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“Optical Guiding” Limits on Extraction Efficiencies of Single-Pass, Tapered Wiggler Amplifiers

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Abstract

Single-pass, tapered wiggler amplifiers have an attractive feature of being able, in theory at least, of extracting a large portion of the electron beam energy into light. In circumstances where an optical FEL wiggler length is significantly longer than the Rayleigh length $\xi_R$ corresponding to the electron beam radius, diffusion losses must be controlled via the phenomenon of optical guiding. Since the strength of the guiding depends upon the effective refractive index $n$ exceeding one, and since $(n-1)$ is inversely proportional to the optical electric field, there is a natural limiting mechanism to the on-axis field strength and thus the rate at which energy may be extracted from the electron beam. In particular, the extraction efficiency for a prebunched beam asymptotically grows linearly with $z$ rather than quadratically. We present analytical and numerical simulation results concerning this behavior and discuss its applicability to various FEL designs including oscillator/amplifier-radiator configurations.

1 Introduction

For over a decade (see, e.g. [1]), single-pass, tapered wiggler free-electron laser (FEL) amplifiers have been suggested as a means to obtain much higher extraction efficiencies of electron beam power into laser light than is normally possible in simple oscillator configurations. A key aspect of the tapered wiggler amplifier is the phenomenon of "optical guiding"[3] which permits both the optical gain length and total wiggler length to be many times that of the optical Rayleigh length. The guiding is caused by the bunched electron beam having an effective refractive index $n$ exceeding one and thus acting as an optical fiber. Experimentally, “gain guiding” was observed in the LLNL Paladin experiment[4] but the electron beam brightness was insufficient to permit meaningful tapering experiments.

While doing extensive modeling in the mid-1980’s for the (then) upcoming Paladin experiment, I (and undoubtedly others) noticed that, well into the saturated gain regime of an FEL amplifier, the optical power grew approximately linearly with $z$, and since $(n-1)$ is inversely proportional to the optical electric field, there is a natural limiting mechanism to the on-axis field strength and thus the rate at which energy may be extracted from the electron beam. In particular, the extraction efficiency for a prepunched beam approximately grows linearly with $z$ rather than quadratically. We present analytical and numerical simulation results concerning this behavior and discuss its applicability to various FEL designs including oscillator/amplifier-radiator configurations.

2 Theoretical Analysis

I first adapt Colson’s normalized FEL parameters to amplifier configurations and the apply them to determine the limits of optical guiding in the saturated gain regime.

2.1 Normalized Variables

Colson [2] introduced a set of normalized quantities for analysis of oscillator FEL’s; with minor adaptation, they also prove useful for analysis of single-pass amplifiers. The normalized, complex RMS electric field $a$ and the normalized current density $j$ may be defined as

$$ a \equiv \frac{2a_{\omega} f_B k_{\omega} L^3}{\gamma_0^2} e \bar{E} $$

$$ j \equiv \frac{8 \pi a_{\omega} f_B^2 (\epsilon a_{\omega} f_B)^2}{\lambda_0 \gamma_0^2 \gamma_0^2 m c^2} = \frac{k_{\omega} L^3 \omega_0^2 a_{\omega}^2 f_B^2}{\gamma_0^2 \epsilon^2} $$

Here $\omega_p$ is the on-axis electron plasma frequency, $a_{\omega}$ is the normalized rms vector potential, $\gamma_0$ is the beam’s initial Lorentz factor, and $f_B$ is the Bessel function difference coupling term for a linearly polarized undulator, and $L$ is a scaling length to be defined below.

Letting $z = z/L$, the FEL field equation in the slowly-varying envelope approximation may be rewritten as

$$ \frac{\partial a}{\partial z} = j (\zeta \sin \theta + i \zeta \cos \theta) + i \frac{L \nabla^2 a}{2k_\omega} $$

where the brackets represent averaging over the particle phases $\theta$ (measured relative to that of a plane wave), $k_\omega$ is the radiation wavenumber, and $\zeta \equiv a_{\omega} f_B \gamma_0^2/d_{\omega}^2 f_B^2 \gamma_0^2$ where the “$o$” refers to the quantity’s initial value. For $a_{\omega} \geq 2$, $\zeta$ varies little from one for trapped particles and for simplicity I drop it in the remainder of the analysis in this section (the particle simulations described in §3 include it implicitly).

Recognizing that $j \propto \rho^2 \equiv \omega_0^2 a_{\omega}^2 f_B^2 / 16 \gamma_0^2 k_\omega^2 \epsilon^2$ where $\rho$ is the Pierce parameter, we judiciously choose $L \equiv \lambda_0 / 4\pi \rho$, which is (approximately) the exponential growth length for the optical electric field. Note that with this particular definition of...
At this point, where, using relation (1), \( |a| = 2 \). To increase the laser power significantly beyond this level, the wiggler must be tapered to reduce the resonant energy \( \gamma_r \) with \( z \). Tapering will work well, however, only if the diffractive losses are not extreme. Following the analysis presented in Scharlemann, Sessler, and Wurtele[3], we expect strong, refractive guiding in both the exponential and saturated gain regions if the “fiber parameter”

\[
V^2 \equiv (n^2 - 1) k^2 R^2
\]

is of order 1 or greater where \( R \) is, approximately, the \( 1/e \) point in a Gaussian profile electron beam or the HWHM in a parabolic profile. From relation (3), the real part of \( n \) is given by

\[
\text{Re}(n) - 1 = \frac{j}{k_R L |a|} \langle \cos \psi \rangle
\]

and, for the usual case of \(( n - 1) \) small, one finds

\[
V^2 = 4 \frac{j}{|a|} \frac{k_R r^2}{2L} \langle \cos \psi \rangle
\]

where \( \psi_i \equiv \theta_i + \phi \), measures the particle longitudinal phase relative to that of the ponderomotive well. With \( j = 2 \) and \( z_R \equiv k_R r^2/2 \) and reasonable values of \( \langle \cos \psi \rangle > 0.5 \) and \( z_R/L > 1 \), optical guiding is strong at the beginning of the saturated gain regime permitting \( |a| \) to grow linearly with \( z \).

Eventually though, when \( |a| \) approaches

\[
a^* \equiv 4 j \langle \cos \psi \rangle \frac{z_R}{L}
\]

\( V^2 \) becomes sufficiently small that optical guiding “fails”, and significant radiation begins to leak transversely beyond \( r = r_s \). At this point, \( |a| \) stays nearly constant with \( z \) and, since the particle deceleration is directly proportional to \( |a| \langle \sin \psi \rangle \), the total power grows linearly with \( z \).

With \( |a| \) constant and assuming constant values of \( \langle \sin \psi \rangle \) and \( \langle \cos \psi \rangle \), it is easy to estimate an upper bound to the energy extraction in the saturated gain regime. Denoting \( \Delta \gamma \) as the mean reduction in the beam energy,

\[
\frac{d}{dz} \left( \frac{\Delta \gamma}{\gamma_0} \right) = -\frac{1}{4\pi} |a| \frac{\lambda_w}{L} \langle \sin \psi \rangle
\]
confine the radiation mode as is usually true for microwave FEL amplifiers, optical guiding physics is no longer critical and the energy extraction rate should be less sensitive to bunching fraction. Although a cursory glance at eq. (9) would suggest (for a constant $b$) that the extraction rate would improve if the beam radius and hence $z_R$ increased, since $\rho^3 \propto n_e \propto 1/r^2_c$, to lowest order this is not so and in general decreasing $\rho$ both increases the gain length in the exponential gain regime and makes the effective energy spread due to emittance even worse. Consequently, as many experimentalists know from painful experience, it is always best to optimize beam quality, even if it means trading off a bit of peak beam current.

## References


