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Author
Nagamiya, S.

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S. Nagamiya and D. J. Morrissey

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The Geometrical Aspect of High-Energy Heavy Ion Collisions

S. Nagamiya and D. J. Morrissey

Nuclear Science Division
Lawrence Berkeley Laboratory, University of California,
Berkeley, California 94720, U. S. A.

ABSTRACT

The total yields of nuclear charge or mass from projectile and target fragments and the fragments from the overlapping region between projectile and target were evaluated based on existing data. These values are compared with simple formulas expected from the participant-spectator model. Agreement is reasonably good, suggesting that the major part of the integrated yields for all reaction products from high-energy heavy-ion collisions are geometrical.
Fragment spectra observed in high-energy heavy-ion collisions have the following general features. At 0° they are peaked at velocities equal to the beam velocity [1-7], while at large angles their spectra are essentially structureless and vary smoothly as a function of the fragment momentum [8-10]. The yields at large angles are dominated by elementary particles such as protons and pions, whereas at 0° many isotopes with mass numbers smaller than that of the beam nucleus are produced. Further, large target residues are produced in relatively large yields and nearly at rest in the laboratory [11].

These observations readily suggest a simple picture of the collision, called the participant-spectator model, which is illustrated in Fig. 1. After the collision the non-overlapping parts of the beam and target nuclei, called the spectators, continue along straight line trajectories. Thus, the beam spectator produces a sharp peak in fragment spectra at 0° with a velocity approximately equal to the beam velocity. On the other hand, in the overlap region, strong interactions between beam and target nucleons cause fragments to be emitted over a wide angular range, these are the participants. Fragments emitted from this region are mainly elementary particles, because the energy transfer involved is much higher than the binding energies of nuclei. In the present paper we give expressions for how many nucleons can be classified as beam and target spectators and participants, and then compare the calculated values with available data. These comparisons will tell us to what extent this simple model explains high-energy heavy-ion collisions.

Let us first estimate how many protons from the beam nucleus become participant protons. If the nuclei have sharp radii, then the
total reaction cross section, $\sigma_{\text{Tot}}$, is given by

$$\sigma_{\text{Tot}} = \pi r_0^2 \left( \frac{A_B^{1/3} + A_T^{1/3}}{A_T} \right)^2 \text{ with } r_0 \sim 1.2 \text{ fm},$$

(1)

where $A_B$ and $A_T$ are mass numbers of beam and target nuclei, respectively.

If a proton inside the beam nucleus hits the target, it becomes a participant proton, but otherwise it remains as a spectator proton. Under the assumption that the nucleus consists of $Z_B$ independent protons and $N_B$ independent neutrons, the average number of participant protons from the beam nucleus can be written [12]

$$<Z_{\text{Parti}}^{\text{Beam}} > = Z_B \frac{\pi r_0^2 A_T^{2/3}}{\sigma_{\text{Tot}}}$$

$$= Z_B \frac{A_T^{2/3}}{(A_B^{1/3} + A_T^{1/3})^2}.$$  \hspace{1cm} (2)

Similarly we have

$$<Z_{\text{Parti}}^{\text{Target}} > = Z_T \frac{A_B^{2/3}}{(A_B^{1/3} + A_T^{1/3})^2}.$$  \hspace{1cm} (3)

The total yield of protons contained in projectile fragments is thus given by the difference between the beam proton number and the average participant proton number multiplied by $\sigma_{\text{Tot}}$,

$$y^{\text{Proj Frag (proton)}} = (Z_B - <Z_{\text{Parti}}^{\text{Beam}}>) \times \sigma_{\text{Tot}}$$

$$= \pi r_0^2 Z_B \left( \frac{A_B^{2/3} + 2A_B^{1/3} A_T^{1/3}}{A_B^{1/3} + A_T^{1/3}} \right).$$  \hspace{1cm} (4)

The total yield of protons contained in target fragments, $y^{\text{Tgt Frag (proton)}}$, can be simply obtained by interchanging suffices B and T in the above formula. The total yield of nucleons contained in projectile fragments, $y^{\text{Proj Frag (nucleon)}}$, similarly can be obtained by replacing $Z_B$ by $A_B$. 
On the other hand, the total yield of protons in the participant region is given by the sum of contributions from the beam and target,

\[ Y_{\text{Parti}}(\text{proton}) = (\langle Z_{\text{Beam}} \rangle + \langle Z_{\text{Target}} \rangle) \times \sigma_{\text{Tot}} \]

\[ = \pi r_0^2 \left( Z_B A_T^{2/3} + Z_T A_B^{2/3} \right). \]  

This above formula (5) has been previously obtained by Hufner et al. [13]

In Figs. 2–4 the above formulas are compared with the data.

Fig. 2 shows the total yield of proton numbers for projectile fragments calculated from the observed isotope yields measured by Lindstrom et al. [2] for $^{12}$C and $^{16}$O beams: namely, the data points were obtained as the sum from the measured values of $\sigma_i$. The total yield is found to be almost independent of incident beam energies. The observed target mass dependence is approximately given by $A_T^{1/4}$ which is nicely predicted by Eq. (4). The absolute values of the observed yields are about 30 percent lower than the predicted ones; however, if we recall that Eq. (4) is an oversimplified formula, we should conclude that the agreement is fair.

In Fig. 3 we show the results of summing over the cross sections for production of $p$, $d$, $t$, and $^3$He observed at angles between $10^\circ \leq \theta \leq 145^\circ$ [14] after extrapolation to $0^\circ$ and $180^\circ$. The ambiguity of the extrapolation is not large, since $(d\sigma/d\theta)_{0^\circ} = (d\sigma/d\theta)_{180^\circ} = 0$. These data are essentially all from the participant region and, thus they should be compared with Eq. (5). We see again the beam-energy independence, and the agreement with the prediction is better than for the sum over projectile charges.

Fig. 4 shows the total yield of nucleons contained as target
fragments calculated from Eq. (4) with the appropriate substitutions for $^{197}$Au target nuclei. The data points were obtained from the observed radioactive isotope yields measured by Morrissey et al. [15] and Kaufman et al. [16]. After fitting the observed $d\sigma/dA$ results, the total yields were obtained by integrating $\int A(d\sigma/dA)dA$. The absolute values of the observed products are less than the predicted ones, which is similar to the previous case of projectile fragments, but again we should conclude that the agreement is fair.

These comparisons of the cross section calculations with the data tell us that the participant-spectator model describes rather well all the general features of high-energy heavy-ion collision cross sections. However, good agreement between the model and measured cross sections is only expected for very high energy reactions where deflection of projectile by Coulomb and nuclear forces is small, making the assumption of straight line trajectories valid. In general, this assumption of straight line trajectories is easier to accept if we can regard the projectile and target as assemblies of independent nucleons. Such an independency of nucleons is meaningful only when the de Broglie wave length of incident nucleon is shorter than the internucleon distance inside that nucleus. This condition sets the lowest beam energies at which the model can be applied at about a few 10's of MeV per nucleon.

A second rather limiting assumption is that of rigid hard sphere nuclei which make clean cuts through each other. That is, the question of whether or not the boundary between participants and spectators is clearly defined. This distinction is particularly difficult to make when the diffuseness of nuclear matter is taken into account [17].
The lack of such a sharp boundary and the presence of a region where "soft" nucleon-nucleon collisions occur would lead to a decrease in the total yield of projectile and target fragments compared to the results shown in Figs. 2 and 4. The boundary layer could be viewed as a region in which macroscopic frictional forces are acting to damp orbital angular momentum into the spectator fragments. Measurements of the $\beta$-decay asymmetry [18] of projectile fragments and $\gamma$-decay multiplicity or radioactive isomeric ratios of target fragments could shed light on this process.

In summary we have taken cross sections for the production of projectile and target fragments and light, high velocity, fragments from high-energy heavy-ion reactions and compared them to the absolute predictions of the simple participant-spectator model. The data and calculations compared very well for all three categories of reaction products. A small overestimation (up to 30 percent) of projectile and target fragments may be due to a simplified formula for the overlap of the two spherical nuclei and the assumption of a sharp boundary between participants and spectators. This work was supported by the Nuclear Science Division of the U.S. Department of Energy, the Yamada Foundation, and the INS-LBL Collaboration Program.
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[12] Equation (2) is an approximate formula and is not exact because the
spherical shape of the beam nucleus is not taken into account.
However, the deviation of equation (2) from the actual numerical
result has been found to be less than 10 percent.

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FIGURE CAPTIONS

Fig. 1. Participant-spectator model. After the collision non-overlapping parts of the beam and target nuclei form the beam spectator and the target spectator, while the overlapping part forms the participant piece.

Fig. 2. Total yield of nuclear charges emitted at $0^\circ$ as compared with the prediction of the participant-spectator model.

Fig. 3. Total yield of nuclear charges emitted at large angles as compared with the prediction of the participant-spectator model. Data at $10^\circ < \theta < 145^\circ$ were used and extrapolated to $0^\circ$ and $180^\circ$.

Fig. 4. Total yield of nucleon numbers observed in isotopes stopped inside the target material as compared with the prediction of the participant-spectator model.
Fig. 1
Total yield of nuclear charges for projectile fragments

\[ \pi r_0^2 Z_B \left( A_B^{2/3} + 2 A_B^{1/3} A_T^{1/3} \right) \]

\[ r_0 = 1.2 \text{ fm} \]

- 2.1 GeV/A $^{16}$O
- 2.1 GeV/A $^{12}$C
- 1.05 GeV/A $^{12}$C

Target mass $A_T$ vs. Total yield of nuclear charges (barn)

Fig. 2
Total yield of nuclear charges at large angles

\[ \pi r_0^2 \left( Z_B A_T^{2/3} + Z_T A_B^{2/3} \right) \]

\( r_0 = 1.2 \text{ fm} \)

- Ar+KCl
- Ne+NaF
- C+C

0.4 GeV/A
0.8 GeV/A
2.1 GeV/A

Fig. 3
Total yield of nucleons for target fragments (Au target)

\[ \pi r_0^2 A_T (A_T^{2/3} + 2 A_T^{1/3} A_B^{1/3}) \]

\( r_0 = 1.2 \text{ fm} \)

8 GeV Ne + Au

11.5 GeV p + Au

Fig. 4