Title
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EXTRACTION AND EXTENSION OF MATRICES IN COMPUTER CALCULATIONS

By

William Hadley Richardson

1. The problems involved here are: The extraction of a submatrix from a matrix; and the extension of a matrix to a matrix of larger dimensions with the original matrix as a given submatrix of the larger and the rest of the larger matrix composed of zero submatrices.

This is generally a trivial problem but of interest in computer calculations where submatrices must be separated from massive matrices which are stored in memory and matrix multiplication subroutines are available.

2. To extract a submatrix \( Y_{22} \) from a matrix \( Y = (Y_{ij}), i, j = (1,2,3) \) (see Figure 1), define a matrix \( X = (X_i) \) such that:

- \( X \) has the same number of rows as \( Y_{22} \).
- \( X_1 = 0 \) with same number of columns as \( Y_{12} \) has rows.
- \( X_2 = I \) with same number of rows and columns as \( Y_{22} \) has rows.
- \( X_3 = 0 \) with same number of columns as \( Y_{32} \) has rows.

Define a matrix \( Z = (Z_j) \) such that:

- \( Z \) has same number of columns as \( Y_{22} \).
- \( Z_1 = 0 \) with same number of rows as \( Y_{21} \) has columns.
- \( Z_2 = I \) with same number of rows and columns as \( Y_{22} \) has columns.
- \( Z_3 = 0 \) with same number of rows as \( Y_{23} \) has columns.

Then \( X Y Z = (Y_{22}) Z = Y_{22} \).
The method is completely general if it is considered that, when \( Y_{22} \) is a corner submatrix, certain of the \( Y_{ij} \) become empty matrices. For instance, if the submatrix, \( Y_{22} \), to be extracted is in the upper left-hand corner the \( Y_{11} \) and \( Y_{12} \) are empty matrices and in consequence \( X_1 \) and \( Z_1 \) are empty matrices.

3. Extending a matrix is a similar operation in reverse. To extend a matrix, \( Y_{22} \), to a larger matrix, \( \mathbf{Y} = (Y_{ij}), i,j = (1,2,3) \), which has the original matrix as a submatrix and the rest of the larger matrix zeros, (see Figure 2), define a matrix \( \mathbf{X} = (X_1) \) such that:

\[
\begin{align*}
X & \text{ has the same number of columns as } Y_{22} \text{ has rows.} \\
X_1 & = 0 \text{ with the same number of rows as } Y_{12}. \\
X_2 & = 1 \text{ with the same number of rows and columns as } Y_{22} \text{ has rows.} \\
X_3 & = 0 \text{ with the same number of rows as } Y_{32}.
\end{align*}
\]

Define a matrix \( \mathbf{Z} = \mathbf{Z}_j \) such that:

\[
\begin{align*}
Z & \text{ has same number of rows as } Y_{22} \text{ has columns.} \\
Z_1 & = 0 \text{ with same number of columns as } Y_{21}. \\
Z_2 & = 1 \text{ with same number of rows and columns as } Y_{22} \text{ has columns} \\
Z_3 & = 0 \text{ with same number of columns as } Y_{23}.
\end{align*}
\]

Then \( XY_{22}Z = (Y_{12})Z = (Y_{ij}) = \mathbf{Y} \).

The generality of method is as stated in paragraph 2 above.
4. Proofs:

4.1. Extractor: \( XYZ = Y_{22} \)

\[ XY = (XY_j) \]

\[ XY_1 = X_1 Y_{11} + X_2 Y_{21} + X_3 Y_{31} \]

\[ = OY_{11} + IY_{21} + OY_{31} \]

\[ = Y_{21} \]

\[ XY_2 = X_1 Y_{12} + X_2 Y_{22} + X_3 Y_{32} \]

\[ = OY_{12} + IY_{22} + OY_{32} \]

\[ = Y_{22} \]

\[ XY_3 = X_1 Y_{13} + X_2 Y_{23} + X_3 Y_{33} \]

\[ = OY_{13} + IY_{23} + OY_{33} \]

\[ = Y_{23} \]

So \( XY = (Y_{21}, Y_{22}, Y_{23}) = (Y_{2j}) \)

Then \( XYZ = (Y_{2j})Z \)

\[ XYZ = Y_{21} Z_1 + Y_{22} Z_2 + Y_{23} Z_3 \]

\[ = Y_{21} O + Y_{22} I + Y_{23} O \]

\[ = Y_{22} \]

Q.E.D.
4.2. Extender: \( XY_{22} Z = Y \)

\( XY = (XY_1) \)

\( XY_1 = X_1 Y_{22} = 0 Y_{22} = 0 \)

\( XY_2 = X_2 Y_{22} = Y_{22} = Y_{22} \)

\( XY_3 = X_3 Y_{22} = O Y_{22} = 0 \)

So \( XY = (0, Y_{22}, 0) = (Y_{12}) \)

Then \( XYZ = (Y_{12}) Z = (XYZ_{1}) \)

\( XYZ_11 = Y_{12} Z_1 = 0 0 = 0 \)

\( XYZ_{12} = Y_{12} Z_2 = 0 I = 0 \)

\( XYZ_{13} = Y_{12} Z_3 = 0 0 = 0 \)

\( XYZ_{21} = Y_{22} Z_1 = Y_{22} 0 = 0 \)

\( XYZ_{22} = Y_{22} Z_2 = Y_{22} I = Y_{22} \)

\( XYZ_{23} = Y_{22} Z_3 = Y_{22} 0 = 0 \)

\( XYZ_{31} = Y_{32} Z_1 = 0 0 = 0 \)

\( XYZ_{32} = Y_{31} Z_2 = 0 I = 0 \)

\( XYZ_{33} = Y_{31} Z_3 = 0 0 = 0 \)

And \( XYZ = \begin{pmatrix} 0 & 0 & 0 \\ 0 & Y_{22}^0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = Y \)

Q.E.D.
\[ \begin{align*}
X & = [0, 1, 0] \\
Y & = [1, 0, 1] \\
Z & = [0, 1, 0]
\end{align*} \]

\[ Y_2 \]

\[ (Y_2_{\text{j}}) \]

\[ \text{FIGURE 1} \]

\[ \text{EXTRACTION} \]
FIGURE 2

EXTENSION