UNIVERSITY OF CALIFORNIA, SAN DIEGO

Building on Flat Land: Dimension in Musical Interaction

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by

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The thesis of Brendan Bernhardt Gaffney is approved, and it is acceptable in quality and form for publication on microfilm and electronically:

Chair

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DEDICATION

For Robert Emmet Gaffney.
Here am I, here is my work — and someone is waiting for the fruits of these fleeting hours. Hands will caress this shimmery surface, a thumb will discover the edge which I am rounding.

—James Krenov
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ABSTRACT OF THE THESIS

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The creation of new means and aids in interacting with digital sound synthesis is of growing necessity in computer music, due to the proliferation of complex synthesis algorithms and control interfaces. Herein, the goal of summing current research areas, highlighting a new possibility, and pursuing this possibility through to implementation is pursued.
Chapter 1

A Need for New Interaction

Computer music and its related fields have experienced explosive growth over the past half-century. The proliferation of dedicated softwares and languages, digital musical instruments and interfaces, and creative output fueled by these new technologies have led to a diverse set of subdisciplines, specializations and practitioners.

Within these subdisciplines, a number of innovators have pioneered the choices available in interacting with digitally created sound. It is in this context that parameter mapping and sonic interaction design have emerged as core concerns in the ongoing creation of computer music paradigms. There are several trends that concern these subdisciplines, and highlight its growing relevancy and importance– the increasing complexity of synthesis algorithms, desire to control these algorithms in real time and the explosion of consumer electronics being repurposed as musical interfaces.

1.0.1 Growing Parametric Complexity of New Synthesis Algorithms

Through the past half-century, sound synthesis algorithms have evolved significantly, influenced by both the advancement of techniques and increasing capability of the computers used in the process. Early sound synthesis algorithms, like subtractive synthesis, relied on relatively few synthesis parameters. Sounds were
specified in terms of both static and changing filter cutoffs, oscillator frequencies and amplitudes, in keeping with the paradigm of electronic sound synthesizers of the time.

In the latter half of the last century, significant advances in computation and technique led to an explosion in complexity of input and computation, and a break from the relative simplicity of classic means of electronic sound synthesis. Algorithms like granular and physical modeling synthesis, and the growing density and complexity of their execution, have led to growing demand on input and control techniques.

Parameter mapping strategies seek to enable a performer or composer to specify or interact with a smaller number of higher level controls, and use these controls to generate and interpolate the lower level synthesis parameters needed for actual computation and generation of sound. This allows for more expansive predetermination of sound spaces, allowing greater flexibility and expressivity of performance, and parametric adaptation to the possible permutations of control-to-sound paradigms in digital musical instruments.

1.0.2 Real Time Control of Digital Sound Synthesis

The extension of flexibility and expressivity of digital musical instruments coupled with growing computational power has led to the extension, even expectation, of the real time capabilities of digitally synthesized sound. The fact that computer music is now often performed live, and not only generated ahead of time and accompanied or played back, has led to a need to design interfaces to facilitate purposeful interaction with the sound synthesis algorithms.

This growing world of real time computer music performance has led to the creation of a vast collection of digital musical instruments. These instruments, often purpose built for specific algorithms or paradigms of digital sound synthesis, take advantage of parameter mapping strategies to break otherwise complex algorithms into high level controllers with a feasible, human scale number of inputs and controls.

Even in real time score advancement and following scenarios, parameter
mapping can be used to aid in simplifying compositional choices, helping to break
down an often tedious process of setting and keeping track of a large number of
states and performance variables.

1.0.3 Availability of Low Complexity Digital Music Control Interfaces

With the proliferation of consumer electronics beginning in the seventies,
coupled with the invention of personal computers, today’s marketplace is rife with
computer interfaces of all sorts. From the QWERTY keyboard to PC gaming
accessories, human computer interaction has flourished as a means of making the
intangible ones and zeroes of the computer real and interactive to their human
manipulators.

For digital music creators, this has meant a large expansion of options ava-
iable to performers, composers and engineers in designing and building interactive
musical scenarios on digital platforms. Game controllers become filter manipula-
tors, a keyboard becomes a score following aid, and the vast market of purpose built
music controllers incorporate the real-world acoustic-like interactions available to
instrumentalists into the world of digital musical instruments.

This growing set of options has meant that the task of matching controller to
algorithm is a growing field of concern for all involved in the process of computer
music creation. Where some controllers allow a simple, one-to-one mapping of
control to algorithm, both the growing complexity of controllers, the emphasis
on real-time control, and the ever-expanding parameter space of new synthesis
algorithms stress the available solutions for parameter mapping.

1.1 The Creation of New Mapping Strategies

To meet the growing challenges of musical interaction in digital music, it is
necessary for those involved in building this interaction to research the possibilities
available to ease and expand these interactions.

These new, more complex parameter mapping strategies can be borrowed
and derived from many fields and techniques. More recently, the ability to derive these models form well known algorithmic means, intuition and emulation of more acoustic-like paradigms of interaction have garnered a fair amount of attention.

Alongside these new derived models, there also lies a great wealth of potential mapping schemes in related fields of research, both closely related like human-computer interaction and distantly like dynamical systems and artificial intelligence.

1.1.1 Deriving Musical Interaction Models

In terms of the derivation of new parameter mapping strategies, there are several new techniques developing alongside those age old techniques in use since the inception of computer music performance.

A growing means of designing new parameter mapping layers between control and algorithm is the use of genetic algorithms and adaptive computer processes. Work done by Laine [1] in creating genetic algorithms that create digital musical paradigms, Wanderley and Miranda’s exploration of genetic and other algorithms in the creation of new digital musical instruments [2] and Dahlstedt’s MutaSynth and work involving genetic algorithms and evolutionary processes in the creation of sound and musical instruments [3] all indicate the vast richness of the inclusion of algorithmic determination of parameters and musical control paradigms.

While old-fashioned one-to-one mappings of control to synthesizer parameter are still overwhelmingly common, the intuitive process of matching control to algorithm in a more complex manner is still being done with the aid of human intuition. Specifically, perception derived control parameters are being used widely to sum the effect of a single parameter’s changing on the more complex timbral changes inherent in complex parameter mapping. Wanderley’s ESCHER system based on IRCAM’s jMax is a great example of this work[4], alongside the work of Arfib et al. [5] and others[6][7].
1.1.2 Borrowing Models

While previous methods can be perceived as the borrowing of tools from psychoacoustics and computer science, both directly related to the computer music field, many disciplines outside those closely related are being looked to for their possible solutions in reducing dimensionality of data, parsing gestures and relating the physical or mechanical worlds to the digital realm of computer sound synthesis.

Most directly, human computer interaction has been a constant source of borrowed models, as its attempts to decode and describe human gestures in terms of control and intent make it uniquely relevant. Once again, Hunt and Wanderley’s work in the evaluation and creation of digital musical instruments is a testament to this method’s effectiveness [8][9].

However, a growing set of disciplines, like that of interaction design, topology and dynamical systems continue to tread into waters concerning those interested in the mapping of human input to sound output. Aside from the work done in the following chapters, statistical techniques like principal component analysis [10] and the development of terminology common across those in the fields of music, science and interaction [11] point to the increasing interdisciplnarity of the analysis of human intent, and the diverse means by which it may be decoded and put to work.

1.2 Goals of This Research

Herein, the goals of this research are to take cues from the preceding pages in identifying a new means by which control may be interpreted for digital sound synthesis. Specifically, catastrophe theory, a subset of dynamical systems theory, will be explored as a means of interpreting human control into more acoustic-behaving algorithmic controls.

After a summation of currently available scenarios of control and parameter mapping, and describing the need for new mapping paradigms, a new technique using simple polynomials will be explored as a potential mapping scheme. This new mapping strategy promises a simple, acoustic-like behavior that may port well
to digital musical instruments.

Then, this new mapping strategy can be verified by looking to existing acoustic-like behaviors in physical-modeling algorithms that may exhibit behaviors similar to this mappings strategy. The hope is, that this technique not only works well, but can be shown to have a direct tie to the behaviors of acoustical systems.
Chapter 2

Acoustics-like Dynamics in Signal Based Models

The quality of a parametric sound synthesis model is not only determined by its produced sound, but also by the richness, depth, and intuitiveness of its control. As is the case with their acoustic counterparts, virtual musical instruments should engage users with music and sonic possibilities, allowing for exploration, discovery, and expression, with increased use, practice, and familiarity. A mapping strategy, therefore, may be evaluated by its “virtuosic ceiling” (potential for maturation with extended use) and its “entry fee” (ease of initial interaction)[12]. Balancing these two attributes is an important aspect in designing a system whereby performative gestures will be translated into synthesis parameters.

Physics-based synthesis models often have a myriad of possible synthesis parameters, offering possibilities in the produced sound akin to their acoustic counterparts. Though the complete set of possible parameters is usually too large to be effectively controlled by the user in realtime, there is usually a subset of “control parameters” that is naturally intuitive, largely because they are physical and relate to acoustic instruments with which the user has some familiarity and experience: blowing harder produces a louder sound; shortening the string produces a higher pitch. In addition to offering a low “entry fee” (ease of use) without requiring additional mapping, a quality physics-based model implements the dynamics of the system (the produced sound being dependent on both the current state of the
model/parameters and their change over time), which also, by nature, offers possibilities that raise the “virtuosic ceiling” (maturation): blowing harder produces not only a louder sound, but also one that is brighter, harsher, detuned, or even the octave above (overblowing).

In signal-based models, the relationship between control and synthesis parameters is far less obvious (to both developer and user), and a mapping strategy is required to achieve a balance between ease of use and maturation. These mappings can be difficult to create, due to both their abstraction from a more obvious linear mapping, and their potential to create densely connected and difficult to debug and describe interactions. Existing strategies have incorporated generative methods to produce these mappings[13, 14, 15] and many have developed taxonomies to enable the decryption and development of these complex mappings[16, 17]. In this work we present an approach to parameter-mapping that, by borrowing concepts and models from catastrophe theory, aims to enrich signal-based models with the inherent complexities/intuitiveness of those that are based on some more natural, physically based musical interaction.

In an attempt to further the current mapping toolset, we have chosen to examine catastrophe theory as a potential set of theorems and models. Work done to extend the toolset available in creating these mappings is valuable to performer, composer and designer alike, as creating new primitives in mapping strategies yields a better set of design choices for the development of new mappings of control to synthesis, and therefore a more dynamic and nuanced interaction between instrument/interface designer, composer and performer.

In Section 2, we will examine catastrophe theory, its models and those attributes that indicate its potential value to parameter mapping development. In Section 3, we discuss its implementation, specifically in code via Pure Data and in a parameter mapping paradigm within frequency modulation synthesis. In Section 4 we discuss the results of these initial implementations, in Section 5 we examine the research to suggest possible topics for expansion and investigation, and in Section 6 we discuss the conclusions derived from our research.
2.1 Catastrophe Theory

René Thom, a twentieth-century French mathematician, developed catastrophe theory as a means of explaining a set of complex singularities in geometry and mathematics.[18][19] Thom’s work inspired many to pursue the conclusions of catastrophe theory, not only in mathematics, but across disciplines. In his book *Catastrophe Theory*, Sir E.C. Zeeman, a British mathematician and champion of the relevancy of catastrophe theory across disciplines, presents several examples of simple, catastrophic systems outside mathematical fields [20]. A number of other researchers have used Thom’s work in modeling a number of sociological[21], economic[22], physical[23], and biological[24] systems.

Catastrophe theory describes simple geometric models to explain systems that yield drastic changes in state in response to slowly changing attributes or parameters. These models have been developed from theorems proposed by Thom, that describe higher-dimensional geometry, specifically that of bifurcating sets of higher-order polynomials. His work concerned itself specifically with the discontinuities yielded by a number of special multi-dimensional geometric equations he termed *elementary catastrophes*, which are classified by the dimensions of their behavior and parameter spaces. The models Thom and Zeeman use to describe these systems are eloquent in that they are simple polynomials, whose real roots yield the stable states of the system, and whose coefficients shape the attributes of the thresholds and surface of the models[25].

While these previous implementations of catastrophe theory have little to prove for our mapping here, they point to the validity of catastrophe theory models in a range of applications and disciplines.

The elementary catastrophe we will concern ourselves with herein will be a lower dimensional model, due to its potential for representation on paper and its relative ease of comprehension and application. The model is the cusp catastrophe, which is described by a simple cubic polynomial, and from a two dimensional control space yields a third, potentially bimodally distributed behavior axis, whose value is dependent on previous states and trajectory through our control space. The cusp is manipulated by adjusting the coefficients of a polynomial, using two
Figure 2.1: The elementary cusp catastrophe. Our variable axes $b$ (splitting factor) and $a$ (normal factor) and behavior axis $x$ are labeled in the control surface $C$, and several trajectories through this control surface are traced both on $C$ and their resulting values for $x$ are traced on the behavior manifold $M$.

of these coefficients as navigational axes of a control space.

### 2.1.1 The Cusp Catastrophe

Catastrophe theory comprises a number of models that relate or map “attributes” to “behavioral” states. One such model, called the cusp catastrophe, is given by

$$c_hx^3 + bc_wx + a = 0, \quad (2.1)$$

where $c_h$ and $c_w$ are used to change the cusp height and width, respectively, and coefficients $a$ and $b$ are input control parameters. The surface $C$ in Figure 2.1 is the control surface created by axes $a$ and $b$, while the manifold cusped surface $M$ (above $C$) is defined by the real roots of (2.1). The positive and negative values of $x$ create the two sheets (upper and lower regions) of $M$.

Since (2.1) is a cubic polynomial, it has three roots. The shaded area on control surface $C$, indicates values of $a$ and $b$ for which all three of these roots are real—*the bifurcating set*. These three real roots define the folded or “cusped”
region of the manifold surface $M$. Outside the shaded region in $C$ lie values for $a$ and $b$ yielding only a single real value for $x$. The two curved lines outlining the shaded area are thus thresholds for which $a$ and $b$ yield single or multiple (bifurcating) values of $x$. Bifurcating values of $x$ appear for values of $b > 0$. For $b < 0$, $x$ increases continuously with $a$. Static coefficients, $c_h$ and $c_w$, effectively scale the coefficients $a$ and $b$, thus skewing the dimensions of the cusp.

Fig. 2.1 shows several trajectories, labeled 1-4, of linearly changing values for $a$ and $b$. Trajectories 1 and 2 on $C$, which originate on either side of the bifurcating set, produce different values for $x$, shown by corresponding trajectories 1 and 2 on $M$, despite a common destination point and similarly changing values of $a$ and $b$. This exemplifies the first of the catastrophe model’s attributes:

**Attribute 1** The behavior resulting from a given set of control values is dependent both on initial conditions and previous behavior.

Trajectories 3 and 4 illustrate the characteristic jumps, or “catastrophes,” after which the models are named, which occur when moving from the bifurcating set to the non-bifurcating set (jumps are illustrated in Fig. 2.1 using dashed lines on $M$ and occur at points on $C$ when the trajectory moves from inside to outside the shaded area). Furthermore, if a trajectory exits across the same threshold from which it entered, i.e. remains on the same “sheet”, no jump occurs. This exemplifies the second of the model’s attributes:

**Attribute 2** Jumps in the value of $x$ occur only upon exiting the bifurcating set onto a new sheet.

### 2.1.2 Applying to Dynamic Systems

Though the cusp model has two input parameters, it generates another two, yielding a total of four possible synthesis/application parameters: $a$, $b$, location $x$ on the cusp manifold surface, and a binary value indicating whether $x$ is on the upper or lower sheet. This increase indicates a potential value in parameter mapping, as it suggests a possible mapping of a simple control space to a more complex dynamic or sound synthesis system.
Any system that exhibits:

1. bimodal distributions of behavior for a dynamic input (relating to Attribute 1),

2. drastic changes in behavior despite slowly changing control parameters (relating to Attribute 2).

is a potential candidate for representation by a catastrophe model. Several such systems exist in music applications. In particular, blowing into the mouthpiece of a saxophone presents an example of an acoustic system that exhibits these two attributes: slowly varying embouchure and blowing pressure (corresponding
to control axes \( a \) and \( b \) for a given fingering, produces a sound that can leap in register/octave—a bimodality in state (Attribute 1) that can result in a jump in \( x \) (Attribute 2). That is, the tendency of the horn to lock into an upper or lower register, based on its previous state, exhibits Attribute 1. The tendency for a horn to jump *catastrophically* in register despite slow changes in control exhibits Attribute 2.

This simple catastrophic model of the saxophone shows the natural and musical behavior of control parameters fed through a cusp model. This nuanced behavior, coupled with the simplicity of the mathematics and rules of behavior, point to a potentially rewarding mapping strategy.

### 2.2 Implementation

In implementing catastrophe theory and polynomial equations in a mapping strategy, we are looking for complexity and capability in expression without diminishing the ability to use an interface effectively and easily. Furthermore, we hope to reward maturation with an interface, providing a more complex and nuanced interaction with the interface over time, more so than previously available without a complex mapping. The cusp model (2.1) is implemented as a Pd external object (written in C) [26], which offers a real-time interactive programming environment popular among computer musicians.

The first step in implementation is to fully understand the effects of manipulating the coefficients of (2.1). Initial tests were run in graphing programs to illustrate the width and height of the cusp for different values of \( c_h \) and \( c_w \) (see Figure 2.2). Following this, implementation is straightforward. First, the Cusp model is coded as a function having four input parameters, two static \((c_w \text{ and } c_h)\) and two dynamic \((a \text{ and } b)\), and two returned values, \( x \) and a binary indicating on which sheet, HIGH or LOW, \( x \) lies. Through experimentation, \( c_h \) was deemed unnecessary as it was *nearly* a scaling of \( x \) that could instead be more effectively and predictably applied as a linear scaling of the output (reducing required inlets in the Pd external to three).
The function uses the cubic polynomial solver in the GNU Scientific Library, as it returns only real values (and not complex values that have nothing to do with surface $M$). This function takes our three coefficients above and three pointers to memory locations in which it stores the returned roots of our equation. It also returns an integer indicating whether there is one or three real roots, effectively indicating whether we are in a bifurcating or non-bifurcating set of values for $a$ and $b$.

Finally, a state variable is used to “remember” on which sheet of the cusp surface $x$ resided in the previous time step, determining which of the roots of $x$, lower or upper, should be returned (the middle value is not considered in these models). Therefore, in this example we have a doubling of possible control parameters: the original $a$ and $b$, plus two more given by the cusp model, $x$ and sheet of $x$.

This code can be further optimized by implementing our own polynomial solver instead of calling an outside function (which itself makes several outside function calls). Furthermore, a number of other techniques can be used to determine the correct root, and some of these may be more optimal. Because this code, wrapped as a Pd external, is computed for every sample, it may be used in wave-shaping and audio-rate modulation, as well as control rate paradigm.

### 2.3 Application and Results

Here, we choose to explore its use in the context of an FM (frequency modulation) synthesizer, to see how acoustic behavior as described in Section 2.2 can be incorporated in a signal-based model. A very simple implementation can be observed in Figure 2.3, where the index of modulation is controlled by both $x$ and the binary HIGH/LOW sheet variable, while the carrier and modulator frequency are controlled by $a$ and $b$, respectively.

The patch illustrated in Fig. 2.3 was used as an experimentation platform for determining the effect of our two generated parameters in very minimal signal-based synthesis system. Frequency modulation was chosen for our familiarity with
its common control mappings and produced sound.

The interface chosen for initial experimentation was a touch sensitive trackpad, which returned an x and y value for a finger moved about its surface. By implementing our mapping with the cusp modeling, we essentially are able to traverse the lower and upper sheets of the model with our finger, and dictate the behavior based on our trajectory across and around the thresholds of the model, much as the paths in Fig. 2.1. This allows nuanced control of the output values, as it is immediately possible for a novice user to locate, empirically, the location of these thresholds and quickly learn to exploit or avoid their happening.

2.3.1 Cusps in Musical Control Paradigms

The cusp in the above patch maps timbre to our cusp model and pitch to our input $a$ and $b$. Several other implementations were made systematically to determine by isolation the effect of cusp models on signal-based synthesis’s most often used parameters, timbre, amplitude and pitch.

In Fig. 2.3 we have mapped our FM timbral parameter, the index of modulation, to the $x$ output by the cusp model. We also tested this same system without the changing pitch, and therefore isolated timbral control with the cusped model. This yields an interesting, pseudo-vocal behavior, jumps in sideband presence and spread affecting a dynamic, albeit it expressively limited, control of timbre.

In other experiments, amplitude and pitch were controlled with the new complex yielded parameters $x$ and sheet of $x$. An interesting result of this experimentation was the effect of the changing $x$ without leaving the current sheet. The effect was to obtain a vernier control of a small subset of the accessible control space, effectively enabling a magnification of the values of $x$ available on a given sheet. When mapping to amplitude, at higher values of $b$, where the sheets are most distant and the values of $x$ therefore more disparate, this amounts to an ability to make nuanced changes in loudness at either a lower piano dynamic or, after jumping sheets, fine adjustments at a higher forte dynamic.

It is in our mapping of this model to pitch that the aforementioned “magnification” of certain subsets of the control space is most notable. The lower
portion of the control space allowed minute control of a lower pitch subset, and after a jump, minute control of a higher pitch subset. As the middle pitches can be accessed by simply decreasing $b$, this introduces a very interesting paradigm of control. A scale running from lowest to highest pitch sets therefore runs in a horseshoe shape, retreating around the bifurcating set of values of $a$ and $b$ and back out to the higher sheet, without encountering catastrophic jumps but increasing the nuance of control at all points. Furthermore, jumps of different sizes between registers can be made easily and with some precision by simply locating the proper crossing point of the threshold to take a path through.

These mappings to signal-based synthesis parameter primitives helped illustrate the value of these models to the expansion of available parameter mapping strategies. To our initial goal of introducing acoustic-like behavior to these simpler signal based models, it points to observed behaviors, like the selective magnification, that may map to acoustic-like paradigms.
2.3.2 Introducing Acoustic-like Behavior to a Signal Model

The main purpose of these experiments is to determine if the two additional parameters generated by the cusp model, $x$ and sheet of $x$, are useful and intuitive synthesis parameters. As previously illustrated, it can be shown that acoustic systems have a tendency to behave like the cusped model, so our aim was to investigate the presence of some more natural or acoustic-like behavior in the mapping.

We can show this behavior by observing Attributes 1 and 2 in the process of using the interface, and determine if they are related as predictable and controllable features to a user.

By default a simple FM synthesis model has no inherent acoustic-like qualities, as FM linearly mapped to the control parameters of a trackpad or other continuous controller is dissimilar from any existing acoustic system. This allows us to track the effect of introducing the cusp mapping, and evaluate it independently of the synthesis algorithm's behavior. This isolation of a mapping is key to evaluating its worth, as many synthesis algorithms behave naturally and effectively without an intermediate mapping between control and synthesis parameters.

First, the implementation of this model effectively enlarges the parameter space of our sound synthesis system, as Attribute 1 shows that a large portion of our control surface has two possible values of $x$. By introducing this bifurcating behavior, like that found in acoustic systems, the parameter space of our interface widens, and therefore a larger portion of the sound space of the synthesis algorithm is available to a performer.

Furthermore, by using cusp-generated parameters $x$ and sheet of $x$ to control the index of modulation, an abstract synthesis parameter without an acoustic analog, we introduced a way of jumping between timbres of the synthesis algorithm. Each sheet of the cusp maps to two different sound spaces, with finer adjustments accessible using $x$, and a user can switch purposefully from one to another. These jumps, described by Attribute 2, introduce a triggered, more dynamic behavior to our previously linear interface.

Also, the complex behaviors of wind instruments discussed in Section 2.2
can be modeled with careful application of the cusp model in mapping control parameters. In experimentation, the sheet of $x$ was mapped to pitch, while $x$ was mapped to index of modulation. This mapping closely resembles the articulation of a single keying of the saxophone, where an increase in embouchre and blowing pressure will push the horn to both jump in octave (a catastrophic jump in pitch) and harshen in timbre (an accompanying increase in $x$).

The selective magnification also has many acoustic analogs. Jumps between registers as described above, with some small, more nuanced adjustments available on either end of these jumps, also closely resembles paradigms present in wind instruments. Again, the jumps can be associated with pitch, but if mapped with proper scaling of $x$ instead of the binary sheet of $x$, small adjustments in intonation can be made in each register with some precision.

By identifying and exploiting these acoustic behaviors in our new mapping, which introduce more complex expression and control in an otherwise simple system, we have increased the potential for engagement and discovery in the process of learning a musical interaction with a digital system. We have done this by relating the interaction with a digital musical instrument to interactions a performer and composer are more likely to have some experience with. Furthermore, we have helped mediate the potential expressiveness of the vast sound space available in signal-based models to a much smaller and simpler control space.

2.3.3 Balancing Complexity and Cost

Another main focus of these experiments is to determine the ability for a user to easily acquire the mapping and behavior of the interface, and if maturation with the system is rewarding over time. While the initial experiments are basic, results indicate that the mapping has the potential to fulfill our two desired features of a new mapping, namely the low entry fee and high virtuosic ceiling.

By scaling and offsetting our input parameters, the bottom half of the trackpad can be kept near-linear, or without bifurcation (by keeping $b < 0$, as shown in Fig. 2.1), and therefore more immediately intuitive, while still allowing the top half to exhibit the more complex bifurcating behavior. By building this
duality into the interface, it is possible for a simple interface to yield both easily accessible and more complex behaviors.

Furthermore, several cusps can be implemented with differing dimensions and locales on the control surface by simply adding more of these models in the intermediary mapping layer. These additional mappings afford the same designed duality in simplicity and complexity. We can therefore introduce the complexity in behavior available with several cusps without accumulating complexity in the lower half of our mapping and eliminating its ease of acquisition. These two conditions satisfy the desire to find mappings both easy to acquire and rich in complexity and nuance that can be acquired over time.

2.4 Conclusions

Catastrophe theory, as laid out by Thom and others, allows us the means by which to extend the currently available tools used in parameter mapping. It does so by supplying models in which a low number of parameters yield new and complex output behaviors.

The cusp model from catastrophe theory is ideal for several reasons. First, it is relatively easy to understand, due to its ease of representation in three dimensions on paper, and its low order polynomial description. It is also easy to implement in code, and easier still to include in a mapping strategy once encapsulated into an external or its equivalent outside of Pd.

The likeness of the cusp model to acoustical systems further extends the implemented mappings, by extending their behaviors, size of control space, and introducing control that is intuitive and nuanced like that of an acoustical system. It allows for an acoustics-like dynamic for a signal based synthesis algorithm, by introducing an intermediary mapping layer. We can see the real mapping benefits of introducing both dynamic jumps in parameter range and the effect of bifurcating control surfaces in both ease of control and likeness to acoustic analogs.

Furthermore, the cusp model, and other polynomials like it, is possible to implement in a non-complicating manner. It can be subtle or drastic, with
or without linear mapping possibilities behind some threshold, and multiplied in number, all potentially without cost to the initial acquisition of the interface’s function and the ease that simple mappings allow novice users to begin making sound in a purposeful manner. It is this introduction of complexity without cost that highlights the possibilities of these equations as tools in the mapping strategies of larger, more complex algorithms.

2.5 Further Research

While this paper focuses on a simple implementation of a lower complexity model of catastrophe theory, there is still more to do in terms of applying these models and evaluating their wider uses and conclusions.

First and foremost, examination of all of catastrophe theory’s models, not simply those more conveniently laid out on paper, is called for. While they do not guarantee the possibility of nuanced or simple behavior like the cusp catastrophe does, their higher level of input and output parameters suggest their potential relevancy. One such model, the butterfly model, is suited for further research, as its surface can also be traced with two parameters, and its coefficients and behaviors are more complex. Initial experimentation with the butterfly model’s behaviors show some promise for parameter mapping.

Second, catastrophe theory itself may well be worth examining in music and sound synthesis outside of parameter mapping. Its relevancy in physics to describe complex behaviors resembling resonance point to its potential use in modeling the behavior of musical instruments, in terms of musical information retrieval or parameter estimation techniques.

In effect, catastrophe theory’s implementation herein has only been the initial stages of applying a theory to a new discipline. The scope of this article was necessarily smaller in scope to more carefully explore a single implementation, and without expansion outwards, this topic is not fully explored or tested.
Chapter 3

Deriving New Manifolds for Parameter Mapping from Acoustic Models

Following work done with the expansion of mapping strategies using polynomial defined manifolds as parameter mapping layers, expanding upon the mapping strategies available for simple control interfaces was shown to be a valid exercise. While borrowing equations and models from other fields proved valid and rewarding, a method by which these mappings could be derived from musical or acoustical systems may serve to make many more strategies available.

To this end, a number of acoustic-like systems, specifically physical models of musical instruments, can be probed and explored for behaviors like that of the borrowed models, therefore both validating the use of borrowed systems and yielding a potential for the specification of new mapping strategies and primitives.

3.1 Nonlinearity in Parameter-Sound Space

Current research demonstrates the existence of non-linearities inherent in many musical systems. It is these non-linearities, like the behavior of vibrating reeds and bowed strings for example, that often introduce the nuance of interaction and extended behavior within a musical instrument. The introduction of mappings
derived from these non-linear systems may serve to introduce a greater intuitiveness of control and interesting behavior into signal-based means of sound synthesis.

It is necessary to look at these nonlinearities and how they arise in acoustical systems to better inform our search for these behaviors.

3.1.1 Acoustical Nonlinearity

Using Fletcher’s *The Nonlinear Physics of Musical Instruments* as a guide, nonlinearity in musical instruments can be found in many, if not all, musical instruments.[27] For this work, the focus will be on the family of single reed instruments, but they arise in all manner of instruments, from bowed string to impulse driven instruments to the human voice.

In the single reed instruments, nonlinearity arises almost entirely in the mouthpiece and reed itself. The resonating bore, aside from very minute turbulence effects, is a simple passive linear system without great complexity. However, the relationship between the pressure difference across the reed and volume flow at the inlet of the pipe of the instrument is richly non-linear.

Some pioneering work has been done to describe these non-linear systems. Work by Dalmont in 2003 served to both describe these behaviors and evaluate the techniques in use to model these systems. Beyond the reed, the mouth, interior of the mouthpiece and the coupling of the whole system have been described in great detail.

Several models have been used to describe the non-linear element of the single reed instrument, the mouthpiece/lips/reed system. First, there are elementary models which describe the reed under steady flow conditions, ignoring dynamic parameters and friction, and simplifying the mechanical response. After this early model, the quasi-stationary model of air flow and the quasi-static model of the reed are more current implementations of the reed system.[28] These models continue to evolve today, and serve to highlight the complexity of the non-linearity in the mouthpiece system. In examining this research, it is clear that we will be primarily investigating the reed and air flow of the mouthpiece for cusp-like behavior.
3.2 Exploratory Work

To discover and pinpoint nonlinearities in acoustic instruments that behave similar to the cusp catastrophe, first the attributes of the cusp must be stated explicitly, and those features identified in the behavior of these instruments. To restate previous points, these are:

1. bimodal distributions of behavior for a dynamic input,

2. drastic changes in behavior despite slowly changing control parameters.

We therefore are looking for acoustic instruments that behave in a nonlinear fashion where both a single given state can yield more than a single output, and a slight change in this state can yield drastic change.

Furthermore, there is a feature that will prove important in identifying these features in a system— that previous states influence the current stable state in the bifurcating regions. In other words, the direction from which one approaches an area with potential bimodality influences the output state once the bimodal state is reached.

To begin, a sampling of instruments with known nonlinearities were identified and examined. Specifically, research on bowed strings[29], the human vocal chords[30] and single reed instruments[31][32] were examined to determine the potential for their behaviors to fit the predicted cusped model.

After researching these possible avenues, the behavior of a blown reed was chosen for its clear illustration of the two aforementioned features. In terms of the saxophone, a single keying, blowing pressure and embouchre tightness can exhibit one of at least two sounding notes, and slight changes in any of these performative parameters can yield a drastic change, a jump in octave or chaotic overblowing. Furthermore, approaching these bimodal areas of performance from higher or lower values for the above performance parameters directly influence the sounding note or timbre.

After considering the difficulty and accuracy of current means of measuring acoustic performative parameters, especially considering the difficulty of measuring
those of a blown saxophone, and the relative difficulty in acquiring the necessary resources (a very technically proficient saxophonist, the measurement system, and a sufficient amount of time to gather the necessary sound samples) it was decided that physical models of the systems would be the easiest means by which to capture the exact behavior of the nonlinearities of a blown reed.

To this end, a number of physical models were explored, and a relatively simple clarinet model was chosen as particularly reactive and unstable, and the fact that many physical models are bounded by “performative” space. In other words, after some exploratory tinkering, it was found that many physical models remain quiet or are entirely noise at the extremes of synthesis parameters, and therefore less useful in locating the exact behavior of their nonlinearities.

3.2.1 Methods

In order to identify the exact behavior of the nonlinearities in a system, it was first important to establish bounds for the parameters of blowing pressure and “jet width,” our substitution for embouchre pressure (discovered through exploratory work to be an effective replacement). Alongside these bounds, appropriate fixed values for the models other input parameters were decided upon, based on their effect on the output.

By fixing all but the two desired input parameters, it allowed to imagine the axes of our cusp catastrophe’s control surface $C$ to be these two parameters, and the third observed sonic behavior of our model to determine the third dimension that would comprise our values for the catastrophe manifold $M$.

The next step was to determine what the measured feature of the sonic output would be, more specifically. After exploring several possibilities like pitch, amplitude and certain spectral features, spectral centroid was decided upon as an effective indication of output in relation to the model output’s octave and timbre.

After fixing the experiment’s parameters, the last methodological step was to determine a means of stepping through the parameter values in such a way that the characteristic jumps of the predicted catastrophe may be observed. In the end, an alternating right to left and left to right incrementing scan of the parameter
Figure 3.1: The method of scanning the sound space through the manipulation of two parameters. The above plane is comprised of our two synthesis parameters, here alpha and beta, and our trajectories through their values indicated by the arrows. The goal of this method is to find certain points, indicated by the $x$ on the graph above, in which a drastic change in state occur. If the underlying pattern of these breaks resembles the curve fitted above, we can infer that the behavior of our model is similar to that of the cusped catastrophe.

space, as described in figure 3.2, was decided upon for its ability to detect the jumps and telltale cusped pattern that was predicted.

Many passes, at various fixed parameter values, we recorded, and both sonograms and measurements of spectral centroid were taken, and then compared to predicted patterns.

3.3 Results

After many passes at low fixed parameter settings, it was quickly determined that the chosen model’s non-linearities laid in more extreme values for fixed parameter settings. This presented two challenges- the first that spectral centroid readings were less salient, as the output was largely chaotic during leaps between stable oscillating states, and that sonograms became less legible in the face of noisy output.

However, in terms of finding the characteristic leaps between stable states,
Figure 3.2: A sample of the sonograms generated, with imposed vertical lines at the points of jumping between octaves. By reversing one file and comparing them side by side, the exact point at which they break can be compared, and therefore determined to be in or out of sync. As seen above, they are significantly out of sync, helping bolster the opinion that these areas of bifurcation may behave much like those of the catastrophic cusp model.

The generated sound samples were compared side by side, with one file reversed. From these comparisons, the moments of stability and jumps could be compared in time, to determine their syncronicity. As expected, the jumps do not match, and show a tendency to remain in their given octave, and hold on before jumping. This illustrates the characteristic inertiac non-linearity that we see in the cusp.

However, the exact shape and features of the bifurcating region was mushc
more difficult to reliably determine, for two distinct reasons. For one, the challenges presented by the chaotic and noisy output of the model in legibility of results and accuracy of spectral centroid.

So, the observed behavior is enough to confirm the existence of non-linearities that fulfill some of the aforementioned attributes of the cusp catastrophe. These are encouraging results, although insufficient to concretely confirm the exact shape and features of the present cusp, as much of the spectral centroid data was too noisy for extrapolation into a manifold. Therefore, the goal of implementing a derived manifold, in the place of the ideal/predicted manifold cannot be realized.

3.4 Further Work

After the successes and shortcomings of the experiment, there is certainly further work to be done, both in expanding the previous experiment and working with alternate models.

First, it is important to solidify the previous results, by reexamining the previous experiment with a different single reed model. While the model used was shown to possess the sought for non-linearities, another model may serve to highlight the existing cusp without the noise and uncertainty found in the current model. This would allow for a clean manifold to be derived and implemented in mapping, and allow the features of the cusp to be accurately described.

Second, and maybe most interesting and expansive, is to recreate the experiment with a number of other types of physical modeling algorithms. Illuminating the parameter-sound space dimensions and behavior of their non-linearities would be a rewarding and potentially useful exercise, both in furthering parameter mapping and characterizing various physical models and their behavior.

Third, to gather and analyze real world acoustic behaviors with the same goals. This would prove to confirm any results gained in this or further experimentation. It would also yield more manifolds that may be implemented as parameter mapping layers.
Bibliography


