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W.J. Swiatecki

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MACROSCOPIC TREATMENT OF NUCLEAR DYNAMICS*

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A qualitative classification of nucleus-nucleus reactions into four types is described, a consequence of the existence of up to three "milestone configurations" that a fusing system may be faced with. These considerations lead to phenomenological formulae for fusion and compound-nucleus cross-sections that may be compared with experiments by the use of "rectilinear cross-section plots." Examples of more specific model calculations of nuclear reactions employing the "Chaotic Regime Dynamics" are described. Some misunderstandings regarding the Wall and Wall-and-Window formulae, underlying this type of dynamics, are discussed in the appendix.

1. INTRODUCTION

We have been urged by the conference organizers to make our presentations understandable to a wide audience and to emphasize the connection between theories and experiments. I will accordingly devote the first half of my talk to some very simple considerations concerning nucleus-nucleus reactions. I will inject the physical ideas one by one and derive the resulting cross-section formulae in a form especially suited to the presentation of empirical data.

The conference is also dedicated to discussing recent theoretical developments and new ideas. In the second part of my talk I will thus mention two recent developments in the more specific domain of macroscopic nuclear dynamics based on the use of the so-called one-body dissipation formulae (the "Chaotic Regime Dynamics"). I will also attempt to clarify, in the appendix, some misunderstandings that seem to have arisen in the past few years in connection with that theory.

The outline of my talk is thus as follows:

Part I, Phenomenological

***Qualitative classification of nucleus-nucleus reactions. (Underlying physics: common sense.)

***Rectilinear cross-section plots. (Physics: centrifugal force is proportional to the square of the angular momentum.)

***"Extra-push" formulae. (Physics: nuclei have Coulomb and surface energies;
nuclear dynamics is dominated by "one-body" dissipation.)

Part II, Chaotic Regime Dynamics

***Examples of numerical calculations of nucleus-nucleus reaction dynamics.
***A new term in the Wall-and-Window dissipation formula, dealing with the mass-asymmetry degree of freedom.
***A miracle when "Fermi Jets" are treated according to the Wall-and-Window formula.

Appendix, Some misunderstandings

***The Pauli exclusion principle and Liouville's theorem.
***Self-consistent drifts and the conservation of linear and angular momenta.

2. QUALITATIVE CLASSIFICATION OF NUCLEUS-NUCLEUS REACTIONS

Think of two nuclei coming together in a nuclear reaction. On qualitative grounds one may expect the existence of up to three milestones in the dynamical evolution of the reaction. The first milestone is obviously the attainment of contact. Up to about the contact point the nuclei can, for rough estimates, be idealized as rigid and the essence of the problem is, approximately, one-dimensional. The separation between the mass centers, \( r \), is the single dominant degree of freedom. In this one-dimensional configuration space the potential energy goes up as \( r \) decreases (Coulomb repulsion). Around contact the nuclear attraction usually overcomes the electric repulsion, producing a maximum in the potential energy—the Coulomb barrier (or contact barrier). In any case the energy needed to achieve contact is the first hurdle that must be overcome in a nuclear reaction.

After contact, the freezing out of all degrees of freedom except the separation \( r \) cannot continue to be a good approximation. In particular, the size of the neck can be expected to be an essential degree of freedom. The shape degrees of freedom of the fragments will also become more important. The only degree of freedom that may, for a while, continue to be approximately frozen—so long as the neck is small—is the mass-asymmetry degree of freedom, describing the relative sizes of the two pieces. This is because it is, in general, difficult to push matter through a small neck. So now we have to think in terms of a dynamical problem in (\( N-1 \)) dimensions, where \( N \) is the total number of relevant degrees of freedom, and the 1 represents the frozen-out asymmetry degree of freedom. The distance coordinate \( r \) is now merely a one-dimensional sub-space of the larger (\( N-1 \)) dimensional space. With all these additional degrees of freedom unfrozen, the contact configuration of rigid fragments (and the associated Coulomb barrier) no longer plays any special role (except as the approximate injection point into the (\( N-1 \)) dimensional space). What has replaced the Coulomb barrier as a "go-or-no-go" configuration is the conditional
**saddle-point configuration** in the \((N-1)\) dimensional space. It is a configuration corresponding to a saddle point in the \((N-1)\) dimensional potential energy hyper-surface. It has the significance of a potential energy barrier separating the configuration of two fragments of given asymmetry from the configuration of a fused, undifferentiated blob, the "mononucleus." This conditional saddle-point is the second milestone configuration that a fusing system may be faced with. I say "may," because for some systems (the lighter ones) overcoming the first hurdle automatically guarantees the overcoming of the second. But for heavier systems that is not so, and one has to give the colliding nuclei an "extra push" over and above the Coulomb barrier in order to send the system over the conditional saddle point.

For systems that have overcome the conditional saddle, the neck will usually continue to grow and, as a result, the asymmetry degree of freedom will also become unfrozen eventually. One is then faced with the full dynamical problem in \(N\) dimensions. The conditional saddle point in the \((N-1)\) dimensional subspace loses its significance. The "go-or-no-go" configuration determining the further evolution of the system is now the **unconditional saddle point.** This is nothing else than the familiar configuration determining the potential energy barrier of a compound nucleus against fission. In the context of a nucleus-nucleus reaction, this saddle-point configuration is the third hurdle that the system may encounter on its way to compound-nucleus formation. Again it is "may" because overcoming the first two hurdles may or may not lead to an automatic overcoming of the third. If not, then an "extra-extra push" will be needed to make a compound nucleus.

Figure 1, taken from ref. 1, illustrates schematically the consequences of these qualitative considerations. The existence of up to three hurdles results in a classification of nucleus-nucleus reactions into four broad categories, shown in the figure. Also, two kinds of extra push are anticipated: one to make two nuclei fuse into a mononucleus (an undifferentiated blob) and one to make a compound nucleus (the stable equilibrium configuration of the compound system).

### 3. RECTILINEAR CROSS-SECTION PLOTS

I will now derive some cross-section formulae that follow from the above qualitative considerations. Let me start by reminding you of the derivation of the standard formula for the reaction cross-section (the cross-section to make nuclei touch), viz.:

\[
\sigma = \pi r^2 \left(1 - \frac{B}{E}\right),
\]  

(1)
BOMBARDING ENERGY

Compound Nucleus Reactions

III ENERGY TO OVERCOME UNCONDITIONAL SADDLE

Mononucleus (Fast Fission) Reactions

II ENERGY TO OVERCOME CONDITIONAL SADDLE

Dinucleus (Deep - inelastic) Reactions

I ENERGY NEEDED TO MAKE CONTACT

Binary \{ Quasi - Elastic \} Reactions

FIGURE 1

Schematic illustration of the relation between three critical energies, four types of nuclear reactions, and two kinds of extra push. This figure is appropriate when the three milestone configurations discussed in the text exist and are distinct. In some situations the critical energies I, II, III may merge (pairwise or all together) squeezing out the regimes corresponding to dinucleus and/or mononucleus reactions. In other situations one or both of the upper boundaries (II and III) may dissolve, making the adjoining regions merge into continuously graduated reaction types.

where $E$ is the bombarding energy, $r$ is now the nuclear center separation at contact and $B$ is the contact energy (Coulomb barrier). To prove eq.(1) you just write down the energy conservation equation at contact

$$E = B + E_t + E_r$$

where $E_r$ is the energy in the radial motion and $E_t$ the tangential energy (at contact). In a head-on collision, $E_t = 0$ and $E_r = E - B$. As the impact parameter increases, $E_t$ will increase and $E_r$ will decrease. At some maximum impact parameter $b$, $E_r$ will be zero (grazing collision) and beyond that the colliding nuclei do not touch. The cross-section for touching is then

$$\sigma = \pi b^2$$

and since $E_t$ is related to the angular momentum $L$ and the impact parameter $b$ by
where $M$ is the reduced mass, we have (using eq. (2) with $E_r = 0$)

$$
\sigma = \frac{\pi r^2 E_t}{E} = \pi r^2 \left(1 - \frac{B}{E}\right)
$$

Consider now the cross-section for a process of fusion that requires an extra push—an extra radial velocity—for its initiation. Part of this extra radial velocity may be needed because of an adverse balance of Coulomb and nuclear forces and part because of the presence of a centrifugal force. If the latter is taken to be proportional to the square of the angular momentum, we may write the extra push velocity, $u_r$, as

$$
u_r \propto \sqrt{E_r} = \alpha + \text{(constant)} L^2
$$

or, using eqs. (4) and (5), as

$$
\sqrt{E_r} = \alpha + \beta \frac{\sigma E}{\pi r^2}
$$

Here $\alpha$ and $\beta$ are constants for a given reacting system, $\alpha$ being a measure of the excess of Coulomb over nuclear forces and $\beta$ a factor in the centrifugal force. Inserting eq. (6) into eq. (2) and using again eq. (5) we have an equation relating implicitly the fusion cross-section $\sigma$ to the center of mass energy $E$:

$$
\sqrt{E - B - \sigma E} = \alpha + \beta \frac{\sigma E}{\pi r^2}
$$

[You may ask: why did I write the extra push velocity $u_r$, rather than the extra push energy $E_r$, as proportional to $\alpha + \beta \frac{\sigma E}{\pi r^2}$? It is a symmetry argument that can be appreciated by an analogy. If you're shooting at a target (the conditional saddle) in the presence of a cross wind (the sum of the Coulomb, nuclear and centrifugal forces in the radial direction) then the correction to your aiming angle is proportional to the wind velocity. (If the wind velocity changes sign so does the correction angle). Now the correction angle is proportional to the velocity component imparted to the bullet in the wind direction (the "radial" direction), so it is the extra velocity and not the extra energy that tracks with the wind velocity.)

Equation (7) may be solved explicitly for $\sigma$, giving
\[ \sigma = \frac{\pi r^2}{E} \left[ \sqrt{\left(\frac{\alpha \beta + 1/2}{\beta^2}\right)^2 - \left(\frac{\alpha^2 + B - E}{\beta^2}\right)} - \left(\frac{\alpha \beta + 1/2}{\beta^2}\right) \right] \] (8)

This is a generalization of the standard equation (1), to which it reduces when \( \alpha = \beta = 0 \) (no extra push). From an experimental point of view, the implicit eq. (7) may be actually more convenient than eq. (8). Just plot the experimentally determined quantity \( \sqrt{E - B - \frac{\sigma E}{\pi r^2}} \) against \( \sigma E/\pi r^2 \). Theory predicts a straight line, with slope \( \alpha \) and intercept \( \alpha \). Note that the physical input into the theory is merely that fusion needs an extra radial velocity push over the Coulomb barrier, the push being in part due to a centrifugal force, proportional to \( L^2 \). This is not much of an assumption, so the resulting formula might be valid for a wide variety of theories. The differences in the theories would show up in the numerical values of \( \alpha \) and \( \beta \), but for many theories the functional form of eq. (7) or (8) is expected to be the same.

It turns out that the quantities

\[ Y \equiv \frac{\sigma E}{\pi r^2} \] (the "energy-weighted reduced cross-section") (9)
\[ X \equiv \sqrt{E - B - \frac{\sigma E}{\pi r^2}} \] (the square root of the "cross-section defect") (10)

have nice properties in a wider context than the analysis of fusion cross-sections. Thus, predicted compound nucleus cross-sections limited by an extra-extra push should also be straight lines in a plot of \( Y \) vs. \( X \). In addition, compound nucleus cross-sections limited by the vanishing of the fission barrier of a rotating nucleus, as well as evaporation residue cross-sections limited by the decrease of the fission barrier below the neutron binding energy, are all predicted to result in straight-line plots of \( Y \) vs. \( X \). For details and examples of comparisons with data (not comprehensive) see refs. 1, 2, 3, 4

4. THE EXTRA-PUSH FORMULAE

So far the only physics that showed up in the algebra was the recognition of the presence of a centrifugal force proportional to the square of the angular momentum. Let us now inject the following additional pieces of physics:

a) The electric repulsion between two fragments near contact is approximately \( Z_1 Z_2 e^2/r^2 \).

b) The nuclear attraction is related to the nuclear surface-energy coefficient \( \gamma \) and, near contact, is proportional to the quantity \( 4\pi \gamma \mathcal{R} \), where \( \mathcal{R} = R_1 R_2/(R_1 + R_2) \). (Nuclear Proximity Force Theorem).5,6)

c) The nuclear dynamics is dominated by "one-body dissipation" (the kind of dissipation expected to be relevant, under certain conditions, for an ap-
proximately independent-particle system like a nucleus\textsuperscript{7,8,9,10,11}).

d) The centrifugal force is approximately proportional to the square of a fraction, $f_L$, of the total angular momentum, divided by the "mass points" of inertia of the nuclear configuration at contact.

From now on it is an exercise in dimensional analysis. The assumptions a) and b) predict that a relevant dimensionless parameter measuring the relative strengths of electric and nuclear forces may be taken as the "effective fissility parameter" $x_e$ defined by

$$x_e = \frac{(Z^2/A)_{\text{eff}}}{(Z^2/A)_{\text{crit}}}$$

where

$$\left(\frac{Z^2}{A}\right)_{\text{eff}} = \frac{4Z_1^2Z_2^2}{A_1^{1/3}A_2^{1/3}(A_1^{1/3} + A_2^{1/3})}$$

and

$$\left(\frac{Z^2}{A}\right)_{\text{crit}} = \frac{40\pi r_0^3\gamma}{3e^2}$$

$$= \frac{40\pi r_0^3\gamma_0(1 - \kappa_s I^2)}{3e^2}$$

(Here $r_0$ is the nuclear radius constant, $\gamma_0$ is the surface energy coefficient of standard nuclear matter and $\kappa_s$ is the surface symmetry coefficient, describing the dependence of $\gamma$ on the relative neutron excess $I \equiv (N - Z)/A$. Assumptions c) and d) predict that, for systems near contact, a natural unit of velocity in the dynamical equations of motion is the quantity $2\gamma/\rho v R$, where $\rho$ is the nuclear mass density and $v$ the mean nucleonic speed. (The "flux factor" $\rho v$ is a characteristic quantity of the one-body dissipation mechanism. For a nuclear Fermi gas one finds

$$\rho v = \frac{27}{32\pi} \left(\frac{\pi}{3}\right)^{1/3} \frac{h}{r_0^4}$$

(14)

For a head-on collision it should therefore be possible to write the extra-push velocity, $-dr/dt$, in the form

$$\frac{-dr/dt}{2\gamma/\rho v R} = \phi (x_e - x_{th})$$

$$\approx a(x_e - x_{th}) + \text{higher powers of } (x_e - x_{th})$$

(15)

In the above, $\phi$ is a universal dimensionless function of the excess of $x_e$ over a universal dimensionless threshold value $x_{th}$. In eq.(15), $\phi$ has been expanded in powers of $x_e - x_{th}$, and $a$ is thus the derivative of $\phi$, another universal
pure number.
Converting the extra-push velocity into an extra-push energy $E_x$, one finds, for head-on collisions,

$$E_x = \frac{1}{2} M \left( \frac{dr}{dt} \right)^2$$

$$= \frac{32}{2025} \left( \frac{3}{\pi} \right)^{2/3} \frac{e^4 m}{\hbar^2} \frac{A_1^{1/3} A_2^{1/3} (A_1^{1/3} + A_2^{1/3})^2}{A} \left[ \left( \frac{2}{A} \right)^{1/3} \frac{r_m}{A_1^{1/3} A_2^{1/3}} \right]^2 .$$

(16)

Here $A = A_1 + A_2$, $m$ is the nucleon mass and $b$ is a pure number, related to $x_{th}$ by

$$b = \frac{40 \pi \gamma_0 r_0^3}{3 e^2} x_{th} .$$

(17)

Eq.(16) is the theoretical prediction of how the quantity $a$ in eq.(8) is expected to vary with $A_1$, $A_2$, $Z_1$, $Z_2$ as one goes from one colliding system to another.

Throwing in the assumption d), one may similarly write down the expected behavior of $b$, required for the discussion of non-central collisions and for the prediction of cross-sections (eq.(8)). One finds

$$b = \frac{32 \sqrt{2}}{45} \left( \frac{3}{\pi} \right)^{1/3} \frac{r_m^{1/3} (A_1^{1/3} + A_2^{1/3})}{\hbar} \frac{f^2 a}{A_1^{1/6} A_2^{1/6} \sqrt{A}} .$$

(18)

So, having injected the four physical assumptions a)-d), one now has a more specific set of cross-section formulae to compare with experiment. The formulae contain three parameters: the threshold parameter $x_{th}$ (or $b$), the slope parameter $a$ and the angular momentum fraction $f$. Fig. 2 shows one of the early comparisons of the predictions with data on fusion cross-sections, from which the following values of the parameters were deduced graphically (by using rectilinear cross-section plots, as explained in section 3);

$$x_{th} = 0.70 \pm 0.02 \ (b = 35.6)$$

$$a = 12 \pm 2$$

$$f = 0.75 \pm 10\% .$$

(19)

Additional such confrontations of theory and experiment are accumulating.
Examples of six reactions, discussed in ref. 1, showing experimental fusion cross sections as a function of center of mass energy. The dashed curve shows the theoretical prediction (eq. (8)). The meaning of the other curves is explained in ref. 1.

There are also related comparisons of the extra push energy $E_X$, as given by eq. (16), with values deduced from measured evaporation-residue cross-sections. 1,13 (Fig. 3) The time may soon be ripe for a comprehensive survey and assessment of where we stand today in such comparisons. As I indicated, the theory contains now the assumption of the one-body dissipation dynamics [item c) above], and a careful confrontation with experiments might begin to shed light on this interesting aspect of nuclear physics. It is also true, however, that items a), b) and d) are rather general and are expected to be common to many different types of theories, all which might lead to similar, even if not identical, functional forms for the quantities $\alpha, \beta$. It may well be that rather precise and comprehensive comparisons of theory and experiment will be needed to extract the really interesting features of nuclear dynamics.
A comparison of experimentally deduced values of the extra-push energy with the trend of the theoretical prediction (approximately eq.(16)). The data are from ref.13.

Another way to make progress is to ask the dynamical theory to predict not only the functional form of the cross-sections but also the absolute values of the parameters $x_{th}$, $a$ and $f$. An example of some early theoretical estimates, using the model I will describe in the next section, gave the following results:

$$x_{th} \approx 0.723$$
$$a \approx 18$$

A theoretical estimate of $f$ in ref.23 gave

$$f \approx 0.85$$

The discrepancy in the value of $a$ may be significant and might eventually throw light either on the limitations of the one-body dissipation dynamics, or on more technical idealizations of the model used to implement the theory.
5. NUMERICAL STUDIES OF REACTION DYNAMICS

In the past several years there have been a number of numerical computer studies of nucleus-nucleus reactions and of fission, based on the Chaotic Regime Dynamics.\textsuperscript{7,10,14,15,16} This is a form of nuclear dynamics, believed to be appropriate for a system composed of approximately independent particles, in the regime of conditions when the particle motions can be considered as chaotic.\textsuperscript{11} The opposite extreme, the Ordered Regime, is characterized by regular particle orbits, usually leading to strong shell effects. In the Chaotic Regime, the nuclear energy is well approximated by a liquid-drop type of expression, and the dissipative forces are believed to be given by the so-called Wall Formula (for convex, compact shapes) and the Wall-and-Window Formula (for strongly necked-in shapes). Inertial effects are often negligible and shell effects are unimportant, so that all properties should be smooth functions of shape and of nuclear type. The Ordered Regime is more complex: both the potential energy and the inertial and dissipative effects are dominated by symmetries (resulting in shell effects) and nuclei are expected to exhibit complex visco-elastic properties (and sometimes superfluidity). The conditions favouring the validity of the simpler Chaotic Regime dynamics are: irregular nuclear shapes to break down symmetries, assisted by a certain amount of residual interactions between the nucleons, and some excitation energy (to break up pairing effects and reduce shell effects). The precise way in which the Ordered Regime goes over into the Chaotic Regime with the destruction of nuclear symmetries is largely an open question at this time. Rapid progress is being made in the theories of the usually abrupt transition from order to chaos in dynamical systems in general, and it is likely that also in the nuclear case this transition will turn out to be relatively abrupt.

As I mentioned, the physical ingredients of the Chaotic Regime Dynamics are typically as follows:

1. Conservative forces derived from a liquid-drop type of potential energy.
2. Dissipative forces described by the Wall and Wall-and-Window formulae.
3. A simple approximation to inertial forces, whose finer features are usually obliterated anyway by dissipative forces. (The so-called Werner-Wheeler approximation to irrotational mass flow is often used.)

The nuclear shapes are usually parameterized in a suitable way, for example as spheres (or spheroids) connected by a quadratic surface of revolution (Fig. 4). In the calculation done in collaboration with J. Blocki,\textsuperscript{16} which I will describe, there are three degrees of freedom specifying the nuclear configuration:
The parameterization of nuclear shapes by two spheres and a third quadratic surface of revolution. This parameterization is used in the comprehensive atlas of nuclear deformation energies in ref. 17 and in current reaction studies based on the Chaotic Regime Dynamics.

1. An asymmetry variable \( \Delta \equiv \frac{R_1 - R_2}{R_1 + R_2} \)

2. A distance variable \( \rho \equiv \frac{r}{R_1 + R_2} \)

3. A "deck" variable \( \lambda \equiv \frac{\ell}{R_1 + R_2} \)

("Deck" stands for "deformation/neck"). Here \( R_1, R_2 \) are the radii of the two spheres and \( \ell \) is the combined thickness of the spherical lenses covered up by the middle quadratic surface of revolution—it is a convenient measure of the deviation of the actual shape from two unconnected, undeformed spheres. The configuration space \( \Delta, \rho, \lambda \) is shown in Fig. 5. Focus first on the vertical section at \( \Delta = 0 \), corresponding to reflection symmetric shapes. Along the bottom edge you have approaching (equal) spheres, which touch at the point \( \rho = 1 \). Along the line inclined at 45° you have non-overlapping portions of intersecting spheres, ending up as a single sphere (properly renormalized, of course) at \( \rho = 0, \lambda = 1 \). As you move to the right along the upper edge you have elongating spheroids. Along the dashed horizontal line at \( \lambda = 1 \) you have elongating cylinders (with hemispherical ends). Above this line you have convex shapes, below the line necked-in shapes. Along the curve labeled "scission" the area of the neck has gone to zero percent of its maximum degree of opening (which is 100% for the cylinder).

Figure 5 also shows sections of the configuration space at \( \Delta = 0.2, 0.4, \ldots \)
The configuration space of the deformation variables $\Delta, \rho, \lambda$, explained in the text.

0.6, 0.8, 1.0. At each asymmetry, the shapes are similar to those I described for $\Delta = 0$, except that one side of the configuration is blown up and the others shrunk. (For example, in place of a cylinder with hemispherical caps one has a portion of a cone capped with portions of unequal spheres). The value of $\rho$ extends to infinity on the right, but the other boundaries in Fig. 5, shown as thicker curves for each asymmetry, are significant: shapes beyond those boundaries are either unphysical or redundant in our parameterization. So our configuration space is confined to a semi-infinite box, with flat bottom and sides and a somewhat peculiarly shaped top. (Only half of the box is shown in Fig. 5. The other half, for negative $\Delta$, is obtained by reflection in the plane $\Delta = 0$). This way of parameterizing nuclear shapes is taken from ref. 17, where comprehensive potential energy maps of nuclear deformation energies are presented.

A dynamical calculation of two colliding nuclei corresponds to a trajectory in the configuration space of Fig. 5, which enters from the right along the bottom of the box, and runs along a line corresponding to the initial asymmetry. When the line labeled "TANGENT SPHERES" is reached, the neck ($\lambda$) and asymmetry ($\Delta$) degrees of freedom begin to change, $\lambda$ very rapidly and $\Delta$ very slowly at first. The trajectory becomes a curve in three dimensions and sometimes bends back and re-emerges on the right as two separating, deformed fragments, and other times bends over to the left and tends to the spherical (compound nucleus) shape. An example of the kind of potential-energy landscape that such a trajectory traverses is shown in Fig. 6, which refers to a section at fixed
An example of a potential energy map, taken from the atlas in ref. 17. The map represents approximately the deformation energy for a colliding and fusing system of $^{208}$Pb and $^{52}$Cr at fixed asymmetry $\Delta$. The energy contours show the deformation energy with respect to the sphere, corresponding to the tip on the left. The spacing between contours is 0.002 times the surface energy of the sphere (695.85 MeV), i.e., about 1.4 MeV. The spherical shape is separated from the two-fragment valley on the lower right by a flat saddle-point structure (with a height of the order of 1 MeV) in the upper left-hand part.

asymmetry corresponding approximately to the system $^{208}$Pb + $^{52}$Cr. In cases when the nucleus re-separates, one finds that sometimes the asymmetry is very similar to the initial one, and this corresponds to deep inelastic scattering, associated with trajectories that did not overcome the conditional saddle. In other cases one observes re-separating fragments with an asymmetry very different from the initial one, ranging practically to symmetry. These reactions are of the "fast-fission" type, in which the conditional saddle was overcome but not the unconditional one. These calculations are currently in progress. Table I shows a sample of results, taken from ref. 16, concerning a few symmetric and asymmetric head-on reactions (specified in the first two columns). The collision energies (above the Coulomb barrier) are specified in the third column. The fourth column gives the "sticking time" (the time from contact to
scission). The fifth and sixth columns give the final fragment masses. The seventh column gives the Coulomb interaction energy between the fragments and the eighth column the translational energy of the fragments at scission. The sum, shown in the last column, is an estimate of the fragments' kinetic energy at infinity.

**Table I**

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There are many features of these calculations that remain to be studied thoroughly and confronted with experiment. I will only draw your attention to one, namely that in the asymmetric reactions there is, as a rule, a fair degree of preservation of the initial asymmetry, of the kind seen in deep inelastic reactions. (The case of \(A_1 = 248\), \(A_2 = 48\) is an exception.) That may seem trivial to you but, in fact, it is a feature that dynamical calculations of the type I have described were completely unable to reproduce until quite recently. It was only last year that we realized\(^{18,19}\) that the Window Formula for dissipation, as used up to then, was incomplete and should have in it an addition-
al term. (Without this term the final asymmetries in most of the reactions in Table I would be practically zero.) If further numerical studies bear this out, then the new term is really the underlying reason for the very existence of deep inelastic reactions in dynamical calculations using the Chaotic Regime Dynamics. I will now describe this new term.

6. THE DISSIPATIVE RESISTANCE AGAINST CHANGES IN MASS ASYMMETRY

Suppose you have two idealized nuclei (with volumes \( V_1, V_2 \) and mass numbers \( A_1, A_2 \)) which are in relative motion and which communicate through a small window of area \( a \). The rate of dissipation of the collective energy when the fragments are assumed to be rigid is given by the Window Formula, and if they are deformable, by the Wall-and-Window formula. But what if, in addition to a relative motion and fragment deformations, the asymmetry degree of freedom is also changing, i.e. if \( \dot{A}_1 \) or \( \dot{V}_1 \) are non-zero? What is the rate of dissipation of energy associated with this degree of freedom? The answer may be written as follows:

\[
- \frac{dE}{dt} = \frac{16 \rho v \dot{V}_1^2}{9 a V_1^2},
\]

where \( \rho v \) is the flux factor already mentioned in section 4 and \( \dot{V}_1 \) is the rate of change of the volume of one of the fragments. The essence of the derivation is as follows: Start with the identities

\[
- \frac{dE}{dt} = \left( - \frac{dE}{dA_1} \right) \frac{dA_1}{dt} = \left[ \left( - \frac{dE}{dA_1} \right) / \frac{dA_1}{dt} \right] \dot{A}_1^2 = \left( \frac{\rho}{m} \right) ^2 \left( - \frac{dE}{dA_1} \right) / \frac{dA_1}{dt} \dot{V}_1^2,
\]

intended to factor out the square of the rate of change of the (volume) asymmetry degree of freedom \( \dot{V}_1 \), to which the rate of change of energy loss, \(-dE/dt\), ought to be proportional in a dissipative process. (Here \( m \) is the nucleon mass.) The cofactor of \( \dot{V}_1^2 \) in eq.(21) is evaluated by considering the case of two containers filled with Fermi gases of nucleons, characterized by nearly equal Fermi energies \( T_1, T_2 \), and associated flux factors \( \rho_1 v_1, \rho_2 v_2 \).

Note that the potential energy cost, \(- \frac{dE}{dA_1}\), associated with particle transfer from container 1 into container 2 is, in the case of simple Fermi gases, given by the difference in the Fermi energies, \( T_2 - T_1 \), i.e.,
Recall also the standard result that for a Fermi gas the one-sided particle flux (per unit area) is \( \frac{1}{4} \frac{\rho}{m} v \), so that

\[
\frac{dA_1}{dt} = \frac{1}{4} \frac{\rho}{m} (\rho_2 v_2 - \rho_1 v_1). \tag{23}
\]

It follows that

\[
- \frac{dE}{dt} = \left( \frac{\rho}{m} \right)^2 \frac{24m}{a} (\rho_2 v_2 - \rho_1 v_1) \approx \frac{4\rho^2}{ma} \frac{d(\rho v)}{dt}. \tag{24}
\]

The denominator in eq. (24) is readily evaluated by recalling that for a Fermi gas the mean particle speed \( v \) is proportional to \( \rho^{1/3} \) and the Fermi energy to \( \rho^{2/3} \). Hence

\[
\frac{d(\rho v)}{dt} = \frac{\rho v}{T} \frac{d(\ln \rho v)}{d(\ln T)} = \frac{\rho v}{T} \frac{d(\ln \rho^{4/3})}{d(\ln \rho^{2/3})} = 2\rho v. \tag{25}
\]

Recalling that for a Fermi gas the mean speed \( v \) is \( \frac{3}{4} \) of the Fermi velocity, so that

\[
T = \frac{1}{2} m \left( \frac{4}{3} v \right)^2 \tag{26}
\]

and substituting in eq. (24), we find

\[
- \frac{dE}{dt} = \frac{4\rho^2}{ma} \frac{1}{2} \frac{m}{3} \left( \frac{4}{3} v \right)^2 \frac{v^2}{2\rho v} = \frac{16}{9} \frac{\rho v}{a} \frac{v^2}{1}. \tag{27}
\]

Because the area of the neck appears in the denominator, this new dissipation term effectively freezes out the asymmetry degree of freedom for small necks.

Eq. (20), derived independently in refs. 18, 19, is new in the context of the one-body dissipation dynamics, but I should stress that the physics it represents is already present in transport treatments of nuclear reactions, in particular in the form used by Randrup in ref. 20. That is presumably why such transport theories had no major difficulties in accounting for the existence of deep inelastic reactions. It seems that the same will be true in our dynamical treatment, now that we have incorporated the properly completed Wall-and-Window Formula.
7. "FERMI JETS" AND THE WINDOW FORMULA

A second recent result I would like to mention concerns a simplification in the treatment of Fermi Jets, resulting from the intimate relation between the Fermi Jets and the Window Formula dissipation. Let me remind you that Fermi Jets (or PEPs, promptly emitted particles) is the name given to a type of pre-equilibrium emission of neutrons or protons in the early stages of a nucleus-nucleus collision, in which the Fermi velocities of the nucleons play a dominant role. \textsuperscript{21,29} Experimentally, pre-equilibrium tails in the energy spectra of nucleons produced in nucleus-nucleus reactions have been known for a long time, but their relation to the specific Fermi jetting mechanism remains unclear. This is partly due to the cumbersome nature of the calculations necessary to characterize the expected properties of the Fermi jets. Such calculations have required, in the past, numerical time integrations over the complex and poorly understood time evolution of the nucleus-nucleus collision dynamics. What we have found recently is that if this time evolution is formulated according to the Chaotic Regime Dynamics, then it tends to cancel out in the calculation of Fermi jets and the question of the collision dynamics can, to a large extent, be bypassed. In simple cases one can even write down closed algebraic formulae for the properties of the jetted particles, the time evolution having cancelled out exactly from the analysis. \textsuperscript{22}

Let me sketch how this miracle comes about in the case of a head-on collision. The calculation of the jetting goes something like this. After contact, a window opens up between the nuclei, and its cross-sectional area, \( a(t) \), will be some complicated function of time. Through this window time-dependent fluxes of particles begin to irradiate from inside the two potential wells. These fluxes are calculable \textit{elementarily} in terms of the instantaneous relative velocity \( u(t) \) and the window area \( a(t) \), to which they are proportional at each instant. A small fraction of the nucleons in these fluxes have velocities high enough for escape and produce the jetting. The remainder is captured in the receptor wells and causes a slowing down of the relative motion of the nuclei according to the Window Formula dissipation. This slowing down is, of course, also proportional to the window area \( a(t) \), so that

\[
\frac{du}{dt} = a(t), \quad (27)
\]

i.e. \( dt = \frac{du}{a(t)} \).

It follows now that if the Fermi jetting, which at each instant is \textit{directly} proportional to \( a(t) \), is integrated over time, and the integration variable is changed from \( dt \) to \( du \), the unknown window-area function \( a(t) \) cancels out and
the result is an elementay integral over u, independent of the collision dynamics!

Since the jetting fluxes as well as the window friction contain no adjustable parameters, the final answers to all calculations with the above idealized model come out as absolute magnitudes, with nothing to adjust. Even the flux factor \( \rho v \) cancels out!

As an example, let me give you the formula for the predicted total number of particles, \( \Delta A \), jetted through both the target \( A_1 \) and projectile \( A_2 \), as a function of the relative velocity \( u_0 \) at the moment of contact in a head-on collision:

\[
\Delta A = \frac{A_1 A_2}{A_1 + A_2} [F(v) - F(n - 1)]
\]

(28)

where

\[
F(x) = (1 - n^2)^2 x n + \frac{8}{3} (1 - n^2) x + (1 + n^2) x^2 - \frac{1}{12} x^4
\]

(29)

In the above, \( v \) is the velocity \( u_0 \) in units of the Fermi velocity \( v_F \) and \( n \) is the nucleon "escape velocity" (also in units of \( v_F \)), given by

\[
n = \sqrt{\frac{T + S}{T}}
\]

(30)

where \( T \) is the Fermi energy and \( S \) is the neutron separation energy in the case of neutron jetting or the proton separation energy, augmented by the proton Coulomb barrier, in the case of proton jetting. Let me stress again that one is able to derive eq.(29) without having solved the dynamical problem of the slowing down of the relative motion of the nuclei.

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What I have described in my talk is how progress is being made along one of the several paths that are being followed in our quest to understand the dynamics of nucleus-nucleus reactions. To some extent this progress consists of simplifying and tidying up the theoretical concepts, so that the essential features may stand out clearly. Apart from conventional elements concerning nuclear, electric and centrifugal forces, the most interesting component of the analysis that I presented is the Chaotic Regime Dynamics, with its one-body dissipation. As time goes on we should be able to specify more closely the range of validity of this elegant idealization.
Appendix. Some misunderstandings

A1. THE EXCLUSION PRINCIPLE AND LIOUVILLE'S THEOREM

A problem that often crops up in discussions of the Wall or Wall-and-Window formulae is the exclusion principle. Doesn't a nucleon bouncing off a moving wall, or trying to enter another nucleus, have to look for an unoccupied state in the Fermi sea of nucleons? Should one not introduce blocking factors that cut out most of the contemplated collisions with the wall and most of the particle transfers, and thus modify drastically the dissipation formulae?

The answer is NO, insofar as one is using a mean-field model, in which the particles are assumed to move in a common (time-dependent) potential well. The reason is that, according to Liouville's theorem, non-interacting particles moving in such a potential automatically make room for each other in phase space. If they are started off with a distribution in phase space that respects the exclusion principle (e.g. 2 particles per $h^3$ of phase space for nucleons) then Liouville assures us that the classical equations of motion will see to it that the exclusion principle will never be violated in the subsequent time evolution. This is obvious from the statement of Liouville's theorem in the form: "representative points in phase space behave like an incompressible fluid." This means that if you follow the motion in phase space of a cluster of a given number of points bounded by a cell of a certain volume then, in the course of the time evolution, this volume will not change (only the shape of the cell may become distorted).

To remind you of the essence of the argument, consider the six-dimensional phase space of a particle moving in an arbitrary (time-dependent) potential, as described by some general Hamiltonian $H(t)$. For a swarm of $N$ such particles—provided they do not interact—we may continue to use the six-dimensional phase space, rather than the full $6N$-dimensional phase space that would otherwise be necessary. The motion of each of the $N$ particles is then represented by the motion of a point in phase space. The velocity of a point in this phase space is given by Hamilton's equations

$$
\frac{\dot{q}_k}{\dot{p}_k} = \frac{\partial H}{\partial q_k}, \quad \frac{\dot{p}_k}{\dot{q}_k} = -\frac{\partial H}{\partial p_k}, \quad (31)
$$

where $q_k$ is one of the generalized coordinates and $p_k$ its conjugate momentum. Consider the two-dimensional sub-space of just $q_k$ and $p_k$ and consider a small rectangular cell defined by $q_k, p_k$ and $q_k + \Delta q_k, p_k + \Delta p_k$, so that the area of the cell is $\Delta q_k \Delta p_k$. Now the velocity of the $q_k$-edge of the cell is $\dot{q}_k$ and of the $(q_k + \Delta q_k)$-edge is $\dot{q}_k + \Delta \dot{q}_k$, so in a time $dt$ the length of the cell in the $q_k$ direction will have changed by
Similarly the length of the cell in the $p_k$ direction will have changed by
\[ \frac{\dot{a}p_k}{ap_k} \Delta p_k dt \ . \]

It follows that the change in the area of the cell is
\begin{align*}
\Delta q_k \left( 1 + \frac{\dot{a}q_k}{aq_k} dt \right) \Delta p_k \left( 1 + \frac{\dot{a}p_k}{ap_k} dt \right) - \Delta q_k \Delta p_k \\
= \Delta q_k \Delta p_k dt \left( \frac{\dot{a}q_k}{aq_k} + \frac{\dot{a}p_k}{ap_k} \right) \\
= \Delta q_k \Delta p_k dt \left( \frac{\dot{a}^2}{aq_k ap_k} - \frac{\dot{a}^2}{ap_k aq_k} \right) \equiv 0 \ . \tag{33}
\end{align*}

It is clear that we can carry through the same argument with all the pairs of conjugate variables and thus arrive at Liouville's result that the volume of the cell in phase space is exactly preserved in virtue of Hamilton's equations of motion. So the particles make room for each other very nicely and do not show any tendency to violate the exclusion principle.

The condition for the validity of this result is that we stay with the mean-field idealization. If the particles interact by (residual) two-body forces then, for the discussion of the effect of such interactions, the exclusion principle is, of course, crucial.

A2. SELF-CONSISTENT DRIFTS AND THE CONSERVATION OF LINEAR AND ANGULAR MOMENTA

The Wall Formula for the rate of dissipation of collective energy looks like this
\[ \frac{dE}{dt} = \rho v \int (\dot{n} - D)^2 d\sigma \ . \tag{34} \]

Here $\dot{n}$ is the normal velocity of the surface element $d\sigma$ of the deforming vessel containing the gas and $D$ is the normal component (at $d\sigma$) of the drift set up in the wall-directed particles of the gas by the motion and deformation of the vessel, as specified by $\dot{n}$.\textsuperscript{7}

The Wall Formula is sometimes referred to as if $D$ were absent, in which case a nonsensical result is arrived at: since $\dot{n}^2$ is a positive definite quantity even for pure translations (or for uniform rotations of a non-spherical vessel) the implication would be that a translating or rotating system would continue
to dissipate energy until it came to rest. (This is sometimes described as a conflict between the Wall Formula and Galilean invariance.) That there is no such paradox implied by eq.(34) is clear: for a uniform translation and rotation the gas will soon acquire a drift that follows faithfully the motion of the vessel, and \( \dot{n} - D \) will be zero.

But now comes the non-trivial question. How does one actually evaluate \( D \) when the container is translating, rotating and deforming at the same time? In general, \( D \) will be a function of position \( \vec{r} \) on the surface and a functional of the state of motion of the surface, as specified by \( \hat{n} \), say \( D = D(\vec{r},\hat{n}) \). In ref. 7 a prescription was given for calculating the drift, based on assuming a plausible functional form for \( D(\vec{r},\hat{n}) \), namely a drift pattern that would be generated by a translation with some velocity \( \vec{V} \) together with a rotation with some angular velocity \( \vec{\Omega} \), i.e. \( D = (\vec{V} + \vec{\Omega} \times \vec{r}) \cdot \hat{n} \), where \( \hat{n} \) is the unit vector along the normal at \( d\sigma \). The quantities \( \vec{V} \) and \( \vec{\Omega} \) are regarded as vector parameters that are determined by the requirement that the linear and angular momenta of the gas should not change in time. This requirement is equivalent to the demand that the total force and total torque exerted by the container on the gas should be zero. You may verify that this is expressed by the two equations

\[
\int [\hat{n} - (\vec{V} + \vec{\Omega} \times \vec{r}) \cdot \hat{n}] \hat{n} d\sigma = 0 \quad \text{(zero force)} \tag{35}
\]

\[
\int (\vec{r} \times \hat{n}) [\hat{n} - (\vec{V} + \vec{\Omega} \times \vec{r}) \cdot \hat{n}] d\sigma = 0 \quad \text{(zero torque)} \tag{36}
\]

These are two simultaneous vector equations for \( \vec{V}, \vec{\Omega} \) which may be written

\[
\vec{A} \vec{V} + \vec{B} \vec{\Omega} = \vec{a} \tag{37}
\]

\[
\vec{C} \vec{V} + \vec{D} \vec{\Omega} = \vec{b} \tag{38}
\]

where \( \vec{A}, \vec{B}, \vec{C}, \vec{D} \) are dyadics and \( \vec{a}, \vec{b} \) are vectors given by

\[
\vec{A} = \int d\sigma \hat{n} \hat{n}, \quad \vec{B} = \int d\sigma \hat{n} \vec{r} \times \hat{n} \tag{39}
\]

\[
\vec{C} = \int d\sigma \vec{r} \times \hat{n} \hat{n}, \quad \vec{D} = \int d\sigma \vec{r} \times \hat{n} \vec{r} \times \hat{n} \tag{40}
\]

\[
\vec{a} = \int d\sigma \vec{r} \times \hat{n}, \quad \vec{b} = \int d\sigma \hat{n} \vec{r} \times \hat{n} \tag{41}
\]

[If one wishes, one may write down the formal general solution of eqs. (37),(38) as

\[
\vec{V} = (\vec{B}^{-1}\vec{A} - \vec{D}^{-1}\vec{C})^{-1}(\vec{B}^{-1}\vec{a} - \vec{D}^{-1}\vec{b}) \tag{42}
\]

\[
\vec{\Omega} = (\vec{C}^{-1}\vec{D} - \vec{A}^{-1}\vec{B})^{-1}(-\vec{A}^{-1}\vec{a} + \vec{C}^{-1}\vec{b}) \tag{43}
\]
Here $\mathbf{B}^{-1}$ denotes the inverse of the dyadic $\mathbf{B}$, etc. In many cases of practical interest, the situation is much simpler, and $\mathbf{v}$ and $\mathbf{s}$ can be read off by inspection directly from eqs.(37),(38).

In the case of the Wall-and-Window formula one has to determine two drifts, $D_1$ and $D_2$, one for each of the two pieces. With the same prescription as before, one has now to determine two pairs of vector parameters, $\mathbf{v}_1, \mathbf{s}_1$ and $\mathbf{v}_2, \mathbf{s}_2$. Again, the condition which determines these parameters is that the linear and angular momenta of the fragments should not change, apart from the expected changes induced by the actual forces (conservative and dissipative) acting between them. This means that there should be no net force or torque exerted on a fragment by the wall part of the Wall-and-Window formula belonging to that fragment.

The bottom line is that, with the drift terms included, there is no "violation of Galilei invariance."

A related misunderstanding that has generated some confusion is the contention that, as it stands, the Wall formula dissipation would always have to vanish identically, because $n$ - $D$ represents the relative motion of the wall and the gas next to it, and this has to vanish by the well-known hydrodynamical boundary condition. (Attempts were actually made to remedy this supposed difficulty by introducing into the wall formula an adjustable parameter). This misunderstanding is based on an incorrect interpretation of $D$ as the drift of all particles near the wall rather than the drift of the wall-directed particles near the wall. Given the correct interpretation of $D$, there is no reason to look for arbitrary modifications of the Wall formula.

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REFERENCES


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