Title
A METHOD FOR PRECISE CHARGE MEASUREMENTS OF RELATIVISTIC LIGHT NUCLEI, Z < 3, IN NUCLEAR TRACK EMULSION

Permalink
https://escholarship.org/uc/item/24w1f6jf

Authors
Bloomer, M.A.
Heckman, H.H.
Karant, Y.J.

Publication Date
1983
Submitted to Nuclear Instruments and Methods

A METHOD FOR PRECISE CHARGE MEASUREMENTS OF
RELATIVISTIC LIGHT NUCLEI, Z ≤ 3, IN NUCLEAR
TRACK EMULSION

M.A. Bloomer, H.H. Heckman, and Y.J. Karant

January 1983

TWO-WEEK LOAN COPY
This is a Library Circulating Copy which may be borrowed for two weeks. For a personal retention copy, call Tech. Info. Division, Ext. 6782.
DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.
A Method for Precise Charge Measurements of Relativistic Light Nuclei, 
$Z < 3$, in Nuclear Track Emulsion

M.A. Bloomer,* H.H. Heckman, and Y.J. Karant

Nuclear Science Division
Lawrence Berkeley Laboratory
University of California
Berkeley, CA 94720

ABSTRACT

A technique for precise charge measurement of particle tracks in nuclear emulsion is described. The method is based on measurements of the lacunarity, i.e., fractional transparency, and/or the fractional opacity in the linear track structure of ionization tracks observed in the developed emulsion. The method yields estimates of charge for relativistic $Z = 1$ and 2 nuclei to a precision of $\pm 0.05$ charge units, in agreement with Barkas' theoretical model of track structure. The technique is applicable up to $Z \approx 6$, but with diminished charge resolution with increasing charge above $Z = 3$. Systematics that affect the accuracy of the method are noted and discussed.

*Based in part on Senior Thesis, Undergraduate Honors Program, Department of Physics, University of California, Berkeley, California

This work was supported by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the U.S. Department of Energy under Contract DE-AC03-76SF0098.
I. Introduction

Nuclear research emulsions are particle detectors composed of silver halide crystals immersed in a gelatin matrix consisting mostly of carbon, hydrogen, nitrogen, and oxygen, with other elements in smaller quantities [1]. The passage of charged particles through the photosensitive halide crystals renders them developable under the action of a chemical reducing agent. The result is a trail of opaque silver "grains" that faithfully record the trajectories and interactions of particles as they traverse the emulsion. Nuclear emulsions have been used extensively in both accelerator and cosmic-ray research because of their compactness of size, acceptance angle, high spatial resolution, and large range of ionization sensitivity. They are ideal for the detection and exploration of completely new particle and nuclear behavior [1]. A contemporary example, and one we shall specifically consider in this report, is the study of nucleus-nucleus collisions of relativistic heavy ions (RHI) and their resulting fragmentation.

Figure 1 is a photomicrograph that shows a typical example of RHI fragmentation in emulsion. A fully stripped $^{56}$Fe nucleus with kinetic energy of 1.88A GeV enters the emulsion and after travelling a few centimeters interacts strongly with an emulsion nucleus. The $^{56}$Fe "primary" nucleus fragments into corresponding "secondary" projectile fragments, which, in turn, will propagate through the emulsion, at near-beam velocity [2], until they also interact with emulsion nuclei, giving rise to "tertiary" fragments, or until they exit the stack of emulsion pellicles. The projectile fragments are emitted in a narrow forward cone corresponding to the low transverse momenta distributions typical of these interactions. The heavily ionizing "black"
tracks emitted almost isotropically from the vertex are the low-energy evaporation products of the target nucleus.

An essential quantity to be determined by measurement in the study of these nuclear interactions in emulsions is the nuclear charge of each of the projectile fragments. Toward this end we have developed a technique that enables one to obtain a precision of 0.05 charge units for charge measurements of $Z = 1$ and $Z = 2$ nuclei at relativistic velocities. The technique can also be applied to nuclei of charge $3 \leq Z \leq 6$, but with decreasing precision as $Z$ increases.

One of the main impetuses to develop a method of precise charge measurement for projectile fragments stems from the observations of anomalously short mean free paths of projectile fragments produced in relativistic heavy-ion collisions [3-13], and the speculation that the enhancement of the interaction cross sections of these fragments might imply the existence of fractionally charged nuclei [14].

The method we shall describe for estimating the charge of high-velocity particles traversing emulsion uses the linear track structure of their ionization tracks as observed in the developed emulsion. This method thus complements a variety of techniques for charge measurements that extract similar information from an ionization track [1]. The method is not applicable to tracks of particles whose rates of ionization are so high as to produce completely saturated, i.e., opaque, tracks. In this latter case, methods of charge analysis commonly used are those that measure $\delta$-ray densities and track widths (see ref. 1 and references therein).
II. Theory of Grain Density and Linear Track Structure

To understand how the charge of a particle can be determined from its track structure, we review how a track is formed in nuclear emulsion. Distributed throughout the volume of the gelatin matrix are silver halide crystals, approximately spherical in shape, with mean diameters that depend on the type of emulsion. In this work we have used Ilford G-5 emulsion with a mean AgBr crystal diameter <D> = 0.28 microns [1]. A charged particle that traverses a halide crystal deposits energy in the crystal by ionization. These excited halide crystals are converted to metallic silver under a process of chemical reduction—the remaining AgBr crystals are then removed and the opaque silver grains form the structure of the track. The probability that a given crystal will develop depends on 1) the emulsion sensitivity, 2) the total energy deposited in the crystal, 3) where the particle traversed the crystal, and 4) the possible contribution of "secondary" ionization from delta rays, which are electrons knocked free from nearby atoms by the primary ionizing particle. This last effect results in "secondary" grains, grains produced by ionizing delta rays and not by the direct intrusion of the ionizing particle. The term primary grain density refers to the density of grains that results from the ionization of crystals actually traversed by the particle, the latter termed the "primary ionization". The energy loss attributable to primary ionization is called the "restricted energy loss".

For nuclei up to Z ≈ 6 the structure of ionization tracks is linear, composed of blobs, i.e., clusters of unresolved grains, gaps, and occasional high-energy delta rays as illustrated in fig. 2. For a given sensitivity, the primary grain density, g, is assumed to be proportional to the restricted energy-loss rate, \( \Phi \),

\[ g = K \Phi \]  

with K = sensitivity factor. (1)
The restricted energy loss can be calculated from the differential energy transfer cross section, \( \frac{d\sigma}{dw} \), via

\[
\varphi = n_e \int_{w_{\text{min}}}^{w'} w \frac{d\sigma}{dw} dw ,
\]

(2)

where \( n_e \) = electron density

\( w' \) = the upper limit of delta-ray energies, \( w \), responsible for producing primary grains, \( \sim 2-5 \) keV.

Under the approximation that delta rays of energy \( w > w' \) do not contribute to the primary ionization, eq. (2) is taken to be exact [1]. In general, the restricted energy-loss rate has the form

\[
\varphi = \frac{Z^2}{\beta^2} f(\beta) ,
\]

(3a)

where \( \beta c \) = particle velocity, \( Z e \) = particle charge, and \( \varphi \) varies slowly with \( \beta \) for regions in the minimum ionization valley of the \( \varphi \) vs \( \gamma - 1 \) graph, fig. 3.

A semi-empirical calculation of \( f(\beta) \) yields the form [1]

\[
f(\beta) = 2\pi r_0^2 mc^2 n_e \left[ \ln \left( \frac{2mc^2}{I^2} \beta^2 \gamma^2 w' \right) - 2\beta^2 - 2C \right]
\]

(3b)

where \( I \) = mean ionization, or excitation, potential

\( m \) = electron rest mass

\( C \) = correction term

\( r_0 \) = classical electron radius

\( n_e \) = density of electrons in stopping material

\( \gamma = (1 - \beta^2)^{-1/2} \)
In practice the actual number of grains in a track segment is not amenable to direct measurement. O'Ceallaigh [15] made the key observation that the gap length distribution is exponential.

\[ H(\geq t) = B \exp(-g't) \]  

(4)

where \( H(\geq t) \) represents the density of gaps greater than or equal to \( t \), and \( g' \) is the slope of the exponential distribution, known as the ionization parameter. Fowler and Perkins [16] confirmed the form of the distribution for a very wide range of ionization parameters and for all gap lengths. They furthermore determined that \( B \), the blob density, or, equivalently, the gap density, is identical to \( H(\geq 0) \) and is governed by

\[ B = g' \exp(-g'a) \]  

(5)

where \( a \) is the mean diameter of a developed grain. It was Barkas [1, 17] who proved that the ionization parameter, \( g' \), in the exponential distribution of gaps is, in fact, the actual grain density, i.e., \( g \equiv g' \).

Some important results follow as a consequence. If we define the lacunarity, \( L \), to be the linear fraction of a track made up of gaps, then

\[ L = -\int_0^\infty \frac{dH}{d\lambda} \, d\lambda = \exp(-ga) \]  

(6)

From Eqs. (5) and (6), it follows that measurements of both \( B \) and \( L \) over a given track segment can be used to estimate the grain density,

\[ g = \frac{B}{L} \]  

(7)

and the mean grain diameter,

\[ a = -\frac{L}{B} \ln L \]  

(8)

As a corollary, the opacity, \( \phi \), that fraction of track made up of blobs, is

\[ \phi = 1 - L. \]
From the above relations, the dependence of $L$ on $\alpha$, $\beta$, $K$, and $Z$ is

$$-\ln L = \left[ \frac{aKf(\beta)}{\beta^2} \right] Z^2.$$  \hspace{1cm} (9)

For a given development of the emulsion $\alpha$ is a constant; if we deal with particles of sufficiently high energy, $\beta$ is also a constant. In this case, the charge of an ionizing particle is simply related to the $L$ of its ionization track by

$$Z = k_0 \sqrt{-\ln L}$$  \hspace{1cm} (10)

where $k_0$ is a proportionality constant that is most conveniently determined empirically. For convenience we define the quantity proportional to charge as $p$,

$$p = \sqrt{-\ln L}$$  \hspace{1cm} (11a)

or $$p = \sqrt{-\ln (1 - \phi)}$$  \hspace{1cm} (11b)

We now have two ionization parameters, $L$ and $\phi$, which are operationally well defined, easy to measure, and related to charge by the simple relation $Z = k_0 \phi$.

The precision, $\delta Z$, with which the charge $Z$ of an ionizing particle can be determined from its ionization track is, from eq. (10)

$$\delta Z = \frac{k_0^2 \delta L}{2Z L},$$  \hspace{1cm} (12)

with

$$\frac{\delta L}{L} = \frac{\sigma_L}{L} N^{-1/2},$$

where $\sigma_L^2$ is the variance of $L$, and $N$ is the number of cells, each of length $S$, over which $L$ is measured. Barkas [1] gives a theoretical upper limit to the quantity $\delta L/L$ for a model based on completely random spacings between individual grains in a blob. It is

$$\frac{\delta L}{L} = \left[ \frac{2a}{\Lambda} \left( - \frac{1-L}{L \ln L} - \gamma \right) \right]^{1/2}.$$  \hspace{1cm} (13)
where $a$ is the mean diameter in microns of the developed grains, and $\Lambda = NS$, the total path length (in microns) over which $L$ is measured.

The G-5 emulsion used in this work has a mean grain diameter $a = 0.62 \mu m$, evaluated via eq. (8). Given that the measured lacunarity of relativistic $Z = 2$ particles is $L \approx 0.70$, i.e., $k_0 \approx 3.1$, we tabulate in table 1 the theoretical estimates for the path lengths $\Lambda$ over which $L$ or $\phi$ measurements must be made to attain a precision of $\delta Z = 0.05$ charge units for charges $1 \leq Z \leq 8$.

The rather surprising result of these calculations is the prediction that, for $Z \leq 4$, charge measurements can be made to an accuracy of $\delta Z \approx 0.05$ charge units by lacunarity (or opacity) measurements in $\leq 1$ mm of track length. Such measurements can be performed by an observer in about 10-15 min per track. For charges $Z > 5$, the path lengths $\Lambda$ required to obtain $\delta Z \approx 0.05$ charge units become excessively long because of the rapidly diminishing values of $L$. The method clearly becomes impractical for charges $Z > 6$ in G-5 emulsion.

It should be pointed out that for $Z > 6$ the values of $L$ are calculated to be a few percent and less. Not taken into account in the calculated values of $L$, hence $\Lambda$, given in table 1, are the contributions of background gaps caused by submicron inhomogeneities in the gelatin-AgBr matrix. As we shall see, these inhomogeneities introduce systematic deviations in $L$, hence $Z$, that ultimately degrade the charge resolution altogether, and thereby the applicability of this technique, for values of $Z > 6$. 
III. Operational Method of Charge Measurement

In this experiment stacks of G-5 emulsion pellicles were exposed at the Bevalac to beam nuclei at an energy of $\sim 2A$ GeV with the entering nuclei travelling parallel to the emulsion surfaces. The processed emulsion pellicles are mounted on glass and scanned under optical microscopes. All charge measurements were performed with an oil objective at high magnification (2500x). We return to our illustration of linear track structure, fig. 2, where, in fig. 2b we have superimposed a replica of a 100-unit reticle on the track as it appears in the field of view of a microscope. We define the sum of all 100 reticle units as a cell. A measurement of charge, then, consists of measuring the opacity or lacunarity for different cells along the particle track until the charge converges to a desired precision, which in most of our charge measurements was $\delta Z = 0.045$ charge units. Referring to fig. 2b,

$$L = \sum_i \xi_i / S$$

$$\phi = \sum_i \phi_i / S$$

(14)

$S =$ length of cell

To measure $L$ or $\phi$ for a given cell, each gap or blob was compared to the superimposed reticle unit, and values of $\xi_i$ or $\phi_i$ were estimated to the nearest half-integer reticle unit, summed over the entire cell length, and finally rounded to the nearest integer reticle unit. This operation is most simply done by counting (e.g. with a hand-held counter) the number of empty reticle units, giving $L$ directly in percent. Under 2500x magnification the cell lengths of the reticles were, by design, 62.2 $\mu$m, so the individual reticle units were 0.62 $\mu$m in length, which, not coincidentally, is the value of the mean diameter of the developed grains, $\alpha$. This congruence of lengths is convenient because it reduces systematic overestimation or underestimation of $L$ (or $\phi$).
In this work the recording and evaluation of the lacunarity (opacity) of particle tracks by the observer was carried out with an HP-41C calculator that functioned as the gap counter and was programmed to give an on-line analysis of the \( L_i(\phi_i) \) data as the measurement proceeded. The observer, on counting the number of reticle units that were empty (full) in the 100-unit cell, entered the values for \( L_i(\phi_i) \) for each cell. After a minimum of 16 cells were counted, the mean value \( \bar{Z} \) of \( Z \) and its standard deviation, \( S_\bar{Z} \), was computed (given a predetermined value of \( k_0 \)) from the measured distribution of \( L_i(\phi_i) \). When the condition \( S_\bar{Z} < 0.045 \) charge units was satisfied, the numerical values of \( \bar{Z} \), \( S_\bar{Z} \) (S.D.), \( \bar{L} \), \( S_{\bar{L}} \) (S.D.), and the differential distribution of \( L_i(\phi_i) \) of the track segment were printed.

For \( Z = 2 \) particles, the requirement \( S_\bar{Z} < 0.045 \) charge units was met by the time 16 cells were measured in about 70% of the measurements. The average number of cells to attain \( S_\bar{Z} < 0.045 \) charge units was 17. By a Monte Carlo calculation we found that these conditions and observations are met when the standard deviation of the opacity distribution, assumed to be Gaussian, is \( S_{\phi} \) (population) = 0.043. With this value of \( S_{\phi} \) the Monte Carlo-simulated value of \( S_\bar{Z} \) one obtains by measuring the charge of \( Z = 2 \) particles for a minimum of 16 cells, under the condition that \( S_{\bar{Z}} \) (obs) \( \leq \) 0.045 charge units, is \( S_\bar{Z} = 0.40 \) charge units.

IV. Measurements and Results

Our initial applications of this method of charge estimation were to perform i) measurements on tracks of relativistic particles of known charge for purposes of calibration and ii) measurements on a sample of \( Z = 2 \) projectile fragments produced by collisions of 1.9A GeV \( ^{56}\text{Fe} \) with emulsion nuclei. The calibration measurements were made on primary beams of \( ^3\text{He} \) at
1.9 A GeV and $^6$Li and $^{12}$C, both at 2.1 A GeV. At these energies the particle velocities are $v = 0.944 \ (\gamma - 1 = 2.013)$ and 0.952 $(\gamma - 1 = 2.267)$, respectively, and the restricted energy-loss rates are near the point of minimum ionization, fig. 3. The velocities of all the beam nuclei were the same within a fraction of a percent and hence possessed a constant rate of ionization over the path lengths of measurement.

All the exposures to the above beams were made within a 8-hr period using Ilford G-5 emulsions from a single production batch number. The pellicles of emulsion were printed with a 1 mm$^2$ grid, mounted on glass, and processed together to ensure the uniformity of development of all emulsions. The processed emulsions exhibited remarkably uniform sensitivity with respect to both lateral and depth gradients. It was found that the charge measurements made in this experiment, in any practical application, would have not required depth or plate-to-plate corrections attributable to differences in development nor the use of individual calibration constants $(k_0)$ of the three observers who contributed data to this work. Of course, to achieve maximum resolution in the charge measurements, such corrections must be invoked. We shall consider in Sec. IV.b a procedure for incorporating corrections caused by depth gradients in the emulsion and temporal variations in the calibration constants $k_0$ and the improvement in the charge resolution gained therefrom.

a) Charge Calibration

The empirical opacity distribution obtained by observer 1 for $^3$He tracks is shown in fig. 4, where the characteristic widths are indicated on both the differential, $f(\phi)$, and integral, $P(\leq \phi)$, distributions of the opacity $\phi$. The observed distribution of the opacity is approximately Gaussian in shape and hence is well described by its mean and standard deviation.
The results of charge measurements on beams of $^3$He, $^6$Li, and $^{12}$C are displayed in fig. 5. These calibration data were augmented by measurements of $Z = 1$ projectile fragments that were produced in collisions between the beam and target nuclei of the emulsion. The charge-calibration measurements were carried out by observer 1, who employed opacity measurements for $Z = 1$, 2, and 3 particles and lacunarity measurements for $Z = 3$ and 6. The data for each charge were corrected for apparent gradients in the measured charges, i.e., $\rho$, correlated with depth in the emulsions, attributable to both processing and optical effects. The corrections to the data were small and had negligible effect on the shape of the $Z_{\text{obs}}$ versus $Z_{\text{beam}}$ data. In all cases, the statistical errors of the measurements are less than the size of the data points shown. The value of $k_0 (=3.191)$ used to calculate the respective charges from the measured values of $\rho_Z$ was that which normalized $\rho_2$ to $Z = 2.00$.

The most apparent feature of the charge-calibration data is that charge measurements via the lacunarity/opacity method vary linearly with $Z_{\text{beam}}$ in the range $1 \leq Z_{\text{beam}} \leq 3$. Beyond $Z = 3$ the lacunarity/opacity, i.e., grain density, begins to enter a plateau region where the measured values of $\rho$, hence $Z$, become increasingly insensitive to increasing rates of ionization, i.e., $Z_{\text{beam}}$. Included in fig. 5 are curves deduced from the work of Fowler and Perkins, presented in fig. 3.21 of ref. 18, who examined the dependence of the grain density $g$, as determined from the reciprocal of the mean-gap-length, on the charge of primary cosmic-ray nuclei. The two curves correspond to measurements in G-5 emulsions of different sensitivities, and both are normalized to $Z_{\text{obs}} = 2$ at $Z = 2$. Clearly, the data represented by these curves also exhibit a breakdown in the linearity between $Z$ and $\sqrt{g}$ for $Z \geq 3$. 
Thus, although the restricted energy-loss rate is calculated to increase as $Z^2$, this increase is not revealed in the linear track structure of an ionization track for $Z > 3$. Operationally, this "saturation of charge" results from what may be termed a "background lacunarity" whereby submicron regions of the emulsion are insensitive (e.g., globules of gelatin that lack halide crystals) and thus never become opaque. However, because of the $Z^2$-dependence of the number of $\delta$ rays produced along a track, it is evident that any linear track structure will eventually be masked completely by the halo of silver grains of the $\delta$-ray tracks enveloping the core of the primary track. It is interesting to note that whereas the track of a 1.9A GeV $^{56}$Fe should have $L \approx 10^{-29}$, assuming $Z = 3.19\sqrt{-\ln L}$, one actually observes $L \sim 10^{-3}$. This demonstrates that the $\delta$-ray halo of an ion having an ionization rate of 676 (dE/dx)$_{\text{min}}$ is still insufficient to mask the background gap structure of the core of the track.

b) $Z = 2$ Projectile Fragments of $^{56}$Fe

As a test of the theory and technique of charge measurement by the lacunarity method, we have selected for examination a sample of 683 secondary He nuclei emitted from $^{56}$Fe interactions within the fragmentation cone, $\phi \geq 6^\circ$. Under the assumption that the $Z = 2$ projectile fragments are emitted at the same velocity as the incident Fe projectile [2], and by requiring that all the interactions of the incident 1.9A GeV $^{56}$Fe nuclei occur within 6 cm of the entrance surface of the emulsion stack, the velocities of the $Z = 2$ fragments were limited to the interval $0.94 < \beta < 0.92$. The standard deviation of $Z_{\text{meas}}$ caused by this dispersion in velocity amounts to only $\sigma \approx 0.004$ charge units at $Z = 2$, a quantity that contributes negligibly to the intrinsic charge resolution expected and obtained in this experiment.
The data on charge measurements of $Z = 2$ projectile fragments were obtained by three observers from seven different emulsion pellicles. We specifically have examined the systematics of these charge measurements as they pertain to the differences between observers, the depth gradients in the sensitivities of the emulsions, and time gradients in the charge measurements by the observers. We found that the corrections to the measured values of $\rho$ attributable to depth and time gradients were satisfactorily accounted for by assuming, for each plate, the linear relationship

$$\rho = A\xi + B(t - t_0) + \rho_0$$

where $\xi = \text{ratio of the distance of the } Z = 2 \text{ track from the glass surface relative to the thickness of the processed emulsion}$, $t = \text{time of measurement}$, $\rho_0 = \text{constant, the value of } \rho \text{ evaluated for } \xi = 0 \text{ and } t = t_0$. In the case where time gradients seemed negligible, $\rho = \rho_0 + A\xi$ was satisfactory.

The results of the charge measurements on the He projectile fragments obtained by measurements of the opacities of their ionization tracks are summarized in table 2. Identified as to observer and plate number are the tabulated values of $k_0(\text{observed}) = \rho/Z$ where no corrections have been applied to $\rho$, with $Z = 2.00$, and the standard deviation of the charge distributions before and after corrections. Although systematic differences between the observers and plates are evident, the fractional standard deviation of $k_0(\text{observed})$ about its mean is only $\pm 1\%$, from which we conclude that the charge of relativistic $Z = 2$ nuclei can be measured to a systematic limited precision of $S_Z \approx 0.02$ charge units. Such values of $S_Z$, obtainable without regard to observer and/or plate in this experiment,
contribute by quadrature only ~20% to the statistical value of $S_Z \leq 0.045$ we have demanded for each individual charge measurement. The observed values of $S_Z$ are seen to vary from 0.042 to 0.057, with a mean of 0.049. The measurements by observer 1 exhibit the presence of both depth and time gradients, whereas these gradients were too small to detect in the data of observers 2 and 3. If the datum from plate 21 is excluded, the mean value $S_Z$ (corrected) becomes $0.047 \pm 0.004$, suggesting that the data from this plate possesses additional systematic errors, e.g., emulsion defects, not apparent in other emulsion plates. If we take $S_Z = 0.047$ as the best estimate of our precision for charge measurement of $Z = 2$ fragments, then the difference, in quadrature, between $S_Z$ and the required (corrected) convergent value of 0.040 is $\pm 0.026$ charge units, a value we take to be indicative of the overall systematic errors in our charge measurements.

Figure 6 presents the corrected charge distribution when the value of the mean charge for each plate is normalized to $Z = 2.00$. The standard deviation $S_Z = 0.049$ is indicated, as are the values of $\bar{Z}$ evaluated from the summed data of each observer, where $k_0 = 3.115$ is the observer-averaged value of $k_0$ corrected for depth gradients only (see table 2).

V. Summary

We have shown that the application of the conventional technique of lacunarity/opacity measurements, coupled with the theoretical treatment by Barkas, on tracks of charged particles in nuclear emulsion leads to precise estimates of charge for $1 \leq Z \leq 3$, with $S_Z = 0.05$ charge units being obtained for $Z = 2$ nuclei. Because of the appearance of submicron regions of insensitivity in the emulsion, the "background of lacunarity" therefrom limits
this technique of charge measurement to tracks that have lacunarities $L \gtrsim 0.25$ (i.e., to about $Z < 4$ in this experiment) when charge resolutions of $S_Z \gtrsim 0.1$ are required. For less critical requirements e.g., $S_Z = 0.5$, the technique can be, and is being, applied to tracks having charges up to $Z = 6$.

The measurements of charge for $Z = 2$ nuclei, after corrections for depth and/or time gradients, have shown that the precision of charge estimation by the lacunarity/opacity technique is well accounted for by Barkas' (model-dependent) expression for $\delta L/L$, eq. (13). (See ref. 1, eq. 9.7.20.) One of the basic reasons that precise charge measurements of light nuclei can be obtained in track lengths $\sim 1-3$ mm by the lacunarity/opacity technique stems from the fact that the $L$ parameter, when compared with parameters of blob/gap densities and mean gap length, has the smallest statistical variance in the determination of the grain density $g$. The variance of $g$ based on measurements of $L$ is comparable to that obtained by the maximum likelihood method, which incorporates all information on the linear track structure of a track (ref. 19, fig. 2.2.2).

Although the measurements of lacunarity/opacity in this work were carried out visually, the measurements of the integral gap lengths of a track segment are clearly amenable to conventional optical-electronic techniques. The incorporation of such techniques for charge measurement coincident with track following by the observer is clearly the next and essential advance in the development of this method of charge measurement in emulsion.

This work was supported by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.
References


Table 1.
Calculated lacunarity $L$ and path length $\Lambda$ over which $L$ must be measured to obtain a precision of $\delta Z = 0.05$ charge units. $k_0$ (eq. 8) is taken to be 3.10.

<table>
<thead>
<tr>
<th>$z$</th>
<th>$L$</th>
<th>$\Lambda_{\mu m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.901</td>
<td>617</td>
</tr>
<tr>
<td>2</td>
<td>0.660</td>
<td>689</td>
</tr>
<tr>
<td>3</td>
<td>0.392</td>
<td>835</td>
</tr>
<tr>
<td>4</td>
<td>0.189</td>
<td>1 126</td>
</tr>
<tr>
<td>5</td>
<td>0.0742</td>
<td>1 740</td>
</tr>
<tr>
<td>6</td>
<td>0.0236</td>
<td>3 194</td>
</tr>
<tr>
<td>7</td>
<td>0.0061</td>
<td>7 230</td>
</tr>
<tr>
<td>8</td>
<td>0.0013</td>
<td>20 760</td>
</tr>
</tbody>
</table>
Table 2.
Results of charge measurements on Z = 2 projectile fragments. The quantity $k_0 = \bar{\nu}/Z(=2)$, eq. 10. $S_Z$ are the standard deviations of the observed and corrected values of the measured charge for Z = 2 projectile fragments.

<table>
<thead>
<tr>
<th>Observer</th>
<th>Plate</th>
<th>$k_0$ (observed)</th>
<th>$S_Z$ (observed)</th>
<th>$S_Z$ (corrected for time/depth gradients)</th>
<th>$k_0$ (corrected for depth gradient) only</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>33</td>
<td>3.106 ± 0.008</td>
<td>0.057</td>
<td>0.053</td>
<td>3.106 ± 0.004</td>
</tr>
<tr>
<td></td>
<td>27</td>
<td>3.165 ± 0.008</td>
<td>0.054</td>
<td>0.048</td>
<td></td>
</tr>
<tr>
<td></td>
<td>26</td>
<td>3.154 ± 0.009</td>
<td>0.052</td>
<td>0.050</td>
<td></td>
</tr>
<tr>
<td></td>
<td>21</td>
<td>3.190 ± 0.010</td>
<td>0.058</td>
<td>0.057</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>19</td>
<td>3.125 ± 0.006</td>
<td>0.047</td>
<td>0.047</td>
<td>3.115 ± 0.006</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
<td>3.155 ± 0.009</td>
<td>0.042</td>
<td>0.042</td>
<td>3.125 ± 0.006</td>
</tr>
<tr>
<td></td>
<td>17</td>
<td>3.110 ± 0.008</td>
<td>0.044</td>
<td>0.043</td>
<td></td>
</tr>
<tr>
<td>ave</td>
<td></td>
<td>3.137 ± 0.031 (SD)</td>
<td>0.052</td>
<td>0.049</td>
<td>3.115</td>
</tr>
</tbody>
</table>
Figure Captions

Fig. 1. Photomicrograph of an interaction of a 1.88A GeV $^{56}$Fe nucleus observed in nuclear emulsion. The incident Fe, on colliding with an Ag(Br) nucleus, fragments into a "jet" of light projectile-fragment nuclei. The large-angle, highly ionizing tracks are low-energy nuclei emitted from the target.

Fig. 2. a) An illustration of the structure of an ionization track as observed in emulsion. Indicated are blobs (a group of unresolved grains), gaps (the distances between blobs), and $\delta$ rays (energetic electrons).

b) Fig. 2a superimposed with a 100-unit reticle as it appears in the microscope image. All individual gaps ($\ell_i$) and blobs ($\phi_j$) are estimated to the nearest 1/2-reticle unit. The sums $\sum \ell_i$ and $\sum \phi_i$ over the length of the reticle are measures of the lacunarity $L$ and opacity $\phi$, respectively.

Fig. 3. Restricted energy loss vs $\gamma - 1$ for singly charged particles [1]. The range of values of the velocity $\beta$ ($c = 1$) for beam and fragment nuclei examined in this experiment are indicated.

Fig. 4. The observed distributions of opacity $\phi$ (observer 1) for $^3$He tracks at 1.9A GeV, a) frequency distribution with mean and standard deviations indicated, b) integral distribution with median $[P(\leq \phi) = 0.5]$ and $\pm 1\sigma$ values (at $P = 0.500 \pm 0.3413$) indicated.

Fig. 5. Measured charge versus true charge of beam nuclei, $Z = 2, 3,$ and 6 at 2A GeV. $Z = 1$ data were obtained using beam fragments. Statistical errors are smaller than the size of the data points. The curves shown are from ref. 18 (see Fig. 3.21), the dashed curve corresponding to emulsion less sensitive than that represented by the solid curve. Data and curves are normalized at $Z = 2$. 
Fig. 6. Distribution of measured charges for $Z = 2$ projectile fragments from $^{56}\text{Fe}$ at 1.88A GeV. The data for each plate are corrected for depth/time gradients and normalized to $Z = 2.00$. Also shown are the values of $\bar{Z}$ obtained by each observer, plate averaged. The values of $k_0$ (corrected for depth only) are given in table 2.
Fig. 4
Fig. 6

\[ S_Z = 0.049 \]
This report was done with support from the Department of Energy. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory, or the Department of Energy.

Reference to a company or product name does not imply approval or recommendation of the product by the University of California or the U.S. Department of Energy to the exclusion of others that may be suitable.