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1. INTRODUCTION

The concept of ether does not appear to be obsolete in today's literature a century after the Michelson-Morley (MM) experiment. One of the reasons for its renaissance is that it provides an intuitive framework to account for the features of the $3^0 K$ cosmic background radiation associated with the hot big-bang cosmological model. Furthermore the electromagnetic (e.m.) field radiated by all the atoms of the universe has been related to the idea of an ethereal medium. This e.m. background is fully equivalent to the zero-point radiation of stochastic electrodynamics, which, in turn, is hoped to be proven equivalent to quantum electrodynamics. Obviously, the same matter that has generated the cosmic fossil radiation must have also produced the zero-point radiation. Thus, in line with the modern cosmological assumption that the universe is isotropic and homogeneous on a large scale, it appears natural to postulate that there is an inertial privileged reference frame, at rest with the ancient matter which emitted the background radiations. These are isotropic in the privileged frame, which must be the center of mass of the universe or the local galaxies.

Historically, the idea of a privileged reference frame has been more readily accepted than the assumption that the speed of light in vacuo is invariant and equal to the universal constant $c$. Indeed, the constancy of the speed of light is highly non-intuitive, particularly when confronted with philosophical concepts of space and time based on the physics of Galileo transformations that implicitly require absolute synchronization of clocks.
Although special relativity (SR) and ether theories seem irreconcilable, Dirac \(^3\) made the suggestion that the ether concept is no longer ruled out by SR, and that good reasons can now be advanced for postulating an ether. Even Einstein, \(^4\) although not going as far as Dirac, has some peculiar statements about ether, e.g.: "... the most appropriate point of view that one could take from the start to conform to such a state of affairs (i.e. SR), seemed to be the following: that the ether does not exist at all... But a more ponderate reflection suggests that the negation of ether is not necessarily required by the principle of special relativity."

We will assume here that a modern ether no longer consists of a mechanical or material medium as conceived in the past century, but rather of a field-like medium like the one of stochastic electrodynamics or, even simpler, of a medium to be identified with the physical space between bodies. The characteristics and properties of this ether, or physical space, will depend on the specific ether theory or model to be considered.

As a basis for the interpretation of the observed \(^5\) cosmic-background radiation anisotropy, and of some data from quasars and compact radio-galaxy radiation, \(^6\) we assume that the earth is an inertial frame moving through the cosmic ether, i.e., moving with respect to the privileged (or absolute) reference frame.

We will consider in this paper some of the best known ether theories, including the Stokes-Planck theory. We will show that some ether theories can either be fully equivalent to SR or different from it, depending on the co-ordinate transformation and synchronization procedure chosen. For this purpose it is helpful to review the ether theories to be considered and analyze the behaviour of the wave equation for different types of co-ordinate transformations. Maxwell's equations will be considered and a tensor formalism will be developed for the Stokes-Planck theory. Finally we will consider some possible tests for these ether theories.

2. Ether Theories

Two possibilities arise with regard to the most relevant feature of physical space, or ether. It can be: a) anisotropic, b) locally isotropic, when considered from a frame \(S(x, y, z, t)\) in motion with respect to the absolute frame \(S_0(x_0, y_0, z_0, t_0)\).

a) With respect to \(S_0,\) space is homogeneous and isotropic everywhere and the speed of light in this cosmic ether will always be equal to \(c.\) However, any inertial frame \(S\) moving with respect to \(S_0\) will detect an anisotropy in the cosmic background radiation and, possibly, a space anisotropy. Consequently, anisotropy of the velocity of light will be associated with the motion of \(S\) through the cosmic ether.

b) A completely anisotropic ether is not a suitable basis for a physical theory. However we can conceive of an ether that is essentially isotropic everywhere in \(S_0,\) save in the proximity of moving massive bodies where it is anisotropic. This ether has the further property that in the reference frame \(S\) of the moving body, the physical space, or ether, is locally isotropic within the immediate surroundings of the body itself. Thus, we can imagine that the cosmic ether is at rest and isotropic everywhere in \(S_0,\) except
near the earth and planets, which "carry a portion of the aether along with them so that the aether close to their surfaces is at rest relatively to those surfaces, till, at no great distance, it is at rest in space." \(^7\) This ether hypothesis was originally formulated by Stokes to reconcile in a simple way the phenomenon of aberration of light from distant stars with the undulatory theory of light. Later it was re-elaborated by Planck\(^8\) in order to answer some criticisms made by Lorentz.\(^9\) A common misunderstanding\(^10\) of the Stokes-Planck theory regarding the interpretation of classical optics experiments has recently been clarified\(^11\) as being due to a mis-interpretation of Stokes' original hypothesis.

3. Co-ordinates Transformations

The most general co-ordinate transformation between inertial frames \(S\) and \(S_0\), can be reduced to the following form if we assume that rods keep the same length after being accelerated to the velocity \(v\) (likewise for clocks):\(^12\)

\[
\begin{align*}
  x &= a(x_0 - vt_0) \\
  y &= b y_0 \\
  t &= d(t_0 - ct_0)
\end{align*}
\]

Without loss of generality we may consider here, for simplicity, the velocity to be only in the \(x\)-direction and omit the \(z\) co-ordinate. It is important to notice that physical predictions based on transformation (1) could be different when related to ether models a) or b) of Section 2.

In order to clarify the role of the various parameters entering (1) when used in physical equations we consider now the transformation properties of the wave equation

\[
\Box \psi = (\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}) \psi = 0 \quad (2)
\]

with its plane-wave solution in the form

\[
\psi = \psi_0 \psi(t - x/c_0) = \psi_0 \psi(t - \frac{x_0 \cos \theta_0}{c} - \frac{y_0 \sin \theta_0}{c}) \quad (3)
\]

Here \(\theta_0\) is the angle between the direction of propagation \(\vec{r}_0\) and the \(x\) axis. By making use of (1) we may transform (3) to

\[
\psi = \psi_0 \psi(t - \frac{x}{c_0} - \frac{y a \sin \theta}{b \sqrt{1 - \frac{c^2}{c^2}}}) = e^{i\omega(t - \frac{r^2}{c^2})} \quad (4)
\]

where

\[
c^* = \frac{c^2 - v \cos \theta_0}{\sqrt{b}} \quad (5)
\]

with \(|r^2| = a |\vec{r_0}|\),

\[
\omega = \omega_0 (1 - \frac{v \cos \theta_0}{c^*}) \quad (6)
\]

and

\[
\sin \theta = \frac{\sin \theta_0 (1 - cv)}{1 - cc} \quad \cos \theta = \frac{\cos \theta_0 (1 - cc/c^2)}{1 - cc} \quad (7)
\]

Still, by means of (1) and of the relation \(\frac{\partial}{\partial x} = \frac{\cos \theta}{c^*} \frac{\partial}{\partial t}\) obtained for plane-wave solution (4), the wave equation (2) takes the form

\[
(a^2 \frac{\partial^2}{\partial x^2} + b^2 \frac{\partial^2}{\partial y^2}) \psi = \frac{1}{c^*} \frac{\partial^2}{\partial t^2} \psi \quad (8)
\]

which is still a wave equation for plane-wave solution (4). Considering Eq. (8) as a generalized form for the wave equation, we can say that it is form invariant with respect to transformations (1), provided that the
wave velocity transforms like (5) in expressions (4) and (8). Here $c^\ast$ represents, as we shall see below, the velocity of propagation of the wave front along the direction $\vec{n} = \vec{r}/|\vec{r}| = \vec{v}/|\vec{v}|$. Had we chosen

$$c^\ast = \frac{a}{d} c$$  \hspace{1cm} (5')

instead of relation (5), in transforming (2) and (3), we would still have obtained (4) and (6) but now with

$$\sin \theta = \sin \theta_0 \frac{a(1 - cv)}{b(1 - v \cos \theta_0/c)}, \quad \cos \theta = \cos \theta_0 (1 - \frac{cc}{\cos \theta_0})$$  \hspace{1cm} (7')

That is, we would have obtained the same frequency dependence (6) but a different propagation direction. In this case only, SR is recovered for $a = d = \gamma$, $(\gamma = (1 - v^2/c^2)^{-1/2})$, $b = 1$, and $c = v/c^2$, leading, by (5'), to the light speed invariance: $c^\ast = c$.

With respect to the wave equation it is interesting to notice that the condition $\cos^2 \theta + \sin^2 \theta = 1$ leads, by (7), to

$$(\cos \theta - cc)^2 + \sin^2 \theta (1 - cv)^2 = (1 - cc)^2$$  \hspace{1cm} (9)

In the case just considered of parameter values leading to the Lorentz transformations (LT), the above Eq. is satisfied for $c = v/c^2$, only for $\theta = 0$. Therefore, (2) cannot be transformed into Eq. (8) by means of the LT. Generally speaking, the wave equation is form invariant under transformations (1) only if the value of $c$ satisfies Eq. (9). By making the simplest choice, $\epsilon = 0$, to satisfy (9), we can realize that the wave equation and its solution (4) might describe a physical reality different from that represented by SR. Indeed, results (5) and (8) are not generally compatible with the LT regardless of the synchronization procedure used to determine the value of $\epsilon$.

4. Anisotropic Ether

We consider now the various implications related to the anisotropic ether hypothesis a) of Sect. 2. First we have to determine the values of the parameters appearing in transformations (1). As seen above, we can choose either (5) or (5') for the velocity transformation.

If we adopt (5'), the condition $\cos^2 \theta + \sin^2 \theta = 1$ gives, using (7'), $\epsilon = v/c^2$, if $c$ has to be independent of $\theta$. Symmetry requirements for (1) (such as group properties) immediately imply $b = 1$ and $a = d = \gamma$, and as shown above, we obtain from (5') $c^\ast = c$, that is SR. Thus, even with this approach, we associate with SR an isotropic space in $S$ as well as in $S_0$. However, the cosmic background radiation is isotropic in $S_0$ but anisotropic in $S$, while the e.m. zero-point radiation has to have a spectral density which is Lorentz invariant. Thus, if we wish to consider SR as a modern ether theory we find that a privileged reference frame can be identified in terms of background radiation anisotropy but never in terms of space anisotropy.

If we adopt (5) in transforming the plane-wave solution (3), then we obtain from (9) $\epsilon = 0$, if $c$ has to be $\theta$-independent. This value of $c$ implies absolute synchronization of clocks between $S$ and $S_0$. Transformations (1), with $c = 0$, are similar to Galileo's transformations, for there is no dependence of time on space. For this reason we shall refer to transformation (1), with $c = 0$, as generalized Galileo transformations (GGT). Note that GGT differ from LT mainly with respect to time simultaneity, and it is for this reason that we are led by GGT to the speed of light anisotropy of expression (5). The values of the parameters to be used in GGT can be determined.
phenomenologically. As shown by Mansouri and Sexl, only the following GGT, with \( a = \gamma, b = 1, \) and \( d = \gamma^{-1} \) are possible candidates:

\[
\begin{align*}
x &= \gamma(x_0 - vt_0) \\
y &= y_0 \\
t &= \gamma^{-1}t_0
\end{align*}
\]  

(10)

We must stress here that the assumptions made by Mansouri and Sexl to obtain transformations (10) are based implicitly on the ether hypothesis, a) considered in this Sect. Transformations (10) were originally derived by Tangherlini and later generalized by Chang and Rombielski. For a two-way-trip optic experiment, like the MM one, performed in \( S \), transformation (10) lead to the same results as SR, since, for such experiments, light turns out to have an average speed \( \bar{c} = c \). Since, for these experiments, it is as if transformations (10) spanned an isotropic space also in \( S \), the equivalence between this ether theory and SR can still be claimed at this point. Indeed, transformation (10) can be considered equivalent to the LT, the only difference being that clocks are absolutely synchronized \( (t = 0) \) in (10), while they are synchronized according to Einstein's procedure in the LT.

If we wish to perform an unambiguous test of this ether theory by means of a one-way-trip optic experiment in \( S \), then clocks will have to be synchronized by slow-clock transport. If, in this case, the outcome of the experiment indicates space anisotropy in \( S \), then the ether theory defined by (10) cannot be considered equivalent to SR. That is, light-speed anisotropy in \( S \), given by (5), cannot in this case be ascribed to the synchronization procedure. In fact, if space were isotropic in \( S \), slow-clock transport would be equivalent to Einstein's synchronization. Thus, \( c = v/c^2 \) and, by (5'), we should have obtained \( c* = c \).

We observe here that for transformations (10), as well as for any GGT, we obtain from (7), \( \theta = \theta_0 \). This indicates that the direction of the wave-front normal \( \hat{n} \) is left unchanged by GGT, while the direction of the ray velocity changes according to the velocity composition law. Neglecting terms in \( v^2/c^2 \), the change is obviously the same as given by SR. Since the ingredient relevant to aberration is the change in direction of the ray velocity, this phenomenon can still be interpreted on account of GGT. However, Eq. (8) turns out to describe a plane wave for which the direction of the wave-front normal does not coincide with the aberrated direction of propagation of the ray velocity. For this reason, we shall refer to (8) as the aberrated-plane-wave equation.

Assuming that transformations (10) span a truly anisotropic space in \( S \), we consider now some of its properties and differences from SR. The composed transformations between frames \( S' \) and \( S \), \( S' \) moving with velocity \( \hat{v}' = (v', 0, 0) \) with respect to \( S_0 \), are

\[
\begin{align*}
x' &= \frac{y'}{y} (x - wt) \\
y' &= y \\
t' &= \frac{y'}{y}, t
\end{align*}
\]  

(11)

where \( w = \gamma^2(v' - v) \). Written in this form, the parameter \( w \) appearing in transformations (11) can be identified with the relative velocity of \( S' \) with respect to \( S \) (but not with that of \( S \) with respect to \( S' \)). We can see from (11) that the lifetime of a decaying particle moving with velocity \( w \) with respect to \( S \), is

\[
T = \frac{Y'}{Y} T^0
\]  

(11')
where T is the lifetime for the particle at rest. Only if terms in \( v^2/c^2 \) are neglected we obtain the same results of SR. The composed transformations can be written in the Rembelinski form

\[
\begin{align*}
    x' &= \gamma_w [(1 - \frac{vw}{c^2}) x - wt] \\
    y' &= y \\
    t' &= \frac{t}{\gamma_w (1 - \frac{vw}{c^2})}
\end{align*}
\]  

(12)

In this case, absolute velocities are connected to w through the relativistic velocity transformations: 

\[
v' = (w - v)/(1 - \frac{vw}{c^2}).
\]

However, the parameter w cannot always be identified with the relative velocity between S' and S. Both transformations (11) and (12) exhibit group properties. Metrics, invariant length, and tensor formalism can be developed to write the Maxwell equations, test charge equations of motion, and field equations for spinor fields in covariant form. Since the metric tensor has the same expression for both (11) and (12), the development of the four-dimensional tensor formalism is similar for both transformations.

We should point out at this stage that the development of electromagnetism from transformations (10) may proceed according to two distinct alternatives. The first is to consider the charge density \( \rho \) and the current density \( \mathbf{J} \) as forming a four-vector. In this case the Maxwell equations transform as in Ref. 17 and 18, and, as pointed out by Chang, a static charge in S would create a magnetic induction \( \mathbf{B} = \mathbf{x} \mathbf{E}/c \). If the test-charge equation of motion in S is written in the same tensorial form as in SR, then, apparently, we are led to the same predictions of SR. Thus, as remarked by Rembelinski, the field \( \mathbf{B} \) produced by a static charge could not be detected experimentally, e.g., with the Trouton-Noble experiment. However, this is true only if the cardinal equations for continuous bodies of SR are incorporated in this ether theory. In fact, in frame S, the Trouton-Noble experiment is interpreted in SR on the basis that the rod supporting the two charges cannot be considered a rigid body. The same assumption is made in the case of the right-angle lever paradox. Still, the relativistic cardinal equations for continua in motion are Lorentz covariant, and it is not generally true that the same SR effects will be predicted if the LT, are substituted by the GTR. This point will be discussed below in relation to e.m. effects.

The second possibility is simply to postulate that \( \rho \) and \( \mathbf{J} \) do not transform as a four-vector. This would imply that a charge in relative uniform motion is not always equivalent to a current and therefore, does not create a magnetic induction field. In this framework, the latter is created only by charges moving in closed paths (closed currents), like in the Rowland experiment. Only in this improbable circumstance can the Trouton-Noble experiment be interpreted without the necessity of introducing into this ether theory the relativistic cardinal equations for moving bodies.

In any event, transformations (10) differ from the LT with regard to time simultaneity. Therefore, even assuming that charge and current transform as a four-vector, currents in motion cannot, by (10), generate charge densities. Indeed, in SR, moving circuits carrying currents give rise to charge polarization because of nonsimultaneity of time in the LT. Thus, although we may assume in this ether theory the same equations of motion for a test charge in frame
\[ S_0 \text{ as in SR, we might not obtain the same results, because physical quantities transform differently for different transformations.} \]

5. Locally Isotropic Ether

We will now consider the case of locally isotropic ether referred to as the Stokes-Planck ether theory. The interpretations of optics experiments on the basis of this ether theory are considered extensively in Ref. 10. We will only point out here that the interpretation of the Mm experiment is evident, since, at the surface of the earth (moving frame S), space is isotropic. Also, the phenomenon of aberration can be interpreted in a simple way for this nonmaterial ether. The moving frame S is approached by light from distant stars traveling at speed \( c \) through the cosmic ether of frame \( S_0 \). According to this theory, as light enters the physical space of frame S, its velocity changes in such a way that it is now equal to \( c \) with respect to S. Since the ether is nonmaterial, the direction of the ray of light does not change as it leaves the cosmic space and enters the local space of frame S. Thus, the ray of light reaching S appears to proceed from the direction it had before entering S-physical space. The apparent direction changes only because of the change of the velocity \( v \) of S with respect to \( S_0 \), thus leading to the phenomenon of aberration.\(^{19}\)

It could be argued that the ether drag might not be total at the surface of the moving body. In this case space would not be completely isotropic within frame S. Let us suppose that the ether is related or made up by e.m. fields, as it would be in the case of stochastic electrodynamics. Then the earth could be carrying a portion of this ether along with its motion as it is carrying along its magnetic field.\(^{22}\) Since there is also a magnetic field of cosmic origin, this would be at rest with respect to the center of mass of the local galaxy which generates it. Therefore there would be an ether wind at the earth surface due to the motion of the earth through the cosmic magnetic field. If we assume the simplest possible relation between ether wind velocity and field intensity we may write \( v_{\text{ether}} = v_E \frac{B}{B_E} \). Here we can take for the earth absolute velocity,\(^{23}\) \( v_E = 400 \text{ km/sec} \), for the cosmic or galactic field,\(^{24}\) \( B_C = 2.5 \times 10^{-6} \text{ G} \), and for the earth magnetic field, \( B_E = 0.5 \text{ G} \) at the surface of the earth, obtaining \( v_{\text{ether}} = 2 \times 10^{-3} \text{ km/sec} \). This value is not detectable even with the sensitivity of \( 10^{-2} \text{ km/sec} \) obtained in recent laser tests of space isotropy.\(^{25}\) In this paper we assume that there is no ether drift (\( v_{\text{ether}} \) can be zero for Stokes-Planck theory)\(^{11}\) and that space is locally isotropic at the earth surface.

If, within a short distance from the earth, space is isotropic, frame S of the earth is locally equivalent to an absolute frame of reference. Indeed, physical phenomena taking place within S-physical space and being described in S, obey the same physical equations for the corresponding phenomena taking place in cosmic space and being described there. Therefore, once the co-ordinate transformations between frame S and frame \( S_0 \) are established, the same transformations hold between S and any inertial frame \( S' \) moving through S-space.

We will now determine the values of the parameters of transformations (1) that are suitable for this ether theory. If we choose \( (S') \) and consider that space is locally isotropic in S, we obtain \( a = d \), and
\[ \varepsilon = \frac{v}{c^2} \text{ from (7')} \] and condition \( \cos^2 \theta + \sin^2 \theta = 1 \). Further symmetry requirements would lead again to the LT. However a complete equivalence between this ether theory and SR would require isotropy of space everywhere in frame S and not only local isotropy.

Let us choose (3), then \( \varepsilon = 0 \) by (19). If we further require that the relative velocities of S with respect to \( S_0 \), and vice-versa, are equal and opposite, we obtain \( a = d \) for our GCT. Still, if space is locally isotropic in S, spatial co-ordinates can be affected by motion but not by its direction, and therefore we can write \( a = b \).

In this case the wave solution (4) assumes the simple invariant form

\[ \psi = \psi_0 e^{i \omega (t - \frac{r}{c^2})} \] (13)

and the equation (2) transforms invariantly as

\[ \Box_{c^2} \psi = (3 \frac{x^2}{c^2} + 3 \frac{y^2}{c^2} - \frac{1}{c^2}) \psi = 0. \] (14)

Expression (13) represents a plane wave travelling in cosmic space as described from frame S. The propagation velocity in the direction \( \mathbf{n} = \hat{r}/|\hat{r}| \) is given by \( c_s = c - v \cos \theta = \hat{c} - \hat{a} \), where \( \hat{a} \) is the actual ray or aberrated-wave velocity evaluated in S. As mentioned above, the direction \( \mathbf{n} \) is unchanged in passing from S to \( S_0 \), while the ray direction definately is.

We now set \( a = b = d = \gamma^{-1} \), since phenomenological considerations suggests choosing in (6) the frequency transformation of SR. For this ether theory a suitable GCT, in agreement with experiment, is therefore given by

\[ x = \gamma^{-1} (x_0 - vt_0) \]

\[ y = \gamma^{-1} y_0 \]

\[ t = \gamma^{-1} t_0 \] (15)

Composed transformations to a frame \( S'' \) moving through the cosmic space with velocity \( \mathbf{v''} = (v'', 0, 0) \) with respect to S show group properties:

\[ x'' = \frac{\gamma}{\gamma'} (x - wt) \]

\[ y'' = \frac{\gamma}{\gamma'} y \]

\[ t'' = \frac{\gamma}{\gamma'} t \] (16)

For transformations (16) the parameter \( w = v'' - v \) is always identifiable with the relative velocity between \( S'' \) and S and vice-versa. This is not true for transformations (11) and, in particular, it is not true for (12). Had we chosen \( b = 1 \) the transformations corresponding to (16) could have been written in the Remielinski form

\[ x'' = \frac{\gamma w}{\gamma''} [(1 - \frac{w^2}{c^2}) \gamma x - wt] \]

\[ y'' = y \]

\[ t'' = \frac{t}{\gamma'' (1 - \frac{w^2}{c^2})} \] (17)

with \( w \) related to the absolute velocities by the transformation formula \( v'' = (v - w)/(1 - \frac{w^2}{c^2}) \), which is the same as in SR. Like transformation (12), transformation (17) belong to the nonlinear realization of the orthochronous Lorentz group. The fact that \( w \) cannot be identified unrestrictedly with the relative velocity between inertial frames seems to us a valid reason for not considering (17) suitable here. The time transformation in (15) and (16) is formally the same as in (10) and (11), still, there is a fundamental difference between the underlying ether hypothesis. The lifetime of a decaying particle at rest on earth will have the same expression
in both ether theories when related to S, i.e. $T_0 = \gamma T^0$. When a decaying particle is moving with respect to the earth but within its locally isotropic space, then, in the Stokes-Planck ether theory, the particle is shielded from the cosmic ether wind. The only relevant velocity is now the particle velocity $\hat{u}$ with respect to the physical space of the earth. Consequently, for this ether theory, the lifetime measured on S is not $T = \gamma T^0$, which differs from that given by (11') derived from the anisotropic ether hypothesis, but is the same as in SR.

We restrict our considerations now to phenomena taking place within the isotropic physical space of frame S. For these, physical equations are written in S in the usual form, as for SR. A plane wave generated in S, and therefore not aberrated, will obey the wave equation $5\psi = 0$, which is derivable from the Maxwell equations written in S in the usual form. For an inertial frame S' moving with velocity $\hat{v}$ with respect to S but within the isotropic S-space, the co-ordinate transformations are formally the same as in (15):

\[ \hat{t}' = \gamma^{-1}(t - \frac{v}{c}t) \]
\[ \hat{c}' = \gamma^{-1}c \]  

(18)

With $x_0 = ct, x_1 = x, x_2 = y, x_3 = z$, the invariant length reads

\[ ds^2 = c^2 dt^2 - dr^2 = dx_0^2 - \gamma^2(dx_1^2 + \frac{v}{c}dx_2^1 + dx_3^2) \].

Writing $ds^2 = g_{uv}dx^u dx^v$, the metric tensor is given by

\[
g_{uv}(\hat{\tau}) = \begin{pmatrix}
1 - \frac{v^2}{c^2} & \frac{v}{c} & \frac{v}{c} & \frac{v}{c} \\
-\frac{v}{c} & -1 & 0 & 0 \\
-\frac{v}{c} & 0 & -1 & 0 \\
-\frac{v}{c} & 0 & 0 & -1 \\
\end{pmatrix}
\]

The contravariant metric tensor $g^{uv}$ has to be derived, if we wish to raise the indices of other tensors. The invariant proper time is given by

\[ d\tau = \sqrt{ds^2/c^2} = dt' \sqrt{1 - (\hat{\tau} + \hat{u})^2/c^2} \].

The Galilean four-momentum is thus

\[ p^\mu = m \frac{dx^\mu}{d\tau} \]

which explicitly reads

\[ p^0 = mc/\gamma \sqrt{1 - (\hat{\tau} + \hat{u})^2/c^2} \]
\[ p^i = mc/\gamma \sqrt{1 - (\hat{\tau} + \hat{u})^2/c^2} \].

Note that $p^0 p_0 = m^2 c^2$ is invariant, and that all Galilean physical quantities reduce to the usual Lorentzian ones in frame S, i.e., for $\hat{\tau} + \hat{u} = 0$.

If $\hat{\tau}$ and $\vec{J}$ are assumed to transform as a four-vector $j^\mu = (\gamma \dot{\tau}, \vec{J})$, then the Maxwell equations can be written in tensorial form "covariant" with respect to transformations (18). As usual the physical electric field $\vec{E}(\hat{\tau})$ and magnetic induction $\vec{B}(\hat{\tau})$ are identified in S with the
components \( F^{0k} \) and \( p^{kj} \) of the antisymmetric tensor \( F^{\mu\nu} \). By transforming \( \tilde{p}^{\mu\nu} \) and the dual tensor \( \tilde{\Sigma}^{\mu\nu} \) by (18) we can write the Maxwell equations in \( S' \) as

\[
F'_{\mu\nu} = \frac{4}{c} j^{\nu}, \quad \tilde{\Sigma}'_{\mu\nu} = 0
\]

which in three dimensions are

\[
\nabla' \cdot \tilde{E} = \gamma^2 \frac{\rho}{c}, \quad \nabla' \cdot \tilde{B} = 0
\]

\[
\nabla' \times (\tilde{E} - \frac{\tilde{v} \times \tilde{B}}{c}) = \gamma^2 \frac{4\pi}{c} j^{\nu} + \frac{1}{c} \frac{\tilde{v}}{\gamma} \times \tilde{E}, \quad \nabla' \times (\tilde{E} + \frac{\tilde{v} \times \tilde{B}}{c}) = -\frac{1}{c^2} \frac{\tilde{v}}{\gamma} \times \tilde{B}
\]

with

\[
j^{\mu}_{\gamma} = \left[ \gamma^{-1} c \rho, \gamma^{-1}(\tilde{J} - \tilde{v} \rho) \right].
\]

Maxwell's equations have to be complemented by the equation of motion for a test charge in \( S' \)

\[
\frac{d \rho_{\mu}}{d\tau} = \frac{\rho}{mc} \frac{\tilde{F}_{\mu\nu} \frac{d \tilde{x}^{\nu}}{d\tau}}{d\tau}
\]

Note that, if frame \( S' \), the tensor components are now connected by the general relation \( p^{\mu\nu} = g^{\mu\nu}(\tilde{v}) F'_{\gamma\delta} g^{\gamma\delta}(\tilde{v}) \). In this framework field equations for spinor fields are exactly the same as for SR, as they are written in \( S \). In \( S' \) the relevant physical quantities will be functions of the matrix \( \mathcal{T}(\tilde{v}) \), defined by \( \tilde{t}^{\mu}_{\mu} = \mathcal{T}^{\mu}_{\gamma}(\tilde{v}) x^{\nu}_{\gamma} \) where \( t^{\mu}_{\mu} \) are Galilean co-ordinates and \( x^{\nu}_{\gamma} \) are Lorentzian co-ordinates. 18

Then, the Dirac equation in \( S' \) takes the form

\[
(i \gamma^{\nu}_{\gamma}(\gamma^0)_{\gamma} \frac{d}{d\tau} - mc) \psi'(\tilde{r}') = 0
\]

where \( \gamma \)-matrices \( \gamma^{\nu}_{\gamma}(\gamma^0) \) and bi-spinors \( \psi'(\tilde{r}') \) are connected to \( \gamma \) and

\[\psi(x') \text{ via } \gamma^{\nu}_{\gamma}(\gamma^0) = \gamma^{\mu}_{\gamma}(\gamma^0) \gamma^{\nu}_{\gamma}(\gamma^0) \text{ and } \psi'(\tilde{r}') = \psi[T(-\tilde{v})x'] \]. We will now extend our considerations to some peculiar e.m. effects which arise from the formalism of this theory. Since the equations of motion for test charge particles are the same as in SR we should expect the same results. Let us now consider a specific phenomenon: the effect of a static electric field on a moving current loop. For SR the predicted effect 26 is that a torque will act on the current loop. Indeed, because of non-conservation of simultaneity in SR, a moving current loop will appear as charge polarized, and this charge polarization will interact with the electric field, thus giving rise to a torque. According to GGT there is conservation of simultaneity and thus, as we can directly infer from (19), a moving current does not generate a charge. Therefore for any ether theory based on GGT there should be no torque acting on a current loop moving through an electric field. The situation is somehow more complicated if the current loop is made of conducting material. In this case the charges induced on the loop by the external electric field will generate with their motion a magnetic induction which in turn acts with a torque on the current loop. 26 The latter effect does not arise if open currents (in this case, the moving induced charges) do not generate a magnetic induction field, i.e. if \( \rho \) and \( \tilde{J} \) do not transform as a four-vector.

In any case, unless the e.m. formalism is substantially modified, the substitution of GGT for LT in e.m. leads, generally speaking, to predictions different from these of SR. For the example just considered, of a non-conducting current loop or magnetic moment in motion through an electric field, the positive effect, interpretable in terms of non-conservation of simultaneity by only SR, 27 can be detected experimentally
by means of magnetic resonance techniques.\textsuperscript{28} However, the predictions related to this type of e.m. effects should properly be made in the framework of a more complete and detailed e.m. formalism, taking into account possible theoretical alternatives suitable for an ether theory.

6. Conclusions

We have considered in this paper some of the conditions for which an ether theory is a possible alternative to SR or an equivalent theory.

In relation to optic phenomena, an anisotropic ether theory, with GCT given by Tangherlini's transformations, turns out to be equivalent to SR simply because clocks are synchronized according to absolute synchronization and not according to Einstein- or slow-clock transport procedure. A locally isotropic ether theory of the Stokes-Planck type cannot be equivalent to SR unless isotropy is extended unlimitedly in space.

In relation to electromagnetic effects, an ether theory based on GCT differs from SR. When a tensorial formalism is developed from GCT, physical quantities, like four-vectors, transform differently than for SR and relativistic effects are not generally reproduced. We have shown that, for an ether theory, the most suitable co-ordinate transformations are the GCT, which differ from LT mainly because of the absolute character of time. In GCT time does not depend on space co-ordinates, and consequently the corresponding Galilean metric tensor possesses a lower symmetry than the Lorentzian one. On theoretical grounds, therefore, SR has a distinct advantage compared to an ether theory based on GCT. However the question whether nature has chosen co-ordinate transformations symmetry as an essential and necessary feature of physical laws or not, has ultimately to be answered by experimental rather than theoretical evidence. In this context we have shown that testing an ether theory vs. SR amounts essentially to testing conservation vs. non-conservation of simultaneity.

Experiments based on some particular e.m. effect could provide a test for an ether theory only if the correspondent e.m. formalism is unambiguously developed. For the purpose of a test, only new experiments on optical phenomena seem to provide, at least at this stage, clearcut results. A possible test for the anisotropic ether theory would consist of a one-way trip optic exp., or of an exp. on aberration anisotropy.\textsuperscript{29} For the locally isotropic, or Stokes-Planck, ether theory the most reliable test would be to perform the MM exp. on a satellite moving close to the earth's surface.\textsuperscript{11}
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