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Essays on Cyclical and Long-Run Labor Market Behaviors

A Dissertation submitted in partial satisfaction
of the requirements for the degree of

Doctor of Philosophy

in

Economics

by

Dongpeng Liu

June 2014

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To my parents and my wife.
ABSTRACT OF THE DISSERTATION

Essays on Cyclical and Long-Run Labor Market Behaviors

by

Dongpeng Liu

Doctor of Philosophy, Graduate Program in Economics
University of California, Riverside, June 2014
Professor Jang-Ting Guo, Committee Co-Chairperson
Professor Richard Min Hon Suen, Committee Co-Chairperson

The dissertation explores the inter-dependence between agents’ educational choices and their labor market outcomes. This individual level of inter-relationship is aggregated under a search and matching framework to analyze the cyclical and long-run labor market behaviors. The dissertation argues that the high volatility of labor market can be attributed to complementarity between high-skill and low-skill workers, as well as the economies of scale of job training for high-skill labor. Then, the dissertation combines a search and matching model with a signaling game to explain agents’ educational choices and the unemployment rate gap between educated and uneducated workers. It also discusses some indirect effects of education and labor market policies. The effect of skill-biased technological change is scrutinized as well. An extension of this model is then used to analyze the growing mismatch of skill in China.
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Chapter 1

Introduction

Since the onset of the most recent financial crisis in late 2007, high unemployment has drawn great attentions from all economic participants, Gallup Polls show that unemployment is consistently among the most important problems facing the US economy.\(^1\) Therefore, questions as (i) why there exists unemployment; (ii) how unemployment fluctuates over time as well as (iii) what factors affect the long-run unemployment rate in US are of vital importance, not only to academic researchers but also to policy makers.

Traditional economic theories with perfectly competitive labor market are incapable of answering the above-mentioned questions. However, the search and matching model, pioneered by the 2010 Nobel Prize Laureates - Diamond, Mortenson and Pissarides (DMP hereafter) is able to successfully explain the coexistence of job vacancies and unemployment observed in the actual data.

Despite the success of DMP models in answering (i), it is incapable of generating

\(^1\)For example, see http://www.gallup.com/poll/167450/unemployment-rises-top-problem.aspx.
quantitatively realistic labor-market volatilities, as pointed out by the important work of Shimer (2005). Subsequent research has been devoted to solve this puzzle, most of which adopts the analytic framework of representative agent models. However, the US data shows that workers with different educational levels face different labor market outcomes. Specifically, high-skill workers (college graduates) are less likely to suffer from unemployment and enjoy a higher real wage. Furthermore, the high-skill unemployment rate is relatively more volatile. A natural question to ask is that whether the heterogeneity of labor and interactions of different labor markets play important roles in determining labor market volatility. Hence, Chapter 2 of my dissertation is intended to generate more quantitatively realistic labor market volatility (compared to a standard DMP model) within the context of a modified DMP model, taking the complementarity between different groups of workers into account. Furthermore, it is argued in this chapter that the economies of scale of job training may further contribute to the large size of the volatility of high-skill labor market.

After taking labor heterogeneity in terms of educational achievements as evidence to support the argument made in Chapter 2, a more fundamental question to answer is the source of the heterogeneity. Specifically, why there exists unemployment rate gap between workers with different educational achievements and what factors drive people to make different educational choices are of vital importance. Consequently, Chapter 3 of the dissertation explores the interaction between agents’ educational choices and their long-run labor market outcomes. With a combination of a signaling game and a search and matching model, Chapter 3 argues that educated agents are high productivity workers and they send the signal about their productivity through their educational choices. This process
can explain why people with more education on average suffer less from unemployment and why some firms set requirements on educational attainment for job applicants. This interaction between educational choices and labor market outcomes also indicate strong policy implications. For example, Chapter 3 argues that higher unemployment benefit will create larger unemployment rate gap between educated and uneducated workers, which in turn results in stronger incentives to pursue education. Furthermore, it also implies that higher educational subsidy will lower the average productivity of both educated and uneducated workers, leading to higher skill-specific unemployment rates. More interestingly, the model presented in Chapter 3 can also be applied to analyze the effects of a skill-biased technological change, which is widely attributed to be the reason causing the deterioration of the labor market outcomes of low-skill workers. Existing literature explicitly or implicitly assumes that the composition of labor force in terms of educational attainments remains stable when the skill-biased technology change hits the economy. However, the model in Chapter 3 allow people to respond to such a change by changing their educational choices. Hence, this model offers much richer implications when used to scrutinize the effects of such fundamental changes.

The model in Chapter 3 assumes that all jobs are essentially the same. That is, no job requires the worker to possess the knowledge obtained from formal education to produce a positive amount of output. However, this assumption can be relaxed to match the reality better and shed light on more topics. In Chapter 4, an extension of Chapter 3 is introduced to explain a set of dramatic changes observed in China’s labor market during the past two decades, including the rising of mismatch of skill, over-education, and
Educational requirement for low-skill jobs. It is assumed that there are two types of jobs, high-skill and low-skill jobs. High-skill job vacancies require knowledge obtained through education and can only be filled by educated workers. Low-skill job vacancies, on the other hand, can be filled by either educated or uneducated workers. Along with capital-skill complementarity, these assumptions imply that skill mismatch does not happen until the proportion of educated workers is large enough. As the proportion is small, only the most productive workers are educated and they all have the incentive to search for a high-skill job. If more people get educated, relatively less productive educated workers will find it beneficial to search in low-skill jobs. This change will also motivate profit-maximizing firms to post some low-skill job vacancies requiring education in order to fully utilize the information revealed by educational choices.

The rest of the dissertation is organized as the following. Chapter 2 discusses how labor heterogeneity, interactions between labor markets and economies of scale for job training contribute to the large labor market volatility. Chapter 3 combines a search and matching model with a signaling game to analyze why there exists unemployment rate gap between educated and uneducated workers as well as why people make different educational choices. The policy implications of the model is also discussed in this chapter. Chapter 4 extends the model presented in Chapter 3 and applies it to explain several key changes observed during the past two decades in China’s labor market. Chapter 5 concludes the dissertation.
Chapter 2

Labor Heterogeneity, Job Training and Labor Market Volatility

2.1 Introduction

The Diamond-Mortensen-Pissarides (DMP) model is now a standard approach to analyze labor market dynamics. The DMP model is capable of explaining why there exists unemployment in the equilibrium as well as the coexistence of job vacancies and unemployed workers. However, as pointed out by Shimer (2005), regardless of the source of aggregate fluctuations, the DMP model can only explain a small proportion of the volatility of labor market variables. DMP model’s incompetence in this aspect is often attributed to the unrealistic smoothness of firms’ profits generated by the model. The overly stable profits would lead to very weak incentives for firms to change recruiting efforts. Hence, employment, unemployment and market tightness generated will not be as volatile as those
observed in the data.

Since the work of Shimer (2005), a large body of research has been done to address this issue, most of which are using representative-agent models.  

Meanwhile, postwar US data shows strong evidence that workers with different educational achievements face very different labor market outcomes. Figure 2.1 shows the seasonally unadjusted monthly unemployment rates from January 1992 to December 2010 for workers with and without a college degree.  

From this point on, for simplicity, workers with a college degree will be called high-skill labor, whereas workers without a college degree will be called low-skill labor.  

The first observation is that low-skill unemployment rate always lies above high-skill unemployment rate. The average high-skill and low-skill unemployment rates over this period are 2.8% and 6.9%, respectively.  

The second observation is that the standard deviation of the logarithm of high-skill unemployment rate is larger than that of low-skill unemployment rate.  

The third observation is that high-skill unemployment and low-skill unemployment rates are 2.3% and 7.4% respectively over this period.

\[\text{For example, Hall (2005), Hall and Milgrom (2008), Hagedorn and Manovakii(2008), Rudanko (2009) and Faccini and Ortigueira (2010).}\]

\[The data is obtained from Bureau of Labor Statistics (BLS) publication, Employment and Earnings.\]

\[The detailed method used to calculate the unemployment rates is provided in the Appendix A.\]

\[This is consistent with some existing papers. For example, Krusell et al (2000).\]

\[This pattern actually holds for all educational levels over that period of time: workers with higher educational achievements are always less likely to be unemployed. In a longer time span, Current Population Survey (CPS) data shows high skill labor are always less likely to be unemployed from 1964 to 2010. In the survey, each respondent is asked about his/her educational level, military status, labor force participation decision and employment status. I use this information to calculate unemployment rates for different groups of workers. The average annual high-skill and low-skill unemployment rates are 2.3% and 7.4% respectively over this period.\]

\[The relative standard deviation of logarithm of high-skill unemployment rate and that of low-skill unemployment rates over this period.\]
unemployment are strongly correlated. After the Hodrick-Prescott (HP) detrending with a smoothing parameter of 1600, the correlation coefficient between detrended quarterly (aggregated from monthly data) logarithm of high-skill and low-skill unemployment rates is 0.77. In addition to the differences in employment, high-skill workers also enjoy the skill premium. Figure 2.2 shows the median of real weekly labor income of high-skill and low-skill workers from 2000Q1 to 2010Q4 calculated based on BLS data and consumer price index (CPI).\(^6\) It can be observed that high-skill workers consistently receive higher real wage. All these facts suggest that different groups of workers face distinct labor market outcomes, which may be the result of labor market partition.

Given these stylized facts, a natural research question to answer is whether the heterogeneity of labor and the interactions between different labor markets contribute to the large labor market volatility. Hence, in this paper, a model economy with two kinds of labors and capital is considered. Specifically, I adopt a production function with capital skill complementarity. In order to highlight the role of labor heterogeneity, other features of the model are kept as close as the standard DMP model.

Xie (2008) develops a similar model. In her research, she was able to replicate large volatility of labor market variables. However, her model was not able to generate unemployment rate is around 4/3. Nevertheless, high-skill unemployment rate is less volatile than low-skill unemployment rate in level. This difference comes from the fact that high-skill unemployment rate is much smaller than that rate of low-skill workers.

\(^6\)The median of real weekly wage for low-skill labor in Figure 2.2 is an approximation. When constructing this series, the author simply takes the average of three subgroups of the low-skill workers without using the population weights. In the longer time span, skill premium also exists. Based on CPS data, the skill premium has been increasing since 1970s.
realistic relative volatility between the two unemployment rates. This is a problem widely shared by models with labor heterogeneity aiming at increasing labor market volatility. For example, in the model developed by Pries (2008), most of the unemployment volatility is created by unrealistically large volatility of low skill labor while the unemployment rate of high skill labor hardly fluctuates. Hagedorn, Manovskii and Stetsenko (2010) are able to replicate aggregate and skill-specific labor market volatility with a model featuring two sorts of capital, labor heterogeneity, investment shock and capital skill complementarity. However, a large degree of the success of their model can be attributed to a method of calibration similar to that was used by Hagedorn and Manovskii (2008), which may lead to unrealistically small bargaining power of workers and a extremely large elasticity of labor supply. I will further compare my model with theirs in detail.

In this paper, a model with labor market heterogeneity, job training and capital skill complementarity is considered. The model manages to magnify labor market volatility, generate realistic relative unemployment volatility and replicate strong correlation between high-skill and low-skill unemployment. The intuition of the increase in volatility and strong co-movement between the two unemployment rates is straightforward. When a positive shock hits the economy, it increases the marginal product of every factor. Hence, firms would post more vacancies and rent more capital. In the next period, both employment and capital stock will increase. Since all inputs are complements, it will encourage firms to post even more vacancies and rent more capital. Therefore, the effect of a TFP shock will be prolonged and amplified. Furthermore, the two types of labors tend to move in the same direction. In contrast, a DMP model does not consider the complementarity between
different groups of labors, which makes the effect of a shock die out much faster.

The fact that most existing literature featuring labor heterogeneity fails to generate realistic skill-specific unemployment volatility is a natural consequence of the settings of the models. High skill labors are more productive and bring higher profits to firms. Hence, the additional asset value a firm gains by hiring a high skill labor is larger than that of hiring a low skill labor. Both Shimer (2005) and Hagedorn and Manovskii (2008) show that smaller asset value of a job leads to larger labor market volatility. As a result of the same productivity shock, the percentage change of the asset value is negatively related to the asset value itself, so are the incentives for firms to change recruiting efforts.

The introduction of economies of scale for job training, however, can solve the problem stated above. There are three key assumptions about job training cost adopted in this paper. First, the job training cost is paid directly by the firm, rather in a form of lower productivity for newly recruited workers, as was assumed by Silva and Toledo (2009). Second, firms only offer formal training to high skill workers. This assumption is a simplification of the findings of several studies. Altonji and Spletzer (1991) show that the amount of training and educational achievements are positively related. Third, per worker training cost is negatively related to firm size. It has been confirmed by various empirical studies that larger firms tend to offer more formal training to employees. The economies of scale of job training was introduced by Black, Noel and Wang (1999) to explain this fact. The intuitions of the economies of scale is as the following. Provided that a firm recruits more workers, it may still offer the same amount orientations and hire the same amount of instructors.
If a positive shock hits the economy, firms will post more vacancies for both high skill and low skill workers. As a result, more output is produced and firms become larger in the next period. As a result, the per worker training cost will become smaller in the next period, which will stimulate firms to post more vacancies for high skill labor. Under this setting, the per worker training cost is anti-cyclical, becoming the second driving force of the fluctuations of high skill labor unemployment. With this effect, volatility of high skill unemployment can exceed that of low skill unemployment, even though the asset value of a high skill job is much larger.

The rest of the chapter is organized as the following. A theoretical model will be presented in section 2.2. Section 2.3 will be used to introduce the calibration methods. I will present the quantitative results of the model with and without training cost and explain why economies of scale for job training are necessary in section 2.4. Section 2.5 will conclude this chapter.

2.2 Model

2.2.1 Demographics

In the model economy, there inhabits a representative household with high-skill and low-skill labors. The size of the household is normalized to one. An index \(i \in \{h, l\}\) is used to show the type of household members, where \(h\) represents high-skill while \(l\) represents low-skill. Denote the numbers of type \(i\) member in the representative household by \(N_i\), where \(N_i \in (0, 1)\) and \(N_h + N_l = 1\). \(N_i\) is also the share of type \(i\) individual of the
total population in the economy. In each period, the household member is endowed with one indivisible unit of time that is either used as working for wage or searching for a job. These two status are mutually exclusive. An individual who is searching for a job is called unemployed. Denote the numbers of working and unemployed type $i$ individual in period $t$ by $n_{i,t}$ and $u_{i,t}$. It follows that $n_{i,t} + u_{i,t} = N_i$.

There are two labor markets in the model economy. Each market is strictly restricted to one type of individuals.

### 2.2.2 Timing For Unemployed Workers

At the beginning of each period, each unemployed worker searches for a job. If a high skill unemployed worker manages to find one, she will accept training from the employer till the end of the period. Starting from the next period, this worker will start to produce output. If a low skill worker manages to find a job, she will start to work for the employer in the next period. This assumption is a simplification of a widely confirmed result that formal training and skill level are positively related. See, for example Altonji and Spletzer (1991) and Osterman (1995).

### 2.2.3 The Representative Household’s Problem

In each period, each household member obtains utility from consumption and disutility from working. The household pools all the resources together. Hence, the consumption of each household member is exactly the same. The expected lifetime utility of
the representative household is given by

\[ E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\varepsilon}}{1-\varepsilon} - \eta_h n_{h,t} - \eta_l n_{l,t} \right\} \right\}, \quad (2.1) \]

where \( \beta \in (0, 1) \) is the subjective discount factor, \( c_t > 0 \) is the household consumption at time \( t \) and \( \eta_h, \eta_l > 0 \) are the marginal disutility of working for high skill and low skill workers respectively.

Assume firms and all the capital are owned by households. In each period, the representative household makes decisions on investment and consumption. In period \( t \), if a type \( i \) member is employed, she receives a wage \( w_{i,t} \). If a member is unemployed, she receives unemployment insurance \( \psi > 0 \). Denote the amount of capital held by the representative household in period \( t \) by \( k^s_t \). \( k^s_0 > 0 \) is given. For each unit of capital, the household receives rental income \( r_t \). Denote the dividend paid by firms in period \( t \) by \( d_t \). The government levies a lump sum tax on the representative household, \( \tau_t \), to finance the unemployment insurance. The budget constraint of the representative household can be written as

\[ c_t + k^s_{t+1} - (1-\delta)k^s_t + \tau_t = w_{h,t} n_{h,t} + w_{l,t} n_{l,t} + \psi(1 - n_{h,t} - n_{l,t}) + r_t k^s_t + d_t, \quad (2.2) \]

where \( \delta \in [0, 1] \) is the depreciation rate.

In period \( t \), a type \( i \) unemployed worker may find a job at a rate \( g_{i,t} > 0 \), which is determined endogenously in equilibrium. Simultaneously, a type \( i \) employed worker faces a constant probability \( x_i \in (0, 1) \) of becoming unemployed. Households take \( g_{i,t} \) and \( x_i \) as given. If successfully finding a job, an unemployed worker will start working in the next period for a firm till exogenous separation happens. Hence, the representative household is
subject to the employment constraint as follows 7

\[ n_{i,t+1} = (1 - x_i)n_{i,t} + g_{i,t}(N_i - n_{i,t}), \quad n_{i,0} \in (0, N_i) \text{ is given} \quad (2.3) \]

Denote the expected lifetime utility of the household in period \( t \) by \( \Phi_t \). Then, the representative household’s problem can be defined recursively as

\[ \Phi_t = \max_{c_t, k_{t+1}^s} \left\{ \frac{c_t^{1-\varepsilon}}{1 - \varepsilon} - \eta_i n_{h,t} - \eta_l n_{l,t} \right\} + \beta E_t \Phi_{t+1}, \quad (2.4) \]

under the constraints of (2.2) and (2.3). If there is an interior solution in every period, then combining the first-order condition with respect to \( k_{t+1}^s \) and the one period ahead version of the envelope condition with respect to \( k_t^s \) yields

\[ \frac{1}{c_t^s} = \beta E_t \frac{1}{c_{t+1}^s} (1 + r_{t+1} - \delta), \quad (2.5) \]

which is the standard consumption Euler equation.

Now consider the envelope condition with respect to type \( i \) workers. It measures the additional benefit the household obtains by having one more type \( i \) member employed rather than unemployed at the current wage. Denote this additional value by \( W_{i,t} \). It follows that

\[ W_{i,t} \equiv \frac{\partial \Phi_t}{\partial n_{i,t}} = \frac{1}{c_t^i} (w_{i,t} - \psi) - \eta_i + \beta (1 - x_i - g_{i,t}) E_t W_{i,t+1}. \quad (2.6) \]

Specifically, \( \frac{1}{c_t^i} (w_{i,t} - \psi) - \eta_i \) is the extra amount of utility obtained in the current period from more consumption but less leisure, while \( (1 - x_i - g_{i,t}) E_t W_{i,t+1} \) is the continuation value of being employed.

\(^7\)This specification means that households do not have control on how many household members are employed. The number of employed members is predetermined at the beginning of each period.
The Representative Firm’s Problem

A representative firm has access to a production technology given by

\[ y_t = f(z_t, k_t^d, n_{h,t}, n_{l,t}) = z_t\left[\alpha n_{l,t}^{1-\frac{\gamma}{\gamma - 1}} + (1 - \alpha) y_{h,t}^{1-\frac{\gamma}{\gamma - 1}}\right]^{\frac{1}{\gamma - 1}}, \]  

(2.7)

and

\[ y_{h,t} = \left[\iota n_{h,t}^{1-\frac{1}{\varepsilon}} + (1 - \iota) (k_t^d)^{1-\frac{1}{\varepsilon}}\right]^{\frac{1}{\varepsilon - 1}}, \]  

(2.8)

where \( z_t > 0 \) is the total factor productivity, \( k_t^d \) denotes the amount of capital rented by the firm, \( \gamma > 0 \) is the elasticity of substitution between low-skill labor and capital, \( \varepsilon \in (0, \gamma) \) is the elasticity of substitution between high-skill labor and capital, \( \alpha \in [0, 1] \) is the relative weight of low-skill workers and \( \iota \in [0, 1] \) is the relative weight of high-skill labor. This two-stage CES production function is similar to the one proposed by Krusell et al. (2000), which is also used by Hagedorn, Manovskii and Stetsenko (2010). Their version of the production function contains two types of capital and (2.7) is nested in it. The first stage of (2.7) is between capital and high-skill labor, which is given by (2.8). The combination of these two factors and low-skill labor forms the second stage, which is given by (2.7). The production function is very flexible and implies various possible interaction among different factor markets. Several studies, including Krusell et al. (2000), Duffy et al. (2004) and Polgreen and Silos (2008), have verified the existence of capital-skill complementarity, which means that \( \gamma \) is greater than \( \varepsilon \). With this feature, compared to the demand for low-skill labor, the demand for high-skill labor will be affected more by capital stock.

The total factor productivity \( z_t \) is assumed to evolve according to,

\[ \ln z_{t+1} = \rho \ln z_t + s_{t+1}, \quad s_t \sim i.i.d. \mathcal{N}(0, \sigma^2), \]  

(2.9)
where $s_t$ is the shock on total factor productivity, $\rho \in [-1,1]$ is the autocorrelation of $z_t$ and $z_0 > 0$ is given, and $\sigma > 0$ denotes the standard deviation of $s_t$.

In each period, firms post vacancies of each type of jobs and rent capital from the capital market. Similar to $g_{i,t}$, the vacancy filling rate is endogenously determined in the equilibrium. The cost of posting each type $i$ vacancy is $q_i$. Denote the rate at which a firm successfully finding a type $i$ unemployed worker to fill a corresponding type vacancy by $\mu_{i,t} > 0$. Firms take the type $i$ vacancy filling rate $\mu_{i,t}$ and job separation rate $x_i$ as given in every period. Hence, a representative firm faces the employment constraint

$$n_{i,t+1} = (1 - x_i)n_{i,t} + v_{i,t}\mu_{i,t}, \quad (2.10)$$

where $v_{i,t}$ is the number of type $i$ vacancies posted by the firm and $n_{i,0} \in (0, N_i)$ is given.

Suppose firms discount future profits with the stochastic discount factor $\tilde{\beta}_t > 0$. Since firms are owned by households, the stochastic discount factor depends on the representative household’s inter-temporal economic decisions. If a household gets one unit of output as dividend income in the next period, the household can obtain extra discounted utility as much as $\beta/c_{t+1}^\epsilon$; however, if the household consume one more unit in the current period, it can get $1/c_t^\epsilon$ as extra utility. Therefore, the stochastic discount is determined by

$$\tilde{\beta}_t = \frac{\beta MU_{t+1}}{MU_t} = \beta(\frac{c_t}{c_{t+1}})^\epsilon. \quad (2.11)$$

Each high skill worker will receive job training immediately after being recruited. The per person cost to train a newly recruited high skill worker is $p_t > 0$. Black, Noel and Wang (1999) uses the economies of scale for job training to explain why large firms tend to offer more intensive job training. I adopt this assumption by assume that $p_t = p(y_t)^\kappa$. 
where \( p > 0 \) is the per worker training cost at the steady state, \( y^* > 0 \) is the steady state output and \( \kappa > 0 \) is the degree of the economies of scale for training. This assumption also means that the per worker training cost is counter-cyclical.

Denote the expected present value of the firm by \( \Pi_t \). Then, the firm’s problem is to maximize its value and it can be defined recursively as

\[
\Pi_t = \max_{k^d,t, v_{h,t}, v_{l,t}} \{ y_t - (r_t k^d_t + w_{h,t} n_{h,t} + w_{l,t} n_{l,t} + q_t v_{h,t} + q_t v_{l,t} + p_t v_{h,t} \mu_{h,t}) + E_t \tilde{\beta} \Pi_{t+1} \}, \tag{2.12}
\]

under the constraints of (2.9) and (2.10). Note that the flow profit of the firm or the dividend paid in period \( t \) is given by

\[
d_t = y_t - (r_t k^d_t + w_{h,t} n_{h,t} + w_{l,t} n_{l,t} + q_t v_{h,t} + q_t v_{l,t} + p_t v_{h,t} \mu_{h,t}). \tag{2.13}
\]

Define \( J_{i,t} \) as the additional value obtained by the firm with one more type \( i \) worker employed. By definition, this is given by

\[
J_{h,t} = \frac{\partial \Pi_t}{\partial n_{h,t}} = M P_{h,t} - w_{h,t} + \kappa v_{h,t} \mu_{h,t} \frac{p_t}{y_t} M P_{h,t} + (1 - x_h) E_t \tilde{\beta} J_{h,t+1}, \tag{2.14}
\]

and

\[
J_{l,t} = \frac{\partial \Pi_t}{\partial n_{l,t}} = M P_{l,t} - w_{l,t} + (1 - x_l) E_t \tilde{\beta} J_{l,t+1}, \tag{2.15}
\]

where \( M P_{i,t} = \partial y_t / \partial n_{i,t} \) is the marginal product of type \( i \) labor. The intuitions of (2.14) and (2.15) are as the following. For each firm, by hiring one more labor, it will obtain \( M P_{i,t} - w_{i,t} \) units of flow output in period \( t \). In addition, when an additional high skill labor is employed, a firm’s output increases, as a result, per worker training cost decreases. \( \kappa v_{h,t} \mu_{h,t} \frac{p_t}{y_t} M P_{h,t} \) measures the total amount of training cost saved by hiring one more high skill labor in period \( t \). \( (1 - x_i) E_t \tilde{\beta} J_{i,t+1} \) is the continuation value of each type \( i \) job.
If an interior solution exists in each period, then the following first-order conditions hold:

\[
\frac{\partial \Pi_t}{\partial v_{h,t}} = \mu_{h,t} E_t \beta_t J_{h,t+1} - q_h - p_t \mu_{h,t} = 0, \tag{2.16}
\]

\[
\frac{\partial \Pi_t}{\partial v_{l,t}} = \mu_{l,t} E_t \beta_t J_{l,t+1} - q_l = 0, \tag{2.17}
\]

and

\[
MP_{k,t}(1 + \kappa v_{h,t} \mu_{h,t} p_t y_t) - r_t = 0, \tag{2.18}
\]

where \( MP_{k,t} = \frac{\partial y_t}{\partial k_t} \) is the marginal product of capital. (2.16) states that the cost of posting one high skill vacancy plus the expected cost of job training should be equated to the discounted expected benefit of posting that vacancy. (2.17) states that the expected discounted gain from posting a low skill vacancy should be just enough to cover the cost of the vacancy. (2.18) is the first-order condition governing how many units of capital to rent. The difference between (2.18) and the standard first order condition is the term \( \kappa v_{h,t} \mu_{h,t} p_t y_t MP_{k,t} \), which is the total amount of training cost saved by hiring one more unit of capital. Eliminating the continuation value in (2.14) and (2.15) using (2.16) and (2.17) yields

\[
J_{h,t} = (1 + \kappa v_{h,t} \mu_{h,t} p_t y_t) MP_{h,t} - w_{h,t} + \left( \frac{q_h}{\mu_{h,t}} + p_t \right) (1 - x_h), \tag{2.19}
\]

and

\[
J_{l,t} = MP_{l,t} - w_{l,t} + q_l (1 - x_l) / \mu_{l,t}. \tag{2.20}
\]

### 2.2.5 Matching Technology and the Determination of Wages

When unemployed workers and vacancies meet in the labor markets, matches will be made. As the two types of workers search for jobs in two separate labor markets, there
are two matching technologies to be considered. Denote the number of matches in type \( i \) labor market in period \( t \) by \( m_{i,t} \). The number of successful type \( i \) matches is determined by a constant-return-to-scale matching function

\[
m_{i,t} = a_i v_{i,t}^{b_i} u_{i,t}^{1-b_i},
\]

(2.21)

where \( a_i \in (0,1) \) is the efficiency of matching for type \( i \) workers and \( b_i \in (0,1) \) is the elasticity of matching with respect to type \( i \) vacancies.

Define the market tightness for market \( i \) in period \( t \) as \( \theta_{i,t} \equiv v_{i,t}/u_{i,t} \). By definition,

\[
\mu_{i,t} = m_{i,t}/v_{i,t} = a_i \theta_{i,t}^{b_i-1},
\]

(2.22)

and

\[
g_{i,t} = m_{i,t}/u_{i,t} = a_i \theta_{i,t}^{b_i}.
\]

(2.23)

Each individual firm and household are small relative to the entire labor market so that they take \( \theta_{i,t}, \mu_{i,t} \) and \( g_{i,t} \) as given.

To determine the wage, Nash bargaining is assumed. After a successful match is made, firms and households bargain over the wages. The firms and households act as if they are maximizing the objective function below,

\[
\Psi_{i,t} = W_{i,t}^{\lambda_i} J_{i,t}^{1-\lambda_i},
\]

(2.24)

When coming to the wage determination, some economists abandon the Nash Bargaining assumption adopted by the standard DMP model in order to generate sticky real wage and larger fluctuations of labor market variables. Since the goal of this paper is to highlight the role of capital-skill complementarity in a standard DMP model, the assumption of Nash Bargaining is maintained. However, it does not necessarily mean that Nash Bargaining is the most reasonable assumption.
where $\lambda_i \in (0, 1)$ can be interpreted as the bargaining power of a type $i$ worker.

The first-order condition of the Nash Bargaining problem is

$$
\lambda_i J_{i,t} \frac{\partial W_{i,t}}{\partial w_{i,t}} + (1 - \lambda_i) W_{i,t} \frac{\partial J_{i,t}}{\partial w_{i,t}} = 0.
$$

(2.25)

Using equations (2.6), (2.14) and (2.15) yields$^9$,

$$
W_{i,t} = \frac{\lambda_i}{1 - \lambda_i c_t^e} J_{i,t}.
$$

(2.26)

Eliminating $W_{i,t+1}$ in (2.6) with the one period ahead version of (2.26) generates

$$
W_{i,t} = \frac{1}{c_t^e} (w_{i,t} - \psi) - \frac{1 - \lambda_i}{\lambda_i} \eta c_t^e + (1 - x_i - g_{i,t}) \frac{\lambda_i}{1 - \lambda_i} E_t \beta c_{t+1}^e J_{i,t+1}.
$$

(2.27)

The combination of (2.26) and (2.27) gives

$$
J_{i,t} = \frac{1 - \lambda_i}{\lambda_i} (w_{i,t} - \psi) - \frac{1 - \lambda_i}{\lambda_i} \eta c_t^e + (1 - x_i - g_{i,t}) \frac{\lambda_i}{1 - \lambda_i} E_t \beta c_{t+1}^e J_{i,t+1}.
$$

(2.28)

With (2.5), (2.11), (2.16) and (2.17), we have

$$
J_{h,t} = \frac{1 - \lambda_h}{\lambda_h} (w_{h,t} - \psi) - \frac{1 - \lambda_h}{\lambda_h} \eta_h c_t^e + (1 - x_h - g_{h,t}) \frac{\eta_h}{\mu_{h,t}} p_t + q_{h,t} \theta_{h,t} + p_t g_{h,t},
$$

(2.29)

and

$$
J_{l,t} = \frac{1 - \lambda_l}{\lambda_l} (w_{l,t} - \psi) - \frac{1 - \lambda_l}{\lambda_l} \eta_l c_t^e + (1 - x_l - g_{l,t}) \frac{q_l}{\mu_{l,t}}.
$$

(2.30)

Combining (2.19) and (2.29) generates the wage equation for high skill workers

$$
w_{h,t} = \lambda_h [M P_{h,t}(1 + \kappa v_{h,t} \mu_{h,t} \frac{p_t}{y_t}) + q_{h,t} \theta_{h,t} + p_t g_{h,t}] + (1 - \lambda_h) (\eta_h c_t^e + \psi).
$$

(2.31)

The intuition of this wage equation is as the following. By hiring an additional high skill worker, the firm can get the additional output $M P_{h,t}$. Above that, by having

$^9$According to equation (2.6), (2.14) and (2.15), we have $\frac{\partial W_{i,t}}{\partial w_{i,t}} = \frac{1}{c_t^e}$ and $\frac{\partial J_{i,t}}{\partial w_{i,t}} = -1$. Plugging these two equations into (2.25) yields (2.26).
each unit of extra output, the per worker training cost decreases by $\kappa \frac{p_t}{y_t}$. Furthermore, there is one fewer high skill worker unemployed; therefore, to make sure that the number of employed workers in the next period is at the desired level, the economy as a whole should spend less output on recruiting. Note that for each unemployed high skill labor, $\theta_{h,t} = \frac{v_{h,t}}{u_{h,t}}$ vacancies are posted. Hence, $q_h \theta_{h,t}$ is the recruiting cost saved per high skill currently employed worker. Meanwhile, the number of newly recruited worker in the current period will decrease by $\theta_{h,t} \mu_{h,t} = g_{h,t}$ if the number of unemployed high skill worker decreases by 1. The corresponding training cost saved, therefore, is $p_t g_{h,t}$. As a result, $MP_{h,t}(1 + \kappa v_{h,t} \mu_{h,t} \frac{p_t}{y_t}) + q_h \theta_{h,t} + p_t g_{h,t}$ is the firm’s total gains of hiring the last worker. $\eta_h c_t^e + \psi$ is the marginal rate of substitution between consumption and leisure plus unemployment insurance. Only if $w_{h,t}$ is greater than $\eta_h c_t^e + \psi$ would the household accept the wage contract. The wage is the weighted average of the two terms and the weight is the bargaining power of high skill workers, $\lambda_h$. Given this wage, the household will get $\lambda_h (J_{h,t} + W_{h,t})$ from the joint surplus while the firm will get $(1 - \lambda_h) (J_{h,t} + W_{h,t})$.

Combining (2.20) and (2.30) yields the wage equation for low skill workers

$$w_{l,t} = \lambda_l (MP_{l,t} + q_l \theta_{l,t}) + (1 - \lambda_l) (\eta_l c_t^e + \psi).$$

(2.32)

The intuitions of (2.32) are very similar to those of (2.31). $MP_{l,t}$ is the amount of additional output a firm can get by hiring one more low skill worker. $q_l \theta_{l,t}$ is the recruiting cost saved in order to maintain the desired level of employment in the next period and $MP_{l,t} + q_l \theta_{l,t}$ measures the total gains of a firm by hiring the last low skill worker. The low skill wage is also a weighted average of total gains of the firm and the reservation wage of a low skill worker.
2.2.6 The Search Equilibrium

The search equilibrium of this model economy consists of sequences of prices \(\{r_t, w_{h,t}, w_{l,t}\}_{t=0}^{\infty}\), household allocation \(\{c_t, k_{t+1}^s\}_{t=0}^{\infty}\), employment \(\{n_{h,t}, n_{l,t}\}_{t=0}^{\infty}\), dividend income \(\{d_t\}_{t=0}^{\infty}\), lump sum tax \(\{\tau_t\}_{t=0}^{\infty}\) vacancies and capital demand \(\{v_{h,t}, v_{l,t}, k_t^d\}_{t=0}^{\infty}\), vacancy filling rates \(\{\mu_{h,t}, \mu_{l,t}\}_{t=0}^{\infty}\) and job finding rates \(\{g_{h,t}, g_{l,t}\}_{t=0}^{\infty}\), such that,

- Given the prices \(\{r_t, w_{h,t}, w_{l,t}\}_{t=0}^{\infty}\), dividend income \(\{d_t\}_{t=0}^{\infty}\) and job finding rates \(\{g_{h,t}, g_{l,t}\}_{t=0}^{\infty}\), household allocation \(\{c_t, k_{t+1}^s\}_{t=0}^{\infty}\) solve the representative household’s problem.

- Given the prices \(\{r_t, w_{h,t}, w_{l,t}\}_{t=0}^{\infty}\) and vacancy filling rates \(\{\mu_{h,t}, \mu_{l,t}\}_{t=0}^{\infty}\), vacancies and capital demand \(\{v_{h,t}, v_{l,t}, k_t^d\}_{t=0}^{\infty}\) solve the firm’s problem. The amount of flow profits or dividend income is determined by (2.13).

- The government keeps a balanced budget; that is, \(\psi(1 - n_{h,t} - n_{l,t}) = \tau_t\).

- Goods market clears; that is, \(c_t + k_{t+1}^s - (1 - \delta)k_t^s + q_nv_{h,t} + q_lv_{l,t} + ptv_{h,t}\mu = f(z_t, z_{h,t}, z_{l,t}, k_t^d, n_{h,t}, n_{l,t})\) for any \(t\).

- Capital market clears; that is, \(k_t^d = k_t^s\).

- Wages \(\{w_{h,t}, w_{l,t}\}_{t=0}^{\infty}\) are determined by (2.31) and (2.32).

- Job finding rates \(\{g_{h,t}, g_{l,t}\}_{t=0}^{\infty}\) are determined by (2.23).

- Vacancy filling rates \(\{\mu_{h,t}, \mu_{l,t}\}_{t=0}^{\infty}\) are determined by (2.22).
2.2.7 Characterizing the Equilibrium

Define \( k_t \) as the equilibrium amount of capital stock. In the dynamic system specified by this paper, there are seven variables \((n_h, n_l, c, k, \theta_h, \theta_l, z)\). Therefore, a system of seven difference equations is needed to solve the model. These include the employment constraints (2.3) or (2.10), the consumption Euler equation (2.5), the budget constraint (2.2) or the goods market clearing condition, the laws of motion of productivity (2.9) and the following equation which can be obtained from the combination of (2.16) and (2.19)

\[
\frac{q_h}{\mu_{h,t}} + p_t = E_t[\tilde{\beta}_t[MP_{h,t+1}(1+\kappa v_{h,t+1}\mu_{h,t+1} \frac{p_{t+1}}{\mu_{h,t+1}}) - w_{h,t+1} + (\frac{q_h}{\mu_{h,t+1}} + p_{t+1})(1-x_h)]],
\]

and the following equation based on (2.17) and (2.20)

\[
q_l/\mu_{l,t} = E_t[\tilde{\beta}_t[MP_{l,t+1} - w_{l,t+1} + q_l(1-x_l)/\mu_{l,t+1}]].
\]

A deterministic steady state with \( z^* = 1 \) can be characterized by

\[
n^*_h = (1-x_h)n_h^* + a_h \theta_h^{b_h}(N_h - n_h^*),
\]

\[
n^*_l = (1-x_l)n_l^* + a_l \theta_l^{b_l}(N_l - n_l^*),
\]

\[
1 = \beta (1 + f'_k(1, k^*, n_h^*, n_l^*)(1 + \kappa \theta_h^{b_h}(N_h - n_h^*)a_h \theta_h^{b_h-1} \frac{p}{f(1, k^*, n_h^*, n_l^*)}) - \delta),
\]

\[
c^* + \delta k^* + q_h \theta_h^*(N_h - n_h^*) + q_l \theta_l^*(N_l - n_l^*) + p \theta_h^*(N_h - n_h^*)a_h \theta_h^{b_h-1} = f(1, k^*, n_h^*, n_l^*),
\]

\[
[1 - \beta(1 - x_h)](q_h + p a_h \theta_h^{b_h-1})
\]

\[
= a_h \theta_h^{b_h-1} \beta \{(1 - \lambda_h)[f'_{n_h}(1, k^*, n_h^*, n_l^*)(1 + \kappa \theta_h^*(N_h - n_h^*)a_h \theta_h^{b_h-1} \frac{p}{f(1, k^*, n_h^*, n_l^*)})
\]

\[
- \eta_h c^* e - \psi] - \lambda_h(q_h \theta_h^* + p a_h \theta_h^{b_h*})
\]
\[
a_t \theta^*(b_t^{-1}) \beta[(1 - \lambda_t)(f'_n(1, k^*, n^*_h, n^*_l) - \eta c^* - \psi) - \lambda_t q_t \theta^*_t] = [1 - \beta(1 - x_t)]q_t, \tag{2.40}
\]

where the variables labeled with an asterisk are those valued at the steady state and \( f'_x(\cdot) \) is the first order partial derivative of \( f(\cdot) \) with respect to \( x \). Since there is no anomaly in the production function or utility function, the steady state is unique.

The dynamic system is then log-linearized around the deterministic steady state to find out the percentage deviation of each variable from its steady state value in each period given the three initial conditions \( n_{h,0}, n_{l,0} \) and \( k_0 \).

### 2.3 Parameterization

In order to pin down the values of the parameters, the length of one period has to be fixed. A proper setting is one month as many labor market variables are at the monthly frequency.

The value of the discount factor \( \beta \) is set to 0.995, which yields a 0.5 percent monthly interest rate net of depreciation. The value of \( \delta \) is set to be 0.0083 such that the annual depreciation rate is 10%. Fallick and Fleischman (2004) show that the monthly total separation rates for college graduates and workers without college degrees are 0.042 and 0.064. I use these numbers as \( x_h \) and \( x_l \), respectively.

Hall and Milgrom (2008) estimate the elasticity of matching with respect to vacancies posted \( (b_t) \) and the number is 0.77. Before that, Blanchard and Diamond (1989) report an estimate of 0.6. Petrongolo and Pissarides (2001) finds that the value should be between 0.3-0.5. Hence, I calibrate the model to have an aggregate matching elasticity of 0.6, which is between the two estimates made by Petrongolo and Pissarides (2001) and Hall
and Milgrom (2005). Furthermore, I also follow Hagedorn, Manovskii and Stetsenko (2010) to calibrate $b_i$ and such that the relative elasticity of job finding with respect to aggregate market tightness is 1.33. Since Krusell et al. (2000), several studies, for example, Duffy et al. (2004) and Polgreen and Silos (2008), estimated the elasticity of substitution between capital equipment and high-skill labor as well as that elasticity between capital equipment and low-skill labor. All the studies confirm the existence of capital-skill complementarity, that is, high-skill labor is more complementary to capital equipment. Since Krusell et al. (2000) and Polgreen and Silos (2008) distinguish capital equipment (machine) from capital structure (architecture) while my research and Duffy et al. (2004) do not, the estimates reported by Duffy et al. (2004) are adopted. Hence, $e$ and $\gamma$ are set to 1.26 and 2.20 respectively. Hosios (1990) shows that, when the bargaining power of workers is equal to the elasticity of matching with respect to unemployment, a search economy achieves Pareto efficiency. Hence, the bargaining powers of both high-skill and low-skill labors are calibrated such that the Hosios condition is satisfied. The bargaining powers calibrated by Hagedorn, Manovskii and Stetsenko (2010) are much smaller. The high and low skill bargaining powers used in their papers are 0.064 and 0.098. However, there is no direct evidence that such small bargaining powers prevail. Second, as pointed out by Shimer and Milgrom (2008), such small bargaining powers will lead to very large labor supply elasticity that far beyond any empirical estimation. It is widely known that small bargaining powers will lead to high labor market volatility. Hence, a large proportion of the generated volatility in their model can be attributed to the very small calibrated bargaining powers. The population share of high-skill labor is set to be 0.351, which is calculated based on CPS data from 1964 to 2010.
In addition to the parameters stated above, there are still twelve parameters ($q_h$, $q_l$, $p$, $\kappa$, $\alpha$, $\iota$, $a_h$, $a_l$, $\eta_h$, $\eta_l$, $\varepsilon$, $\psi$) to be fixed. Therefore I use these parameters to match twelve targets simultaneously. The first two targets are the ratio between vacancy posting costs and the steady state wages. According to Silva and Toledo (2009) and Milgrom and Hall (2008), the average cost of posting one vacancy is about 9.1 days of the average wage. Hence, I set $q_h$ as large as 9.1 days of steady state wage of high skill labors. Similarly, $q_l$ is set to be 9.1 days of steady state wage of low skill labors. The third target is the elasticity of training with respect to establishment size. Black, Noel and Wang (1999) find it to be 0.36. The forth target is the ratio between steady state training cost and steady state high skill wage. Silva and Toledo (2009) argues that the per worker direct cost of training is about 6% of annual wage while the indirect cost is about 8% of annual wage. Hence, I set $p$ 1.68 times as large as steady state high skill wage. The fifth and sixth targets are the steady state unemployment rates for the two types of labor. According to the data used to plot figure 2.1, the average unemployment rates are 2.8% and 6.9% for high-skill labor and low-skill labor, respectively. The seventh target to match is the aggregate market tightness. The average market tightness from December 2000 to February 2011 is 0.45.\footnote{The market tightness is calculated based on the ratio of new job openings and the total number of unemployed workers. Source of DATA: Job Openings and Labor Turnover Survey (JOLTS).} The eighth target is capital’s share of income in the steady state, which is 0.36. This value corresponds to the estimate reported by Kydland and Prescott (1982). In contrast to a standard real business cycle model, 0.64 is not labor’s share of income, but the sum of share of labor income, profits and vacancy posting costs. The ninth target is the wage
ratio between high and low-skill workers in the steady state. The number is set to be 1.984, which was used by Hagedorn, Manovskii and Stetsenko (2008) and is consistent with CPS data. The tenth target is relative volatility between wage and labor productivity. Bureau of Labor Statistics constructs Real average weekly earnings of production and non-supervisory employees. The volatility of the logarithm of this variable is about the same size of the volatility of the logarithm of labor productivity. The eleventh target is the relative volatility between consumption and output. I choose $\varepsilon$ such that the volatility of logarithm of consumption is 0.65 time as large as the volatility of logarithm of output. Following Shimer (2005), I set the unemployment benefit as 40% of the steady state average wage. With the twelve conditions, the twelve parameters left can be solved simultaneously.

2.4 Quantitative Results

In this section, I will present two versions of the model developed in this paper. The first version of the model does not consider the training cost, that is $p = 0$. The second version of the model considers the anti-cyclical training cost for high skill labor. Both versions of the model are calibrated to match the volatility of average product of labor.

Though the length of one period in the model is one month, there is no monthly frequency data for GDP and APL. Therefore, when comparing the performances of the model with data, I aggregate the data generated to quarterly frequency.

The first column of table 2.1 presents the quarterly post war US data of labor market variables and will be used as the benchmark.\footnote{The original data of $u$ and $v$ are in monthly frequency. I aggregate them into quarterly frequency by...}
all of the variables are of the same time span. 12 All the variables in the tables hereafter are reported in logarithms as deviations from the HP trend with a smoothing parameter 1600.13 Note that all variables are seasonally adjusted. u is constructed by BLS based on CPS data. The help-wanted advertising index constructed by the Conference Board is used as the number of vacancies v.14 By definition, θ is the ratio between v and u. APL is the real average output per worker in the non-farm business sector, which is constructed by BLS. Real GDP y is obtained from National Income and Product Accounts (NIPA). Skill specific unemployment rates uh and ul are constructed from Employment and Earnings between 1992Q1 to 2010Q4. w is real average weekly earnings of production and non-supervisory employees constructed by BLS and is available from 1964Q1 to 2010Q4. wh and wl represent the median real wages for high-skill and low-skill labor respectively. BLS provides data on the median of weekly nominal earnings of people over 25 with different educational achievements. wh and wl are constructed based on that data set and CPI15.

This dataset covers the period from 2000Q1 to 2010Q416.

taking the simple average. s.d. stands for standard deviation and a.c. stands for autocorrelation.

12 Data on u, v, θ, APL and y is collected from 1951Q1 to 2003Q1.
13 Some studies instead use 10^5 as the smoothing parameter. See Shimer (2005), Silva and Toledo (2009) and Faccini and Ortigueira (2010). A larger smoothing parameter makes the trend part of a variable more stable. Therefore, by construction, it will make the cyclical part of a variable more volatile. However, this will not change any conclusion qualitatively.
14 The Conference Board has stopped updating help-wanted advertising index since March 2005. A new series, help-wanted advertising online, is provided instead. This new series shows the total amount of help-wanted advertisements posted online, which is not directly comparable to the original index.
15 wl is constructed as the simple average of the median real wages of three subgroups of workers without a college degree.
16 The time span of skill specific wages is very short. I use them to show the relative volatility of the two
For each simulation presented, I create 100 samples of 750 periods, that is, 100 samples of 250 quarters.

In order to set a benchmark, a standard DMP model with a Cobb-Douglas production function is examined. The capital’s share is calibrated to 0.36 as well. The vacancy elasticity of matching $b$, bargaining power $\lambda$, subjective discount rate $\beta$, separation rate $x$ are in accordance with the values used in section 2.3. $a$ is calibrated such that the average unemployment rate equals to the 5.5%. $\eta$ is calibrated such that the average of market tightness is 0.45. $q$ is calibrated such that the vacancy posting cost is 9.1 days of normal state wage. $\psi$ is also set to be 40% of the steady state wage. The standard deviation and quarterly autocorrelation generated by the standard DMP model are presented in the second column of table 2.1. As expected, the standard DMP model can only account for a small proportion of labor market volatility.\(^{17}\) Furthermore, for the sake of completeness, the second order moments generated by a DMP model with economies of scale for job training is shown in the third column of table 2.1.

2.4.1 Specification 1: the Model without Training Costs

In this subsection, the training costs are not considered, that is, $p$ is set to zero. To generate the quantitative results, $\rho$ and $\sigma$ are calibrated to match the autocorrelation \(^{17}\) However, the volatility generated is larger than Shimer (2005) and some other studies, which may results from differences of calibration methods. Nevertheless, the differences in term of volatility between the model with two types of labor and the standard DMP model is very large, which can be attributed to the mechanism introduced in this study.
of detrended and the volatility of detrended $\ln APL_t$. The parameter values used for the simulations in this specification are reported in table 2.3 and table 2.4.

The fourth column of table 2.1 presents the simulation results of the model without considering training cost. By comparing the second and fourth column of table 2.1, I can show that this specification generates significantly larger volatility of detrended logarithm of aggregate unemployment rate compared to the DMP model. The intuition of this result is of the following. When there is a positive shock hitting the economy, the demand for high-skill labor, low-skill labor and capital will all increase. In the next period, the marginal product of each factor may increase further due to the facts that more other factors are employed and all factors are complements, which in turn increase the demand for all factors further. This will magnify the impact of one single shock and make it last longer.

The major problem of this specification is that the ratio between the standard deviation of detrended $\ln u_{h,t}$ and $\ln u_{l,t}$ is much smaller than the data. The data shows that the standard deviation of detrended $\ln u_h$ is about 1.3 times as large as that of the detrended $\ln u_{l,t}$. Nevertheless, the standard deviation of detrended $\ln u_{h,t}$ generated by this specification is only about 70% of the standard deviation of detrended $\ln u_l$. This problem is shared by some other studies, for example, Pries (2008), which also consider skill heterogeneity.

The problem stated above is a consequence of the model setting. When labor heterogeneity is considered, there also exists differences in the asset values of different types of jobs ($J_{h,t}$ or $J_{l,t}$). The value of a high skill job is larger than the value of a low skill job. Shimer (2005) and Hagedorn and Manovskii (2008) have shown that the generated volatility
is negatively related to the asset value of a job. Given a certain shock, a larger asset value of a job means that the percentage change of the asset value is smaller. Note that in each period, a firm equates the cost of posting a vacancy with the expected discounted benefit of that vacancy. Hence, a smaller percentage change in the asset value of a job means a smaller change in vacancy filling rate\(^\text{18}\), which in turn leads to smaller change in market tightness. Hence, the volatility of vacancies and unemployment will also be smaller. This explains why the volatility of high-skill unemployment, vacancies and market tightness are unrealistically small.

### 2.4.2 Specification 2: the Model with Training Costs

In order to fix the problem of relative volatility between high skill and low skill market variables, the training costs are considered. Altonji and Spletzer (1991) finds that the intensity of training is positively related to skill level. For the purpose of this paper, I assume that only high skill labor receive formal job training. Black, Noel and Wang (1999) argue that there exists economies of scale in term of job training. At the business cycle frequency, this means that the per worker cost of training is anti-cyclical. The fifth column of table 2.1, summarizes the performance of the model with the training costs considered. All the variables are reported in logarithms as deviations from the HP trend with a smoothing parameter 1600. The parameter values used for the simulations in this specification are reported in table 2.5 and table 2.6.

The standard deviation of unemployment rate generated is also much larger than

\(^{18}\text{See (2.16) and (2.17).}\)
what can be generated by the standard DMP model. The intuitions of the larger labor market volatility are very similar to those discussed in section 2.4.1.

A major difference between the results of this specification and specification without job training is that detrended $\ln u_{h,t}$ is more volatile than the detrended $\ln u_{l,t}$. The ratio between the two numbers is qualitatively correct and quantitatively much closer to the reality, which can be attributed to the anti-cyclical training costs.

Under this setting, the high-skill labor market volatility is magnified for two reasons. First, when recruiting new high skill workers, the firm not only needs to consider the vacancy posting cost, but also the training costs. Comparing (2.16) and (2.17), we can find that if $E_t J_{h,t+1}$ and $E_t J_{l,t+1}$ changes by the same amount, $\mu_{h,t}$ will fluctuate more than $\mu_{l,t}$ due to the existence of the training costs. Second, due to the economies of scale, the per worker training cost is anti-cyclical. Consider a positive shock hitting the economy. As the MPL of both high skill and low skill labors increase, firms will hire more workers and produce more output. Hence, the firm size increases and the per worker training cost decreases. Consequently, firms are willing to post even more vacancies.

2.4.3 Sensitivity Analysis: Common Matching Elasticity

It is widely known that matching elasticity and workers’ bargaining power used can affect labor market fluctuations generated by a search and matching model. Specifically, larger matching elasticity tend to create larger labor market volatility, holding all other things equal.

Hence, as a sensitivity analysis, the simulations for the two specifications are re-
done without having skill-specific matching elasticity. Table 2.2 summarizes the results. Note that the first 3 columns of table 2.2 are exactly the same with those in table 2.1. The last two columns are simulation results based on a common matching elasticity of 0.6, and therefore, a common bargaining power of 0.4 (satisfying the Hosios condition). The models without skill-specific matching elasticity and bargaining power do exhibit smaller relative volatility compared to the results presented in table 2.1. Nevertheless, it is obvious that the economies of scale for job training can still dramatically increase the volatility of high skill labor market and make the relative volatility qualitatively realistic.

2.5 Conclusions

This paper argues that skill heterogeneity and the training costs, when integrated into an otherwise standard Diamond-Mortensen-Pissarides model, would result in a better performance in terms of labor market volatility. In order to highlight this point, the assumption of Nash bargaining is maintained. The model presented in this paper can generate about 60% of total volatility of aggregate unemployment rate. The mechanism leading to this improvement is the complementarity between different types of workers. Both the model with and without training costs can dramatically increase the size of labor market fluctuations. Nevertheless, if training cost is not considered, the model generates unrealistically small relative standard deviation of the detrended $\ln u_h$ and $\ln u_l$. The introduction of training costs and economies of scale for training can fix this problem and create quantitatively realistic relative volatility between high-skill and low-skill labor market variables.
Table 2.1: US labor market data and simulation results

<table>
<thead>
<tr>
<th>US Data</th>
<th>Homogeneous Labor w/o $p_t$</th>
<th>Homogeneous Labor w/ $p_t$</th>
<th>Heterogeneity w/o $p_t$</th>
<th>Heterogeneity w/ $p_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>s.d. a.c.</td>
<td>s.d. a.c.</td>
<td>s.d. a.c.</td>
<td>s.d. a.c.</td>
<td>s.d. a.c.</td>
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<tr>
<td>$u$</td>
<td>0.125 0.870</td>
<td>0.033 0.757</td>
<td>0.047 0.803</td>
<td>0.081 0.806</td>
</tr>
<tr>
<td>$v$</td>
<td>0.139 0.904</td>
<td>0.039 0.684</td>
<td>0.048 0.700</td>
<td>0.063 0.589</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.259 0.896</td>
<td>0.071 0.766</td>
<td>0.081 0.809</td>
<td>0.137 0.791</td>
</tr>
<tr>
<td>$APL$</td>
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<td>0.013 0.765</td>
<td>0.013 0.765</td>
<td>0.013 0.765</td>
</tr>
<tr>
<td>$y$</td>
<td>0.016 0.836</td>
<td>0.015 0.774</td>
<td>0.015 0.775</td>
<td>0.017 0.800</td>
</tr>
<tr>
<td>$w$</td>
<td>0.013 0.881</td>
<td>0.013 0.772</td>
<td>0.013 0.808</td>
<td>0.013 0.764</td>
</tr>
<tr>
<td>$u_h$</td>
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<td>- -</td>
<td>0.063 0.743</td>
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<tr>
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<tr>
<td>$w_l$</td>
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</table>

Table 2.2: US labor market data and simulation results (common matching elasticity)

<table>
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<th>Homogeneous Labor w/ $p_t$</th>
<th>Heterogeneity w/o $p_t$</th>
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<td>s.d. a.c.</td>
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</tr>
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<td>0.047 0.803</td>
<td>0.071 0.808</td>
</tr>
<tr>
<td>$v$</td>
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<td>0.048 0.700</td>
<td>0.069 0.577</td>
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<tr>
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</tr>
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<td>0.013 0.765</td>
<td>0.013 0.765</td>
<td>0.013 0.765</td>
</tr>
<tr>
<td>$y$</td>
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<td>0.015 0.774</td>
<td>0.015 0.775</td>
<td>0.017 0.800</td>
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<tr>
<td>$w$</td>
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<td>0.013 0.808</td>
<td>0.013 0.767</td>
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</tr>
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</tr>
<tr>
<td>$\lambda_h$</td>
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<td>Hosios condition</td>
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<td>Hosios condition</td>
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<tr>
<td>$x_l$</td>
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<td>Fallick and Fleischman (2004)</td>
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<td>$b_h$</td>
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<td>relative matching elasticity</td>
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<td>low-skill unemployment rate 6.9%</td>
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<tr>
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<td>0.3923</td>
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<td></td>
</tr>
<tr>
<td>$q_l$</td>
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<td>9.1 days of low skill wage</td>
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<tr>
<td>$\psi$</td>
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<td>40% of average wage</td>
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Table 2.3: Parameter Values (the model without training costs)

<table>
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<tr>
<th>Parameter</th>
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Table 2.4: Parameters of the TFP process (the model without training cost)
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<td>$\beta$</td>
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<td>Monthly interest rate 0.5%</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>2.2045</td>
<td>Duffy et al. (2004)</td>
</tr>
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<td>$e$</td>
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<td>Duffy et al. (2004)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.0083</td>
<td>Annual depreciation rate 10%</td>
</tr>
<tr>
<td>$\lambda_h$</td>
<td>0.2579</td>
<td>Hosios condition</td>
</tr>
<tr>
<td>$\lambda_l$</td>
<td>0.4436</td>
<td>Hosios condition</td>
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<tr>
<td>$x_h$</td>
<td>0.0420</td>
<td>Fallick and Fleischman (2004)</td>
</tr>
<tr>
<td>$x_l$</td>
<td>0.0640</td>
<td>Fallick and Fleischman (2004)</td>
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<td>relative matching elasticity 1.33</td>
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<td>Wage premium 1.98</td>
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<td>$a_l$</td>
<td>0.9417</td>
<td>low-skill unemployment rate 6.9%</td>
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<td>$q_h$</td>
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<td>9.1 days of high skill wage</td>
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<td>9.1 days of low skill wage</td>
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<td>elasticity of training wrt firm size 0.36</td>
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<tr>
<td>$\psi$</td>
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<td>40% of average wage</td>
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Table 2.5: Parameter Values (the model with training costs)

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<td>$\sigma$</td>
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Table 2.6: Parameters of the TFP process (the model with training cost)
Figure 2.1: Skill-Specific Unemployment Rates
Figure 2.2: Skill-Specific Median of Weekly Wages
Chapter 3

Education as Unemployment Insurance: A Model with Endogenous Educational Requirement for Job Application and Its Policy Implications

3.1 Introduction

Most existing studies, for example, Spence (1973), Stiglitz (1975) and Riley (1979), attribute the incentive of undertaking education to the wage premium. However, these
studies ignored a second incentive that is of vital importance. As pointed out by Kiefer (1985), education could alleviate the problem of unemployment by reducing the duration and possibility of it. These observations are supported by US data. Figure 3.1 exhibits the unemployment rate of high school graduates ($u_n$), the unemployment rate of college graduates\(^1\) ($u_e$), and aggregate unemployment rate of the two types combined. It clearly shows that college education affects the probability of unemployment. Furthermore, Pissarides (1981) argues that unemployment would reduce the return from labor market participation and increase the demand for education. These interactions between unemployment probability and agents’ educational choices will be thoroughly investigated in this paper. Not only will this study shed light on the decision making process of pursuing education, but it will also create a framework that can be applied to analyze related policy implications.

A very important fact of labor market is that employers often require a certain level of educational achievement to apply for a job. Existing literature, for example, Makenna (1996) attributes this phenomenon to the adaptability issue. In other words, it is assumed that uneducated workers can only produce a limited varieties of goods or services while educated workers are able to adapt themselves to any sort of jobs. As a result, firms must request education for certain kinds of jobs. Similar assumption is used by Albrecht and Vroman (2002). However, under a lot of scenarios, we can observe that firms may require relatively high educational achievements for relatively simple jobs. For instance, some banks require a college degree when recruiting bank tellers, even though historically a GED would be sufficient.

\(^1\)Workers with graduate degrees are not included, as the model presented in this paper will only discuss one educational choice - whether to pursue college education. Taking graduate education into consideration will dramatically complicate the model, yet not provide significantly more insights.
suffice for this position.

This paper, therefore, tries to answer the following questions under the framework of a search and matching model: (1) How agents’ educational choices and their labor market outcomes, especially unemployment probabilities, depend on each other; (2) Why there exists educational requirement for job application when adaptability is not the issue; (3) What are the policy implications; and (4) How this interdependence shed light on the inequality of labor market outcomes.

There are several existing studies taking into consideration of the interdependence of education and unemployment, most of which are representative agent models. These models, for instance, Laing, Palivos and Wang (1995) and Charlot, Decreuse and Granier (2005) cannot generate different educational choices and heterogeneous unemployment probabilities. Several heterogeneous agent models on this issue have been developed as well. Makenna (1996) provides a model with heterogeneous labor and sequential labor search. In that paper, educated workers are less likely to be unemployed because they are adaptable to more types of jobs while uneducated workers are suitable for only one type. Eggert, Krieger and Meier (2010) develop a model in which unemployment is a result of efficiency wage, discussing how unemployment gap stimulated education and migration. Vesala (2004) develops a similar model with signaling and labor market search in which wage premium is the only reason driving workers to pursue education. However, none of these papers endogenizes educational requirement for job application.

In this paper, a signaling game and search model are combined to explain the interdependence of educational choices and unemployment probability. When making edu-
cational choices, agents compare the costs versus the benefits of pursuing education. In the economy, residents are different in terms of their endowed ability and it takes less effort for a “smarter” individual to undertake education. As a result, the utility cost of education for “smarter” ones is lower. On the benefit’s side, it can be observed that educated workers tend to earn higher wages and are less likely to be unemployed. Hence, these facts are taken as the benefits of pursuing education. Consequently, only “smarter” individuals will find it worth pursuing education, as the benefits can outweigh their relatively small costs. In other words, there exists a threshold level of ability only above which agents will pursue education. Hence, education itself is a signal of higher ability and firms will have the belief that educated workers are the more productive ones. By pursuing education, more productive individuals send a good signal to the firms in the economy.

The argument above also reveals why educational requirement for job application exists. Suppose all workers have the same preference and their unemployment benefits are the same. Consequently, the reservation wages required by the two groups of workers are the same.\(^2\) If entrepreneurs and workers split the joint benefit from matching according to Nash Bargaining, entrepreneurs can expect higher post-match profits if they hire educated workers. Since educational achievements are observable, entrepreneurs are willing to post relatively more vacancies requiring education. This is why firms specify educational

\(^2\)In reality, the unemployment benefits are not the same among different workers. They are positively related to wage income previously earned. However, since each state sets its minimum and maximum unemployment benefit, the replacement ratio is negatively related to previous wage income. A constant unemployment benefit will catch this point and dramatically simplify the analysis without changing qualitative results.
requirements, even if uneducated workers can do the same job as their educated peers. If entrepreneurs do not post vacancies requiring different levels of educational requirements, they are not using up all of the information available and therefore, are not maximizing profits.

The model proceeds to show why undertaking education can reduce the probability of being unemployed, or why the unemployment rate of educated workers is smaller. Since entrepreneurs would post relatively more vacancies for educated workers, the labor market for them would be less "tight". Therefore, it takes less time for educated unemployed workers to find a job, which lowers the unemployment rate of educated workers. The realized equilibrium coincides with the observations of the residents and their ex-anti belief about the benefits of undertaking education. Note that the argument made here is only consistent with data if the observed job finding rate for college graduates exceeds that rate for high school graduates, which is confirmed by Farber (2004).

Given the interdependence between educational choices and labor market outcomes, we should expect policies designed to affect educational or labor market directly should also have an impact on the other market. However, these policy implications are often ignored by the policy makers as well as economists. The model developed in this paper, therefore, is used to address two specific indirect policy implications. First, the model predicts that higher unemployment benefit will motivate more people to pursue education. Second, the paper also argues that higher educational subsidy would cause both skill-specific unemployment rates to rise, though the effect on the aggregate unemployment rate is ambiguous. The intuitions of the policy implications are as the following.
As the unemployment benefit increases, so does the reservation wage. As a result, firms’ profits will decrease, leading entrepreneurs to post fewer vacancies. Consequently, the labor markets become tighter and unemployment rates will rise. These results are similar to the Diamond-Mortensen-Pissarides (DMP) model. Nevertheless, unlike the DMP model, the effects of the change in unemployment benefit on different groups of workers are asymmetric. Since unemployment benefit takes a smaller proportion of wages of educated workers, the percentage increase in their wages, and consequently, the percentage decrease in profits of firms hiring educated workers, are smaller. Hence, educated workers suffer relatively less from the tightened labor market. As the unemployment rate of uneducated workers increases by a larger amount, the incentive motivating individuals to undertake education, unemployment rate gap, becomes stronger and more individuals would be willing to pursue education. Hence, an increase in unemployment benefit tends to improve the overall quality of workers.\(^3\) Existing literature would overestimate the costs of unemployment benefit without taking into consideration its impact on educational choices. Through the mechanism stated above, a change in unemployment benefit will also affect wage inequality. These aspects are also analyzed in this paper.

As agents are offered higher educational subsidy, more people are willing to pursue education, which lowers the threshold ability of undertaking education. The quality of the signal sent by pursuing education drops. Meanwhile, not pursuing education becomes a

\(^3\)This argument holds when unemployment benefit is around the current level. Obviously, when unemployment benefit is very high, people will not care about unemployment rate because they can still enjoy very high level of consumption when unemployed. Hence, if unemployment benefit is very high, a further increase of it will lead to a decrease in educational effort.
worse signal. The consequences are that the expected ability of both uneducated and educated workers decreases. Entrepreneurs will respond to this change by posting fewer vacancies for both groups of workers, leading to higher skill specific unemployment rates. However, since educated workers now take a larger proportion of the total population and they are the group of agents with lower unemployment rate, the aggregate unemployment rate may decrease under the presence of higher skill specific unemployment rates.

Since 1970s, it has been observed that the real wage of workers with less education decreased both relatively and absolutely, even though the supply of uneducated workers decreased. Furthermore, the unemployment rate of uneducated workers increased over the same time period. To explain these facts, skill-biased technological change is introduced. Existing literature emphasizes on the demand side effects of this change. A new mechanism on how the skill-biased technological change affects real wage and unemployment rate of uneducated workers is introduced in this paper.

The rest of the chapter is organized as the following. Section 3.2 presents the theoretical model. Parameterization of this model is discussed in Section 3.3. Section 3.4 analyzes the impacts of changes in unemployment benefit and educational subsidy in detail. The consequences of a skill-biased technological change on the labor market outcomes of uneducated workers are presented in section 3.5. Section 3.6 concludes the chapter.
3.2 Model

3.2.1 Basic Settings

Consider an economy without aggregate uncertainty. It is inhabited by $N$ residents who are different only in terms of their ability when they are born. Call the residents born in period $\tau$ generation $\tau$. The ability of type $i$ resident of any generation, $z_i$, is a continuous variable and is bounded by $[z_l, z_h]$. The type of residents is solely defined by their endowed ability. For simplicity, assume the ability of a resident follows a uniform distribution. Denote the unconditional cdf of $z_i$ by $F(\cdot)$. In each period, each resident may die with a probability of $\delta$, where $\delta \in (0, 1)$. Meanwhile, $\delta N$ new residents are born. Hence, the population of the economy remains constant over time. Assume the death and birth of residents are independent from ability. Consequently, the distribution of ability of the population is invariant over time. Denote the subjective discount rate of a resident by $\beta \in [0, 1]$. Hence, after considering the probability of death, the effective discount rate is $\rho = \beta(1 - \delta)$.

When a resident is born, she has a one-time chance to determine whether to undertake education. Each resident is endowed with one unit of time per period, which has three mutually exclusive alternative usages: undertaking education, searching for a job or working for wage. If undertaking education, this resident will stay in school for $k$.  

\footnote{This assumption would make the theoretical analysis much simpler. This assumption suffices for the purpose of comparative steady state analysis. Note that a resident will only undertake education if the lifetime utility of being educated is larger than the life-time utility of not being educated. At the steady state, if undertaking education is better for a resident in the second period, it must also be better for her in the first period. Hence, there is no reason for a resident to postpone her educational choice.}
periods unless she unfortunately dies before graduation. Her labor productivity will become $p_i = (1 + e)z_i$ as she graduates, where $e > 0$. She then starts to search for a job in the $k + 1^{th}$ period. If not undertaking education, her labor productivity is $p_i = z_i$ and she starts to search for a job immediately after her birth. Call residents actively searching for a job unemployed workers. In each period, an unemployed worker can search for a job only once. The society has an ex-ante belief on the probabilities of uneducated and educated unemployed workers for successfully finding a job each period. If the resident manages to find a job in the current period, she will start to work in the next period. If the resident is currently working, there is a probability $x$ that the job will be destructed. Call $x$ separation rate.\footnote{In a search and matching model, steady state unemployment rate depends on per period job separation rate and the probability of finding a job. To highlight the mechanism presented in this paper, I assume that educated and uneducated workers are subject to the same job separation rate. Fallick and Fleischman (2004) show that the monthly total separation rates for college graduates and high school graduates are 0.046 and 0.066. The relatively small difference between the two separation rates are not enough to explain the large difference between the two unemployment rates.}

In the model economy, there exist many "small" firms, which are owned and created by entrepreneurs with the same subjective discount rate $\beta$. Each firm consists of only one job position. Call unfilled job positions vacancies. Some vacancies require educated workers while the others do not. Denote the number of vacancies requiring and not requiring education in period $t$ by $v_{e,t}$ and $v_{n,t}$ respectively. In each period, the society has ex-ante beliefs on the probabilities for each type of vacancies being filled. If a job vacancy is filled by a type $i$ resident in the current period, the firm would start to produce $p_i$ from the
next period until the (job) firm is destructed. Before matching, firms cannot observe each job candidate’s ability or productivity. Nevertheless, they can observe each job candidate’s educational status. After matching and before the two parties start to negotiate the wage, a firm can immediately observe its worker’s productivity without any uncertainty. This assumption can greatly simplify the model without changing the qualitative result. A very similar assumption is adopted by Pries (2008).

3.2.2 The Resident’s Problem

In period \( t \), the flow utility of a type \( i \) resident of generation \( \tau \) is

\[
\phi^\tau_{i,t} = \begin{cases} 
  c^\tau_{i,t} - \gamma(z_h - z_i) & \text{if in school} \\
  c^\tau_{i,t} & \text{if unemployed} \\
  c^\tau_{i,t} - \eta & \text{if working for wage}
\end{cases},
\]

(3.1)

where \( c^\tau_{i,t} \) is the consumption of a type \( i \) resident of generation \( \tau \) measured in period \( t \) and \( \gamma, \eta > 0 \). From this point on, superscripts following any variable indicate the generation of the worker being discussed. The flow utility function implies that one’s disutility derived from education depends on her ability, which is a straightforward and commonly used assumption in signaling games. Intuitively, if a resident is endowed with a low level of ability, she might feel frustrated more frequently during education, or it may take her more time to prepare for exams.

Assume that all goods are perishable and there is no financial market. Hence, residents consume all their income in each period. Denote the labor income of a type \( i \) resident of generation \( \tau \) in period \( t \) by \( w^\tau_{i,t} > 0 \). Depending on her educational status, \( w^\tau_{i,t} \)
either equals to $w_{i,n,t}^\tau$ if the worker is uneducated, or $w_{i,e,t}^\tau$ if the worker is educated. If a resident is unemployed, regardless of her educational achievement, she will receive $b > 0$ as unemployment benefit from the government. If a resident is undertaking education, her consumption would be $s > 0$, which is the educational subsidy the agent receives from the government. Hence, the flow utility can be rewritten as

\[
\phi_{i,t}^\tau = \begin{cases} 
  s - \gamma (z_h - z_i) & \text{if undertaking education} \\
  b & \text{if unemployed} \\
  w_{i,t}^\tau - \eta & \text{if working for wage}
\end{cases}.
\]  

Consider a type $i$ resident of generation $\tau$. If she chooses not to pursue education, then her life-time utility in period $t$ can be written recursively as

\[
\Phi_{i,n,t}^\tau = b + \rho \left[ \mu_{n,t} \Psi_{i,n,t+1}^\tau + (1 - \mu_{n,t}) \Phi_{i,n,t+1}^\tau \right],
\]

where $\Phi_{i,n,t}^\tau$ is the life-time utility of a type $i$ resident of generation $\tau$ measured in period $t$ if she is uneducated and unemployed, $\Psi_{i,n,t+1}^\tau$ is her life-time utility in period $t + 1$ if she is uneducated but employed, $\mu_{n,t}$ is the ex-ante job finding probability for an uneducated worker in period $t$. The first term of (3.3) is the flow utility. Since the resident is searching for a job in period $t$, there is a probability of $\mu_{n,t}$ that she can find a job and start working next period. If she successfully finds a job, her lifetime utility in period $t + 1$ would be $\Psi_{i,n,t+1}^\tau$. However, there is also a probability of $1 - \mu_{n,t}$ that she fails to secure a job, in which case her life-time utility would be $\Phi_{i,n,t+1}^\tau$. After discounting, the continuation value of being unemployed in period $t$ would be $\rho [\mu_{n,t} \Psi_{i,n,t+1}^\tau + (1 - \mu_{n,t}) \Phi_{i,n,t+1}^\tau]$.

For any $t > \tau$, $\Psi_{i,n,t}^\tau$ can be written recursively as

\[
\Psi_{i,n,t}^\tau = w_{i,n,t}^\tau - \eta + \rho [(1 - x) \Psi_{i,n,t+1}^\tau + x \Phi_{i,n,t+1}^\tau].
\]
The term $w_{i,n,t}^\tau - \eta$ in (3.4) is the flow utility. As the resident is working in period $t$, the firm hiring her may still exist with a probability of $1 - x$ in period $t + 1$, under which scenario her life-time utility in period $t + 1$ would be $\Psi_{i,n,t+1}^\tau$. However, there is also a chance that the job is destructed in period $t + 1$ with a probability of $x$. If this happens, her life-time utility in period $t + 1$ would become $\Phi_{i,n,t+1}^\tau$. After discounting, the continuation value of being employed in period $t$ is $\rho[(1 - x)\Psi_{i,n,t+1}^\tau + x\Phi_{i,n,t+1}^\tau]$.

Now, consider the same resident of generation $\tau$. If she chooses to undertake education, then her life-time utility when she is born can be written as

$$\Omega_i^\tau = \sum_{t = \tau}^{\tau+k-1} \rho^{t-\tau}[s - \gamma(z_h - z_i)] + \rho^k \Phi_{i,e,\tau+k+1}^\tau,$$

(3.5)

where $\Omega_i^\tau$ is the life-time utility of a type $i$ resident of generation $\tau$ measured when she is born, if the worker pursues education. For any $t \geq \tau + k + 1$ and $\Phi_{i,e,t}^\tau$ is her lifetime utility in period $t$ if she is unemployed and educated. Similar to (3.3), $\Phi_{i,e,t}^\tau$ can be written recursively as

$$\Phi_{i,e,t}^\tau = b + \rho[\mu_{e,t}\Psi_{i,e,t+1}^\tau + (1 - \mu_{e,t})\Phi_{i,e,t+1}^\tau],$$

(3.6)

where $\Psi_{i,e,t}^\tau$ is the life-time utility of an educated type $i$ resident of generation $\tau$ measured in period $t$ if she is employed, and $\mu_{e,\tau}$ is the ex-ante job finding probability for an educated worker in period $t$. Similar to (3.4), $\Psi_{i,e,t}^\tau$ can be written recursively as

$$\Psi_{i,e,t}^\tau = w_{i,e,t}^\tau - \eta + \rho[(1 - x)\Psi_{i,e,t+1}^\tau + x\Phi_{i,e,t+1}^\tau].$$

(3.7)

Under the setting of this model, each resident has only one choice to make - whether to undertake education when born. This decision is made based on which option would give
the resident a higher life-time utility. Hence, a type $i$ resident who is born in period $\tau$ will undertake education if $\Omega^r_i > \Phi^r_{i,n,\tau}$. As to be discussed later on, under common parameter values, there exists a threshold ability $\bar{z}^\tau$ for each generation $\tau$, such that if this type $i$ generation $\tau$ resident’s ability is greater than $\bar{z}^\tau$, she will pursue education.

3.2.3 Firms

In each period, entrepreneurs open vacancies for educated and uneducated workers. Following standard search and matching settings, I assume that there is a constant vacancy posting cost. Denote the cost of posting one vacancy as $q$. Given the assumption on the timing of information revelation, we know that the flow profit as well as the value of a filled job position remain unknown when the vacancy is posted. Hence, the expected, rather than realized value, of newly created firms should be used to calculate the benefit of posting vacancies. In period $t$, if a vacancy is posted, it might be filled by an uneducated worker with a probability $g_{n,t}$, or by an educated worker with a probability $g_{e,t}$. Call $g_{n,t}$ and $g_{e,t}$ the vacancy filling rates. In the next period, the newly recruited worker will start to produce output. Denote the expected values of a newly created firm hiring an uneducated and educated worker, measured in period $t+1$, as $J_{n,t+1}$ and $J_{e,t+1}$ respectively. Hence, depending on the educational achievement of the worker hired, the benefit of posting one vacancy is $\rho g_{n,t} J_{n,t+1}$ or $\rho g_{e,t} J_{e,t+1}$, which is to be compared with $q$. The question that remains is to find out the value of $J_{j,t+1}$, which can be solved in three steps.

The first step is to calculate the realized value of a firm.

Consider a firm hiring a type $i$ resident of generation $\tau$, given that the resident is
uneducated. The realized asset value of this firm in period $t$, $J_{i,n,t}$, therefore, can be written recursively as

$$J_{i,n,t} = z_i - w_{i,n,t} + \rho(1 - x)J_{i,n,t+1}, \quad (3.8)$$

where $z_i - w_{i,n,t}$ in (3.8) is the flow profit of this firm. In the next period, the firm may still exist with a probability of $1 - x$. After discounting, the continuation value of the firm is $\rho(1 - x)J_{i,n,t+1}$.

Now, suppose the firm hires an educated type $i$ generation $\tau$ worker. Then, the realized asset value of the firm hiring her in period $t$, $J_{i,e,t}$, can be written as

$$J_{i,e,t} = (1 + e)z_i - w_{i,e,t} + \rho(1 - x)J_{i,e,t+1}. \quad (3.9)$$

The realized value of a firm depends on the type and generation of the employee, which are unknown when the vacancies are posted. Hence, in order to calculate $J_{j,t}$, we need to take expectation of $J_{i,j,t}$ with respect to $i$ and $\tau$.

Therefore, the second step is to take expectation of $J_{i,j,t}$ with respect to $z_i$.

Assume that entrepreneurs have an ex-ante belief that there is a threshold level of ability $\bar{z}_\tau$, such that any resident of generation $\tau$ would choose to undertake education if and only if her ability is greater than $\bar{z}_\tau$.

Define the expected value of firms in period $t$ whose employees are uneducated residents of generation $\tau$ as $J_{n,t}$. It follows that

$$J_{n,t} = E_{z_i}(J_{i,n,t}|z_i \leq \bar{z}_\tau). \quad (3.10)$$

Similarly, define the expected value of firms in period $t$ whose employees are educated
residents of generation $\tau$ as $J_{\tau,t}^\tau$. It follows that

$$J_{\tau,t}^\tau = E_{\tau} (J_{\tau,t}^\tau | \tau_i > \bar{\tau}). \quad (3.11)$$

Naturally, the third step is to take expectation of $J_{\tau,t}^\tau$ with respect to $\tau$.

Denote the numbers of unemployed uneducated and educated residents in period $t$ by $u_{n,t}$ and $u_{e,t}$, respectively. Denote the numbers of generation $\tau$ unemployed uneducated and educated residents in period $t$ by $u_{n,\tau,t}$ and $u_{e,\tau,t}$, respectively. It is obvious that $u_{n,t} = \sum_{\tau=-\infty}^{t} u_{n,\tau,t}$ and $u_{e,t} = \sum_{\tau=-\infty}^{t} u_{e,\tau,t}$.\(^6\)

Workers recruited in period $t$ will start working in period $t+1$. Since the probability of filling a type $j$ vacancy is independent from the generation of potential job candidate, the probability that a newly created firm hires a generation $\tau$ worker is $u_{\tau,j,t}^\tau / u_{j,t}$. Hence, it follows that

$$J_{n,t+1} = \sum_{\tau=-\infty}^{t} \frac{u_{n,\tau,t}^\tau}{u_{n,t}^\tau} J_{\tau,n,t+1}, \quad (3.12)$$

and

$$J_{e,t+1} = \sum_{\tau=-\infty}^{t} \frac{u_{e,\tau,t}^\tau}{u_{e,t}^\tau} J_{\tau,e,t+1}. \quad (3.13)$$

Assume that the market for posting job vacancies is perfectly competitive. That is, no one can earn positive profit by just posting vacancies. Based on this assumption, the vacancy posting cost should be equal to the discounted expected value of the firm in the next period, multiplied by the probability of successfully filling the vacancy. Hence,

$$q = \rho g_{n,t} J_{n,t+1}, \quad (3.14)$$

\(^6\)Since education takes $k$ periods to complete. We must have $u_{e,t} = \sum_{\tau=-\infty}^{t-k} u_{e,\tau,t}^\tau$. However, by definition, for any $\tau > t - k$, $u_{e,\tau,t}^\tau = 0$. Hence, $u_{e,t} = \sum_{\tau=-\infty}^{t} u_{e,t}^\tau$. 

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and

\[ q = \rho g_{e,t} J_{e,t+1}. \tag{3.15} \]

Based on the discussions above and those in section 3.2.4 and 3.2.5 below, we can conclude that \( J_{e,t+1} > J_{n,t+1} \). Therefore, entrepreneurs should be willing to accept a lower vacancy filling rate if vacancies are filled by educated workers. However, this also means that entrepreneurs must explicitly require educational achievement for some of the vacancies.

Suppose they only post one type of vacancies without educational requirement and accept a vacancy filling rate \( g_t \) between \( g_{e,t} \) and \( g_{n,t} \). It means that they are not using up all of the available information. One entrepreneur can exploit this opportunity by posting vacancies requiring education and accept a lower vacancy filling rate. All educated workers will only apply for these jobs as they would enjoy a higher job finding rate \(^7\). Consequently, the entrepreneur who post vacancies with educational requirement can earn positive economic profits and all the others will make economic losses. Within the framework of the model, this is why educational requirement for job application exists. Denote the number of employed uneducated and educated workers of generation \( \tau \) in period \( t \) by \( n_{n,t}^\tau \) and \( n_{e,t}^\tau \). It follows that

\[ n_{n,t}^\tau = \delta N(1 - \delta)^{t-\tau} F(\bar{z}^\tau) - u_{n,t}^\tau, \tag{3.16} \]

and

\[ n_{e,t}^\tau = \begin{cases} \delta N(1 - \delta)^{t-\tau}[1 - F(\bar{z}^\tau)] - u_{e,t}^\tau, & \text{if } t > \tau + k \\ 0, & \text{if otherwise} \end{cases}. \tag{3.17} \]

\(^7\)As is shown in section 3.2.4, a job finding rate and the corresponding vacancy filling rate are negatively related.
Denote the numbers of employed uneducated and educated residents in period $t$ by $n_{n,t}$ and $n_{e,t}$, respectively. Obviously, $n_{n,t} = \sum_{\tau=-\infty}^{t} n_{n,\tau}$ and $n_{e,t} = \sum_{\tau=-\infty}^{t} n_{e,\tau}$.

3.2.4 Matching Technology and The Determination of Wages

For each type of workers and vacancies, there is a constant-return-to-scale matching technology. Assume the matching functions exhibit a Cobb-Douglas form. Denote the numbers of matches made in period $t$ for uneducated and educated workers as $m_{n,t}$ and $m_{e,t}$, respectively. Hence, it follows that

$$m_{n,t} = \varphi v_{n,t}^{\alpha} u_{n,t}^{1-\alpha},$$

and

$$m_{e,t} = \varphi v_{e,t}^{\alpha} u_{e,t}^{1-\alpha},$$

where $\varphi > 0$ is the matching efficiency and $\alpha \in (0,1)$ is the elasticity of matches with respect to vacancies. Denote the market tightness for uneducated and educated workers by $\theta_{n,t} = v_{n,t}/u_{n,t}$ and $\theta_{e,t} = v_{e,t}/u_{e,t}$, respectively.

Assume that the wage is determined by a Nash Bargaining process. In each period, after the matches are made and the productivity of newly recruited workers is revealed, workers and firms start to negotiate the wage. Consider a type $i$ resident of generation $\tau$ and the firm hiring her. Depending on whether this resident chose to undertake education or not, they negotiate a wage as if to maximize $(\Psi_{i,j,t} - \Phi_{i,j,t})^\lambda (J_{i,j,t})^{1-\lambda}$, where $\lambda \in (0,1)$ and the subscript $j \in \{e, n\}$. The first-order condition is

$$(1 - \lambda)(\Psi_{i,j,t} - \Phi_{i,j,t}) = \lambda J_{i,j,t}.$$

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Define $S^{r}_{i,j,t} = J^{r}_{i,j,t} + \Psi^{r}_{i,j,t} - \Phi^{r}_{i,j,t}$ as the joint surplus when the match is made, given the worker is uneducated or educated. By (3.20), it follows that

$$\Psi^{r}_{i,j,t} - \Phi^{r}_{i,j,t} = \lambda S^{r}_{i,j,t}.$$  

(3.21)

Hence, $\lambda$ is the bargaining power of workers.

### 3.2.5 The Existence of a Separating Equilibrium

The intuitions of the existence of a separating equilibrium are straightforward. It is obvious that for all residents in all periods, no matter educated or not, the reservation wage should be $b + \eta$. Since firms believe that only workers with ability greater than a threshold level would undertake education and education itself could enhance a worker’s productivity, they must also believe that the joint surplus obtained through a match with an educated worker is greater than the joint surplus obtained through a match with an uneducated worker. Since firms can get a constant proportion of the joint surplus through the bargaining process, the value of firms hiring educated workers would also be higher. Consequently, by (3.12) and (3.13), entrepreneurs are willing to post relatively more vacancies requiring education, which in turn implies that educated workers enjoy a higher job finding rate and are less likely to be unemployed.

Meanwhile, as a result of the less tight labor market, educated residents can earn higher wages. From the residents’ point of view, the benefits of education is a higher wage and a lower unemployment probability, whereas the major cost of education is the disutility from education. Since the disutility from education decreases as the ability increases, the cost of education is negatively related to the ability, implying that there exists a threshold
level of ability for the residents, if a resident’s ability is greater than the threshold, she would like to pursue education since the benefits of education would exceed the costs of education. Hence, the ex-post equilibrium outcome would be consistent with the ex-ante belief of the firms, which results in a separating equilibrium.

3.2.6 The Steady State

3.2.6.1 Equilibrium at the Steady State

At the steady state, variables such as $w$, $\theta$, $J$, $\Psi$, $\Phi$, $\Omega$ and $S$, are sensitive to neither the generation of workers nor the time period. Hence, denote the steady state lifetime utility for an educated type $i$ resident measured when she is born by $\Omega_i$. $X_{i,j}$ is used to represent the steady state values of the corresponding variables for a type $i$ resident, where $X \in \{w, J, \Psi, \Phi, S\}$ and $j \in \{n, e\}$. Furthermore, let $\mu_j$ and $g_j$ denote the job finding rates and vacancy filling rates at the steady state. Denote the steady state level of $\theta_{j,t}$ as $\theta_j$.

Consider a type $i$ worker who chooses not to undertake education. Based on the steady state version of (3.3) and (3.4), it follows that

$$\Phi_{i,n} = \frac{b[1 - \rho(1 - x)] + \rho\mu_n(w_{i,n} - \eta)}{(1 - \rho)[1 - \rho(1 - x - \mu_n)]},$$

(3.22)

and

$$\Psi_{i,n} - \Phi_{i,n} = \frac{w_{i,n} - \eta - b}{1 - \rho(1 - x - \mu_n)}.$$ 

(3.23)

(3.23) exhibits the net benefit of being employed for an uneducated type $i$ worker at the steady state.

Now, consider a type $i$ resident who chooses to undertake education instead. A
simple manipulation of the steady state version of (3.6) and (3.7) yields

\[ \Phi_{i,e} = \frac{b[1 - \rho(1 - x)] + \rho \mu_e(w_{i,e} - \eta)}{(1 - \rho)[1 - \rho(1 - x - \mu_e)]}, \]  
(3.24)

and

\[ \Psi_{i,e} - \Phi_{i,e} = \frac{w_{i,e} - \eta - b}{1 - \rho(1 - x - \mu_e)}. \]  
(3.25)

(3.25) shows the net benefit of being employed for an educated type \( i \) worker at the steady state.

Plugging (3.24) into the steady state version of (3.5) yields

\[ \Omega_i = \frac{1 - \rho^{k-1}}{1 - \rho} [s - \gamma(z_h - z_i)] + \rho^k \frac{b[1 - \rho(1 - x)] + \rho \mu_e(w_{i,e} - \eta)}{(1 - \rho)[1 - \rho(1 - x - \mu_e)]}. \]  
(3.26)

Therefore, a type \( i \) worker would like to undertake education if and only if

\[
\frac{1 - \rho^{k-1}}{1 - \rho} [s - \gamma(z_h - z_i)] + \rho^k \frac{b[1 - \rho(1 - x)] + \rho \mu_e(w_{i,e} - \eta)}{(1 - \rho)[1 - \rho(1 - x - \mu_e)]} > \frac{b[1 - \rho(1 - x)] + \rho \mu_n(w_{i,n} - \eta)}{(1 - \rho)[1 - \rho(1 - x - \mu_n)]}.
\]

Any type \( i \) resident knows that \( w_{i,e} \) and \( w_{i,n} \) are functions of \( z_i \). Hence, when considering the threshold level of ability to undertake education, explicit forms of \( w_{i,e} \) and \( w_{i,n} \) are needed.

Now, consider firms’ behavior. Based on the steady state version of (3.8) and (3.9), simple algebra shows that

\[ J_{i,n} = \frac{z_i - w_{i,n}}{1 - \rho(1 - x)}, \]  
(3.27)

and

\[ J_{i,e} = \frac{(1 + e)z_i - w_{i,e}}{1 - \rho(1 - x)}. \]  
(3.28)
Plugging (3.23), (3.25), (3.27) and (3.28) into the steady state version of (3.20) yields

\[
w_{i,n} = \frac{\lambda[1 - \rho(1 - x - \mu_n)]z_i + (1 - \lambda)[1 - \rho(1 - x)](\eta + b)}{1 - \rho(1 - x) + \lambda \rho \mu_n} \tag{3.29}
\]

and

\[
w_{i,e} = \frac{\lambda[1 - \rho(1 - x - \mu_e)](1 + e)z_i + (1 - \lambda)[1 - \rho(1 - x)](\eta + b)}{1 - \rho(1 - x) + \lambda \rho \mu_e} \tag{3.30}
\]

Hence, the steady state equilibrium wages are positively related to the job finding rates and \(z_i\). These results are consistent with the DMP model. Note that \(z_i\) can only affect wages of type \(i\) workers directly while a change in steady state skill-specific job finding rate will affect all workers with the same educational choices. The implications of these results will be discussed in detail in section 3.5. Plugging (3.29) and (3.30) into (3.22) and (3.24) yields

\[
\Phi_{i,j} = A_j z_i + B_j + C_j, \tag{3.31}
\]

where subscript \(j \in \{n, e\}\) and

\[
A_n = \frac{\lambda \rho \mu_n}{(1 - \rho)[1 - \rho(1 - x) + \lambda \rho \mu_n]},
\]

\[
B_n = \frac{\rho \mu_n(1 - \lambda)[1 - \rho(1 - x)](\eta + b)}{(1 - \rho)[1 - \rho(1 - x - \mu_n)][1 - \rho(1 - x) + \lambda \rho \mu_n]},
\]

\[
C_n = \frac{b[1 - \rho(1 - x)] - \rho \mu_n \eta}{(1 - \rho)[1 - \rho(1 - x - \mu_n)]},
\]

\[
A_e = \frac{\lambda \rho \mu_e(1 + e)}{(1 - \rho)[1 - \rho(1 - x) + \lambda \rho \mu_e]},
\]

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\[ B_e = \frac{\rho \mu_e (1 - \lambda)[1 - \rho(1 - x)](\eta + b)}{(1 - \rho)[1 - \rho(1 - x - \mu_e)][1 - \rho(1 - x) + \lambda \rho \mu_e]}, \]

and

\[ C_e = \frac{b[1 - \rho(1 - x)] - \rho \mu_e \eta}{(1 - \rho)[1 - \rho(1 - x - \mu_e)]}. \]

Plugging (3.31) into (3.26) yields

\[ \Omega_i = D_e z_i + G_e, \quad (3.32) \]

where

\[ D_e = \frac{1 - \rho^{k-1}}{1 - \rho} \gamma + \rho^k A_e, \]

and

\[ G_e = \frac{1 - \rho^{k-1}}{1 - \rho} (s - \gamma z_h) + \rho^k (B_e + C_e). \]

As long as \( D_e - A_n > 0 \), there exists a

\[ \bar{z} = \frac{B_n + C_n - G_e}{D_e - A_n}, \quad (3.33) \]

such that whenever \( z_i > \bar{z} \), a type \( i \) worker would choose to pursue education and vice versa. Note that \( D_e - A_n \) is not necessarily greater than zero. If the subjective discount factor is too small, or the probability of death and the time it takes to pursue education are too large, then agents may not be patient enough to take advantage of higher future wage and lower unemployment rate. However, if the effectiveness of education, \( e \), is large enough, then workers will have strong incentives to undertake education. Generally speaking, \( \bar{z} \) is negatively related to \( e, \gamma, s, \rho \) and \( b \) and positively related to \( k \). How \( b \) and \( s \) affect \( \bar{z} \) is going to be scrutinized in section 3.4.
If the economy is in equilibrium, it must be the case that the ex-ante and ex-post beliefs on job finding rates match each other. Hence, by definition, $\mu_n = \varphi \theta_n^\alpha$ and $\mu_e = \varphi \theta_e^\alpha$. Similarly, in the equilibrium, $g_n = \varphi \theta_n^{\alpha-1}$ and $g_e = \varphi \theta_e^{\alpha-1}$. As a result, $\bar{z}$ is a function of $\theta_n$ and $\theta_e$ in the equilibrium. Therefore, the free-entry conditions become

$$q = pg_n(\theta_n)E_{z_i}(J_{i,n}|z_i \leq \bar{z}(\theta_n, \theta_e)),$$

(3.34)

and

$$q = pg_e(\theta_e)E_{z_i}(J_{i,e}|z_i > \bar{z}(\theta_n, \theta_e)),$$

(3.35)

which can be used to solve for $\theta_n$ and $\theta_e$ in the equilibrium. All other endogenous variables can be solved using equilibrium $\theta_n$ and $\theta_e$.

**Proposition 1** Holding $\rho, \eta, b, x, \lambda, e, q, \varphi$ and $\alpha$ constant, $\theta_e$ is an increasing function of $\bar{z}$, given the existence of $\bar{z} \in [z_l, z_h]$.

**Proof.** Suppose the proposition does not hold.

Hence, there exist $\bar{z}_0, \bar{z}_1 \in [z_l, z_h]$ such that $\bar{z}_0 < \bar{z}_1$, $q = pg_e(\theta_{e,0})E_{z_i}(J_{i,e}|z_i > \bar{z}_0)$, $q = pg_e(\theta_{e,1})E_{z_i}(J_{i,e}|z_i > \bar{z}_1)$ and $\theta_{e,0} \geq \theta_{e,1}$. Now, consider $J_{i,e}$. The value of $J_{i,e}$ is determined by $z_i, \bar{z}, \theta_e$. Specifically, according to (3.30) and (3.28)

$$J_{i,e}(z_i; \theta_e) = \frac{(1 - \lambda)[(1 + e)z_{i,e} - \eta - b]}{1 - \rho(1 - x) + \lambda \mu_e}.$$  

(3.36)

Hence,

$$E_{z_i}(J_{i,e}|z_i > \bar{z}_0) = \frac{(1 - \lambda)[(1 + e)(\bar{z}_0 + z_h)/2 - \eta - b]}{1 - \rho(1 - x) + \lambda \mu_e(\theta_{e,0})},$$

and

$$E_{z_i}(J_{i,e}|z_i > \bar{z}_1) = \frac{(1 - \lambda)[(1 + e)(\bar{z}_1 + z_h)/2 - \eta - b]}{1 - \rho(1 - x) + \lambda \mu_e(\theta_{e,1})}.$$  

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Since \( \mu_e \) is an increasing function of \( \theta_e, \mu_e(\theta_{e,0}) \geq \mu_e(\theta_{e,1}) \). As is assumed, \( \bar{z}_0 < \bar{z}_1 \). Hence, 
\[ E_{z_i}(J_{i,e}|z_i > \bar{z}_1) > E_{z_i}(J_{i,e}|z_i > \bar{z}_0) \]. Therefore, \( g_e(\theta_{e,1}) < g_e(\theta_{e,0}) \). Since \( g_e \) is a decreasing function of \( \theta_e, \theta_{e,1} \) must be greater than \( \theta_{e,0} \). Contradiction.

**Proposition 2** Holding \( \rho, \eta, b, x, \lambda, q, \varphi \) and \( \alpha \) constant, \( \theta_n \) is an increasing function of \( \bar{z}, \) given the existence of \( \bar{z} \in [z_l, z_h] \).

**Proof.** Suppose the proposition does not hold.

Hence, there exist \( \bar{z}_0, \bar{z}_1 \in [z_l, z_h] \) such that \( \bar{z}_0 < \bar{z}_1 \), \( q = \rho g_n(\theta_{n,0})E_{z_i}(J_{i,n}|z_i \leq \bar{z}_0) \), 
\[ q = \rho g_n(\theta_{n,1})E_{z_i}(J_{i,n}|z_i \leq \bar{z}_1) \] and \( \theta_{n,0} \geq \theta_{n,1} \). Now, consider \( J_{i,n} \). The value of \( J_{i,n} \) is determined by \( z_i, \bar{z}, \theta_n \). Specifically, according to (3.30) and (3.28)
\[ J_{i,n}(z_i; \theta_n) = \frac{(1 - \lambda)(z_{i,n} - \eta - b)}{1 - \rho(1 - x) + \lambda \mu_n}, \] (3.37)

Hence,
\[ E_{z_i}(J_{i,n}|z_i \leq \bar{z}_0) = \frac{(1 - \lambda)(\bar{z}_0 + z_i)/2 - \eta - b}{1 - \rho(1 - x) + \lambda \mu_n(\theta_{n,0})} \],

and
\[ E_{z_i}(J_{i,n}|z_i \leq \bar{z}_1) = \frac{(1 - \lambda)(\bar{z}_1 + z_i)/2 - \eta - b}{1 - \rho(1 - x) + \lambda \mu_n(\theta_{n,1})} \],

Since \( \mu_n \) is an increasing function of \( \theta_n, \mu_n(\theta_{n,0}) \geq \mu_n(\theta_{n,1}) \). As is assumed, \( \bar{z}_0 < \bar{z}_1 \). Hence, 
\[ E_{z_i}(J_{i,n}|z_i \leq \bar{z}_1) > E_{z_i}(J_{i,n}|z_i \leq \bar{z}_0) \]. Therefore, \( g_n(\theta_{n,1}) < g_n(\theta_{n,0}) \). Since \( g_n \) is a decreasing function of \( \theta_n, \theta_{n,1} > \theta_{n,0} \). Contradiction.

These propositions are very useful to analyze the labor market outcomes caused by changes in educational subsidy or skill-biased technological shocks. The intuitions of the propositions are as the following. As \( \bar{z} \) increases, less residents are undertaking education.
The expected productivity of both educated and uneducated workers will increase. As a result, the expected profits entrepreneurs can earn by hiring educated or uneducated workers will both increase. Consequently, entrepreneurs are willing to post relatively more vacancies for both groups of workers and both labor markets will be less tight.

### 3.2.6.2 Unemployment Rates in the Steady State

Consider uneducated workers. Let $N_n$ be the steady state population of uneducated labor force. Obviously, $N_n = NF(z)$. According to the dynamics of unemployment of uneducated workers, it must follow that

$$u_n = u_n(1 - \mu_n(\theta_n))(1 - \delta) + \delta N_n + x(1 - \delta)(N_n - u_n).$$

(3.38)

The first term on the right hand side of (3.38) measures the number of uneducated workers unemployed in the last period who are still alive and unemployed in the current period. The second term characterizes the number of newly born residents who choose not to undertake education and start to search for a job immediately. The third term measures the number of currently alive uneducated workers who lost their jobs in the last period. Hence, the steady state unemployment rate for uneducated workers is

$$u_n = \frac{\delta + x(1 - \delta)}{1 - (1 - x - \mu_n(\theta_n))(1 - \delta)}. \tag{3.39}$$

If $\delta = 0$, then (3.39) will collapse to $x/(x + \mu_n)$, which is the steady state unemployment rate in the DMP model.

Consider educated workers. Denote the steady state population of educated labor
force by $N_e$. It follows that

$$N_e = N[1 - F(\bar{z})] - N[1 - F(\bar{z})] \delta \sum_{j=0}^{k-1} (1 - \delta)^j.$$  \hfill (3.40)

The first term on the right hand side of (3.40) measures the total amount of residents whose ability is greater than the threshold ability. The second term measures the amount of residents who are currently undertaking education. The steady state unemployment rate of educated workers is characterized by

$$u_e = \frac{\delta + x(1 - \delta)}{1 - (1 - x - \mu_e(\theta_e))(1 - \delta)}.$$  \hfill (3.41)

One purpose of this paper is to explain the unemployment rate gap between educated and uneducated workers. Proposition 3 shows the existence of the gap.

**Proposition 3**  The steady state unemployment rate of uneducated workers is higher than the steady state unemployment rate of educated workers.

**Proof.** According to (3.39) and (3.41), I need to show that $\mu_e(\theta_e) > \mu_n(\theta_n)$. Suppose not, that is $\mu_e(\theta_e) \leq \mu_n(\theta_n)$. Hence, $\theta_e \leq \theta_n$. According to (3.36) and (3.37), it can be shown that

$$E_{z_i}(J_{i,e}|z_i > \bar{z}) = \frac{(1 - \lambda)[(1 + e)(\bar{z} + z_h)/2 - \eta - b]}{1 - \rho(1 - x) + \lambda \mu_e(\theta_e)},$$

and

$$E_{z_i}(J_{i,n}|z_i \leq \bar{z}) = \frac{(1 - \lambda)[(\bar{z} + z_l)/2 - \eta - b]}{1 - \rho(1 - x) + \lambda \mu_n(\theta_n)}.$$  

Therefore, $E_{z_i}(J_{i,e}|z_i > \bar{z}) > E_{z_i}(J_{i,n}|z_i \leq \bar{z})$. To satisfy (3.34) and (3.35), $g_n(\theta_n)$ must be greater than $g_e(\theta_e)$. Hence, $\theta_e > \theta_n$. Contradiction. Hence, $\mu_e(\theta_e) > \mu_n(\theta_n)$ and $\frac{u_e}{N_e} > \frac{u_n}{N_n}$.

$\blacksquare$
3.3 Parameterization

In a search model, a worker searching for a job will be unemployed for at least one period. Hence, the length of each period should be short enough and it is set to be 10 days, or 1/3 month. Accordingly, the discount factor $\beta$ is set to be 0.999, such that the annual discount factor is 0.96. Assume that the length of a person’s life follows the Poisson distribution. Then $\delta$ is set to be $4.42 \times 10^{-4}$, such that the life expectancy in the model is 62 years. Since higher education is to be discussed, $k$ is set to be 146, implying that it takes a student four years to complete her college education. Normalize the lower bound of ability, $z_l$, and the total population, $N$, to 1.

The bargaining power $\lambda$ for both types of workers are set to be 0.5. According to Hosios (1990), a search economy reaches Pareto Optimality when the sum of bargaining power of workers and the vacancy elasticity of matching is 1. Following this condition, I set $\alpha$ to be 0.5. The job separation rate is obtained from Job Openings and Labor Turnover Survey (JOLTS). The average job separation rate from December 2000 to March 2011 is 3.6%. Hence, $x$ is calibrated to 0.012.

In addition to the parameters stated above, there are still eight of them to be pinned down. Therefore, eight calibration targets are needed. Among them, six are based on the data of the Current Population Survey (CPS) from year 1992 to year 2011. In order to focus on the choice of whether to pursue college education, all the observations with educational achievements less than a high school diploma or higher than a bachelor’s degree. Economic agents enter into the model at the age of 18. Hence, the probability corresponds to an average total life expectancy of 80 years.
degree are dropped. The first calibration target is the aggregate unemployment rate. During this period, the average unemployment rate is 5.4%. The second calibration target is the proportion of individuals with high school diploma but without a college degree, which is 62.2%. The third target is the ratio of average weekly wages between high school graduates and workers with bachelor’s degrees. The weekly wage is calculated as the last year’s wage income divided by the number of weeks employed last year.\footnote{In the original data set, only the number of weeks unemployed was provided. Number of weeks employed last year is calculated as the difference between 52 and number of weeks unemployed last year. This is an approximation because the calculation rules out the possibility that a worker was not in the labor force during a certain period last year.} It turns out that the wage ratio given by CPS is 0.57. The forth target is the ratio between average weekly unemployment benefit and average weekly wage of high school graduates, which is 0.52 over this period.\footnote{To calculate the average weekly unemployment benefit, I dropped the observations without any unemployment benefit income first, and then divided unemployment benefit income by the number of weeks unemployed.} The fifth calibration target is the ratio between average weekly educational subsidy and average weekly unemployment benefit, which turns out to be 0.33.\footnote{To calculate the average weekly educational subsidy, I dropped all observations with zero income from educational assistance, then divided educational assistance by 52.} The sixth calibration target is ratio between the standard deviation of weekly wage income and average weekly wage of high school graduates. I found this ratio to be 1.37.

The seventh calibration target is the average market tightness. According to the data from JOLTS, the average market tightness from December 2000 to March 2011 is 0.44. According to Silva and Toledo (2009) and Milgrom and Hall (2008), the average cost of
posting one vacancy is about 9 days of the average wage. Hence, as the last calibration target, $q$ is set to be 90\% of the average wage, since one period is set to be 10 days.

Table 3.1 summarizes the values of the parameters.

3.3.1 Performance of the Model

To demonstrate the performance of this model in terms of explaining the unemployment gap, the steady state unemployment rates of the two groups of workers generated by the model are compared with those from data. This comparison is summarized in table 3.2. Since the two unemployment rates are not calibration targets, it shows that the mechanism presented in this paper can explain 64\% of the difference between the two unemployment rates. The overestimate of $u_e$ and underestimate of $u_n$ might be attributed to the assumption that job separation rates for the two labor markets are the same. Nevertheless, some studies show that the separation rates are different. Fallick and Fleishman (2004) find that the monthly total separation rates for workers with college degree and high school graduates are 4.6\% and 6.6\%, respectively. However, these numbers are not appropriate for the purpose of this paper, because total separation rate is calculated based on separations caused by unemployment, leaving the labor force, and employer switching. Only the first scenario is modeled in this paper. To see the performance of the model when education-specific separation rates are considered, I set the monthly separation rate for uneducated workers to be 4.04\% and that for educated workers to be 2.83\%, such that the aggregate monthly separation rate is still 3.6\% and the ratio between the two education-specific separation rates are the same with the ratio of the two total separation rates reported by Fallick.
and Fleischman (2004). Under this setting, the model can explain 97% of the difference between the two unemployment rates. This result is summarized in table 3.3.

3.4 Policy Experiments

3.4.1 Unemployment Benefit

The first policy to be examined is a change in unemployment benefit, while holding all other parameters constant. Specifically, consequences of a change in unemployment benefit on educational choices and wage inequality are scrutinized. For the purpose of this section, the values of the variables at the steady state are calculated for each value of unemployment benefit between 0.556 to 0.876.

3.4.1.1 Educational Choice

Figure 3.2 exhibits the proportion of residents choosing to be educated at the steady state for a given value of unemployment benefit. It is clear that as unemployment benefit increases, more residents will undertake education. The intuitions of this result are as the following.

In the DMP model, with a more generous unemployment benefit, workers will ask for higher wages when bargaining. Hence, firms will earn fewer profits and have weaker incentives to post vacancies, which will result in higher unemployment rate. The argument also holds in this model. However, the effects of the change in $b$ are asymmetric. Since unemployment benefit takes up a larger proportion of wage income for uneducated workers, the percentage change of the wages of uneducated workers is larger as well. Consequently,
the expected asset value of firms hiring uneducated workers decreases by a larger percentage value. Hence, uneducated workers suffer from a larger increase in unemployment rate compared to educated workers. Since one motivation of getting more education is to decrease the probability of unemployment, the value of education increases when the difference between the two unemployment rates increases. As a result, more workers opt to undertake education. For simplification, I assume all workers receive the same unemployment benefit regardless of educational choices. If this assumption is relaxed, the result will not change qualitatively, as long as the replacement ratio of unemployment benefit is a decreasing function of wage income.

To exhibit this point, figure 3.3 presents the unemployment rate of uneducated workers subtracted by that of educated workers for each value of unemployment benefit. The upward slope of the curve clearly shows that it is the unemployment gap that motivates more residents to undertake education when unemployment benefit increases. Actual data is consistent with the prediction. During the past decades, the real unemployment benefit per month per person saw sharp increases during recessions. Simultaneously, the unemployment rate gap between the two groups of workers also tends to increase.

A very important implication of this result is that the standard search and matching model will overestimate the cost of an increase in unemployment benefit. In the DMP model, unemployment rate increases due to the increase of $b$, which lowers output and consumption. However, when educational choices are considered, an increase in unemployment benefit leads to a compositional change of labor force, reducing the output loss. The effects of this change are two-fold. First, the average productivity of workers improves when $b$
increases. This effect is similar to the result of Acemoglu and Shimer (2000). However, their mechanism focuses on the demand side of the labor market. They argue that higher unemployment benefit encourages firms to post more high quality jobs, leading to higher productivity of employed workers. Second, when more residents undertake education, the group of workers with lower unemployment rate becomes larger, which partially cancels the increase in unemployment rate due to higher unemployment benefit. Figure 3.4 exhibits the total consumption of the economy given different levels of $b$. It can be concluded that the decline of consumption predicted by the standard search model is reversed.

### 3.4.1.2 Wage Inequality

Several empirical studies, for example, Koeniger, Leonardi and Nunziata (2007) and Mooi-Reci (2011), show that a more generous unemployment benefit tends to lower wage inequality. This result is also confirmed in this paper. Figure 3.5 exhibits the negative relationship between wage Gini coefficient and unemployment benefit.

Note that the Gini coefficient generated by the model is about 0.18 in the benchmark steady state. Nevertheless, according to United Nations Development Programme (2006), the wage Gini coefficient in US is 0.4.\(^\text{12}\) The large difference between the two numbers can be attributed to at least two reasons. First, the target groups of workers are high school graduates and college graduates, which are in the middle of the spectrum of educational achievements. Hence, extreme values of wage income are not generated by the model. If this model is further expanded to allow more than two levels of education, the Gini coefficient generated by the model will be lower.

Gini coefficient should be magnified. Second, the model in this paper does not consider frictional wage inequality in addition to ability inequality. Nevertheless, Hornstein, Krusell and Violante (2011) argues that frictional wage inequality is relatively large.

The reason why wage inequality decreases as unemployment benefit increases is two-fold. First, as stated above, a change in unemployment benefit will lead to a larger percentage change of the wages of uneducated workers. Second, several types of uneducated workers would switch their educational choices and enjoy the increase in wage, due to the productivity gain and a less tight labor market, when unemployment benefit becomes more generous.

The most interesting implication of the decrease in wage inequality due to the more generous unemployment benefit is the importance of taking unemployment probability into account. When it comes to model the incentive of undertaking education, if we only consider wage premium, then we should have predicted that fewer people would like to pursue education, given that wage premium narrows as unemployment benefit rises, which contradicts with the prediction made in section 3.4.1.1.

3.4.2 Educational Subsidy

3.4.2.1 Unemployment Rates and Wages

The following proposition discusses what happens when educational subsidy changes.

**Proposition 4** An increase in educational subsidy leads to higher steady state unemployment rates for both educated and uneducated workers. For any newly born individual that would have the same educational choice given the original and new levels of educational
subsidy, they will earn lower wages in the steady state.

Proof. According to (3.33), when \( s \) increases, \( \bar{z} \) will decrease, that is, more residents are going to undertake education. According Proposition 1 and 2, both \( \theta_e \) and \( \theta_n \) are increasing functions of \( \bar{z} \). Hence, \( \theta_e \) and \( \theta_n \) will decrease due to the increase of \( s \). As characterized by (3.39) and (3.41), \( \frac{u_e}{N_e} \) and \( \frac{u_n}{N_n} \) are negatively related to corresponding market tightness. Hence, \( \frac{u_e}{N_e} \) and \( \frac{u_n}{N_n} \) will increase due to the increase of \( s \). Based on (3.29) and (3.30), a worker’s wage is negatively related to market tightness. Unless, a newly born worker would have different educational choices, the decrease of \( \theta_e \) and \( \theta_n \) will lead to lower wages. ■

The intuition of Proposition 4 is straightforward. Unlike unemployment benefit that affects educational choices indirectly by changing the unemployment rate gap, educational subsidy can directly alter the incentives of undertaking education. The threshold ability of undertaking education decreases as \( s \) is raised. Such a dip in the threshold ability means that the expected productivity of educated workers will decrease, as the additional workers who undertake education are less productive compared to those educated workers before the change in educational subsidy. As a result, entrepreneurs will expect lower profits from educated workers and a lower \( E_{z_t}(J_{i,e}|z_t > \bar{z}(\theta_n, \theta_e)) \). Hence, the free entry condition predicts a tighter labor market for educated workers, resulting in higher unemployment rate. It has been shown in (3.30) that the equilibrium wage of an educated worker is positively related to job finding rate and market tightness. As a result of a tighter labor market, the equilibrium wage of an originally educated worker will decrease.
Now, consider the changes in the labor market for uneducated workers. A decline in the threshold ability of undertaking education is also bad news for uneducated workers. Those who switch from being uneducated to educated are the most productive ones in the initial uneducated group. Hence, entrepreneurs will also expect a lower $E_{z_i}(J_{i,e}|z_i > \bar{z}(\theta_n, \theta_e))$ as a result of an increase in educational subsidy, which means that $g_n$ becomes larger and the market is tighter for uneducated workers. Similar to educated workers, the uneducated workers will also experience an increase in unemployment rate and a decrease in wages. The changes of skill specific unemployment rates are summarized in figure 3.6 and figure 3.7.

Given the arguments stated above, all those types of workers that have the same educational choices before and after the increase in educational subsidy are worse-off. However, the types of educated workers who originally would not have pursued education will be better-off, as they suffer less from unemployment and earn higher wages. Another very important implication of the arguments is that unemployment rate in this model is not determined by the quantity of workers of each group, but the quality of workers. Even though the supply of uneducated workers decreases, they suffer higher unemployment probability and lower real wage.

Though both skill-specific unemployment rates increase, the aggregate unemployment rate does not necessarily rise. The reason why this seemingly unreasonable result emerges is the compositional change of labor force in terms of educational achievements. The fraction of educated workers increases. Hence, the weight of the group of workers with lower unemployment rate becomes larger. This compositional change of labor force may
reduce, neutralize or even reverse the effects of the increase in skill-specific unemployment rates, depending on the choice of parameter values. Figure 3.8 presents the simulated aggregate unemployment rate for each value of educational subsidy. It turns out that, based on the calibrated parameters, the aggregate unemployment rate decreases as educational subsidy increases.

3.4.2.2 Social Welfare

In this part, I show that an increase in educational subsidy will lead to an improvement of social welfare in the long run, based on the simulation results shown below.

In the previous sections, the goods market clearing condition is not explicitly stated. To proceed, I need to specify the budget constraint of the government and use it to construct the resource constraint.

Assume that each resident in the economy has to pay a lump sum tax \( \tau_t \) in period \( t \) regardless of her labor force status or employment status. The taxes collected are used to pay for the unemployment benefit and educational subsidy and the government runs a balanced budget in each period. Obviously, this tax scheme will not distort any incentive, and it will not change any educational choice. Hence, the labor market outcomes obtained from sections 3.2 and 3.3 can be transferred to this part directly. Assume that an entrepreneur’s flow utility function equals her consumption. Since all profits are obtained by entrepreneurs. It is thus obvious that the steady state goods market clearing condition is

\[
tc = y - q(v_e + v_n),
\]

where \( tc \) is steady state total consumption, \( y \) is steady state output, \( v_e \) and \( v_n \) are steady
state vacancies for educated and uneducated workers. Provided with the production function and distribution of ability, I can show that

$$y = \frac{1}{2}(z_l + \bar{z})(N_n - u_n) + \frac{1}{2}(1 + e)(\bar{z} + z_h)(N_e - u_e).$$  

(3.43)

Since the utility functions of all agents (residents and entrepreneurs) in the economy are linear in consumption, the summation of the flow utility of all agents at the steady state, $\chi$, can be written as

$$\chi = tc - \eta(N_n + N_e - u_n - u_e) - \frac{1}{2}\gamma(z_h - \bar{z})N[1 - F(\bar{z})]\delta \sum_{j=0}^{k-1}(1 - \delta)^j.$$  

(3.44)

The second term of (3.44) is the total disutility from working. $\frac{1}{2}\gamma(z_h - \bar{z})$ is the average disutility from education and $N[1 - F(\bar{z})]\delta \sum_{j=0}^{k-1}(1 - \delta)^j$ is the number of residents currently undertaking education. Hence, the third term of (3.44) shows the total disutility from education. The social welfare, $\Delta$, at the steady state can be characterized as

$$\Delta = \frac{\chi}{1 - \rho}.$$  

(3.45)

Since $\Delta$ is a monotonic transformation of $\chi$, I will concentrate my analysis on $\chi$. As stated above, when $s$ increases, more residents choose to undertake education and aggregate unemployment rate decreases, both of which lead to more total output. As a result, there is a utility gain from more consumption (the first term of $\chi$). However, as unemployment decreases, disutility from working increases (the second term of $\chi$). Since $\bar{z}$ goes down due to the increase of $s$, the average disutility from education increases. Together with an increase in the number of residents undertaking education, total disutility from education should also increase (the third term of $\chi$).
According to the simulation results, the positive impact from the first term of (3.44) as a result of a higher educational subsidy outweighs the negative impact from the last two terms. Hence, based on this model, a higher educational subsidy actually improves the welfare of the society as a whole. Figure 3.9 shows the positive relationship between unemployment benefit and social welfare.

3.5 Skill-biased Technological Change

Since 1970s, uneducated workers’ real wage has decreased both absolutely and relatively compared to the real wages of educated workers even though the supply of educated labor increased dramatically. One explanation is that technology improvements are biased to educated workers such that the demand for uneducated workers decreased relatively. The model presented in this paper can also be used to analyze the effect of a skill-biased technological change. Existing literature emphasizes on the fact that demand for low skill labor drops. Under the search and matching framework, Acemoglu (1999) develops a model with two types of labor with different productivity and show that separating equilibrium only emerges when the productivity difference or the number of high skill labor are large enough. He argues that a skill-biased technological change would increase the productivity gap and therefore, the economy would switch from a pooling equilibrium to separating equilibrium. The labor market will be divided into two sub-markets and the demand for low skill worker falls. As a result, unemployment rate of low skill workers increases and they receive lower real wages.

My model, however, will give a new explanation on how skill-biased technological
change impacts on uneducated workers. To model skill-biased technological change, suppose there is a productivity bonus for educated workers. That is, \( p_i = (1 + \varepsilon)(1 + \varepsilon)z_i \) if type \( i \) workers undertake education, where \( \varepsilon > 0 \) is the size of a onetime technological change.

Hence, the wage of a type \( i \) educated worker becomes

\[
 w_{i,e} = \frac{\lambda[1 - \rho(1 - x - \mu_e)](1 + \varepsilon)(1 + \varepsilon)z_i + (1 - \lambda)[1 - \rho(1 - x)](\eta + b)}{1 - \rho(1 - x) + \lambda\rho\mu_e} 
\]

\[
= (1 + \varepsilon)(1 + \varepsilon)z_{i,e} - \frac{(1 - \lambda)[1 - \rho(1 - x)][(1 + \varepsilon)(1 + \varepsilon)z_{i,e} - \eta - b]}{1 - \rho(1 - x) + \lambda\mu_e}. 
\]

Acemoglu (1999) assumes that there is an exogenously given constant share of workers that are of low skill type. Unlike him, I assume that workers can choose their educational achievements endogenously. Hence, in addition to the demand side effect, a skill-biased technological change also has a supply side effect.

If a skill-biased technological change hits the economy, the productivity of educated workers increases. Holding all other variables and parameters constant, the profits a firm obtains by hiring an educated worker rise. Consequently, entrepreneurs are willing to post more vacancies for educated workers, making the job finding rate for educated workers larger. According to (3.46), both the productivity bonus and the larger job finding rate lead to higher wages for educated workers. Furthermore, they also face a lower unemployment rate as a result. Both effects of the technological change provide a stronger incentive for workers to undertake education and the number of uneducated worker declines. This compositional change of uneducated workers indicates a drop in \( \overline{z} \). This is a very important supply side effect of the technological change, which is not considered by Acemoglu (1999). Hence, entrepreneurs expect the average productivity of uneducated workers to go down, posting fewer vacancies for them, leading to a larger unemployment rate for uneducated
workers. The average wage of uneducated workers is also strongly affected by the supply side effect. Note that the steady state wages are partially determined by the productivity of workers. The originally most productive types of uneducated workers are also those who earned the highest wages in the group. As they change their educational choice, the average wage certainly drops.

The following proposition formalizes the effects of such a skill-biased technological change on uneducated workers.

**Proposition 5** When there is a positive skill-biased technological change, in the steady state, more residents undertake education. The steady state unemployment rate of uneducated workers will increase, the steady state wage of each uneducated worker and the steady state average wage of uneducated workers will decrease.

**Proof.** Technically, a skill-biased shock is equivalent to an increase in the effect of education. Hence, consider an increase of $e$ in (3.33). This change will lead to a smaller $\bar{z}$, which means more residents will undertake education as $1 - F(\bar{z})$ is the steady state proportion of residents undertaking education. According to Proposition 2, $\theta_n$ is an increasing function of $\bar{z}$. Hence, $\theta_n$ will decrease. Based on (3.39), it can be concluded that the unemployment rate of uneducated workers will increase. According to (3.29), $w_{i,n}$ is an increasing function of $\theta_n$. Hence, the wage of each type of individuals, who still choose not to be educated after the change in $e$, decreases. The average wage of uneducated worker
can be written as

\[
\begin{align*}
\frac{\partial w_n}{\partial \bar{z}} &= \frac{\lambda[1 - \rho(1 - x - \mu_n)](z_l + \bar{z})/2 + (1 - \lambda)[1 - \rho(1 - x)](\eta + b)}{1 - \rho(1 - x) + \lambda \rho \mu_n} \\
\frac{\partial w_n}{\partial \theta_e} &= 0.
\end{align*}
\] (3.47)

The first line of (3.47) shows that \( \frac{\partial w_n}{\partial \bar{z}} > 0 \). The second line of (3.47) shows that \( \frac{\partial w_n}{\partial \theta_e} > 0 \). Since both \( \bar{z} \) and \( \theta_e \) decreases as the result of an increase in \( e \), the average wage of uneducated workers decreases.

The average real wage and unemployment rate of uneducated workers given various level of skill-biased technological change are presented in figure 3.10 and figure 3.11.

3.6 Conclusion

This paper considers a combination of search model and a signaling game. Based on this model, by undertaking education, workers send a positive signal to entrepreneurs. As a result, firms are more willing to post vacancies and hire educated workers, which means educational achievement is required for job application by entrepreneurs to distinguish high-productivity workers from their low-productivity peers. Benefiting from a less tight labor market, educated workers enjoy higher wages and suffer less from unemployment, which motivates residents to undertake education. This model can explain about 64% of unemployment rate gap between educated and uneducated workers when a single job separation rate is used. When education-specific job separation rates are taken into consideration, the model can explain almost all the unemployment rate gap.

If unemployment benefit is increased, both skill specific unemployment rates will
increase. The model predicts that, however, the unemployment gap will increase as the consequence. Hence, the higher unemployment benefit stimulates more residents to undertake education, meaning that the DMP model overestimates the cost of unemployment benefit.

Higher educational subsidy encourages more workers to undertake education, lowering the expected ability of both groups of workers. Both skill-specific unemployment rates, therefore, will increase. However, as the weight of educated workers becomes larger, aggregate unemployment rate will decrease. The additional utility brought by extra consumption will outweigh the additional disutility from extra working and education, which means the welfare effect of a higher educational subsidy is positive.

When the economy encounters a skill-specific technological change, firms will post more vacancies for educated workers, creating a stronger incentive to undertake education. There are two consequences due to this change. First, the average quality of uneducated workers is lower. Second, the labor market for uneducated workers becomes tighter. Both of them tend to increase the unemployment rate of uneducated workers and lower their real wage, which is consistent with economic data since 1970s.
### Table 3.1: Values of Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>total population</td>
<td>1.00</td>
</tr>
<tr>
<td>( \beta )</td>
<td>discount factor</td>
<td>.999</td>
</tr>
<tr>
<td>( \delta )</td>
<td>probability of death</td>
<td>( 4.42 \times 10^{-4} )</td>
</tr>
<tr>
<td>( k )</td>
<td>periods needed for education</td>
<td>146</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>workers’ bargaining power</td>
<td>.500</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>elasticity of matching</td>
<td>.500</td>
</tr>
<tr>
<td>( x )</td>
<td>job separation rate</td>
<td>.012</td>
</tr>
<tr>
<td>( \eta )</td>
<td>disutility from working</td>
<td>.091</td>
</tr>
<tr>
<td>( b )</td>
<td>unemployment benefit</td>
<td>.726</td>
</tr>
<tr>
<td>( z_l )</td>
<td>lower bound of ability</td>
<td>1.00</td>
</tr>
<tr>
<td>( z_h )</td>
<td>upper bound of ability</td>
<td>2.40</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>rate of disutility from education</td>
<td>.374</td>
</tr>
<tr>
<td>( e )</td>
<td>productivity return of education</td>
<td>.203</td>
</tr>
<tr>
<td>( \varphi )</td>
<td>matching efficiency</td>
<td>.338</td>
</tr>
<tr>
<td>( q )</td>
<td>vacancy posting cost</td>
<td>1.86</td>
</tr>
<tr>
<td>( s )</td>
<td>educational subsidy</td>
<td>.242</td>
</tr>
</tbody>
</table>

### Table 3.2: Unemployment Rates (common separation rate)

<table>
<thead>
<tr>
<th>Employment Rate</th>
<th>Model</th>
<th>Data (92-11)</th>
</tr>
</thead>
<tbody>
<tr>
<td>educated workers</td>
<td>3.80%</td>
<td>2.91%</td>
</tr>
<tr>
<td>uneducated workers</td>
<td>6.40%</td>
<td>7.00%</td>
</tr>
<tr>
<td>difference</td>
<td>2.60%</td>
<td>4.09%</td>
</tr>
</tbody>
</table>

### Table 3.3: Unemployment Rates (with education-specific job separation rates)

<table>
<thead>
<tr>
<th>Employment Rate</th>
<th>Model</th>
<th>Data (92-11)</th>
</tr>
</thead>
<tbody>
<tr>
<td>educated workers</td>
<td>2.91%</td>
<td>2.91%</td>
</tr>
<tr>
<td>uneducated workers</td>
<td>6.89%</td>
<td>7.00%</td>
</tr>
<tr>
<td>difference</td>
<td>3.98%</td>
<td>4.00%</td>
</tr>
</tbody>
</table>
Figure 3.1: Unemployment Rates
Figure 3.2: Steady State Proportion of Educated Agents for Each Level of Unemployment Benefit
Figure 3.3: Steady State Unemployment Rate Gap for Each Level of Unemployment Benefit
Figure 3.4: Steady State Total Consumption for Each Level of Unemployment Benefit
Figure 3.5: Steady State GINI Coefficient for Each Level of Unemployment Benefit
Figure 3.6: Steady State Unemployment Rate of Uneducated Agents for Each Level of Educational Subsidy
Figure 3.7: Steady State Unemployment Rate of Educated Agents for Each Level of Educational Subsidy
Figure 3.8: Steady State Aggregate Unemployment Rate for Each Level of Educational Subsidy
Figure 3.9: Steady State Social Welfare for Each Level of Educational Subsidy
Figure 3.10: Steady State Wage Rate of Uneducated Agents for Each Level of Educational Subsidy
Figure 3.11: Steady State Unemployment Rate of Uneducated Agents for Each Level of Educational Subsidy
Chapter 4

On the Higher Unemployment Rate of College Graduates, Higher Educational Requirement for Job Application, and Overeducation in China

4.1 Introduction

It has been well observed that recent college graduates in China suffered from the changes in labor market conditions. Specifically, current college graduates, on average, are
more likely to be unemployed and earn lower wages compared to their predecessors. Historically, China’s labor market was very segmented. College graduates only search in high-skill labor market whereas low-skill job vacancies are filled with workers without college degrees. However, recently, some college graduates search for low-skill jobs. Though almost all firms with low-skill job vacancies historically accept job applications from any applicant, some of them start to require their job applicants to have higher educational achievements. Generally speaking, overeducation at the college level and skill mismatch emerged in the same period of time. Simultaneously, since year 1998, the supply of higher education in China has experienced extremely fast growth and the number of college graduates skyrocketed. According to the data provided by the ministry of education of China P.R., 358,373 students were awarded a Bachelor’s degree in 1998. This number became 6,361,179 in year 2012.

1A very extreme case is that over 3000 Chinese college graduates competed for 457 positions of street sweepers in the city of Harbin. Among them, 25 job applicants have Master’s degrees. See http://www.scmp.com/comment/blogs/article/1076065/thousands-college-graduates-vie-jobs-street-sweepers-china

2For example, it has been widely reported that several Chinese universities require the job applicants for security guard jobs to possess master’s degrees. See http://edu.sina.com.cn/kaoyan/2014-01-17/0759407591.shtml and http://www.sn.xinhuanet.com/2013-11/29/c_118347173.htm.

3Overeducation and skill mismatch are often discussed together in existing literature. Specifically, a lot of studies define overeducation as underutilization of the skills attained through the process of education. For example, see Clogg and Shockey (1984) and Burris (1983).

4The data is available at the website of the Ministry of Education, PRC. See http://www.moe.edu.cn/publicfiles/business/htmlfiles/moe/moe_577/200505/3067.html

5The data is available at the website of the Ministry of Education, PRC. See http://www.moe.edu.cn/publicfiles/business/htmlfiles/moe/s7382/201305/152553.html
This paper develops the model proposed by Liu (2012) further to link and explain all the facts stated above with a search and matching framework. Unlike the existing assignment models and job competition models addressing the issue of overeducation, for example, Sattinger (1993) and Moen (1999), this paper focuses on the interactions between individuals’ educational choices and the average productivity of educated workers. Furthermore, this paper also put an emphasis on the role played by unemployment. The intuitions of this paper are as the following.

Consider an economy in which there exists many individuals with different productivity. These individuals, when born, determine whether they undertake education or not. Each educated worker suffers disutility from education, which is negatively related to the productivity of the worker. Consequently, only high-productivity individuals will choose to be educated. Entrepreneurs in the economy post high-skill and low-skill job vacancies. A high-skill job requires capital input and specific knowledge. Hence, only educated workers are adaptive to high-skill jobs. However, both educated and uneducated workers can fill low-skill job vacancies. Entrepreneurs do not know which specific worker will fill a job vacancy when it is posted. Consequently, entrepreneurs determine the number of vacancies posted based on the expected productivity of different groups of workers. However, through an interview process, the productivity of the job applicant is revealed to the entrepreneur after the match happens.

If the disutility from education is very large, only the most productive individuals will choose to be educated, which is a very small proportion of the whole population. As a result, they will only search in the high-skill labor market. Furthermore, only uneducated
workers searches in the low-skill labor market and no educational requirement in that market is necessary. As the supply of higher education increases, it is easier for individuals to pass the college entrance exam and graduate. In other words, the disutility from education drops. Therefore, more individuals will undertake education and the average productivity of educated workers drops. Consequently, firms post relatively fewer high-skill job vacancies and unemployment rate of educated workers rises. Since firms have to pay costs for capital involved with high-skill jobs, relatively less productive educated workers will find it hard to earn a high real wage, which motivates these workers to search in the low-skill labor market and compete with uneducated workers. When both educated and uneducated workers search in the low-skill labor market, entrepreneurs have an incentive to divide the low-skill market further into two sub-markets to maximize total profits. In one of the sub-markets, entrepreneurs would require all job applicants to have received education. Thus, educational requirement for job application emerges in the low-skill labor market even though the duties of the jobs remain unchanged. In other words, unlike the model proposed by Liu (2012) in which the endogenous educational requirement always exists, it would only exist when the number of educated workers is large enough.

The reason why the division of low-skill labor market takes place is as the following. Since the educational choice of an individual serves as the signal of her productivity, entrepreneurs know that the educated workers are more productive compared to their un-educated peers. Instead of posting two types of vacancies for different groups of workers, if an entrepreneur posts only one type of vacancies based on the average productivity of all unemployed workers searching in the low-skill labor market, then the entrepreneur is not
using up all the information available and will fail to maximize the value of her firms.

Hence, many observed changes in China’s labor market outcomes, including higher educational requirement for job application, higher unemployment rate of college graduates, and college graduates’ searching in low-skill labor market, might be triggered by the dramatic increase in the supply of higher education. However, none of these observed changes would take place if the increase in the supply of higher education does not alter the average productivity of college educated workers. Essentially, it is the decrease in their average productivity due to the larger number of college educated workers that results in the variations of China’s labor market.

The rest of the chapter is organized as the following. Section 4.2 presents the general form of the model. In this section, the educational choices of all individuals as well as the labor market search decisions made by educated workers are considered. It is assumed that at least some of the educated workers search in the low skill labor market. The conditions that lead to all educated workers to search in the high skill labor market are derived in section 4.3. Section 4.4 evaluates the impacts of an increase in the supply of higher education. Section 4.5 concludes the chapter.

4.2 Model with the Division low-skill Labor Market

4.2.1 Basic Settings

Consider a model economy with no aggregate uncertainty. \( N \) individuals who are born with different levels of productivity reside in the economy. A generation \( \tau \) individual
is born in period $\tau$. The type of an individual is entirely defined by her endowed productivity. Denote the productivity of a type $i$ individual of any generation by $p_i$, which is a continuous variable bounded by $[p_l, p_h]$. The unconditional cdf and pdf of $p_i$ are $F(\cdot)$ and $f(\cdot)$, respectively. The rate of death for each individual is $\delta$, where $\delta \in (0, 1)$. Hence, $\delta N$ individuals die in each period. To keep total population constant, assume that $\delta N$ individuals are born every period. The subjective discount rate of an individual is $\beta \in (0, 1)$. The effective discount rate, after taking the probability of death into consideration, therefore, is $\rho = \beta(1 - \delta)$.

When born, an individual has a one-time chance to determine whether to undertake education or not. This is a simplifying assumption. Since this paper focuses on analyzing long-run equilibrium outcomes of the labor markets and individuals’ educational choices, this assumption suffice for the purpose of comparative steady state analysis. A utility maximizing individual will only choose to be educated if the life-time utility as an educated worker is at least as large as the life-time utility of an uneducated worker. At the steady state, if an individual finds herself better-off by undertaking education in the second period, she must also find herself better-off by doing so as soon as she is born. Consequently, she would not be willing to postpone her education.

If an individual decides to undertake education, she will suffer a disutility, which is negatively related to her productivity. It follows that the disutility of undertaking education for a type $i$ individuals given by $d(e) + \gamma(p_h - p_i)$, where $\gamma$ is a positive constant, $e$ is the supply of higher education and $d'(e) < 0$. The negative relationship between $p_i$ and disutility of education is the same assumption used in signaling games such as Arrow (1973)
and Spence (1973). As to be discussed later, this assumption is the key for the existence of educational requirement in the low-skill labor market. For simplicity, assume that education does not take time. This assumption will not change the intuitions and qualitative conclusions of the model.

Each individual is endowed with one unit of time every period. There are two mutually exclusive alternative usages of time: searching for a job or working for wage. Call an individual actively searching for a job unemployed worker. An unemployed worker can search for a job only once in one labor market in each period. This is also a simplifying assumption. It can be shown that, even searching in multiple job markets are allowed, except those workers whose productivity is exactly as large as a certain threshold value, the optimal behavior for an unemployed worker is to search in one labor market that yields the highest expected life-time utility. An unemployed worker can receive a constant amount of unemployment benefit $b > 0$ from the government regardless of her productivity, educational achievement, previous wage, working history or unemployment duration. The society has an ex-ante belief on the probabilities of successfully finding a job in each period. If an unemployed worker successfully finds a job in the current period, she will start to work in the next period. The per period disutility from working is $\eta$, where $\eta > 0$. If an unemployed individual fails to find a job, she will continue to search. A currently employed worker faces a constant probability $x \in (0,1)$ of job destruction. Call $x$ job separation rate.

Entrepreneurs In the model economy have the same subjective discount rate $\beta$ and death probability $\delta$. These entrepreneurs create and own many ”small firms”, each of which only consists of one job position. Call unfilled job positions vacancies.
Essentially, there are two types of production technologies, and thereby, two types of firms. High-skill firms use both labor and capital as inputs. Each high-skill firm employs \( k > 0 \) units of capital. Assume that a high-skill job position can only be filled with educated workers due to some special knowledge required to operate the capital of the firm. In other words, the educational requirement made by high-skill firms are exogenously given. Denote the number of high-skill job vacancies in period \( t \) by \( v_{h,t} \). The other firms are low-skill firms. Each low-skill firm only uses labor as inputs.\(^6\) low-skill firms can employ both educated and uneducated workers. Hence, there is no exogenous educational requirement for job applicants of low-skill firms. However, as to be explained later, it is possible that endogenous educational requirements arise in the low-skill labor market. On contrary, most existing studies, for instance, Makenna (1996) and Charlot et al. (2005), assume that educational requirements hinges on the adaptability issue only, without considering the possibility of endogenous educational requirements. Generally speaking, an entrepreneur is not using up all available information if she only posts one type of vacancy in the low tech labor market. Denote the numbers of low-skill job vacancies requiring and not requiring education in period \( t \) by \( v_{c,t} \) and \( v_{n,t} \), respectively. At the beginning of each period, entrepreneurs decide how many vacancies of each type should be posted. The cost of posting one vacancy of any type is \( q \), where \( q > 0 \).

\(^6\)Alternatively, it can be assumed that each low-skill job requires a certain amount of capital \( k_l < k \). This assumption is consistent with the capital-skill complementarity, which has been observed in many countries. See Krusell et e (2000) and Duffy et al (2004). This assumption would not change the qualitative conclusion of this paper.
filling each type of vacancies. If a low-skill job vacancy is filled with a type \( i \) individual, then the output of the firm per period would be \( p_i \) until job is destructed. If a type \( i \) individual is hired by a high-skill firm, the firm would produce \((1 + s)p_i\) units of output until job separation takes place, where \( s \) is the markup of the worker’s productivity due to the use of capital and \( s > 0 \).

Before matching, entrepreneurs do not know who would fill job vacancies. Therefore, when determining how many vacancies of each type to post, entrepreneurs can only base their decisions on the expected quality of each type of job applicants. However, after matching happens, a firm can immediately observe applicants’ productivity through an interview process. Then, the firms and newly recruited workers negotiate the wage and split the joint surplus from matching based on Nash bargaining process. This is also a simplifying assumption, which is very similar to the one adopted by Pries (2008). Instead of this assumption, a learning process, like those introduced by Jovanovic (1979 a, b 1984), can be specified. However, it would only dramatically complicate the model without changing the qualitative results. The bargaining power of workers is set to be \( \lambda \in (0, 1) \).

### 4.2.2 Individuals’ Problem

For the purpose of section 4.2, assume that at least some educated unemployed workers search for jobs in the low-skill labor market. It will be verified that this can be an equilibrium outcome. The possibility that all educated unemployment workers search in the high-skill labor market is discussed in section 4.3.

Essentially, a newly born individual make consecutive choices. First, she needs to
determine whether to undertake education. The worker will immediately search for a job in the low-skill labor market if she chooses not to be educated. If a worker determines to be educated, she then needs to decide whether to search for a job in the high-skill labor market or low-skill labor market. The decision process is presented in figure ??.

Given the decision process of a typical individual, I solve the individuals’ problem with backward deduction. The first step to solve this problem is to determine whether an educated worker shall search for jobs in the high-skill labor market or the low-skill labor market by comparing the expected life-time utility under the two scenarios. The second step is to compare the lifetime utility of an uneducated worker versus the larger lifetime utility obtained from the first step, in order to determine whether this individual should undertake education at the first place.

Assume all goods are perishable and workers do not have access to the financial market. Consequently, individuals consume all their income in each period. The flow utility of a generation $\tau$ type $i$ individual is set to be

$$\phi_{i, t}^{\tau} = \begin{cases} b & \text{unemployed} \\ w_{i, t}^{\tau} - \eta & \text{employed} \end{cases}$$

(4.1)

where $w_{i, t}^{\tau} > 0$ is the wage income of the generation $\tau$ type $i$ individual measured in period $t$. From this point on, I use a superscript and a group of subscripts to illustrate the type, educational choice and labor market status of a certain individual as the following. The superscript represents the generation of an individual. Furthermore, the type of an individual (given by her productivity) is represented by subscript $i$. Subscript $h$ means the worker is educated and searches or works in the high-skill market. Subscript $e$ means the
worker is educated and searches or works in the low-skill market. Subscript $n$ means the worker is uneducated. If this individual is uneducated, then $w_{i,t}^n = w_{i,n,t}^n$. For an educated individual, $w_{i,t}^e$ either equals to $w_{i,e,t}^e$ if she works in the low-skill labor market, or $w_{i,h,t}^e$ if she works in the high-skill market.

From this point on, I use $\Phi$ to represent the life-time utility of an unemployed worker, whereas $\Psi$ is the life-time utility of an employed worker. For example, $\Phi_{\tau,i,e,t}^e$ is the life-time utility of an unemployed generation $\tau$ type $i$ individual, measured in period $t$, if she is educated and searching in the low-skill labor market. Similarly, $\Psi_{\tau,i,h,t}^e$ is the life-time utility of an employed type $i$ individual of generation $\tau$, if she is educated and working for a high-skill firm.

For any $t > \tau$, if this type $i$ generation $\tau$ individual is educated and currently searching in the high-skill labor market in period $t$, then her current life-time utility $\Phi_{\tau,i,h,t}^e$, can be derived recursively as

$$
\Phi_{\tau,i,h,t}^e = b + \rho[\mu_{h,t}\Psi_{\tau,i,h,t+1}^e + (1 - \mu_{h,t}) \max(\Phi_{\tau,i,e,t+1}^e, \Phi_{\tau,i,h,t+1}^e)], \quad (4.2)
$$

where $\mu_{h,t}$ represents the current probability for this worker to successfully find a job in the high-skill labor market. The society takes $\mu_{h,t}$ as given. The first term of (4.2) is the flow utility of the individual. The rest of (4.2) is the continuation value, which measures the discounted value of her expected life-time utility measured in the next period. Similarly, it follows that

$$
\Psi_{\tau,i,h,t}^e = w_{i,h,t}^e - \eta + \rho[(1 - x)\Psi_{\tau,i,h,t+1}^e + x \max(\Phi_{\tau,i,e,t+1}^e, \Phi_{\tau,i,h,t+1}^e)]. \quad (4.3)
$$

If, however, this educated generation $\tau$ type $i$ individual is searching in the low-skill labor market, then her current life-time utility $\Phi_{\tau,i,l,t}^e$, can be derived recursively as

$$
\Phi_{\tau,i,l,t}^e = w_{i,l,t}^e - \eta + \rho[1 - \rho]\max(\Phi_{\tau,i,e,t+1}^e, \Phi_{\tau,i,l,t+1}^e)]. \quad (4.4)
$$

where $\rho$ is the discount factor and $\eta$ is the wage in the low-skill labor market. The society takes $\rho$ and $\eta$ as given. The first term of (4.4) is the flow utility of the individual. The rest of (4.4) is the continuation value, which measures the discounted value of her expected life-time utility measured in the next period. Similarly, it follows that

$$
\Psi_{\tau,i,l,t}^e = w_{i,l,t}^e - \eta + \rho[(1 - x)\Psi_{\tau,i,l,t+1}^e + x \max(\Phi_{\tau,i,e,t+1}^e, \Phi_{\tau,i,l,t+1}^e)]. \quad (4.5)
$$

If, however, this uneducated generation $\tau$ type $i$ individual is searching in the low-skill labor market, then her current life-time utility $\Phi_{\tau,i,n,t}^n$, can be derived recursively as

$$
\Phi_{\tau,i,n,t}^n = w_{i,n,t}^n - \eta + \rho[1 - \rho]\max(\Phi_{\tau,i,n,t+1}^n, \Phi_{\tau,i,l,t+1}^n)]. \quad (4.6)
$$

where $\rho$ is the discount factor and $\eta$ is the wage in the low-skill labor market. The society takes $\rho$ and $\eta$ as given. The first term of (4.6) is the flow utility of the individual. The rest of (4.6) is the continuation value, which measures the discounted value of her expected life-time utility measured in the next period. Similarly, it follows that

$$
\Psi_{\tau,i,n,t}^n = w_{i,n,t}^n - \eta + \rho[(1 - x)\Psi_{\tau,i,n,t+1}^n + x \max(\Phi_{\tau,i,n,t+1}^n, \Phi_{\tau,i,l,t+1}^n)]. \quad (4.7)
$$

If, however, this uneducated generation $\tau$ type $i$ individual is searching in the low-skill labor market, then her current life-time utility $\Phi_{\tau,i,n,t}^n$, can be derived recursively as

$$
\Phi_{\tau,i,n,t}^n = w_{i,n,t}^n - \eta + \rho[1 - \rho]\max(\Phi_{\tau,i,n,t+1}^n, \Phi_{\tau,i,l,t+1}^n)]. \quad (4.8)
$$

where $\rho$ is the discount factor and $\eta$ is the wage in the low-skill labor market. The society takes $\rho$ and $\eta$ as given. The first term of (4.8) is the flow utility of the individual. The rest of (4.8) is the continuation value, which measures the discounted value of her expected life-time utility measured in the next period. Similarly, it follows that

$$
\Psi_{\tau,i,n,t}^n = w_{i,n,t}^n - \eta + \rho[(1 - x)\Psi_{\tau,i,n,t+1}^n + x \max(\Phi_{\tau,i,n,t+1}^n, \Phi_{\tau,i,l,t+1}^n)]. \quad (4.9)
$$
labor market in period $t$, then her current life-time utility

$$\Phi_{t,e,t}^\tau = b + \rho[\mu_{e,t} \Psi_{t,e,t+1}^\tau + (1 - \mu_{e,t}) \max(\Phi_{t,e,t+1}^\tau, \Phi_{t,h,t+1}^\tau)],$$

(4.4)

where $\mu_{e,t}$ represents the current probability for this educated worker to successfully find a job in the low-skill labor market. The society also takes $\mu_{e,t}$ as given. Obviously, it can be shown that

$$\Psi_{t,e,t}^\tau = w_{t,e,t}^\tau - \eta + \rho[(1 - x)\Psi_{t,e,t+1}^\tau + x \max(\Phi_{t,e,t+1}^\tau, \Phi_{t,h,t+1}^\tau)].$$

(4.5)

Note that an educated individual will only search in the high-skill labor market in period $t$ if $\Phi_{t,h,t}^\tau \geq \Phi_{t,e,t}^\tau$.

If a type $i$ individual of generation $\tau$ chooses not to be educated, it follows that

$$\Phi_{t,n,t}^\tau = b + \rho[\mu_{n,t} \Psi_{t,n,t+1}^\tau + (1 - \mu_{n,t}) \Phi_{t,n,t+1}^\tau],$$

(4.6)

and

$$\Psi_{t,n,t}^\tau = w_{t,n,t}^\tau - \eta + \rho[(1 - x)\Psi_{t,n,t+1}^\tau + x \Phi_{t,n,t+1}^\tau)],$$

(4.7)

where $\mu_{n,t}$ represents the current probability for this uneducated worker to successfully find a job in the low-skill labor market. Hence, a generation $\tau$ type $i$ worker would only undertake education if

$$\Phi_{t,n,\tau}^\tau \leq \max(\Phi_{t,e,\tau}^\tau, \Phi_{t,h,\tau}^\tau) - [d(e) + \gamma(p_h - p_i)].$$

Due to the negative relationship between disutility from education and an individual’s productivity, it can be shown that an individual would only undertake education if her productivity is high enough. The details on this point will be discussed later on.
4.2.3 Entrepreneurs’ Problem

Entrepreneurs post high-skill and low-skill vacancies in each period. Since entrepreneurs do not know who will fill the vacancies when they are posted, the expected, rather than the realized value of the vacancies, should be used to compare with the vacancy posting costs, when entrepreneurs determine how many vacancies to open.

As stated earlier, it is assumed that at least some of the educated unemployed workers search for jobs in the low-skill labor market. This assumption will be relaxed in the next section. Assume entrepreneurs have an ex-ante belief that educated workers are more productive than educated workers. It will be shown later that this is consistent with the ex-post long run equilibrium result. Consequently, entrepreneurs have an incentive to post some of the low-skill job vacancies with an educational requirement, even though the production technology used does not require specific knowledge like that of a high-skill firm.

Intuitively, suppose that, based on the average productivity of all unemployed workers in the low-skill labor market, all but one entrepreneurs post only one type of vacancies without any educational requirement in this market. Then, the last entrepreneur can post a certain amount of low-skill job vacancies requiring education, such that, (1) educated workers find it easier to find a job if they only apply for those positions posted by the last entrepreneur, and (2) the value of the posted job positions is higher than the vacancy posting cost. As discussed later, by multiplying discounted expected value of a filled vacancy and the probability of successfully filling that vacancy, one can obtain the vacancy’s value. The first goal can be achieved when the sub-market created by the last entrepreneur is less tight for educated workers compared to the low-skill labor market as
a whole. Under normal assumptions of a search and matching model, it means that the probability of filling those vacancies with educational requirement is smaller.\footnote{This is true given any HD1 matching technology.} However, the second goal can be achieved simultaneously if the ex-ante belief of the entrepreneurs is correct. That is, educated workers are more productive. Specifically, since all workers share the same unemployment benefit as well as disutility from working, the realized joint surplus from matching when hiring an educated worker, is larger than that when the employee is uneducated. Consequently, the expected value of a job position filled by an educated worker must be higher. This is why the value of a vacancy requiring education can be still larger than the vacancy posting cost even though it is more difficult to be filled. As all educated workers searching jobs in the low-skill labor market are attracted to the sub-market created by the last entrepreneur, only uneducated workers whose productivity is lower than the pooled average will apply for the positions without educational requirement, causing all the other entrepreneurs suffering economic losses.

Denote the probabilities of filling the three types of vacancies (high-skill, low-skill with educational requirement, and low-skill without educational requirement) posted in period $t$ by $g_{h,t}$, $g_{e,t}$ and $g_{n,t}$, respectively. Note that entrepreneurs take these probabilities as given. If a vacancy is filled in period $t$, the newly recruited worker starts to produce in period $t + 1$. Denote the expected value of a high-skill firm in period $t$ by $J_{h,t}$. Similarly, Denote the expected values of low-skill firm hiring educated and uneducated workers in period $t$ by $J_{e,t}$ and $J_{n,t}$ respectively. Entrepreneurs determine whether to post one more vacancy by comparing the expected benefits versus the cost. Take a high-skill job vacancy
as an example. An risk-neutral entrepreneur would only post a high-skill job vacancy in period $t$ if $q \leq pg_{h,t}J_{h,t+1}$. Hence, the intrinsic problem is to pin down $J_{h,t+1}$, $J_{e,t+1}$ and $J_{n,t+1}$.

Consider the realized value of each type of firms first. If a high-skill firm employs a generation $\tau$ type $i$ individual, then it follows that the realized asset value of this firm in period $t$, $J_{i,h,t}^\tau$, can be written recursively as

$$J_{i,h,t}^\tau = (1 + s)p_i - rk - w_{i,h,t}^\tau + \rho(1 - x)J_{i,h,t+1}^\tau,$$  \hspace{1cm} (4.8)

where $r > 0$ is the capital rental rate.\footnote{Since the focus of this paper is on the long run behavior of the labor market, $r$ is taken as exogenously given and its determination is not explicitly modeled. Alternatively, it can be assumed that all physical capital is rented from a group of owners and they charge for a rental rate equal to the average of marginal product of capital. Simultaneously, the entrepreneurs take all the risks related to the productivity of the worker operating the capital. Such an assumption would not change the conclusion of the paper.} Note that $(1 + e)p_i - rk - w_{i,h,t}^\tau$ is the flow profit earned by this firm in period $t$. In the next period, the firms value will be $J_{i,h,t+1}^\tau$ and the probability that this firm still exists is $1 - x$. Hence, $\rho(1 - x)J_{i,h,t+1}^\tau$ is the continuation value of the firm.

Similarly, the realized values of low-skill firms in period $t$ hiring educated and uneducated can be calculated as

$$J_{i,j,t}^\tau = p_i - w_{i,j,t}^\tau + \rho(1 - x)J_{i,j,t+1}^\tau,$$  \hspace{1cm} (4.9)

where subscript $j \in \{e, n\}$. Note that capital is not an input for low-skill firms. Therefore, $rk$ does not appear in (4.9).

Let $f_{h,t}^\tau(p_i)$ be the conditional pdf of generation $\tau$ individuals’ productivity, given
that these workers search in the high-skill labor market. Following the same manner, we can also define \( f_{e,t}^\tau(p_i) \) and \( f_{n,t}^\tau(p_i) \) as the conditional pdfs of generation \( \tau \) educated and uneducated individuals’ productivity, given that these workers search in low-skill labor market. Then the expected values of firms hiring generation \( \tau \) workers in period \( t + 1 \) are

\[
J_{j,t+1}^\tau = \int_{p_i}^{p_h} \int_{J_{j,t+1}^\tau(p_i)} f_{j,t}^\tau(p_i) dp_i,
\]

where subscript \( j \in \{h, e, n\} \).

Denote the numbers of educated individuals searching in high-skill and low-skill job markets in period \( t \) by \( u_{h,t} \) and \( u_{e,t} \), respectively. Let \( u_{h,t}^\tau \) and \( u_{e,t}^\tau \) be the numbers of generation \( \tau \) unemployed educated individuals among them. Similarly, Denote the number of unemployed uneducated individuals in period \( t \) by \( u_{n,t} \) and let \( u_{n,t}^\tau \) be the current number of generation \( \tau \) unemployed individuals. According to the law of large numbers,

\[
J_{j,t+1} = \sum_{\tau = -\infty}^{t} \frac{u_{j,t}^\tau}{u_{j,t}} J_{j,t+1}^\tau,
\]

where subscript \( j \in \{h, e, n\} \).

Assume free entry to the labor market. Consequently, in the equilibrium, no entrepreneur should be able to make profits by simply posting any type of job vacancies. The free entry conditions, therefore, imply that

\[
q = \rho g_{j,t} J_{j,t+1},
\]

where subscript \( j \in \{h, e, n\} \).
4.2.4 Matching Technology and the Determination of Wages

Assume that the matching technology exhibits constant return to scale. Specifically, the matching function follows a Cobb-Douglas form. Denote the numbers of the three types of matches (matches in high-skill labor market, matches between educated workers and vacancies requiring education, matches between uneducated workers and low-skill vacancies not requiring education) made in period $t$ by $m_{h,t}$, $m_{e,t}$ and $m_{n,t}$, respectively. Specifically,

$$m_{j,t} = \varphi v_{j,t}^\alpha u_{j,t}^{1-\alpha},$$  \hspace{1cm} (4.13)

where subscript $j \in \{h, e, n\}$ and $\varphi > 0$ is the matching efficiency. $\alpha \in (0, 1)$ is the elasticity of matches with respect to vacancies. Define $\theta_{j,t} = v_{j,t}/u_{j,t}$ as the market tightness for type $j$ labor market. Obviously, the equilibrium job finding rates ($\mu_{h,t}$, $\mu_{e,t}$ and $\mu_{n,t}$) as well as equilibrium vacancy filling rates ($g_{h,t}$, $g_{e,t}$ and $g_{n,t}$) are determined by the market tightness. For example, $\mu_{n,t} = m_{h,t}/u_{h,t} = \varphi \theta_{h,t}^\alpha$ and $g_{h,t} = m_{h,t}/v_{h,t} = \varphi \theta_{h,t}^{\alpha-1}$. Hence, a job finding rate is positively related to the corresponding market tightness, whereas a vacancy filling rate is negatively related to the corresponding market tightness.

As stated in section 4.2.1, the wage is determined by a Nash Bargaining process and the bargaining power of workers is $\lambda$. Hence, the first-order conditions of the bargaining problems for the three types of the matches are

$$(1 - \lambda)(\Psi_{i,j,t}^r - \Phi_{i,j,t}^r) = \lambda J_{i,j,t}^r,$$  \hspace{1cm} (4.14)

where subscript $j \in \{h, e, n\}$.

\footnote{Depending on the the type of the matches, individuals and entrepreneurs negotiate the wages to maximize $(\Psi_{i,j,t}^r - \Phi_{i,j,t}^r)^\lambda (J_{i,j,t}^r)^{1-\lambda}$, where subscript $j \in \{h, e, n\}$.}
4.2.5 The Steady State

4.2.5.1 Steady State Educational and Searching Choices

Since the distribution of productivity is invariable and all individuals of the same type make the same choice at the steady state, time period and generation do not need to be considered when deriving the steady state equilibrium. Hence, from this subsection on, all notations without generation superscript and time period subscript represent the steady state level of the corresponding variables.

First, given the condition that educated workers search in both labor markets, consider which of them will search in the low-skill labor market.

If a type $i$ educated individual chooses to search in the type $j \in \{h,e\}$ labor market, based on the steady state version of (4.2) and (4.3), it can be shown that

$$\Phi_{i,j} = \frac{b}{1 - \rho} + \frac{\rho \mu_j (w_{i,j} - \eta - b)}{(1 - \rho)[1 - \rho(1 - x - \mu_j)]},$$

and

$$\Psi_{i,j} - \Phi_{i,j} = \frac{w_{i,j} - \eta - b}{1 - \rho(1 - x - \mu_j)}.$$

Equation (4.16) is the net benefits of being employed for this individual, provided that she works in the high-skill or low-skill labor market. Note that a worker would only choose to search in the high-skill labor market if $\Phi_{i,h} \geq \Phi_{i,e}$. Individuals take $\mu_h$, $\mu_e$ and $x$ as given, however, they know that the negotiated wages depend on their own productivity. Hence, to show who actually choose to search in the low-skill labor market, we first need to pin down the equilibrium wage at the steady state.
The steady state version of (4.8) and (4.9) yield

\[ J_{i,h} = \frac{(1 + s)p_i - r k - w_{i,h}}{1 - \rho(1 - x)}, \tag{4.17} \]

and

\[ J_{i,e} = \frac{p_i - w_{i,e}}{1 - \rho(1 - x)}. \tag{4.18} \]

By combining (4.16), (4.17), (4.18) and the steady state version of (4.14), it can be shown that

\[ w_{i,h} = \frac{\lambda[1 - \rho(1 - x - \mu_h)][(1 + s)p_i - r k] + (1 - \lambda)[1 - \rho(1 - x)][\eta + b]}{1 - \rho(1 - x) + \lambda\rho\mu_h} \tag{4.19} \]

and

\[ w_{i,e} = \frac{\lambda[1 - \rho(1 - x - \mu_e)]p_i + (1 - \lambda)[1 - \rho(1 - x)][\eta + b]}{1 - \rho(1 - x) + \lambda\mu_e} \tag{4.20} \]

Plugging (4.19) and (4.20) into (4.15) yields

\[ \Phi_{i,h} = \frac{[(1 + s)p_i - r k - \eta - b]\lambda\rho\mu_h}{(1 - \rho)[1 - \rho(1 - x) + \lambda\mu_h]} + \frac{b}{1 - \rho}, \tag{4.21} \]

and

\[ \Phi_{i,e} = \frac{[p_i - \eta - b]\lambda\rho\mu_e}{(1 - \rho)[1 - \rho(1 - x) + \lambda\mu_e]} + \frac{b}{1 - \rho}. \tag{4.22} \]

Hence, it can be shown that if \( \frac{(1 + s)\mu_h}{1 - \rho(1 - x) + \lambda\mu_h} > \frac{\mu_e}{1 - \rho(1 - x) + \lambda\mu_e} \), educated workers whose productivity is lower than \( p \) will search in low-skill labor market, whereas workers whose productivity is higher than \( p \) will search in high-skill labor market. In other words, given that the existence of a separation equilibrium, individuals searching in high-skill
market are more productive than those educated workers searching in low-skill labor market.

The threshold level of productivity, $\bar{p}$, satisfies

$$\begin{align*}
\bar{p} &= \frac{(1 + s)\mu_h}{1 - \rho(1 - x) + \lambda \mu_h} - \frac{\mu_e}{1 - \rho(1 - x) + \lambda \mu_e} \\
&= \frac{\mu_h (r k + \eta + b)}{1 - \rho(1 - x) + \lambda \mu_h} - \frac{\mu_e (\eta + b)}{1 - \rho(1 - x) + \lambda \mu_e}. 
\end{align*}$$

Intuitively, if a worker works in a high-skill labor market, she enjoys a productivity gain proportional to her productivity due to the usage of capital as a second input. Meanwhile, the firm also has to pay a fixed capital rental cost before the joint surplus is shared between the entrepreneur and the worker. If the productivity of this certain worker is high enough, then the potential increase in wage due to the proportional productivity gain can dominate the potential decrease in wage due to the fixed capital rental cost. Hence, only the most productive educated workers would search in the high-skill labor market.

Next, it can be shown that educated workers are more productive than uneducated workers. Given that those educated workers who search in low-skill labor market are less productive compared to those who search in high-skill labor market, I only need to compare uneducated workers with those educated workers searching in low-skill labor market.

Combining the steady state version of (4.6) and (4.7) yields

$$\Phi_{i,n} = \frac{b}{1 - \rho} + \frac{\rho \mu_n (w_{i,n} - \eta - b)}{1 - \rho (1 - x - \mu_n)},$$

and

$$\Psi_{i,n} - \Phi_{i,n} = \frac{w_{i,n} - \eta - b}{1 - \rho (1 - x - \mu_n)},$$

Since the steady state value of a firm hiring uneducated individual is

$$J_{i,n} = \frac{p_i - w_{i,n}}{1 - \rho (1 - x)}$$
Based on (4.24), (4.25), (4.26) and (4.14), it can be shown that

\[
w_{i,n} = \frac{\lambda[1 - \rho(1-x - \mu_n)]p_i + (1 - \lambda)[1 - \rho(1-x)](\eta + b)}{1 - \rho(1-x) + \lambda \rho \mu_n}
\]

\[= \frac{p_i - (1 - \lambda)[1 - \rho(1-x)][p_i - \eta - b]}{1 - \rho(1-x) + \lambda \mu_n}.
\]

Plugging (4.27) into (4.24) yields

\[
\Phi_{i,n} = \frac{[p_i - \eta - b] \lambda \rho \mu_n}{(1 - \rho)[1 - \rho(1-x) + \lambda \mu_n]} + \frac{b}{1 - \rho}.
\]

Note that a type \( i \) individual will only undertake education if \( \Phi_{i,n,\tau} \leq \max(\Phi_{i,e,\tau}, \Phi_{i,h,\tau}) - [d(e) + \gamma(p_h - p_i)] \). Assume that individuals’ have an ex-ante belief that they can earn higher wages and suffer less from unemployment if undertaking education\(^{10}\). Hence, it can be shown that there exists a threshold value \( \hat{p} \)\(^{11}\), such that individuals with a productivity lower than \( \hat{p} \) will not undertake education while individuals whose productivity higher than \( \hat{p} \) would choose to be educated, where \( \hat{p} \) satisfies

\[
\frac{\mu_e}{1 - \rho(1-x) + \lambda \rho \mu_e} - \frac{\mu_n}{1 - \rho(1-x) + \lambda \rho \mu_n} + \frac{\gamma(1 - \rho)}{\lambda \rho} \hat{p} = \frac{(d(e) + \gamma p_h)(1 - \rho)}{\lambda \rho} + (\eta + b)\left[\frac{\mu_e}{1 - \rho(1-x) + \lambda \mu_e} - \frac{\mu_n}{1 - \rho(1-x) + \lambda \mu_n}\right].
\]

Intuitively, since the utility cost of education is negatively related to an individual’s productivity, an individual will only find that the utility cost of education smaller than the given utility gains if she is relatively more productive.

\(^{10}\)This ex-ante belief is consistent with the equilibrium result. This argument is verified in the section 4.2.5.2.

\(^{11}\)Technically, this threshold level exists given that \( \frac{\lambda \rho \mu_e}{1 - \rho(1-x) + \lambda \rho \mu_e} - \frac{\lambda \rho \mu_n}{1 - \rho(1-x) + \lambda \rho \mu_n} + \gamma > 0 \). However, according to proposition 1, it can be shown that in the equilibrium, this condition is always satisfied.
4.2.5.2 Steady State Equilibrium

Since \( g_h, g_e, g_n, \mu_h, \mu_e, \) and \( \mu_n \) are all functions of \( \theta_h, \theta_e, \theta_n, \mu_p, \mu_\bar{p}, \mu_\overline{p} \). The steady state equilibrium can be pinned down by the values of the five endogenous variables \( \{\theta_h, \theta_e, \theta_n, \mu_p, \mu_\bar{p} \} \).

First, the steady state equilibrium level of \( \theta_h \) and \( \theta_e \) can be solved if the value of \( \mu_\bar{p} \) is known. Plugging (4.19), (4.20) and (4.27) into (4.17), (4.18) and (4.26) yields

\[
J_{i,h} = \frac{(1 - \lambda)(1 + s)p_i - rk - \eta - b}{1 - \rho(1 - x) + \lambda \mu_h},
\]

(4.30)

\[
J_{i,e} = \frac{(1 - \lambda)[p_i - \eta - b]}{1 - \rho(1 - x) + \lambda \mu_e},
\]

(4.31)

and

\[
J_{i,n} = \frac{(1 - \lambda)[p_i - \eta - b]}{1 - \rho(1 - x) + \lambda \mu_n}.
\]

(4.32)

Based on (4.30), (4.31), and (4.32), the steady state version of (4.12) follows that

\[
q = \rho g_h \frac{(1 - \lambda)(1 + s)E(p_i | p_i \leq \bar{p} \leq p_h) - rk - \eta - b}{1 - \rho(1 - x) + \lambda \mu_h},
\]

(4.33)

\[
q = \rho g_e \frac{(1 - \lambda)[E(p_i | \hat{p} \leq p_i < \overline{p}) - \eta - b]}{1 - \rho(1 - x) + \lambda \mu_e},
\]

(4.34)

and

\[
q = \rho g_n \frac{(1 - \lambda)[E(p_i | \overline{p} \leq p_i < \hat{p}) - \eta - b]}{1 - \rho(1 - x) + \lambda \mu_n}.
\]

(4.35)

Hence, the solution to (4.33), (4.34), (4.35), (4.23) and (4.29) characterize the steady state equilibrium of the model discussed in this section. Denote the unemployment rates of uneducated individuals and educated individuals in the low-skill labor market by \( ur_n \) and \( ur_e \), respectively. Proposition 1 shows the benefit of undertaking education and why some individuals are willing to accept the utility cost of education. Specifically, it indicates that,
even when an educated worker does not enjoy the benefit of a high-skill job, she can still be better-off compared to her uneducated peers.

**Proposition 6** In the steady state, $\mu_e > \mu_n$, $ur_e < ur_n$ and $w_{i,e} > w_{i,n}$.

**Proof.** First, I need to show that $\mu_e > \mu_n$. Suppose this is not true. Then, $\mu_e \leq \mu_n$. According to the steady state version of (4.13), it means $\theta_e \leq \theta_n$. Equations (4.31) and (4.32) yield

$$J_e = \frac{(1 - \lambda)\{E(p_i|p_i \leq \hat{p} \leq p_i) - \eta - b\}}{1 - \rho(1 - x) + \lambda \mu_e},$$

and

$$J_n = \frac{(1 - \lambda)\{E(p_i|p_i \leq \hat{p} \leq p_i) - \eta - b\}}{1 - \rho(1 - x) + \lambda \mu_n}.$$

Obviously, $E(p_i|\hat{p} \leq p_i \leq \bar{p}) > E(p_i|p_i \leq \hat{p})$. Hence, $J_e > J_n$. According to (4.34) and (4.35), it implies that $g_e < g_n$. Hence, $\theta_e > \theta_n$. Contradiction. Therefore, $\mu_e > \mu_n$.

In the steady state, the unemployment rate of each group of workers remains constant and it follows that

$$x(1 - ur_j) = \mu_j ur_j,$$

where $j \in \{n, e, h\}$. Therefore,

$$ur_j = \frac{x}{x + \mu_j}.$$  \hspace{1cm} (4.36)

Since $\mu_e > \mu_n$, $w_{e} < w_{n}$.

It is obvious that $w_{i,h} > w_{i,e}$ according to (4.20) and (4.27).
4.2.5.3 Comparative Steady State Analysis

This subsection is devoted to analyze some important features of the model economy, which are summarized by proposition 2.

Proposition 7 In the steady state equilibrium, \( d \) and \( \hat{p} \) are positively related.

Proof. The statement is true according to (4.29) ■

4.3 Pooling Equilibrium for Educated Workers

4.3.1 Existence of the Pooling Equilibrium for Educated Workers

In the previous section, it is assumed that the division of low-skill labor market exists. Consequently, there are two threshold levels of productivity. Individuals with a productivity of \( \hat{p} \) or higher are going to undertake education and educated individuals with a productivity higher than \( \bar{p} \) search for jobs in high-skill labor market. In this section, keeping all other assumptions, I first investigate the possibility the scenario under which a pooling equilibrium exists for educated workers. In other words, whether all educated individuals search in the same labor market can be a plausible steady state equilibrium.

Given the existence of a separating equilibrium for educated workers, it must be the case that the solved \( \hat{p} \) is smaller than \( p \). Otherwise, the pooling equilibrium exists. To show that it is possible that the \( \hat{p} \) solved based on (4.29) can be larger than the \( p \), I first verify that there exists an upper limit of \( \bar{p} \).

Proposition 8 If an educated type \( i \) individual’s productivity \( p_i \) is greater than \( rk/s \), then
in the steady state equilibrium, the type $\iota$ individual will only search in the high-skill labor market.

**Proof.** Suppose the statement is not true. That is, there exists at least one type $\iota$ individual that searches in the low-skill labor market when the economy is in the steady state equilibrium and $p_\iota > r_k/s$. Hence, by definition, $\bar{p} > r_k/s$ and $E(p_\iota | \bar{p} \leq p_\iota \leq p_h) > r_k/s$. Consequently, $(1 + s)E(p_\iota | \bar{p} \leq p_\iota \leq p_h) - r_k > E(p_\iota | \bar{p} \leq p_\iota \leq p_h) > E(p_\iota | \bar{p} \leq p_\iota \leq \bar{p})$, which implies that

$$J_h > \frac{(1 - \lambda)[E(p_\iota | \bar{p} \leq p_\iota \leq p_h) - \eta - b]}{1 - \rho(1 - x) + \lambda \mu_h}.$$ 

Suppose $\mu_h < \mu_e$, which means $\theta_h < \theta_e$. Then, $J_h > J_e$. To satisfy the (4.33) and (4.34), it follows that $g_h > g_e$ and $\theta_h > \theta_e$. Contradiction. Hence, $\mu_h > \mu_e$.

Now, consider this type $\iota$ individual. Since $(1 + s)p_\iota - r_k > p_\iota$, it can be shown that

$$w_{\iota,h} > \frac{\lambda[1 - \rho(1 - x - \mu_h)]p_\iota + (1 - \lambda)[1 - \rho(1 - x)](\eta + b)}{1 - \rho(1 - x) + \lambda \mu_h}$$

$$= p_\iota - \frac{(1 - \lambda)[1 - \rho(1 - x)][p_\iota - \eta - b]}{1 - \rho(1 - x) + \lambda \mu_h}$$

$$> p_\iota - \frac{(1 - \lambda)[1 - \rho(1 - x)][p_\iota - \eta - b]}{1 - \rho(1 - x) + \lambda \mu_e}$$

$$= w_{\iota,e},$$

where the third and fourth line are based on the conclusion that $\mu_h > \mu_e$.

since $\mu_e > \mu_n$. According to (4.15),

$$\Phi_{\iota,j} = \frac{b}{1 - \rho} + \frac{\rho(w_{\iota,j} - \eta - b)}{(1 - \rho)[1 - \rho(1 - x) + \rho] \mu_j},$$

Hence, $\Phi_{\iota,h} > \Phi_{\iota,e}$. In other words, this type $\iota$ individual would be better-off if she chooses to search in the high-skill labor market and searching in the low-skill labor market cannot
be an equilibrium result. Contradiction. Therefore, the type \( i \) individual will only search in the high-skill labor market. ■

Proposition 3 means that an upper limit of \( \overline{p} \) is \( rk/s \). This result is very straightforward. The productivity gain a high-ability individual enjoys due to the usage of capital can outweigh the cost of capital she uses, leading to higher wages. Furthermore, she would suffer less from unemployment if they search in high-skill labor market. Hence, there is no reason for the most productive workers to search in low-skill labor market.

Consider an increase in \( d \). According to proposition 1, it can be shown that

\[
\frac{\mu_e}{1-\rho(1-x) + \lambda \rho \mu_e} - \frac{\mu_n}{1-\rho(1-x) + \lambda \rho \mu_n} > 0.
\]

Hence, as \( d \) becomes sufficiently large, \( \hat{p} \) solved based on (4.29) would ultimately reach the upper limit of \( \overline{p} \), implying that the searching choices of educated workers converge. As the number of educated individuals decrease, the average quality of educated workers are high and they all find that searching in high-skill labor market leave them better-off. Hence, the pooling equilibrium, instead of separating equilibrium, will exist for educated individuals.

4.3.2 Steady State Equilibrium

Given that all educated workers search for jobs in the high-skill labor market, the benefit of undertaking education is positively related to an individual’s productivity. Meanwhile, the cost of education, by assumption, is still negatively related an individual’s productivity. Hence, in the equilibrium, educated workers are more productive compared
to uneducated workers. Under this scenario, \( p \) no longer exists and \( \hat{p} \) is determined by

\[
\hat{p} = \frac{(1 + s)\mu_h}{1 - \rho(1 - x) + \lambda \mu_h} - \frac{\mu_n}{1 - \rho(1 - x) + \lambda \mu_n} + \frac{\gamma(1 - \rho)}{\lambda \rho} \hat{p} \tag{4.37}
\]

Furthermore, \( w_{i,h}, w_{i,n}, \Phi_{i,h}, \Phi_{i,n}, \Psi_{i,h}, \Psi_{i,n}, J_{i,h} \) and \( J_{i,n} \) follow exactly the same forms compared to those derived in section 4.2. The free entry conditions for high-skill and low-skill labor markets become

\[
q = \rho g_h (1 - \lambda) \frac{(1 + s)E(p_i | \hat{p} \leq p_i \leq p_h) - rk - \eta - b}{1 - \rho(1 - x) + \lambda \mu_h}, \tag{4.38}
\]

and

\[
q = \rho g_n (1 - \lambda) \frac{E(p_i | p_i < \hat{p}) - \eta - b}{1 - \rho(1 - x) + \lambda \mu_n}. \tag{4.39}
\]

**Proposition 9** Given that all educated individuals search for jobs in high-skill job market, \( d \) and \( \hat{p} \) are positively related.

**Proof.** The statement is true according to (4.37).

**Proposition 10** Given that all educated individuals search for jobs in high-skill job market, steady state unemployment rate for educated individuals is negatively related to \( \hat{p} \), when \( \rho \), \( \eta \), \( b \), \( x \), \( \lambda \), \( s \), \( q \), \( \varphi \), \( r \) and \( k \) are held constant.

**Proof.** Suppose the proposition does not hold. Consider a pair of threshold productivity \( \hat{p}_0 \) and \( \hat{p}_1 \) such that \( \hat{p}_0 < \hat{p}_1 \). Let \( ur_{h,0} \) and \( ur_{h,1} \) represent the unemployment rates of the high-skill labor market corresponding to \( \hat{p}_0 \) and \( \hat{p}_1 \). Hence, \( ur_{h,0} \leq ur_{h,1} \). Denote the market tightness of high-skill labor market corresponding to \( \hat{p}_0 \) and \( \hat{p}_1 \) by \( \theta_{h,0} \) and \( \theta_{h,1} \), respectively. Hence, according to (4.36), \( \mu_h(\theta_{h,0}) \geq \mu_h(\theta_{h,1}) \) and \( \theta_{h,0} \geq \theta_{h,1} \).
Accordingly, it means that

\[
\frac{(1 - \lambda)(1 + s)E(p_i|\hat{p}_0 \leq p_i \leq \hat{p}_h) - rk - \eta - b}{1 - \rho(1 - x) + \lambda \mu_h(\theta_{h,0})} < \frac{(1 - \lambda)(1 + s)E(p_i|\hat{p}_1 \leq p_i \leq \hat{p}_h) - rk - \eta - b}{1 - \rho(1 - x) + \lambda \mu_h(\theta_{h,1})}.
\]

Hence, according to (4.38) and (4.39), \( g_h(\theta_{h,0}) > g_h(\theta_{h,1}) \), which implies that \( \theta_{h,0} < \theta_{h,1} \). Contradiction. Therefore, \( ur_{h,0} > ur_{h,1} \). ■

**Proposition 11** Given that all educated individuals search for jobs in high-skill job market, the mean value of the steady state wage of all educated individuals is positively related to \( \hat{p} \), when \( \rho, \eta, b, x, \lambda, s, q, \varphi, r \) and \( k \) are held constant.

**Proof.** According to proposition 5 and the wage equation (4.19), the statement is true. ■

### 4.4 Effects of an Increase in the Supply of Higher Education

It has been shown that the model presented in this paper can lead to a separating steady state equilibrium or a pooling steady state equilibrium for educated workers, depending on the value of of \( d \) and other model parameters. In this section, based on the analysis in section 4.2 and section 4.3, it is argued that the increase of the supply of higher education can be the reason of the transition from the pooling equilibrium to the separating equilibrium, and therefore, responsible for many observed labor market changes, including the overeducation of some college graduates.

When the supply of higher education is very low, students have to make a very significant effort in order to pass the college entrance examination. In other words, \( d \) is
very large. According to the analysis in section 4.3.1, $\hat{p}$ can be very large and all educated individuals will search in the high-skill job market. As the supply of higher education rises, $d$ gradually decreases. Hence, more individuals choose to undertake education and $\hat{p}$ drops. At the beginning, the decrease in $d$ is not too large and all educated individuals still all for jobs in the high-skill market. According to proposition 5 and proposition 6, the unemployment rate rises and average wage of educated individuals decreases. This explains the deterioration of the labor market outcomes for educated workers. Up to this stage, all educational requirements are still exogenous.

While the supply of higher education continues to grow, $d$ decreases further and $\hat{p}$ will fall below the upper limit of $\bar{p}$. Consequently, the separating equilibrium for educated individuals exists. That is, some educated individuals start to search in the low-skill job market because they would have earned lower wages if they search in high-skill labor market. This explains the fact that college graduates in China start to search for jobs that they previously are not interested in. Entrepreneurs know that educated workers are more productive than uneducated workers. Hence, by hiring educated workers for low-skill job positions, entrepreneurs can earn higher profits. As a result, entrepreneurs are willing to post relatively more low-skill job vacancies for educated individuals, which means they have to distinguish two types of low-skill job vacancies. Consequently, the educational requirement for low-skill jobs emerges, even though the nature of the jobs does not require the knowledge obtained through college education. This explains why educational requirement for job application spread from high-skill jobs to jobs previously not requiring college education.
It might be argued that the increase in the number of educated workers is caused by an increase in income rather than the decrease in disutility from education. If tuition remains stable and is incorporated into the model, an increase in income will motivate more people to pursue education. However, before mid 1990s, college education in China was free. Nowadays, students have to pay a tuition ranging from several hundred to several thousand dollars per year. Hence, even with the increase in income, we should still observe fewer people to undertake college education if the change in financial conditions is the only driving force. Therefore, the change in disutility and the change from education, caused by the increase in the supply of higher education, must have played a very important role in determining agents’ educational choices.

4.5 Conclusions

The dramatic increase in the supply of higher education, and more importantly, the resulting decrease of the average productivity of educated individuals, might be a key reason that explains the transition from a pooling to a separating equilibrium in the low skill labor market, lower well-being of college educated workers, overeducation and higher educational requirement in the low-skill labor market. Initially, college graduates compose of a very small proportion of the population and they are the most productive ones. The optimal behaviors of these individuals are to only search in the labor market that require specific knowledge they obtained through higher education. As more individuals undertake higher education, the average quality of the college graduates deteriorates, resulting in the increase of unemployment rate and the drop of real wage for college graduates. Furthermore, when
the number of educated workers is significantly larger, the optimal behavior for educated individuals with relatively lower productivity is to search in the job market historically not requiring higher education. Consequently, overeducation and skill mismatch emerges. Entrepreneurs respond to this change by posting two kinds of vacancies, one requiring job applicants being college graduates whereas the other one does not. By doing this, entrepreneurs use up all the information about job applicants’ productivity that can be deduced from their educational choices. Hence, educational requirement arises in the labor market in which knowledge obtained through higher education is not essential.
Chapter 5

Conclusions

The DMP model has been widely used to analyze behaviors of labor market variables, specifically, unemployment. This dissertation applies several modified DMP models to explain and discuss a wide variety of cyclical and long-run labor market phenomena.

Chapter 2 shows that incorporating heterogeneity of labor and economies of scale for job training into an otherwise standard DMP model fits significantly better with post-war US data. In particular, the model economy exhibits considerably more volatile unemployment, vacancy and labor market tightness compared to a standard model. Furthermore, it also generates quantitatively more realistic relative volatility between high-skill and low-skill unemployment rates compared to other works with labor heterogeneity.

Chapter 3 of the dissertation combines a search and matching model with a signaling game to analyze the interrelationship between labor market outcomes and educational choices, featuring endogenous educational requirement for job application. It also sheds light on relevant policy implications. The model explains at least 64% of the unemployment rate
difference between college and high school graduates. If heterogeneous job separation rates are considered, the model can explain up to 97% of the unemployment rate gap. It predicts that higher unemployment benefit encourages individual to pursue education. It is also predicted that both skill-specific unemployment rates increase if higher educational subsidy is provided. Furthermore, the model suggests that a new explanation on why skill-biased technological changes lower the wages of uneducated workers and raise their unemployment rate.

Chapter 4 of the dissertation extends the model presented in Chapter 3 to explain the following well observed recent changes in China’s labor market outcomes. First, some firms require their job applicants to have higher educational achievements even though the job duties essentially remain the same. Second, current college graduates suffer more from unemployment compared to their predecessors. Third, college graduates search for low-skill jobs, which indicates over-education at the college level. Fourth, current college graduates earn lower wages. The author attributes these changes to the decrease of the expected productivity of college graduates, which in turn is triggered by the significant increase in the supply of higher education.
Appendix A

Appendix to Chapter 2

The high-skill unemployment rate and low-skill unemployment rate are calculated based on the BLS publication, Employment and Earnings, as well as BLS website. The labor force statistics for people who are 25 years old are collected from the BLS website \(^1\) and the those for people who are between 16 and 24 are collected from Employment and Earnings\(^2\). Both data sources are based on Current Population Survey.

Based on BLS categorization by educational attainment, people who are at least 25 years old fall into the following categories: less than high school, high school graduates, less than a bachelor’s degree (associate degree or some college but no degree) and college graduates. The first three categories are low-skill labor in this paper. For each category, statistics as civilian labor force and number of unemployed are provided. Denote the number of unemployed workers of each category as \(U_{25LH}, U_{25HG}, U_{25LB}\) and \(U_{25CG}\), respectively.

\(^1\)http://data.bls.gov/pdq/querytool.jsp?survey=ln

\(^2\)The title of the table is: Employment status of the civilian noninstitutional population 16 to 24 years of age by school enrollment, educational attainment, sex, race, and Hispanic or Latino ethnicity.
tively. Denote the civilian labor force of each category as $L_{25LH}$, $L_{25HG}$, $L_{25LB}$ and $L_{25CG}$, respectively.

Meanwhile, workers between 16 and 24 years old fall into the following categories: students, less than high school, high school graduates, less than a bachelor’s degree and college graduates. The first four categories are low-skill labors. For each category, statistics as civilian labor force and number of unemployed are provided. Denote the number of unemployed workers of each category as $U_{16S}$, $U_{16LH}$, $U_{16HG}$, $U_{16LB}$ and $U_{16CG}$, respectively. Denote the civilian labor force of each category as $L_{16S}$, $L_{16LH}$, $L_{16HG}$, $L_{16LB}$ and $L_{16CG}$, respectively.

Hence, low-skill and high-skill unemployment rates are given by

$$u_l = \frac{U_{25LH} + U_{25HG} + U_{25LB} + U_{16S} + U_{16LH} + U_{16HG} + U_{16LB}}{L_{25LH} + L_{25HG} + L_{25LB} + L_{16S} + L_{16LH} + L_{16HG} + L_{16LB}},$$

and

$$u_h = \frac{U_{25CG} + U_{16CG}}{L_{25CG} + L_{25CG}}.$$
Bibliography


