Dynamic Aperture Study for the NLC Main Damping Rings

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Abstract

A sufficiently large acceptance is critical for the NLC Main Damping Rings (MDR) as the high power carried by the beams demands very high injection efficiency. Chromatic sextupoles and wiggler insertions (needed for rapid damping) are substantial sources of nonlinearities limiting the dynamic aperture. We report on the techniques we are using for analysis of single-particle beam dynamics in the presence of wiggler fields with significant nonlinear components. We demonstrate that our approach gives results in good agreement with experimental data when applied to the BL11 wiggler in SPEAR2, and discuss the present status of studies for the NLC MDR.

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Abstract
A sufficiently large acceptance is critical for the NLC Main Damping Rings (MDR) as the high power carried by the beams demands very high injection efficiency. Chromatic sextupoles and wiggler insertions (needed for rapid damping) are substantial sources of nonlinearities limiting the dynamic aperture. We report on the techniques we are using for analysis of single-particle beam dynamics in the presence of wiggler fields with significant nonlinear components. We demonstrate that our approach gives results in good agreement with experimental data when applied to the BL11 wiggler in SPEAR2, and discuss the present status of studies for the NLC MDR.

INTRODUCTION
The Main Damping Rings [1] of the NLC are designed to reduce the incoming normalized beam emittances from 150 µm to 3 µm horizontally and 0.02 µm vertically, in a time consistent with the machine repetition rate of 120 Hz. The short damping times needed (around 5 ms) are achieved in a 300 m lattice by use of over 60 m of wiggler with peak field 2.1 T. The wiggler provides 90% of the synchrotron radiation energy loss in the ring. The high average injected beam power of 60 kW means that the constraints imposed by Maxwell’s equations were therefore produced, with tighter limits on the numerical quality, and this has led to a significantly revised estimate on the size of the dynamic octupole component, which is found close to the pole faces. New field data have been produced, with tighter limits on the numerical quality, and this has led to a significantly revised estimate on the size of the dynamic octupole component. This experience has emphasized for us the importance of field data of very high quality for calculating the beam dynamics in the field of a device such as a wiggler.

FIELD FITTING
The first step in our analysis of beam dynamics in a wiggler is to fit the numerical field data with an analytic series. This will allow the generation of a dynamical map in Taylor form, using a differential algebra code. Appropriate expressions (satisfying Maxwell’s equations) can be given in either cylindrical polar co-ordinates, or rectangular Cartesian co-ordinates. In cylindrical co-ordinates, the field components can be written:

\[ B_\rho = \sum_{mn} \alpha_{mn} I_n(m\rho) \sin(m\phi) \cos(nk_z) \]
\[ B_\phi = \sum_{mn} \alpha_{mn} \frac{m}{nk} I_m(n\rho) \sin(m\phi) \cos(nk_z) \]  
\[ B_z = -\sum_{mn} \alpha_{mn} I_m(n\rho) \sin(m\phi) \sin(nk_z) \]  

In Cartesian co-ordinates the components are:

\[ B_x = -\sum_{m,n} c_{mn} \frac{mk}{k_y} \sin(mk_x) \cos(nk_z) \sinh(k_{y,mm}y) \]
\[ B_y = \sum_{m,n} c_{mn} \cos(mk_x) \cos(nk_z) \cosh(k_{y,mm}y) \]  
\[ B_z = -\sum_{m,n} c_{mn} \frac{nk}{k_y} \cos(mk_x) \sin(nk_z) \sinh(k_{y,mm}y) \]

The cylindrical representation (1) reflects the periodicity in the azimuthal and longitudinal co-ordinates. In the Cartesian representation (2), there is an artificial periodicity in the horizontal co-ordinate; nonetheless, it is possible by using a small enough value for \( k_y \) to obtain a very good fit to the field in the region of interest for the beam dynamics. The Cartesian representation is more convenient for use in beam dynamics codes, including the differential algebra codes we shall use to produce the dynamical map.

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In practice, we find the most robust fitting method is to identify the coefficients of the cylindrical fit, $\alpha_{mn}$, by a two-dimensional Fourier analysis of the radial field component on the surface of a cylinder, and then calculate the coefficients $c_{mn}$ of the Cartesian fit using the following relationship:

$$\alpha_{mn} = \frac{2(-1)^m}{(nk)^{m+1}} \sum_{k=-n}^{n} \sum_{q=0}^{\infty} k^{m+1} (m^2 + q^2)^{k} c_{m,n}$$  \hfill (3)$$

The coefficients $c_{mn}$ can be found directly by a Fourier analysis of the vertical field component on some plane of fixed $y$; however the implicit periodicity of the field imposes a lower limit on the value of $k_c = \pi 2 \chi_{\text{max}}$, where $x_{\text{max}}$ is the limit of the range of the numerical field data.

By making a cylindrical fit, and then using (3) to calculate the $c_{mn}$, we can choose a smaller value of $k_c$ and obtain a better fit over a wider range.

As an example, we consider the SPEAR2 BL11 wiggler. A model of this device is provided in the distribution of the magnet code RADIA [4], and we have used this model to generate a numerical field map for one periodic section of the wiggler. As an indication of the quality of the field data, Figure 1 shows the divergence and curl of the field, calculated from the field data, on a plane 6 mm above the mid-plane. The peak field is 2 T, and the period is 175 mm; the full-gap between the upper and lower poles is 16 mm. We observe some deviation from zero over a narrow longitudinal region close to the edge of the pole, where the vertical field is falling rapidly.

We fitted the field by first calculating the cylindrical coefficients using a Fourier analysis of the radial component of the field on a cylinder of radius 6 mm, taking mode numbers up to 9 azimuthally, and 39 longitudinally. We then calculated the Cartesian coefficients using equation (3), with $k_c = 2\pi / 192$ mm$^{-1}$. Figure 2 shows the residual of the fit to the vertical field component on the mid-plane, and on a plane 6 mm above the mid-plane. On the mid-plane, the residual is less than a Gauss everywhere; close to the field tip, the residual is a few tens of Gauss.

**CALCULATING THE DYNAMICAL MAP**

Having obtained a good fit of an analytical expression for the field to the numerical field data, a Differential Algebra (DA) code can be used to integrate the equations of motion in the field to produce a dynamical map in Taylor form. We have used the code COSY [5] for this; other differential algebra codes are available. COSY includes a $7^{th}$ order Runge-Kutta integrator that solves the exact equations of motion for a relativistic particle in magnetic field. Because the integration is not explicitly symplectic, the result is a map that has (small) symplectic errors in some of the lower-order terms. For comparison, we have also implemented an explicit symplectic integrator [6] that solves Hamilton’s equations with the small-angle approximation for the Hamiltonian.

![Figure 1: Field divergence and curl of the SPEAR2 BL11 wiggler field map, on a plane 6 mm above the mid-plane.](image1)

![Figure 2: Residual of a fit to the SPEAR2 BL11 wiggler field map.](image2)

![Figure 3: Coefficients in the 5th order map for $x$ through one period of the BL11 wiggler, calculated with Runge-Kutta integration (blue points), and an explicit symplectic integrator (red crosses). Green lines divide terms of different order.](image3)

**TUNE SHIFTS FROM BL11 WIGGLER**

After the BL11 wiggler was installed in SPEAR2, strong nonlinear effects were observed. Measurements were made of the tune shift with amplitude [2]. We used an interpolated Fourier-Hanning technique to determine...
numerically the effect of the BL11 wiggler on the dynamics in a model of the SPEAR2 lattice, using the map produced using COSY. We found a linear horizontal tune shift of −0.010, in excellent agreement with the experimental observation of −0.009. We found a tune shift with amplitude of −4.5x10^{-5} mm^{-2}, smaller than the experimentally observed value of −7.4x10^{-5} mm^{-2}. However, studies based on numerical integration through a field model produced in TOSCA, also reported in [2] underestimated the tune shift by the same amount; it appears likely that the discrepancy is due to an error in the model of the magnet, rather than in the analysis. We note that the theoretical studies in [2] calculated only the dynamic octupole component, equivalent to a single term in the map. The techniques presented above give a complete model of the dynamics.

DYNAMICS IN THE MDR WIGGLER
The NLC MDR wiggler has a peak field of 2.1 T, a period of 270 mm, and a gap of 18 mm. A field map has been produced using TOSCA. Figure 4 shows the divergence and curl of the field, calculated on a horizontal plane 6 mm above the mid-plane. The deviation from the ideal value of zero has a similar pattern to the SPEAR2 BL11 wiggler, but is a little larger.

Figure 5 shows residuals of the fit to the MDR wiggler, using the same procedure and same number of modes as for the SPEAR2 BL11 wiggler. The relatively large residuals on the mid-plane may be due to deviations from the constraints of Maxwell’s equations.

Finally, Figure 6 shows the dynamic aperture in coordinate space and in tune space for the NLC MDR with a linear wiggler model, and with a 5th order map from the Runge-Kutta integration through the field in COSY. The colors of the points indicate the diffusion rate in tune space on a logarithmic scale. Particles were tracked for 512 turns. At the observation point, the injected rms beam size is 0.8 mm horizontally, and 0.25 mm vertically. The wiggler has some small impact on the tune shifts, but the nonlinear dynamics are dominated by the sextupoles. The results need to be confirmed using a more accurate field map.

The off-energy dynamics in the lattice remain a concern, with little margin for the 1% full-width energy spread on the incoming beam.

Figure 5: Residual of a fit to the NLC MDR wiggler field map.

Figure 6: Dynamic aperture and frequency map for the NLC MDR with linear wiggler model (top) and 5th order map for each wiggler period (bottom).

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