both the mechanical properties of solids and the structural integrity of flawed engineering components, the development of fracture mechanics has presented the materials scientist and the mechanical engineer with a powerful means to characterize quantitatively the macroscopic fracture behavior of materials. On the one hand, the use of fracture mechanics has permitted the materials scientist to conduct meaningful comparisons between different materials on the role of many mechanical, microstructural and environmental factors in influencing conditions for the initiation and growth of cracks. In essence, it has provided a continuum-mechanics framework for the presentation of laboratory test data in order to evaluate the toughness of materials. To the engineer, on the other hand, fracture mechanics has provided methodology to utilize such data (which invariably are derived from small-sized samples) to predict, with a fair degree of certainty, the structural integrity of larger components in service, and specifically to aid in the analysis of service failures. Furthermore, this is achieved without any recourse to formulating microstructural models for the complex fracture mechanisms involved.

The essential premise of this approach has been the realization that all materials contain defects and incipient flaws, such that for conservative lifetime predictions the time spent in crack initiation must be considered to be minimal. Thus, the expected life of a given component is assessed in terms of the time, or number of loading
cycles, required to propagate the largest undetected crack to failure. An assumption often is made for the initial crack size, or it is estimated from proof tests or the limit of resolution of the non-destructive evaluation technique; the final crack size is estimated from the fracture toughness (e.g., $K_{IC}$, $J_{IC}$, etc.), limit load or some other design requirement. This approach, known as defect- or damage-tolerant design, is now in widespread use, particularly for safety-critical structures, such as are encountered frequently in the nuclear and aerospace industries.

According to this approach, the anticipated life of a component depends primarily on the time spent in propagating sub-critical cracks to the critical size which constitutes failure. However, there are several mechanisms by which this can occur. The principal processes are by the coalescence of microvoids (i.e., ductile fracture) under monotonic loading, by fatigue under cyclic loading, by creep crack growth at elevated temperatures, and by environmentally-assisted mechanisms, such as stress corrosion cracking and corrosion fatigue, in the presence of active (e.g., corrosive or hydrogen-containing) environments.

In the current article, an overview is presented of the various major sub-critical cracking processes, both with respect to the macroscopic aspects of relevant fracture mechanics used to globally characterize crack advance, and the microscopic aspects of the local fracture criteria and the role of microstructure. The complex role of environment, specifically involving slow crack growth by such
mechanisms as hydrogen-assisted cracking, stress corrosion cracking and corrosion fatigue, is not explicitly discussed in this review. For further information on this topic, the reader is referred to the proceedings of a recent conference (Gangloff, ed., 1984). We begin with a brief summary of continuum fracture mechanics.

FRACTURE MECHANICS CHARACTERIZATION OF CRACK GROWTH

Linear Elastic Fracture Mechanics (LEFM)

The basic features of fracture mechanics used to globally correlate slow crack advance begin with characterizing the stress and deformation fields, local to the region at the crack tip. This is achieved principally through asymptotic continuum mechanics analyses where the functional form of the local singular field is determined within a scalar amplitude factor whose magnitude is calculated from a complete analysis of the applied loading and geometry. For the linear elastic behavior of a nominally stationary crack subjected to tensile (Mode I) opening, the local crack tip stresses $\sigma_{ij}$ can be characterized in terms of the $K_I$ singular field (Williams, 1957; Irwin, 1958):

$$\sigma_{ij}(r,\theta) = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta) + O(r^{3/2}) + \ldots$$

$$+ \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta), \quad \text{as } r \to 0, \quad (1)$$

where $K_I$ is the Mode I stress intensity factor, $r$ the distance ahead of the tip, $\theta$ the polar angle measured from the crack plane, and $f_{ij}$
a dimensionless function of \( \Theta \). Similar expressions exist for cracks subjected to pure shear (Mode II) and anti-plane strain (Mode III). Provided this asymptotic field can be considered to "dominate" the local crack tip vicinity over a region which is large compared to the scale of microstructural deformation and fracture events involved, the scalar amplitude factor \( K_I \) can be considered as a single, configuration-independent parameter which uniquely and autonomously characterizes the local stress field ahead of a linear elastic crack and can be used as a correlator of crack extension (Irwin, 1958).

For cracks subjected to cyclically varying loads, \( K_I \) must be defined at the extremes of the cycle, such that a maximum and a minimum stress intensity, \( K_{\text{max}} \) and \( K_{\text{min}} \), respectively, for a particular crack length, \( a \), can be computed. According to the original analysis of Paris and Erdogan (1963), the crack growth increment per cycle in fatigue \( (\frac{da}{dN}) \) can be described in terms of a power law function of the range of \( K_I \), given by the stress intensity range \( \Delta K \), i.e.,

\[
\frac{da}{dN} = C \Delta K^m ,
\]

where \( C \) and \( m \) are experimentally determined scaling constants.

One of the principal limitations of this approach, specifically the criterion that \( K_I \) is a valid description of the crack tip field, is that a state of small-scale yielding must exist. From Eq. 1, it is apparent that as \( r \) tends to zero, stresses become infinite at the tip. In reality, however, such stresses are limited by local crack
tip yielding, which occurs over a region ahead of the crack tip known as the plastic zone size $r_y$. Calculations of the extent of this region vary depending upon the mode of applied loading and the geometry of the body (Rice, 1968a), but a rough estimate for $r_y$ for a monotonically-loaded crack can be taken as:

$$r_y \approx \frac{1}{2\pi} \left( \frac{K}{\sigma_0} \right)^2,$$

where $\sigma_0$ is the yield strength of the material. Provided this extent of local plasticity is small compared with the extent of the $K_I$-field, which itself is small compared to the overall dimensions of the body (including the crack length), the plastic zone can be considered as merely a small perturbation in the linear elastic field and $K_I$ crack tip dominance can be preserved. In general, this situation, known as small-scale yielding, exists where the plastic zone size from Eq. 3 is at most one fifteenth of the in-plane dimensions of crack length ($a$) and remaining ligament depth ($b$).

The local yielding ahead of fatigue cracks is somewhat more complex due to the presence of reversed plasticity. However, following the analysis of Rice (1967) for a cyclically-stressed elastic-perfectly plastic solid, plastic superposition of loading and unloading stress distributions can be used to compute the extent of plastic zones ahead of the crack. On loading to $K_{\text{max}}$, a monotonic or maximum plastic zone is formed at the crack tip of dimension (from Eq. 3):

$$r_{\text{max}} \approx \frac{1}{2\pi} \left( \frac{K_{\text{max}}}{\sigma_0} \right)^2.$$
However, on unloading from $K_{\text{max}}$ to $K_{\text{min}}$, superposing an elastic unloading distribution of maximum extent $-2\sigma_0$ results in a region ahead of the crack tip where residual compressive stresses, of magnitude $-\sigma_0$, will exist. This region is known as the cyclic plastic zone size $r_\Delta$ and is approximately one fourth of the size of the monotonic zone, i.e.:

$$r_\Delta = \frac{1}{2\pi} \left( \frac{\Delta K}{2\sigma_0} \right)^2,$$

where, strictly speaking, $\sigma_0$ is now the cyclic yield strength. Once again, the correlation of $K_I$ to crack extension by fatigue will be a valid approach provided small-scale yielding conditions apply, namely $r_{\text{max}}$ small compared to the in-plane dimensions of the cracked body.

The numerical values of the stress intensity factors at the crack tip, i.e., $K_I$, $\Delta K$, etc., remain undetermined from the asymptotic analyses, yet can be computed from the overall geometry and applied loading conditions. In fact, solutions for $K_I$ applicable to a wide variety of situations are now tabulated in handbooks (e.g., Tada and others, 1973).

**Nonlinear Elastic Fracture Mechanics**

Global characterization of crack tip fields using LEFM becomes unsuitable where small-scale yielding conditions do not apply, i.e., when the plastic zone at the tip of the crack becomes comparable with crack length, i.e., when $a \sim r_y$. Since the use of $K_I$ singular fields is no longer appropriate in such instances, alternative asymptotic
analyses have been developed to define the crack tip stress and strain fields in the presence of more extensive local plasticity (for a recent review, see Ritchie, 1983). Based on the deformation theory of plasticity (i.e., non-linear elasticity), the asymptotic form of these local fields, for non-linear elastic power-hardening solids of constitutive law:

\[
\frac{\bar{\varepsilon}_p}{\varepsilon_0} = \alpha \left( \frac{\bar{\sigma}}{\sigma_0} \right)^N,
\]

is given by the so-called HRR singularity as, in the limit of \( r \to 0 \) (Hutchinson, 1968; Rice and Rosengren, 1968):

\[
\sigma_{ij}(r,\theta) = \sigma_0 \left[ \frac{J}{\alpha \sigma_0 \varepsilon_0 \lambda r} \right]^{N+1} \tilde{\sigma}_{ij}(\theta,N), \quad \varepsilon_{ij}^p(r,\theta) = \varepsilon_0 \left[ \frac{J}{\alpha \sigma_0 \varepsilon_0 \lambda r} \right]^{N+1} \tilde{\varepsilon}_{ij}^p(\theta,N), \quad u_i(r,\theta) = \varepsilon_0 r \left[ \frac{J}{\alpha \sigma_0 \varepsilon_0 \lambda r} \right]^{N+1} \tilde{u}_i(\theta,N),
\]

where \( N = 1/n \) is the work hardening exponent, \( \bar{\sigma} \) and \( \bar{\varepsilon}_p \) are the equivalent stress and plastic strain, \( \sigma_0 \) and \( \varepsilon_0 \) are the yield stress and yield strain, \( I_N \) a numerical constant weakly dependent upon \( N \), and \( \tilde{\sigma}_{ij}(\theta,N) \), \( \tilde{\varepsilon}_{ij}^p(\theta,N) \), and \( \tilde{u}_i(\theta) \) are normalized stress, strain and displacement functions of \( \theta \) and \( N \) and of whether plane stress or plane strain conditions prevail. The amplitude of this field is the J-integral (Rice, 1968). Analogous to \( K_I \), \( J \) uniquely and autonomously characterizes the crack tip field under elastic-plastic
conditions provided some degree of strain hardening exists, and thus can be used to correlate crack extension. Further, for small-scale yielding, J can be directly related to the strain energy release rate G, and hence K_I, i.e.,

\[ J = G = \frac{K_I^2}{E'} \quad \text{(linear elastic)}, \quad (8) \]

where \( E' \) is the appropriate elastic modulus (= E for plane stress, or \( E/(1 - \nu^2) \) for plane strain) and \( \nu \) is Poisson's ratio.

It should be noted here that the HRR singularity (Eq. 7) and the J-integral are defined strictly for a non-linear elastic solid, where stress is proportional to current strain, rather than the more physically realistic elastic-incrementally plastic solid, where stress is proportional to strain increment (Fig. 1). Provided the crack remains stationary and is subjected only to a monotonically increasing load, plastic loading will not depart radically from proportionality (i.e., the stress and strain components stay in the same proportion to one another) and this approach is appropriate. However, for growing cracks where regions of elastic unloading and non-proportional plastic flow will be embedded within the J-dominated field (Hutchinson and Paris, 1979), behavior is not properly modelled by such non-linear elasticity and this poses certain restrictions on the fracture mechanics characterization of crack growth.

An alternative treatment of elastic-plastic crack initiation and growth, which is not subject to restrictions required by non-linear elasticity, is to utilize the concept of crack tip opening
displacement or CTOD (Knott, 1973). From Eq. 7, it is apparent that the opening of the crack faces at $r \to 0$ varies as $r^{1/N+1}$, such that this separation can be used to define the CTOD ($\delta_t$) as the opening where $45^\circ$ lines emanating back from the crack tip intercept the crack faces, i.e., for proportional loading (Shih, 1981):

$$\delta_t = d_N(\varepsilon_0, N) J/\sigma_0 \text{ (non-linear elastic)},$$

$$= \frac{K_I^2}{\sigma_0 E} \text{ (linear elastic)},$$

(9)

where $d_N$ is a proportionality factor ($\sim 0.3$ to 1) dependent upon the yield strain $\varepsilon_0$, the work hardening exponent $N$, and whether plane stress or plane strain is assumed. Since $\delta_t$, like $J$, can be taken as a measure of the intensity of the elastic-plastic crack tip fields, it also is feasible to correlate crack initiation and crack advance to $\delta_t$.

It is important to realize, however, that all such continuum mechanics characterizations of the crack tip fields do not necessitate detailed quantitative microscopic models to be known for the individual fracture events. The use of such analyses, incorporating field parameters such as $K_I$ and $J$ as "driving forces" for crack extension, is thus independent of the specific micromechanisms of crack advance.
Fracture Mechanics of Creeping Solids

Fracture mechanics characterizations of crack tip fields also have been applied to the problem of crack growth in a creeping solid at elevated temperatures. As described in detail by Riedel (1984), for a solid with a power-law viscous constitutive law of:

$$\dot{\varepsilon}_p = B(\sigma)^N_\text{p},$$

the HRR singularity (Eq. 7) will have the rate-dependent form, given, as $r \to 0$, by (Rice and Riedel, 1980):

$$\varepsilon_{ij}(r,\theta) = \left(\frac{C^*}{N^* r}\right)^{(N^*+1)/2} \varepsilon_{ij}(\theta),$$

where $\dot{\varepsilon}_p$ is the equivalent plastic strain rate, and $C^*$ is the viscous analogue of $J$. Provided the bulk of the material deforms predominately by linear and non-linear viscous creep, i.e., in accord with Eq. 10, $C^*$ similarly can be applied as an appropriate field parameter to globally correlate to crack advance at elevated temperatures.

In more brittle materials, however, the elastic deformation at the crack tip still can predominate. Under these conditions, creep crack growth can be correlated to the stress intensity factor $K_I$. However, unlike behavior at low homologous temperatures, the dominant crack tip fields can change with time, such that at a given instant, consideration must be given to all fields associated with the local modes of deformation, i.e., elastic deformation ($K_I$-field), elastic-plastic deformation ($J$ or $C^*$-field) and those associated with primary creep etc., to decipher which is the dominant field, i.e., which characterizes the stress and deformation conditions over a distance
from the crack tip comparable with the scale of fracture events (Riedel, 1984; Bassani and McClintock, 1981).

SLOW CRACK GROWTH BY DUCTILE FRACTURE

Crack Initiation Toughness

Slow crack growth by ductile fracture proceeds via a microscopic process involving the initiation and growth of voids formed around particles. From a continuum perspective, the initiation of such cracking can be characterized by the well-accepted criteria $K_I = K_{IC}$, $J = J_{IC}$ or $\delta_t = \delta_{IC}$, where $K_{IC}$, $J_{IC}$ and $\delta_t$ describe the (crack initiation) fracture toughness of the material.

Microscopically, such fracture events are modelled in terms of some local failure criterion applied over a characteristic dimension of the microstructure ($l_0^*$) representing the scale of fracture events. For the initiation of ductile fracture, the simplest criterion involves the critical crack opening displacement exceeding half the mean void-initiating particle spacing ($d_p$) (McClintock, 1969; Rice and Tracey, 1969; Rice and Johnson, 1970), i.e.,

$$\delta_{IC} \sim (0.5 \text{ to } 2)d_p \quad (12)$$

This model is based on the notion that, in non-hardening materials, microvoid coalescence would take place when the void sites first are enveloped by the intense strain region at the crack tip, i.e., at distance $x \sim 2\delta_t$ from the tip. From Eq. 9, this model for the fracture toughness implies:
although it is rare to find the toughness to increase directly with increasing strength.

An alternative approach to modelling the initiation of slow crack growth via microvoid coalescence has been to utilize a stress-modified critical strain criterion (McClintock, 1958; Mackenzie and others, 1977; Ritchie and others, 1979), similar to the well-known critical stress criterion (Ritchie, Knott and Rice, 1973) for brittle cleavage fracture (Fig. 2). Here, at \( J = J_{IC} \), the local equivalent plastic strain \( \varepsilon_p \) must exceed a critical fracture strain or ductility \( \varepsilon_f^*(\sigma_m/\bar{\sigma}) \), specific to the relevant stress state \( (\sigma_m/\bar{\sigma}) \), over some characteristic distance \( l_0^* \) comparable with the mean spacing \( (d_p) \) of the void initiating particles. \((\sigma_m \text{ and } \bar{\sigma} \text{ here are the hydrostatic and equivalent stress, respectively})\). The model is shown schematically in Fig. 2b. If the near-tip strain distribution \( \varepsilon_p \) from Eq. 7 is considered in terms of distance \( (r = x) \) directly ahead of the crack, normalized by the crack opening displacement, \( \delta_t \), as:

\[
\varepsilon_p = \left( \frac{J}{\sigma_0 r} \right)^{1/n+1} \cdot c_1 \frac{\delta_t}{x},
\]

with \( c_1 \) of order unity, then the crack initiation criterion of \( \varepsilon_p \) exceeding \( \varepsilon_f^*(\sigma_m/\bar{\sigma}) \) over \( x = l_0^* \approx d_p \) at \( J = J_{IC} \) now implies a ductile fracture toughness of (Ritchie and others, 1979; Ritchie and Thompson, 1984):

\[
J_{IC} \sim \sigma_0 l_0^*.
\]
Unlike the critical CTOD criterion (Eq. 13), this latter microscopic criterion (Eq. 15) involving a critical strain ahead of a crack tip now implies that the toughness $J_{IC}$ for ductile fracture is proportional to the product of strength and ductility, which is more physically realistic.

Owing to the extreme complexity of crack tip fracture events, such microscopic models must only be considered as first order. Their use, therefore, is not generally for toughness prediction in engineering design, where invariably $K_{IC}$ or $J_{IC}$ are measured by experiment. However, they do provide a semi-quantitative basis for the understanding of how microstructural factors in a material can contribute to the toughness. For ductile fracture, $\varepsilon_f^*$ and $l_0^*$ from Eq. 15 clearly are the relevant parameters. Whereas $l_0^*$ is related simply the the mean spacing of the void-initiating particles ($d_p$), $\varepsilon_f^*$ is a function of both the size and spacing of the particles, the stress-state ($\sigma_m/\bar{\sigma}$) and the strain hardening exponent, $n$. For example, analysis by Rice and Tracey (1969), for the rate of void expansion in the triaxial stress field ahead of a crack in a non-hardening material, suggests that:

$$\frac{dR_p}{R_p} = 0.28 \frac{d\varepsilon}{\varepsilon} \exp(1.5 \frac{\sigma_m}{\bar{\sigma}}) ,$$

(16)
where $R_p$ is the void radius. For an array of void initiating particles of diameter $D_p$ and mean spacing $d_p$, setting the initial void radius to $D_p/2$ and integrating to the point of ductile fracture initiation gives an expression for the fracture strain, $\varepsilon_f^*$, as:

$$\varepsilon_f^* = \frac{2\ln\left(d_p/D_p\right)}{0.28 \exp(1.5 \sigma_m / \bar{\sigma})} \quad (17)$$

Earlier analysis for a strain hardening material containing cylindrical holes similarly suggests (McClintock, 1958):

$$\varepsilon_f^* = \frac{2\ln\left(d_p/D_p\right)(1 - n)}{\sinh\left[(1 - n)\left(\sigma_a + \sigma_b\right)/(2\bar{\sigma}/\sqrt{3})\right]} \quad (18)$$

where $\sigma_a$ and $\sigma_b$ are the transverse stress components.

Although both analyses consider the fracture strain to be limited by the simple impingement of the growing voids and thus tend to overestimate $\varepsilon_f^*$ by ignoring prior coalescence due to shear banding by strain localization, they do indicate the dependence of toughness on stress-state ($\sigma_m/\bar{\sigma}$), strain hardening ($n$) and purity ($d_p/D_p$). For example, a large effect of stress-state (i.e., triaxiality) is predicted; from Eqs. 17,18, $\varepsilon_f^*$ would be expected to be reduced by an order of magnitude ahead of a sharp crack compared to an unnotched plane strain condition. Increasing strain hardening, conversely, can enhance $\varepsilon_f^*$, particularly at high triaxiality. The benefits of increased purity (i.e., increased particle spacing $d_p$), however, only are pronounced at low $D_p/d_p$ ratios due to the
logarithmic terms in Eqs. 17, 18. For example, reducing the volume fraction of inclusions by three orders of magnitude, say, from 0.001 to 0.000001, merely doubles $\varepsilon_f^*$ (McClintock, 1977).

Crack Growth Toughness

When cracks grow, the elastic crack tip stress and deformation fields remain unchanged and therefore can be characterized still by the stress intensity factor $K_I$. However, the elastic-plastic fields for such non-stationary cracks may become altered due to the enclave of prior plastic zones left in the wake of the crack tip (Chitaley and McClintock, 1971; Rice and Sorensen, 1978). This follows because in the near-tip vicinity of a growing tensile crack there are regions of elastic unloading and non-proportional plastic loading (Fig. 3), both of which are inadequately described by the deformation theory of plasticity upon which $J$ characterizations are based (e.g., Eq. 7). The macroscopic description of slow crack growth by ductile fracture relies, however, on the assumption that these non-proportional terms can be ignored.

Following the deformation theory of Hutchinson and Paris (1979), which utilizes the incremental form of the HRR singularity:

$$d\varepsilon_{ij}(r, \theta) = \Delta \varepsilon_0 \left[ \frac{J}{\alpha \sigma_0 \varepsilon_0^* N} \right]^{N+1} \frac{N}{N+1} \frac{dJ}{g_{ij} + \frac{da}{r} h_{ij}}$$ (19)

where

$$h_{ij}(\theta) = \frac{N}{N+1} \cos \theta g_{ij}(\theta) + \sin \theta \frac{\partial}{\partial \theta} g_{ij}(\theta)$$
the regions of elastic unloading, which comparable with the scale of crack advance (\(\Delta a\)), and non-proportional plastic loading are assumed to be embedded within the HRR J-controlled singularity field of radius \(R\) (Fig. 3). The argument for J-controlled growth then relies on the fact that these regions remain small compared to \(R\), such that the singularity field can be said to be controlling. For the region of elastic unloading to be small, the increment of crack extension (\(\Delta a\)) must be small compared to \(R\), whereas for the region of non-proportionality to be small, J must increase rapidly with crack extension. With reference to Eq. 19, this means that the first term, corresponding to proportional load increments, must dominate the second term, corresponding to non-proportional load increments, i.e.:

\[
\frac{dJ}{da} \gg \frac{J}{R},
\]

and \(\Delta a \ll R\). \hspace{1cm} (20)

Eq. 20 usually is expressed as a single specimen size requirement for J to uniquely characterize crack growth as:

\[
\omega \equiv \frac{b}{J_{IC}} \left( \frac{dJ}{da} \right) \gg 1,
\]

(21)

where \(b\) is the uncracked ligament. From numerical calculations (Shih and others, 1979), the parameter \(\omega\) must exceed 10 for deep-cracked single-edge-notch bend geometries (i.e., for the Prandtl field) and \(\sim 100\) for center-cracked tension geometries (for \(N \approx 10\)).
To provide a continuum measure of the toughness of slowly growing ductile cracks, this analysis is applied to experimental measurements of crack growth resistance curves, based either on CTOD, i.e., $\delta_R(\Delta a)$ curves, or more usually on $J$, i.e., $J_R(\Delta a)$ curves, as shown in Fig. 4. Crack initiation then is assessed as before as $J = J_{Ic}$, whereas crack growth is assessed in terms of the slope $dJ/da$. The latter normally is expressed non-dimensionally as the so-called tearing modulus (Paris and others, 1979):

$$T_R = (E'\sigma_0^2)dJ/da . \quad (22)$$

Crack instability thus is characterized when the tearing force ($T = (E'\sigma_0^2)dJ/da$) exceeds $T_R$. Analogous procedures have been developed with the crack opening displacement where crack growth toughness is evaluated in terms of the crack opening angle (CTOA = $d\delta/da$), defined as the slope of the $\delta_R(\Delta a)$ curve normalized with respect to the yield strain $\sigma_0/E'$ (Shih and others, 1979), i.e.,

$$T_R = \frac{E'}{\sigma_0^2} \frac{dJ}{da} = \frac{E'}{\sigma_0} \frac{d\delta}{da} = \frac{CTOA}{Yield \ Strain} . \quad (23)$$

Whereas crack initiation toughness values (i.e., $K_{IC}$, $J_{IC}$, etc.) are by far the most widely utilized in engineering design and analysis, it has been noted that, in many high toughness structural materials, stable crack growth can occur at $J$ values some 5 to 10 times the initiation $J_{IC}$ value, prior to instability, e.g., Fig. 5 after Wilson (1979). For this reason, in certain applications,
particularly in the nuclear industry, it has been reasoned correctly that evaluation of the toughness of such materials using crack initiation parameters is overly conservative. Accordingly, for these situations, the recent trend has been to additionally consider crack growth parameters such as $dJ/da$, CTOA or the tearing modulus $T_R$ for both engineering fracture mechanics design and metallurgical toughness evaluation (e.g., Ritchie and Thompson, 1984).

The major practical drawbacks with this approach are the limitations of section size, which are defined in Eq. 21 and imposed by the restrictions of the deformation theory of plasticity, i.e., through the use of stationary crack fields such as HRR. For example, for a 25 mm thick 1-T compact specimen in plane strain, deformation $J$-controlled growth is only a reality for the first 1.5 to 2.0 mm of crack extension (i.e., where $\Delta a < 0.06b$), whereas for a similar sized center-cracked tension specimen, it is valid merely for the initial 0.5 mm or so of a 25 mm ligament (i.e., where $\Delta a < 0.016b$). This means that for further crack extension in a given material, $J$ will no longer provide a valid description of crack growth, with the shape of the $J_R(\Delta a)$ resistance curve, and hence $T_R$, differing with specimen geometry and with varying ligament size in a given geometry.

Although certain continuum analyses based on modified $J$ parameters have been proposed to overcome this problem (Ernst, 1983), the most fundamental approach to characterizing the stress and deformation conditions for slow ductile crack growth is through the use of the recently-derived flow theory solutions for actual non-
stationary cracks. Such solutions are not as yet ameanable to a macroscopic single-value parameter description of crack advance, but do provide insight for microscopic descriptions (Ritchie and Thompson, 1984; Rice and others, 1980).

Exact asymptotic analysis by Drugan, Rice and Sham (1982), together with earlier analyses by Rice and Sorensen (1978), for quasi-static plane strain Mode I crack advance in an elastic-perfectly plastic solid, have shown that the local stresses ahead of a non-stationary crack are unchanged from the Prandtl field for a stationary crack, except behind the crack tip where differences of the order of 10% result from the presence of a wedge of elastic unloading between approximately $\theta = 112$ to 162 deg (Fig. 6).

However, because of the distinctly non-proportional straining of elements above and below the plane of the growing crack, compared to the predominately proportional plastic straining of material elements near a stationary crack tip, the strains are smaller ahead of the non-stationary crack. Specifically, the strains decay as $1/r$ ahead of a stationary crack in an elastic-perfectly plastic solid (or $(1/r)^{N/N+1}$ in a power hardening solid), whereas for a non-stationary crack, the strain singularity is weaker, decaying as a function of $\ln(1/r)$. For an advancing tensile crack, the equivalent plastic shear strain distribution at small radial distances $r$ is given, in the limit of $r \to 0$, as (Rice and others, 1980):

$$\gamma_p = \frac{m}{\sigma_0} \frac{dj}{da} + \frac{1.88(2 - \nu) \sigma_0}{\varepsilon} \ln\left(\frac{L}{r}\right) ,$$  \hspace{1cm} (24)
where the parameters m and L are unset by the asymptotic analysis, although L can be identified with the extent of the plastic zone size $r'_y$ of the moving crack, i.e.:

$$r'_y = \frac{E J}{\sigma_0^2} = (0.11 \text{ to } 0.13) \frac{E J}{\sigma_0^2} \quad (25)$$

Similar to the equivalent relationship from deformation theory (Eq. 19), the first term of Eq. 25 represents the effect of proportional plastic strain increments due to crack tip blunting of the stationary crack whilst the second term represents the effect of additional non-proportional plastic strain increments caused by the extension of the crack.

Unlike the continuum analysis for the slowly growing ductile crack, where the terms in the expression for crack tip strains representing the actual movement of the crack were ignored (Eqs. 19, 20), using Eq. 25 for the equivalent microscopic analysis we can assume that these non-stationary terms actually dominate. Local criteria for crack growth by microvoid coalescence have been proposed in terms of a critical crack tip opening angle (Green and Knott, 1975) or the attainment of a critical crack opening displacement $\delta_p$ at some microstructurally significant distance $l_o^*$ behind the crack tip (Rice and Sorensen, 1978), as illustrated in Fig. 7. However, by applying the same criterion as that utilized to predict the initiation of the stationary crack, i.e., the attainment of a critical strain $\varepsilon_f^*$ radially across a characteristic distance $l_o^*$
ahead of the crack tip (Eq. 15), only now incorporating the relevant strain distribution for the growing crack (Eq. 24), comparative estimates of the fracture toughness at initiation and instability are given as (Ritchie and Thompson, 1984):

\[ J_{lc} = \left( \frac{E}{\sigma_0^2} \right) l_0^* \left( \frac{\varepsilon_f^*}{\varepsilon_0} \right) \]  

(\text{initiation}) \quad , \quad (26) \\

\[ J_{ss} = \left( \frac{E}{\sigma_0^2} \right) l_0^* \exp \left[ \frac{0.6(1 + \nu)}{(2 - \nu)} \cdot \left( \frac{\varepsilon_f^*}{\varepsilon_0} \right) \right] \]  

(\text{instability}) \quad , \quad (27) \\

where the initiation result, at \( J = J_{lc} \) is restated from Eq. 15 and the instability result, at the plateau of the \( J_R(\Delta a) \) resistance curve where \( J = J_{ss} \) as \( dJ/da \to 0 \), is derived from Eq. 24 assuming sufficient plasticity during crack advance for the non-stationary term to dominate.

Eqs. 26 and 27, representing microscopic estimates for the ductile fracture toughness in the presence of initiating or slowly growing cracks, indicate that toughness is promoted by higher ductility (large \( \varepsilon_f^* \)), lower strength (low \( \sigma_0 \)) and more widely spaced particles (large \( l_0^* \)). However, these effects of microstructure can be enhanced for the growing crack since toughness there is an exponential function of the ratio of fracture to yield strain, rather than a direct function for crack initiation. This larger influence of microstructure on the growing crack can be appreciated by comparing \( J_{lc} \) and \( J_R(\Delta a) \) data, as in Fig. 5, for example, which shows R-curves for A516 Grade 70 steels following various steelmaking
processes to control the inclusion content (Wilson, 1979). It is apparent that the additional calcium treatments (CaT), which control the volume fraction and shape of oxides and sulphides compared to conventional vacuum degassing (CON), become progressively more significant with increasing crack extension. According to the simple modelling result in Eqs. 26, 27, the enhanced influence of microstructure on the growing crack can result simply from the different strain distributions ahead of stationary and slowly running cracks (c.f., Eqs. 7, 14 with Eq. 24) rather than from any differences in the local fracture mechanisms.

Finally, consideration of this analysis for the toughness of the initiating and slowing growing ductile crack can provide a useful rationalization of the existence of a crack resistance curve. Since at fixed J, crack tip strains are reduced by the advance of the crack, to maintain an identical strain-controlled local fracture criterion ahead of the crack tip must involve an increase in the applied driving force. Therefore, crack extension by microvoid coalescence must require increasing J conditions to be sustained.

SLOW CRACK GROWTH BY FATIGUE

Slow crack growth by fatigue must be considered as the principal cause of in-service failures in engineering structures and components, either as a result of pure mechanical loading or in conjunction with sliding and friction between surfaces (fretting fatigue), rolling contact between surfaces (rolling contact fatigue),
aggressive environments (environmentally-assisted or corrosion fatigue) or elevated temperatures (creep-fatigue). Although the process of fatigue failure consists of several distinct phenomena involving cyclic hardening or softening, microcrack initiation and coalescence, and macrocrack growth, it is the latter process which is of most importance currently for defect-tolerant design.

As described above, fatigue crack growth under small-scale yielding conditions can be correlated with the nominal stress intensity range $\Delta K$ (e.g., in Eq. 2), with the variation between $\frac{da}{dN}$ and $\Delta K$ being sigmoidal in shape over a wide range of growth rates (Fig. 8). The simple power law relationship (Eq. 2) provides a reasonable description of behavior in the so-called intermediate range of growth rates between typically $10^{-6}$ and $10^{-3}$ mm/cycle. At higher growth rates, however, where final instability is approached and the onset of static fracture modes such as cleavage and fibrous fracture accelerates propagation rates, Eq. 2 underestimates behavior. Conversely, it overestimates growth rates at lower $\Delta K$ levels approaching a fatigue threshold range $\Delta K_{TH}$, below which long cracks appear dormant or advance at experimentally-undetectable rates (Ritchie, 1979).

Since the majority of a fatigue lifetime must be spent where the crack is growing most slowly, it generally is the near-threshold regime which dominates life. However, it is primarily in this regime that behavior can become non-unique under certain crack size, geometry and loading conditions (Ritchie, 1984). For example, it is
now realized that when crack sizes are small (i.e., \( \leq 1 \) \( \text{mm} \)), as is generally the case in many service components, their behavior may involve accelerated growth rates, even below the threshold, as shown schematically in Fig. 9 and reviewed recently by Suresh and Ritchie (1984a). Moreover, these effects can be accentuated by environmental factors (Gangloff, 1981; Gangloff and Ritchie, 1984). The limiting dimensions for such non-unique behavior appear to be where crack sizes approach the scale of the microstructure or the scale of local plasticity, or where cracks simply are physically small (i.e., \( \leq 1 \) \( \text{mm} \)) (e.g., Lankford, 1984; Suresh and Ritchie, 1984a).

The importance of the small crack and near-threshold behavior cannot be overemphasized. This is well illustrated in Fig. 10 which shows actual defect tolerant fatigue lifetime predictions, in the form of defect size, \( a \), versus life, for a Canadian sour gas pipeline (Vosikovsky and Cooke, 1978). Although the pipeline is predicted to last in excess of 85 years, based on the extension of an assumed 0.5 mm initial flaw to a critical crack length of 4 mm under an operational spectrum of applied loads, the first 70 years of this time is spent with the crack both smaller than 1 mm and propagating in the near-threshold regime.

**Mechanisms of Fatigue Crack Growth**

Although fatigue crack growth data are generally described in terms of \( \Delta K \), computed from geometry, crack size and applied loading, the local driving force experienced at the crack tip may differ from
the nominal (far field) $\Delta K$ where factors such as cyclic plasticity, crack deflection and crack closure perturb the near-tip field. Crack closure in particular is most relevant at very low, near-threshold growth rates where the spread of plasticity is limited to the extent that invariably plane strain conditions prevail with CTOD and plastic zone sizes both small compared to microstructural size-scales.

**Plasticity-induced closure.** Fatigue crack closure originally was considered to arise solely from the elastic constraint, of material surrounding the plastic zone enclave in the wake of the crack tip, on material elements previously plastically stretched at the tip (Elber, 1970). The resulting interference between crack surfaces can lead to a reduction in crack driving force from the nominal $\Delta K$ value to some lower effective value, $\Delta K_{\text{eff}}$, actually experienced at the tip, viz:

$$
\Delta K_{\text{eff}} = K_{\text{max}} - K_{\text{CL}}
$$

(28)

where $K_{\text{CL}}$ is the closure stress intensity representing the point of first contact between the crack surfaces during unloading. Closure arising from cyclic plasticity, generally referred to as **plasticity-induced closure**, is most prevalent under plane stress conditions (Lindley and Richards, 1974), and thus is more significant at higher stress intensities, rather than at near-threshold levels.

At near-threshold levels, several other sources of closure have been shown recently to assume greater importance (Suresh and Ritchie,
1984b). As schematically illustrated in Fig. 11, these mechanisms involve the wedging action of crack flank corrosion deposits (Ritchie and others, 1980; Stewart, 1980; Suresh and others, 1981) and fracture surface asperities (Walker and Beevers, 1979; Minakawa and McEvily, 1981; Suresh and Ritchie, 1982), coupled with significant crack tip shear displacements (Davidson, 1981), fluid-induced pressure between the crack walls (Endo and others, 1972; Tzou and others, 1985a, 1985b), and compression between the crack surfaces resulting from certain metallurgical phase transformations. Since a detailed description of these alternative closure mechanisms has been the subject of a recent review (Suresh and Ritchie, 1984b), only a brief summary is presented here.

Oxide-induced closure. Crack closure arising from crack surface corrosion deposits, generally referred to as oxide-induced closure, is promoted in moist, oxidizing environments when the size-scale of the debris becomes comparable with crack tip opening displacements (Suresh and others, 1981). Notable examples are the crack surface oxides and calcareous deposits formed during corrosion fatigue in structural steels tested, respectively, in water and seawater (Suresh and Ritchie, 1983a; Scott and others, 1983), and the chromic oxides formed during creep-fatigue in Ni-based superalloys (Yuen and others, 1984). Simple quantitative modelling, based on the concept of a rigid wedge inside a linear elastic crack, suggests that the closure which results from such deposits depends upon the thickness of the
oxide film, s, the location of its peak thickness from the crack tip, 
$2\pi$, Poisson's ratio, $\nu$, and Young's modulus, $E$, i.e. (Suresh and others, 1982):

$$K_{cl} = \frac{sE}{4\sqrt{\pi}z (1 - \nu^2)} \quad (29)$$

This relationship clearly shows that deposits in the immediate vicinity of the crack tip will have a dominating influence in the development of closure by this mechanism (Suresh and Ritchie, 1983b).

In lower strength materials, particularly in low carbon steels, the extent of the corrosion debris can be significantly enhanced at low load ratios from fretting oxidation processes (Benoit and others, 1981) between the crack walls, leading to a greater accumulation of deposits and hence to more closure. This can produce surprising results, such as observations in lower strength alloy steels of near-threshold growth rates at $R = 0.05$ being significantly faster in dry helium gas (Suresh and others, 1981), and slower in water or steam, compared to behavior in room air. Since susceptibility to hydrogen embrittlement is not large in these steels, at the high frequencies and low growth rates characteristic of near-threshold conditions, such results simply can be interpreted in terms of less corrosion deposits being generated in the dry atmosphere and more in the wet environment, thereby governing the extent of oxide-induced closure (Suresh and Ritchie, 1984b). With steels of higher tensile strength (Toplosky and Ritchie, 1981) and in the majority of aluminium alloys (Suresh and others, 1984; Carter and others, 1984; Zaiken and
Ritchie, 1985), the degree of fretting oxidation between crack surfaces appears much reduced, with the result that, except in very oxidizing environments, the thickness of the fracture surface oxide films remains small compared to the crack opening displacements such that the contribution to closure from this mechanism becomes negligible.

**Roughness-induced closure.** A more general source of crack closure arises from the wedging action of fracture surface asperities, where crack tip opening displacements are small and where significant Mode II crack tip shear displacements occur (Walker and Beevers, 1979; Minakawa and McEvily, 1981; Suresh and Ritchie, 1982). Such roughness-induced closure thus is promoted at near-threshold levels, particularly since crack advance in this regime tends to occur via a single shear type mechanism (i.e., involving Mode II + I displacements akin to Forsyth's Stage I (Forsyth, 1962) when the extent of crack tip plasticity does not exceed the characteristic microstructural dimensions. This induces a faceted or crystallographic mode of crack growth (Fig. 12), which is most prevalent in coherent particle hardened (planar slip) systems (e.g., underaged aluminium alloys (Suresh and others, 1984; Carter and others, 1984; Zaiken and Ritchie, 1985) and Ni-based superalloys (Brown and others, 1984)), thereby enhancing closure from increased asperity contact (Fig. 13 from Schulte and others, 1980). Significant roughness-induced closure also can be generated in
certain duplex microstructures where crack paths can be made to meander from frequent crack deflection (Suresh, 1983) at the harder phase (e.g., in ferritic-martensitic dual-phase steels (Minakawa and others, 1982; Dutta and others, 1984) and B-annealed titanium alloys (Yoder and others, 1978; Gerdes and others, 1984)).

The magnitude of the contribution from the roughness-induced mechanism appears to depend upon the degree of fracture surface roughness and the extent of the Mode II crack tip displacements. For example, from simple two-dimensional geometric modelling of the process, the non-dimensional closure stress intensity at the point of first asperity contact has been derived to be (Suresh and Ritchie, 1982):

$$K_{cl} = \sqrt{\frac{2\gamma u}{1 + 2\gamma u}} \cdot K_{max} \quad (30)$$

where $\gamma$ is the measure of surface roughness taken as the ratio of height to width of the asperities and $u$ is the ratio of Mode II to Mode I crack tip displacements. Although only a first-order model, experimental results in a range of ferrous and non-ferrous alloys have been found to be in reasonable agreement with this relationship (e.g., Brown and others, 1984; Dutta and others, 1984).

**Fluid-induced closure.** Crack closure also can be generated in liquid environments through the hydrodynamic action of fluid within the crack. Such fluids can generate an internal fluid pressure, which acts to oppose the closing, and to a lesser extent the opening,
of the crack under cyclic loading (Endo and others, 1972; Tzou and others, 1985a, 1985b). The magnitude of this internal pressure distribution, \( p(x) \), shown schematically in Fig. 14, is a function of several factors, including the absolute viscosity of the fluid, \( \eta_0 \), the magnitude of crack opening, \( h \), the closing velocity of the crack walls, \( \dot{\theta} \), and most importantly the degree of fluid penetration, \( d \), into the crack. Thus, for a fatigue crack of length \( a \), where the pressure \( p(x) \) at distance \( x \) from the crack mouth is given by (Tzou and others, 1985a):

\[
p(x) = \frac{6\eta_0 a^3}{h} \log[1 - (x/a)] \dot{\theta}, \quad d/a = 1 . \quad \text{(31a)}
\]

or

\[
p(x) = 6\eta_0 \frac{<h>}{<h>^3} x(d - x), \quad d/a < 1 . \quad \text{(31b)}
\]

and where the extent of fluid penetration, \( d \), can be assessed from capillary flow arguments, in terms of the fluid surface tension, \( \gamma_L \), and wetting angle, \( \beta \), as:

\[
d^2(t) = \left( \frac{\gamma_L \cos \beta}{3\eta_0} \right) \int_0^t <h>(t)dt , \quad \text{(32)}
\]

estimates of the "effective" closure stress intensity, \( K_{\text{max}}^* \), can be computed and superimposed onto the applied stress intensities to derive the variation in \( \Delta K_{\text{eff}} \) (Fig. 15). Predicted values for a 2.25Cr-1Mo pressure vessel steel, fatigued at a nominal \( \Delta K \) of 10 MPa\m\(\sqrt{m} \) (\( R = 0.05 \)) in oils of varying viscosity (Tzou and others,
1985b), are shown as a function of crack length in Fig. 16 (Tzou and others, 1985a). It is apparent from Fig. 15 that the closure effect is comprised of two parts: the fluid opposes both the closing of the crack, which gives an effective increase in $K_{\text{min}}$ by an amount $K_p$, and the opening of the crack, which gives a smaller effective decrease in $K_{\text{max}}$ by an amount $K_q$, where

$$K^*_{\text{max}} = K_q + K_p \quad (33)$$

The decreasing effect with decreasing crack length in Fig. 16 is characteristic of all closure mechanisms and is the basis of current ideas on the role of closure in influencing the growth of short cracks (Suresh and Ritchie, 1984a).

In general, fluid-induced closure should be promoted in higher viscosity liquids, but the development of a fluid pressure quickly saturated (Fig. 16) and further is offset by the slower penetration kinetics of highly viscous fluids into the crack. The maximum contribution to closure from this mechanism should be such that $K^*_{\text{max}} \rightarrow K_{\text{max}}$ (i.e., $\Delta K_{\text{eff}} \rightarrow 0$) as $n \rightarrow \infty$. However, for the majority of viscous fluids (i.e., $n \leq 10^5$ cS), values of $K^*_{\text{max}}$ tend to saturate close to the mean stress intensity in the cycle, due to the minimal changes in $K_q$, such that the maximum extent of closure is generally of the order of $K_{CI}/K_{\text{max}}$ of 0.5. Thus, the hydrodynamic wedge mechanism must be regarded as a less potent mechanism of closure, compared to that generated by cyclic plasticity, corrosion.
deposits and fracture surface asperities, where considerably larger values of $K_c/K_{\text{max}}$ are possible.

**Crack deflection.** Fatigue crack growth behavior also can be influenced markedly by the process of crack deflection (Suresh, 1983). Although Mode I $da/dN$ data invariably are analysed assuming a linear crack orientated perpendicular to the plane of maximum tensile stress, cracks frequently deviate from this path due to load excursions, environmental effects or interaction with specific microstructural features. The result of such deflection, whether associated with a simple kink, twist or more complex bifurcation, is a reduction in the local Mode I driving force. For example, two-dimensional linear elastic analysis for a crack subjected to both shear and tensile loads which undergoes a simple kink at angle $\theta$ to the crack plane, gives solutions for the local Mode I and Mode II stress intensities at the tip of the deflected crack, $k_1$ and $k_2$, in terms of the nominal stress intensities, $K_I$ and $K_{II}$ and angular functions $a_{ij}(\theta)$ as (Bilby and others, 1977):

$$
\begin{align*}
  k_1 &= a_{11}(\theta)K_I + a_{12}(\theta)K_{II} \\
  k_2 &= a_{21}(\theta)K_I + a_{22}(\theta)K_{II}
\end{align*}
$$

(34)

such that the effective near-tip driving force can be considered as:

$$
  k_{\text{eff}} = (k_1^2 + k_2^2)^{1/2}
$$

(35)

For a simple $45^\circ$ deflected crack, where the length of the branch is
small compared to the crack length, solutions to Eqs. 34 and 35 suggest roughly a 20% reduction in local Mode I stress intensity factor resulting from the deflection (Suresh, 1983).

The effect of crack deflection is several fold. Not only is the local Mode I crack driving force reduced and the length of crack path increased, but the resultant Mode II shear promotes roughness-induced crack closure under cyclic loading conditions. Accordingly, microstructures which enhance tortuosity in crack path generally show excellent resistance to near-threshold crack growth. As discussed below, coherent particle hardened and duplex microstructures are notable examples in this regard.

Role of Microstructure

Many microstructural effects observed on fatigue crack propagation can be linked to a prominent role of crack closure, particularly at near-threshold levels. Of these effects, the principal microstructural variables affecting closure and low growth rate behavior appear to be grain size, precipitate type and distribution, slip characteristics and, in duplex structures, the proportion and morphology of the two phases. In many instances, optimizing these variables for maximum fatigue resistance can have the opposite effect on other mechanical properties, such as toughness, ductility and resistance to $\Delta K_{TH}$ values yet this generally reduces fracture toughness and the initiation-controlled fatigue limit (Ritchie, 1977). Similarly, coherent particle hardening, which
induces planar slip, is beneficial for near-threshold fatigue resistance as it leads to crystallographic crack paths which promote crack deflection and roughness-induced closure, yet the resulting strain localization can be very detrimental to both ductility and toughness (Zaiken and Ritchie, 1985). Near-threshold fatigue crack propagation behavior in dual-phase steels and in precipitation hardened aluminum alloys provides illustrative examples of these effects, as discussed below.

Behavior in duplex microstructures. Ferritic-martensitic dual-phase steels provide an excellent metallurgical system to improve fatigue crack growth resistance through the generation of tortuous crack paths by crack deflection at interfaces, leading to enhanced roughness-induced closure (Minakawa and others, 1982; Dutta and others, 1984). Recent studies on duplex microstructures in Fe-2Si-0.1C steel (Dutta and others, 1984), in particular, have shown that by modifying the proportion and primarily the morphology of the ferrite and martensite phases through intercritical heat treatment, increases in the $\Delta K_{TH}$ value by up to a factor of two can be readily obtained without loss in strength (Fig. 17). Such marked increases in crack growth resistance, shown in Fig. 17 specifically for step quenched (SQ) and intercritical annealed (IA) structures in Fe-2Si-0.1C, are associated with measured increases in closure and can be attributed to the production of meandering crack paths from frequent
deflection at ferrite-ferrite and ferrite-martensite interfaces (Fig. 18) (Suresh, 1983; Dutta and others, 1984).

As pointed out several years ago (Minakawa and others, 1982), the benefits of this approach in dual-phase steels are that very high thresholds can be obtained without lowering tensile strength. In fact, in the intercritical annealed Fe-2Si-0.1C steel, the $\Delta K_{TH}$ value of almost 20 MPa$\sqrt{m}$, with a yield strength of 600 MPa, is believed to represent the highest ambient temperature threshold ever reported and certainly represents the highest combination of fatigue threshold and yield strength measured to date in ferrous alloys (Fig. 19) (Dutta and others, 1984).

**Behavior in precipitation hardening systems.** Similar enhancements in near-threshold fatigue crack propagation resistance through the deflection of the crack path can be achieved by modifying the nature of the slip mode in precipitation hardening alloys. In aluminum alloys, for example, underaged microstructures generally show higher thresholds and lower near-threshold growth rates than peak and overaged microstructures, as shown for I/M 7150 alloy in Fig. 20 (Zaiken and Ritchie, 1985). Such differences are reduced at high load ratios and are virtually non-existent at higher growth rates above $10^{-6}$ mm/cycle. The increasing resistance to near-threshold crack growth with decreased aging once more can be associated with a measured increase in crack closure and attributable mechanistically to a greater propensity for crack path deviation, and
hence in rougher fracture surfaces, in the lightly aged structures (Fig. 21) (Suresh and others, 1984; Carter and others, 1984; Zaiken and Ritchie, 1985). This follows because underaged microstructures are hardened primarily by the shearing of small coherent precipitates resulting in heterogeneous deformation (i.e., planar slip), which promotes crystallographic crack paths (Hornbogen and Zum Gahr, 1976). The added benefit of such a deformation mode is that slip at the crack tip is occurring on fewer slip systems, thereby raising the degree of slip reversibility which lessens the crack tip damage per cycle (Garrett and Knott, 1975; Hornbogen and Zum Gahr, 1976; Antolovich and Campbell, 1982). Conversely, in more heavily aged systems where the mode of hardening becomes one of Orowan bypassing around larger semi-coherent or incoherent (non-shearable) precipitates, the resulting homogeneous deformation (i.e., wavy slip) generates a far more planar fracture surface due to the larger number of finer slip steps. This leads to less roughness-induced closure and less slip reversibility resulting in more crack tip damage per cycle.

As mentioned above, such microstructural factors, which provide increased fatigue crack growth resistance at near-threshold levels, actually may be detrimental to other mechanical properties. For example, the planar slip characteristics of coherent particle hardened microstructures, which are so potent in generating superior fatigue properties, can lead simultaneously to inferior fracture toughness from a greater tendency for strain localization. This is
particularly evident in aluminum-lithium alloys where the increased coherency between lithium-containing intermetallics and the matrix can result in exceptionally good fatigue crack propagation resistance, through enhanced crack path tortuosity, yet at the same time can produce extremely low toughness values (Balmuth and Schmidt, 1981).

SLOW CRACK GROWTH BY CREEP

At high homologous temperatures, sub-critical crack growth can occur additionally via creep mechanisms. Here, unlike crack extension by fatigue, a sharp crack tip can only be maintained if growth rates are sufficiently rapid for the crack to keep pace with the spread of damage in the creep process zone ahead of the tip (Sadananda, 1984). Such damage primarily takes place by the nucleation and growth of cavities and microcracks ahead of the main crack, in grain boundaries often weakened by oxidation or other modes of environmental attack. The growth of these cavities has been modelled in terms of combinations of grain boundary diffusion and matrix creep, aided by such mechanisms as grain boundary sliding, intergranular oxidation and impurity segregation to the boundary region (e.g., Dimelfi and Nix, 1977; Argon and others, 1980; Riedel and Rice, 1980; Bassani, 1984).

The kinetics of the growth of creep cracks are not solely a function of the rate of the nucleation, growth and coalescence of grain boundary cavities. Such accumulation of damage must be
balanced by the relative kinetics of stress relaxation, involving both creep deformation of material ahead of the crack tip and blunting at the tip (Argon and others, 1984). Thus, creep crack growth rates can be considered to be controlled by the competition between crack tip blunting and creep relaxation of the crack tip stress fields, which reduces stresses, counteracted by damage in the form of cavity and microcrack growth, which increases stresses (Sadananda, 1984).

Aside from the modelling of the local mechanisms of creep crack propagation, one of the major complexities of this topic has been the computation of the crack tip stress and deformation fields in a creeping material. The description of such fields is vital, not only for such microscopic modelling, but, more importantly from the engineering perspective, for the definition of characterizing parameters to correlate to crack extension rates. Much experimental work has been performed to date in attempts to find which parameter provides the best fit to creep crack growth rate \(\frac{da}{dt}\) data (e.g., Sadananda and Shaninian, 1981). The most notable of these parameters include \(K_I\), based on linear-elastic deformation (Eq. 1), \(J\), based on time-independent non-linear elastic deformation (Eq. 7), \(C^*\), based on time-dependent creep deformation (Eq. 11), and the net section or equivalent stress. From an experimental viewpoint, however, no one field parameter has been found to provide the ideal normalization of \(\frac{da}{dt}\) data, over an extended range of stresses, times and temperatures in different materials.
As noted above, rationalization of this problem has come from crack tip continuum mechanics analyses of creeping solids which define, with respect to time, the dominant crack tip fields. Specifically, this has initially involved finding asymptotic continuum mechanics solutions of the crack tip fields for each relevant mode of deformation, i.e., elastic, plastic, primary creep, and secondary creep, as a function of time. Then, since these fields all exist simultaneously in different regions ahead of the crack tip and can change with time due to stress relaxation and blunting, it is necessary to determine which of them describes, at a given instant in time, the stress and strain conditions over the region where the fracture events are occurring, i.e., which field, hence which characterizing parameter, dominates the creep zone ahead of the crack tip (Riedel and Rice, 1980; Bassani and McClintock, 1981; Riedel, 1984; Bassani, 1984).

In brittle materials, or simply after short times, the characterizing parameter generally is found to be the stress intensity factor $K_I$, where crack growth is sufficiently fast such that the damage is maintained within the linear elastic crack tip field. With more ductile materials, or at longer times, the singular plastic field, which is embedded or engulfs the elastic field, may provide a better description of behavior within the creep zone; in which case $J$ will be the dominant characterizing parameter. With further time, creep deformation can result in crack tip regions governed by other singular stress fields. These fields now can be
described by the rate-dependent form of the HRR singularity (Eq. 11), with $C^*$ as the time-dependent amplitude of the field and hence the potential characterizing parameter. Where primary creep dominates the strain rates in the vicinity of the crack tip, the crack tip stresses will be given by $C^*$ defined in terms of a primary creep constitutive law (analogous to Eq. 10). Similarly, if secondary creep rates dominate, $C^*$ is defined in identical fashion only utilizing the secondary creep constitutive law (Riedel, 1984; Bassani, 1984).

Since all these fields can exist simultaneously, finding the appropriate one which describes stress and deformation within the creep zone involves approximate field matching. Studies of this type by Riedel and Rice (1980) have shown that for times less than a characteristic time $t_c$, where the extent of the creep zone equals the $K_I$-field, given by:

$$t_c = \frac{K_I^2(1 - v^2)}{E(N + 1)C^*} \quad (36)$$

$K_I$ is likely to be the controlling parameter for any observable crack growth. If cracking occurs only after longer times, $J$ or $C^*$ are likely to apply.

Finally, it should be noted that $K_I$, $J$ and $C^*$ are all steady-state parameters which assume that stresses in the crack tip region have reached their steady-state level before creep crack growth commences (Sadananda, 1984). Since this situation often is not met
under experimental conditions, a large degree of scatter must be expected in much da/dt data, particularly during the initial stages of propagation.

SUMMARY AND CONCLUDING REMARKS

In this review, an attempt has been made to highlight major characteristics of slow crack growth, by such mechanisms as ductile fracture, fatigue and creep cracking, with emphasis on both the fracture mechanics description of macroscopic behavior and the mechanistic and microstructural aspects of microscopic behavior. In modern engineering lifetime prediction, the topic is of vital importance as damage-tolerant procedures invariably assess remaining life in a structure or component primarily on the basis of the rate of such sub-critical crack growth. The effect of environment in influencing such crack growth has not been explicitly treated in the present article although it must be appreciated that, in many instances, it can play a dominant role.

In terms of continuum descriptions of crack tip fields and the definition of characterizing parameters for crack extension, it is clear that substantial progress has been made in recent years with slow ductile crack growth in the non-linear elastic descriptions of crack extension, i.e., J-controlled growth, and in the asymptotic elastic-perfectly plastic solutions for non-stationary tensile cracks. Similarly, for crack growth in creeping materials at high homologous temperatures, it has become possible to define crack
growth characterizing parameters, as a function of time, for the sequence of dominant singular fields at the crack tip arising from time-independent elastic and plastic deformation and from time-dependent primary and secondary creep deformation. Furthermore, such advances in the macroscopic description of crack growth by these processes have been matched, in general, by an improved mechanistic understanding of the role of microstructure and of the local crack tip failure mechanisms involved.

In contrast to ductile and creep crack growth, the macroscopic and microscopic understanding of fatigue crack growth is far less advanced. Aside from the fact that the basic mechanism of cyclic crack advance involving striation formation still is not precisely understood, the "fundamental" characterizing parameter, \( \Delta K \), is based still on the superposition of monotonic, linear elastic, stationary fields (Rice, 1967). Not surprisingly, \( \Delta K \) has been shown to have problems in uniquely characterizing growth rates in many important situations, such as with small cracks (Gangloff, 1981; Gangloff and Ritchie, 1984; Lankford, 1984; Suresh and Ritchie, 1984a), multiaxial loading (de los Rios, 1984), and where significant crack closure is present (Suresh and Ritchie, 1984b). These situations constitute breakdowns in the fracture mechanics similitude concept whereby cracks of differing length, subjected to the same nominal "driving force," are presumed to extend at equal rates (Ritchie and Suresh, 1983): a concept which currently forms the basis scaling of laboratory data to predict component life.
From a microscopic viewpoint, however, the description of fatigue crack growth in the mechanistic terms of crack closure, and crack deviation, has provided an excellent rationalization of the majority of experimental observations. It also has been important in the development of microstructures with vastly improved resistance to crack extension, principally through the enhancement of closure forces. Improved interpretations of the effects of variable amplitude loading on fatigue crack growth have resulted from a consideration of these factors, and they may provide the critical link needed to unify the classical stress/strain-life and defect tolerant (long crack da/dN) analyses of fatigue.

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Fig. 1. Idealized constitutive behavior, of equivalent stress \( \bar{\sigma} \) as a function of equivalent plastic strain \( \bar{\varepsilon}_p \), for a) non-linear elastic material conforming to deformation plasticity theory, and b) incrementally-plastic material conforming to flow theory of plasticity.
Fig. 2. Schematic idealization of microscopic fracture criteria pertaining to i) critical stress-controlled model for cleavage fracture (RKR) and ii) critical stress-modified critical strain-controlled model for microvoid coalescence.
Fig. 3. Schematic representation of the near-tip conditions for a non-stationary crack relevant to the definition of $J$-controlled growth. (After Hutchinson and Paris, 1979)
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Fig. 5. Experimental data showing $J_R(\Delta a)$ resistance curves for several heats of A516 Grade 70 plain carbon steel plate ($\sigma_0 \sim 260$ MPa). Sulphide and oxide non-metallic inclusions have been controlled by both conventional techniques (CON) using vacuum degassing and calcium treatments (CaT). Note how modifying the inclusion distribution has a more significant effect on crack growth compared to crack initiation. (After Wilson, 1979)
Fig. 6. Comparison of local stresses $\sigma_{ij}$ ahead of the crack tip in plane strain as a function of angle $\theta$ for a) stationary crack based on Prandtl field, and b) non-stationary crack based on exact solution for $\nu = 0.3$ of the same field, which contains an elastic unloading sector. (After Drugan, Rice and Sham, 1982.) Note how the stress distribution is unchanged by the growing crack, except for $\theta > 110^\circ$. 

$\sigma_{yy} = (2 + \pi) k$

$\sigma_{zz} = (1 + \pi) k$

$\sigma_{xx} = \pi k$

$\sigma_{xy}$

--- Stationary crack
Prandtl Field

--- Exact Growing
Crack Solution,
$\nu = 0.3$

--- Elastic Sector
Fig. 7. Idealization of stable crack growth by microvoid coalescence showing a) blunted crack tip, b) crack growth to next inclusion based on constant CTOA (φ) or on critical CTOD (δ_p) distance (l_0^* \sim d_p) behind the crack tip, c) morphology of resulting fracture surface, and d) fractographic section (after Knott, 1983) through ductile crack growth via coalescence of voids in free-cutting mild steel (after Thompson and Ritchie, 1984).
Fig. 8. Schematic variation of fatigue crack growth rates \( \frac{da}{dN} \) with stress intensity range \( \Delta K \), showing primary regimes of growth rate mechanisms.
Fig. 9. Schematic variation of $\frac{da}{dN}$ with $\Delta K$ for short and long fatigue cracks, showing "anomalous" short crack behavior.
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Fig. 11. Schematic illustration of primary mechanisms of fatigue crack closure. (After Suresh and Ritchie, 1984a).
Near-Threshold: $r_y < d_g$
(Stage I, Modes II+I)

Higher Growth Rates: $r_y > d_g$
(Stage II, Mode I)

Fig. 12. Crack path profiles at near-threshold and at higher growth rates. (After Suresh and Ritchie, 1982)
Fig. 13. Illustration of roughness-induced crack closure showing asperity contact during fatigue of underaged X-7075 aluminum alloy. (After Schulte and others, 1980)
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Fig. 17. Fatigue crack growth behavior in Fe-2Si-0.1C ferritic-martensitic dual-phase steel. (After Dutta, Suresh and Ritchie, 1984)
Fig. 18. Fatigue crack growth profiles in Fe-2Si-0.1C dual-phase steel. (After Dutta, Suresh and Ritchie, 1984)
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Fig. 20. Fatigue crack growth in I/M 7050 aluminum alloy as a function of aging treatment. (After Zaiken and Ritchie, 1985)
Fig. 21. Fatigue crack profiles for a) underaged, b) peak aged and c) overaged 7150 aluminum alloy showing the more tortuous crack path in the coherent-particle hardened underaged microstructure. (After Zaiken and Ritchie, 1985)
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