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ROLE OF NONEQUILIBRIUM WATER VAPOR DIFFUSION IN GEOTHERMAL ENERGY STORAGE SYSTEMS IN THE VADOSE ZONE

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Abstract: Although siting of geothermal energy storage systems in the vadose zone may be beneficial due to the low heat losses associated with the low thermal conductivity of unsaturated soils, water phase change and vapor diffusion in soils surrounding geothermal heat exchangers may play important roles in both the heat injection and retention processes that are not considered in established design models for these systems. This study incorporates recently-developed coupled thermo-hydraulic constitutive relationships for unsaturated soils into a coupled heat transfer and water flow model that considers time-dependent, nonequilibrium water phase change and enhanced vapor diffusion to study the behavior of geothermal energy storage systems in the vadose zone. After calibration of key parameters using a tank-scale heating test on compacted silt, the ground response during 90 days of heat injection from a vertical geothermal heat exchanger

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followed by 90 days of ambient cooling was investigated. Significant decreases in degree of
saturation and thermal conductivity of the ground surrounding the vertical geothermal heat
exchanger were observed during the heat injection period that were not recovered during the
cooling period. This effect can lead to a greater amount of heat retained in the ground beyond that
estimated in conduction-based design models.

INTRODUCTION

An important challenge facing society is the storage of energy collected from renewable
sources. One such application is the storage of heat collected from solar thermal panels in the
subsurface so that it can be harvested later (Claesson and Hellström 1981; Nordell and Hellström
2000; Chapuis and Bernier 2009). A practical mode of heat injection into the subsurface involves
circulation of a heated carrier fluid through a closely-spaced array of closed-loop geothermal heat
exchangers in boreholes to reach ground temperatures ranging from 35 to 80 °C (Sibbitt et al.
2012; Başer et al. 2016a; McCartney et al. 2017). Unsaturated soils in the vadose zone are ideal
thermal energy storage media because low heat losses can be expected due to the low thermal
conductivity of unsaturated soils (McCartney et al. 2013). The mode of heat transfer during
injection of heat into unsaturated soils is complex as it may be coupled with thermally-induced
water flow in either liquid or vapor forms along with latent heat transfer associated with phase
change. However, most design models for geothermal heat storage systems focus on ground
temperature changes during heating and do not consider coupled heat transfer and water transport
(Claesson and Hellström 1981; Eskilson 1987). Although some recent studies on geothermal
energy storage systems highlighted the importance of considering coupled heat transfer and water
flow in their performance evaluation (Catolico et al. 2016; Moradi et al. 2016), the impact of water
vapor diffusion and phase change in unsaturated soils during heat injection on the heat retention
This paper presents simulations of the response of a low-permeability, low activity, incompressible, unsaturated silt layer surrounding a single geothermal heat exchanger to understand the impacts of considering water vapor diffusion and water phase change on the transient heat injection and retention processes. Comparison of the simulation results from the coupled heat transfer and water flow model with a simpler heat transfer model without water vapor diffusion or phase change permits an evaluation of the importance of these heat transfer mechanisms in simulating geothermal energy storage systems in the vadose zone.

BACKGROUND

Most models of heat transfer from geothermal heat exchangers employ analytical solutions to the heat equation assuming conduction is the primary mechanism of heat transfer, using constant thermal properties that do not consider the effects of changes in degree of saturation expected during heat transfer in unsaturated soils (e.g., Kavanaugh 1985; Eskilson 1987). Analytical solutions have been developed for geothermal heat exchanger geometries including the infinite line source (Ingersoll and Plass 1948; Beier et al. 2014), finite line source (Acuña et al. 2012; Lamarche and Beauchamp 2007), hollow cylinder source (Ingersoll et al. 1954; Gehlin 2002), finite plate source (Ciriello et al. 2015), and one- and two-dimensional solid cylinder sources (Tarn and Wang 2004). Although numerical simulations of geothermal heat exchangers have also been performed, most have also considered conduction as the primary mechanism of heat transfer (Ozudogru et al. 2015; Welsch et al. 2015; Başer et al. 2016a). While these conduction-based analytical models and numerical simulations may be practical for the design of geothermal heat exchangers in dry or saturated low permeability soils, they may not be practical for design of those in unsaturated soils due to the potential for convective heat transfer associated with thermally-
induced liquid water or water vapor flow, which may result in irreversible changes in behavior during cyclic heat injection and extraction (or ambient cooling). Further, the thermal properties of unsaturated soils are highly dependent on the degree of saturation, even when conduction is assumed to be the primary mode of heat transfer (e.g., Farouki 1981; Côté and Konrad 2005; Smits et al. 2013; Lu and Dong 2015). Conduction-only models may also not be practical for use in saturated soils with high permeability due to the potential for thermally-induced convection of water from buoyancy effects (Catolico et al. 2016).

Because the properties of water in liquid and gas forms are dependent on temperature, heat transfer in the unsaturated soils in the vadose zone leads to thermally induced water flow through soil. Specifically, temperature dependency of the density of liquid water $\rho_w$ (Hillel 1980), dynamic viscosity of liquid water $\mu_w$ (Lide 2001), surface tension of soil water $\sigma$ (Saito et al. 2006), relative humidity at equilibrium $R_{h,eq}$ (Philip and de Vries 1957), saturated vapor concentration in the gas phase $c_{v,sat}$ (Campbell 1985), vapor diffusion coefficient in air $D_v$ (Campbell 1985), and the latent heat of water vaporization $L_w$ (Monteith and Unworth 1990) may lead to thermally-induced water flow through unsaturated soils. The movement of water in soil caused by thermal and hydraulic gradients and the associated impacts on heat transfer have been studied experimentally for more than 100 years (Bouyoucos 1915; Lewis 1937; Smith 1943; Gurr et al. 1952; Baladi et al. 1981; Shah et al. 1983; Ewen 1988; Gens et al. 1998, 2007, 2009; Cleall et al. 2011; Smits et al. 2011; Moradi et al. 2015, 2016; Başer et al. 2016b). Some general observations from these studies are: (1) heat transfer occurs in unsaturated porous media by conduction, convection in both liquid and gas phases, and latent heat transfer associated with water phase change; (2) water movement due to a temperature gradient is controlled by both vaporization/condensation processes as well as development of suction gradients caused by changes in water properties with temperature (i.e.,
density, viscosity, solid-liquid contact angle) and drying effects; (3) vapor diffusion may occur at
greater rates than predicted by Fick’s law, (4) the magnitude of thermally induced water flow
depends on the initial degree of saturation; and (5) the times required to reach steady-state
distributions in degree of saturation and temperature may be different depending on the coupling
between the thermal and hydraulic properties of a given soil.

The governing equations for coupled heat transfer and flow of water in liquid and vapor forms
have been investigated for unsaturated porous media in nondeformable conditions (Philip and de
Vries 1957; Ewen and Thomas 1989; Thomas and King 1991; Thomas and Sansom 1995; Thomas
et al. 2001; Smits et al. 2011), deformable conditions (Thomas and He 1996; Thomas et al. 1996),
and in the presence of pore fluids containing salts or chemicals (Cleall et al. 2007; Olivella et al.
and vapor forms in nondeformable unsaturated soils are based on the model of Philip and de Vries
(1957), who proposed the “liquid island” theory as an explanation for observations from studies
like Gurr et al. (1952) that vapor diffusion occurred at a faster rate than predicted by Fick’s law.
Their theory is a pore-scale explanation where local thermal gradients is assumed to be higher
across microscopic air-filled pores than the global thermal gradient across a soil element, and
where water vapor diffusion is enhanced by evaporation and condensation from water held
between soil particles by capillarity (liquid islands), effectively increasing the area available for
vapor diffusion through a soil element. They implemented their pore-scale theory on a
macroscopic scale by extending the vapor diffusion theory of Penman (1940) through inclusion of
a soil-specific enhancement factor to correct the vapor diffusion rate calculated from Fick’s law.
Cass et al. (1984) found that the enhancement factor approaches 1.0 (no enhancement) for dry soils
and increases significantly with increasing degree of saturation. Although the model of Philip and
de Vries (1957) has been used in many coupled heat transfer and water flow problems in nondeformable soils, their model does not account for convective transport in the gas or liquid water phases, nonequilibrium phase change, vapor dispersion, or sensible heat dispersion in the liquid phase (Smits et al. 2011). Of these issues, consideration of nonequilibrium phase change in the model is expected to lead to more accurate identification of the appropriate vapor enhancement factor for a given soil (Smits et al. 2011). Lozano et al. (2008) observed that phase change may become the process limiting evaporation at low saturations rather than vapor diffusion as classically believed.

In the model of Philip and de Vries (1957), it is assumed that the water in liquid and gas phases are in equilibrium, which means that phase change occurs instantaneously in response to a change in vapor pressure. However, experimental studies have identified that time is required for liquid water to volatilize in response to a change in vapor pressure in a pore resulting from vapor diffusion in response to gradients in vapor pressure and/or temperature (Bénet and Jouanna 1982; Armstrong et al. 1994; Chammari et al. 2008, Bénet et al. 2009). To account for this in a model of coupled heat transfer and water flow, a source term for the liquid/gas phase change rate is added to the mass balance equations of liquid and vapor that is based on irreversible thermodynamics, first order reaction kinetics, or the kinetic theory of gases and contains a fitting coefficient that can calibrated for a given soil (Bénet and Jouanna 1982; Bixler 1985; Zhang and Datta 2004). Smits et al. (2011) adopted the source term of Bixler (1985) because it was derived from the kinetic theory of gases and is thus inherently temperature dependent. In the model of Bixler (1985), the vaporization rate is proportional to the difference between local equilibrium vapor pressure and local partial vapor pressure and the difference between the local degree of saturation and residual saturation. Smits et al. (2011) compared predictions of coupled heat transfer and water flow from
equilibrium and nonequilibrium models, and found major differences in the early stages of the flow process, with greater differences for soils with initially lower degrees of saturation. Smits et al. (2011) and Trautz et al. (2015) also found that nonequilibrium models provide a better match to experimental data from column tests involving evaporation from fine sand with a heated surface than the model of Philip and de Vries (1957), indicating that the nonequilibrium assumption for phase change may better capture the transient process of thermally-induced drying.

**MODEL CALIBRATION**

**Model Description**

A non-equilibrium, non-isothermal, and coupled heat transfer and water flow numerical model developed by Smits et al. (2011) and extended by Moradi et al. (2016) was used to consider the behavior of an unsaturated soil layer during heating and cooling of a single vertical geothermal heat exchanger. The governing equations and primary variables used in the formulation are given in Table 1. Calibration of the model requires soil-specific quantification of the parameters for the thermo-hydraulic constitutive relationships governing water retention, hydraulic conductivity, thermal conductivity, and volumetric heat capacity, as well as estimates of parameters $a$ and $b$ in Equations (4) and (5) that govern the rates of vapor diffusion and phase change, respectively.

The model used in the simulations incorporates recently-developed thermo-hydraulic constitutive relationships for unsaturated soils (Lu and Dong 2015; Baser et al. 2016c). The experimental approach used by Lu and Dong (2015) was used to obtain the data for calibration of these coupled thermo-hydraulic constitutive relationships. Lu and Dong (2015) used a modified form of the transient-release and imbibition method (TRIM) of Wayllace and Lu (2012) that included a dual-needle thermal probe to measure the thermal conductivity and volumetric heat capacity during monotonic drying of different unsaturated soils under isothermal conditions.
TRIM uses an inverse analysis to estimate the parameters of the soil-water retention curve (SWRC) and hydraulic conductivity function (HCF) given by van Genuchten (1980). These parameters include $\alpha_{vG}$, which represents the inverse of the air entry suction in the SWRC, $N_{vG}$, which represents the pore size distribution in the SWRC, and $k_{sw}$, which is the hydraulic conductivity of saturated soil. The value of $k_{sw}$ obtained from a test at room temperature can be used to calculate the intrinsic permeability $\kappa$ in Equation (1). Although the saturation-dependent relative permeability to water (the HCF) was assumed not to vary with temperature, the hydraulic conductivity of the unsaturated soil will vary with temperature because the dynamic viscosity and density of water vary with temperature according to the relationships presented in Lide (2001) and Hillel (1980), respectively. The relative permeability to gas was not measured in this study, but was assumed to equal $k_{rg}=1-k_{rw}$. The temperature-dependent surface tension $\sigma$ relationship presented by Saito et al. (2006) was used in the temperature correction for capillary pressure of Grant and Salehzadeh (1996), given as follows:

$$P_c(T) = P_c(T_{ref})[\sigma(T)/\sigma(T_{ref})]$$

where $\sigma$ is the surface tension (N/m), $T$ is the temperature (K), and $T_{ref}$ is the initial reference temperature of 293.15 K.

Lu and Dong (2015) defined a thermal conductivity function (TCF) that can capture transitions in the thermal conductivity in the capillary, funicular, and pendular water retention regimes of the SWRC, given as follows:

$$\frac{\lambda - \lambda_{dry}}{\lambda_{sat} - \lambda_{dry}} = 1 - \left[1 + \left(\frac{S_{rw}}{S_f}\right)^m\right]^{1/m-1}$$

where $\lambda_{dry}$ and $\lambda_{sat}$ are the thermal conductivities of dry and saturated soil specimens, respectively, $S_f$ is a parameter representing the degree of saturation at the onset of the funicular regime, and $m$
is a parameter related to the pore fluid network connectivity. Lu and Dong (2015) correlated the
dparameters of the TCFs and SWRCs of several soils and found that the $m$ parameter in the TCF is
related to the pore-size parameter $N_{vG}$ in the SWRC model of van Genuchten (1980), and can be
estimated to be $3.0-0.2N_{vG}$. Evaluation of the form of Equation 8 indicates that the thermal
conductivity will not reduce to the value of $\lambda_{sat}$ when $S_e=1$, so Lu and Dong (2015) treated $\lambda_{sat}$ as
a fitting parameter. Although Smits et al. (2013) observed that the TCF may vary with temperature,
this temperature dependency is likely due to vapor diffusion and phase change that was not
accounted for in their simulations. Because the simulations in this study account for vapor
diffusion and phase change explicitly, the TCF and VCHF measured at 20 °C were used in the
coupled heat transfer and water flow simulations.

Başer et al. (2016c) presented trends in the volumetric heat capacity of compacted silt during
monotonic drying and found that it also depends on the degree of saturation in a similar manner to
the thermal conductivity, and defined a volumetric heat capacity function (VCHF) that has the
same form as the THF of Lu and Dong (2015), as follows:

$$\frac{C_v - C_{v, dry}}{C_{v, sat} - C_{v, dry}} = 1 - \left[ 1 + \left( \frac{S_m}{S_f} \right)^m \right]^{-1/m-1}$$  \hspace{1cm} (9)

where $C_{v, dry}$ and $C_{v, sat}$ are the volumetric heat capacities of dry and saturated soil, respectively, and
are similarly treated as fitting parameters, and $S_f$ and $m$ are the same parameters as in Equation (8).
Başer et al. (2016c) found that this model and the assumptions regarding the parameters provided
a good match to the volumetric heat capacity data measured in the TRIM tests on different soils
performed by Lu and Dong (2015) that were not reported in their paper due to its focus on the
thermal conductivity.
Calibration of Thermo-Hydraulic Constitutive Relationships

The soil investigated in this study is Bonny silt, which is classified as ML (inorganic silt) according to the Unified Soil Classification System (USCS), and has a specific gravity of 2.65. Silt was selected for this evaluation because it is not expected to deform significantly during changes in temperature or degree of saturation, and its low activity of 0.33 (plasticity index of 4 divided by clay size fraction of 12%) indicates that it will not have significant diffuse double layer effects which could complicate thermo-hydraulic analyses. The silt specimens used in the calibration process were prepared using compaction at a gravimetric water content of 13.7% to a dry unit weight of 14.0 kN/m³, which correspond to an initial degree of saturation of 0.42 and a porosity of 0.46. For reference, the optimum water content and the maximum dry unit weight corresponding to the standard Proctor compaction effort are 13.6% and 16.3 kN/m³ respectively.

The SWRC and HCF (in terms of the relative permeability to water) obtained from the modified TRIM test on compacted Bonny silt are shown in Figure 1(a) along with the model parameters. The shape of the SWRC indicates that an appreciable amount of water will be retained in the soil several meters above the water table under hydrostatic conditions. The compaction conditions for these curves are the same as those mentioned above, even though Lu and Dong (2015) report a different porosity due to a lower value of $G_s$ used in their calculations. An intrinsic permeability of $1.27 \times 10^{-14}$ m² was calculated from the hydraulic conductivity of saturated soil of $1.24 \times 10^{-7}$ m/s and the values of water viscosity and density at 20 °C (293.15 K). The TCF and VHCF for Bonny silt are shown in Figure 1(b), along with the TCF and VHCF parameters in Equations (8) and (9). The value of $m = 2.62$ measured in the experiment for Bonny silt was used in the simulations, which is lower than the value of 2.68 obtained from the correlation between m and $N_{vG}$ of Lu and Dong (2015). The experimental value still reflects the coupling between the
thermo-hydraulic properties as they were defined in the same test. The thermal conductivity ranges from 1.25 to 0.37 W/mK for saturated to dry conditions, respectively, while the volumetric heat capacity ranges from 2.75 to 1.30 MJ/m³K for saturated to dry conditions, respectively. The variations in these parameters with degree of saturation indicate that changes in heat retention within a storage volume may occur if a soil experiences drying during heat injection.

**Calibration of Vapor Diffusion and Phase Change Parameters**

To define the parameters $a$ and $b$, a tank-scale heat injection experiment was performed in an instrumented layer of compacted Bonny silt, which was then simulated using the parameters from the thermo-hydraulic constitutive relationships defined in Figure 1. A schematic of the experimental setup is shown in Figure 2. Bonny silt was compacted in 9 lifts in a cylindrical aluminum container having a diameter of 550 mm and a height of 477 mm. A 215 mm-long cylindrical cartridge heater having a diameter of 10 mm was used as the heating source. During heat injection, a temperature control unit was used to impose a constant temperature boundary condition of 60 °C on the heating rod. To monitor changes in temperature and degree of saturation during heating of the soil, ten 5TM dielectric sensors manufactured by Decagon Devices of Pullman, WA were placed at the locations shown in Figure 2. After all the lifts and sensors were placed, the top of the soil layer was covered with several layers of plastic wrap to minimize loss of water vapor to the laboratory air. The top and sides of the tank were then wrapped in insulation, and thermocouples were used to monitor the temperatures of the boundaries of the tank. The soil had an initial temperature of 23.5 °C and an initial degree of saturation of 0.42.

In the simulations of the tank-scale tests, no mass flux boundary conditions were applied for both liquid water and vapor flow for all boundaries of the tank. The top boundary was thermally insulated, convective heat flux boundaries were defined for the side boundaries to consider heat loss from the tank, a constant temperature boundary condition was used for the heating rod, and a
constant temperature of 18 °C was applied at the bottom of tank. The system of partial differential
 equations in Table 1 was solved simultaneously using COMSOL Multiphysics software. Results
from the numerical analyses were then compared with the experimental results to calibrate the
parameters $a$ and $b$. Comparisons of predicted and measured time series of temperature and degree
of saturation inferred from dielectric sensor #3 are shown in Figures 3(a) to 3(d). Sensor #3 was
selected as the primary location for calibration of the model as it is near the center of the heating
rod and is relatively close to the heat exchanger. The predicted time series in these figures include
curves for different values of the fitting parameters $a$ and $b$. The parameters are observed to have
a greater effect on the change in degree of saturation as they control the rates of vapor diffusion
and water phase change. Simulations from a model where no vapor diffusion or phase change is
considered (hereafter referred to as the “no vapor” case) are also shown in Figures 3(a) and 3(b),
which indicate slower increases in temperature to lower magnitudes at this location as well as a
negligible change in degree of saturation. Values of $a = 30$ and $b = 5 \times 10^{-7}$ s/m$^2$ were found to best
fit the Bonny silt data based on visual inspection, a similar approach used by Smits et al. (2011).

To evaluate the calibration, the spatial distributions of temperature and degree of saturation
along Transects B and A at the end of the heating from the numerical simulations and the
experiments are shown in Figures 4(a) through 4(d). In most of the cases, the predicted profiles
show good agreement with the measured data, except in the case of the degree of saturation
measured by the sensor nearest the edge of container in Figure 4(b). This sensor may have
malfunctioned due to the compaction process. Overall, the comparisons between model predictions
and experimental results in Figures 3 and 4 indicate that the calibrated values of $a$ and $b$ can be
assumed to be representative of Bonny silt under these compaction conditions.
EVALUATION OF VAPOR DIFFUSION AND PHASE CHANGE AROUND A FIELD-SCALE GEOTHERMAL HEAT EXCHANGER

Scenario Considered

The main goal of this study is to use the calibrated parameters to understand the changes in the behavior of a layer of unsaturated Bonny silt surrounding a geothermal heat exchanger during a heating and cooling cycle representative of geothermal energy storage systems. Although geothermal heat storage systems typically involve an array of geothermal heat exchangers with spacings as close at 1.5 m (Baser et al. 2016a), this study focuses on the changes in soil behavior around a single geothermal heat exchanger. This choice simplifies the boundary conditions and permits evaluation of the relative effects of the different heat transfer mechanisms. It is possible that the close spacing between geothermal heat exchangers may lead to different distributions in temperature and degree of saturation than those observed in this evaluation due to interactions between heat exchangers, but the simpler scenario of a single vertical geothermal heat exchanger is evaluated in this paper to help establish the impact of a heating-cooling cycle on the distributions in temperature and degree of saturation in the surrounding unsaturated silt layer.

The vertical geothermal heat exchanger investigated in this study has a length of 25 m and a radius of 0.04 m, embedded at a depth of 1 m from the surface. The embedment is consistent with the practice of installing geothermal heat exchangers below the frost depth. Even though this scenario could be investigated using an axisymmetric analysis, a 3D simulation was performed for a rectangular domain so that the domain could be modified to incorporate additional geothermal heat exchangers in future studies. The quarter domain having a height of 30 m and a width of 10 m with the geothermal heat exchanger along one edge is shown in Figure 5. The entire domain was assumed to be uniform and isotropic, and the soil was discretized into 101,073 elements (394,394...
degrees of freedom) with finer elements around the heat exchanger. The hydraulic and thermal boundary conditions for the models are also shown in Figure 5. For liquid water and gas flow, Neumann boundary conditions (no mass flux) were assumed for all boundaries except the bottom boundary, which was set to be a constant head boundary condition corresponding to the water table. For heat transfer, a constant temperature that represents an average mean subsurface soil temperature of 21 °C was applied at the bottom while at the outer boundaries the temperature varied with depth. No flux boundary conditions were applied to the planes of symmetry. The size of the domain was selected to be large enough that a constant temperature and zero fluid flux could be assumed on the outer vertical boundaries.

The initial conditions are shown in the color bars in Figure 5. The initial ambient temperature of the domain was assumed to be a function of depth until a certain depth of 9 m from surface, and this initial temperature profile is a representative of early summer months in San Diego (specifically May 2015). A hydrostatic initial condition was assumed, so the soil along the length of the heat exchanger is unsaturated with initial degrees of saturation ranging from 0.50 to 0.21 depending on the height from the water table. Two locations of interest that will be investigated further are noted in Figure 5(b) having different initial degrees of saturation.

During heat injection, a constant heat flux of 50 W/m² was applied to the outer boundary of the geothermal heat exchanger. This heat flux was converted to a volumetric heat source to obtain the value of Q in Equation (6). Although the magnitude of heat flux used in this study is representative of average value in geothermal energy storage systems (Acuña et al. 2012; Welsch et al. 2015; McCartney et al. 2017), a constant heat flux is not expected in a system where solar thermal panels are the heat source. In these cases, the input temperature from the solar thermal panels will remain relatively constant, which means that the heat flux will decrease with time as
the subsurface warms (Welsch et al. 2015). Although use of a constant heat flux will lead to greater increases in ground temperature than those expected when using solar thermal panels as the heat source, it provides a simple boundary condition for evaluating the roles of different heat transfer mechanisms in unsaturated soils.

**Heat Transfer and Water Flow Evaluation**

The temperature time series at a distance of 0.05 m from the geothermal heat exchanger and a depth of 8.5 m from the surface (i.e., $S_{r0} = 0.25$) is shown in Figure 6(a). After the 90-day heat injection period, the ground temperature reached a maximum value of 45.6 °C at this location. For comparison, the temperature time series from the model where no vapor diffusion or phase change is considered (the no vapor case) is also shown in Figure 6(a). In addition to showing a slower rate of increase in temperature, a lower maximum temperature of 36.3 °C was observed for the no vapor case. After the heating injection period, the heat flux was set to 0 W/m$^2$ and the soil was allowed to cool ambiently. After 180 days from the start of the simulation (90 days after the end of heat injection), the temperature decreased to 22.8 °C for the model with vapor diffusion and phase change and to 21.8 °C for the no vapor case. Although the gradients for heat loss are higher in the model with vapor diffusion and phase change, more heat is retained at this location after the 180-day cooling period due to the decrease in degree of saturation of the soil during heat injection due to vapor diffusion and latent heat transfer observed in Figure 6(b). Specifically, at the end of the heat injection period, a decrease in degree of saturation of 0.14 at this depth was observed for the model with vapor diffusion and phase change, while a negligible decrease in degree of saturation of 0.01 was observed for the no vapor case. The greater decrease in the degree of saturation for the model with vapor diffusion and phase change led to a decrease in thermal conductivity according to the TCF (from 0.84 to 0.49 W/mK), which will be assessed in more
Another interesting observation from Figure 6(b) is that at the end of the 90-day cooling period, only 16.5% of the decrease in degree of saturation observed during heat injection was recovered, indicating that the drying near the heat exchanger was permanent from a practical point of view. This may have an impact on subsequent heat injection and cooling cycles, and may be one of the reasons that an increase in the ground temperature is observed after several cycles of heat injection and extraction in practice (Sibbitt et al. 2012).

Radial distributions in temperature at a depth of 8.5 m from the surface at the ends of the heat injection and cooling periods are shown in Figure 7(a). Heat injection led to a notable change in temperature up to a distance of about 3 m from the heat exchanger. The temperature at the location of the heat exchanger was nearly 10 °C greater when vapor diffusion and latent heat transfer was considered than the case when it was not, and the temperature at the end of the ambient cooling period was greater throughout the zone of influence. A decrease in degree of saturation was only observed within approximately 1 m from heat exchanger for the model with vapor diffusion and phase change, as shown in Figure 7(b). A slight decrease in degree of saturation was observed near the heat exchanger for the no vapor case due to thermally-induced liquid flow. The zone of influence for temperature changes is greater than the zone of influence for degree of saturation changes for the conditions evaluated. For Bonny silt, this indicates that an overlap in the effects of different heat exchangers may be observed for the typical geothermal heat exchanger spacing of 1.5 m in geothermal energy storage systems.

Profiles of temperature with depth at horizontal distances of 0.05 m and 0.20 m from the heat exchanger at the end the 90-day heat injection period are shown in Figure 8(a). The temperature profiles varied nonlinearly with depth and had a maximum value at a depth of 4.5 m from the surface. For comparison, the temperature profiles for the no vapor case show more uniform
distributions in temperature with depth at the end of the heat injection period. The difference in temperature observed with depth in both models is due to the thermo-hydraulic properties with depth associated with the variations in initial degree of saturation with depth shown in Figure 5. A significant decrease in the in degree of saturation with depth is observed at both horizontal distances for the model with vapor diffusion and phase change in Figure 8(b), while only a slight decrease was observed for the no vapor case. Profiles of temperature after the ambient cooling are shown in Figure 8(c), with the profiles at horizontal distances of 0.05 m and 0.20 m overlapping. Although most of the heat injected has dissipated away from the heat exchanger, a greater amount of heat was retained in the soil close to the heat exchanger for the model with vapor diffusion and phase change. The profiles of degree of saturation at the end of the cooling period shown in Figure 8(d) only show slight increases from the profiles observed in Figure 8(b).

The impact of the initial degree of saturation on heat transfer and water flow can be investigated by evaluating the transient response at different depths in the soil profile, which have different initial degrees of saturation. Time series of temperature at depths of 8.5 m and 24.5 m from the surface at a horizontal distance of 0.05 m from the heat exchanger corresponding to initial degrees of saturation of 0.25 and 0.50 are shown in Figure 9(a). Increases in temperature of 45.6 and 42.3 °C at the end of the heat injection period were observed when the initial degree of saturation was doubled from 0.25 to 0.50. However, decreases in degree of saturation of 0.14 and 0.35 were observed at the end of the heat injection period for the same depths, as shown in Figure 9(b). The greater decrease in degree of saturation for the initially wetter soil is likely due to the availability of water to evaporate from the region near the heat exchanger. The horizontal zone of influence of the change in temperature is similar for the two depths as shown in Figure 9(c), but the horizontal zone of influence of the change in degree of saturation was greater at the depth of 8.5 m as shown
in Figure 9(d). This is consistent with observations that dryer initial conditions lead to greater zones of influence of vapor diffusion (e.g., Smits et al. 2011). The soil with $S_{r0}=0.50$ also shows a slight wetting front due to the movement of water away from the heat exchanger.

**Assessment of Heat Transfer Mechanisms and Effects of Coupled Flow**

Profiles of thermal conductivity and volumetric heat capacity that correspond to the profiles of degree of saturation in Figure 8(b) are shown in Figures 10(a) and 10(b), respectively. Despite the nonlinear decrease in degree of saturation along the length of the heat exchanger, a comparatively uniform decrease of approximately 0.3 W/mK is observed at 0.05 m from the heat exchanger. A more nonlinear decrease in thermal conductivity is observed further away from the heat exchanger at 0.20 m. The shapes of the profiles of the volumetric heat capacity are the same as those for the thermal conductivity due to the same parameters used in Equations (8) and (9), but because of the range of the two relationships for Bonny silt the volumetric heat capacity decreased by as much as 25% while the thermal conductivity decreased by as much as 70%. This is a positive finding for geothermal energy storage in similar soil deposits, as it means that lower heat losses can be expected without a significant reduction in the quantity of heat stored.

The vapor diffusion and latent heat transfer that results in the drying around the heat exchanger also leads to a suction gradient that may result in liquid water flow back toward the heat exchanger. Horizontal profiles of suction profiles at a depth of 8.5 m ($S_{r0}=0.25$) at different times are shown in figure 11(a). Large increases in suction are observed within 0.6 m from the heat exchanger, with decreases in suction beyond that point. Despite the large gradient associated with the suction distribution at the end of the heating period, the suction did not return to its original distribution during the ambient cooling period. This may have been due to the order of magnitude decrease in
the hydraulic conductivity (adjusted for temperature effects) shown in Figure 11(b), indicating that
more time may be needed for liquid flow to occur than permitted in the 90-day cooling period.

Vapor concentrations (kg/m$^3$) near the heat exchanger normalized by the equilibrium vapor
concentration (kg/m$^3$) are shown in Figure 12(a) for Bonny silt with initial degrees of saturation
of 0.25 and 0.50 (depths of 8.5 and 24.5 m). When the normalized vapor concentration is greater
or equal to 0.75, the phase change in the soil can be assumed to be near equilibrium (Lozano et al.
2008). For an initial degree of saturation of 0.25, the normalized vapor concentration soon after
the start of heating was smaller than this limit. The normalized vapor concentration decreased to
0.63 at the end of the heating period, indicating that use of the nonequilibrium model was justified.
For an initial degree of saturation of 0.5, the normalized vapor concentration was 0.82 soon after
the start of heating and remained above 0.75 indicating that an equilibrium phase change
assumption may be valid for initially wetter soils. The time series in Figure 12(a) indicate that the
phase change process did not reach steady state conditions by the end of the heat injection period.
Horizontal profiles of the normalized vapor concentrations at the end of heating shown in Figure
12(b) indicate that lower vapor concentrations were present near the heat source and had a similar
zone of influence to the degree of saturation in Figure 7(b). Despite the higher magnitudes of
normalized vapor concentration, greater changes in normalized vapor concentration with
horizontal distance are observed for the initially wetter soil ($S_0=0.5$), which may be the reason for
the greater change in degree of saturation at this location. Vertical profiles of normalized vapor
concentration at the end in Figure 12(c) are similar to those for the degree of saturation in Figure
8(b). A higher vapor concentration was observed close to the surface because of the lower initial
degrees of saturation and higher temperatures and because of upward movement of water vapor
due to buoyancy effects.
Horizontal profiles of the latent heat transfer rate, calculated as the product of $L_w R_{gw}$ in Equation (6), at the end of heat injection for soil at depths corresponding to initial degrees of saturation of 0.25 and 0.50 are shown in Figure 13(a). While a positive latent heat transfer rate was higher near the heat source indicating evaporation, a very slight value less than zero was observed at a distance about 1 m away from the heat exchanger. This indicates that condensation is occurring in the soil at lower temperatures further from the heat source. A comparison between the total thermal energy injected into the geothermal heat exchanger and the total latent heat (i.e., the total energy associated with phase change calculated by integrating the product of $L_w R_{gw}$ over the volume of the domain and over time) is shown in Figure 13(b). The total energy associated with phase change at the end of 90 days was 44 MJ, which is approximately 24% of the total heat injected into the system of 180 MJ. This indicates that an appreciable amount of the thermal energy injected into the system leads to phase change and further justifies the need to accurately account for nonequilibrium phase change effects in unsaturated soils. The total latent heat appears to have stabilized over the heat injection period, which may be because of the decrease in degree of saturation and reduction in availability of water to change phase near the heat exchanger.

**CONCLUSIONS**

A model that includes a recently-developed set of thermo-hydraulic constitutive relationships was used to understand the roles of vapor diffusion and phase change on the coupled heat transfer and water flow in a fine-grained, non-deformable unsaturated silt layer initially under hydrostatic conditions surrounding a vertical geothermal heat exchanger during heat injection and ambient cooling. In general, the modeling results confirm the importance of considering vapor diffusion and water phase change in simulations of geothermal heat exchangers in unsaturated soils, as well as the relevance of considering nonequilibrium phase change in initially drier soil layers. Although
quantitative conclusions from the simulations are specific to the given soil and geometry investigated, several conclusions can be drawn regarding the use of geothermal heat exchangers in geothermal energy storage systems in unsaturated soils, including:

- A greater rate of increase in temperature and greater magnitude of temperature change were observed in the soil near the heat exchanger during heat injection in a model that includes enhanced vapor diffusion and phase change. This conclusion indicates that conduction-only design models may underestimate the heat injection response of geothermal energy storage systems in the vadose zone.

- The heat retained in the soil near the heat exchanger during an ambient cooling period was greater when considering vapor diffusion and phase change, despite the greater thermal gradient compared to a model with no vapor. This was found to be due to the decrease in thermal conductivity associated with drying during heat injection. The drying during heat injection can be considered permanent for practical purposes within the time frame of ambient cooling considered. This may partially be due to the decrease in hydraulic conductivity due to thermally-induced drying, leading to a negligible amount of liquid water flow back toward the heat exchanger during ambient cooling.

- Although reductions in both thermal conductivity and volumetric heat capacity are observed during thermally induced drying of the soil near the geothermal heat exchanger, the percentage reductions in thermal conductivity were greater. This indicates that the greater amount of heat retention can be expected in unsaturated soils during ambient cooling, but the maximum possible heat stored will not decrease by as large of an amount.

- The zone of influence of changes in temperature was observed to be greater than the zone of influence of changes in degree of saturation for the silt under investigation, but both zones of
influence are appreciable enough that overlap is expected in geothermal energy storage systems with closely-spaced geothermal heat exchangers (i.e., 1.5 to 2.0 m).

- The normalized vapor concentrations in the initially drier soil near the ground surface were below the limit at which nonequilibrium phase change is expected to occur, justifying the use of this more advanced modeling approach. The vapor concentration gradient was greater in the initially wetter soil deeper in the profile.

- The initial degree of saturation was observed to influence both heat transfer and water flow in the model with vapor diffusion and phase change, with the greatest change in the degree of saturation occurring for soil with initially higher degrees of saturation.

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### Table 1. Equations used in the numerical analyses

<table>
<thead>
<tr>
<th>Equation</th>
<th>Number</th>
<th>Reference</th>
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<tr>
<td><strong>Nonisothermal liquid flow governing equation:</strong></td>
<td></td>
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<tr>
<td>$nS_{\text{rw}} \frac{\partial \rho_w}{\partial t} + n \rho_w \frac{dS_w}{dP_c} \frac{\partial P_c}{\partial t} + \nabla \cdot \left[ \rho_w \left( \frac{k_{\text{rw}} \kappa}{\mu_w} \right) \nabla \left( P_w + \rho_w g z \right) \right] = -R_{gw}$</td>
<td>(1)</td>
<td>(Bear 1972; Moradi et al. 2016)</td>
</tr>
<tr>
<td>$n$=porosity (m$^3$/m$^3$), $S_w$=degree of water saturation (m$^3$/m$^3$), $\rho_w$=temperature-dependent density of water (kg/m$^3$) (Hillel 1980), $t$=time(s), $P_c$=P$_w$-P$_g$=capillary pressure (Pa), P$_w$=pore water pressure (Pa), P$<em>g$=pore gas pressure (Pa), $k_w$=relative permeability function for water (m/s); $\kappa$=intrinsic permeability (m$^2$); $\mu_w$=temperature-dependent water dynamic viscosity (kg/(ms)) (Lide 2001), $g$=acceleration due to gravity (m/s$^2$) $R</em>{gw}$=Phase change rate (kg/m$^3$s)</td>
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| **Nonisothermal gas flow governing equation:**                           |        |           |
| $nS_{\text{rg}} \frac{\partial \rho_g}{\partial t} + n \rho_g \frac{dS_g}{dP_c} \frac{\partial P_c}{\partial t} + \nabla \cdot \left[ \rho_g \left( \frac{k_{\text{rg}} \kappa}{\mu_g} \right) \nabla \left( P_g + \rho_g g z \right) \right] = R_{gw}$ | (2)    | (Bear 1972; Moradi et al. 2016) |
| $S_g$=degree of gas saturation (m$^3$/m$^3$), $\rho_g$=temperature-dependent density of gas (kg/m$^3$) (Smits et al. 2011), $k_g$=relative permeability function for gas (m/s); $\mu_g$=temperature-dependent gas dynamic viscosity (kg/(ms)) |

| **Water vapor mass balance equation:**                                   |        |           |
| $n\left( \frac{\partial \rho_g S_{gw}}{\partial t} \right) + \nabla \cdot \left( \rho_g u - D_e \frac{\partial \rho_g}{\partial v} \nabla w \right) = R_{gw}$ | (3)    | (Smits et al. 2011) |
| $D_e=D_v \tau$=effective diffusion coefficient (m$^2$/s), $D_v$=diffusion coefficient of water vapor in air (m$^2$/s) (Campbell 1985), $w_{\text{v}}$=mass fraction of water vapor in the gas phase (kg/kg), $\tau=n^{1/3} S_{rg}^{7/3} \eta$=tortuosity (Millington and Quirk 1961) |

| **Enhancement factor for vapor diffusion, $\eta$:**                       |        |           |
| $\eta = a + 3S_{\text{rw}} - (a-1) \exp \left\{ \left[ 1 + \frac{2.6}{T_c} \right] S_{\text{rw}} \right\}$ | (4)    | (Cass et al. 1984) |
| $a$=empirical fitting parameter, $f_c$=clay content |

| **Nonequilibrium gas phase change rate, $R_{gw}$ (kg/m$^3$s):**            |        |           |
| $R_{gw} = \left( \frac{b S_{\text{rw}} RT}{M_w} \right) (\rho_{veq} - \rho_v)$ | (5)    | (Bixler 1985; Zhang and Datta 2004; Moradi et al. 2016) |
| $b$=empirical fitting parameter (s/m$^2$), $R$=universal gas constant (J/molK), $\rho_{veq}$=c$_{w,eq}$=equilibrium vapor density (kg/m$^3$), $T$=Temperature (K), $\rho_v$=p$_w$=vapor density (kg/m$^3$), $M_w$=molecular weight of water (kg/mol) |

| **Heat transfer energy balance:**                                         |        |           |
| $\left( \rho C_p \right) \frac{\partial T}{\partial t} + \nabla \cdot \left( (\rho u) C_{pw} u + (\rho g C_{pg})_{g} u + T - (\lambda \nabla T) \right) = -L_w R_{gw} + Q$ | (6)    | (Whitaker 1977; Moradi et al. 2016) |
| $\rho$=total density of soil (kg/m$^3$), $C_p$=specific heat of soil (J/kgK), $C_{pw}$=specific heat capacity of water (J/kgK), $C_{pg}$=specific heat capacity of gas (J/kgK), $\lambda$=thermal conductivity (W/mK), $L_w$=latent heat of water vaporization (J/kg), $u_w$=water velocity (m/s), $u_g$=gas velocity (m/s), $Q$=heat source term (W/m$^3$) |
LIST OF FIGURE CAPTIONS

FIG. 1. Hydraulic and thermal constitutive relationships and relevant parameters for Bonny silt:
(a) SWRC and HCF; (b) TCF and VHCF

FIG. 2. Cross-sectional elevation view of the experimental setup for model calibration with
dielectric sensor locations

FIG. 3. Predicted and measured time series from the tank-scale heating test: (a) Soil temperatures
for different values of $a$; (b) Changes in degree of saturation for different values of $a$; (c)
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the end of a 90-day heat injection period; (b) Degree of saturation at the end of a 90-day
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**FIG. 9.** Effect of initial degree of saturation: (a) Time series of temperature; (b) Time series of degree of saturation; (c) Radial temperature profiles; (d) Radial degree of saturation profiles

**FIG. 10.** Thermal property evaluation at the end of heat injection: (a) Thermal conductivity profiles; (b) Volumetric heat capacity profiles

**FIG. 11.** Liquid water flow evaluation: (a) Horizontal profiles of suction at different times; (b) Horizontal profiles of hydraulic conductivity at different times

**FIG. 12.** Vapor concentrations normalized by the equilibrium vapor concentration: (a) Time series; (b) Horizontal profiles at the end of heating; (c) Vertical profiles at the end of heating

**FIG. 13.** (a) Distributions of latent heat transfer rate at depths corresponding to initial degrees of saturation of 0.25 and 0.50; (b) Comparison of the total heat injected with the energy due to phase change
Figure 1

- SWRC
- HCF

(a) Degree of saturation, $S_{rw}$

- $\alpha_{VG} = 0.09$ kPa$^{-1}$
- $N_{VG} = 1.6$
- $n = 0.43$
- $\theta_r = 0.03$
- $k_{sat} = 1.24 \times 10^{-7}$ m/s

(b) Degree of saturation, $S_{rw}$

- $C_{vsat} = 2.75$ MJ/m$^3$K
- $C_{vdry} = 1.32$ MJ/m$^3$K
- $\lambda_{sat} = 1.28$ W/mK
- $\lambda_{dry} = 0.37$ W/mK
- $S_f = 0.145$
- $m = 2.62$
Figure 2

Surface covered with plastic wrap and insulation

Heater controller

Decagon 5TM dielectric sensor

Cartridge heater

Transect C

Transect B

Transect A

Compacted Bonny silt

Aluminum tank (insulated on sides)
Figure 7

(a) Ground temperature (°C) vs. Distance from heat source (m)

- \( q = 50 \text{W/m}^2, S_{r_0} = 0.25 \)
- \( a = 30, b = 5 \times 10^{-7} \text{s/m}^2, t = 90 \text{d} \)
- No vapor, \( t = 90 \text{d} \)
- \( a = 30, b = 5 \times 10^{-7} \text{s/m}^2, t = 180 \text{d} \)
- No vapor, \( t = 180 \text{d} \)

(b) Degree of saturation (m³/m³) vs. Distance from heat source (m)

- \( q = 50 \text{W/m}^2, S_{r_n} = 0.25 \)
- \( a = 30, b = 5 \times 10^{-7} \text{s/m}^2, t = 90 \text{d} \)
- No vapor, \( t = 90 \text{d} \)
- \( a = 30, b = 5 \times 10^{-7} \text{s/m}^2, t = 180 \text{d} \)
- No vapor, \( t = 180 \text{d} \)
Figure 10

(a) Thermal conductivity (W/mK)

- Depth from surface (m)
  - x=0.05m, No vapor
  - a=30, b=5x10^{-7} s/m²

- x=0.20m
  - a=30, b=5x10^{-7} s/m²

- q=50 W/m², t=90d

(b) Volumetric heat capacity (MJ/m³K)

- Depth from surface (m)
  - x=0.05m, No vapor
  - x=0.20m, No vapor
  - a=30, b=5x10^{-7} s/m²

- q=50 W/m², t=90d
Figure 11

(a) Suction (kPa) vs. Distance from heat source (m)

- $q = 50 \text{W/m}^2$, $S_{r0} = 0.25$
- $a = 30$, $b = 5 \times 10^{-7} \text{s/m}^2$
- $t = 0 \text{d}$
- $t = 90 \text{d}$
- $t = 180 \text{d}$

(b) Hydraulic conductivity (m/s) vs. Distance from heat source (m)

- $q = 50 \text{W/m}^2$, $S_{r0} = 0.25$
- $a = 30$, $b = 5 \times 10^{-7} \text{s/m}^2$
- $t = 0 \text{d}$
- $t = 90 \text{d}$
- $t = 180 \text{d}$
Figure 12

(a) Normalized vapor concentration (cv/cveq) vs. time elapsed (d)

- \( q = 50 \text{ W/m}^2 \), \( x = 0.05 \text{ m} \)
- \( a = 30 \), \( b = 5 \times 10^{-7} \text{ s/m}^2 \)
- \( S_r = 0.25 \)
- \( S_r = 0.50 \)

(b) Normalized vapor concentration (cv/cveq) vs. distance from heat source (m)

- \( q = 50 \text{ W/m}^2 \), \( t = 90 \text{ d} \)
- \( a = 30 \), \( b = 5 \times 10^{-7} \text{ s/m}^2 \)
- \( S_r = 0.25 \)
- \( S_r = 0.50 \)

(c) Normalized vapor concentration (cv/cveq) vs. depth from surface (m)

- \( q = 50 \text{ W/m}^2 \), \( t = 90 \text{ d} \)
- \( a = 30 \), \( b = 5 \times 10^{-7} \text{ s/m}^2 \)
- \( x = 0.05 \text{ m} \)
- \( x = 0.20 \text{ m} \)
Figure 13

(a) Latent heat transfer rate (W/m$^3$)

Distance from heat source (m)

- $q=50$ W/m$^2$, $t=90$ d
- $a=30$, $b=5 \times 10^{-7}$ s/m$^2$
- $S_{r0}=0.25$
- $S_{r0}=0.50$

(b) Thermal energy (J)

Time elapsed (d)

- Total heat
- Latent heat

$q=50$ W/m$^2$

$a=30$, $b=5 \times 10^{-7}$ s/m$^2$