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Simultaneous Offers and
The Inefficiency of Bargaining:
A Two-Period Example

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Key words: forward induction, simultaneous bargaining

Abstract

It is shown that the Pareto optimal outcomes in a two period simultaneous move bargaining model violate forwards induction rationality when the players are sufficiently patient. This bargaining model describes a situation where a principal is represented by an agent whose flexibility is restricted. Hence, a bargaining process with such agents can create costly delays. The result also provides another example of the power of forwards induction and stability.

JEL Classification: 026
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1. Introduction

In the recent literature on noncooperative bargaining, starting from Rubinstein’s [23] seminal paper, players make alternating offers in sequence, until agreement is reached about the division of a “pie,” which is discounted over time. Rubinstein showed that in the unique subgame perfect equilibrium of the perfect information game the first player’s offer is accepted by the second player. More recent papers [1, 4, 6, 10-13, 24, 26, 29] have focussed on introducing incomplete information to the model. These papers have imposed various refinements and have then shown that the equilibrium involves delay,¹ which acts as a screening device.

This paper analyzes a bargaining game with imperfect, but complete, information and with simultaneous, rather than alternating, moves. The model is a twice repeated Nash [20] noncooperative bargaining game. Both players tender offers simultaneously, and if their offers agree then the pie is allocated accordingly. If no agreement is reached they try again, whereupon if agreement is obtained the (discounted) pie is split, while if once again no agreement is attained the game ends without any division, or trade, occurring.

This model is obviously very specialized. It does, however, incorporate one feature of bargaining which the model of alternating moves fails to capture. This feature is that the bargaining parties may come to the table with predetermined expectations and offers, and will not be very influenced (in the course of one meeting) by what the opponent says. This may be because incorporating the information revealed by the opponent takes more time than is spent in one meeting, for example if the bargaining party must confer with its principal if (significant) deviations from its offer are to be accepted.²

A difficulty with the model of this paper is the extremely large set of subgame perfect (Selten [25]) equilibrium. Even after imposing the fairly strong notion of forwards induction (Kohlberg and Mertens [17]), as captured in stability
(op cit), the equilibrium set remains large. However, qualitative conclusions are obtained. The main objective in this paper is to examine the set of Pareto efficient outcomes which are supported as stable outcomes, and in particular how this set changes with the discount rate. The main result is that as the players become more patient, less Pareto efficient outcomes remain, until a point beyond which only inefficient outcomes remain.\(^3\) Thus, patience causes delay.\(^4\) This also suggests that delegating bargaining to agents who must confer with the principal can cause inefficiencies.\(^5\) This necessity of inefficiency seems to me an important characteristic of noncooperative bargaining. In particular, this form of delay does not act as a device to screen among exogenously determined types, but as a method of indicating (endogenous) "stubbornness" (cf. [2]).

This note is motivated above as clarifying certain aspects of bargaining situations. Another motivation is that this work contributes to the understanding of forwards induction and stability, and as such adds to [2, 12, 21, and 28] which have examined the role of forwards induction and stability in selecting among equilibria in other games of imperfect information. Arguments based on forwards induction restrict beliefs about an opponent's "type" according to the possible gains to that "type" from deviating. Incomplete information bargaining models use such arguments [1, p. 349; 6, p. 202] where a player's "type" is a characteristic of his/her utility function (as in [15], and adverse selection models). Here the same intuition applies, where "type" refers to a strategy choice (as in moral hazard models). Thus, to the extent that the results here seem unreasonable they may suggest that further work is needed to justify the use of forwards induction in other models (e.g. [2]).
2. The Model and Results

The (noncooperative) bargaining model of Nash [19] is specified by a utility possibility frontier, denoted by a non increasing function \( f: [0,1] \rightarrow [0,1] \), and a disagreement point \( d = (0, 0) \). (The strategies and payoffs are determined in such a way that (as long as utilities are bounded) \( f \) and \( d \) can be normalized to have these values by taking positive affine transformations.) The (first period) strategies of players 1 and 2 are to specify a utility allocation in \([0, 1]\) for player 1, denoted by \( s_1 \) and \( s_2 \) respectively. If \( s_1 \leq s_2 \) then player 1 receives \( s_1 \) and 2 receives \( f(s_2) \), while if \( s_1 > s_2 \) they both receive 0.

In this paper a two stage discrete version of this model is examined. Players are only allowed to suggest amounts in \( S = \{m/n : m = 1, \ldots, n-1\} \), for some \( n \). Furthermore, when \( s_1 > s_2 \) the players proceed to a second round. The second round strategies, denoted \( t_1 \) and \( t_2 \) respectively, are thus functions in \( T = \{f: S \rightarrow S\} \). A fully specified strategy of a player is then a pair in \( S \times T \). When the players choose strategies \( (s_1, t_1) \) and \( (s_2, t_2) \) respectively their payoffs are as follows. If in the first round \( s_1 \leq s_2 \), then (as in the one stage game) they receive \( s_1 \) and \( f(s_2) \). Otherwise \( s_2 > s_1 \), and if \( t_1(s_2) \leq t_2(s_1) \) (i.e. if 1's second period suggestion, after observing \( s_2 \) in the first period, is compatible with 2's second period suggestion after observing \( s_1 \) in the first period) they receive \( \delta_1 t_1(s_2) \) and \( \delta_2 t_2(s_1) \) respectively, where \( \delta_1 \) is player i's discount rate. If their second period suggestions are not compatible (i.e. \( t_1(s_2) > t_2(s_1) \)) they both receive the disagreement allocation of 0. Henceforth discount rates in \( \{\delta : s = s'/\delta \quad \text{for some} \quad s, s' \in S\} \) are ruled out. This avoids (knife-edge) indifference between a first and a second period allocation, and thus serves the same purpose as tie breaking assumptions such as (A1) in [1] and (b-3) in [24]. The main result of this paper follows.
In order to state the main result of this paper we review the definition of a stable outcome. If a set of Nash equilibrium is stable [17, p.1027] and each of these Nash equilibria induces the same probability distribution on endpoints of the game, then this distribution is called a stable outcome [5, pp. 189-192].

PROPOSITION 1: The set of Pareto efficient and stable outcomes is the set

\[ K = \{ (s_1, f(s_1)) : s_1 > \delta_1 \text{ and } f(s_1) > \delta_2 \}. \]

Furthermore, even if \( K \) is empty there exists a stable outcome.

Proof: The proof is an application of the forwards induction properties of stability [17, Proposition 6]. First it is demonstrated that any Pareto efficient outcome which is not in \( K \) is not stable. Consider the set of Nash equilibrium, denoted by \( A \), leading to any Pareto efficient outcome not in \( K \), say the allocation \( (s_1, f(s_1)) \) where \( s_1 \leq \delta_1 \). (The case where \( f(s_1) \leq \delta_2 \) is similar.) For any \( (s_2, t_2) \in A \) and any \( s_1', s_2 \leq \delta_1 \) and \( t_2 (s_1') < s_1' / \delta \) since otherwise 1 would deviate to \( s_1' \).

Therefore, for any \( s_1' > s_1 \), a strategy for player 1 in \( \{ (s_1', t_1) : t_1 (s_2) < s_1' / \delta_1 \text{ for some } s_2 \in S \} \) is never a weak best reply against any strategy for player 2 which is in \( A \). So consider the sub (matrix) game where these strategies are deleted. In this subgame, the strategies for player 2 in \( \{ (s_2, t_2) : t_2 (s_1') \leq s_1' / \delta_1 , s_1' > s_1 \} \) are weakly dominated. Now consider the further sub (matrix) game obtained by deleting these weakly dominated strategies. In this game none of the equilibria in \( A \) are Nash equilibria, since player 1 would prefer to deviate from \( s_1 \) to the next higher amount in \( S \), to which 2 would respond with more than \( s_1 / \delta_1 \).

Theorem 6 in [17] implies that if \( A \) is a stable set, then there is an element in \( A \)
which is an equilibrium in the sub (matrix) game which remains after such deletions. Thus \( A \) is not a stable set.

Next it is shown that any outcome in \( K \) is stable. In fact, these outcomes are hyperstable. In what follows let \( r_i \) denote strategies for the full game, namely pairs \((s_i, t_i)\), for player \( i \). Let \((r_1, r_2) = ((s_1, t_1), (s_2, t_2))\) be an equilibrium which determines an outcome in \( K \), and let \( A \) be the set of Nash equilibrium which yield that outcome. For any \( r_2' \) such that \((r_1, r_2)\) and \((r_1, r_2')\) yield the same payoffs for 2, \((r_1, r_2')\) is a Nash equilibrium. This is because the same payoffs can be obtained for 2 only if the players reach agreement in the first period. The payoffs for outcomes in \( K \) are greater than any possible second period payoff for either player. Thus player 1 cannot gain by deviating from \( r_1 \). So \( A \) includes all strategy pairs in any row of the game which yield player 2 the same payoff as \((r_1, r_2)\) and all strategy pairs in any column which yield player 1 the same payoff as \((r_1, r_2)\). Set \( \epsilon \) to be less than the smallest difference among payoffs in the game, excluding equalities. If any of the payoffs in the normal form are then perturbed by an amount less than \( \epsilon \), one of the equilibria in \( A \) will still be a Nash equilibrium.

Finally, it is necessary to argue that a stable outcome exists when \( K \) is empty. Since this game has a non generic extensive form, the claim does not follow from the generic arguments in Kohlberg and Mertens [17]. However, a completely mixed equilibrium can be constructed for this game, which is sufficient. This construction is straightforward.

QED

The characterization result in Proposition 1 has several nice properties. It demonstrates inefficiencies in the bargaining process which are a direct consequence of the players’ patience, since for large enough \( \delta_1 \), \( K \) is empty.
On the other hand, as one player's discount rate is increased, some efficient outcomes which are less favorable for that player become unstable. So, in a sense, each player benefits from increased patience -- until a point beyond which they both lose. It is also noteworthy that the delay is not a signal of a player's preferences, but of his or her intent to play "tough." The result also shows how forwards induction rules out efficient allocations which are "extremely one sided."

In [23] if the discount rates are equal, then, as the player's impatience decreases, the equilibrium allocation converges to the Nash [19] cooperative bargaining solution. Thus, a noncooperative basis for the Nash solution is provided. We show next that in our model qualified support is obtained for a minor modification of the Kalai and Smorodinsky [16] solution (denoted MKS). When discount rates are equal, if any equilibrium outcome is Pareto efficient, then the MKS outcome is supported in an equilibrium (and no other Pareto efficient outcome has this property). That is, as the set of stable equilibrium outcomes that are Pareto optimal shrinks, it converges to the MKS outcome, before disappearing entirely. The KS solution to a normalized bargaining game with a convex utility possibility set is the allocation determined by the intersection of the diagonal with the Pareto frontier. Since this solution does not apply to the discrete version of the bargaining model (because this version is not convex) the solution must be modified slightly in order to retain the property that it is strongly Pareto optimal. The MKS solution is defined to be any point on the Pareto frontier which weakly dominates the KS solution and is not itself weakly dominated.

COROLLARY 1: Let \( \delta_1 = \delta_2 = \delta \). Either the MKS solution is efficient and stable, or the efficient and stable set is empty. Furthermore, no other outcome has this property for all discount rates.
Proof: Let \((s', \delta')\) be the point on the diagonal which intersects the Pareto frontier, and let the MKS solution be \((s_1, s_2)\). For any \(\delta < \delta', s_1 > \delta\), and \(s_2 > \delta\). Hence the MKS solution is in \(K\). For \(\delta\) sufficiently close to \(\delta'\), only the MKS solution satisfies the last two inequalities. Finally if \(\delta > \delta'\), \(K\) is empty. QED

An example is now provided to illustrate the problems which arise in generalizing the previous results to a model with more than two periods.\(^7\) The example shows that stability (in fact hyperstability) allows for efficient outcomes in a three-period model even when \(\delta\) is large. It is worthwhile noting that the forwards induction used above may be weaker than that assumed in incomplete information bargaining models (cf [1, p. 349]). Nevertheless in the present context it is not clear how the assumptions may be strengthened in order to extend the results to \(n > 2\) periods. Despite the difficulties in attaining such an extension I believe that the intuition that delegating bargaining can cause delays by creating (signalling) incentives to play tough is of interest.

Consider a three-period game where in every period each player can either say low or high. Agreement is reached in the first period in which at least one player says low. If agreement is reached then low yields the payoff of 1, and high yields 2, while disagreement throughout the three periods yields zero. The discount rate is assumed to be \(\delta\) for both players. This determines the payoff matrix in Figure 1, where L; HL; HHL; HHH indicate: low in the first period; high in the first and low in the second period; high in both the first and second periods and low in the third; and high in all three periods for player 1 (and lower-case letters are used for player 2).
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<thead>
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<td>8,2δ</td>
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Figure 1

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Figure 2

It is easy to see that in the two-period version of this game (see Figure 2) the outcome (2,1) is not stable (hl is never a weak best reply against any strategy of player 1 which is used in a Nash equilibrium that yields (2,1); HH is weakly dominated in the sub (matrix) game remaining after hl is deleted; l is weakly dominated after HH is deleted; and then HL is dominated so (2,1) is not an equilibrium outcome after these deletions hence cannot be stable). Intuitively, by rejecting the outcome (2,1) player 2 indicates that s/he will be demanding the high payoff in the second period. In the three-period model such a rejection is ambiguous -- player 2 could be indicating a demand for the high payoff in either the second or the third period. Formally it is proven below that (2,1) is a hyperstable outcome of the game in Figure 1.

PROPOSITION 2: In the three-period model of Figure 1 the outcome (2,1) is hyperstable.
PROOF: First the set of Nash equilibria, denoted $M$, which yields $(2,1)$ is described.

$$M = \{ \langle \alpha, HL; \beta, HHL; 1-\alpha-\beta, HHH \rangle, l \rangle : 2\delta \alpha + \delta^2 (1-\alpha) \leq 1, 2\delta \alpha + 2\delta^2 \beta \leq 1, \alpha \in [0,1], \beta \in [0,1], \alpha + \beta \in [0,1] \}$$

where $\langle \alpha, HL; \beta, HHL; 1-\alpha-\beta, HHH \rangle, l \rangle$ denotes the strategy pair consisting of player 1 playing HL (respectively HHL, HHH) with probability $\alpha$ (respectively $\beta$, $1-\alpha-\beta$) and player 2 playing $l$. In particular $\langle (1-\delta^2)/(2\delta-\delta^2), HL; (2\delta-1)/(2\delta-\delta^2), HHH \rangle, l \rangle \in M$ and $\langle (1-\delta^2)/(2\delta-\delta^2), HL; (2\delta-1)/(4\delta-2\delta^2), HHL; (2\delta-1)/(4\delta-2\delta^2), HHH \rangle, l \rangle \in M$. Denote the strategy of player 1 in the former equilibrium by $q_1$ and in the latter by $r_1$.

Next it is argued that if any collection of payoffs in the matrix of Figure 1 are perturbed by some collection of $\epsilon$'s (where the $\epsilon$'s are sufficiently small) then the perturbed game has a Nash equilibrium which is close to one of the equilibria in $M$. Only the essential parts of this argument are provided below.

Step 1: $L$ and $hl$ can be ignored since in no equilibrium close to $M$ will either be assigned positive probability.

Step 2: Assume that the only perturbations are to player 1’s payoffs in the cells corresponding to $\langle HL, l \rangle$, $\langle HH, l \rangle$, and $\langle HHH, l \rangle$, and denote these perturbations by $\epsilon_1$, $\epsilon_2$, and $\epsilon_3$ respectively. Perturbations of other payoffs in the matrix can be dealt with using arguments which involve the same steps and approximating
equilibria as below -- the arguments are similar but more tedious, hence are omitted.

Step 3: If $\varepsilon_3 \geq \varepsilon_1$ and $\varepsilon_3 \geq \varepsilon_2$ then $<\text{HHH}, l> \in M$ is a Nash equilibrium of the perturbed game.

Step 4: If $\varepsilon_1 > \varepsilon_3$ and $\varepsilon_1 \geq \varepsilon_2$ then there is an equilibrium of the perturbed game close to $<q_1, l> \in M$. In particular $<q_1, (1-\mu, 1; \mu, hhl)>$ is an equilibrium of the perturbed game where $\mu$ solves:

\[(2 + \varepsilon_1)(1-\mu) + \delta \mu = (2 + \varepsilon_3)(1-\mu) + 2\delta^2 \mu \geq (2 + \varepsilon_2)(1-\mu) + \delta^2 \mu.\]  

(1)

This equation makes player 1 indifferent between HL and HHH with both preferred to HHL. Furthermore as $(\varepsilon_1, \varepsilon_3) \rightarrow (0, 0)$, then $\mu \rightarrow 0$ so this equilibrium is close to $M$.

Step 5: If $\varepsilon_2 > \varepsilon_3$ and $\varepsilon_2 > \varepsilon_1$ then either there is an equilibrium of the perturbed game close to $<q_1, l> \in M$, or there is an equilibrium of the perturbed game close to $<r_1, l> \in M$. The former holds if there is a $\mu \in [0,1]$ such that (1) is satisfied. The latter is the case if there are $\nu, \eta \in [0,1]$ with $\nu + \eta \in [0,1]$ such that

\[(2 + \varepsilon_1)(1-\nu-\eta) + \delta \nu + \delta \eta = (2 + \varepsilon_3)(1-\nu-\eta) + 2\delta^2 \nu = (2 + \varepsilon_2)(1-\nu-\eta) + \delta^2 \nu + \delta^2 \eta.\]  

(2)
For $\mu$ solving Eq. (1), we can proceed as in Step 4. For $\nu$, $\eta$ solving Eq. (2), if player 2 plays $r_2 = (1-\nu, \eta, 1; \nu, hhl; \eta, hhh)$ then player 1 is indifferent among HL, HHL and HHL so $<r_1, r_2>$ is an equilibrium of the perturbed game. It is easily verified that as $(\epsilon_1, \epsilon_2, \epsilon_3) \rightarrow (0,0,0)$ then $\nu \rightarrow 0$ and $\eta \rightarrow 0$, and it can also be demonstrated, by comparing equations (1) and (2) that at least one of these equations can be solved with $\mu, \nu, \eta$, and $\nu + \eta$ in $[0,1]$. QED
References


Endnotes

1. Gul, Sonnenschein and Wilson [14] have shown that for a general class of these models (excluding Admati and Perry [1]) the delay disappears as the time between offers goes to zero.

2. Another feature of the simultaneous offer model is that it is ex ante symmetric, whereas there is a procedural asymmetry in the alternating move model.

3. In fact there exist subgame perfect equilibria which strongly Pareto dominate all the stable outcomes in a game with sufficiently patient players. This seems to raise doubts regarding the practice of focussing on Pareto dominating Nash equilibria (for example in the literature on renegotiation proof equilibrium [3, 8, 22, 27]).

4. This delay of course disappears as the time between the stages disappears, since only a two stage model is examined (cf. Gul, Sonnenschein and Wilson [14]).

5. Delegation to agents who play a game instead of the original players is shown by Fershtman, Judd, and Kalai [9] to lead to cooperation.

6. The example actually involves a minor modification of the previous model.
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