Singlet Extensions of the MSSM with $Z_4^R$ Symmetry

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We discuss singlet extensions of the MSSM with $Z_4^R$ symmetry. We show that holomorphic zeros can avoid a potentially large coefficient of the term linear in the singlet. The emerging model has both an effective $\mu$ term and a supersymmetric mass term for the singlet $\mu_N$ which are controlled by the gravitino mass. The $\mu$ term turns out to be suppressed against $\mu_N$ by about one or two orders of magnitude. We argue that this class of models might provide us with a solution to the little hierarchy problem of the MSSM.

1. Purpose of This Paper

The $Z_4^R$ symmetry [1, 2] provides us with compelling solutions of the $\mu$ and proton decay problems of the minimal supersymmetric extension of the standard model (MSSM). This symmetry appears anomalous, but the anomaly is cancelled by the (discrete) Green-Schwarz (GS) mechanism [3] in such a way that it does not spoil gauge coupling unification (see, e.g., [4] for a discussion). More precisely, if one extends the MSSM by a symmetry (continuous or discrete) that solves the $\mu$ problem and (i) demands anomaly freedom (while allowing GS anomaly cancellation), (ii) demands that the usual Yukawa couplings and the Weinberg operator be allowed, (iii) demands consistency with $SO(10)$ grand unification, and (iv) demands precision gauge coupling unification, then this $Z_4^R$ is the unique solution [2] (see also [5] for an alternative proof). By relaxing (iii) to consistency with $SU(5)$, one obtains four additional symmetries [6]. Further, $Z_4^R$ can be thought of as a discrete remnant of the Lorentz symmetry of compact extra dimensions; that is, it has a simple geometric interpretation and can arise in explicit string-derived models with the precise MSSM matter content [7]. The charge assignment is very simple: MSSM matter superfields have $Z_4^R$ charge 1 while the Higgs superfields have 0, and the superpotential $W$ carries $R$ charge 2.

However, if one attempts to construct singlet extensions of the $Z_4^R$ MSSM, one faces the problem that the presence of superpotential coupling of the singlet $N$ to the Higgs bilinear $H_uH_d$ implies that also a linear term in the singlet is allowed by all symmetries. In more detail, since the Higgs bilinear has $Z_4^R$ charge 0, the singlet $N$ needs to carry charge 2 in order to match the $Z_4^R$ charge 2 of the superpotential. Then the desired term $W \subset NH_uH_d$ is allowed. However, in this case one might expect to have a problematic, unsuppressed linear term in $N$ in the (effective) superpotential,

$$W_{\text{eff}} \supset \Lambda^2 N,$$

with $\Lambda$ of the order of the fundamental scale. In order to forbid this linear term, one may try to add a new symmetry. It is quite straightforward to see that an ordinary symmetry cannot forbid this linear term and be, at the same time, consistent with criteria (i)–(iv) above: in order to forbid the linear term, the singlet $N$ needs to carry a nontrivial charge under the new symmetry. But, as we want the term $NH_uH_d$, this implies that also $H_uH_d$ carries a nontrivial charge.
Consequently, the new symmetry would yield a solution to the $\mu$ problem. However, this is not possible: as stated above, one can prove that (under our assumptions) the unique solution to the $\mu$ problem is $Z_4^R$, and this symmetry does not forbid the linear term.

In this paper, we take an alternative route and describe how one can get rid of the linear term (1) by employing holomorphic zeros [8] associated with an additional pseudoanomalous $U(1)$ gauge symmetry.

2. Forbidding the Linear Term in the $Z_4^R (G)$ NMSSM

2.1. Setup. Consider a singlet extension of the MSSM with a singlet $N$ and an additional $Z_4^R \times U(1)_{\text{anom}}$ symmetry. $U(1)_{\text{anom}}$ is pseudoanomalous $U(1)$ symmetry, whose anomaly is cancelled by the GS mechanism. Such $U(1)$ factors often arise in string compactifications and are accompanied by nontrivial Fayet-Iliopoulos (FI) term [9] $\xi$, which arises at 1-loop [10]. The FI term of the $U(1)_{\text{anom}}$ is assumed to be cancelled by a nontrivial vacuum expectation value (VEV) of a “flavon” $\phi$, which carries negative $U(1)_{\text{anom}}$ charge and $Z_4^R$ charge 0. Without loss of generality, we can normalize $U(1)_{\text{anom}}$ such that $\phi$ has charge $-1$ and $\xi > 0$. (Of course, in true string-derived models the situation is usually more complicated: in approximately 500 out of a total of 11940 MSSM-like models from [11] the FI term can be cancelled with one field only. In all other models, one would have to identify $\phi$ with an appropriate monomial of MSSM singlet fields (see Appendix A for details)). For the sake of definiteness, we assume that

$$\epsilon := \frac{\langle \phi \rangle}{M_p} \sim \sin \theta_{\text{Cabibbo}} \sim 0.2,$$

(2)

where the Planck scale $M_p$ is identified with the “fundamental scale.” In this case, $U(1)_{\text{anom}}$ can be used as Froggatt-Nielsen symmetry [12] to explain the flavor structure of quarks and leptons. However, this assumption is not crucial for the subsequent discussion, yet this is what one gets in explicit orbifold compactifications of the heterotic string which exhibit the exact MSSM spectrum at energies below the compactification scale.

Further, also the anomaly of $Z_4^R$ is assumed to be cancelled by the GS mechanism with the GS axion being contained in the dilaton or another superfield, which we will denote by $S$. Since the mixed $U(1)_{\text{anom}} - G_5^SM$ and $Z_4^R - G_5^SM$ anomalies are universal, the GS mechanism does not interfere with the beautiful picture of MSSM gauge coupling unification (see, e.g., [4]). The “nonperturbative” term $e^{-bs}$ carries the same $Z_4^R$ charge as the superpotential, namely, 2. It might be thought of as some nonperturbative hidden sector (see, e.g., [13]). Further, $e^{-bs}$ will also carry positive $U(1)_{\text{anom}}$ charge $s > 0$ such that holomorphic zeros get lifted by “nonperturbative” terms. More details on the charge of $e^{-bs}$ can be found in Appendix B (see, e.g., [6, 14]). In more detail, we demand that

$$\mathcal{W}_{\text{hid}} \sim M_p^2 \left( \frac{\phi}{M_p} \right)^s e^{-bs}$$

(3)

be allowed, which is equivalent to the statement that $e^{-bs}$ carries $U(1)_{\text{anom}}$ charge $s > 0$. (Note that $s$ may also be fractional even if the charges of all “fundamental” fields are integer, for instance, if one assumes that $\mathcal{W}_{\text{hid}}$ is given by the Affleck-Dine-Seiberg superpotential [15]. Examples for such terms can be found, e.g., in [13,].) $\mathcal{W}_{\text{hid}}$ may be thought of as gaugino condensate [16] or some other nonperturbative physics, such as the one discussed in [17], which is involved in spontaneous supersymmetry breaking. We discuss this in more detail in Appendix B. Inserting the $\phi$ VEV we obtain

$$\mathcal{W}_{\text{hid}} \rightarrow \phi \rightarrow M_p^2 m_{3/2}$$

(4)

in Planck units. (Note that (3) is not the “full” hidden sector superpotential. One must, of course, make sure that $\phi$ does not attain an $F$-term VEV, and one needs to cancel the vacuum energy. A detailed discussion of these issues is, however, beyond the scope of the present paper.) This implies, in particular, that

$$\left( e^{-bs} \right) \sim \frac{m_{3/2}}{M_p} e^{-s}.$$

(5)

That is, $R$ symmetry breaking is controlled by the gravitino mass, as it should be, and due to the presence of $U(1)_{\text{anom}}$ we obtain a Froggatt-Nielsen-like [12] modification of the terms. However, in contrast to the usual Froggatt-Nielsen mechanism, it yields in our setup an enhancement rather than a suppression factor for the lifting of the holomorphic zeros by nonperturbative effects.

2.2. Charges and Allowed Terms in the Superpotential. We summarize the $U(1)_{\text{anom}}$ and $Z_4^R$ charges in Table 1.

Below the $U(1)_{\text{anom}}$ breaking scale set by the $\phi$ VEV, we wish to have a nontrivial $\mu$ term at the nonperturbative level; that is,

$$\mathcal{W}_{\text{eff}} \supset M_p e^{-bs} \left( \frac{\phi}{M_p} \right)^{s+h} H_u H_d.$$

(6)

This implies

$$s + h \geq 0.$$  

(7)

We will then get effective

$$\mathcal{W}_{\text{eff}} \supset M_p e^{-bs} \left( \frac{\langle \phi \rangle}{M_p} \right)^{s+h} H_u H_d =: \mu H_u H_d$$

(8)

with $\mu \sim m_{3/2} e^{-h}$.

Next, we wish to couple the singlet $N$ to the Higgs bilinear. We hence demand that

$$n + h \geq 0.$$  

(9)
such that

\[
\mathcal{W}_{\text{eff}} \supset \left( \langle \phi \rangle / M_P \right)^{n+h} N H_u H_d \sim e^{n+h} N H_u H_d
\]

\[
= \lambda N H_u H_d \quad \text{with } \lambda \sim e^{n+h}.
\]

Now we wish to forbid the linear term in \( N \) at the perturbative level. This can be achieved with holomorphic zeros [8], which amounts in our setup to demanding that

\[
\frac{1}{n} < 0.
\]

This implies, in particular, that the cubic term in \( N \) is also forbidden.

Of course, this all works only if we make sure that \( \phi \) rather than \( N \) cancels the FI term. This might be achieved by postulating that the soft mass squared of \( N \) is positive while the one of \( \phi \) is negative; that is,

\[
m_{\phi}^2 < 0,
\]

\[
m_{N}^2 > 0.
\]

Full justification of such an assumption would require deriving the setting from some UV complete construction such as a string model. This is, however, beyond the scope of this paper.

We further obtain nonperturbative terms which are linear or quadratic in \( N \) if \( n + 2s \geq 0 \) or \( 2n + s \geq 0 \), respectively. Altogether we have

\[
n + h \geq 0
\]

\[
\quad \iff \text{coupling } \lambda \text{ between } N \text{ and } H_u H_d \text{ with } \lambda \sim e^{n+h},
\]

\[
n < 0 \quad \iff \text{suppress linear term in } N,
\]

\[
s + h \geq 0
\]

\[
\quad \iff \mu \text{ term with } \mu \sim M_p e^{n+h} e^{-bs} \sim e^{bs} m_{3/2}.
\]

\[
n + 2s \geq 0
\]

\[
\quad \iff f^2 N \text{ term with } f \sim M_p e^{(n+2s)/2} e^{-bs} \sim e^{n+2s} m_{3/2},
\]

\[
2n + s \geq 0
\]

\[
\quad \iff \mu N m_N^2 \text{ term with } \mu N \sim M_p e^{2n+s} e^{-bs} \sim e^{2n+s} m_{3/2},
\]

\[
3n + 2s \geq 0
\]

\[
\quad \iff \kappa N^3 \text{ term with } \kappa \sim e^{3n+2s} (e^{-bs})^2 \sim e^{3n+s} m_{3/2} / M_p^2
\]

where the coefficient \( \kappa \) of the cubic term is generically highly suppressed. Not all conditions on \( \{n, h, s\} \) are independent; for example, if the quadratic term is allowed also, since \( s > 0 \), the linear term will be present.

There are many possible values that satisfy all the constraints; for instance, \( \{n, h, s\} = \{-1, 1, 2\} \), which gives us

\[
\lambda \sim 0 \quad (1),
\]

\[
\mu \sim e m_{3/2},
\]

\[
\mu_N \sim 1 / e^3 m_{3/2},
\]

\[
f \sim 1 / \sqrt{e} m_{3/2}.
\]

That is, the (holomorphic) \( \mu \) term is roughly two orders of magnitude smaller than \( \mu_N \), which might be favorable in view of the so-called “little hierarchy problem.”

Note also that the effective superpotential

\[
\mathcal{W}_{\text{eff}} = f^2 N + \mu H_d H_u + \lambda N H_d H_u + \mu_N N^2
\]

admits two solutions to the \( F^- \) and \( D \)-term equations, the first one being (recall that \( n < 0 \))

\[
\langle N \rangle = -\frac{\mu}{\lambda} \sim -\epsilon |n| m_{3/2},
\]

\[
\langle H_u \rangle = \langle H_d \rangle = \frac{\sqrt{2\mu \mu_N - \lambda f^2}}{\lambda} \sim e^{-b_{3/2}} m_{3/2}.
\]

Here one has electroweak symmetry breaking prior to supersymmetry breaking, and the Higgs VEV may be subject to cancellations since both \( \mu \mu_N \) and \( \lambda f^2 \) are of the order \( \epsilon^{3n-h} \), for example, \( \epsilon^{-1} \) in our example. The second solution is

\[
\langle N \rangle = -\frac{f^2}{2\mu_N} \sim -\frac{1}{2} \epsilon^{|n|} m_{3/2},
\]

\[
\langle H_u \rangle = \langle H_d \rangle = 0
\]

with unbroken electroweak symmetry for unbroken supersymmetry.

2.3. Discussion. In summary, we find that the \( \mathbb{Z}_R^3 \times U(1)_{\text{anom}} \) charge assignment of Table 1 yields an effective superpotential,

\[
\mathcal{W}_{\text{eff}} = f^2 N + \mu H_d H_u + \lambda N H_d H_u + \mu_N N^2,
\]

with all the dimensionful parameters \( \mu, \mu_N \), and \( f \) of the order of the gravitino mass \( m_{3/2} \). This description is valid below the U(1)$_{\text{anom}}$ breaking scale, which is set by the flavon VEV \( \langle \phi \rangle \). In particular, the linear term in the singlet is sufficiently suppressed. In contrast to the original (G)NMSSM [18], here,

(i) there is (essentially) no cubic term in \( N \);

(ii) there is a suppressed linear term in \( N \). (Note that, unlike in [18], we cannot shift the singlet in order to eliminate the linear term because the point \( N = 0 \) is special as it denotes the point of unbroken \( \mathbb{Z}_R^3 \).)
The scheme leads to certain predictions and expectations:

1. Forbidding the linear term by holomorphic zeros implies the absence of a perturbative cubic term in $N$.

2. Further, we obtain the “little hierarchies” (recall that $n < 0$)

$$
\mu \sim \epsilon^{n+2|\eta|} \mu_N, \\
f \sim \frac{\mu}{\epsilon^{n+|\eta|/2}}.
$$

(19)

2.4. Further Applications. Clearly, this method of avoiding a linear term in a gauge singlet may find further applications. For instance, in model building one sometimes introduces so-called “driving fields” in order to “explain” a certain structure of flavon VEVs. Here, one may forbid too large tadpole terms in the same way as we have discussed above.

3. Discussion

We have discussed how to build singlet extensions of the MSSM with $Z^N_2$ symmetry. We have shown that a potentially large linear term in the singlet can be avoided by using holomorphic zeros. The resulting model has a $\mu$ term, a supersymmetric mass of the order of the gravitino mass $m_{3/2}$, as well as a coefficient of an effective linear term in the singlet of the order $m_{3/2}^2$. $\mu$ is expected to be one or two orders of magnitude smaller than $\mu_N$. This might be viewed as the first step towards a solution to the little hierarchy problem; that is, explain why the electroweak scale is at least one order of magnitude smaller than the soft supersymmetric terms. Obtaining a complete solution requires the derivation of our setting from a UV complete model, which allows us to compute various terms precisely. This, however, is beyond the scope of this paper.

Appendices

A. Cancellation of the FI Term

In this appendix, we discuss how the FI term gets cancelled by a single monomial $M$. The generalization to the case of several monomials is straightforward. We consider a monomial of chiral superfields $\phi_i$, which are assumed to be standard model singlets,

$$
M = \prod_i \phi_i^{n_i},
$$

(A.1)

with $n_i \in \mathbb{N}$. $M$ is constructed to be gauge invariant with respect to all gauge symmetries except the “anomalous” $U(1)_{anom}$. In a supersymmetric vacuum one then has

$$
\langle \phi_i \rangle = v_i,
$$

(A.2)

where $v$ is determined from the requirement that the FI term $\xi > 0$ in the $D$-term potential of the anomalous $U(1)_{anom}$ gets cancelled. That is,

$$
0 = D_{anom} = \xi + \sum_i Q^{(i)}_{anom} |\langle \phi_i \rangle|^2
$$

(A.3)

that is,

$$
\nu = \sqrt{\frac{\xi}{\sum_i Q^{(i)}_{anom} n_i}}.
$$

(A.4)

On the other hand, the “anomalous” charge of the monomial $M$ is

$$
Q_{anom}(M) = \sum_i Q^{(i)}_{anom} n_i < 0.
$$

(A.5)

Hence, we obtain

$$
\langle \phi_i \rangle = \sqrt{n_i} \nu \sqrt{\frac{\xi}{Q_{anom}(M)}}.
$$

(A.6)

That is, if one compares the cases in which (i) the FI term $\xi$ is cancelled by a single field and (ii) the FI term is cancelled by a monomial, there are $\sqrt{n_i}$ factors that enhance the flavon VEVs somewhat in case (ii).

B. Nonperturbative Terms in the Superpotential

In this appendix we discuss how to compute the $U(1)_{anom}$ charge of the nonperturbative term $e^{-i\alpha}$ in the case that the anomaly of $U(1)_{anom}$ is cancelled via the universal Green-Schwarz mechanism. We follow the notation of Appendix A.2 in [6].

The Kähler potential of the dilaton $S$ reads

$$
K(S, S^\dagger, V) = - \ln \left(S + S^\dagger - \delta_{GS} V \right).
$$

(B.1)

Then, under $U(1)_{anom}$ gauge transformations with gauge parameter $\Lambda(x)$, the $U(1)_{anom}$ vector field $V$ and the dilaton $S$ shift according to

$$
V \mapsto V + \frac{i}{2} \left( \Lambda(x) - \Lambda(x)^\dagger \right),
$$

(B.2a)

$$
S \mapsto S + \frac{i}{2} \delta_{GS} \Lambda(x),
$$

(B.2b)

such that $K(S, S^\dagger, V)$ is invariant. Furthermore, in order to cancel the cubic anomaly $A_{U(1)_{anom}^3}$, the constant $\delta_{GS}$ has to satisfy

$$
\delta_{GS} = \frac{1}{2\pi^2} A_{U(1)_{anom}^3} = \frac{1}{6\pi^2} \text{tr} Q_{anom}^3.
$$

(B.3)
where the trace sums over the $U(1)_{\text{anom}}$ charges of all matter superfields. Consequently, one can define a charge $s$ for the nonperturbative term,

$$e^{-bS} \rightarrow e^{-i\Lambda(x)}e^{-bS},$$

(B.4)

with $b > 0$ and the charge $s$ is given by

$$s = Q_{\text{anom}}(e^{-bS}) = \frac{b}{2\delta_{GS}} = \frac{b}{12\pi^2} \text{tr} Q_{\text{anom}}^3.$$ (B.5)

Depending on $\text{tr} Q_{\text{anom}}^3$ the charge $s$ of $e^{-bS}$ can be positive or negative. On the other hand, in certain string-derived models, in which the Green-Schwarz mechanism is universal, one has the relation

$$\text{tr} Q_{\text{anom}}^3 = \frac{1}{8} \text{tr} Q_{\text{anom}}^3,$$ (B.6)

using the fact that the generator of $U(1)_{\text{anom}}$ is normalized to 1/2. Then one obtains

$$s = \frac{b}{96\pi^2} \text{tr} Q_{\text{anom}}^3.$$ (B.7)

We have chosen $U(1)_{\text{anom}}$ such that the FI term $\xi$ is positive; that is,

$$\xi = \frac{g}{192\pi^2} \text{tr} Q_{\text{anom}} > 0;$$ (B.8)

see Appendix A. Consequently, the $U(1)_{\text{anom}}$ charge of the nonperturbative term $e^{-bS}$ is positive as well; that is,

$$s = \frac{2b}{g}\xi > 0.$$ (B.9)

For instance, in the case of a condensing $SU(N_c)$ group with $N_f < N_c$ fundamental and antifundamental “matter” fields, $Q$ and $\overline{Q}$, one has (see, e.g., [13, Equation (2.7)] of the published version)

$$W_{\text{hid}} \supset (N_c-N_f)\frac{\Lambda}{(3N_c-N_f)}(\frac{\text{det} M}{(N_c-N_f)})^{(3N_c-N_f)}$$

$$+ \left( \frac{\phi}{M_p} \right)^{9/3} m_t^7 m_t,$$ (B.10)

where $\Lambda$ denotes the renormalization group invariant scale and carries charge $Q_{\text{anom}}(\Lambda) = N_f(q+\bar{q})/(3N_c-N_f)$. $\bar{q}$ and $\bar{q}$ are the “anomalous” charges of $Q$ and $\overline{Q}$, respectively. Inserting the VEV of the mesons $M_t = Q \overline{Q}$ (see [13, Equation (2.13)]), one obtains a term of the form (3).

**Conflict of Interests**

The authors declare that there is no conflict of interests regarding the publication of this paper.

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