Title
The LIFO/FIFO Choice as a Signal of Future Costs

Permalink
https://escholarship.org/uc/item/2gp3p3fj

Authors
Bar-Yosef, Sasson
Hughes, Patricia J.
Venezia, Itzhak

Publication Date
1991-03-01
THE LIFO/FIFO CHOICE AS A SIGNAL OF FUTURE COSTS

by

Sasson Bar-Yosef*
Patricia J. Hughes**
and
Itzhak Venezia***

March 1991

* Hebrew University
** University of Southern California
*** University of California, Los Angeles
THE LIFO/FIFO CHOICE AS A SIGNAL OF FUTURE COSTS

Abstract

We explain the puzzling empirical evidence on the inventory accounting choice through a management signalling argument. We assert that firms with lower nominal production costs than other firms have relatively less to gain from the tax advantages associated with LIFO adoption. For these firms, future nominal production costs are lower due to increased efficiency or technological improvements. Because of asymmetric information about future costs, and the moral hazard problem of direct communication, the managers of the more efficient firms are able to credibly communicate their production superiority by using the FIFO accounting method.

We show in this paper that, if firms can choose the fraction of their inventory which is allocated to FIFO and LIFO, then a separating equilibrium exists in which firms with lower production costs signal their greater efficiency by allocating a higher fraction of their inventory to FIFO.
THE LIFO/FIFO CHOICE AS A SIGNAL OF FUTURE COSTS

I. INTRODUCTION

The question of why firms choose to use the FIFO inventory accounting method during inflation has attracted much attention. In times of inflation, when nominal costs are rising, there should be obvious tax advantages from using LIFO. Yet many firms continue to use FIFO\(^1\), and some firms have abandoned LIFO and switched back to FIFO\(^2\). Moreover, the tax consequences of a switch from FIFO to LIFO (under inflation) suggest that, in the absence of asymmetric information, it should be associated with a positive stock price reaction. However, Brown [1980], Ricks [1982], and Hand [1991] show that LIFO adoptions may be accompanied by negative stock price changes.

Sunder [1976] analyzes the conditions under which it is advantageous for a firm to adopt LIFO in the absence of asymmetric information or moral hazard. Sunder's results, however, imply a corner solution: FIFO or LIFO will be used to value 100% of a firm's inventory. Biddle and Lindahl [1982] provide empirical support to this "tax hypothesis". More recently, however, Dopuch and Pincus [1988] report that "LIFO is rarely used for the entire inventory holding of firms; in fact, the typical percentage in our sample was less than 75%." (p. 35). Furthermore, Dopuch and Pincus do not find a statistically significant difference in the average effective tax-rate variable as affecting the LIFO decision.\(^3\)

---

\(^1\) According to Accounting Trends and Techniques [1989], close to 2/3 of the 600 firms surveyed reported the use of FIFO to some extent.

\(^2\) See Johnson and Dhaliwal [1988].

\(^3\) Their results are consistent with those reported by Lee and Hsieh [1985].
In light of these observations, the firm's accounting (and tax) inventory decision is puzzling. Specifically, noting the above empirical results, the following questions have not been satisfactorily addressed:

a) Noting that during inflationary periods some firms use FIFO while others partially adopt LIFO, is there an inducement to the firm's management, in addition to the tax hypothesis, to use the inventory accounting choice as a way of communicating private information about the value of the firm?

b) Is there an optimal percentage of inventory for which to adopt LIFO?

c) If there is an optimal percentage, what factors determine it?

Several papers suggest that the accounting method choice may serve as a signal of inside information about investment and production activities (e.g., Gonedes and Dopuch [1974], Dye [1988], Hughes and Schwartz [1988], Hughes, Schwartz and Thakor [1991], Jung [1989] and Bar-Yosef and Sen [1990]). Hughes and Schwartz suggest that the FIFO choice is a way in which the manager of a high quality firm can communicate this private information because the firm can better afford the reduction in cash flows under the threat of bankruptcy than a lower quality firm. Hughes, Schwartz and Thakor [1988] extend this idea and observe that firms use more than one signal to communicate their values. They show that if firms use capital structure and accounting method as signalling devices, then firms would pool on inventory method and then separate within those pools through capital structure choices. Each firm selects the most efficient signalling mechanism with which to reveal its value.
by trading off the costs of FIFO with the costs of debt. Managers of high-quality firms who find signalling with debt under LIFO to be too costly (because they must employ very high debt levels to discourage mimicry) may find it optimal to impair firm value sufficiently with FIFO so that signalling can be achieved with lower debt.

These theoretical results show that the FIFO choice may be consistent with value maximization and optimal capital structure choice. There is nothing unique, however, about the FIFO choice that makes it a more effective signal than any other "cash burning device". It is therefore the purpose of this article to provide a unique signalling explanation for the use of FIFO. We develop a theoretical model which yields results that provide answers to the questions outlined above. In our model, the firm's manager uses the inventory accounting method choice to signal his/her beliefs about future production costs.

We assume that there is an asymmetry of information between managers and investors concerning future production costs, that managers cannot directly communicate this information without moral hazard, and that managers are delegated to make the inventory accounting choice. While prices and costs rise with inflation, nominal production costs will rise less for the more efficient firms. This implies that the tax advantages from LIFO for these latter firms are not as large as the tax advantages for the less efficient

---

4 See Lee and Hsieh [1985] and Dopuch and Pincus [1988] for empirical evidence relating to this hypothesis.

5 Jung [1989] also provides a model where the FIFO/LIFO choice signals costs. In his model, however, the uninformed party is the firm’s competition and the signal is used to deter entry. In our model there is an information asymmetry between managers and investors.
firms (i.e., firms with higher production costs). The more efficient firms can then communicate their higher value by using the FIFO accounting method.

We show that if firms can choose the fraction of their inventory valued by FIFO and LIFO, then a separating equilibrium exists where firms with lower production costs signal their superiority by using FIFO for a higher fraction of their inventory.

The paper is organized as follows. We present the model in Section II, and prove the existence of the separating equilibrium. Section III provides comparative statistics. Concluding remarks are given in Section IV.

II. THE MODEL

We extend Sunder's [1976] model to include differential production costs by considering an economy in which firms are identical except for their future production costs. That is, all firms purchase and sell the same constant number of units of an identical product at identical selling prices. The firm's FIFO value of inventory at the current time is $X_0$, the FIFO value at the end of a future period $t$ is $X_t$, and the firm (or its inventory) is liquidated at some final future date $T > t$. Costs of production are expected to change in the future due to inflation and technological improvements. We denote the constant (for all firms) inflation rate as $i$ and the constant (for one firm) change in production costs as $c$. While we do not specify the sign of $c$, it is likely to be negative due to technological improvements and/or

---

6 In a different context, Bar-Yosef and Sen [1990] suggest that the inventory accounting choice may affect the firm's net cash flow as it may affect the firm's procurement decision. In their model, however, the inventory accounting choice was not delegated to management.
increased efficiency of the production process over time. Therefore, the
effective rate of change in production costs is \((1+i)(1+c)\). For analytical
purposes, we assume that nominal costs grow by \((1+i+c)^7\). We also assume a
constant nominal discount rate \(r\), and that the corporate tax rate is \(r\). Under
these assumptions it can be shown that:

**Proposition 1:** As \(T\) becomes very large, the net present value of the taxes
saved by using LIFO as compared with FIFO, denoted by \(L\), is:

\[
L = \frac{\tau X_0 i + c}{r - i - c}.
\]  

(1)

**Proof:** From our assumptions it follows that under FIFO the value of
inventories at the end of period \(t\) is:

\[
X_t = X_0 (1+i)^t.
\]  

(2)

During year \(t\), the value of FIFO inventory grows by \((X_t - X_{t-1})\). The value of
LIFO inventory remains \(X_0\) each period because production must equal sales when
both are constant over time\(^8\).

The difference between the LIFO cost of goods sold and the FIFO cost of
goods sold is therefore \((X_t - X_{t-1})\), and FIFO taxes will be higher than LIFO
taxes by \(\tau (X_t - X_{t-1})\) for \(t = 1,2,\ldots,T-1\). In year \(T\), all taxes saved with LIFO
are paid. It hence follows that:

\[
L = \tau \sum_{t=1}^{T-1} \frac{(X_t - X_{t-1})}{(1+r)^t} - \tau \frac{(X_{T-1} - X_0)}{(1+r)^T}.
\]  

(3)

After substitution of (2) into (3), the latter equation becomes:

---

\(^7\) Note that the effective change in costs is less than \(1+i\) if \(c<0\).

\(^8\) Otherwise, over time, inventories will grow to infinity or become zero.
\[ L = \tau \sum_{t=1}^{T-1} \frac{X_0 ((1+i+c)^t-(1+i+c)^{t-1})}{(1+r)^t} - \tau X_0 \frac{((1+i+c)^T-1)}{(1+r)^T}. \]  

After rearrangement of terms and algebraic manipulation we obtain:

\[ L = \tau X_0 \frac{(1+i+c)^T}{(1+r)^T} - \tau X_0 \frac{\sum_{t=1}^{T-1} ((1+i+c)^t-1)}{(1+r)^t} - \tau X_0 \frac{((1+i+c)^T-1)}{(1+r)^T}, \]  

where \( \rho \) is the present value factor of the real production costs, i.e.

\[ \rho = \frac{1+i+c}{1+r}. \]  

Equation (1) is obtained after letting \( T \) go to infinity in (5), and assuming that the real discount rate is higher than its production cost growth rate (i.e., \( r > c+i \), or that \( \rho < 1 \)).

Q.E.D.

Proposition 1 gives the increase in a firm's value due to switching from FIFO to LIFO for valuing all of its inventory. If the firm allocates only a certain proportion \( \alpha \) of its inventory to LIFO, then the increase in value will be \( \alpha L \), or from (1)

\[ \alpha L = \alpha \tau X_0 \frac{(i+c)}{(r-i-c)}. \]  

The above results will next be used in order to establish the value of the firm.

**Proposition 2**: The value of the firm under full information if it allocates a proportion \( \alpha \) of its inventories to LIFO is given by
\[ V^m(c, \alpha) = (1-r)S - X_0 \frac{1 - r - \alpha r(i+c)}{(r-i-c)}. \]  

(8)

**Proof:** The value of the firm under full information is the present value of its sales less the present value of production costs and taxes. Denoting by \( T_F \) and \( T_L \) the present value of taxes that the firm would have paid under FIFO and LIFO, respectively, and by \( S \) the present value of sales, the value of the firm is:

\[ V^m = S - \frac{X_0}{(r-i-c)} - (1-\alpha)T_F - \alpha T_L, \]

(9)

where \( X_0/(r-i-c) \) denotes the present value of production costs.\(^9\) After rearranging terms, (9) becomes:

\[ V^m(c, \alpha) = S - \frac{X_0}{(r-i-c)} - T_F + \alpha(T_F-T_L). \]

(10)

The value of \((T_F-T_L)\) from (1) is substituted into (10), obtaining.

\[ V^m(c, \alpha) = S - \frac{X_0}{(r-i-c)} - T_F + \alpha \frac{rX_0(i+c)}{(r-i-c)}. \]

(11)

Since the present value of FIFO taxes is

\[ T_F = r(S - \frac{X_0}{(r-i-c)}), \]

(12)

substituting (12) into (11) and rearranging gives the result.

Q.E.D.

The tax benefits from LIFO exist when nominal costs are growing, or when \( c > -i \). Since it is clear from equation (8) that \( \partial V^m/\partial \alpha > 0 \) if \( c > -i \), \( \alpha = 1 \) is the optimal choice under full information. In a perfect market, a firm will choose LIFO to value 100% of its inventory, and the corresponding market value of the firm will be

\[ \]

\(^9\) For simplicity we disregard all other operating expenses and assume that other inventory costs and benefits are zero.
\[ v^m = (1 - \tau)S - X_0 \left( \frac{1 - r (1 + c)}{r - 1 - c} \right). \]

If, however, different firms have unique and unobservable costs of production, then investors cannot correctly value individual firms. Investors are likely to know only that the real growth rate in production costs is bounded from above and below by \( c \) and \( \bar{c} \) respectively (i.e., \( c < c < \bar{c} \)). While it is unlikely that investors have direct knowledge of \( c \), the manager who is operating the firm may know \( c \). However, moral hazard problems may preclude the manager from directly revealing \( c \) to investors. We argue that the manager can communicate the value of \( c \) through his or her choice of \( (1 - \alpha) \), the proportion of inventory valued under FIFO. The manager's (correct) valuation of the firm is expression (8), and the value that investors infer from the signal \( \alpha \) is given by:

\[ V^I[c(\alpha), \alpha] = (1 - r)S - X_0 \left( \frac{1 - r - \alpha r c(\alpha)}{r - 1 - c(\alpha)} \right). \]  

(13)

In (13), \( c(\alpha) \) represents the investors' inference of the unobservable costs from the observed signal \( \alpha \).

We assume that the objective function of the managers is given by:

\[ \max_{\alpha} \phi^m(\alpha) = \lambda V^I[c(\alpha), \alpha] + (1 - \lambda)V^m(c, \alpha), \]  

(14)

where \( 0 \leq \lambda \leq 1 \). That is, the compensation of the manager is proportional to a weighted average of the value of the firm as seen by the investors and the "true" value of the firm.\(^\text{10}\) This objective function is proportional to:

\[ \max_{\alpha} \phi(\alpha) = V^I[c(\alpha), \alpha] + \beta V^m(c, \alpha), \]  

(15)

\(^{10}\) This type of objective function has been widely used in the literature (see, for example, Ross [1977], Miller and Rock [1985], and Hughes and Schwartz [1988]).
where $\beta = (1-\lambda)/\lambda$.

The manager will choose the signal $\alpha$ in order to maximize compensation, or maximize $\phi(\alpha)$ in (15). A separating equilibrium will be attained if an inference function $c(\alpha)$ can be determined such that, for all $c$,

(i) $c(\alpha) = c$, and

(ii) $\alpha$ maximizes $\phi(\alpha)$ in (15).\(^\text{11}\)

In what follows we derive a cost inference function which satisfies these conditions.

**Proposition 3:** There exists a separating equilibrium, which satisfies the conditions for Riley's reactive equilibrium, such that, for any $c \leq c \leq \tilde{c}$, the firm will choose an $\alpha(c)$ from which the investors will infer that the growth in costs is $c(\alpha)$. This signal $\alpha$ will maximize the managers compensation and will be fully revealing. The function $c(\alpha)$ is given by:

$$c(\alpha) = \frac{Kr}{K + (1-\tau-\alpha r)^{1+g}} - i,$$

where

$$K = \frac{(c+1)}{(r-c-i)} (1-\tau-\tau r)^{1+g}.\quad (17)$$

The optimal signal $\alpha$ is given by:

$$\alpha(c) = \frac{1-\tau}{\tau r} - \left\{ \frac{(c+1)/(r-c-i)}{(c+1)/(r-c-i)} \right\}^{1/(1+g)} \frac{1-\tau}{\tau r} - 1\right\}$$

where we assume that $(1-\tau)/\tau r > 1$.

**Proof:** The proof consists of showing that there exists an inference function $c(\alpha)$ that will induce separation.

\(^{11}\) Other necessary technical conditions are later discussed.
The manager chooses \( \alpha \) so as to maximize (15). The first order condition to the manager's problem is:

\[
\frac{d\phi(\alpha)}{d\alpha} = -x_0 \left\{ \frac{[r-i-c(\alpha)][-\tau(i+c(\alpha)) - \alpha c'(\alpha)]}{[r-i-c(\alpha)]^2} + \frac{c'(\alpha)[1-\tau-ar(i+c(\alpha))]}{[r-i-c(\alpha)]^2} - \frac{\beta r(i+c)}{r-i-c} \right\} = 0.
\]

(19)

Since \( c(\alpha) = c \) in equilibrium, \( c(\alpha) \) can be substituted for \( c \) in (19). This substitution and some rearranging of terms yields the following differential equation:

\[
c'(\alpha) = \frac{\tau(1+\beta)[r-i-c(\alpha)][i+c(\alpha)]}{1-\tau-ar}
\]

(20)

The solution of this differential equation is given (for the proof see Appendix A) by equation (16), where \( K \) is a constant of integration and is obtained as follows. Since there is no benefit to be gained from signalling for that firm with the highest possible growth in costs, the firm with \( c = \bar{c} \) will choose \( \alpha = 1 \) in equilibrium. It therefore follows, by substituting 1 for \( \alpha \) and \( \bar{c} \) for \( c \) in (16) that:

\[
c(\alpha=1) = \frac{Kr}{K+(1-\tau-r\bar{r})}, \quad i = \bar{c}.
\]

(21)

The solution for \( K \) from (21) is given by equation (17). The optimal value of \( \alpha \) as given in equation (18) is obtained by solving equation (17) for \( \alpha \), replacing \( c(\alpha) \) by \( c \), and substituting for \( K \) from (17). Riley (1979) provides additional conditions that must be satisfied for a separating equilibrium to be viable when there is a continuum of types. Since it is shown in Appendix B that these conditions hold our proof is now complete.

Q.E.D.

Proposition 3 demonstrates that the firm can signal its quality (i.e. its growth/decline in costs) using the proportion of inventory allocated to
LIFO/FIFO as a signal.

In the following section comparative statics analysis will examine the effect of several exogenous parameters on the value of the equilibrium signal.

III. COMPARATIVE STATIC ANALYSIS

According to equation (18), the proportion of inventory allocated to LIFO, $\alpha$, depends on the following parameters: (a) $c$, the rate of change in real costs; (b) $\lambda$, the weight the manager assigns to investors' inference from the signal; (c) $r$, the tax rate; (d) $\bar{c}$, the maximum growth in real costs; (e) $i$, the inflation rate, and (f) $r$, the discount rate.

Proposition 4: The choice of $\alpha$ (from equation (18)) is such that it is:

(a) increasing in the growth in costs: $\partial \alpha / \partial c > 0$;

(b) decreasing in the weight placed on the market inference of value:

$\alpha / \partial \lambda < 0$;

(c) increasing in the tax rate: $\partial \alpha / \partial r > 0$;

(d) decreasing in the maximum possible growth rate in costs: $\partial \alpha / \partial \bar{c} < 0$;

(e) decreasing in the rate of inflation: $\partial \alpha / \partial i < 0$; and

(f) increasing in the discount rate: $\partial \alpha / \partial r > 0$.

Proof: See Appendix C.

The basic idea in the signalling argument is that more efficient firms (i.e., firms with lower $c$) lose less from FIFO than the less efficient firms (i.e., those with a higher $c$). The more efficient firms would then signal their quality by allocating more of their inventory to FIFO rather than LIFO. It follows then, that in order for signalling to be effective, the proportion of inventory allocated to FIFO, $1-\alpha$, should be a decreasing function of $c$. 

11
Indeed (a) establishes this result.

The emphasis that a manager gives to investors' beliefs in the objective function (14) is represented by \( \lambda \). The higher is \( \lambda \), the more the manager cares about the inferences made from \( \alpha \). It follows that the higher \( \lambda \), the greater is the amount of taxes that the manager will be willing to sacrifice to signal their quality; and hence the higher will be the proportion of inventories allocated to FIFO, and the lower the proportion \( \alpha \) allocated to LIFO. The result in (b) agrees with this intuition.

The higher the tax rate \( r \), the more costly it is to signal. We would then expect less inventory allocated to FIFO (which is costly in tax terms) and more to LIFO. Result (c) that \( \partial \alpha / \partial r > 0 \) confirms this idea.

Since a firm that does not signal (i.e., \( \alpha = 1 \)) is assumed to be the worst firm, a higher \( \bar{c} \) implies that a firm has more to lose by not signalling. A higher \( \bar{c} \) will hence simply be a stronger incentive to signal low costs and hence a higher allocation to FIFO and a lower allocation to LIFO. This is the result obtained in (d).

FIFO is more costly under inflation. We might therefore expect that a higher inflation rate will be associated with more inventory allocated to LIFO. However, an increase in inflation has another offsetting effect. When inflation increases, then the maximum nominal costs \((\bar{c}+i)\) that a firm might incur also rise. An increase in inflation has, therefore, an additional effect similar to that of an increase in \( \bar{c} \), which tends to decrease the proportion allocated to LIFO. It turns out that this effect is stronger than the former one, and increasing inflation will be accompanied by less inventory allocated to LIFO and more to FIFO. This result is surprising, but may explain why the empirical correlation between LIFO adoptions and inflation found by Stevenson
[1987] and Biddle and Lindahl [1982] is not as strong as one might expect in
the absence of signalling.

Finally, when the discount rate $r$ is high, the present value of taxes
which are deferred under LIFO becomes small. Therefore the tax costs of FIFO
are relatively greater when $r$ is high, implying that more inventory will be
allocated to LIFO for a high discount rate. Result (f) is consistent with
this intuition.

IV. CONCLUSION

We have shown that firms may signal their production costs with their
LIFO/FIFO choice. Firms with lower production costs suffer less tax
disadvantages from using FIFO than firms with higher production costs. The
more efficient firm can signal its value by allocating a higher fraction of
inventory to FIFO. Since there is no demand for signalling when future
production costs are predictable and certain, we expect to see higher FIFO
usage in industries where production costs are uncertain. Examples of such
industries are the electronic equipment industry, business equipment and
drugs. In these industries, innovations render future costs to be highly
dependent on R&D results which in turn are very speculative. The model
presented herein suggests that such firms will tend to use more FIFO than
industries where production innovations are less frequent such as
merchandising, foods, beverages, tobacco, paper, etc. Indeed, in a survey of
accounting methods it was found that only 21% of the firms in the electronic
equipment industry used LIFO, 41% in the electric equipment industry, and 57%
in drugs. On the other hand, 100% of department stores, 100% of beverage
companies and 75% of tobacco used LIFO.\textsuperscript{12}

Hand [1991] provides empirical results which lend support to our model in a unique empirical test of the stock market reaction to both LIFO adoptions and non-adoptions. He obtained a sample of FIFO firms that announced they were considering adopting LIFO. He then examines the market reaction to the subsequent earnings announcement in which the inventory method choice was first disclosed. After controlling for unexpected earnings, he finds a statistically significant positive reaction to the announcements for those firms remaining at FIFO and a statistically significant reaction to the announcements of LIFO adoption. He concludes that the LIFO/FIFO choice may be informative about future inflation and that investors are not naively reacting to changes in reported income numbers.

\textsuperscript{12} See Accounting Trends and Techniques [1989].
APPENDIX A

Solution of the differential equation (20)

The differential equation is separable and hence can be solved by standard techniques (For example, see Boyce and Diprima [1970]). For this, note that the equation can be written as:

\[ M(\alpha)\,d\alpha = -N(c)\,dc, \quad (A1) \]

where

\[ M(\alpha) = \frac{1}{[1-r_0r^r]}, \quad \text{and} \quad (A2) \]

\[ N(c) = -\frac{1}{[r(1+\beta)(c+i)(r-i-c)]}. \quad (A3) \]

The solution to (A1) is the implicit function:

\[ H_1(\alpha) + H_2(c) = K_0, \quad (A4) \]

where \( K_0 \) is an arbitrary constant and

\[ H_1(\alpha) = H_1(\alpha_0) + \int_{\alpha_0}^{\alpha} M(t)\,dt, \quad (A5) \]

\[ H_2(c) = H_2(c_0) + \int_{c_0}^{c} N(t)\,dt. \quad (A6) \]

Substituting (A2) and (A3) into (A5) and (A6), and carrying on the integration yields the implicit equation:

\[-\frac{1}{r} \ln[1-r_0r^r] - \frac{1}{r(1+\beta)} \ln[(c+i)/(r-i-c)] = -K_0r. \quad (A7)\]

Rearranging terms in (A7), and defining \( K = \exp \left[ -K_0(1+\beta)/r \right] \) yields the result.

Q.E.D.
APPENDIX B

Proof that Riley's conditions hold

Riley's conditions (Riley [1979] pp. 334-335) are the following:

(i) The unobservable attribute is distributed on \((c, \bar{c})\) according to a
strictly increasing distribution function.

(ii) The two functions
\[
\phi(c, \alpha) = \lambda V^I(c(\alpha), \alpha) + (1-\lambda) V^m(c, \alpha), \text{ and}
\]
\[V^m(\cdot)\]
are infinitely differentiable in all variables.\(^{13}\)

(iii) \(\frac{\partial \phi}{\partial V^I} > 0.\)

(iv) \(\frac{\partial \phi}{\partial c} < 0.\)

Note that the sign in (iv) is opposite from that in Riley because the
quality variable \(c\) is a cost term, and therefore not a utility-
increasing attribute as is the case with Riley.

(v) \(\frac{\partial}{\partial c} \left(\frac{\partial \phi}{\partial (-\alpha)}\right) > 0.\)

In (v), the signal is \(-\alpha\) because \(1-\alpha\) is the proportion of inventory on
FIFO. Again, for the same reason as in (iv), the sign is opposite that
of Riley.

(vi) \(\phi\) has a unique maximum over \(\alpha.\)

Conditions (i) to (iii) are easy to verify. Condition (iv) is verified by
taking the derivative of (8) with respect to \(c:\)

\[
\frac{\partial V^m}{\partial c} = -X_0 \frac{1-r-\alpha r}{(r-i-c)^2}
\]

\(^{13}\) For the purposes of this paper the existence of second derivatives
suffices.
Since we have assumed that $1-\tau-\alpha r > 0$, then $\partial v^m/\partial c < 0$, and hence $\partial \phi/\partial c < 0$.

From equations (19) and (81) it follows that

$$\frac{\partial}{\partial c} \left( \frac{\partial \phi/\partial (-\alpha)}{\partial \phi/\partial v^l} \right) = \tau r > 0.$$

Therefore (v) is also satisfied. Condition (vi) can be proven by showing that the $\alpha$ which maximizes $\phi(\alpha)$ is unique. For this, the second derivative $\partial^2 \phi(\alpha)/\partial \alpha^2$ (obtained by differentiating (19)) must be examined. It can be shown that this derivative is negative and the proof is complete.

Q.E.D.
APPENDIX C

Proof of Proposition 4

From (18), α can be written as:

\[ \alpha = B - A^\lambda (B-1), \]  

(C1)

where

\[ B = (1-r)/rr > 1, \]

and

\[ A = \frac{[(c+i)/(r-c-i)]}{[(c+i)/(r-i-c)]}. \]  

(C2)

The proof consists of taking the derivatives of α with respect to c, λ, r, c, i, and r.

To show (a),

\[ \frac{\partial \alpha}{\partial c} = -(B-1)\lambda (\partial A/\partial c)A^{\lambda-1}, \]  

and

\[ \frac{\partial A}{\partial c} = \frac{c+i}{r-c-i} - \frac{(c+i)-(r-c-i)}{(c+i)^2}, \]

(C3)

(C4)

and hence \( \frac{\partial \alpha}{\partial c} > 0 \), which establishes (a).

To show (b),

\[ \frac{\partial \alpha}{\partial \lambda} = -(B-1)A^{\lambda} \ln A. \]  

(C5)

Since B > 1 and A > 1, it follows that \( \frac{\partial \alpha}{\partial \lambda} < 0 \), thereby establishing (b).

To show (c),

\[ \frac{\partial \alpha}{\partial r} = (1-A)\partial B/\partial r, \]  

and

\[ \frac{\partial B}{\partial r} = -1/rr < 0. \]

(C6)

(C7)

Since A > 1, it follows that \( \frac{\partial \alpha}{\partial r} > 0 \) and (c) is proved.

In order to prove (d),

\[ \frac{\partial \alpha}{\partial c} = -\lambda (\partial A/\partial c)A^{\lambda-1}(B-1), \]  

(C8)

\[ \frac{\partial A}{\partial c} = \frac{r-c-i}{c+i} \cdot \frac{\partial [((c+i)/(r-c-i))]}{\partial c}. \]
\[
= \frac{r-c-i}{c+i} \cdot \frac{(r-\bar{c}-i)+(\bar{c}+i)}{(r-\bar{c}-i)^2} > 0,
\]
and therefore \(\delta a/\delta \bar{c} < 0\), thus proving (d).

Condition (e) becomes
\[
\delta a/\delta i = -\lambda A(\delta A/\delta i)\Lambda^{-1}(B-1).
\] (C10)

The sign of \((\delta A/\delta i)\) is examined by analyzing the sign of \(\delta \ln A/\delta i\).

\[
\delta \ln A/\delta i = \frac{1}{c+i} - \frac{1}{c+i} + \frac{1}{r-c-i} - \frac{1}{r-c-i} = \frac{c+i-\bar{c}-i}{(c+i)(c+i)} + \frac{r-c-i-(r-\bar{c}-i)}{(r-\bar{c}-i)(r-c-i)}
\]
\[
= \frac{c-\bar{c}}{(c+i)(c+i)} + \frac{\bar{c}-c}{(r-\bar{c}-i)(r-c-i)} = (\bar{c}-c)\left\{\frac{1}{(c+i-r)(c+i+r)} - \frac{1}{(c+i)(c+i)}\right\}
\]
\[
= (\bar{c}-c)\left\{\frac{1}{(c+i-r)(c+i+r)} - \frac{1}{(c+i)(c+i)}\right\} > 0.
\] (C11)

Since \(\delta \ln A/\delta i > 0\), \(\delta A/\delta i\) is positive and it follows from (C10) that \(\delta a/\delta i < 0\), and (e) is proved.

To show (f),
\[
\delta a/\delta r = \delta B/\delta r - \lambda A^{-1} \frac{\delta A}{\delta r}(B-1) - A^{-1} \frac{\delta B}{\delta r}
\] (C12)

Note that:
\[
\delta B/\delta r = -(1-r)/rr^2 < 0,
\] (C13)
\[
\delta A/\delta r = \frac{c+i}{c+i} \cdot \frac{(r-\bar{c}-i)-(r-c-i)}{(r-\bar{c}-i)^2} < 0.
\] (C14)

Since \(\lambda > 0\) and \(A > 1\), it follows that \(\delta a/\delta r > 0\) and (f) is established.

Q.E.D.
REFERENCES


