Title
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Publication Date
1998-07-18
Feedback from Stock Prices to Cash Flows

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We are grateful to two anonymous referees, Andres Almazan, René Stulz, and Hong Yan for insightful and constructive comments that have greatly improved the paper. We also thank another anonymous referee, seminar participants at the University of Chicago, the University of Colorado, the University of California at Berkeley, DePaul University, the University of Maryland, Stanford University, the University of Texas, the Federal Reserve Board of Governors, and the Federal Reserve Bank at Chicago, for helpful comments, and Ashley Wang for excellent research assistance.
Abstract

Feedback from Stock Prices to Cash Flows

This paper explores how financial market prices directly influence a firm’s cash flows. Feedback from financial market prices to cash flows arises when a firm’s non-financial stakeholders, e.g., its customers, employees, and suppliers, make decisions that are contingent on the information revealed by the price. When there are complementarities across these stakeholders, such feedback leads to cascades in which relatively small stock price moves trigger substantial changes in asset values. The paper analyzes the relation between such feedback effects and parameters such as the cost of information acquisition, the volume of liquidity trading, the volatility of the value of existing projects, the risk aversion of liquidity suppliers, and the precision of managerial information releases.
Introduction

Traditional valuation models take as given an investment’s cash flow pattern, which, along with a discount rate, determines the price or value of the investment. Based on these models, we expect large stock price movements to be associated with important news about either future cash flows or discount rates. However, in reality, it is often quite difficult to attach significant economic news to large stock price movements.

This paper explores the notion that stock prices affect, as well as reflect, future cash flows. Feedback of this sort is relevant whenever stock prices influence the behavior of economic agents. For instance, existing research suggests that stock prices affect corporate investment choices,¹ and these investment choices may, in turn, affect existing investments. To illustrate this point, if the stock market places higher values on oil reserves, exploration activities increase, resulting in more oil and lower oil prices in the future. In this case, where information from stock prices influences the behavior of competitors, there is negative feedback that dampens stock price movements. In contrast, as we will discuss in this paper, when stock prices provide information to a firm’s customers, employees, or suppliers, there is positive feedback from stock prices to future cash flows, which magnifies stock price movements.

Our model of positive feedback includes two important elements. The first element relates to the interdependence among agents that creates potential coordination problems. Instances could include a computer producer and its customers who rely on the continuing support of their supplier, or alternatively, the producers of complementary products, such as Microsoft software and Intel chips, who are affected by each other’s investment choices.² Our model could also describe the producer of a product with inherent network externalities, like a computer operating system, that becomes more valuable as more people use it. In each of these cases there is the possibility of coordination failures and self-fulfilling fragile equilibria. This aspect of our model is closely related to the bank runs literature

¹See for example Fishman and Hagerty (1989), Bradley, Khanna, and Slezak (1994), and, more recently, Subrahmanyan and Titman (1999) for models along these lines.
²See Titman (1984) and Scitovsky (1954) for an elaboration of these arguments.
(e.g., Diamond and Dybvig (1983)) as well as other models with complementarities and coordination failures (e.g., Shleifer (1986) and Cooper and John (1988)).

The second element of our model relates to the way that stock prices convey information, an issue that has been extensively analyzed in the rational expectations literature beginning with Grossman (1976). We depart from this literature by explicitly considering how stock prices influence the behavior of a class of agents other than those who trade the stock. Specifically, we examine how a firm’s stock price can influence the decisions of the firm’s non-investor stakeholders (e.g., its customers, employees, and suppliers).

To understand how these two elements interact consider first the situation of a hypothetical personal computer firm with a proprietary operating system. Suppose that for some reason (either because of positive fundamental information, liquidity shocks, or changes in investor sentiment) some large investors aggressively buy shares of the company’s stock. The resulting price increase causes customers of the firm to positively update their estimate of the value of the firm’s operating system and thereby adopt the system. This, in turn, causes more customers to do the same, increasing the value of the operating system, because of the network effects, and hence, the value of the firm. In general, the initial favorable stock return helps a firm attract the best employees, enhance its reputation with customers, and make it a more attractive joint venture partner.\(^3\) In this sense, the idea that “success breeds success” is a clear implication of our setting.

Of course, positive feedback may also have a reverse effect. For example suppose blockholders sell stock (again for information or liquidity reasons). A class of firms, for example, software companies, seeing this price drop surmise that the growth prospects for the proprietary PC have slipped and decide to place less emphasis on developing new products for the PC. This, in turn, leads some potential customers, who observe the drop in interest by software writers, to choose to buy a different computer, decreasing the firm’s cash flows.

\(^3\)Tom Meredith, the former CFO of Dell Computer Corporation, claimed in a recent discussion with one of the authors that Dell’s rising stock price was one of its sources of comparative advantage for precisely these reasons.
To understand the factors that affect feedback, and how feedback is likely to influence managerial choices, we develop a simple model of a firm with assets in place and a growth opportunity. Because of complementarities, the value of the growth opportunity is partially determined by the perceived value of the assets in place, which is, in turn, influenced by the firm's stock price. In particular, when the perceived value of the assets in place is high, the firm can more easily attract employees to develop its growth opportunities, and because of complementarities between employees, the new employees make it easier to attract additional employees.

To highlight our main points, we consider extreme cases where changes in the perceived value of the firm's assets in place trigger either a positive cascade, where the growth opportunity becomes extremely valuable, or a negative cascade, where the growth opportunity is lost. As we show, the probability of triggering either a positive or negative cascade is determined by a number of factors. First, since the perceived value of the assets in place are likely to be highly correlated with their actual values, a cascade is more likely to be triggered when the value of the assets in place is very volatile. In addition, a cascade is more likely when there are greater complementarities across the firm's stakeholders.

The firm's information and trading environment also affects the importance of feedback and the probability of triggering a cascade. If the risk aversion of liquidity providers (e.g., market makers) are very high, or if the variance of uninformed noise or liquidity trading is high, these feedback effects will be lower if the number of informed investors is held fixed. However, if the number of informed investors is determined endogenously this result can be reversed. For instance, we show that when the market maker is risk neutral, that increasing the volume of liquidity trading stimulates the entry of informed investors, which makes the price more informative and increases the importance of the feedback effect. For a similar reason, decreasing the cost of information acquisition increases feedback effects.

The latter point relates to the incentives of management to actively promote their stocks and be accessible to outside analysts. This issue was discussed in a Wall Street Journal article (May 9, 2000, p. B1) about the fact that Lou Gerstner, CEO of IBM, provides very little access to outside analysts. The article states:
Mr. Gerstner takes the high ground, telling analysts that his time is best spent with customers and employees. Critics say that this sounds noble, but it doesn’t work in the new world of technology stocks....If CEOs can succeed in being heard above the marketplace, they can attract momentum investors and their stock price can become a competitive weapon in and of itself.

In the same article, PaineWebber’s Young says “If you win the minds of investors, it tends to help you win in the marketplace as well...Creating a buzz around your company is what the most effective CEOs are doing...”

Our model sheds light on the above quotes by demonstrating how managers can influence feedback effects by changing the precision of public information releases or reducing the cost of information acquisition, e.g., by easing the access of outside analysts to their firm. Basically, the precision of the public information release and the cost of information acquisition influence the volatility of the conditional expectation of the stakeholder and thus influences the likelihood of a cascade. We show that the manager will seek to add noise to the public information release and increase the information acquisition cost when he wants to reduce the likelihood of a negative cascade but seek to increase the precision and ease outside analyst access when he wishes to increase the likelihood of a positive cascade. Thus, our analysis indicates that firms that wish to prevent a negative cascade (e.g., mature firms who are concerned about losing a well-established work force) would tend to reduce focus on the stock price, whereas firms that wish to promote a positive cascade would do the reverse by promoting analyst coverage.

The final issue we consider relates to the concerns expressed by managers of emerging growth companies about short-selling and manipulation. For example, Appleby (1996) quotes the president of Columbia/HCA as saying that the excessive short position in the stock was “very, very misleading.” Green (1997) recounts that short-selling in Fuisz Technologies was so “ferocious” that the CEO, Richard Fuisz started receiving calls from “scared investors who think I was either jailed by the Internal Revenue Service or incarcerated by the FBI.” Palmeri (1994) reports how the CEO of Seitel, Inc. (which sells seismic
data to firms engaged in oil and gas exploration) put out a press release criticizing short-sellers and asking Seitel shareholders to “call their brokers and request that their Seitel shares not be lent out for the purposes of short-selling.” Loomis (1996) describes how a fledgling firm, Panax Pharmaceutical, was brought almost to bankruptcy by aggressive short-selling by three small brokerage houses.\(^4\)

The concerns expressed by these individuals contrasts with the existing academic literature on market manipulation which suggests that manipulation is generally not profitable in standard financial market settings.\(^5\) However, our analysis indicates that these concerns are relevant when stock price moves can be magnified because of their effects on the firm’s operations. Indeed, we show that when complementarities across a firm’s stakeholders are sufficiently strong, an equilibrium without manipulation does not exist.

Our analysis suggests that managers can combat manipulation strategies of this type by indulging in insider trading in a direction opposite to that of the manipulator. Such strategies are a rationale for a legal form of insider trading and may help explain active trading by insiders on a regular basis (see Seyhun, 1990, 1992). Open-market transactions such as stock repurchases can also help to address the preclusion of cascades.

The paper is organized as follows. Section 1 describes the economic setting. Section 2 describes the workers’ problem in a simple setting. Section 3 analyzes the workers’ decision in a noisy financial market. Section 4 analyzes endogenous entry by informed traders, while Section 5 discusses the opportunity for price manipulation in our setting.

\(^4\)Aside from these specific examples, other writings in the press also indicate that visible firms such as Wal-Mart and Corning are concerned about the possibility that a declining stock price could make it more difficult to retain their employees (see Markowitz (1995) and Hemmerick and Williams (1992)). Representatives of developing economies have also expressed concern about the effect of financial manipulators on their markets. See Mohamad (1997) and “Intervention Puts Hong Kong’s Image at Risk,” Bruce Knecht and Erik Guyot, *The Wall Street Journal*, August 17, 1998, p. A8, and “(Russia Needs a New Currency)...and Hong Kong Needs to Defend the one it has,” *The Wall Street Journal*, August 20, 1998, p. A8.]

\(^5\)For manipulation by informed investors, see Fishman and Hagerty (1995) and John and Narayanan (1997). For manipulation in a cash-settled futures contract, see Kumar and Seppi (1992). Allen and Gorton (1995) derive an equilibrium with manipulation in a Glosten and Milgrom (1985) setting where the probability of liquidity sales is greater than that of liquidity buys. In general, profitable manipulation in these models require situations where markets are more liquid when investors are unwinding their trades than they are when the original trades are made.
Section 6 considers the precision of public information by the manager. Section 7 concludes by briefly reviewing some of the implications of the analysis. All proofs, unless otherwise stated, appear in the appendix.

1 The Economic Setting

We first consider a single firm with symbiotic relationships with other agents that we will call stakeholders. These stakeholders make decisions that affect the firm’s value, and then receive a payoff that is contingent on that value. The term stakeholder is generically taken to apply to agents who receive a benefit from being associated with the firm, such as the firm’s employees, customers, lenders, and suppliers. We illustrate the case for positive feedback with an example where the relevant stakeholders are potential consumers who are interested in purchasing a product with network externalities, i.e., a product whose value increases when more of the product is purchased, and the case of negative feedback with employees who can leave the firm for outside employment opportunities. What is important for the analysis is that there are externalities associated with the stakeholder’s decision. The value derived from being a stakeholder depends on the participation of other stakeholders.

As we will show, the externalities associated with stakeholder decisions create situations where firm values are fragile, meaning that an initial shock to cash flows can have a substantial economic effect by triggering a cascade. In a positive cascade, “success breeds success,” in that favorable information induces new stakeholders to do business with the firm at favorable terms, creating additional increases in value. On the other hand, in the negative cascade, bad news leads one of the stakeholders to terminate its relation with the

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6For discussions of adoption externalities, see Dybvig and Spatt (1983) and Milgrom and Roberts (1995). Fletcher (1996), Hutheesing (1994), and Cusumano, Mylonadis, and Rosenbloom (1992) discuss the network externalities which allowed the VHS format to become the industry standard over the technologically superior betamax.

7Becker (1991) also discusses a model with network externalities in which cascades result from small changes in product prices. Our definition of fragility is similar to that used by Becker (1991), in the sense that with feedback effects, the equilibrium is such that for any given set of parameters, a minor shock to fundamentals or to uninformative trades triggers a cascade.
firm, which in turn lowers the value to the other stakeholders, leading some of them to also defect, further reducing firm values and triggering further rounds of defection.\textsuperscript{8}

The firm consists of “assets in place” and a “growth opportunity,” whose value is determined by the value of the assets in place as well as the availability of its key employees or other stakeholders such as customers and suppliers. Production choices are made in period 1, and in period 2, the payoffs from the assets in place and the growth opportunity are realized. The period 2 payoff on the assets in place is given by $F + \delta$, where $F$ is the ex ante mean and $\delta$ is a zero mean, normally distributed random variable and the payoff from the growth opportunity is $G$. Outside investors can become informed by collecting information about $\delta$.

For concreteness, consider a firm with $N$ stakeholders who have specific human capital that is difficult to replicate. Each stakeholder, who is ex ante identical, obtains $\rho_1$ times the payoff from the assets in place and $\rho_2$ times the payoff from the growth opportunity, $G$; their payoff, denoted by $\Pi$, is thus given by

$$\Pi = \rho_1(F + \delta) + \rho_2G.$$ \hfill (1)

The stakeholders make a strategic decision of whether to associate themselves with the firm or not which we analyze in detail below.

To simplify our analysis we initially assume that the entrepreneur issues public claims on the cash flow of the firm’s assets in place rather than the whole firm (i.e., the assets in place plus the growth opportunity). This implies that only the assets in place are publicly traded.\textsuperscript{9} This assumption has no substantive effect on the results since there is a deterministic relation in this model between the cash flows of the assets in place and the cash flows of the entire firm.\textsuperscript{10} However, as we will see, this relation is non-linear.\textsuperscript{9}

\footnote{Teoh (1997) presents a related model in which information disclosure triggers a similar type of cascade. In contrast to Teoh (1997), we explicitly incorporate securities market trading, which allows us to shed light on the role of partially informative stock prices in inducing cascades (in particular, cascades can be triggered by noise, and managers have incentives to take actions that affect price informativeness).}

\footnote{We relax this assumption in Section 5.}

\footnote{In our structure, we assume that the growth opportunity depends on the realization of $\delta$ only through the actions of stakeholders. While this simplifies the exposition, allowing the growth opportunity to also depend directly on $\delta$ will not alter our main results.}
which implies that the cash flow of the total firm is not normally distributed, precluding the possibility of obtaining a closed-form solution to the security market equilibrium in a model where a claim on the firm’s total cash flow is sold.

2 The Stakeholders’ Decision in a Simple Setting

In our setting, the stakeholders’ future payoffs are contingent on the firm’s future profits. We thus consider a situation where the $N$ stakeholders are ordered by their reservation wages $\bar{w}_m, m = 1, \ldots, N$, with $\bar{w}_1$ being the highest reservation wage. As already mentioned, the parameter $G$ is influenced by how many stakeholders associate themselves with the firm. In particular, if all $N$ stakeholders are involved in the firm, $G = \bar{G}$. If $m$ stakeholders do not associate with the firm, $G = \bar{G} - mr$, where $r$ is positive, and $\bar{G} > mr$.

We assume in this section that $\delta$ is directly observed by a competitive market maker, who thus sets the price of the assets in place equal to $\bar{F} + \delta$.

2.1 Positive Cascades

In this subsection we present a case where a positive cascade arises. We will assume that our interdependent stakeholders are consumers of a product with network externalities rather than employees whose marginal productivities require the inputs of their co-workers.

For illustrative purposes, we assume that the firm has two divisions; one that produces an operating system and the second that produces a software product that is used with the operating system. There are $N$ potential users of this software combination whose utility obtained from the software increases with the number of other users adopting the product as well as with the reliability of the operating system. Let us interpret the variable $\bar{F} + \delta$ as an index of the reliability of the operating system, so that $\rho_1(\bar{F} + \delta)$ in (1) measures that portion of the users gain that is associated with using an operating system with a specified level of quality. In this setting $G = \bar{G} - mr$ measures that portion of the utility from using the software that depends on the number of other users. In addition, the quantities $\bar{w}_i, i = 1, \ldots, N$, can be interpreted as the utility from adopting an alternative product,
which we assume can be different for different individuals.

Now, consider the condition

$$\rho_1(\bar{F} + \delta) + \rho_2[\bar{G} - (N - 1)r] < \bar{w}_m$$

(2)

for \(m = 1, \ldots, N - 1\) so that none of the agents with reservation gains \(w_1\) through \(w_{N-1}\) want to adopt the product. Then, if the conditions

$$\rho_1(\bar{F} + \delta) + \rho_2[\bar{G} - (N - 1)r] > \bar{w}_N,$$

(3)

and

$$\rho_1(\bar{F} + \delta) + \rho_2[\bar{G} - (N - k - 1)r] > \bar{w}_{N-k},$$

(4)

for \(k = 1, \ldots, N - 1\) hold, we have an equilibrium where the agent with the reservation gain \(w_N\) adopts the product, which allows the other \(N - 1\) agents to also adopt the product because of the positive externality induced by the first agent’s adoption.

Thus, a realization of \(\delta\), which signals an increased expected quality of the operating system, triggers a positive cascade, where some individuals are initially led to buy the software product based on the increase in the expected reliability of the operating system, and then others are induced to buy the software as its user base grows from these initial purchases.\(^{11}\)

### 2.2 Negative Cascades

In this subsection, we provide an instance of a negative cascade. We assume that the stakeholders are employees of the firm. For exogenous reasons (e.g., cash constraints) the firm cannot commit to paying the stakeholders a fixed wage, but must instead offer them a wage contract that is contingent on the future opportunities of the business. Specifically, both future pay and advancement opportunities are better if the firm is growing and prospering. We assume that agents have equal productivity and have unobservable reservation wages which have identical ex ante expected values but which differ ex post. Since the

\(^{11}\)An analogy here is the recent Linux phenomenon where the success of Red Hat’s IPO positively affected the sales of Corel software that uses the Linux operating system.
employees are identical ex ante as well as ex post on all observable dimensions, they must be paid the same.

The latter assumption rules out the possibility that the employee with a higher reservation price can credibly signal his reservation price to his employer in an effort to increase his compensation, for instance, by revealing an outside offer, and then asking for additional compensation. In reality, while it may be possible to reveal an outside wage offer, an employee’s reservation wage is determined by other considerations (e.g., location and work conditions) that are more difficult to observe. Of course, in a setting where the relevant stakeholders are customers rather than employees it is more natural to assume that the reservation prices are unobservable. For instance, we would not expect a Windows customer to be able to obtain a discount from Microsoft by demonstrating that many of his or her colleagues have switched to Linux.

The firm in this case has a retention problem if future profits are expected to decline. In particular, depending on their reservation wage rates, some employees will quit if sufficiently unfavorable information about the firm is revealed, and by doing so, create costs that are born by the employees that remain. As we show, these costs can potentially create a negative cascade where all employees quit in response to the defection of one employee.

We now characterize several possible equilibria that arise in this setting. One equilibrium arises when

$$\rho_1(\bar{F} + \delta) + \rho_2 \bar{G} > \bar{w}_1.$$  \hfill (5)

In this case, all employees continue to work for the firm. However, if the condition

$$\rho_1(\bar{F} + \delta) + \rho_2 \left[ \bar{G} - (N - 1)r \right] < \bar{w}_N$$  \hfill (6)

holds, we have an equilibrium in which all $N$ employees quit the firm. Thus, if (5) and (6) hold simultaneously, multiple equilibria exist. The value of the firm in the equilibrium where employees quit $(\bar{F} + \delta + G - (N - 1)r)$ is smaller than the equilibrium where all employees continue to work for the firm $(\bar{F} + \delta + G)$.

For most of the analysis that follows we assume that if there are two possible equilibria, the good equilibrium prevails. However, since employees ignore the negative externalities
associated with their decisions, the fragility illustrated by these multiple equilibria still exist when we restrict the analysis to only the Pareto dominant equilibria. To understand this, note that if \( \delta \) is sufficiently low that the highest reservation wage employee quits,

\[
\rho_1(F + \delta) + \rho_2G < \bar{w}_1,
\]

(7)
even if all other employees prefer staying with the firm conditioned on all others staying, i.e., if

\[
\rho_1(F + \delta) + \rho_2G > \bar{w}_m
\]

(8)
and

\[
\rho_1(F + \delta) + \rho_2\left[\bar{G} - (m - 1)r\right] < \bar{w}_m
\]

(9)
for \( m = 2, \ldots, N \), we have an equilibrium in which the employees all quit even though all except the first employee would be better off if they could commit to staying with the firm. Indeed, ex ante, before the employees observe their reservation wages, they will in this case want to commit not to quit.\(^{12}\)

If we assume that employees are unable to make binding commitments, then the observability of \( \delta \) plays an important role. Thus, if \( \delta \) is not observable, and if the following inequality holds

\[
\rho_1F + \rho_2\bar{G} > \bar{w}_m
\]

(10)
for all \( m \), the kind of cascade described above will not occur. Hence, in this situation, individuals can be made worse off as a result of information conveyed by financial market prices about \( \delta \). Indeed, all of the employees may prefer to commit not to see \( \delta \) to avoid a possible cascade when \( \delta \) is observed (see the appendix for a formal argument that demonstrates the above point).

\(^{12}\)It is easily checked that (7), (8), and (9) simultaneously hold under a large parameter set. Consider, for instance, the case where \( N = 2 \). Then these conditions are satisfied by choosing \( \rho_1 = \rho_2 = 1, \bar{w}_1 = 10, \bar{w}_2 = 3, F = 0, \delta = 0, \bar{G} = 5, \) and \( r = 3 \).
2.3 The Likelihood of Cascades and Volatility

As the following proposition demonstrates, the parameter set under which either positive or negative cascades obtain is larger when the externalities associated with an employee or a firm quitting (or a customer buying) is higher. Further, the presence of network externalities magnifies fluctuations in asset values:

**Proposition 1**  1. Consider two values of \( r, r_1 \) and \( r_2 \), with \( r_2 > r_1 \). For any given realization of \( \delta \), the equilibria with positive and negative cascades obtain under a larger parameter set when \( r = r_2 \) than when \( r = r_1 \).

2. The value of the firm is more volatile when network externalities across stakeholders are present \( (r > 0) \) than when they are not present \( (r = 0) \).

The first part of the above proposition implies that the cascading phenomenon is more likely to obtain when \( r \), which represents the costs borne by other agents when a particular agent chooses to leave the firm or to not adopt the product, is large.\(^{13}\) The second part of the proposition demonstrates that complementarities across a firm’s stakeholders increase the ex ante volatility of firm values. For both positive and negative cascades, perceptions of success affect success, and as we will show, stock prices create as well as reflect those perceptions.

3 The Stakeholders’ Decision and Partially Revealing Stock Prices

An important lesson from the noisy rational expectations literature of Grossman (1976) and others is that stock prices convey as well as reflect information. This section extends this line of research to a setting where the information generated in financial markets is

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\(^{13}\)Note that by changing the marginal contribution of each agent to the total gain (represented by the parameter \( r \)), we can change the magnitude of the feedback effect to any desired level. For this reason we do not calibrate our model numerically, but expect it to be more important when stakeholder/worker contributions are significant determinants of final output, or when there are significant network externalities.
conveyed to the firm’s non-financial stakeholders as well as to its investors. As we show, stock prices play an important role in initiating the kind of cascades discussed in the prior sections. Our analysis of a partially revealing rational expectations equilibrium allows us to address how managers and investment bankers can take actions to affect the likelihood of such cascades by influencing market parameters that determine the informativeness of the stock price.

3.1 The Securities Market Equilibrium

We first consider the case where the investment choices of firms and the decisions of stakeholders are fixed. The analysis in this subsection is also limited to the valuation of the firm’s assets in place, which trade separately from its growth opportunities. The next subsection extends the model to analyze the process by which information in the financial markets influences stakeholder decisions and how this in turn affects the value of its growth opportunities.

In this model, stock prices are set by competitive market makers who may or may not be risk averse. These market makers expect to earn zero expected utility conditional on their information set. Other individuals trade either because they have information or because they have exogenous liquidity needs, or possibly for irrational reasons such as those in Black (1986). We assume that the total liquidity or noise demand in period 1 is a zero-mean normally distributed random variable, \( z \). For tractability, we assume that the informed traders are risk-neutral. There are \( k \) informed traders who learn the realization of \( \delta \) just prior to trade in period 1. All random variables are independently normally distributed with zero mean. Throughout the paper, we assume that at least one agent trades on information. Market makers observe only the total (net) order flow from the informed and liquidity traders, which is denoted by \( Q \). We assume that the market makers are risk averse and possess CARA utility with a risk aversion coefficient of \( A \). We assume that a single market maker takes the entire order flow and impose the condition that he earns the ‘autarky’ utility (the utility he would obtain by not making the market), which is normalized to zero for convenience.
The following lemma describes the unique linear equilibrium of the model. In this lemma, and throughout the paper, \( v_X \) denotes the variance of the random variable \( X \).

**Lemma 1** There exists a unique linear equilibrium in which the price takes the form \( P = F + \zeta Q \) where \( Q \) is the total order flow and the value of \( \zeta \) is given by

\[
\zeta = \frac{Av_8}{4} + \sqrt{\left(\frac{Av_8}{4}\right)^2 + \frac{kv_8}{(k+1)^2v_z}}. \tag{11}
\]

The volatility of the price is given by

\[
\text{var}(P) = \frac{kv_8}{k+1} + \frac{R^2v_8^2v_z}{8} + \frac{Rv_8}{2} \left[ \frac{R^2v_8^2v_z^2}{16} + \frac{kv_8v_z}{(k+1)^2} \right]^\frac{1}{2}. \tag{12}
\]

Note that the illiquidity parameter \( \zeta \) is positively related to the volatility of final value \((\delta)\) and the risk aversion of market makers. In addition, consistent with intuition, \( \zeta \) is negatively related to the variance of noise or liquidity trade, \( z \). Equation (12) indicates that the variance of the price depends on \( v_z \) if and only if the market maker is risk averse. Thus the assumption of risk averse market makers allows liquidity traders to affect volatility.

### 3.2 The Stakeholders and Partially Revealing Securities Market Prices

To simplify the stakeholders’ problem consider the case where the following inequality holds:

\[
\bar{w}_1 < \bar{w}_2 + \rho_2 r < \bar{w}_3 + 2\rho_2 r < \ldots < \bar{w}_N + \rho_2 (N-1)r. \tag{13}
\]

Under the above condition, if the stakeholder with the reservation wage \( \bar{w}_1 \) quits, a cascade is triggered where the firm’s other stakeholders also leave the firm. Examining this case simplifies the exposition, since it allows us to analyze only the decision of one stakeholder who leaves the firm if the expected payoff is less than his reservation wage \( \bar{w}_1 \). The additional condition for the cascade to be triggered is the market price be sufficiently low, i.e.,

\[
\rho_1[F + E(\delta|P)] + \rho_2 G < \bar{w}_1, \tag{14}
\]
which ensures that the stakeholder with reservation wage \( \bar{w}_1 \) leaves the firm.

The analogous conditions for a positive cascade are

\[
\bar{w}_N + \rho_2 N r > \bar{w}_{N-1} + \rho_2 (N-1) r > \ldots > \bar{w}_1 + r,
\]

and

\[
\rho_1 [\bar{F} + E(\delta|P)] + \rho_2 (\bar{G} - N r) > \bar{w}_N,
\]

which ensures that the stakeholder with the reservation wage \( \bar{w}_N \) joins the firm and all others do so as well.

Denote \((\bar{w}_1 - \rho_2 \bar{G})/\rho_1 \equiv \bar{w} \) and \(E(\delta|P) \equiv Y\), with \(\bar{F} - \bar{w} > 0\). Then condition (14) can be written as

\[
Y < \bar{w} - \bar{F}.
\]

The ex ante probability of a negative cascade, denoted by \( p \), is given by

\[
p = N \left[ \frac{\bar{w} - \bar{F}}{\text{std}(Y)} \right].
\]

Further, defining \( \bar{w}' = (\bar{w}_N + \rho_2 N r - \rho_2 \bar{G})/\rho_1 \), with \( \bar{w}' - \bar{F} > 0 \), the equivalent condition for a positive cascade is obtained by simply reversing the direction of inequality (17) and replacing \( \bar{w} \) by \( \bar{w}' \). Thus, the corresponding probability \( p' \) for a positive cascade is

\[
p' = N \left[ \frac{\bar{F} - \bar{w}'}{\text{std}(Y)} \right].
\]

The probability of a cascade of either type depends on \( \text{std}(Y) \). The following proposition relating the probability of a cascade to exogenous parameters is proved in the appendix.

**Proposition 2**

1. The probability of a cascade is increasing in the variance of the stock payoff, \( v_8 \).

2. When market makers are risk-neutral, changes in the variance of noise trading have no effect on the probability of a cascade.

3. When market makers are risk averse, an increase in the variance of noise trading or an increase in market maker risk aversion decreases the ex ante probability of a cascade.
The first item of the proposition indicates that an increase in the volatility of asset value, $\delta$, increases the sensitivity of prices to information and thus increases the probability of a cascade. This result suggests that the cascades are more likely to obtain for firms whose stocks are more volatile, such as firms in nascent economies or firms in growth-oriented industries.\footnote{When stakeholder contribution to output is unobservable, the stock price drop caused by the quitting of the first stakeholder in response to a large noise trade could increase the incentive for other stakeholders to quit; this phenomenon is independent of the strategic complementarities we explore. We do not analyze this possibility in our paper for reasons of tractability, but note that this phenomenon may also be relevant in causing cascades. We thank a referee for mentioning this point.}

The next two items of the proposition describe how changes in the expected volume of noise trading\footnote{Note that the variable $v_z$ is related to the volume of noise trade, $|z|$, through the relationship $E(|z|) = \sqrt{2v_z/\pi}$.} affect the stakeholders’ decision. When market makers are risk-neutral, an increase in the variance of noise trading causes informed traders to scale up their trades to exactly offset the increased magnitude of the noise trades, so that price volatility remains unchanged. As a result, an increase in $v_z$ has no influence on the probability of a cascade. When market makers are risk averse, however, an increase in the variance of noise trading is not exactly offset so price volatility does increase. In this case, prices become less informative which reduces the probability of a cascade. An increase in market maker risk aversion also makes prices less informative which decreases the probability of a cascade.

To further elaborate on these last points consider a firm whose stock is currently priced at 20. Given the current variance of the noise trades and the risk aversion of the market maker, a negative cascade is triggered if the price drops below 15. In this setting, what happens when either the variance of noise trades increase or the market maker risk aversion increases?

In this case, the price variance increases implying that the probability of the price falling below 15 increases. However, \textit{if the stakeholders recognize that the price is less informative a cascade will no longer be triggered when the price falls below 15.} Therefore, the price must fall below some endogenous lower barrier before a cascade is triggered. What our proposition demonstrates is that when the noise trading volatility or market
maker risk aversion increases, this lower barrier falls sufficiently so that the probability that the price crosses the barrier to trigger a cascade decreases.

It should be noted that this last result assumes that the stakeholders observe both the noise trading volatility and the market maker’s risk aversion. In the next section we show that our conclusions change when this assumption is relaxed.

3.3 Stakeholders With Limited Information

In this section, we demonstrate how the possibility of feedback can cause investment bankers to consider decisions that affect stock price volatility. These decisions include how much of IPO to place with sophisticated institutional investors versus retail investors who may be more or less likely to sell when other investors are panicking. These considerations may also influence whether an investment banker lists the issue on the NYSE or on Nasdaq, which allows the investment banker to enhance the liquidity and efficiency of the stock through its role as the stock’s market maker.

As we show in this section, unanticipated deviations from policies that promote both efficiency and liquidity will increase the probability of a cascade. To illustrate this we consider a scenario where an agent (say, an investment banker) takes an action (e.g., positive promotion of the stock through analyst reports) that affects the volatility of the noise trades. While the stakeholders do not directly observe the action, they have beliefs about the action that may or may not be rational. As we show, these unobserved choices will either reduce or increase the probability of a cascade depending on whether they reduce or increase the volatility of the noise trades.

16 While all of our results apply to both positive and negative cascades we concentrate more on negative cascades in this subsection. This is because we believe that investment bankers and entrepreneurs are likely to be more concerned about taking actions to prevent a negative cascade than to enhance the likelihood of a positive cascade. Of course there are cases in which the reverse could be true. Indeed, we have had conversations with investment bankers who have suggested that part of the rationale for drastically underpricing internet IPOs has to do with creating a positive “buzz” that will help the firm attract a critical mass of stakeholders.


18 Our logic here is closely related to the Lucas (1976) critique which discussed how expectations affect
To consider this issue, we denote the stakeholder’s belief regarding the variance of noise trading as $v_{z1}$ and the actual variance as $v_z$. We assume that stakeholders take the liquidity parameter as given. We then have the following proposition:

**Proposition 3** Consider the case where the stakeholders have a given estimate of the variance of noise trading. Then, the ex ante probability of a cascade is increasing in the true variance of noise trading.

The proposition indicates that if the beliefs of stakeholders regarding the estimate of noise trading is held fixed, an increase in the true variance of noise trading will cause them to attribute price moves too often to fundamental information and thus increase the probability of a cascade.

The above result has potential implications to the institutional environment of IPO’s. As we mentioned earlier, a potential benefit of listing on Nasdaq relative to the NYSE is that the investment banker that takes the firm public, acting in its role as the market maker for the stock, can provide liquidity to the market and dampen the volatility of its stock. If we assume that a better-funded market maker behaves as though he is less risk averse, then it follows from equation (12) that the actions of a better-funded market maker will cause prices to be less volatile.\(^{19}\)

If the risk aversion of the market maker is observable, then firms can weigh the benefits and costs of more efficient pricing to determine the benefits of a well-capitalized market maker. However, it is probably unrealistic to assume that the stakeholders of a firm can observe the market maker’s risk aversion. As we show in the preceding proposition, if the stakeholders’ beliefs about the volatility is independent of the market maker’s risk

\[^{18}\text{the efficacy of policy choices. Lucas argues that monetary policy affects employment only to the extent that money supply changes differ from expectations. Similarly, we argue that increased noise trading increases the probability of a cascade only when expectations about the level of noise trading is held constant.}\]

\[^{19}\text{Brennan (1986) provides an analogous rationale for price limits by arguing that they suppress extreme moves and thus prevent default from investors holding margined positions. Similarly, Chowdhry and Nanda (1998) consider an economy with two equilibria with high and low price volatility. They argue that the imposition of margin requirements precludes the high volatility equilibrium by limiting the size of investors’ positions.}\]
aversion, then the entrepreneur will prefer an investment bank that will provide a less risk averse market maker for its stock.\footnote{In our model, the market maker is precluded from taking actions that directly reduce the probability of a cascade. If the market making firm is also an underwriter acting in the interest of the issuing firm, the market maker may support the price by “leaning against the wind” in order to prevent a cascade. This allows a viable market for the firm’s shares to be sustained.}

4 Endogenous Entry by Informed Traders

In the analysis thus far, we have taken the number of informed traders to be fixed. We now use the basic framework of Section 3.2 to analyze how endogenous entry of informed traders affects the stakeholders’ decision. As we show, in our setting more informative security prices make the stakeholder’s decision more sensitive to the stock price and increase the ex ante probability of a cascade, so that factors that stimulate the collection of information increase the impact of feedback on asset values.

In this section, we endogenize the number of informed investors by assuming that each informed investor can observe $\delta$ at a cost of $c$. Thus, the number of informed traders is determined endogenously as that number which makes the per capita profits from becoming informed equal the cost of collecting information.

Consider now the effect of an increase in the variance of uninformed trading on the probability of a cascade. From Section 3, when market makers are risk neutral, increasing the amount of uninformed trading, for a given number of informed agents, has no effect on the information content of the stock price, because informed traders scale up their trading activity proportionately in response to this increase. However, an increase in $v_z$ causes more informed agents to enter the market. The second effect increases the information content of the price, which, in turn, increases the probability of a cascade.

With risk averse market makers, the effect of an increase in $v_z$ is ambiguous. The increase in the number of informed traders that occurs in response to an increase in $v_z$ tends to increase the probability of a cascade. However, $v_z$ now has a direct effect on the noisiness of the conditional expectation, as discussed in the previous section, and this
effect tends to reduce the probability of a cascade. Thus, under risk aversion of market makers, the effect of an increase in $v_z$ on the probability of a cascade is ambiguous. When market maker risk aversion is small, the effect of an increase in the number of informed agents dominates so that an increase in the amount of noise trading increases the number of informed agents and thus increases the probability of a cascade.

We also consider how the cost of information acquisition affects the probability of a cascade. Increasing the cost of information acquisition, $c$, will decrease the number of informed agents, which always decreases the probability of a cascade regardless of whether market makers are risk averse or not.

The above discussion can be summarized by the following proposition, which is proved in the appendix.

**Proposition 4** Under endogenous information acquisition, the following results hold:

1. When market makers are risk neutral, an increase in the variance of noise trading increases the number of informed investors, which increases the informational efficiency of the price, and thereby increases the ex ante probability of a cascade.

2. If market makers are risk averse and information acquisition is endogenous, the effect of an increase in the variance of noise trading on the probability of a cascade is ambiguous. For sufficiently small values of market maker risk aversion, the probability of a cascade is increasing in the variance of noise trading.

3. The probability of a cascade is decreasing in the cost of information acquisition, $c$, regardless of whether market makers are risk averse or risk neutral.

Thus, with endogenous entry of informed agents, an increase in the variance of liquidity or uninformed trading always increases the probability of a cascade if market maker risk aversion is sufficiently small. Negative cascades will ensue when conditions (13) and (14) hold, while positive ones will occur when (15) and (16) hold. The discussion in Sections 2.2 and 2.1 on when negative or positive cascades will occur applies here as well.
Overall, the results of this section suggest that cascades are likely to be positively related to changes in the variance of noise trading and that this relation is likely to be especially strong when market makers are well-capitalized. The cost of acquiring information also affects the probability of a cascade; when information costs are low, prices are more informative and the probability of a cascade is higher. The latter result suggests that if managers of nascent firms wish to increase the likelihood of a positive cascade, they will adopt strategies that decrease the cost of information acquisition, such as facilitating analyst access to the firm. We return to this issue in Section 6.

5 The Opportunity for Manipulation

The analysis thus far has shown that trades in the financial market have feedback effects if stakeholders condition their exit decisions on noisy financial market prices. Up to this point we have assumed that investors are small and can thus ignore the possibility that their orders can influence market prices. In this section we explore how feedback can create an opportunity for an uninformed investor, who is large enough to influence prices, to profit by manipulating market prices. Specifically, we will examine the efficacy of a strategy that initially establishes a short position in the growth opportunity, and then places an order to short the assets in place that is large enough to trigger a negative cascade. Although we focus on the case of negative cascades in this section, this is purely for illustrative purposes, and our analysis also applies to positive cascades.

Unfortunately, solving for an equilibrium with an uninformed manipulator would be quite complicated in this setting. Since the market maker would not knowingly trade against a manipulator, it would have to be the case that the manipulator would follow a mixed strategy, randomly choosing when to manipulate and how much of the security to buy or sell.\textsuperscript{21} We will instead carry out a less ambitious task, which is to show that

\textsuperscript{21}Derivation of an equilibrium with manipulation in an unrestricted microstructure setting is a difficult problem in general. Thus, existing papers on manipulation solve the problem in specialized settings. In particular, Fishman and Hagerty (1995) consider a setting with mandatory disclosure of insider trades, Kumar and Seppi (1992) require a cash-settled futures contract, Allen and Gorton (1995) require a setting where the probability of liquidity sales is greater than that of liquidity buys, and Jarrow (1992) requires
if manipulation of markets is not restricted, under some conditions any equilibrium must involve manipulation. That is, while we are unable to characterize the equilibrium, we are able to show that under reasonable parameter constellations, an equilibrium without manipulation cannot exist.

As we discussed earlier, for tractability reasons we cannot price the firm’s growth opportunity when there is informed trading, since the value of the growth opportunity is not normally distributed. To get around this problem, we assume that an initial round of trading exists where there is no private information. In this round, the manipulator takes a short position in the firm’s growth opportunity. In the second round of trading there is informed trading, as in the previous sections. In this round, the manipulator takes a short position in the assets in place that is large enough to be likely to drive the price to the level that triggers the cascade. In a final round, the positions are unwound at their expected values. Although the manipulator loses money when he shorts the assets in place, this loss is more than offset by the profit made on the short position in the growth opportunity if the exit of the stakeholder causes a sufficiently large drop in the value of the growth opportunity.

To formally illustrate the potential for manipulation we require three dates, 0, 1, and 2. Just prior to date 1, the informed trader observes the signal $\delta$. For simplicity, we assume that the market makers for both the assets-in-place and the growth opportunity are risk-neutral. The assumption of risk-neutral market making allows us to abstract from the complications induced by the dynamic problem of risk averse market makers, which is not the focus of our analysis in this section.

We assume that the market maker has the same information set as the stakeholders and thus conditions his trades on the stakeholders’ decision in setting the period 1 price of the growth opportunity. In other words, the market maker understands how prices affect decisions and how these decisions in turn affect firm values. Let the ex ante probability of the stakeholder leaving be denoted by $p$. The ex ante (period 0) price of the growth opportunity
opportunity is

\[ P_{C0} = p(G - Nr) + (1 - p)G = G - pNr, \]

whereas the period 0 price of the assets in place is \( \bar{F} \). The period 1 equilibrium price of the assets-in-place is given by \( P = \bar{F} + \zeta Q \), where \( Q \) is again the order flow and \( \zeta \) is obtained from (11), with \( A = 0 \).

To prove our proposition we will propose that the market makers believe that prices cannot be manipulated. Within this setting we then introduce a large uninformed investor and show that the investor in fact profits from manipulation under these beliefs. A realistic way to justify this scenario is to make a distinction between “small” traders, who do not believe they can affect cash flows through their actions, and “large” traders, who think they can do so. In essence, we show that if market makers believe that only “small” traders exist, a “large” trader will profit from manipulation. The proposition we prove in the appendix is as follows.

**Proposition 5** If the stakeholder’s productivity (represented by the parameter \( r \)) is sufficiently high and agents do not detect the presence of the manipulator, the expected profits to the manipulator are positive. Thus, if there are no legal restrictions that preclude manipulation, an equilibrium with no manipulation does not exist.

In our model, while it is true that the manipulator does not incur market impact costs when he reverses his trades, his manipulation profits arise only because of the real effects he causes. In fact, if the exit of the stakeholder has no effect on firm values, i.e., if \( r = 0 \), then the profits of the manipulator are negative. This feature of our model distinguishes it from other models of manipulation such as Jarrow (1992), Kumar and Seppi (1992), and Allen and Gorton (1995). In these models, the manipulator’s profits arise because the price impact of the initial trades are assumed to be greater than the price impact when the trades are reversed.

As we noted in the introduction, firms with strong network externalities have expressed concerns about manipulation and have an incentive to take actions to combat manipulation.
strategies that may trigger negative cascades. In the appendix we consider a case where the market maker incorrectly attaches zero probability that an investor exists who can manipulate prices. In the equilibrium we consider, a large investor will in fact manipulate prices if the firm’s management does nothing to prevent manipulation. The manager, however, by taking the opposite side of the trade, can neutralize the overall impact of manipulation and prevent a negative cascade from occurring. This suggests that insider trading activity or share repurchases can be a means by which managers can reduce or even eliminate the impact of manipulation strategies of the above type. An interesting implication of this argument is that such activity should be more common in firms where negative cascades are more of a concern; for example, in firms where there is a lot strategic complementarities across existing employees, e.g., computer software firms.

6 Transparency

In the last section we discussed the possibility that managers repurchase shares to offset potential manipulators. In this section we explore other ways in which feedback can potentially affect managerial choices. Specifically, we consider various choices that can affect what we will call the transparency of a firm. These choices include the quality of the auditors the firm may use and the degree of access they may provide to outside analysts, thereby influencing the precision or informativeness of its public announcements. As we show, a manager can influence the likelihood that a cascade will occur by his or her choice of the precision of information releases.

We extend the model developed in Section 4 by assuming that at date 1, the firm reveals a public signal, $\delta + \epsilon$, where $\epsilon$ is a zero-mean, normally distributed variable which is independent of all other random variables. In order to obtain closed-form solutions, we assume that market makers are risk-neutral, though numerical simulations indicate that our results continue to obtain for the case of a risk averse market maker (the equilibrium conditions for this case are provided in the appendix).

The following lemma describes the analog of Lemma 1 in the presence of a public signal.
Lemma 2  In the setting with a public signal, there exists a unique linear equilibrium where the price is given by $P = F + \xi(\delta + \epsilon) + \zeta Q$, where $Q$ is the order flow. The constants $\xi$ and $\zeta$ are given by

$$\xi = \frac{v_8}{v_8 + (k + 1)v_\epsilon}, \quad (19)$$

and

$$\zeta = \frac{v_\epsilon}{v_8 + (k + 1)v_\epsilon} \sqrt{\frac{k v_8}{v_\zeta}}. \quad (20)$$

One can see that the illiquidity of the market, measured by $\zeta$, is increasing in the variance of noise in the public signal. A noisier public signal makes informed traders more aggressive which makes the market more illiquid. Also note that the weight given to the public signal in the price, $\xi$, is decreasing in the variance of noise in the public signal and in the number of informed agents, which is consistent with intuition.

The next result we state describes the probability of a cascade in terms of the variance of noise in the public signal.

Proposition 6  The probability of the cascade is decreasing in the variance of the noise in public information $v_\epsilon$.

Intuitively, when $v_\epsilon$ increases, the stock price provides a noisier signal of $\delta$ and thereby decreases the probability of a cascade. This intuition is basically correct, but it ignores the fact that an increase in $v_\epsilon$ increases the incentive to collect information, implying that the number of informed investors will increase. However, as the appendix shows, the increase in the number of informed investors dampens the effect of an increase in $v_\epsilon$, but does not reverse the effect.

Now, suppose the manager of the firm can control the precision of the public signal. For example, the manager may have some leeway in the choice of accounting information systems as well as the quality of the auditors that are hired. Under what conditions will the managers choose a system that results in more versus less precise signals?

Whether the manager wants more or less precision depends on whether the expected costs of a negative cascade outweigh the expected benefits of a positive cascade. Within
the context of our model, this is determined by whether the conditions for a negative cascade ((13) and (14)) or those for a positive cascade ((15) and (16)) hold. When the conditions for a positive cascade hold, the manager would like the public signal to be more precise in order to increase the likelihood that the feedback effect will occur. When the conditions for a negative cascade hold, the manager would prefer a less precise signal.

To formalize this notion, define \( \tau_e \equiv 1/v_e \) as the precision of the public information signal. Suppose the manager can increase the precision of the public signal at a cost given by a continuous function \( C(\tau_e) \), with \( C'(\tau_e) > 0, C''(\tau_e) > 0, C'(0) = 0 \) and \( C'(\infty) = \infty \). The following proposition is proved in the appendix.

**Proposition 7**

1. Suppose that the condition \( \rho_1 F + \rho_2 [G - (N - 1)r] < \bar{w}_N \) holds so that the firm is in an equilibrium where none of the stakeholders joins the firm. Then, if the network externality between stakeholders, measured by the parameter \( r \), is sufficiently high, there exists a unique level of the precision of public information \( \tau_e \), which maximizes the firm’s ex ante value of the firm. This choice of \( \tau_e \) optimizes the tradeoffs that increasing precision is costly but increases the probability that the firm will move to an equilibrium where all \( N \) stakeholders work for the firm.

2. Alternatively, suppose that the condition \( \rho_1 F + \rho_2 G > \bar{w}_1 \) holds so that the firm is in an equilibrium where all stakeholders work for the firm. If \( r \) is sufficiently high, in order to avoid a negative cascade the manager optimally sets \( \tau_e = 0 \), i.e., does not release a public signal. This choice of \( \tau_e \) maximizes the ex ante value of the firm.

3. When \( r = 0 \) (network externalities are not present), the only consideration is that increasing precision is costly, so the manager optimally sets \( \tau_e = 0 \).

The above analysis can easily be extended to examine the incentives of managers to be either more or less accessible to outside analysts. If managers are more accessible, then the cost of acquiring information about the firm is lower and, in equilibrium, more analysts will cover the firm, increasing the precision of the stock price signal. Again, a firm’s incentive to improve the precision of the stock price signal depends on the likelihood of positive
and negative cascades, which in turn, are determined by the maturity of the firm and the importance of network externalities. The appendix shows that results similar to those in Proposition 7 can be obtained for the cost of information acquisition. Specifically, under reasonable restrictions on the cost function, there exists a level of information acquisition cost that maximizes firm value when the condition in part 1 of the proposition holds. Under the condition in part 2, the firm has an incentive to keep the cost of information acquisition as high as possible.

Our results indicate that strategies to reveal public information can vary considerably across firms and even across the life of a firm depending on whether the firm is concerned about a positive cascade or a negative cascade. For example, a nascent firm concerned about attracting stakeholders may increase the precision of public information and increase analyst access to the firm, whereas a mature firm that is very concerned about retaining existing stakeholders may choose to reduce this precision.

The incentive for managers to use the precision of public information releases to affect the likelihood of a cascade is higher, the stronger the complementarities across stakeholders, and the larger the number of stakeholders. This is illustrated in Figures 1 and 2, which show that under the condition in Part 1 of Proposition 7, the optimal precision chosen by the manager is increasing in the degree of complementarity, $r$, and the number of stakeholders, $N$.

It should be noted, however, that the above propositions assume that the stakeholders observe the precision of the firm’s stock price. Analogous to the arguments in Section 3.3, the implications of our model are quite different when the precision of the firm’s stock price is not observable. To understand this, consider again the firm’s choice of auditors. If auditor quality is known to be higher, stock prices are more informative, which, in turn, implies that the probability of a cascade is higher. However, the relation between auditor quality and the probability of a cascade is reversed when investors observe the quality

\[ C(\tau) = g\tau^{1.1} \]

with $g = 0.01$. For Figure 1, $N$ is fixed at 3 and for Figure 2, $r$ is fixed at 0.3. Qualitatively similar results were obtained for different parameter ranges around the assumed values.

\[22\] The base parameter values for the figures are $v_h = k = \tilde{G} = \tilde{w}_N = \rho_1 = 1$, $\tilde{F} = 0.1$, and $\rho_2 = 0.3$. The cost function used is $C(\tau) = g\tau^{1.1}$ with $g = 0.01$. For Figure 1, $N$ is fixed at 3 and for Figure 2, $r$ is fixed at 0.3. Qualitatively similar results were obtained for different parameter ranges around the assumed values.
of the auditor but stakeholders do not. In this case, an increase in auditor quality can lower the probability of a cascade because it lowers the volatility of the firm’s stock price, making it less likely that an extreme stock price movement will trigger a cascade.

7 Summary and Possible Extensions

Economists have become increasingly interested in models with increasing returns to scale and network externalities. An important implication of many of these models is that equilibria can be fragile, and minor differences between firms, or just luck, can be responsible for major successes as well as major failures. This fragility arises because perceptions can be self-fulfilling. The perception of success can generate success and vice versa.

Given the substantial literature on the information content of stock prices it seems natural to consider how a firm’s stock price can affect how the firm is perceived by its customers, suppliers, and employees and how this, in turn, influences their decisions. As we show, in the presence of complementarities, stock prices can directly influence a firm’s cash flows, and this has a number of important implications that we consider in detail. The main implications of our analysis are as follows:

- Our analysis indicates that feedback is likely to be most important when complementarities or network externalities are high, but relationships between a firm and its stakeholders are not well established, and when uncertainty about the cash flows of existing projects is high. Hence, the emerging e-commerce and Internet industries should be especially sensitive to the effect of feedback, which is consistent with the anecdotal evidence we mentioned in the introduction.

- The likelihood of feedback is increasing in the risk tolerance of liquidity providers and decreasing in the variance of uninformed noise trading. This result has implications for entrepreneurs and investment bankers of newly public firms. In particular, the result implies that the likelihood of positive feedback is enhanced by choosing relatively well-capitalized market makers to make markets in new issues, and by plac-
ing a greater proportion of an IPO with sophisticated institutional investors versus relatively less well-informed retail investors.

- Since the feedback effect is related to the informativeness of the firm’s stock price, concerns about feedback influence the quality of a firm’s financial statements and the degree of access provided to outside analysts that cover the firm. As we show, young firms, wishing to attract stakeholders, have an incentive to increase the precision of their information releases as well as analyst access. However, our analysis also indicates that more mature firms wishing to avoid losing key stakeholders such as customers and employees want to expend fewer resources to increase the transparency of their financial statements. This is consistent with the anecdotal evidence mentioned in the introduction that less-established companies like Sun have a greater incentive to provide access to outside analysts than mature firms like IBM.

- In the presence of feedback, there is an incentive to manipulate stock prices in order to create a feedback effect and profit from the resulting change in firm value. Indeed, our analysis indicates that when there is feedback from market prices to cash flows, there must be some possibility of manipulation in equilibrium. If managers believe that outside investors are trying to negatively manipulate their stock, they may choose to initiate a repurchase program or even trade on their own accounts to offset the effect of the manipulator.

Our analysis also raises a number of additional questions about how feedback can affect managerial choices that provides possible avenues for new research. In particular, feedback may have important implications that relate to the following strands of literature:

- Managerial myopia and financial signaling: If managers believe that feedback is important, they may choose projects that pay off more quickly in the hopes of reporting higher earnings numbers, thereby either preventing a negative cascade or creating a positive cascade. The signaling models that address these issues\(^\text{23}\) require that

\(^{23}\text{See, for example, Miller and Rock (1985), Stein (1989), and Brennan (1990).}\)
managers have a direct incentive to increase their firms’ current share prices, (e.g., the firm is issuing shares or the manager is selling some of his or her shares). With feedback, a firm’s current share price directly affects its intrinsic value, so that managers have an incentive to signal even when acting in the long-term interests of their firms.

- The role played by investment banks in taking firms public: Recent research has explored the three separate roles that investment banks play for newly public firms. In addition to taking the firm public, they serve as the market maker for the firm’s stock and provide analyst coverage. In particular, our model provides a framework for examining why underwriters who can provide better capitalized market making are more attractive to issuers, and why investment bankers wish to attract possibly uninformed trading volume by publicizing the stock.

- When firms go public and how the new issues are priced: An interesting recent book by Lewis (1999) provides anecdotes that suggest that the possibility of generating the kind of positive cascade described in our model may explain the recent trend to take Silicon Valley companies public very early in their life cycles and substantially underprice the new issues. The idea is that by substantially underpricing the IPOs, the underwriter draws attention to the potential wealth being created by these firms, thereby attracting individuals and firms with complementary products and skills.

The last item is preliminary, since we have not developed a theory of the pricing of new issues. However, such a theory may be worth pursuing, since it could potentially generating much clearer empirical tests than existing information-based models of underpricing. Evidence of a relation between the extent to which a new issue is underpriced and the importance of network externalities for the business would provide strong evidence that feedback plays an important role in the new issues market. Although a careful empirical

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24See Ellis, Michaely, and O’Hara (2000) for a discussion of issues that arise when the underwriter is also the market maker and Michaely and Womack (2000) for a discussion of issues that arise when the underwriter provides analyst coverage.
study is needed, recent evidence of the extreme underpricing of internet IPOs tends to support this implication.
Appendix

Discussion of Precommitment to not Observe $\delta$ (Page 11): Suppose there are two employees and that $r = \bar{w}_1 - \bar{w}_2$. Consider a case where $\delta$ is observed and assume convenience that $\sigma_{\delta} = 1$. Then, the first employee leaves if $\delta < \bar{w}_1 - \bar{F}$, and stays otherwise. His payoff is given by $\bar{F} + \delta$ if $\delta \geq \bar{w}_1 - \bar{F}$, and $\bar{w}_1$ otherwise. Now suppose that information about $\delta$ is not observed. In this case the employee receives $\bar{F} + \delta$. The difference between the payoff when $\delta$ is not and is observed is a random variable which equals zero if $\delta \geq \bar{w}_1 - \bar{F}$ and $\bar{F} + \delta - \bar{w}_1$ otherwise. Denote this random variable as $f_1(\delta)$ (note that the variable is nonpositive), and its expectation as $\mu_1$. It follows from Johnson and Kotz (1970, p. 81) that

$$\mu_1 = (\bar{F} - \bar{w}_1)\Phi(\bar{w}_1 - \bar{F}) - \frac{1}{\sqrt{2\pi}}\exp\left[-0.5(\bar{w}_1 - \bar{F})^2\right],$$

(21)

where $\Phi$ denotes the standard normal distribution.

Now consider the second employee. His payoff differential when $\delta$ is not and is observed is given by $\bar{F} + \delta - \bar{w}_2$ if $\delta < \bar{w}_1 - \bar{F}$ and zero otherwise. This variable can be positive or negative (it is negative if $\delta < \bar{w}_2 - \bar{F}$ and positive if $\bar{F} - \bar{w}_2 < \delta < \bar{F} - \bar{w}_1$). Denote this variable as $f_2(\delta)$. The expected payoff differential is the expectation of $f_2(\delta)$ (denoted by $\mu_2$). As in (21), $\mu_2$ is given by

$$\mu_2 = (\bar{F} - \bar{w}_2)\Phi(\bar{w}_1 - \bar{F}) - \frac{1}{\sqrt{2\pi}}\exp\left[-0.5(\bar{w}_1 - \bar{F})^2\right].$$

(22)

When $\bar{w}_1 = \bar{F}$, the above expression equals

$$0.5(\bar{F} - w_2) - \frac{1}{\sqrt{2\pi}}.$$

Since all the functions in (22) are continuous, $\mu_2$ is positive if $\bar{F}$ is sufficiently large relative to $\bar{w}_2$, and $\bar{w}_1$ is sufficiently close to $\bar{F}$. Thus, the second employee prefers that information about $\delta$ not be observed under a non-null parameter set.

Suppose that ex ante, an employee does not know whether he will obtain a reservation wage $\bar{w}_1$ or $\bar{w}_2$ and that the two outcomes are equally likely. Then the ex ante expected
payoff differential of each employee will be

\[ 0.5(\mu_1 + \mu_2) = \left[ \bar{F} - 0.5(w_1 + w_2) \right] \Phi(\bar{w}_1 - \bar{F}) - \frac{1}{\sqrt{2\pi}} \exp \left[ -0.5(\bar{w}_1 - \bar{F})^2 \right]. \]  

(23)

Again, this expectation will also be positive if \( \bar{F} \) is sufficiently large relative to \( w_2 \) and if \( F \) is sufficiently close to \( w_1 \). Thus, if \( F = w_1 \), the right-hand side of (23) reduces to

\[ 0.25(F - \bar{w}_2) - \frac{1}{\sqrt{2\pi}}, \]

which is positive if \( F - \bar{w}_2 \) is sufficiently large. Thus, both employees prefer that information about \( \delta \) not be released under a non-null parameter set and, under this parameter set, they will therefore be willing to precommit to not observe \( \delta \).

**Proof of Proposition 1:** The first part of the proposition follows directly from the observation that (5) and (9), as well as (2) and (4) hold under a larger parameter set for any given \( \delta \) when \( r = r_2 \) than when \( r = r_1 \). The second part of the proposition can be proved as follows. When \( r = 0 \), the variance of firm value is always \( v_\delta \). Now, since \( \delta \) is normally distributed, for any non-zero value of \( r \), at least one of the probabilities \( p_i \) attached to the events that the growth opportunity will be worth \( G - ir, i = 1, \ldots, N \) must be non-zero. The variance of asset values in this case is

\[ v_\delta + r^2 \left[ \sum_{i=2}^{N+1} (i - 1)^2 p_i^2 \right] + \left[ \sum_{i=2}^{N+1} (i - 1)p_i \right]^2, \]

which is greater than \( v_\delta \). □

**Proof of Lemma 1:** We assume that each informed trader uses a linear strategy and submits an order of the form \( \kappa(\delta) \). The expected utility \( E[U] \) which this market maker obtains by making the market can be written in the mean-variance fashion

\[ E[U] = E[Q(P - F)|Q] - \frac{A}{2} \text{var}[Q(P - F)|Q]. \]

(24)

It is easy to show that the unique linear equilibrium is characterized by the market maker using a linear rule and the informed following symmetric linear strategies. Substituting for a linear pricing rule \( P = \bar{F} + \zeta Q \) in equation (24) and setting the RHS of this equation to zero yields

\[ \zeta = \nu + \frac{A}{2} \text{var}[\delta|Q], \]

(25)
where $\nu$ is the regression coefficient in the forecast of $\delta$ on $Q$.

Each informed trader $i$ maximizes $E[(F - P)x_i|\delta]$, where $x_i$ is his order. This expected profit function can be written as

$$E(x_i(\delta - \zeta(x_i + (k - 1)\kappa \delta + z))|\delta).$$

Maximizing this expression with respect to $x_i$ yields

$$x_i = \frac{\delta(1 - \zeta(k - 1)\beta)}{2\zeta}.$$  \hspace{1cm} (25)

In the symmetric equilibrium

$$\kappa = \frac{\delta(1 - \zeta(k - 1)\beta)}{2\zeta},$$

so that

$$\kappa = 1/((k + 1)\lambda).$$

Now, if $u$ and $v$ are two independent and normally distributed random variables, each having a mean of zero, then it is a standard result (see, e.g., DeGroot (1986)) that

$$\text{var}(u|u + v) = \frac{\sigma_u^2 \sigma_v^2}{\sigma_u^2 + \sigma_v^2}. \hspace{1cm} (26)$$

Noting again that $Q = k\kappa \delta + z$, substituting $\kappa = 1/[(k + 1)\zeta]$ into (25), and using (26) yields the following quadratic equation for $\zeta$,

$$2\zeta^2 v_z (k + 1)^2 - R v_\delta v_z \zeta (k + 1)^2 - 2k v_\delta = 0,$$

the unique positive root of which yields (11). \Box

**Proof of Proposition 2:** Substituting for $P = \bar{F} + \zeta Q$ where $Q$ is the total order flow, we find that condition (17) becomes

$$\frac{k(k + 1)v_\delta}{k^2 v_\delta + \zeta^2 (k + 1)^2 v_z} \left( \frac{k\delta}{k + 1} + \zeta z \right) < \bar{\omega} - \bar{F}. \hspace{1cm} (27)$$

Now, it is evident that both $p$ and $p'$ are increasing in $\text{std}(Y)$, which is given by taking the standard deviation of the left-hand side of (27), and can therefore be written as

$$\text{std}(Y) = \left[ \frac{k^2 v_\delta^2}{k^2 v_\delta + \zeta^2 (k + 1)^2 v_z} \right]^{\frac{1}{2}}. \hspace{1cm} (28)$$

34
The first part of the proposition follows from the fact that when \( R = 0 \), \( \zeta^2 v_z \) is independent of \( z \) since \( \zeta \) then equals \([kv_\delta/(k + 1)^2v_z]^{1/2}\). Noting that \( \zeta^2 v_z \) is increasing in \( v_z \) and \( R \) (from (11)), we obtain the second part of the proposition. Finally, the third part follows by substituting writing the reciprocal of the right-hand side of (28) as

\[
\left[ \frac{1}{v_\delta} + \frac{\zeta^2(k + 1)^2v_z}{k^2v_\delta^2} \right]^{1/2}.
\]

Substituting for \( \zeta \) from (11) into the above expression shows that the expression is decreasing in \( v_\delta \) so that \( \text{std}(Y) \) is increasing in \( v_\delta \). \( \square \)

**Proof of Proposition 3:** We have

\[
Y_s \equiv E_s(\delta|P) = \frac{k(k + 1)v_\delta}{k^2v_\delta + (k + 1)^2\zeta^2v_z1} \left( \frac{k\delta}{k + 1} + \zeta z \right),
\]

where the subscript \( s \) denotes the expectation under the beliefs of the stakeholder. The variance of \( Y_s \) is monotonically related to the ex ante probability of a cascade. This variance is given by

\[
\text{var}(Y_s) = k^2v_\delta^2 \left[ \frac{k^2v_\delta + (k + 1)^2\zeta^2v_z}{\{k^2v_\delta + (k + 1)^2\zeta^2v_z\}^2} \right].
\]

Note that the probability of a cascade is again given by (28) (with \( Y \) replaced by \( Y_s \)), where \( \text{std}(Y_s) \) is the square root of the right-hand side of (30). From (11), \( \zeta^2 v_z \) is increasing in \( v_z \), whereas \( \zeta \) is decreasing in \( v_z \). Thus the numerator of the right-hand side of (30) is increasing in \( v_z \) while the denominator is decreasing in \( v_z \). This implies that \( \text{std}(Y_s) \) is increasing in \( v_z \), so that the probability of a cascade is increasing in \( v_z \). \( \square \)

**Proof of Proposition 4:** First consider Part (1) of the proposition. Recall that the probability of the stakeholder leaving the firm is monotonically increasing in \( \text{std}(Y) \) where \( Y = E(\delta|P) \). Since the market makers are risk-neutral they set the price \( P \) equal to the expectation of \( \delta \) conditional on all available information, so that \( Y = P \). For notational convenience, let \( V \equiv \text{var}(Y) \). When market makers are risk-neutral, \( V \) is given by substituting \( R = 0 \) in (12):

\[
V = \frac{kv_\delta}{k + 1}.
\]
From (11), we have
\[ \zeta = \left[ \frac{k v_k}{(k + 1)^2 v_z} \right]^{\frac{1}{2}}. \]  
(32)

Now, the ex ante expected profit of each informed trader is given by \( \pi = E[(\hat{F} + \delta - P)x] \), where \( x = \kappa \delta \) is the order of the informed trader. Substituting \( P = \hat{F} + \zeta Q \) where \( Q \) is given by \( k \kappa \delta + z \), we have
\[ \pi = \frac{v_k}{(k + 1)^2 \zeta}. \]

The equilibrium entry condition is \( \pi = c \). Letting \( \sigma_\delta = v_\delta^\frac{1}{2} \), \( \sigma_z = v_z^\frac{1}{2} \), and substituting for \( \zeta \) from above, the entry condition is then given by
\[ \frac{\sigma_z \sigma_\delta}{(k + 1)^2} = c. \]
(33)

In this case, the probability of a cascade is monotonically related to the variance of the price. Now, we can write the derivative of the variance of the price with respect to \( \sigma_z \) as
\[ \frac{dV}{d\sigma_z} = \frac{\partial V}{\partial k} \frac{dk}{d\sigma_z} + \frac{\partial V}{\partial \sigma_z}. \]
(34)

Now, from (33), we have
\[ \frac{dk}{d\sigma_z} = -\frac{\partial \pi / \partial \sigma_z}{\partial \pi / \partial k} = \frac{2k(k + 1)}{1 + 3k}. \]

Since, from (31), \( \partial V / \partial \sigma_z = 0 \) and \( \partial V / \partial k > 0 \), we have \( dV / d\sigma_z > 0 \). From (18), the probability of a cascade is increasing in std(\( Y \)), which equals \( V^{\frac{1}{2}} \). The probability of a cascade is therefore increasing in \( v_z \).

To prove the second part of the proposition, again denote \( V \equiv \text{var}(Y) \), and observe from (11) and (28) that
\[ V^{-1} = \text{var}(Y)^{-1} = 1 + \frac{(k + 1)^2 v_\delta^2 A^2 v_z}{8k^2} + \frac{v_\delta}{k} + \frac{Av_\delta}{2} \sqrt{\frac{(k + 1)^4 A^2 v_\delta^2 v_z}{k^4 16} + \frac{(k + 1)^2}{k^3} v_\delta v_z}, \]
(35)
which is decreasing in \( k \) and increasing in \( \sigma_z \), so that \( \partial V / \partial \sigma_z < 0 \), whereas \( \partial V / \partial k > 0 \). Further,
\[ \frac{dk}{d\sigma_z} = -\frac{\partial \pi / \partial \sigma_z}{\partial \pi / \partial k}. \]
(36)
where
\[ \pi = \frac{v_k}{(k + 1)^2 \zeta}. \]  
(37)

Substituting for \( \zeta \) from (11), it follows that \( \frac{dk}{d\sigma_z} > 0 \). Now, we know that
\[ \frac{dV}{d\sigma_z} = \frac{\partial V}{\partial k} \frac{dk}{d\sigma_z} + \frac{dV}{d\sigma_z}. \]  
(38)

From (35), we have \( \partial V/\partial \sigma_z < 0 \) and \( \partial V/\partial k > 0 \). From (37) and (11), we have \( dk/d\sigma_z > 0 \).

Thus, the right-hand side of (38) is of ambiguous sign because the first term of this expression is positive whereas the second term is negative. As \( A \to 0 \), the second term goes to zero, whereas the first term remains positive. This indicates that for sufficiently small values of market maker risk aversion, the entry effect will dominate and increases in \( v_z \) will increase the probability of a cascade.

For proving the part (3) of the proposition, first note that the expected profit of each informed agent is given by
\[ E[k\delta(\delta - \zeta k\delta)], \]
which equals
\[ \frac{v_k}{(k + 1)^2 \zeta}. \]

Substituting for \( \zeta \) from Equation (11), we find that the profits are decreasing in \( k \). So the effect of increasing the cost of information acquisition is to decrease the number of informed agents.

It remains to be shown that \( V \) is increasing in \( k \). To see this, note from (28) that the sign of the derivative of \( V \) with respect to \( k \) is the negative of the sign of \( \zeta(k + 1)/k \). Substituting for \( \zeta \) from Equation (11), \( \zeta(k + 1)/k \) equals
\[ \frac{Av_k(k + 1)}{4k} + \sqrt{\left( \frac{Av_k(k + 1)}{4k} \right)^2 + \frac{v_k}{kv_z}}. \]

Since all of the individual terms in the above expression are decreasing in \( k \), the probability of a cascade is increasing in \( k \), and in turn, decreasing in the cost of information acquisition, \( c \). \( \square \)
Proof of Proposition 5: Equation (11) with \( R = 0 \) implies that

\[
\zeta = \left[ \frac{kv_\delta}{(k+1)^2v_z} \right]^{\frac{1}{2}}
\]  

(as in (32)), and \( p \) (from (18)) is given by

\[
p = N \left( \frac{\bar{w} - \bar{F}}{\text{std}(Y)} \right) = N \left( \frac{\bar{w} - \bar{F}}{(kv_\delta/(k+1))^{\frac{1}{2}}} \right) .
\]  

Let \( P_c \) be the critical value of the price below which the stakeholder with reservation wage \( \bar{w}_1 \) leaves the firm. From (17), and since the price \( P = E(\delta|Q) \) because of risk-neutrality of market makers, we have \( P_c = \bar{w} - \bar{F} \). Given that the price is a linear (non-stochastic) function of the order flow, from (27), we can write the critical value of the order flow \( Q_c \) mapping on to \( P_c \) as

\[
Q_c = \left[ \frac{k(k+1)v_\delta}{k^2v_\delta + \zeta^2(k+1)^2v_z} \right]^{-1} (\bar{w} - \bar{F}).
\]  

Suppose a potential manipulator shorts \( X \) shares (i.e., trades \(-X\) shares) of the growth opportunity at the price \( P_{G_0} \). Then he shorts \( X \) shares of the assets in place in period 1. The stakeholder will quit if \( P < P_c \). This condition is equivalent to the condition \( Q - X < Q_c \). The probability of this event occurring, denoted by \( p'' \), is

\[
p'' = N \left( \frac{Q_c + X}{[(k+1)v_z]^{\frac{1}{2}}} \right) ,
\]  

where we have used the result from (31) that \( \text{var}(Q) = (k+1)v_z \) under risk-neutrality of market makers, which follows by noting that \( Q \) equals the price divided by \( \zeta \) and dividing the expression in (31) by \( \zeta^2 \), where \( \zeta \) is given by (32).

The price impact of the trade in period 1 is given by \( E(\delta - \zeta(Q - X))(-X) = \zeta X^2 \). The net expected profit of the manipulator is then given by

\[
\pi_m = [p''(\bar{G} - Nr) + (1 - p'')\bar{G} - (\bar{G} - Npr)](-X) - \zeta X^2 = (p'' - p)NXr - \zeta X^2 ,
\]  

where \( \zeta \) is given by (39), \( p \) is given by (40), and \( p'' \) is given by (42). Note that \( r \) does not appear in the last term of the expression above. Further, \( p'' > p \) because \( p \) can be written as \( \left[ \frac{Q_c}{[(k+1)v_z]^{\frac{1}{2}}} \right] \) which is clearly less than the right-hand side of (42). Thus the
expression on the right-hand side of (43) is positive if \( r \) is sufficiently high. If the expected profits from manipulation are positive, and agents are not restricted from manipulation, it is evident that a candidate equilibrium without manipulation cannot be sustained.

To see how an equilibrium can be sustained under the market maker belief that there is no manipulator, note that the last expression in (43) is quadratic in \( X \). This implies that the optimal \( X \) and the corresponding expected profit to the manipulator, superscripted by *’s, can easily be shown to equal \( X^* = (p'' - p)r/(2\zeta) \) and \( \pi_m^* = (p'' - p)^2r^2/(2\zeta) \). Thus, if the manipulator is allowed to choose the quantity \( X \), there exists an optimal level of \( X \) under which his expected profit is always positive. This level of \( X \) describes the equilibrium level of manipulation. The market value of \( \zeta \) in this equilibrium remains equal to that given by (32). Since all variables that determine the optimal \( X \) are public knowledge, managers can take an opposing position and offset the manipulator’s trades. □

**Proof of Lemma 2:** Assume that each informed trader uses a strategy linear in \( \delta \) of the form \( \kappa \delta \). The informed trader maximizes

\[
E(x(\delta - \xi(\delta + \epsilon) - \zeta(x + (k-1)\beta\delta + z))|\delta),
\]

which implies that

\[
x = \frac{\delta(1 - \xi - \zeta(k-1)\kappa)}{2\zeta}.
\]

Thus, in the symmetric equilibrium

\[
\kappa = \frac{1 - \xi}{(k + 1)\zeta}.
\]

(44)

The price \( P = E(\delta|\delta + \epsilon, k\kappa\delta + z) \) Equating coefficients by explicitly calculating this conditional expectation, we have

\[
\xi = \frac{v_\delta v_z}{k^2\kappa^2 v_\delta v_\epsilon + v_z(v_\delta + v_\epsilon)},
\]

(45)

and

\[
\zeta = \frac{k\kappa v_\delta v_z}{k^2\kappa^2 v_\delta v_\epsilon + v_z(v_\delta + v_\epsilon)}.
\]

(46)

Substituting for \( \kappa \) from (44) into the equation for \( \zeta \), we find that

\[
\zeta^2 = \frac{k\kappa v_\delta(1 + k\xi)(1 - \xi)}{v_z(1 + k)^2(v_\delta + v_\epsilon)}.
\]

(47)
However, from (44), we also have
\[ \kappa^2 = \frac{(1 - \xi)^2}{\zeta^2(1 + k)^2}. \]
Substituting for \( \zeta^2 \) from (46) into the above expression and substituting the resulting expression for \( \kappa^2 \) into the expression for \( \xi \) in (45) yields
\[ \xi = \frac{v_b(1 + k\xi)}{(1 + k)(v_b + v_e)}, \]
from where we obtain (19). Substituting for \( \xi \) into (47) we obtain (20). \( \Box \)

**Proof of Proposition 6:** The probability of a cascade is monotonically related to \( \text{var}[E(\delta|\delta + \epsilon, k\kappa \delta + z)] \). Now,
\[ \text{var}[E(\delta|\delta + \epsilon, k\kappa \delta + z)] = \text{var}[\xi(\delta + \epsilon) + \zeta(k\kappa \delta + z)] = v_b(1 + k\zeta \kappa)^2 + \xi^2 v_e + \zeta^2 v_z. \]
Substituting for \( \xi \) and \( \zeta \) from (2), we have
\[ \text{var}[E(\delta|\delta + \epsilon, k\kappa \delta + z)] = \frac{v_b(kv_e + v_b)}{v_b + (k + 1)v_e}, \] which is monotonically decreasing in \( v_e \).

Now, the above case is that where the informed agents is held fixed. Under endogenous information acquisition one also has to account for the effect of \( v_e \) on the equilibrium number of informed agents.

The ex ante expected profits per informed agent, \( \pi \), are given by
\[ \pi = E[(\kappa \delta - \xi(\delta + \epsilon) - \zeta(k\kappa \delta + z)]], \]
and, after substituting for the equilibrium parameters from the proof of Lemma 2, can be shown to equal the following closed-form expression
\[ \pi = \frac{v_e}{v_b + (k + 1)v_e} \sqrt{\frac{v_b v_z}{k^2}}. \]

The derivative of the number of informed agents with respect to \( v_e \) is given by
\[ \frac{dk}{dv_e} = -\frac{\partial \pi/\partial v_e}{\partial \pi/\partial k}. \]
and equals
\[ \frac{2k\nu_\delta}{\nu_\delta[v_\delta + (3k + 1)v_\varepsilon]} \].

Now, the derivative of \( V \equiv \text{var}[E(\delta|\delta + \epsilon, k\kappa\delta + z)] \) with respect to \( v_\epsilon \) which is monotonically related to the probability of a cascade, equals
\[
\frac{dV}{dv_\epsilon} = \frac{\partial V}{\partial k} \frac{dk}{dv_\epsilon} + \frac{\partial V}{\partial v_\epsilon}.
\]

Substituting for the various derivatives, we have
\[
\frac{dV}{dv_\epsilon} = -\frac{v_\varepsilon^2}{[v_\delta + (k + 1)v_\epsilon][v_\delta + (3k + 1)v_\epsilon]},
\]
which is negative, completing the proof. \( \square \)

**Proof of Proposition 7:** If the condition in the proposition holds, then in period 0, the firm is in an equilibrium where none of the \( N \) stakeholders joins the firm. Consider two mutually exclusive possibilities at the time of trade in period 1. First consider the possibility that
\[
\rho_1(\bar{F} + E(\delta|P, \delta + \epsilon) + \rho_2(\bar{G} - (N - 1)r) < \bar{w}_N.
\]
In this case the firm remains in the bad equilibrium. If the reverse is true, then at least the stakeholder with reservation wage \( \bar{w}_N \) joins the firm. But if the conditions
\[
\rho_1(\bar{F} + E(\delta|P, \delta + \epsilon) + \rho_2(\bar{G} - (N - i)r) < \bar{w}_{N-i+1}
\]
hold for \( i = 2, \ldots, N \) then all others join the firm as well. Define \( \Delta w_j = \bar{w}_j - \bar{w}_{j-1} \) for \( j = 1, \ldots, N \). Then the above conditions are equivalent to the condition \( r > \Delta w_j \) for \( j = 1, \ldots, N \), which will be true if \( r \) is sufficiently high. The control variable for the firm manager is \( \tau_\epsilon \). The manager thus maximizes
\[
V_f(\tau_\epsilon) - C(\tau_\epsilon),
\]
where \( V_f \) is the value of the firm, and therefore sets \( V'_f(\tau_\epsilon) = C'(\tau_\epsilon) \). Using (1), it follows that
\[
V'_f(\tau_\epsilon) = \frac{Nr\phi(\beta/V)V''(\tau_\epsilon)}{V^2},
\]
where $\phi(\cdot)$ is the standard normal density evaluated at $\cdot$, $\beta$ is a positive constant independent of $\tau_c$. This implies that $V_f'(\tau_c)$ is non-negative. From (50), we have

$$V_f'(\tau_c) = \frac{v_b^2}{[\tau_c v_b + (k + 1)]v_b[3k + 1]}.$$ 

It follows then that $V_f'(0) > C_f'(0) = 0$ and $V_f'(\infty) = 0 < C_f'(\infty)$. Further, $V_f''(\tau_c) - C''(\tau_c) < 0$. This implies that the equation $V_f'(\tau_c) = C'(\tau_c)$ has a unique solution, which is a maximum of the function $V_f'(\tau_c) - C(\tau_c)$. The proposition thus follows. In the case of part 2 of the proposition, it follows that $V_f'(\tau_c)$ is nonpositive. Hence the optimal solution is to set $\tau_c = 0$. When $r = 0$, the firm value is always $F + \delta + G$ so the manager cannot influence ex ante firm value by changing $\tau_c$, hence the third part follows.

Assume alternatively that the control variable is the cost of information acquisition, as opposed to the precision of the public signal. Specifically, define $\alpha = 1/c$ and assume a cost function $C(\alpha)$ with $C'(\alpha) > 0$, $C''(\alpha) > 0$, $C'(0) = 0$ and $C'(\infty) = \infty$. This captures the notion that decreasing the cost of information acquisition by increasing analyst coverage, for example, requires an investment in effort at an increasing rate as the cost of information acquisition drops. Now, we have

$$\frac{dV}{dc} = \frac{\partial V}{\partial k} \frac{dk}{dc}, \quad (52)$$

From (49) and (48), and noting that

$$dk/dc = -\frac{\partial(\pi - c)/\partial c}{\partial(\pi - c)/\partial k},$$

the equation (52) reduces to

$$\frac{dV}{dc} = -\frac{2v_b^\frac{1}{2}v_k \frac{2}{k}}{v_b^\frac{1}{2}v_b + (3k + 1)v_k}.$$ 

Since $\alpha = 1/c$, from the above expression, it can easily be shown that $V'(\alpha) < 0$ and $V''(\alpha) < 0$. The rest of the proof that there exists a unique level of $\alpha$ that maximizes firm value when the condition in part 1 of Proposition 7 and that the optimal $\alpha = 0$ for the case in part 2 of Proposition 7 follows that in the case where $\tau_c$ is the control variable for the manager. □
Public Information and Market Maker Risk Aversion: A simple modification of the proof of Lemma 1 shows that the equations describing the equilibrium with risk aversion of market makers are as follows:

\[ \kappa = \frac{1 - \xi}{(k + 1)\zeta} \]

\[ \zeta = \frac{kkv_8v_e + (R/2)v_8v_e v_z}{k^2\kappa^2 v_8 v_e + v_z(v_8 + v_e)} \]

and

\[ \xi = \frac{v_8v_z}{k^2\kappa^2 v_8 v_e + v_z(v_8 + v_e)} \]

Closed-form solutions are not possible in this case.
References


Figure 1
Optimal Precision of Public Information Release vs. Degree of Complementarity

Figure 2
Optimal Precision of Public Information Release vs. Number of Stakeholders