ESSAYS IN LENDING FRictions, THE LABOR MARKET AND MONETARY POLICY

A dissertation submitted in partial satisfaction of the requirements for the degree of

DOCTOR OF PHILOSOPHY

in

ECONOMICS

by

David Florian Hoyle

December 2015

The Dissertation of David Florian Hoyle is approved:

______________________________
Professor Carl E. Walsh, Chair

______________________________
Professor Johanna L. Francis

______________________________
Professor Kenneth M. Kletzer

______________________________
Dean Tyrus Miller
Vice Provost and Dean of Graduate Studies
# Table of Contents

List of Figures v
List of Tables vii
Abstract viii
Acknowledgments ix

1 Introduction 1

2 Gross credit flows and unemployment in a New Keynesian framework - with Johanna Francis-

2.1 Introduction 4

2.2 Empirical evidence on gross credit flows, unemployment and financial shocks 11

2.2.1 Data Description 11

2.2.2 Cyclical properties of gross credit flows and gross job flows 14

2.2.3 A Simple VAR 17

2.3 The model economy 21

2.3.1 Households 22

2.3.2 Firms 30

2.3.3 A decentralized loan market 35

2.3.4 Government 53

2.3.5 Aggregation and Market Clearing 55

2.4 Computation and simulations 61

2.4.1 The non-stochastic steady state 62

2.4.2 Calibration 64

2.4.3 Equilibrium dynamics: Monetary policy and financial shocks 72

2.5 Conclusion 87
3 Monetary policy operating procedures, lending frictions, and employment
- with Christopher Limnios and Carl Walsh-

3.1 Introduction ........................................... 89
3.2 The Model ............................................. 96
  3.2.1 Households ......................................... 99
  3.2.2 The loan market ................................... 102
  3.2.3 The interbank market ............................... 123
  3.2.4 The central bank .................................. 133
  3.2.5 Market Clearing and the Aggregate equilibrium .... 137
3.3 Computation and simulations .......................... 144
  3.3.1 Calibration ........................................ 144
  3.3.2 Model experiments ................................. 148
3.4 Conclusions .......................................... 158

4 A comparative assessment of labor productivity during the Great Recession and the early 2000s recession: How choosier have employers become?
- with Christopher Limnios -

4.1 Introduction ........................................... 162
4.2 Model of the labor market .......................... 169
  4.2.1 Value functions .................................. 169
4.3 Empirical implementation ............................ 175
  4.3.1 Parameterization ................................ 176
  4.3.2 Results ........................................... 178
  4.3.3 Explanation of the results: What has changed? .... 179
4.4 Conclusion ........................................... 183

Bibliography .............................................. 185

A Appendix to chapter 2 ................................. 200
  A.0.1 Summarizing the non-linear equilibrium conditions .... 203
  A.0.2 Introducing absconding and credit rationing into the loan contract 208
  A.0.3 The loan contract ................................ 210

B Appendix to chapter 3 ................................. 215
  B.0.1 Characterization of the aggregate non-linear equilibrium .... 215
List of Figures

2.1 Gross credit flows: Commercial lending .............................................. 17
2.2 Net job creation last two recessions .................................................... 18
2.3 Responses to a financial (EBP) shock .................................................. 20
2.4 Responses to a financial (EBP) shock .................................................. 21
2.5 Model responses to a monetary policy shock: A ..................................... 77
2.6 Model responses to a monetary policy shock: B ..................................... 78
2.7 Model responses to a monetary policy shock: C ..................................... 79
2.8 Model responses to a monetary policy shock: D ..................................... 81
2.9 Model responses to a financial shock: A .............................................. 83
2.10 Model responses to a financial shock: B ............................................. 84
2.11 Model responses to a financial shock: C ............................................. 85
2.12 Model responses to a financial shock: D ............................................. 86
3.1 Model responses to a payment shock: A .............................................. 148
3.2 Model responses to a payment shock: B .............................................. 150
3.3 Model responses to a payment shock: C .............................................. 151
3.4 Model responses to a payment shock: D .............................................. 152
3.5 Model responses to a payment shock: E .............................................. 153
3.6 Model responses to a financial shock: A .............................................. 154
3.7 Model responses to a financial shock: B .............................................. 156
3.8 Model responses to a financial shock: C .............................................. 157
3.9 Model responses to a financial shock: D .............................................. 158
3.10 Model responses to a financial shock: E ............................................. 159
4.1 The time path of the regression coefficient for Okun’s law resulting by “rolling” a sliding interval of 100 observations. 95% confidence bands included. ................................................................. 164
4.2 The time path for the residuals from the Okun’s Law regression. Note the pattern of residual trajectories during all of the NBER recessions and how the pattern was reversed during the Great Recession. ............... 165
4.3 **Source:** Authors regression residuals. Data series for the civilian unemployment rate and the long-term and short-term natural rate of unemployment come from FRED. GDP data comes from FRED, while potential GDP is the Hodrick-Prescott filtered series of the same GDP data with standard $\lambda = 1600$.

4.4 **Source:** Same as figure 2.

4.5 Time series of the model-implied level of $\tilde{\pi}$.

4.6 Implied series (in percentage deviation) for the firm’s implicit cost to posting a vacancy.

4.7 The series for the St. Louis Financial Stress Index and our series for $v$-hat. **Source:** FRED
List of Tables

2.1 Parameters taken from the data and conventional values from the literature 66
2.2 Steady state targets .................................................. 67
2.3 Calibrated parameters to be consistent with steady state targets .... 71
2.4 Steady state values .................................................... 72

3.1 Parameter values taken from the data ................................. 145
3.2 Steady state targets ..................................................... 146
3.3 Calibrated parameter consistent with steady state targets ......... 147

A.1 Descriptive statistics for credit and employment .................... 201
A.2 Standard Deviations and Correlations ............................. 202
Abstract

ESSAYS in lending frictions, The Labor Market and Monetary Policy

by

David Florian Hoyle

The Great Recession of 2008-09 in the U.S. was characterized by high and persistent unemployment and lack of bank lending due to liquidity issues. The recession was preceded by a housing crisis that quickly spread to the banking and broader financial sectors. We attempt to account for the depth and persistence of unemployment by considering the relationship between credit and firm hiring explicitly. To do so, we introduce search and matching frictions to characterize the dynamics of credit markets in a series of macroeconomic models that present different types of distortions in the labor market as well as in the interbank market. We obtain a novel propagation and amplification mechanism of a financial crisis associated to an inefficiency wedge that affects the aggregate production function of the economy and depends directly on credit conditions.
I would like to express my sincere gratitude to my advisers Prof. Carl E. Walsh and Prof. Johanna L. Francis for their continuous support, motivation and patience during all the stages of this dissertation. I also wish to thank Chris Limnios for collaboration and comments.

Two professors change my way of thinking about economics, the social sciences and mathematics during my years as an undergraduate student: Oscar Dancourt and Ramon Garcia-Cobian. In their own way, Oscar and Ramon encouraged me to pursue doctoral studies. I am truly grateful to both of them.

I gratefully acknowledge the funding sources that made my Ph.D. work possible. I was funded by the Fulbright Scholarship Program for my first two years and by the Central Reserve Bank of Peru during all my doctoral studies. I was awarded UCSC funding for graduate studies for years 2 through 5.

My time at UCSC was made enjoyable in large part due to the many friends and groups that became a part of my life. I am grateful for time spent with friends such as Linh, Jacopo, Yabin, Chris, Nick, Yuhan, Kevin, Maggie, Pia, Manuel, Arsenyi, Benjamin, Ciril and so on.

Lastly, I would like to thank my family for all their love and encouragement. For my mom, Nelly who raised me with a love for knowledge and supported me in all my pursuits. And most of all for my loving, supportive, encouraging, and patient wife, Gaby, whose faithful support during all the stages of my doctoral studies is so
appreciated. Thank you.
Chapter 1

Introduction

The Great Recession of 2008-09 in the U.S. was characterized by high and persistent unemployment and lack of bank lending due to liquidity issues. The recession was preceded by a housing crisis that quickly spread to the banking and broader financial sectors. In the second chapter of this dissertation, we attempt to account for the depth and persistence of unemployment by considering the relationship between credit and firm hiring explicitly. To understand this relationship, we first use a simple structural vector autoregression framework and historical data on gross credit flows in the U.S. economy, to compare the effects of financial versus monetary policy shocks on credit and unemployment. We find that financial shocks generate a more persistent response in unemployment than monetary policy shocks. We find that financial shocks are propagated through gross credit flows to labor productivity due to the linkage between credit and employment. In order to investigate these links, we develop a New Keynesian model with nominal rigidities in wages and prices and unemployment augmented
by a banking sector characterized by search and matching frictions with endogenous credit destruction. We assume that credit contracts are negotiated bilaterally as a Nash bargaining protocol between borrowers and lenders. In the model, financial shocks are propagated and amplified through significant variation in the credit inefficiency gap arising from search and matching frictions, over the cycle. Depending on the size of the financial shock, the model is able to generate a deep and prolonged recession with a substantial prolonged increase in unemployment. A calibrated version of this model is able to reproduce the conditional responses of unemployment, labor productivity, inflation and gross credit flows to a financial shock for the U.S. economy.

The third chapter studies a channel system for implementing monetary policy when bank lending is subject to frictions. These frictions affect the spread between the policy rate and the loan rate. We show how the width of the channel, the nature of random payment flows in the interbank market and the presence of frictions in the loan market affect the propagation of financial shocks that originate in the interbank market as well as in the loan market. We study the transmission mechanism of two different financial shocks: 1) An increase in the volatility of the payment shock that banks face once the interbank market is closed and 2) An exogenous termination of loan contracts that directly affect the probability of survival of credit relationships. Both financial shocks are propagated through the interaction of the marginal value of having excess reserves as collateral relative to other bank assets, the real marginal cost of labor for all active firms and the reservation productivity that selects the mass of producing firms. Our results suggest that financial shocks produce a reallocation of bank assets towards
excess reserves and intensive and extensive margin effects over employment. The aggregation of those effects produce deep and prolonged recessions that are associated to fluctuations in an inefficiency wedge that appears in the aggregate production function of the economy. We show that this wedge depends on credit conditions and on the mass of producing firms.

Finally, in the forth chapter, we analyze the implied productivity of existing and new employment matches during the wake of both the early 2000s recession and the Great Recession by using a variation of the Mortensen-Pissarides model of decentralized labor markets. We find that surviving and new employment matches during the Great Recession exhibit a productivity level 2.16 times higher than those during the early 2000s recession. We show that the increased idiosyncratic productivity threshold in the newly-formed matches was largely the result of the increase in the labor financing costs facing firms originating from disruptions to credit markets during the Great Recession.
Chapter 2

Gross credit flows and unemployment in a New Keynesian framework

- with Johanna Francis -

2.1 Introduction

The Great Recession and slow recovery was characterized domestically by a deep and prolonged decline in GDP, high and persistent unemployment, an increase in financial volatility, and a decline in overall bank lending including commercial and industrial lending. The net decline in bank lending in all loan categories—including to consumers, to firms, and for real estate related reasons—was a novel feature of the Great Recession among previous post-Volker recessions.

These characteristics highlight the potentially critical relationship between bank credit and unemployment but the propagation mechanism from the financial crisis
to unemployment is not clear. Some recent evidence points to the fact that borrowers and lenders form long term credit relationships which when disrupted take time to re-form (Chodorow-Reich 2014). For large firms, as Adrian, Colla, and Shin (2012) demonstrate, banking relationships may be less important as these firms have multiple sources of debt finance and so compensated for the decline in bank lending by bond issuance.

In this paper, we examine a potential mechanism for monetary or financial shocks to impact the employment decisions of firms and thereby aggregate unemployment indirectly through their effect on the bank loans. We model frictions in the loan market using a search and matching framework. Interestingly, we find that frictions in lending impact unemployment through a variety of mechanisms in our model. First, they change the number of firms with profitable projects who can produce by altering the productivity cutoff. Second, they impact the continuation probability for individual credit contracts. If the continuation probability declines resulting in the separation of firms and banks, firms need to search again for a credit contract before being able to hire workers and engage in production. Third labor productivity is affected by the credit frictions.

Before we develop our theoretical model, we consider the effect of financial shocks on unemployment and output and the amplification of such shocks through their effect on gross credit flows and labor productivity in a vector auto-regression (VAR) framework.¹ In this simple VAR, financial shocks are more persistent, producing

¹Credit flows can be decomposed into two parts, credit creation and credit destruction, the difference of which is net credit growth. The pair of credit creation and destruction are referred to as ‘gross credit
larger movements in unemployment and output than monetary policy shocks. We use this suggestive result as motivation to introduce gross credit flows via credit market frictions into a New Keynesian model with nominal rigidities and unemployment. By introducing credit frictions with endogenous credit destruction into this well studied model, we provide additional insights on movements in employment and output following a financial shock. Notably, allowing for endogenous credit destruction permits us to calculate movements in gross flows in a theoretical setting.

The literature on the relationship between credit and macroeconomic aggregates is large and growing. Herrera, Kolar and Minetti (2014), Craig and Haubrich (2013) and Contessi and Francis (2015) were some of the first papers to consider the relationship between macro-aggregates and gross credit flows, particularly banking flows. These papers look at gross lending flows rather than changes in net credit availability using a methodology developed in David, Haltiwanger and Schuh (1996) more commonly used for understanding labor flows, thereby accounting for simultaneous increases and decreases in lending across banks. They also find that sizable gross credit flows coexist at business cycle frequencies. This literature emphasizes the existence of heterogeneous patterns of credit creation and contraction at any phase of the business cycle. For example, Dell’ Ariccia and Garibaldi (2005) and Contensi and Francis (2013) find that in the U.S., gross credit flows are much more volatile than GDP and investment by an order of magnitude. The dynamic patterns of credit flows found in this empirical literature are consistent with those predicted by search models in which the interaction of shocks.
generates simultaneous credit expansions and contractions.

On the theoretical side, there is also a growing literature that discusses the role of matching frictions in credit markets and its associated amplification and propagation mechanism. Some examples are Becsi, Li and Wang (2005), Becsi, Li and Wang (2013), den Haan, Ramey and Watson (2003), Wasmer and Weil (2004), and Petrosky-Nadeu and Wasmer (2014). These authors build highly stylized macroeconomic models that incorporate an aggregate matching function to characterize the search-and-matching process between borrowers and lenders in the loan market. The models in Becsi, Li and Wang (2005), Wasmer and Weil (2004), and Petrosky-Nadeu and Wasmer (2014) all employ a period by period Nash bargaining protocol when solving for the credit contract. den Haan, Ramey and Watson (2003) consider an agency-cost type of model with exogenous matching rates while Becsi, Li and Wang (2013) introduce credit rationing and asymmetric information into the Nash bargaining protocol. Several recent papers, including Petrosky-Nadeu (2014) and Petrosky-Nadeu and Wasmer (2014) study matching frictions in both credit and labor markets simultaneously using a double search approach.

In this paper, we incorporate a search-and-matching process between borrowers (firms) and lenders (banks) into a New Keynesian model with sticky wages and unemployment. The financial contract and the bank loan interest rate are an outcome of a Nash bargaining protocol. Idiosyncratic shocks to the firms’ productivity level determine the rate of endogenous match separation. That is, banks with funds and firms with projects search for partners in the credit market. Banks obtain funds by raising
retail deposits in a competitive market. To produce, an individual firm must obtain external funding first, which can only be obtained if the firm is matched with a bank. Unmatched banks search for lending opportunities while unmatched firms search for a bank to finance their wage bill in advance of production. Matched banks and firms decide whether to maintain or sever their credit relationship, depending on the idiosyncratic productivity of the firm’s project. If the firm and the bank choose to cooperate, a loan contract is created and Nash bargaining determines how the joint surplus of the match is shared. The conditions of the loan contract are characterized by a match-specific loan principal and a credit interest rate. In equilibrium, there is a productivity threshold (reservation productivity) such that only those firms with an idiosyncratic productivity level above this threshold are able to produce. Thus, aggregate equilibrium is characterized by a distribution of active production units as well as a distribution of match-specific loan rates.

The search and matching friction in the loan market produces an endogenous inefficiency wedge that appears as an additional input in the aggregate production function. This inefficiency wedge depends on the aggregate probability of continuation for a loan contract as well as on the mass of active producing firms. Financial shocks as well as other aggregate shocks are transmitted and amplified by a first order effect in this credit inefficiency gap that affects aggregate labor productivity and technology.

Our model presents a cost channel of monetary policy similar to Ravenna and Walsh (2006), in which firms must finance wage payments in advance of production. The standard implication of the cost channel is that the relevant cost of labor is affected
by the interest rate firms pay on loans. However, when the loan rate is the outcome of a bargaining process, as it is in our model, its role is to split the surplus between the borrower (the firm) and the lender (the bank). In this context, the loan rate is irrelevant for the firm’s employment decision which ultimately is a consequence of the Nash bargaining solution and the conditions of the credit contract. In our model, there is still a cost channel but it depends on the opportunity cost of funds to the bank, not the interest rate charged on the loan. Therefore, changes in monetary policy will influence this outside opportunity cost and affect the real marginal cost of production, employment and the equilibrium spread between the average rate on bank loans and the policy interest rate. By the same token, the threshold productivity level, below which the firm is unable to obtain financing, depends on monetary policy. By allowing for entry and exit by banks as well as for active and inactive firms, monetary policy has effects on employment and output on both extensive and intensive margins. The latter arises as a reduction in the cost of funds for banks makes it optimal for firms with access to credit to expand employment. The former arises because the lower cost of finance will make it profitable for more firms produce.

We model unemployment similarly to Blanchard and Gali (2010), Gali (2010), Gali (2011) and Gali, Smets and Wouters (2012) as a reinterpretation of the labor market in the standard New Keynesian model with staggered wage setting as in Erceg, Henderson and Levine (2000). In this context, each household member is indexed by the labor type she is specialized in and by a particular labor disutility index that her labor-type generates if she is employed. Therefore, labor is differentiated and each household
member has market power to set its wage according to a Calvo pricing scheme. The labor force is introduced as an aggregate participation condition for the marginal labor supplier in each labor type category. The existence of market power and wage rigidities produce a gap between aggregate labor demand and the labor force. This gap is related to the difference between the prevailing aggregate real wage and the average disutility of labor expressed in terms of consumption and is positively related to the unemployment rate. Credit conditions affect the marginal cost of labor as well as aggregate labor demand by producing an extensive and intensive margin effect (discussed below). The labor force is also affected by credit conditions via the aggregate marginal rate of substitution of the marginal supplier of labor.

We first provide some motivating empirical evidence on the business cycle properties of gross credit flows as well as the relationship between gross credit flows and unemployment as a conditional response to a financial shock. For the latter we use a simple recursive vector auto-regression. We then develop a New Keynesian model with price and wage frictions as well as search and matching frictions in the loan market to match the decline in output and employment following monetary policy and financial shocks and elucidate the mechanisms through which shocks affecting credit are propagated to the labor market and productivity. The appendix includes an extension of the model to the case where firms can divert loans to unproductive uses.
2.2 Empirical evidence on gross credit flows, unemployment and financial shocks

In this section we present two sets of empirical evidence regarding job flows and credit. The first is a set of unconditional moments and business cycle properties of gross credit flows and gross job flows. We consider the properties of gross job flows in this section even though our model uses net flows as it is useful to consider how changes in either job creation or destruction can generate similar net job creation. Net job creation is ultimately related to employment flows which we consider explicitly in our theoretical framework. The second is the set of conditional responses of gross credit flows, inflation, employment and unemployment to a financial shock within a recursively identified VAR. We use these two sets of analyses to motivate the theoretical model we develop below.

2.2.1 Data Description

The macroeconomic data used in our analysis is available from public sources. Real GDP $Y$, the GDP deflator, the measure of employment (which is defined as the log of workers in the non-farm business sector and denoted by ln $N$), average labor productivity calculated as ln $\frac{Y}{N}$, the unemployment rate and the federal funds rate $FFR$, are all downloaded from the FRED repository at the Federal Reserve Bank of St. Louis. The remaining variables—credit flows, job flows, and the financial shock—are described below.
The excess bond premium (EBP), sometimes referred to as the ‘GZ credit spread’ is taken from Gilchrist and Zakrajšek (2012). It is constructed from the credit spread of U.S. non-financial corporate bonds over Treasury bills and decomposed into two parts. The first is a component that captures systematic movements in the default risk of individual firms measured using Merton (1974) distance to default model. The second is a residual component, the excess bond premium, which is the variation in the average price of bearing exposure to U.S. corporate credit risk that is not otherwise compensated for by the expected default premium. The EBP has been used in many recent papers as a measure of financial risk (for example, Boivin, Kiley and Mishkin (2010) and Christiano, Motto and Rostagno (2013). It is a good measure of unanticipated changes in financial markets.

The last set of data we use is a measure of credit availability in the economy derived from information in the Reports of Income and Condition. The Reports of Income and Condition, known as the Call Reports, must be filed every quarter by every bank and savings institution overseen by the Federal Reserve (or i.e., those who hold a charter with the Federal Reserve). These reports contain a variety of information from banks’ income statements and balance sheets. We use quarterly reported total loans and lending to commercial and industrial enterprises to create measures of credit creation and destruction. Call Report data is available beginning in 1979Q1. We use an additional 24 quarters of historical data from Craig and Houbrich (2013). Lending data is then linked to data from the National Information Center (NIC) on mergers and acquisitions during this period. Using the M&A data from the NIC, we can
remove the bias that might arise from counting lending activity at both the acquired
and acquiring bank (See Contessi and Francis (2013) for a full discussion of how these
data are compiled). We then remove seven investment banks and credit card companies
that acquired commercial bank charters during the 2008-09 recession and financial crisis.

In order to determine ‘gross credit flows’ we use a technique first adapted from
the labor literature in Dell’Ariccia and Garibaldi (2005) for credit flows. Define \( l_{i,t} \) : as
total loans for bank \( i \) in quarter \( t \). Let \( g_{i,t} \) be the credit growth rate for bank \( i \) between
\( t \) and \( t - 1 \), adjusted for any mergers or acquisitions. Then we can define:

\[
POS_t = \sum_{i \mid g_{i,t} \geq 0} N \sum_{i} \alpha_{i,t} g_{i,t}
\]

\[
NEG_t = \sum_{i \mid g_{i,t} < 0} N \sum_{i} \alpha_{i,t} |g_{i,t}|
\]

where

\[
\alpha_{i,t} = \left( 0.5 \left( \frac{l_{i,t} + l_{i,t-1}}{2} \right) \right)
\]

and \( \frac{l_{i,t} + l_{i,t-1}}{2} \) is a measure of the average loan portfolio size of bank \( i \) between period \( t \)
and \( t - 1 \) and where \( \sum_{i=1}^{N} l_{i,t-1} \) is the loan portfolio of the banking system in the previous
period.

Given these measures of credit creation (POS) and credit destruction (NEG),
we can define net lending as \( NET = POS - NEG \).
2.2.2 Cyclical properties of gross credit flows and gross job flows

Although we primarily consider net job flows in this paper by focusing on employment and unemployment, rather than gross job flows, the data on gross job flows is useful to consider when investigating the relationship between credit and unemployment. Table (1) provides the means and standard deviations of lending and job flow variables for our entire sample (1973Q1 to 2012Q4) and three sub-periods—the Great Moderation, 1984Q1 to 2007Q2; the Great Recession, 2007Q3 to 2009Q2; and the post-Recession period, 2009Q3 to 2012Q4. Three features of these summary statistics are notable. First, the mean of loan creation plus loan destruction is significantly larger during the Great Recession than during other periods and this is driven by an increase in loan destruction. The sum of loan creation and destruction is a measure of ‘churning’ in the banking sector. Second, the mean value for net loan creation during each sub-period is positive, though very small during the post-Great Recession period. Third, excess loan creation (the difference between the SUM and the absolute value of the NET) is the largest during the Great Moderation. Excess loan creation (EXC) measures credit reallocation in excess of what is required to accommodate a change in net credit. A positive change in net credit can occur because of an increase in loan creation or a decrease in loan destruction or some combination and similarly for a negative change.

We also find that mean job creation is higher during the Great Moderation than any other period, while job destruction is higher during the Great Recession than other sub-periods implying that there was a significant amount of reallocation of workers
across firms and potentially industries during this period. Interestingly, the mean of net job creation is negative during both the Great Moderation and the Great Recession.

In Table (2) we provide relative standard deviations of bank lending flows (total loan creation and destruction) and job flows (job creation and job destruction) as well as their correlation with the cyclical component of either real GDP or unemployment rates for our entire sample period (1973Q1 to 2012Q4) and three sub-periods (detailed above). We use the HP-filtered log-level of each variable in determining their relative standard deviations and correlations.

We find that loan creation and destruction are much more volatile than real GDP and that the volatility of loan destruction increased significantly during the post Great Recession. Job creation and job destruction are much less volatile than unemployment. Job creation and destruction are approximately 5 and 9 times more volatile than real GDP and this relative volatility widens during the Great Recession. Job creation is approximately one quarter as volatile as loan creation and even less volatile post-Great Recession. Job destruction is approximately one third as volatile as loan destruction, though it is more volatile during the Great Recession and much less post the recession.

In terms of correlations, we find that the cyclical component of loan creation is positively correlated with real GDP in all periods, though the correlation is much strong post-recession than during any other period. Loan creation is essentially acyclic during the Great Moderation. Loan destruction, as expected, is counter-cyclical, though the counter-cyclicality is primarily driven by the Great Recession and post Great Recession period. The sum of loan creation and destruction, a measure of general loan turnover,
is acyclic.

Job creation is negatively correlated with unemployment, primarily during the Great Recession but positively correlated post recession. This last unusual result could be due to the fact that unemployment did not begin a constant decline until 2010Q4 despite the fact that job creation began to increase, relatively smoothly, beginning in 2009Q2. Job destruction is positively correlated with unemployment during our entire sample and throughout each subsample, though it is most strongly correlated during the Great Recession. The sum of job creation and job destruction, a measure of labor market churn, is positively correlated with unemployment throughout the sample and sub-samples, aside from during the Great Moderation when it is not significant. The positive correlation is strongest during the Great Recession.

Figure (1) displays gross credit flows for commercial and industrial lending during our sample period. During recessions (shaded bars on the figure), typically credit destruction increases while credit creation declines. Similarly, gross job flows demonstrate a cyclical pattern, where job creation slows during recessions and job destruction increases to some degree.

During the Great Recession, job destruction increased by much more than during previous recessions (percentage wise) and was a much larger contributor to the increase in unemployment than during previous recessions. The job creation rate was also depressed for much longer which reduced the job finding rate more significantly than in previous recessions. Figure (2) focuses on the last two recessions, depicting net job creation from the mid-1990s through 2012. Net job creation declines dramatically
2.2.3 A Simple VAR

We use the following standard reduced form VAR:

\[ Z_t = A(L)Z_{t-1} + v_t \]

where \( A(L) = A_1 + A_2 L + \cdots + A_p L^p, p < \infty, \) \( v_t \) are the reduced form residuals which are related to the structural shocks \( \epsilon_t \) via the structural matrix \( A_0 \) where \( \epsilon_t = A_0 v_t \) and with variance covariance matrix \( E[v_tv_t'] = V. \) We consider two shocks, monetary policy and financial shocks. Both are identified via contemporaneous restrictions. As in Christiano, Eichenbaum, and Evans (1999), variables ordered before the Federal Funds rate in the VAR do not respond contemporaneously to monetary policy shocks. Similarly, using
Figure 2.2: Net job creation last two recessions

Note: Source: Authors’ calculations based on Business Employment Dynamics data from the U.S. Census.

the same identification as Gilchrist and Zakrajsek (2012), variables ordered before the financial shock do not respond contemporaneously to financial shocks.

The data used in our SVAR analysis is the following:

\[ Z_t = \left[ \Delta \ln \frac{Y_t}{N_t}, \Delta \ln N_t, U_t, \pi_t, EBP_t, i_t, GCF_t \right] \]

The variables include the growth rate of labor productivity \( \Delta \ln \frac{Y_t}{N_t} \), the growth rate of employment \( \Delta \ln N_t \), the unemployment rate \( U_t \), GDP deflator as a measure of inflation \( \pi_t \), the excess bond premium \( EBP_t \) as a measure of the financial shock,
the Federal Funds rate, \( i_t \), as the measure of the monetary policy stance and a pair of gross credit flows rates (credit creation and credit destruction rate) denoted by \( GCF_t = [POS_t, NEG_t] \). Each data series is seasonally adjusted using X-12 ARIMA procedure. We have not explicitly taken into consideration quantitative easing enacted by the Federal Reserve, therefore our measure of the monetary policy stance under-represents the looseness of monetary policy. Since monetary policy is not the focus of this paper, we have not used various measures of the ‘shadow’ Federal Funds rate (see for example, Krippner (2013) among others).

As noted above, the VAR is identified using a recursive ordering such that the last ordered variable responds contemporaneously to all shocks. This implies that the model is exactly identified.

Figures 3 and 4 present the impulse responses to a one standard deviation increase in the EBP as the measure of a financial shock. The first figure shows the responses of labor productivity, employment, unemployment and inflation to a financial shock. The responses are consistent with Gilchrist and Zakrajsek (2012). Employment and unemployment as well as inflation respond sluggishly—and persistently—to the shock, with unemployment peaking at roughly 6 quarters post shock and returning to baseline within 12 to 14 quarters. The next figure depicts the response of credit. There are two notable features of this panel of impulse responses. First, the initial response of credit creation to a financial shock (1 standard deviation increase in the EBP), is to increase. Note that due to the ordering of the VAR, credit flows can respond contemporaneously to financial and monetary shocks.
The increase in credit creation is likely due to firms’ and consumers’ draw down of credit commitments, such as lines of credit, which creates an initial increase in lending even though few new loans are actually extended. Credit destruction initially falls as well but the drop is not statistically significant. This surprising decline in credit destruction initially is also possibly driven by firms’ and consumers’ use of credit lines. After the initial impact, credit creation declines quickly and credit destruction rises. Credit creation reaches a trough after approximately four quarters, but rises slowly over the next ten quarters to return to baseline so that a full recovery in lending activity is not observed for three to four years following a financial shock. Similarly, credit destruction rates peak at approximately four quarters but slowly return to baseline after approximately ten quarters.
2.3 The model economy

The model economy is populated by households, banks, firms, and a central bank. Households supply labor to firms, hold cash and bank deposits, and purchase final output in the goods market. Firms seek financing, hire labor financed by bank loans and produce output. Banks accept deposits, sometimes hold reserves with the central bank and finance the wage bill of firms. The central bank pays interest on reserve deposits for those banks that were not able to extend a loan. Three aspects of the model are of critical importance. First, it is assumed that households cannot lend directly to firms. While this type of market segmentation is taken as exogenous, one could easily motivate it by assuming informational asymmetries under which households
are unable to monitor firms while banks are able to do so. This asymmetry also forces firms to make up-front payments to workers to secure labor. Second, lending activity involving firms and banks occurs in a decentralized market characterized by random matching. And third, we assume all payment flows must be settled at the end of each period. At the beginning of each period, aggregate shocks are realized and households deposit funds with a bank. The market for deposits is competitive and all banks offer the same interest rate on deposits. In the lending market, firms seek funding to make wage payments. Firms are subject to aggregate and idiosyncratic productivity shocks and these determine whether it is profitable for a firm to operate and, if it is, at what scale. If a firm is not already matched with a bank, it must seek out a new lender. Similarly, banks not already matched with a firm must search for borrowers. After the loan market closes, firms and workers produce and households consume, while not matched banks deposit its funds with the central bank and receive an interest rate that matches the interest rate on deposits. After these markets close, all net payment flows are settled. Therefore, loans are not risky and there is no possibility of default. At the end of the period, the bank transfers all its profits (positive or negative) to the representative household. Banks receive repayment from firms.

2.3.1 Households

Each household has a continuum of members. Following Gali (2011), each household member is represented by the unit square and indexed by \((i, j) \in (0, 1)^2\). Where \(i\) denotes the type of labor service in which a given household member is spe-
cialized and \( j \) determines the dis-utility from work for each household member. The dis-utility from work is given by \( \chi_{t,j}^{\varphi} \) if employed and zero otherwise with \( \chi_t \) being an exogenous preference shifter. As it is standard in the unemployment literature, we assume full risk sharing of consumption among household members. Utility from consumption is separable and logarithmic in a CES index of the quantities consumed of the different goods available. Given separability of preferences between consumption and dis-utility from work full risk sharing implies

\[
C_t(i,j) = C_t \quad \forall i,j
\]

where \( C_t(i,j) \) is the consumption for a household member specialized in labor type \( i \) and having dis-utility of work \( \chi_{t,j}^{\varphi} \). Each household member has a period utility function given by

\[
U(C_t, j) = \log C_t - 1_t(i,j) \chi_{t,j}^{\varphi}
\]

aggregating across all household members yield the household period utility function denoted by \( U(C_t, N_t(i), \chi_t) \) and given by:

\[
U(C_t, N_t(i), \chi_t) = \log C_t - \int_0^1 \int_0^1 1_t(i,j) \chi_{t,j}^{\varphi} djdi
\]

\[
= \log C_t - \chi_t \int_0^{N_t(i)} \int_0^1 j^{\varphi} djdi
\]

\[
= \log C_t - \chi_t \int_0^{N_t(i)} \frac{\varphi+1}{1+\varphi} di
\]
where $N_t(i)$ is the fraction of household members specialized in labor type $i$ who are employed during the period. In other words, $N_t(i)$ is the employment rate or aggregate demand during period $t$ among workers specialized in labor type $i$.

**The CIA and Budget constraint**  We assume the household enters the period with money holdings given by $M_{t-1}$ and deposits a fraction of its money holdings, denoted by $D_t$ in the bank. Each household receives a lump-sum transfer $P_tT_t$ from the government. Employed household members are paid in advance their labor income. The household labor income given by $\int_0^1 W_t(i) N_t(i) di$. The household uses its labor income and its money holdings net of deposits to buy a continuum of final goods facing the following CIA constraint expressed in nominal terms

$$\int_0^1 P_t(j) C_t(j) \, dj \leq M_{t-1} + T_t - D_t + \int_0^1 W_t(i) N_t(i) \, di$$

where $P_t$ denotes the aggregate price index of the final good. Aggregate household consumption is given by the standard CES aggregator

$$C_t = \left( \int_0^1 C_t(j)^{\frac{\sigma_p-1}{\sigma_p}} \, dj \right)^{\frac{\sigma_p}{\sigma_p-1}}$$

The end of the period money holdings are given by

$$M_t = (1 + i_t) D_t + \Pi_t^b + \Pi_t^f + \Pi_{t-1} + T_t - D_t + \int_0^1 W_t(i) N_t(i) \, di - \int_0^1 P_t(j) C_t(j) \, dz$$

where $i_t$ is the nominal net interest rate on deposits, $\Pi_t^b + \Pi_t^f + \Pi_{t-1}$ are nominal profits transferred by banks, intermediate and final good producers. Household consumption
optimization imply the following demand schedule for each differentiated good $j$:

$$C_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon_p} C_t$$

such that

$$\int_0^1 P_t(j) C_t(j) \, dz = P_t C_t,$$

where $P_t$ is the final goods price index (i.e., the retail price index)

$$P_t = \left( \int_0^1 P_t(j)^{1-\epsilon_p} \, dj \right)^{\frac{1}{1-\epsilon_p}}$$

**The household problem**

The problem for the representative household is to maximize the expected value of its lifetime utility subject to the CIA constraint and the sequence of budget constraints. The representative household problem is given by

$$\max \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t - \chi t \int_0^1 N_t(i)^{\bar{\pi}+1} \frac{1}{1+\bar{\pi}} \, di \right\}$$

s.t

$$P_t C_t \leq M_{t-1} + P_t T_t - D_t + \int_0^1 W_t(i) N_t(i) \, di$$

$$M_t = (1 + i_t) D_t + \Pi_t^b + \Pi_t^f + M_{t-1} - D_t + \int_0^1 W_t(i) N_t(i) \, di - P_t C_t$$

taking as given the distribution of wages $\{W_t(i)\}_{\forall i}$ and employed household members for each labor type $\{N_t(i)\}_{\forall i}$, the household optimality condition is given by

$$\frac{1}{C_t} = \beta E_t \left\{ \frac{1 + \bar{i}_t}{1 + \bar{\pi}_{t+1}} \frac{1}{C_{t+1}} \right\}$$
where the marginal utility of consumption is equal to

\[
\frac{1}{C_t} = \lambda_t + \mu_t
\]

and \(\lambda_t\) and \(\mu_t\) are the multipliers associated to the budget constraint and the CIA constraint. In this context the stochastic discount factor is distorted by the nominal interest rate and it is given by

\[
\Delta_{t,t+1} = \beta \left( \frac{1 + i_t}{1 + i_{t+1}} \frac{C_t}{C_{t+1}} \right)
\]

**Wage setting**  Workers specialized in a given type of labor, reset their nominal wage with probability \(1 - \theta_w\) each period. Following Erceg at al 2000, when re-optimizing wages during period \(t\), workers choose a wage \(W^*_t\) in order to maximize their household utility taking as given all aggregate variables. Household workers of type \(i\) face a sequence of labor demand schedules of the form

\[
N_t(i) = \left( \frac{W_t(i)}{W_t} \right)^{-\epsilon_w} \int z N_t(z) \, dz
\]

where \(W_t\) denote the aggregate wage index

\[
W_t = \left( \int_0^1 W_t(i)^{1-\epsilon_w} \, di \right)^{\frac{1}{1-\epsilon_w}}
\]

and \(\int z N_t(z) \, dz\) denotes the aggregate labor demand across all active intermediate good producers indexed by \(z\). The wage setting optimization problem for the household workers of type \(i\) is specified as

\[
\max_{W_t} \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} (\beta \theta_w)^k \left\{ \log C_{t+k} - \frac{1}{1 + \phi} \int \frac{N_{t+k}^z}{1 + \phi} \, dz \right\} \right\}
\]
\[ N_{t+k|t} = \left( \frac{W_t^*}{W_{t+k}} \right)^{-\varepsilon_w} \int \limits_z N_{t+k}(z) \, dz \]

as well as subject to the CIA and budget constraint. Notice that \( N_{t+k|t} \) denotes the quantity demanded in period \( t + k \) of a labor type whose wage was last reset in period \( t \).

The first order condition for wage setting is

\[
E_t^\infty \sum_{k=0}^\infty (\beta \theta_w)^k \left\{ \left( \frac{W_t^*}{P_{t+k}} - \frac{\varepsilon_w}{(\varepsilon_w - 1)} MRS_{t+k|t} \right) \left( \frac{N_{t+k|t}}{C_{t+k}} \right) \right\} = 0
\]

where \( MRS_{t+k|t} \) denotes the marginal rate of substitution between consumption and employment for a labor type worker whose wage is reset during period \( t \) and it is given by

\[
MRS_{t+k|t} = C_{t+k} \chi_{t+k} \left( N_{t+k|t} \right)^\nu
\]

under Calvo wage setting, the aggregate wage index in real terms is

\[
1 = \theta_w \left( \frac{w_{t-1}}{w_t} \right)^{1-\varepsilon_w} + (1 - \theta_w) \left( \frac{w_t^*}{w_t} \right)^{1-\varepsilon_w}
\]

The recursive formulation of the wage setting optimality condition expressed in terms of the real wage is given by the following equations:

\[
f_{1,t} = \left( \frac{\varepsilon_w}{\varepsilon_w - 1} \right) f_{2,t}
\]

\[
f_{1,t} = (w_t^*)^{1-\varepsilon_w} (w_t)\varepsilon_w \frac{N_t}{\Delta^w_t} + \beta \theta_w E_t \left( \frac{1}{\Pi_{t+1}} \right)^{1-\varepsilon_w} \left( \frac{w_t^*}{w_{t+1}} \right)^{1-\varepsilon_w} f_{1,t+1}
\]

\[
f_{2,t} = \chi_t \left( \frac{w_t^*}{w_t} \right)^{-\varepsilon_w(1+\varphi)} \left( \frac{N_t}{\Delta^w_t} \right)^{(1+\varphi)} + (\beta \theta_w) E_t \left( \frac{1}{\Pi_{t+1}} \right)^{-\varepsilon_w(1+\varphi)} \left( \frac{w_t^*}{w_{t+1}} \right)^{-\varepsilon_w(1+\varphi)} f_{2,t+1}
\]

where \( w_t^* \) is the optimal real wage, \( f_{1,t} \) and \( f_{2,t} \) are auxiliary variables and \( \Delta^W_t \) is the wage dispersion index given by

\[
\Delta^W_t = \int \limits_0^1 \left( \frac{W_t(j)}{W_t} \right)^{-\varepsilon_w} dj
\]

which can be written recursively.
\[ \Delta_t^w = \theta_w \left( \frac{w_{t-1}}{w_t} \frac{1}{\Pi_t} \right)^{-\epsilon_w} \Delta_{t-1}^w + (1 - \theta_w) \left( \frac{w^*_t}{w_t} \right)^{-\epsilon_w} \]

**Unemployment dynamics**

Following Gali (2010), consider the dis-utility of work for a worker specialized in type \( i \) labor

\[ \chi_t^{ij} \]

Using household welfare as a criterion and taking as given current aggregate labor market conditions, the worker will be willing to work in period \( t \) if and only if the real wage is higher or equal than the dis-utility of labor relative to the marginal value of income \( \lambda_t \), that is

\[ \frac{W_t(i)}{P_t} \geq \frac{\chi_t^{ij}}{\lambda_t} = C_t \chi_t^{ij} \]

since \( \lambda_t = \frac{1}{(1+i_t)C_t} \), the above condition can be written as

\[ \frac{W_t(i)}{P_t} \geq (1 + i_t) C_t \chi_t^{ij} \]

Let \( L_t(i) \) be the marginal supplier of type \( i \) labor, that is, a worker of type \( i \) that is indifferent between working or being unemployed. The marginal supplier of type \( i \) labor satisfy the following condition

\[ \frac{W_t(i)}{P_t} = (1 + i_t) C_t \chi_t (L_t(i))^{\overline{\varphi}} \]

The labor force is defined as integrating over all the marginal suppliers, that is, inte-
grating over all the workers that are willing to work at the margin:

\[ L_t = \int_0^1 L_t(i) \, di \]

and the aggregate supply of labor is defined as

\[ w_t = (1 + i_t^d) \, C_t \chi_t \left( L_t \right)^\psi \]

where \( w_t \) denotes the aggregate real wage of the economy.

On the other hand, the unemployment rate is defined as

\[ U_t = 1 - \frac{N_t}{L_t} \]

with \( N_t \) being aggregate employment which corresponds to the following index

\[ N_t = \int z \int_0^1 N_t(i, \omega_{z,t}) \, didz \]

where as explained below, \( N_t(i, \omega_{z,t}) \) is the demand for labor type \( i \) by the intermediate producer \( z \). The main result of the model is to show that \( N_t \) depends on the endogenous probability of finding a credit relationship, the number of firms that are actually producing during the period, the average productivity of those firms and the dispersion of wages generated in the labor market. In this model, the number of firms producing during the period is the number of firms that are in an active loan contract with a bank. Therefore, unemployment fluctuations are related to credit conditions as well as to the stance of monetary policy.
2.3.2 Firms

The production side of the model is characterized by a two-sector structure that distinguish between intermediate and final good producers as in Walsh (2003, 2005). Firms in the intermediate good sector need to be in a credit relationship with a bank before production takes place. Only the subset of intermediate good producers that obtain funding will hire workers and produce in a competitive market taking the price of the good they produce as given. Each producing firm in the intermediate good sector hire a continuum of workers that include all the different types of labor services of a household. On the other hand, firms in the final good sector purchase the intermediate good and costlessly transform it into a continuum of differentiated final goods that are sold to the household in a market characterized by monopolistic competition with price rigidities in the form of restrictions to the frequency of their price-setting decisions. The separation between a final and intermediate good sectors simplify the difficulties associated with having a producing firm setting its price and bargaining with a bank at the same time.

2.3.2.1 Final good producers

There is a continuum of monopolistically competitive firms indexed by $j$, each producing a differentiated final good. All firms in the final good sector have access to the following technology:

$$Y_t^f (j) = X (j)$$
where $X_t(j)$ is the quantity of the single intermediate good used to produce the final good variety $j$. Final good producers purchase $X(j)$ from intermediate good producers in a competitive market at the common price $P^I_t$ and sell their output directly to households as a differentiated final good. Each final good producer face the following demand schedule obtained from the household decision problem

$$Y_t^f(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\epsilon_p} C_t$$

where $C_t$ is the aggregate final demand for final or consumption goods.

**Price setting** Prices for final goods are sticky as in Calvo (1983). Let $1 - \theta_p$ be the probability that a firm adjusts its price each period. The nominal total cost for a final good producer of variety $j$ is

$$TC^m_t(j) = P_t^I X(j)$$

with nominal marginal cost $MC^m_t(j) = P_t^I$. In this framework, a firm able to set its price at time $t$ will maximize its value subject to the sequence of demand curves it faces. All intermediate good producers who set prices in period $t$ will choose the same price, denoted by $P^*_t$ since they face an identical problem given by

$$\max_{P^*_t} E_t \sum_{k=0}^{\infty} (\theta_p)^k \Delta_{t,t+k} \left\{ P^*_t Y_{t+k|t}^f - TC^m_t(Y_{t+k|t}^f) \right\}$$

s.t

$$Y_{t+k|t}^f = \left( \frac{P^*_t}{P_{t+k}} \right)^{-\epsilon_p} C_{t+k} \quad \text{for } k = 0, 1, 2, ..., \$$

where $\Delta_{t,t+k}$ is the household stochastic discount factor and $Y^f_{t+k|t}$ denotes the demand faced by the firm consistent with the households’ optimality condition with respect to
each final good variety. The resulting first order condition to the price setting problem implies

\[ E_t \sum_{k=0}^{\infty} (\theta_p)^k \Delta_{t,t+k} Y_{i+k|t} \left\{ P_t^* - \frac{\epsilon_p}{\epsilon_p - 1} M C_{t+k|t}^n \right\} = 0 \]

taking into account that the nominal marginal cost for every final good producer is \( P_t^l \), the above optimality condition can be written as

\[ E_t \sum_{k=0}^{\infty} (\theta_p)^k \Delta_{t,t+k} Y_{i+k|t} \left\{ \frac{P_t^*}{P_{t+k}} - \frac{\epsilon_p}{\epsilon_p - 1} \frac{P_t^l}{P_{t+k}} \right\} = 0 \]

The term \( \frac{\epsilon_p}{\epsilon_p - 1} \) is the desired price markup over the marginal cost \( P_t^l \) in the absence of constraints in the frequency of price adjustment. Final good producers obtain nominal profits, \( \Pi_t^f (j) \), at the end of the period of \( \Pi_t^f (j) = P_t (j) Y_t^f (j) - P_t^l X_t (j) \). The competitive monopolistic structure together with Calvo nominal price rigidites imply the following aggregate price index \( P_t \) for the final good:

\[ P_t^{1-\epsilon_p} = \theta_p (P_{t-1})^{1-\epsilon_p} + (1 - \theta_p) (P_t^*)^{1-\epsilon_p} \]

The recursive formulation of the optimal price setting equation is

\[ g_{1,t} = \left( \frac{\epsilon_p}{\epsilon_p - 1} \right) g_{2,t} \]

\[ g_{1,t} = \Delta_{t,t} C_t \Pi_t^* + \theta_p E_t \left( \frac{\Pi_t^*}{\Pi_t^{n+1}} \right) g_{1,t+1} \]

\[ g_{2,t} = \Delta_{t,t} \frac{1}{\mu_t} C_t + \theta_p E_t g_{2,t+1} \]

where \( \Pi_t^* \) is the optimal inflation while \( g_{1,t} \) and \( g_{2,t} \) are auxiliary variables specified in order to write the recursive formulation of the optimal price setting condition.
2.3.2.2 Intermediate good producers

We assume that intermediate good producers need to find external funding before attempting to produce. A firm with funds is able to operate a production technology and produce an homogeneous intermediate good indexed by $z$ in a perfectly competitive market. Nominal total costs for a firm producing an intermediate good consist in total labor cost $R_t W_t N_t (\omega_{z,t})$ plus a fixed cost of production $P_t^I x^f$.

In this subsection, we first describe the technology and labor demand for a firm that have obtained the necessary external funds to produce the intermediate good. In the next section, we describe the loan market and the decisions that each intermediate good producer have to take when searching for external funds or after obtaining a bank loan.

**Technology and labor demand** If an intermediate good producer is matched with a bank, it is endowed with the following technology:

$$y_t (\omega_{z,t}) = \xi_{pf} A_t \omega_{z,t} N_t (\omega_{z,t})^\alpha$$

where $\xi_{pf}$ is a scale technology parameter that serve for calibration purposes, $A_t$ is the aggregate productivity level, $\omega_{z,t}$ is a firm-specific idiosyncratic productivity level drawn from a uniform distribution function $G(\omega)$ with support $[\underline{\omega}, \bar{\omega}]$, and $N_t (\omega_{z,t})$ is the firm’s employment index given by

$$N_t (\omega_{z,t}) = \left( \int_0^1 N_t (i, \omega_{z,t}) \frac{\epsilon_w}{\epsilon_w - 1} \, di \right)^\frac{\epsilon_w}{\epsilon_w - 1}$$
with \( N_t(i, \omega_z) \) being the demand for labor type "\( i \)" by firm "\( z \). Firm's cost minimization, taking wages as given, imply the following demand for labor type "\( i \)":

\[
N_t(i, \omega_{z,t}) = \left( \frac{W_t(i)}{W_t} \right)^{-\varepsilon_w} N_t(\omega_{z,t}) \quad \text{for all } i
\]

The aggregate demand for labor type \( i \) is obtained by aggregating \( N_t(i, \omega_z) \) across all producing firms

\[
N_t(i) = \int z N_t(i, \omega_z) \, dz = \left( \frac{W_t(i)}{W_t} \right)^{-\varepsilon_w} \int z N_t(\omega_z) \, dz
\]

where \( \int z N_t(\omega_z) \, dz \) denotes the aggregate labor index of all producing intermediate good firms during period \( t \) and \( W_t \) denotes the aggregate wage index. Labor costs have to be paid in advance of production to the household and can only be funded by taking loans with banks before attempting to produce. The nominal loan that each intermediate good producer have to obtain in order to produce is given by their specific wage bill during period \( t \):

\[
L_t(\omega_{z,t}) = \int z W_t(i) N_t(i, \omega_{z,t}) \, di = W_t N_t(\omega_{z,t})
\]

We assume that at the end of the period, loans are paid bank to the bank with a gross interest rate \( R^I_t(j, \omega_{z,t}) \) and no default occurs. End of the period nominal profits for an intermediate good producer with funding during period \( t \) is given by

\[
\Pi^I_t(\omega_{z,t}) = P^I_t(y_t(\omega_{z,t}) - x^f) - R^I_t(j, \omega_{z,t}) W_t N_t(\omega_{z,t})
\]
2.3.3 A decentralized loan market

We assume that the process of finding a credit partner is costly in terms of time and resources, leading to the existence of sunk costs at the time of trading and a surplus to be shared between lenders (banks) and borrowers (firms in the intermediate goods sector). Search and matching frictions prevent instantaneous trading in the loan market, implying that not all market participants will end up matched at a given point in time. We allow for both exogenous and endogenous destruction of credit matches, and a matching technology that determines the aggregate flow of new credit relationships over time as a function of the relative number of lenders and borrowers searching for credit partners. Upon a successful match, bilateral Nash bargaining between the parties determine the firm’s employment level and the way the match surplus is shared.

We assume a continuum of banks and firms with the number of banks seeking borrowers varying endogenously and being determined by a free entry condition to the loan market. We assume that banks have a constant returns to scale technology for managing loans, so we can treat each loan as a separate match between a bank and a firm. Each intermediate good producer is endowed with one project and is either searching for funding or involved in an ongoing credit contract with a bank. If a firm is matched with a bank, then the bank extends the necessary funds to allow the firm to hire workers and produce.

In appendix C, we introduce the possibility that borrowers may abscond with the funds obtained from banks. Given this, banks introduce an incentive compatibility
constraint in the loan contract such that the value of the net surplus for an intermediate
good producer in a credit contract is greater or equal to the value of absconding. We
assume, that in the case of absconding the firm is able to produce and generate profits
but does not repay the bank. The bank is able to recover an exogenous fraction of the
profits made by the intermediate good producer whom is not allowed to participate in
the loan market anymore. The optimal credit contract in this scenario is characterized
by credit rationing. A bank will prefer to loan only a fraction of its funds and park the
rest of it as excess reserves at the central bank. Subsequent versions of the paper will
include calibration and simulations related to this extension of the model.

2.3.3.1 The matching process

Firms searching for external funds, $f_t$, are matched to banks seeking for bor-
rowers, $b_t^u$, according to the following constant return to scale matching function

$$m_t = \mu f_t^\nu (b_t^u)^{1-\nu}$$

The function $m_t$ is strictly concave with constant return to scale and determines the
flow of new credit contracts during date $t$; $\mu$ is a scale parameter that measures the
productivity of the matching function and $0 < \nu < 1$ is the elasticity of match arrival
with respect to the mass of searching firms.

Matching rates The variable $\tau_t = f_t/b_t^u$ is the measure of credit market tightness.
The probability that a intermediate good producer with an unfunded project is matched
with a bank seeking to lend at date \( t \) is denoted by \( p^f_t \) and is given by

\[
p^f_t = \mu \tau^\nu_t^{-1}
\]

Similarly, the probability that any bank seeking borrowers is matched with an unfunded entrepreneur at time \( t \) is denoted by \( p^b_t \) and is given by

\[
p^b_t = \mu \tau^\nu_t
\]

Since \( \tau_t = p^b_t / p^f_t \), a rise in \( \tau_t \) implies it is easier for a bank to find a borrower relative to a firm finding a lender and so corresponds to a tighter credit market. An increase (decrease) in \( \tau_t \) reduces the expected time a bank (firm) must search for a credit partner, lowering the bank’s (firm’s) expected pecuniary search costs. Since \( \tau_t = f_t / b^u_t = p^b_t / p^f_t \), at any date \( t \) the number of newly matched banks must equal the number of newly matched firms: \( p^b_t b^u_t = p^f_t f_t \).

**Separations and the evolution of loan contracts** Credit relationships (i.e., loan contracts) end for exogenous reasons with probability \( \delta \). But also contractual parties engaged in a credit relationship that survive this exogenous separation hazard may also decide to dissolve the contract depending on the realization of the productivity of the firm’s project, taken to be \( A_t \omega_{z,t} \), where \( A_t \) is the aggregate component common to firms and \( \omega_{z,t} \) is a firm-specific idiosyncratic productivity shock with distribution function \( G(\omega_{z,t}) \). The decision to endogenously dissolve a credit relationship is characterized by an optimal reservation policy with respect \( \omega_{z,t} \) and denoted by \( \tilde{\omega}_t \). If the realization of \( \omega_{z,t} \) is above the reservation firm-specific productivity both parties agree to continue
the credit relationship and the entrepreneur is able to produce if it has also survive the exogenous separation hazard. On the contrary, If the realization of $\omega_{z,t}$ is below $\tilde{\omega}_t$, both parties choose to dissolve the credit relationship. The probability of endogenous termination of a credit match is $\gamma_t (\tilde{\omega}_t) \equiv \text{prob} (\omega_{z,t} \leq \tilde{\omega}_t) = G (\tilde{\omega}_t)$ while the overall separation rate is $\delta + (1 - \delta) \gamma_t (\tilde{\omega}_t)$. Existence and uniqueness of the optimal reservation policy $\tilde{\omega}_t$ are shown in the appendix.

Let $f_{t-1}^m$ be the measure of intermediate good producers that enter period $t$ matched with a bank. Of those, $(1 - \delta) f_{t-1}^m$ firms survive the exogenous hazard and a fraction $\gamma_t$ of the survivals receive idiosyncratic productivity shocks that are less than $\tilde{\omega}_t$ and so do not produce. The number of intermediate good producers that actually produce in period $t$, therefore, is $(1 - \delta)(1 - \gamma_t) f_{t-1}^m$. The number of firms in a credit relationship at the end of period $t$, denoted by $f_t^m$, is given by the number of firms producing during time $t$ plus all the new matches formed at time $t$. Then, the evolution of $f_t^m$ is expressed as

$$f_t^m = \varphi_t (\tilde{\omega}_t) f_{t-1}^m + m_t$$

where $\varphi_t (\tilde{\omega}_t)$ is the overall continuation rate of a credit relationship defined to be:

$$\varphi_t (\tilde{\omega}_t) = (1 - \delta)(1 - \gamma_t (\tilde{\omega}_t))$$

and $1 - \varphi_t (\tilde{\omega}_t) = \delta + (1 - \delta) \gamma_t (\tilde{\omega}_t)$ denotes the overall separation rate. We normalize the total number of intermediate good producers in every time period to one and assume that if a credit relationship is exogenously separated at time $t$, both parties
will start searching immediately during the period. If the credit relationship survives
the exogenous separation hazard but then endogenously separates, then both parties
must wait until next period in order to start searching for a credit partner again. This
assumption implies that the number of firms seeking finance during period $t$, which we
have denoted by $f_t$, is equal to the number of searching firms at the beginning of time
$t$, $(1 - f^m_{t-1})$ plus the number of firms that started the period matched with a bank but
were exogenously separated $(\delta f^m_{t-1})$. Therefore,

\[ f_t = 1 - (1 - \delta) f^m_{t-1}. \]

Notice that there are still some firms that have been endogenously separated but cannot
search in period $t$. These firms are unmatched but waiting to start searching again next
period.

**Credit Creation and Credit Destruction** Our timing assumption implies that
the fraction $p_f \delta f^m_{t-1}$ of matched intermediate good producers that were exogenously
separated during time $t$, are able to find a new credit relationship within the same
period of time. Then, credit creation, $CC_t$, is defined to be equal to the number of
newly created credit relationships at the end of time $t$ net of the number of exogenous
credit separations that are successfully re-matched in a given period. That is

\[ CC_t = m_t - p_f \delta f^m_{t-1}. \]

The credit creation rate, $cc_t$ is

\[ cc_t = \frac{m_t}{f^m_{t-1}} - p_f \delta. \]
On the other hand, credit destruction, $CD_t$, is defined as the total number of credit separations at the end of time $t$, $(1 - \varphi_t(\tilde{\omega}_t))f^m_{t-1}$ net of the number of exogenous credit separations that are successfully re-matched in a given period. Thus,

$$CD_t = (1 - \varphi_t(\tilde{\omega}_t)) f^m_{t-1} - p^f_t \delta f^m_{t-1}$$

The credit destruction rate, $cd_t$, is

$$cd_t = (1 - \varphi_t(\tilde{\omega}_t)) - p^f_t \delta.$$ 

Finally, the net credit growth rate is defined as

$$cg_t = cc_t - cd_t$$ 

### 2.3.3.2 Intermediate good producers and the loan market

In our setting, a credit relationship is a contract between bank ”j” and intermediate good producer ”z” that allows the latter to operate an specific production technology, hire workers and pay their wage bill in advance of production. As long as, the credit contract prevails, the firm will receive sufficient external funds to pay workers in advance of production every period of time. After selling its output to the final good producers, the firm will repay its debt with the bank and transfer all remaining profits to the household. Therefore, as in De Fiore and Tristani (2012) we abstract from the endogenous evolution of net worth by assuming firms do not accumulate internal funds after repaying their debt.
Value functions If the intermediate good producer obtains financing its instantaneous real profit flow is

\[ \pi^I_{t}(\omega_{z,t}) = \frac{P^l_t}{P_t}(y_t(\omega_{z,t}) - x^f) - R^l_t(j,\omega_{z,t})w_tN_t(\omega_{z,t}) \]

where \( \frac{P^l_t}{P_t} \) is the price of the intermediate good expressed in terms of the final good price index and \( w_t = \frac{W_t}{P_t} \) is the real wage index. The loan principle expressed in real terms is the wage bill of the firm given by \( l_t(j,\omega_{z,t}) = w_tN_t(\omega_{z,t}) \). The loan contract requires the repayment of the total debt with the bank \( R^l_t(j,\omega_{z,t})w_tN_t(\omega_{z,t}) \) within the same period. It is useful to define the mark-up of final goods over intermediate goods prices as \( \mu^p_t = \frac{P_t}{P^l_t} \) and express \( \pi^I_{t}(\omega_{z,t}) \) as

\[ \pi^I_{t}(\omega_{z,t}) = \frac{y_t(\omega_{z,t}) - x^f}{\mu^p_t} - R^l_t(j,\omega_{z,t})w_tN_t(\omega_{z,t}) \]

Profits depend on the status of the intermediate good producer, that is, depend if the firm is searching for external funds or if it is producing. A firm searching for external funds can not produce and obtains zero real profits since we assume that there are no extra search costs when a producer is searching for funding. Under these assumptions, the firm is characterized by two value functions or Bellman equations: The value of being matched with a bank and able to produce at date \( t \), denoted by \( V^FP_t(\omega_{z,t}) \) and the value of searching for external funds at date \( t \), denoted by \( V^FN_t(\omega_{z,t}) \), both measured in terms of current consumption of the final good. \( V^FP_t(\omega_{z,t}) \) is given by

\[ V^FP_t(\omega_{z,t}) = \pi^I_{t}(\omega_{z,t}) + E_t \Delta_{t,t+1} \left\{ \delta V^FN_{t+1} + (1 - \delta) \int_{\omega} \max(V^FP_{t+1}(\omega_{z,t+1}),V^FN_{t+1})dG(\omega) \right\} \]

where \( \Delta_{t,t+1} = \beta \lambda_{t+1}/\lambda_t \) is the household stochastic discount factor. The value of producing is the flow value of current real profits (firm’s real cash flow) plus the expected
continuation value. At the end of the period, the credit relationship is exogenously dissolved with probability $\delta$, and the firm must seek new financing. With probability $(1 - \delta)$, the firm survives the exogenous separation hazard. In the latter case, only those firms receiving an idiosyncratic productivity realization $\omega_{z,t+1} \geq \tilde{\omega}_{t+1}$ will remain matched and produce during next period. Firms with $\omega_{z,t+1} < \tilde{\omega}_{t+1}$ endogenously separate from their bank and obtain $V_{t+1}^{FN}$.

The value of searching for external funds ($V_{t}^{FN}$) for a firm at date $t$ expressed in terms of current consumption is

$$V_{t}^{FN} = p_{t} E_t \Delta_{t,t+1} \left[ \delta V_{t+1}^{FN} + (1 - \delta) \int \max(V_{t+1}^{FP}(\omega), V_{t+1}^{FN}) dG(\omega) \right] + (1 - p_{t}) V_{t+1}^{FN}$$

where $p_{t}$ is the probability of matching with a bank. Notice that we assume matches made in period $t$ do not produce until $t + 1$. With probability $(1 - p_{t})$, the firm does not match and must continue searching for external funds during next period’s loan market.

Under Nash bargaining, the reservation productivity level $\tilde{\omega}_{t}$ that triggers endogenous separation is determined by the point at which the joint surplus of the match is equal to zero. Next period, if $\omega_{z,t+1} < \tilde{\omega}_{t+1}$, both parties agree to end the relationship. The probability of endogenous separation is $\gamma_{t+1}(\tilde{\omega}_{t+1}) = G(\tilde{\omega}_{t+1}) = \text{prob} (\omega_{z,t+1} \leq \tilde{\omega}_{t+1})$. Given existence and uniqueness of $\tilde{\omega}_{t+1}$, the integral term on the expected continuation value is
\[
\int \max(V_{t+1}^{FP}(\omega_{z,t+1}), V_{t+1}^{FN}) dG(\omega) = \gamma_{t+1}(\tilde{\omega}_{t+1}) V_{t+1}^{FN} \\
+ (1 - \gamma_{t+1}(\tilde{\omega}_{t+1})) \int_{\tilde{\omega}_{t+1}}^{\infty} V_{t+1}^{FP}(\omega_{z,t+1}) \frac{dG(\omega)}{1 - \gamma_{t+1}(\tilde{\omega}_{t+1})}.
\]

Therefore, the firm value functions can be written as

\[
V_{t}^{FP}(\omega_{z,t}) = \pi_{t}^{I}(\omega_{z,t}) + E_{t} \Delta_{t,t+1} \left\{ (1 - \varphi_{t}(\tilde{\omega}_{t+1})) V_{t+1}^{FN} + \varphi_{t}(\tilde{\omega}_{t+1}) \int_{\tilde{\omega}_{t+1}}^{\infty} V_{t+1}^{FP}(\omega_{z,t+1}) \frac{dG(\omega)}{1 - \gamma_{t+1}(\tilde{\omega}_{t+1})} \right\}
\]

and

\[
V_{t}^{FN} = E_{t} \Delta_{t,t+1} \left\{ p_{t}^{f} \left[ (1 - \varphi_{t}(\tilde{\omega}_{t+1})) V_{t+1}^{FN} + \varphi_{t}(\tilde{\omega}_{t+1}) \int_{\tilde{\omega}_{t+1}}^{\infty} V_{t+1}^{FP}(\omega_{z,t+1}) \frac{dG(\omega)}{1 - \gamma_{t+1}(\tilde{\omega}_{t+1})} \right] \right\} \\
+ \left( 1 - p_{t}^{f} \right) V_{t+1}^{FN}
\]

Let the surplus to a producing firm be defined as \(V_{t}^{FS}(\omega_{z,t}) = V_{t}^{FP}(\omega_{z,t}) - V_{t}^{FN}\), then the intermediate producer surplus of being in a credit relationship can be written as

\[
V_{t}^{FS}(\omega_{z,t}) = \pi_{t}^{I}(\omega_{z,t}) + \left( 1 - p_{t}^{f} \right) E_{t} \Delta_{t,t+1} \varphi_{t}(\tilde{\omega}_{t+1}) \int_{\tilde{\omega}_{t+1}}^{\infty} V_{t+1}^{FS}(\omega_{z,t+1}) \frac{dG(\omega)}{1 - \gamma_{t+1}(\tilde{\omega}_{t+1})}
\]

### 2.3.3.3 Banks and the loan market

There is an infinite mass of banks indexed by \(j\) that are owned by the representative household. Banks collect deposits from households and invest them into loans with firms. The deposit market is assumed to be a centralized competitive market while
the loan market is a decentralized market characterized by search, matching and bilateral bargaining. At any time, banks operating in the loan market may be involved in a credit contract with a particular firm or may be seeking for potential borrower. Banks decide to enter the loan market to search for potential borrowers until the expected cost of extending a loan is equal to its expected benefit. At this point, banks will be indifferent between searching for projects or only operate in the deposit market. Due to the decentralized nature of the loan market, some banks may not end up with loans in their portfolio. If this is the case, the bank will deposit its funds with the central bank as excess reserves and receive an interest rate that matches the interest rate on deposits, leaving the bank with negative profits due to search costs. All uncertainty is revealed before loans are extended: loans are made and paid back during the same period. Therefore, loans are not risky and there is no possibility of default. At the end of the period, the bank transfers all its profits (positive or negative) to the representative household.

A bank can only form a credit relationship with one firm. If a bank is in a credit relationship with a firm, it can not search for a different bank until separation occurs. Bank $j$’s balance sheet expressed in real terms is

$$\chi_t (j) l_t (j, \omega_{zt}) + (1 - \chi_t (j)) \frac{E R_t (j)}{P_t} = \frac{D_t (j)}{P_t}$$

where $\chi_t (j)$ is an indicator function taking the value of "1" if bank $j$ extends a loan $l_t (j, \omega_{zt})$ to a firm whose idiosyncratic productivity $\omega_{zt}$ exceeds a cut-off level and "0" otherwise, $E R_t (j)$ represents nominal excess reserves held with the central bank in case
\( \chi_t(J) = 0 \) and \( D_t(J) \) are household deposits. In equilibrium there will be a measure of banks with positive loans and a measure of banks with no loans but holding excess reserves instead. Notice that when the bank extends a loan \( (\chi_t(J) = 1) \) the bank balance sheet implies that the bank lend out all its resources \( l_t(J, \omega_{z,t}) = \frac{D_t(J)}{P_t} \) which means that there is not credit rationing since there is no default risk. We introduce the possibility of asymmetric information and credit rationing in section # below.

**Profits** A bank searching for a borrower will incur in a search cost \( \frac{P_I}{P_t} \kappa \) measured in units of the final good and earn zero profits. The current flow of profits of a bank with household deposits \( D_t(J) \) can be written as

\[
\pi^b_t(J) = \chi_t(J) R_l^d(j, \omega_{z,t}) l_t(j, \omega_{z,t}) + (1 - \chi_t(J)) \left( R_t^e \frac{ER_t(j)}{P_t} - \kappa \right) - R_t d \frac{D_t(J)}{P_t}
\]

where \( R_l^d(j, \omega_{z,t}) \) is the bilateral bargained gross loan rate between bank \( j \) and firm \( z \), \( R_t^e \) is the gross interest rate on excess reserves and \( R_t^d \) is the gross deposit rate. The problem of a bank is to maximize its current profits subject to its balance sheet. Optimality with respect to deposits requires that every period \( (R_t^e - R_t^d) D_t(J) = 0 \). Since household deposits are always positive in equilibrium, the bank will choose to collect deposits until the gross interest rate on excess reserves is equal to the gross interest rate on deposits, that is \( R_t^e = R_t^d = R_t \). Substituting the bank’s balance sheet and the optimality conditions with respect to \( D_t(J) \) into the profit function yields

\[
\pi^b_t(J) = \chi_t(J) \left( R_t^e(j, \omega_{z,t}) - R_t \right) l_t(j, \omega_{z,t}) - (1 - \chi_t(J)) \frac{\kappa}{\mu_t}^2
\]
which can also be written as

\[
\pi_t^b (j) = \begin{cases} 
\pi_t^b (j, \omega_{zt}) = (R_t^l (j, \omega_{zt}) - R_t) l_t (j, \omega_{zt}) & \text{if extends a loan to firm } \omega_{zt} \\
- \frac{\kappa}{\mu_t} & \text{otherwise}
\end{cases}
\]

**Value functions** Each period, when the loan market opens, a bank may be in a credit relationship with a firm or searching for potential borrowers. If a bank is matched with a firm whose idiosyncratic productivity realization exceeds \( \bar{\omega}_t \), bank profits will equal

\[
\pi_t^b (j, \omega_{zt}) = \left( R_t^l (j, \omega_{zt}) - R_t \right) l_t (j, \omega_{zt})
\]

where \( R_t^l (j, \omega_{zt}) - R_t \) is the spread between the interest rate on the bank’s loan to a firm with idiosyncratic productivity \( \omega_{zt} \) and the bank’s opportunity cost of funds \( R_t \) given by the deposit interest rate. The determination of \( R_t^l (j, \omega_{zt}) \) is explained below as the result of Nash bargaining between the bank and the intermediate good producer. The loan size is given by the labor costs of firm \( z \), that is \( l_t (j, \omega_{zt}) = w_t N_t (\omega_{zt}) \).

A bank searching for a borrower will incur in a search cost \( \kappa \), measured in current consumption units and will earn zero current profits while operating in the loan market. Under these assumptions the problem of a bank can be characterized by two value functions (Bellman equations): The value of lending to a firm at date \( t \), denoted by \( V_t^{BL} (\omega_{zt}) \) and the value of searching for a potential borrower at date \( t \), denoted by \( V_t^{BN} \). Both value functions are measured in terms of current consumption of the final good and are given by
\[ V_{t}^{BL}(\omega_{z,t}) = \pi_{t}^{b}(\omega_{z,t}) \]

\[ + E_{t}\Delta_{t,t+1}\left\{ (1 - \varphi_{t+1}(\tilde{\omega}_{t+1})) V_{t+1}^{BN} + \varphi_{t+1}(\tilde{\omega}_{t+1}) \int_{\tilde{\omega}_{t+1}}^{\infty} V_{t+1}^{BL}(\omega_{z,t+1}) \frac{dG(\omega)}{1 - \gamma_{t+1}(\tilde{\omega}_{t+1})} \right\} \]

and

\[ V_{t}^{BN} = -\frac{\kappa}{\mu_{t}^{p}} \left\{ p_{t}^{b} \left( (1 - \varphi_{t+1}(\tilde{\omega}_{t+1})) V_{t+1}^{BN} + \varphi_{t+1}(\tilde{\omega}_{t+1}) \int_{\tilde{\omega}_{t+1}}^{\infty} V_{t+1}^{BL}(\omega_{z,t+1}) \frac{dG(\omega)}{1 - \gamma_{t+1}(\tilde{\omega}_{t+1})} \right) \right\} \]

\[ + (1 - p_{t}^{b}) V_{t+1}^{BN} \]

The value of extending a loan is the current value of real profits plus the expected continuation value. A bank that extends a loan to a firm with idiosyncratic productivity \( \omega_{z,t} \) at date \( t \) will continue financing the same firm at time \( t + 1 \) with probability \( \varphi_{t}(\tilde{\omega}_{t+1}) \). In this event, the bank obtains the future expected value of lending conditional on having \( \omega_{z,t+1} \geq \tilde{\omega}_{t+1} \) given by \( \int_{\tilde{\omega}_{t+1}}^{\infty} V_{t+1}^{BL}(\omega_{z,t+1}) (1 - \gamma_{t+1})^{-1} dG(\omega) \). The credit relationship will be severed at time \( t + 1 \) with probability \( \delta_{t} + 1 - \varphi_{t}(\tilde{\omega}_{t+1}) \) and the bank obtains a future value of \( V_{t+1}^{BN} \). On the other hand, the value of a bank searching for a borrower at date \( t \) is given by the flow value of the search costs plus the continuation value. A searching bank faces a probability \( 1 - p_{t}^{b} \) of not being matched during time \( t \), obtaining a future value of \( V_{t+1}^{BN} \) but a probability \( p_{t}^{b} \) of being matched. If a searching bank ends up being matched with a firm at time \( t \), then at the beginning of period \( t + 1 \) will face a probability of separation before extending the loan.
Free entry condition In equilibrium, free entry of banks into the loan market ensures that \( V^{BN}_t = 0 \) for all \( t \). Using this in \( V^{BN}_t \), the free entry condition can be written as

\[
\frac{\kappa}{\mu_t p_t} = E_t \Delta_{t,t+1} \varphi_{t+1} (\tilde{\omega}_{t+1}) \int_{\tilde{\omega}_{t+1}} V^{BL}_{t+1}(\omega_{z,t+1}) \frac{dG(\omega)}{1 - \gamma_{t+1} (\tilde{\omega}_{t+1})}
\]

Banks will enter the loan market until the expected cost of finding a borrower \( \frac{\kappa}{\mu_t p_t} \) is equal to the expected benefit of extending a loan to a firm with idiosyncratic productivity \( \omega_{z,t+1} \geq \tilde{\omega}_{t+1} \). If the expected cost of extending a loan is lower than the corresponding expected benefits, banks will enter the loan market to search for borrowers and the probability that a searching bank finds a borrower will fall, up to the point where equality of the above condition is restored. Note that free entry of banks into the loan market modifies the value function \( V^{BL}_t(\omega_{z,t}) \) as follows

\[
V^{BL}_t(\omega_{z,t}) = \pi^b_t (\omega_{z,t}) + E_t \Delta_{t,t+1} \varphi_{t+1} (\tilde{\omega}_{t+1}) \int_{\tilde{\omega}_{t+1}} V^{BL}_{t+1}(\omega_{z,t+1}) \frac{dG(\omega)}{1 - \gamma_{t+1} (\tilde{\omega}_{t+1})}
\]

The net surplus for bank extending a loan to a firm with productivity \( \omega_{z,t} \) is

\[
V^{BS}_t(\omega_{z,t}) = \pi^b_t (\omega_{z,t}) + \frac{\kappa}{\mu_t p_t}
\]

2.3.3.4 Employment and the loan contract: Nash bargaining

At any point in time, a matched firm and bank that survive the exogenous and endogenous separation hazards engage in bilateral bargaining over the interest rate and loan size to split the joint surplus that results from the match. This joint surplus is defined as \( V^{JS}_t(\omega_{z,t}) = V^{FS}_t(\omega_{z,t}) + V^{BS}_t(\omega_{z,t}) \) and it is given by
\[ V_t^{FS}(\omega_{z,t}) = \frac{y_t(\omega_{z,t}) - x^f}{\mu_t^p} - w_t R_t N_t(\omega_{z,t}) \]
\[ + \left( 1 - p_t^f \right) E_t \Delta_{t,t+1} \varphi_{t+1}(\tilde{\omega}_{t+1}) \int_{\tilde{\omega}_{t+1}} V_{t+1}^{FS}(\omega_{z,t+1}) \frac{dG(\omega)}{1 - \gamma_{t+1}(\tilde{\omega}_{t+1})}. \]

Let \( \bar{\eta} \) be the firm’s share of the joint surplus, and \( 1 - \bar{\eta} \) the banks’. The Nash bargaining problem for an active credit relationship is

\[ \max \left\{ R_t(j,\omega_{z,t},t)(\omega_{z,t}) \right\} \left( V_t^{FS}(\omega_{z,t}) \right)^{\bar{\eta}} \left( V_t^{BS}(\omega_{z,t}) \right)^{1-\bar{\eta}} \]

where \( V_t^{FS}(\omega_{z,t}) \) and \( V_t^{BS}(\omega_{z,t}) \) are defined above. The first order conditions imply the following optimal sharing rule:

\[ \bar{\eta} V_t^{BS}(\omega_{z,t}) = (1 - \bar{\eta}) V_t^{FS}(\omega_{z,t}) \]

and an employment condition that sets the marginal product of labor equal to a markup \( \mu_t^p \) over the marginal cost of labor inclusive of the bank’s opportunity cost when extending a loan to an intermediate good producer:

\[ \alpha \xi^{pf} A_t \omega_{z,t} N_t^*(\omega_{z,t})^{\alpha-1} = \mu_t^p w_t R_t \]

Notice that \( w_t R_t \) is expressed in terms of the final good and it has to be transformed back in terms of the intermediate good as it is the marginal product of labor.

The optimal loan size negotiated between credit partners is

\[ l_t^*(j,\omega_{z,t}) = \left( \frac{\alpha \xi^{pf} A_t \omega_{z,t}}{\mu_t^p w_t^p R_t} \right)^{1/(1-\alpha)} \]
with an equilibrium loan interest rate

\[ R^l_t(j, \omega_{z,t}) = \frac{1}{l^*_t(j, \omega_{z,t})} \left( 1 - \eta \right) \left( \frac{y^*_t(\omega_{z,t}) - x^f}{\mu^p_t} \right) + \eta \left( R_t w_t N^*_t(\omega_{z,t}) - \frac{\kappa p^f}{\mu^p_t \mu^b_t} \right) \]

The above conditions imply that firm \( z \) will produce \( y^*_t(\omega_{z,t}) \) units of the intermediate good and employ \( N^*_t(\omega_{z,t}) \) workers, given by:

\[ y^*_t(\omega_{z,t}) = \left( \xi_p A_t \omega_{z,t} \right)^{\frac{1}{1-\alpha}} \left( \frac{\alpha}{\mu^p_t w_t R_t} \right)^{\frac{\alpha}{1-\alpha}} \]

\[ N^*_t(\omega_{z,t}) = \left( \frac{\alpha \xi p^f A_t \omega_{z,t}}{\mu^p_t w_t R_t} \right)^{\frac{1}{1-\alpha}} \]

The effect of the nominal interest rate on the cost of labor is generally referred to as the cost channel of monetary policy (Ravenna and Walsh (2006)). Normally, the relevant interest rate is taken to be the interest rate the firm pays on loans taken to finance wage payments. Here, the loan interest rate simply ensures the joint surplus generated by a credit relationship is divided optimally between the firm and the bank, with the relevant interest rate capturing the cost channel being \( R_t \), the bank’s opportunity cost of funds. Even though firms will face different interest rates on bank loans, since the loan rate depends on the firms idiosyncratic productivity realization, the interest cost relevant for labor demand is the same for all firms.

2.3.3.5 The optimal reservation policy: Endogenous separations

The joint surplus of a credit relationship can be written explicitly as a function of the idiosyncratic productivity shock \( \omega_{z,t} \) in order to facilitate the characterization of
the loan market equilibrium as follows

\[ V_t^{JS}(\omega_t) = \frac{1}{\mu^p_t} \left( 1 - \alpha \right) \left( \xi^{pf} A_t \omega_t \right)^{\frac{\alpha}{1-\alpha}} \left( \frac{\alpha}{\mu^p_t w_t R_t} \right)^{1-\alpha} \left( 1 - \alpha \right) \left( \xi^{pf} A_t \right)^{\frac{\alpha}{1-\alpha}} - x^f + \frac{\kappa}{p_t} \left( \frac{1 - \eta p_t^f}{1 - \eta} \right) \]

The optimal reservation policy with respect to the idiosyncratic productivity shock implies that

\[ if \quad \omega_{i,t} \leq \tilde{\omega}_{i,t} \implies V_t^{JS}(\omega_{i,t}) \leq 0 \]

\[ if \quad \omega_{i,t} > \tilde{\omega}_{i,t} \implies V_t^{JS}(\omega_{i,t}) > 0. \]

Since the joint surplus is increasing in the firm’s idiosyncratic productivity, there exists a unique threshold level \( \tilde{\omega}_t \) defined by

\[ V_t^{JS}(\tilde{\omega}_t) = 0 \]

such that the joint surplus is negative for any firm facing an idiosyncratic productivity \( \omega_{i,t} < \tilde{\omega}_t \). The optimal threshold level \( \tilde{\omega}_t \) is

\[ \tilde{\omega}_t = \left( \frac{1}{\alpha^\alpha (1 - \alpha)^{1-\alpha}} \left( \frac{\mu^p_t w_t R_t}{\xi^{pf} A_t} \right)^{\alpha} \right) \left[ x^f - \left( \frac{1 - \eta p_t^f}{1 - \eta} \right) \frac{\kappa}{p_t} \right]^{1-\alpha} \]

Since \( \tilde{\omega}_t \) is independent of \( i \), the cutoff value is the same for all firms and banks. Moreover, it is decreasing in aggregate productivity \( A_t \) so that a positive aggregate productivity shock means the number of credit matches that separate endogenously falls and more matched firms produce. The cutoff value is increasing in the cost of labor \( (w_t R_t) \), the firm’s fixed cost \( (x^f) \) and the markup of final good over intermediate good prices \( (\mu^p_t) \).

The bank’s opportunity costs of funds \( R_t \) influences the level of economy activity at both the extensive and intensive margins. A rise in \( R_t \) increases the threshold
level of the idiosyncratic productivity of firms that generate a positive joint surplus. As a consequence, fewer firms obtain financing and produce. This is the extensive margin effect. Conditional on producing, firms equate the marginal product of labor to $w_t R_t$, so a rise in $R_t$ reduces labor demand at each level of the real wage. This is the intensive margin effect. Both channels work to reduce aggregate output as $R_t$ rises. In addition, credit market conditions reflected in $p^f_t$ and $p^b_t$ directly affect the extensive margin; a rise in $\tau_t$ (a credit tightening) increases $\tilde{\omega}_t$ and fewer firms obtain credit. Both interest costs measured by $R$ and credit conditions measured by $\tau$ matter for employment and output.

2.3.3.6 Characterizing the loan market equilibrium

The loan market partial equilibrium can be characterized by a system of two equations for two variables: The credit market tightness $\tau_t$ and the reservation productivity level $\tilde{\omega}_t$. The evolution of credit market tightness is obtained by using the free entry condition, the Nash bargaining sharing rule and the definition of the joint surplus of a credit relationship and it is given by the following equation:

\[
\frac{\kappa}{\mu_t \mu \tau_t} - E_t \Delta_{t,t+1} \tilde{\varphi}_{t+1} (\tilde{\omega}_{t+1}) \left(1 - \eta \mu_T \tilde{\varphi}_{t+1}^{-1}\right) \frac{\kappa}{\mu_{t+1} \mu \tau_{t+1}^2} \\
= (1 - \eta) E_t \Delta_{t,t+1} \frac{1}{\mu_{t+1}} \left(1 - \alpha \right) \frac{Y_{t+1}}{f_{t+1}^m} - \varphi_{t+1} (\tilde{\omega}_{t+1}) x^I
\]

The second equation is given by the optimal reservation productivity level $\tilde{\omega}_t$.
written as a function of \( \tau_t \):

\[
\left[ \alpha (1 - \alpha)^{1-\alpha} \xi^{p\bar{A}} \xi \right]^{1-\alpha} = \left( \frac{\mu w R}{1 - \tilde{\eta}} \right)^{1-\alpha} \left[ x^f - \left( \frac{1 - \eta \mu \tau^{\nu-1}}{1 - \tilde{\eta}} \right) \right]
\]

At the steady state, the equations for \( \tau_t \) and \( \tilde{\omega}_t \) become

\[
\kappa \left( 1 - \beta \varphi \left( \tilde{\omega} \right) \left( 1 - \eta \mu \tau^{\nu-1} \right) \right) = \mu \tau^{\nu} \left( 1 - \tilde{\eta} \right) \beta \left( 1 - \alpha \right) \frac{Y^I}{m} - \varphi \left( \tilde{\omega} \right) x^f
\]

and

\[
\left[ \alpha (1 - \alpha)^{1-\alpha} \xi^{p\bar{A}} \xi \right]^{1-\alpha} = \left( \mu w R \right)^{1-\alpha} \left[ x^f - \frac{\kappa}{1 - \tilde{\eta}} \left( \frac{1 - \eta \mu \tau^{\nu-1}}{\mu \tau^{\nu}} \right) \right]
\]

respectively.

### 2.3.4 Government

**Central bank budget constraint** There are no government bonds in this economy but the central bank pays the same interest rate as the banks’ deposit rate on excess reserves. Therefore, the central bank’s budget constraint is given by:

\[
i_t ER_t + RCB_t = M_t - M_{t-1}
\]

where \( RCB_t \) denotes the central bank transfers to the treasury and \( M_t \) the money supply in the economy. Aggregate excess reserves, \( ER_t \), are obtained by integrating among the measure of banks that were not able to extend loans to intermediate good producers during the time period, that is

\[
ER_t = \int_j \left( 1 - \chi_t \left( j \right) \right) \frac{ER_t \left( j \right)}{P_t} dj
\]
where as explained above, $\chi_t(j)$ is an indicator function taking the value of "1" if the bank extends a loan and "0" if the bank have to park its funds as excess reserves with the central bank.

**Treasury budget constraint** Since there are no government purchases, the treasury budget constraint is defined as $RCB_t = P_t T_t$

**Consolidated government budget constraint** Combining the above two constraints for the government sector yields the following consolidated government budget constraint:

$$M_t - M_{t-1} = P_t T_t + i_t ER_t$$

Therefore, government total expenditures are funded by changes in the money supply. Notice that $T_t$ can be positive or negative in the sense that it can be treated as a tax or transfer to the households.

**Monetary policy** We assume that the central bank follows an exogenous growth rate for the nominal supply of money given by

$$M_t = (1 + \theta_t) M_{t-1}$$

where $\theta_t$ denotes the nominal money growth given by

$$\left( \frac{\theta_t}{\bar{\theta}} \right) = \left( \frac{\theta_{t-1}}{\bar{\theta}} \right)^{\rho_0} \exp \left( \varepsilon_t^\theta \right)$$
Notice that in this case, the nominal interest rate on deposits $R_t$ will be an endogenous variable clearing the market for real money balances. Aggregating the CIA constraint, together with the government budget constraint, the aggregate balance sheet of banks as well as the aggregate equilibrium in the loan market yields an equation that characterizes the market for real money balances.

### 2.3.5 Aggregation and Market Clearing

Market clearing in the final goods market requires demand to equal supply for each final good which implies:

$$C_t (j) = Y_t^f (j) \quad \text{for all } j$$

Using the same CES aggregator for final consumption goods than the one used for final goods yields the following aggregate equilibrium condition:

$$C_t = Y_t^f$$

The aggregate production function for firms in the final good sector is obtained by aggregating across all final good producers the individual production function and taking into account that the demand schedule is consistent with household optimization. Therefore, the following condition must hold:

$$C_t \Delta_t^p = X_t$$
where $X_t = \int X(j) \, dj$ and $\Delta^p_t = \int \left( \frac{\mu(j)}{\mu_t} \right)^{-\epsilon^p} \, dj$ is the price dispersion.

Recall that firms and goods in the intermediate good sector are indexed by the idiosyncratic productivity of each active producer. The equilibrium condition in this market is given by

$$X_t(\omega_{z,t}) = y_t(\omega_{z,t}) \quad \text{for all } z$$

where the demand for each intermediate good is denoted by $X_t(\omega_{z,t})$ and comes from the final good producers. Aggregating across $z$ yields the following market clearing condition

$$X_t = \int y_t(\omega_{z,t}) \, dz$$

$$\equiv Y_t^I$$

where $Y_t^I$ denotes the aggregate supply of intermediate goods and it is given by the total number of producing firms $(1 - \delta) (1 - \gamma_t(\tilde{\omega}_t)) f_{t-1}^m$ times their average output, that is

$$Y_t^I = (1 - \delta) (1 - \gamma_t(\tilde{\omega}_t)) f_{t-1}^m E[y_t^*(\omega_{z,t}) \mid \omega_{z,t} \geq \tilde{\omega}_t]$$

where $E[y_t^*(\omega_{z,t}) \mid \omega_{z,t} \geq \tilde{\omega}_t] = \int y_t^*(\omega_{z,t}) \frac{g(\omega) \, d\omega}{(1 - \gamma_t)}$ is average output. Assuming that $g(\omega)$ is a uniform distribution allow us to calculate explicitly the truncated expectation of $y_t^*(\omega_{z,t})$ and compute $Y_t^I$ as:

$$Y_t^I = (1 - \delta) \alpha \frac{\alpha}{\epsilon^p A_t} \left( \frac{\mu_t}{\mu_t u_t R_t} \right)^{1-\alpha} \left( \frac{(\bar{\omega})^k - (\tilde{\omega}_t)^k}{k (\bar{\omega} - \omega)} \right) f_{t-1}^m$$

Where $k = \frac{2-\alpha}{1-\alpha}$. Notice that $Y_t^I$ depends directly in the number of firms matched with a bank at the beginning of the period $f_{t-1}^m$, on the probability that a credit contract
survives during the period \((1 - \delta) \left( \frac{(\bar{\omega})^k - (\tilde{\omega}_t)^k}{k(\bar{\omega} - \bar{\omega})} \right)\) and on the aggregate productivity shock. On the other hand, \(Y_t^I\) depends inversely on the gross interest rate set by the central bank \(R_t\), the real wage \(w_t\) and the price-markup \(\mu_t^p\).

Market clearing in the labor market requires

\[
N_t = \int \int_0^1 N_t(i, \omega_z, t) \, di \, dz
\]

which can be expressed as

\[
N_t = \Delta^\mu \left( \int \! N_t(\omega_z, t) \, dz \right) = (1 - \delta) \left( \frac{\alpha \xi p f A_t}{\mu_t^p w_t R_t} \right)^{\frac{1}{\alpha}} \left( \frac{(\bar{\omega})^k - (\tilde{\omega}_t)^k}{k(\bar{\omega} - \bar{\omega})} \right) f_{t-1}^m \Delta^w
\]

Notice that combining the above equations for \(Y_t^I\) and \(N_t\) and letting \(F_t = (1 - \delta) \left( \frac{(\bar{\omega})^k - (\tilde{\omega}_t)^k}{k(\bar{\omega} - \bar{\omega})} \right) f_{t-1}^m\), yields an expression for the aggregate production function in the intermediate good sector given by

\[
Y_t^I = \xi p f A_t (F_t)^{1 - \alpha} \left( \frac{N_t}{\Delta^w} \right)^\alpha
\]

where

\[
N_t = \left( \frac{\alpha \xi p f A_t}{\mu_t^p w_t R_t} \right)^{\frac{1}{1 - \alpha}} F_t \Delta^w
\]

the assumption that \(\omega_z, t\) follows a uniform distribution with support \(\omega, \bar{\omega}\) implies a total continuation rate given by \(\varphi_t(\tilde{\omega}_t) = (1 - \delta) \left( \frac{\bar{\omega} - \tilde{\omega}_t}{\bar{\omega} - \bar{\omega}} \right)\).

The deposit market equilibrium implies households having deposits in all active banks, therefore in the aggregate equilibrium \(D_t = \int D_t(j) \, dj\) must hold. Since all
active intermediate good producers take loans to cover their wage bill, market clearing in the loan market requires

\[ l_t^* (j, \omega_{z,t}) = w_t N_t^* (\omega_{z,t}) \quad \text{for all } z \]

Aggregating the above condition across all active intermediate good producers and taking into account the wage heterogeneity causes by the wage rigidity assumption, yields the following expression for aggregate loans:

\[ l_t = \frac{w_t N_t}{\Delta w} \]

Notice that the balance sheet for a lending bank implies that

\[ l_t (j, \omega_{z,t}) = d_t (j) \]

where \( d_t (j) \) denotes real deposits at bank \( j \). Aggregating the above condition across all active intermediate good producers and all lending banks yields an aggregate relation between loans and deposits:

\[ l_t = \varphi_t (\tilde{\omega}_t) f_{t-1}^m d_t \]

The above equation means that in the aggregate loans are a fraction of deposits. The specific fraction is endogenous and given by the measure of active credit contracts during period \( t \) and given by \( \varphi_t (\tilde{\omega}_t) f_{t-1}^m \). By the same token, the aggregate level of excess reserves is the fraction of real deposits that banks were not able to lend out to firms, that is

\[ er_t = (1 - \varphi_t (\tilde{\omega}_t) f_{t-1}^m) d_t \]
Intermediate good producers, final good producers and banks transfer their profits to the household at the end of each period. The aggregate real transfer received by the household from banks and each type of firm is given by

$$\pi_t^b = \left(R_t^l - R_t\right) l_t - b_t^k \kappa$$

$$\pi_t^I = \frac{Y_t^I}{\mu_t} - R_t^l l_t - \varphi_t \left(\tilde{\omega}_t\right) f_{t-1}^m x_t$$

$$\pi_t^f = C_t \left(1 - \frac{\Delta_t^b}{\mu_t}\right)$$

Equilibrium in this economy takes into consideration the aggregate balance sheet for banks

$$l_t + e\varepsilon_t + \xi_{bs} = d_t$$

where $\xi_{bs}$ is a fixed residual that represents, at the steady state, all assets of the banking sector that are not loans or excess reserves with the central bank. Taking into account all the aggregate equilibrium conditions and budget constraints, the aggregate resource constraint in this economy is characterized by

$$C_t = Y_t^f = Y_t^I - \left(b_t^k \kappa + \varphi_t \left(\tilde{\omega}_t\right) f_{t-1}^m x_t\right)$$

Therefore equilibrium in the final good market requires that consumption equals aggregate household income which, in turn, is equal to aggregate production of the intermediate good net of aggregate search and fixed costs. On the other hand, aggregating the CIA constraint, together with the government budget constraint, the aggregate balance sheet of banks as well as the aggregate equilibrium condition in the
loan market yields the following equilibrium condition for the real money balances market:

\[ C_t = m_t - (1 + i_t) er_t \]

The above equilibrium condition implies that the aggregate supply of real money balances is allocated to aggregate consumption as well as to paying back all the excess reserves that the banking sector hold with the central bank. The aggregate marginal rate of substitution, \( MRS_t \), is

\[ MRS_t = C_t \chi_t (N_t) \]

Finally, we define the average spread of interest rates (average credit spread) as the difference between the average loan rate and the bank’s opportunity cost of funds, given by the deposit rate. This interest rate spread is given by:

\[ \frac{R^l l_t - R^d l_t}{l_t} = \frac{1}{l_t} \left[ \left( (1 - \eta) (1 - \alpha) \right) \frac{1}{\mu^l t} \frac{Y^I_t}{\tilde{m}_t} \right] - \left( (1 - \eta) \frac{\mu^f t}{\mu^l t} \right) \]

where the terms \( R^l l_t \) and \( R^d l_t \) are obtained by computing the following conditional expectations:

\[ R^l l_t = E \left[ R^l_t (j, \omega_{z,t}) l^*_t (\omega_{z,t}) \mid \omega_{it} \geq \tilde{\omega}_t \right] \]

and

\[ R^d l_t = E \left[ R^d_t (\omega_{it}) \mid \omega_{it} \geq \tilde{\omega}_t \right] \]
2.4 Computation and simulations

The non-linear system of equations that characterize the dynamic equilibrium of the model is summarized below. The vector of endogenous variables $X_t$ is given by the following 39 variables classified according to:

1. Prices and real wages (11 variables):

$$X_{1,t} = [\Pi_t, \Pi^*_t, w_t, w^*_t, \bar{g}_{1,t}, \bar{g}_{2,t}, f^1_t, f^2_t, \mu^p_t, \Delta^p_t, \Delta^w_t]$$

2. Real variables (7 variables):

$$X_{2,t} = [Y^I_t, Y^f_t, C_t, N_t, U_t, L_t, \Delta_{t,t+1}]$$

3. "Monetary policy" variables (4 variables):

$$X_{3,t} = [m_t, er_t, R_t]$$

4. Credit market variables (13 variables):

$$X_{4,t} = [l_t, d_t, \tau_t, p^b_t, p^f_t, b^u_t, f^m_t, f_t, F_t, \bar{\omega}_t, \varphi_t(\bar{\omega}_t), cd_t, cc_t]$$

5. Auxiliary definitions for calibration purposes (4 variables):

$$X_{5,t} = [LS_t, FCS_t, \hat{t}_t, \hat{er}_t]$$

where $LS_t$ denotes the labor share of GDP, $FCS_t$ is the fixed cost of production share of GDP, $\hat{t}_t$ is aggregate loans as a fraction of total deposits and $\hat{er}_t$ is aggregate excess reserves as a fraction of total deposits.
The vector of endogenous variables is defined as:

\[ X_t = [X_{1,t}, X_{2,t}, X_{3,t}, X_{4,t}, X_{5,t}] \]

We solve the model using a standard perturbation method applied to a first order approximation around the non-stochastic steady-state of the model. In the appendix associated to this chapter we summarize the non-linear equilibrium conditions that characterize the aggregate equilibrium of the model. Next we explain the computation of the steady-state as well as the calibration procedure for the unknown parameters of the model.

2.4.1 The non-stochastic steady state

We assume that at the steady state the growth rate of real money balances is zero. This assumption together with the Euler equation evaluated at the steady state imply a gross inflation rate of \( \Pi = 1 \) and a gross nominal interest rate of \( R = \frac{1}{\beta} \).\(^2\) It is straightforward to notice that the price and wage index equations together with the price and wage dispersion equations evaluated at the steady state with zero net inflation imply no relative price and wage distortions. Then, at the steady state the following holds: \( \Pi^* = 1, \Delta^p = 1, \ w^* = w \) and \( \Delta^w = 1 \). Similarly, the optimal price setting equation evaluated at the steady state yields the following constant markup of final goods over intermediate goods prices:

\(^2\)If the model is closed with a Taylor rule instead of a money growth rule then, at the steady state the gross nominal interest rate is given by \( R = \left( \frac{4}{3} \right) (\Pi)^{\phi_p} \) while the Euler equation implies \( R = \frac{4}{3} \). Since \( \phi_p > 1 \) then at the steady state \( \Pi = 1 \) and \( R = \frac{1}{\beta} \).
\[ \mu^p = \frac{\epsilon_p}{\epsilon_p - 1} \]  

(\text{SS1})

By the same token, the wage setting equation evaluated at the steady state yields a constant markup of the real wage over the aggregate marginal rate of substitution:

\[ w = \left( \frac{\epsilon_w}{\epsilon_w - 1} \right) MRS \]  

(\text{SS2})

where \( MRS = C\chi N\overline{\varphi} \) is the aggregate marginal rate of substitution. Equation SS2 together with the aggregate labor force equation and the unemployment rate definition evaluated at the steady state, imply the following relationship between the unemployment rate and the elasticity among labor types \( \epsilon_w \):

\[ \left( \frac{1}{1 - U} \right) \overline{\varphi} = \left( \frac{\epsilon_w}{\epsilon_w - 1} \right) \]  

(\text{SS3})

Notice that if we parameterize \( \overline{\varphi} \) and target a particular value for the unemployment rate at the steady-state, we obtain a value for the \( \epsilon_w \) parameter. Combining the resource constraint together with the aggregate CIA constraint imply: \( Y^f = C = m - R (er) \) where \( Y^f \) is given by

\[ Y^f = \left( A\xi^{pf} \right) \left( \frac{\alpha}{\mu^p w R} \right)^{\frac{1}{1-\alpha}} (1-\delta) \left( \frac{\overline{\varphi}^{k} - (\overline{\omega})^{k}}{k (\overline{\varphi} - \overline{\omega})} \right) f^m - \left( \frac{1 - (1 - \delta) f^m}{\tau} \right) \kappa + \varphi (\overline{\omega}) f^m x^f \]  

(\text{SS4})

The aggregate labor demand together with the aggregate “credit” input denoted by \( F \) imply that the following equation must hold at the steady state:

\[ N = (1 - \delta) \left( \frac{\alpha A\xi^{pf}}{\mu^p w R} \right)^{\frac{1}{1-\alpha}} \left( \frac{\overline{\varphi}^{k} - (\overline{\omega})^{k}}{k (\overline{\varphi} - \overline{\omega})} \right) f^m \]  

(\text{SS5})
Finally, the equilibrium in the loan market, evaluated at the steady-state, can be reduced to the following set of equations:

\[
\alpha^\alpha (1 - \alpha)^{1 - \alpha} A^{\xi_{pf}} \bar{\omega}^{1 - \alpha} = (\mu^p w R)^{\alpha} \left[ x^f - \left( \frac{1 - \eta \mu \tau^{-1}}{1 - \bar{\eta}} \right) \frac{\kappa}{\mu \tau} \right] \quad (SS6)
\]

\[
(1 - \beta \varphi (\bar{\omega}) (1 - \bar{\eta} \mu \tau^{-1})) \frac{\kappa}{\mu \tau} \quad (SS7)
\]

\[
= (1 - \bar{\eta}) \beta \left( (1 - \alpha) \frac{(A^{\xi_{pf}})^{1 - \alpha}}{\mu^p w R^{\alpha}} \frac{\alpha}{\mu^p w R} \frac{(1 - \delta)}{f^m} \left( \frac{(\bar{\omega})^k - (\bar{\omega})^k}{k(\bar{\omega} - \bar{\omega})} \right) f^m - \varphi (\bar{\omega}) x^f \right)
\]

\[
(1 - \varphi (\bar{\omega}) + (1 - \delta) \mu \tau^{-1}) f^m = \mu \tau^{-1} \quad (SS8)
\]

The steady-state of the model can be partitioned in two blocks. The first block of equations can be solved recursively and consists of equations D1-D20 evaluated at the steady-state. The second block of equations constitute a simultaneous system of equations that incorporate equations SS4-SS8 together with equations D27 and D34 evaluated at the steady-state. We calibrate the following subset of nine parameters: \( x^f, \kappa, \xi_{pf}, \alpha, x^f, [\omega, \bar{\omega}] \) to be consistent with specified targets for the following endogenous variables: \( U, N, Y^f, \varphi (\bar{\omega}), cd, FCS, LS, \frac{1}{\delta} \) and \( \frac{\epsilon_w}{\delta} \). The strategy is explained in more detail in the next section.

**2.4.2 Calibration**

In order to compute the model's equilibrium, we must assign values to the following list of parameters:

- Preferences: \( \beta, \varphi, \chi \)
- Technology: \( A, \xi_{pf}, \alpha, x^f, [\omega, \bar{\omega}] \)
• Search technology and the loan market: \( \mu, \nu, \delta, \kappa, \xi^{bs}, \eta \)

• Price and wage stickiness: \( \theta_p, \theta_w, \epsilon_p, \epsilon_w \)

• Monetary policy: \( \theta_{ss} \)

We parameterize the following subset of parameters according to convention:

The subjective discount factor is set to \( \beta = 0.99 \) which is consistent with a steady-state real interest rate of 1 percent per quarter. We normalize the common component level of technology to be \( A = 1 \) as well as the support for the idiosyncratic productivity to be \([\omega, \overline{\omega}] = [0, 1]\). The parameters determining the degree of price and wage stickiness are set to imply average duration of one year, that is \( \theta_p = \theta_w = 0.75 \). The latter is set in a way consistent with much of the microeconomic evidence on wage and price setting.\(^3\) The elasticity of substitution among final goods is set to be \( \epsilon_p = 9 \), implying a steady state price markup of \( \mu^p = 1.125 \) or 12.5% and the inverse of the Frisch labor supply elasticity is set to be \( \overline{\varphi} = 5 \) which corresponds to a Frisch elasticity of 0.2.

Additionally, we fix values for two of the six parameters related to the loan market: The firm’s share in the Nash bargain problem is assumed to be \( \eta = 0.7 \) which is nearly close to the 0.68 value used by Petrosky and Wasmer (2014). It is worth to mention that Petrosky and Wasmer (2014) calibrate \( \eta \) from calculating the financial sector’s share of aggregate value added using date and the corresponding value added definition in their model. Due to lack of direct evidence we assume that the Hosios condition holds at the steady state and set the elasticity of the matching function at

\(^3\)See, for example, Nakamura and Steinson (2008).
ν = 0.7. In the appendix of this document, we present different robustness checks for ν and η in all the experiments and results we report. We include the following range of values: \( v \in [0.6, 0.8] \) and \( \eta \in [0.5, 0.8] \) for two reasons: When \( v < 0.6 \) the linear approximation of the dynamic equations of the model does not satisfy the Blanchard and Khan (1982) rank condition. On the other hand, if \( \eta < 0.5 \), solving for the non-linear steady state yields imaginary roots. Both restrictions on the range of values for \( \nu \) and \( \eta \) may be an indication that there is no equilibrium in the loan market or that the loan market collapses such as in Becsi, Li and Wang (2005) and Becsi, Li and Wang (2005).

In the next table we summarize the parameter values described above:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Bank’s search costs</td>
<td>0.99</td>
</tr>
<tr>
<td>$A$</td>
<td>Matching function scale parameter</td>
<td>1.0</td>
</tr>
<tr>
<td>$[w, \overline{w}]$</td>
<td>Exogenous probability of separation</td>
<td>[0,1]</td>
</tr>
<tr>
<td>$\theta_p$</td>
<td>Calvo parameter for price setting</td>
<td>0.75</td>
</tr>
<tr>
<td>$\theta_w$</td>
<td>Calvo parameter for price setting</td>
<td>0.75</td>
</tr>
<tr>
<td>$\epsilon_p$</td>
<td>Elasticity of substitution among final goods</td>
<td>9.0</td>
</tr>
<tr>
<td>$\overline{\varphi}$</td>
<td>Inverse of Frisch Elasticity</td>
<td>5</td>
</tr>
<tr>
<td>$\overline{\eta}$</td>
<td>Firm’s Nash bargaining share</td>
<td>0.7</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Matching function elasticity</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Table 2.1: Parameters taken from the data and conventional values from the literature

A total of nine parameters of the model are calibrated to be consistent with a set of nine endogenous targets that we specify below. These parameters are classified as follows:

- Calibrated loan market parameters: The search cost faced by a bank \( \kappa \), the scale parameter of the aggregate matching function \( \mu \), the exogenous probability of
credit destruction $\delta$ and the residual term on the aggregate banks’ balance sheet $\xi^{bs}$.

- Calibrated technology parameters: The elasticity of labor and the scale parameter in the aggregate production function for intermediate goods $\alpha$ and $\xi^{pf}$ respectively as well as the fixed cost of producing the intermediate good $x^f$.

- Calibrated preference parameters: The preference shifter $\chi$ and the elasticity among labor types, $\epsilon_w$.

In the next table, we report the steady state targets that we use to calibrate the above subset of parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U$</td>
<td>Unemployment rate</td>
<td>0.05</td>
</tr>
<tr>
<td>$N$</td>
<td>Employment</td>
<td>0.59</td>
</tr>
<tr>
<td>$Y^f$</td>
<td>GDP</td>
<td>1</td>
</tr>
<tr>
<td>$\varphi(\tilde{\omega})$</td>
<td>Overall continuation rate</td>
<td>0.7</td>
</tr>
<tr>
<td>$c_d$</td>
<td>Credit destruction rate</td>
<td>0.029</td>
</tr>
<tr>
<td>$\varphi(\tilde{\omega})f^m x^f$</td>
<td>Fixed cost share of GDP</td>
<td>0.1</td>
</tr>
<tr>
<td>$\frac{\tilde{Y}^N}{\tilde{Y}^L}$</td>
<td>Labor share of GDP</td>
<td>2/3</td>
</tr>
<tr>
<td>$\frac{L}{\tilde{L}}$</td>
<td>Loan to deposits ratio</td>
<td>0.63</td>
</tr>
<tr>
<td>$\frac{ER}{D}$</td>
<td>Excess reserves to deposits ratio</td>
<td>0.015</td>
</tr>
</tbody>
</table>

Table 2.2: Steady state targets

Following Gali (2011) we target an unemployment rate of $U = 0.05$ and aggregate employment of $N = 0.59$ at the steady-state. Given this, equation SS3 above, implies an elasticity of substitution among labor types of $\epsilon_w = 4.4205$ which in turn is associated with an average wage markup of 29 percent. Due to little direct evidence, we
assume a 10 percent share of the fixed cost of production in GDP and target a steady-state overall probability of continuation for a credit relationship of 70 percent, that is we set $FCS = 0.1$ and $\varphi(\tilde{\omega}) = 0.7$. The latter is consistent with findings reported in Chodorow-Reich (2014) for banking relationships in the U.S. syndicated loan market. Specifically, Chodorow-Reich (2014) finds that after controlling for a bank’s average market share, a bank that served as the prior lead lender of a private borrower has a 71 percent point greater likelihood of serving as the new lead lender in the same loan contract. Given that the scale technology parameter $\xi_{pf}$ is chosen in order to normalize the steady-state level of GDP to unity (i.e., $Y^f = 1$ in equation SS4), we can solve for $x^f$ to be consistent with the steady-state target imposed on the fixed cost share of GDP, which is given by

$$\frac{\varphi(\tilde{\omega}) f^m x^f}{Y^f} = 0.1$$  \hspace{1cm} (SS9)

We target a steady-state loan to deposit ratio of $\frac{l^d}{a} = 0.63$ by using quarterly data on commercial and industrial loans as well as saving deposits for all U.S. commercial banks during the great moderation period which is assumed to be between 1985 and 2007. The steady-state target for the loan to deposit ratio $\frac{l^d}{a}$, together with the steady-state target for $\varphi(\tilde{\omega})$ explained above, allow us to obtain the steady-state level for the measure of firms in a credit relationship ($f^m$) by using the relationship between loans and deposits that arises when aggregating the balance sheet of those banks that are able to lend out their available funds to intermediate good producers. This condition at the steady-state
is given by:
\[ \frac{l}{d} = \varphi (\tilde{\omega}) f^m \]  

Clearly, equations SS9 and SS10 together with the specified steady-state targets for \( FCS, \varphi (\tilde{\omega}), \frac{l}{d} \) and \( Y^f \) are consistent with \( f^m = 0.9 \) and \( x^f = 0.1587 \). Therefore, the steady-state of the model implies that 90 percent of producing firms (intermediate good producers) are in a credit contract with a bank. The parameter \( x^f = 0.1587 \) is consistent with a 10 percent fixed cost of production share of GDP, a 70 percent probability of overall continuation for a credit relationship and a 90 percent measure of firms in a credit relationship.

We target a labor share of GDP at the steady-state of 2/3, that is \( LS = \frac{wN}{Y^f} = \frac{2}{3} \). The latter definition together with the equilibrium condition in the loan market evaluated at the steady-state, \( l = wN \) yields a steady-state value for the real wage equal to \( w = 1.13 \) and aggregate real loans of \( l = 2/3 \). Then, given the steady-state target on the loan to deposit ratio, we obtain the steady-state value of aggregate real deposits to be \( d = 1.0582 \). We use the average of all reserve balances with federal reserve banks during the great moderation period and the average of all saving deposits at U.S commercial banks during the same period of time in order to set the ratio of aggregate excess reserves to aggregate deposits to be 1.5 percent. The aggregate level of reserves consistent with the specified target for \( \frac{er}{d} = 0.015 \) and the steady-state level of aggregate deposits obtained before is \( er = 0.0159 \). The resource constraint of the economy implies consumption at the steady-state to be \( Y^f = C = 1 \) while the aggregate CIA constraint can be solved for the steady-state level of real money balances
m given our parametrization of \( R = 1.0101 \) and the steady-state level of aggregate excess reserves that we have already obtained. Thus, at the steady-state, the supply of real money balances must be allocated into consumption and interest rate payments on excess reserves,

\[
m = C + (R) er
\]  \hspace{1cm} (SS11)

with \( m = 1.016 \).

The labor force equation evaluated at the steady-state implies:

\[
w = C \chi (L)^\varphi
\]  \hspace{1cm} (SS12)

given the parameterization of \( \varphi \) and the steady-state values for \( w,C \) and \( L \) obtained above, we can solve consistently for the preference shift parameter to be \( \chi = 12.2297 \). Notice that the calibration of \( \chi \) is also consistent with the optimal price setting equation evaluated at the steady-state, equation SS2 above, and therefore, it is consistent with the steady-state level of employment that we are targeting \( (N = 0.59) \). The stochastic discount factor evaluated at the steady-state yields \( \Delta = \beta = 0.99 \).

The aggregate balance sheet of banks evaluated at the steady-state allow us to obtain the residual term as a fraction of deposits as \( \xi^{bs} = 1 - \frac{l}{d} - \frac{er}{d} = 0.3550 \).

Following Contessi and Francis (2013), we target an average quarterly credit destruction rate of 2.9 percent during the great moderation period. The credit destruction rate implied by the model and evaluated at the steady-state is

\[
\begin{align*}
\text{cd} &= 0.029 = 1 - \varphi (\tilde{\omega}) - \mu r^\nu \delta \\
\end{align*}
\]  \hspace{1cm} (SS13)
Given the above targets and parameter calibration, equations SS4-SS8 together with equations SS13 and SS14 can be solved for the following set of parameters $\kappa, \mu, \delta, \xi^f, \alpha$ as well as for the corresponding steady-state values for $\bar{\omega}$, $\tau$. Equation SS14 is given by the steady-state probability of continuation for a credit contract:

$$\varphi(\bar{\omega}) = (1 - \delta) \left( \frac{\omega - \bar{\omega}}{\bar{\omega} - \bar{\omega}} \right)$$  \hspace{1cm} (SS14)

Solving the system of equations formed by equations SS4-SS8 and SS13-SS14 yields the baseline calibration for the remaining parameters of the model: $\kappa, \mu, \delta, \xi^f$ and $\alpha$. The next table summarizes the calibrated parameters of the model that are solved to be consistent with the steady state targets specified above.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>Bank’s search costs</td>
<td>0.1469</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Matching function scale parameter</td>
<td>0.9662</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Exogenous probability of separation</td>
<td>0.2281</td>
</tr>
<tr>
<td>$\xi^f$</td>
<td>Production function scale parameter</td>
<td>2.8855</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Labor elasticity of production function</td>
<td>0.6684</td>
</tr>
<tr>
<td>$\xi_{bs}$</td>
<td>Residual term on aggregate bank’s balance sheet</td>
<td>0.3550</td>
</tr>
<tr>
<td>$\pi^f$</td>
<td>Fixed cost of production</td>
<td>0.1587</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Preference parameter for dis-utility of labor</td>
<td>12.2297</td>
</tr>
<tr>
<td>$\epsilon_w$</td>
<td>Elasticity of substitution among labor types</td>
<td>4.4205</td>
</tr>
</tbody>
</table>

Table 2.3: Calibrated parameters to be consistent with steady state targets

The above results imply that $k = \frac{2 - \alpha}{1 - \alpha} = 4.0158$. The steady state values for a group of endogenous variables of the model are summarized in the following table:
### Table 2.4: Steady state values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Pi$</td>
<td>1</td>
<td>$\Pi^*$</td>
<td>1</td>
</tr>
<tr>
<td>$\Delta^w$</td>
<td>1</td>
<td>$\Delta^p$</td>
<td>1</td>
</tr>
<tr>
<td>$Y^I$</td>
<td>1.1334</td>
<td>$Y^f$</td>
<td>1</td>
</tr>
<tr>
<td>$F$</td>
<td>0.1730</td>
<td>$C$</td>
<td>1</td>
</tr>
<tr>
<td>$b^u$</td>
<td>0.2273</td>
<td>$m$</td>
<td>1.0160</td>
</tr>
<tr>
<td>$f$</td>
<td>0.3053</td>
<td>$R$</td>
<td>1.0101</td>
</tr>
<tr>
<td>$p^f$</td>
<td>0.8843</td>
<td>$w$</td>
<td>1.1299</td>
</tr>
<tr>
<td>$p^p$</td>
<td>1.1880</td>
<td>$\mu^p$</td>
<td>1.1250</td>
</tr>
<tr>
<td>$U$</td>
<td>0.0500</td>
<td>$L$</td>
<td>0.6211</td>
</tr>
<tr>
<td>$N$</td>
<td>0.5900</td>
<td>$\varphi(\bar{\omega})$</td>
<td>0.7000</td>
</tr>
<tr>
<td>$f^m$</td>
<td>0.9000</td>
<td>$\bar{\omega}$</td>
<td>0.0931</td>
</tr>
<tr>
<td>$l$</td>
<td>0.6667</td>
<td>$d$</td>
<td>1.0582</td>
</tr>
<tr>
<td>$\tau$</td>
<td>1.3433</td>
<td>$cr$</td>
<td>0.0159</td>
</tr>
</tbody>
</table>

#### 2.4.3 Equilibrium dynamics: Monetary policy and financial shocks

We interpret the dynamic responses to different shocks focusing on two main equations. The optimal hiring rule for all active credit matches, given by:

$$
\alpha \xi^{pf} A_t \omega_{z,t} N_t^* (\omega_{z,t})^{\alpha-1} = \mu^p w_t R_t; \forall \omega_{z,t} > \bar{\omega}_t
$$

and the reservation productivity level written in terms of the real marginal cost $MC_t$ and the credit market tightness $\tau_t$:

$$
\bar{\omega}_t = \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \left( \frac{(MC_t)^\alpha}{A_t} \right) \left[ x^f - \frac{\kappa}{1-\eta} \left( \mu^p \tau _t^{1-v} - \eta \mu^p \tau _t \right) \right]^{1-\alpha}
$$

Notice that the first equation is the result of the Nash bargaining protocol over the joint surplus generated by a credit contract. Therefore, conditional on surviving, each credit contract will determine the loan size and hire workers consistent to the point where the marginal product of labor $MPL_t (\omega_{z,t}) = \alpha \xi^{pf} A_t \omega_{z,t} N_t^* (\omega_{z,t})^{\alpha-1}$ is equal to the real
marginal cost of labor expressed in terms of the intermediate good \( MC_t = \mu_t^0 w_t R_t \). Notice that the term \( w_t R_t \) is expressed in terms of the final good thus in order to express the real marginal cost in terms of the intermediate good, the term \( w_t R_t \) have to be multiplied by the mark-up, \( \mu_t = \frac{P_t}{P_t^I} \) where \( P_t^I \) is the intermediate good price index. Changes in this equation generate an intensive margin effect since it holds only for those credit matches that have survived the exogenous as well as endogenous separation hazards.

The second equation is obtained by setting the joint surplus for a credit contract to zero. The reservation productivity, \( \bar{\omega}_t \), that results, is a productivity threshold that select the subset of firms that are able to obtain funds, hire workers and produce during the period. This threshold productivity generates an extensive margin effect whenever it responds to aggregate macroeconomic shocks. Notice that \( \bar{\omega}_t \) has two main determinants: \( MC_t \) and \( \tau_t \). In a partial equilibrium setting, an increase in \( MC_t \) will raise the reservation productivity taking as given credit market tightness, \( \tau_t \). By the same token, our benchmark calibration implies that given \( MC_t \) constant, an increase in \( \tau_t \) will produce an increase in the reservation productivity. The main transmission mechanism of aggregate shocks in this economy goes through changes in \( \bar{\omega}_t \) which ultimately is a consequence of movements in the joint surplus of a credit match, \( V_t^{JS}(\omega_{z,t}) \). Fluctuations in \( MC_t \) and \( \tau_t \) affect \( V_t^{JS}(\omega_{z,t}) \): An increase in \( MC_t \) given \( \tau_t \) constant reduces the joint surplus of an active credit contract while a tighter credit market (higher \( \tau_t \)), given \( MC_t \) constant will also end up reducing this joint surplus. General equilibrium effects will determine simultaneously all the variables.
2.4.3.1 Credit inefficiency wedge and labor productivity

Recall that aggregation in the final good’s market yields

\[ Y_f^I = Y^I_t - \left( b^\kappa_i + \varphi_i (\tilde{\omega}_t) f_{t-1}^m x^f \right) \]

where \( Y^I_t \) is the economy’s aggregate production function for intermediate goods and it is given by:

\[ Y^I_t = \xi pf A_t F_t^{1-\alpha} \left( \frac{N_t}{\Delta w_t} \right)^\alpha \]

with \( F_t \) being an endogenous component of technology that depends on credit market conditions as well as on the reservation productivity level which is given by:

\[ F_t = (1 - \delta) \left( \frac{(\omega)^k - (\bar{\omega})^k}{k (\omega - \bar{\omega})} \right) f_{t-1}^m \]

Through this section, we define the credit inefficiency wedge as the endogenous component of technology that is not related to employment, given by: \( (F_t)^{1-\alpha} \). Notice that without credit market frictions, \( F_t = 1 \) and aggregate output in the intermediate sector would be: \( Y^I_t = \xi pf A_t (\frac{N_t}{\Delta w_t})^\alpha \). In this latter case, the only inefficiency that appears after aggregation is the one related to presence of wage rigidities. But, under the assumption of search and matching frictions in the loan market, this inefficiency wedge depends on the aggregate probability of continuation of a credit contract as well as on the mass of active credit contracts. Both, depending ultimately on the common reservation productivity threshold. Clearly, in this model, credit conditions affect this inefficiency wedge, generating amplification effects to any shock that hit the economy.
On the other hand, labor productivity in the intermediate good sector is given by:

\[ LP_t = \frac{Y_t^I}{N_t} = \frac{\xi \rho_f A_t}{(\Delta^w_t)^\alpha} \left( \frac{F_t}{N_t} \right)^{1-\alpha} \]

In our model, labor productivity is also affected by the credit wedge. If the loan market is a Walrasian centralized market then \( F_t = 1 \) and credit conditions do not affect labor productivity. Credit market frictions in the form of search and matching frictions generate inefficient fluctuations of labor productivity, employment and intermediate output as well as final output. This of course, translates into inefficient fluctuations in the unemployment rate given the interaction of wage rigidities, market power, and the labor force participation condition that characterize the labor market. Financial shocks are propagated and amplified by the endogenous response of this credit inefficiency ‘input’ term.

Finally, we can define total factor productivity, \( TFP_t \), as all the terms in the aggregate production function that are not associated with the labor input, that is:

\[ TFP_t = \xi \rho_f A_t (F_t)^{1-\alpha} \]

The inefficiency associated with the presence of credit frictions also affects the evolution of total factor productivity by making it responsive to aggregate shocks as long as credit conditions, summarized by the term \( F_t \), also respond to those same shocks. Therefore, total factor productivity is also subject to inefficient endogenous fluctuations that then propagate and amplify aggregate shocks.
2.4.3.2 The effects of a monetary policy shock

Figures (2.5)-(2.8) present the equilibrium responses of several variables of interest to an expansionary monetary policy shock under the assumption that the central bank follows an exogenous money growth rule. An expansionary monetary policy shock corresponds to a 0.25 percent quarterly increase, on impact, in the rate of nominal money growth. On impact, the nominal interest rate increases slightly (20 basis points) since the model does not engender a liquidity effect as we assume a CIA constraint and a logarithmic period utility function. The existence of money demand in our model is a consequence of the cash-in-advance structure assumed.

Monetary policy is transmitted through the standard interest rate channel as well as through impacts on the intensive and extensive margin associated with the interaction among the working capital channel, search and matching frictions in the loan market, and the presence of a match-specific productivity level. The standard interest rate channel of monetary policy works through changes the long run real interest rate and its consequent effect on consumption, output employment, unemployment and inflation. Although the nominal interest rate increases on impact (see figure (2.5)), it falls quickly below zero and remains negative for almost 10 quarters, due to the presence of nominal rigidities and their influence on inflation expectations.

Recall that conditional on surviving, a credit contract requires the firm to equalize the marginal product of labor to the real marginal cost of labor expressed in terms of the intermediate good. Monetary policy shocks alter this optimality condition
generating an intensive margin effect. On the other hand, the extensive margin effect is generated by the persistent decline of reservation productivity, $\tilde{\omega}_t$, which expands the measure of active firms. The aggregation of both margin effects is summarized in the evolution of the credit inefficiency wedge given by the term $F_t$ that appears in the aggregate production function for the intermediate good sector as well as on the aggregate labor demand. The interaction of monetary policy and credit frictions together with heterogeneous productivity at the firm level, generates a very persistent increase in $F_t$ that lasts for approximately 16 quarters (see figure (2.6)).

This additional monetary policy transmission mechanism, through $F_t$, reinforces the standard interest rate channel since it creates better credit conditions for firms and banks by raising the joint surplus of a credit match thus generating a persis-
Figure 2.6: Model responses to a monetary policy shock: B

Note: Source: Authors’ calculations based model simulation

tent decline in the measure of credit market tightness, $\tau_t$, the average spread of interest rates, and the average loan interest rate. Expansionary monetary policy reduces the real marginal cost of hiring a new worker for all producing firms, raising aggregate labor demand at each level of the real wage, and thus employment and production for all active production units (the intensive margin effect of monetary policy). As noted earlier, an expansionary monetary policy does not generate a liquidity effect—this implies the nominal interest rate will increase initially. However the marginal cost of labor expressed in terms of the intermediate good will decrease because the price mark-up ($\mu_t$) exhibits a strong decline that overpowers the initial increase in $R_t$ as well as the persistent increase in $w_t$. Notice that the effect over the real wage occurs due to the increase in aggregate labor demand and is also present in the standard New Keynesian model with wage rigidities. As noted earlier, the expansion of real money balances
also reduces the reservation productivity level for those credit matches generating non-negative joint surpluses. As a consequence, more firms are able to obtain external funds, hire workers and produce (the extensive margin of monetary policy) so there will be fewer firms searching for external funds \( (f_t) \). Thus the firm finding rate increases, see figure (2.7).

Figure 2.7: Model responses to a monetary policy shock: C

Note: Source: Authors’ calculations based model simulation

Free entry of banks to the loan market implies more banks will enter, inducing a temporary increase in the measure of banks searching for projects/borrowers, \( b^u_t \) and decreasing the bank finding rate on impact and for two subsequent quarters. As the reservation productivity, \( \tilde{\omega}_t \), declines the overall continuation rate for credit contracts increases by almost 20 basis points on impact and remains persistently high for more than six quarters. Therefore credit conditions improve for currently active firms as well as for potential firms—those with profitable projects—as can be seen through the persis-
tent decline in \( \tau_t \) and corresponding increase in the measure of active credit contracts, \( f_t^m \) (see figure (2.7)).

These new credit conditions translate into a persistent decline in the credit destruction rate as well as a persistent increase in the credit creation rate which together create a net increase in aggregate loans and deposits. The decline in credit destruction is larger and more persistent than the increase in credit creation on impact. This well-known feature of bank credit is generated by the relative ease with which banks can moderate their current contracts compared to negotiating new contracts (see figure (2.6)). The increase in the probability of credit contract continuation impacts credit destruction immediately while in order for new credit to be negotiated, banks need to enter.

Nominal price and wage rigidities also contribute to generating the persistent decline in the unemployment rate and an expansionary effect over all aggregate macroeconomic variables such as consumption, employment, and output (see figure (2.8)). Another feature of the modeling framework is that the expansion in economic activity as well as the improved credit conditions in the loan market induce the central bank to automatically reduce its loans to the banking sector. In this model, central bank lending to the banking sector corresponds to excess reserves, \( er_t \), that banks hold with the central bank. The decline in \( er_t \) (see figure (2.6)) is a consequence of the reduction in the overall continuation rate of credit contracts and the free entry of banks into the loan market. That is, in equilibrium, there will be fewer banks requiring loans from the central bank to cover the interest rate on deposits.
2.4.3.3 The effects of a financial shock

Figures (2.9)-(2.12) illustrate the dynamics responses of a number of aggregate variables to a negative (bad) financial shock. In our modeling context, a financial shock is defined as an unexpected persistent increase in the exogenous separation rate for credit contracts, $\delta_t$. Recall, the overall continuation rate of a credit contract is defined as $\varphi_t(\tilde{\omega}_t) = (1 - \delta_t)(1 - \gamma_t(\tilde{\omega}_t))$ where its exogenous component $\delta_t$ follows an AR(1) process given by:

$$\delta_t - \delta = \rho_\delta (\delta_{t-1} - \delta) + \varepsilon_\delta^t$$

Our calibration procedure is consistent with a steady-state value for $\delta$ of 0.2163.

In our model, a financial shock implies that a fraction of existing credit con-
tracts are exogenously terminated due to the decline in the overall continuation rate of credit relationships. The impact of such an increase in the separation rate qualitatively matches the impact of a rise in the excess bond premium in our VAR results discussed in the motivation section above, where we use a one standard deviation in crease in the excess bond premium as a measure of a financial shock. After a negative financial shock, there will be a larger mass of intermediate good producers searching for funds, \( f_t \) as well as a larger mass of banks searching for profitable projects to fund, denoted by \( b_t^n \).

From the point of view of a bank, the expected value of searching for a project/borrower turns out to be temporally negative following a negative financial shock, inducing banks to exit the loan market until the expected value of searching for borrowers increases. Since banks are able to exit the loan market, the measure of firms searching for funds after a financial shock will be larger than the measure of banks searching for borrowers inducing an increase in the measure of credit market tightness, \( \tau_t \). Moreover, the mass of intermediate good producers separated from their previous credit contract, are not able to exit the market as is the case of banks, but are only able to search for external funding in order to attempt production in the future. Therefore, credit market conditions tighten from the point of view of borrowers, exhibited by a decline in the firm’s finding rate, \( p_{f}^t \). The bank’s finding rate, \( p_{b}^t \) increases because banks are able to exit the market when its surplus from looking for firms becomes negative. As a result of the increase in the bank finding rate but decline in the firm finding rate, the overall loan market tightens from the perspective of borrowers.

Notice that when banks are separated and exit the loan market, the central
bank must automatically increase excess reserves, $er_t$, to compensate for the fact that banks have no remaining assets to pay interest on households’ deposits.

The transmission of the financial shock is reinforced by a decline in the joint surplus to a credit match which is a consequence of the large persistent raise in the reservation productivity level $\tilde{\omega}_t$. The latter, induces an even more pronounced and persistent fall on the overall continuation rate, $\varphi_t (\tilde{\omega}_t)$, that adds to the one generated by the initial shock to the exogenous separation rate, $\delta_t$. The mass of firms and banks that start the period in a credit contract but also survive the higher separation rate that occurs after a financial shock, decide to raise their reservation productivity threshold due to the fall in the joint surplus of a credit relationship and the consequent tighter credit market (higher $\tau_t$). The dynamics associated to the reservation productivity
level produce an extensive margin effect associated to a selection effect that reduces the subset of firms able to obtain external funds, hire workers and produce. The tighter credit conditions that occur after a negative financial shock are also reflected in a persistent decline of the mass of firms engaged in a credit contract, $f_{t-1}^m$ and a significant reduction on the aggregate amount of loans, $l_t$. These new credit conditions translate to a persistent raise of the average spread of interest rates as well as a raise in the average loan rate. However, the financial shock generates an intensive margin effect that partially off sets the extensive margin effect. This intensive margin effect is related to those credit contracts -firm and bank pairs- that survive the financial shock but adjust their existing loan contract by changing the conditions that characterize their bilateral bargaining protocol. Specifically, a financial shock reduces the real marginal cost of labor expressed in terms of the intermediate good, $\mu_t^p w_t R_t$ for all active intermediate good.

Figure 2.10: Model responses to a financial shock: B

Note: Source: Authors’ calculations based model simulation
producers, inducing a small recovery on the aggregate labor demand that is not enough to off set the negative extensive margin effect of a financial shock. At the aggregate level, a persistent negative financial shock generates a negative response on employment, labor productivity and total factor productivity. Such responses are a consequence of aggregating the intensive and extensive margin effects described above. The deep and prolonged recession that occurs after a financial shock is characterized by a persistent decline of the unemployment rate which is explained by the interaction between price and wage rigidities, the labor force condition together with the search and matching frictions of the loan market. By the same token, aggregate loans decline even though households increase their deposits with banks as a response to the tightening of credit conditions. In this sense, a financial crises also generates an increase in savings as well as
a decline in aggregate expenditures and bank lending. Thus, a financial shock, modeled as an exogenous increase in the separation rate of credit contracts, induces a persistent recession in terms of GDP, the unemployment rate, consumption, labor productivity and total factor productivity. In this model, the propagation of the financial shock goes through persistent changes in the economy wide reservation productivity, the overall continuation rate as well as changes in the marginal cost of labor expressed in terms of the intermediate good. All of these changes produce a significant tightening of aggregate credit conditions that are finally reflected in a persistent fall of the endogenous component of the aggregate total factor productivity, labor productivity and a persistent raise of the average interest rate spread. Finally, an exogenous increase in the credit separation rate, leads to negative response of the inflation rate which is associated to the persistent fall in the real marginal cost. The real supply of money increases causing

Figure 2.12: Model responses to a financial shock: D

Note: Source: Authors’ calculations based model simulation
the nominal interest rate to fall below zero for the first two quarters. After that, the fall in the real money demand (fall in consumption) more than compensate the increase in the real money supply and the nominal interest rate raises.

2.5 Conclusion

The Great Recession and slow recovery was characterized by high and persistent unemployment, a decline in overall bank lending including a decline in lending to firms (commercial and industrial lending). The net decline in bank lending across all loan types (including to consumers, to firms, and for real estate related reasons) was a novel feature of the Great Recession as it had not occurred in any previous post-Volker recession. These characteristics of the recession are suggestive of a the potentially critical relationship between bank credit and unemployment.

We find that the effect of a financial shock on unemployment and output is amplified through its effect on gross credit flows and labor productivity in a vector autoregression (VAR) framework. We use this suggestive result as motivation to introduce gross credit flows via search and matching frictions in the credit market into a New Keynesian model with nominal rigidities and unemployment. By introducing credit frictions with endogenous credit destruction into this well studied model, we provide additional insights on movements in employment and output following a monetary policy and a financial shock.

Allowing for endogenous credit destruction permits us to calculate movements
in gross flows, as opposed to net credit, in a theoretical setting. Our model accounts for a credit inefficiency wedge that comes as the endogenous component of aggregate technology that is not related to employment. We have shown that in the absence of credit market frictions, this inefficiency disappears. In the presence of credit frictions this inefficiency wedge depends on the aggregate probability of continuation of a credit contract as well as on the mass of active firms. Both, depending ultimately on the common reservation productivity threshold. Thus, in our model, credit conditions affect this inefficiency wedge, generating amplification effects to any shock that hit the economy. We implement a financial shock as a persistent exogenous decline in the probability of continuation of credit contracts.

Credit market frictions in the form of search and matching generate inefficient fluctuations of labor productivity, employment and output. This of course, translates into inefficient fluctuations in the unemployment rate given the presence of wage rigidities, market power and the labor force participation condition that characterize the labor market. Financial shocks are propagated and amplified by the endogenous response of this credit inefficiency ”input” term, producing intensive and margin effects as well as higher average interest rate spreads together with tighter credit conditions. The simulations of the model are similar to the ones we obtain from the VAR analysis.
Chapter 3

Monetary policy operating procedures, lending frictions, and employment

- with Christopher Limnios and Carl Walsh-

3.1 Introduction

How does the central bank’s operating procedure affect the transmission process of monetary policy? In the 20 years prior to the financial crisis beginning in 2007, this question was little examined. With major central banks directly targeting the interbank interest rate, this single interest rate was viewed as the sole link between actions of the central bank and the real economy. And how the central bank managed discount borrowing and whether it paid interest on reserves were implicitly deemed irrelevant to
understanding how changes in the target for the interbank rate affected real economic activity. This view was most explicit in standard new Keynesian models in which the policy interest rate was the sole interest rate appearing in the model and monetary aggregates, including bank reserves, could be ignored.

The financial crisis, the renewed recognition that financial markets are subject to frictions, the constraint imposed by the zero lower bound on the policy interest rate, and the adoption of new procedures for affecting reserve supply call for a reexamination of the links between the central bank’s operating procedures in the interbank market, the availability of credit, and the impact of monetary policy on the real economy.

In this paper, we examine these links in a model in which banks hold reserves to meet random fluctuations in settlements, and the central bank pays interest on reserves, lends reserves at a penalty rate, and can independently affect the quantity of reserves and the level of interest rates. Banks make loans to firms in credit markets characterized by matching frictions, and interest rates on loans are set in bilateral bargaining between banks and firms.

The type of monetary policy operating procedure we analyze is often called a corridor or channel system of interest rate control. Such a system is employed by several central banks (e.g., the Reserve Bank of New Zealand) and is the type of system the U.S. Federal Reserve seems likely to employ when interest rates return to historically more normal levels. In a channel system, a central bank offers a lending facility, whereby commercial banks are permitted to borrow against collateral from the central bank at an interest rate that is above the target rate (the penalty or ceiling rate) and a
deposit facility, whereby banks can earn overnight interest on their excess reserves at a rate that is below the target rate (the floor rate). The ceiling and floor rates form an interest rate channel (or corridor). In reality, many central banks use what is known as a symmetric channel system for monetary policy implementation, in which the ceiling and floor rates are the same number of basis points (the width) above and below the target rate. The symmetric channel systems used by various central banks differ in many respects. For example, the Bank of England and the ECB institute a relatively wide channel framework with a spread of 100 basis points on each side of the target. Australia and Canada, in contrast, operate narrow channels with a spread of only 25 basis points above and below their targets.¹

There is a small existing literature on channel systems. Woodford (2000, 2001, 2003) discusses how to conduct monetary policy with a vanishing stock of money using the framework of a channel system. Whitesell (2006) evaluates reserves regimes versus channel systems.² Berentsen and Monnet (2006, 2008) develop a general equilibrium framework of a channel system and investigate optimal policy. Berentsen, Marchesiani, and Waller (2010) show that a positive spread between the policy rate and the interest rate paid on reserves is optimal. The uncertainty facing banks in these papers arises from a Diamond-Dybvig environment in which depositors are revealed, ex post, to be either patient or impatient. Thus, banks must hold excess reserves to insure against a net payment drain from the entire banking system. In contrast, we assume uncertainty arises from the random distribution of payment flows among banks that result in some

¹ Australia and Canada have no reserve requirements.
² These models are also discussed in Walsh (2010).
banks facing a net outflow while others experience a net inflow. However, the net flow aggregated across the banking system is always zero. Other work related to elements of channel systems include Gaspar, Quiros and Mendizabal (2004), Guthrie and Wright (2000), and Heller and Lengwiler (2003).

In contrast to these papers, we focus on the links between the implementation of monetary policy under a channel system of interest-rate control and credit spreads in the market for bank loans in the face of lending frictions. These lending frictions are captured by a simple search-and-matching framework, with lending interest rates determined by Nash bargaining between lenders (banks) and borrowers (firms). In this environment, the joint surplus to the bank and the firm depends, in part, on the structure of the interbank market as the structure of the interbank market affects the outside opportunity of the bank.

The paper is also closely related to another stream of literature—frictions in credit markets. Most work on credit market frictions has rightly focused on issues related to informational asymmetry and moral hazard issues. The literature on this is large; early examples include Carlstrom-Fuerst (1997), Bernanke, Gertler, and Gilchrist (1999), and Kiyotaki and Moore (1997), while more recent papers include Gertler and Kiyotaki (2010), Gertler and Karadi (2010), and del Negro, Eggertsson, Ferrero, and Kiyotaki (2011).

Another type of frictions in financial markets is what is referred to as "search and entry frictions" (Becsi et al, 2000), which represents the search or negotiating costs related to the initial participation of firms into credit markets. Recent empirical
evidence using U.S disaggregated bank-level data by Contesi and Francis (2011 and 2013), Craig and Haubrich (2006) Dell’ Ariccia and Garibaldi (2005) and Herrera, Kolar and Minetti (2007/2011) suggest that sizable gross credit flows coexist at the business cycle frequency. This literature emphasizes the existence of heterogeneous patterns of credit creation and contraction at any phase of the business cycle. For example, Dell’ Ariccia and Garibaldi (2005) find that in the United States, gross credit flows are by an order of magnitude more volatile than GDP and investment. The empirical dynamics patterns of credit flows found in this literature are consistent with those predicted by search models in which the interaction of shocks generates simultaneous occurrence of credit expansions and contractions.


In this paper, we incorporate a search-and-matching process between borrowers (firms) and lenders (banks). The financial contract and the credit interest rate are an outcome of a Nash bargaining, and idiosyncratic shocks to the entrepreneur’s productivity level determine the rate of endogenous match destruction. That is, banks with funds and firms with projects search for partners in the credit market. Banks obtain
funds to finance firms by raising retail deposits. To produce, an individual firm must be matched with a bank; to lend, an individual bank must be matched with a firm. Unmatched banks search for lending opportunities; unmatched firms search for a bank to finance their production. As in den Haan, Ramey, and Watson (2003), Wasmer and Weil (2004), and Beauburn-Diant and Tripier (2009), matched banks and firms decide whether to maintain or sever their credit relationship, depending on the productivity of the firm’s project. If the firm and the bank choose to cooperate, Nash bargaining determines the loan rate that determines how the joint surplus of the match is shared between the bank and the firm.

We extend the search and matching model of credit frictions to incorporating a second stage where banks operate in a centralized bond and interbank market. The nature of the interbank market – the central bank’s use of a corridor system – affects the bargaining process involving banks and firms and affects directly the equilibrium interest rate on loans. Thus, the model consists of an interbank market that involves banks and the monetary authority, and a loan market in which banks and firms participate. Banks need to meet their need for settlement balances in the interbank market and, besides interbank lending, banks can deposit excess reserves at the central bank or borrow reserves through a standing facility. The structure of the interbank market affects the lending decisions of banks in the loan market, and the resulting spread between the average lending rate and the central bank’s policy rate depends on this matching process, the nature of Nash bargaining, and structure of the interbank market and monetary policy operating procedures.
A further contribution of the present paper pertains to the cost channel of monetary policy (Ravenna and Walsh 2006) in which, because firms must finance wage payments in advance of production, the relevant cost of labor is affected by the interest rate firms pay on loans. However, when the loan rate is the outcome of a bargaining process, its role is to split the surplus between the borrower (the firm) and the lender (the bank). It is irrelevant for the firm’s employment decision which is made to maximize the joint surplus. There is still a cost channel but it depends on the opportunity cost of funds to the bank, not the interest rate charged on the loan, and therefore it too is dependent on the structure of the interbank market. Changes in the policy interest rate, the penalty for borrowing reserves from the central bank, the interest rate paid on reserve deposits at the central bank, the supply of bank reserves by the central bank, and the volatility of settlement payment flows all influence this outside opportunity and therefore affect the equilibrium spread between the average rate on bank loans and the policy interest rate.

Finally, by assuming individual firms are subject to stochastic productivity shocks, and the threshold productivity level below which the firm is unable to obtain financing depend on these same factors characterizing the interbank market. In particular, we show that a rise in interbank volatility increases the credit spread and, by raising the threshold productivity level the firm needs to obtain financing, reduces the number of firms able to obtain loans. Similarly, monetary policy has effects on employment and output on both extensive (the fraction of firms receiving loans) and intensive (the size of loans conditional on obtaining one) margins. The latter arises as a reduction in
the cost of funds for banks makes it optimal for firms with access to credit to expand employment. The former arises because the lower cost of finance will make it profitable banks to lend to more firms.

The remainder of the paper is arranged as follows. Section 3.2 presents the basic model setting, describing first the loan market, the bargaining solution that determines the interest rate on loans and the evolution of the number of bank-firm matches. Then, the reserve market under a corridor system is presented together with a description of the consolidated government budget constraint, monetary policy as well as the characterization of the aggregate equilibrium. The results of a numerical analysis are described in section 3.3. Numerical simulations are thought to understand the main transmission mechanism of aggregate shocks that affect the economy. Specifically, two types of financial shocks are studied under a neutral policy response: An increase in the volatility of the payment shock that banks face after the interbank market closes and an exogenous increase in the termination rate of loan contracts. In future versions of the paper, policy experiments, where the central bank responds by using different instruments, will be included. Finally, conclusions are given in section 3.4.

3.2 The Model

The model economy is populated by households, banks, firms, and a central bank. Households supply labor to firms, hold cash and bank deposits, and purchase final output in the goods market. Firms seek financing, hire labor financed by bank
loans and produce output. Banks accept deposits, hold reserves and bonds, and finance the wage bill of firms. The central bank pays interest on reserve deposits and charges a penalty rate on lending to banks.

Three aspects of the model are of critical importance. First, it is assumed that households cannot lend directly to firms. While this type of market segmentation is taken as exogenous, one could easily motivate it by assuming informational asymmetries under which households are unable to monitor firms while banks are able to do so. This asymmetry also forces firms to make up-front payments to workers to secure labor. Second, lending activity involving firms and banks occurs in a decentralized market characterized by random matching. And third, we assume all payment flows must be settled at the end of each period and that individual banks face idiosyncratic and uninsurable risk arising from random end-of-period settlement flows.

At the beginning of each period, aggregate shocks are realized and households deposit funds with a bank. The market for deposits is competitive and all banks offer the same interest rate on deposits. In the lending market, firms seek funding to make wage payments. Firms are subject to aggregate and idiosyncratic productivity shocks and these determine whether it is profitable for a firm to operate and, if it is, at what scale. If a firm is not already matched with a bank, it must seek out a new lender. Similarly, banks not already matched with a firm must search for borrowers. After the loan market closes, firms and workers produce and households consume, while banks can participate in the interbank market, investing deposits net of loans into riskfree bonds, lending to or borrowing from other banks, and holding deposits with the central bank.
After these markets close, all net payment flows are settled. Banks receive repayment from firms, but since firm receipts arise from households with deposits at different banks, some banks experience a net payment outflow, others an inflow. Banks with a shortage of funds must borrow from the central bank’s standing facility; those with an excess of funds can deposit these with the central bank.

The market for bank loans is characterized by search and matching frictions. While these frictions are more commonly employed in the analysis of labor markets (e.g., Mortensen and Pissarides 1994), this approach has also been used to model credit market frictions; examples include Diamond 1990, Acemoglu 2001, den Haan, Ramey, and Watson (2003), Wasmer and Weil (2004), Becsi, Li and Wang (2005), Beauburn-Diant and Tripier 2009 and Petrosky and Wasmer (2013). In contrast to Beauburn-Diant and Tripier, we allow employment and loan size to vary across firms due to firm-specific idiosyncratic productivity shocks. Firms must finance their wage bill, which introduces a cost channel (Ravenna and Walsh 2006) for monetary policy. Wasmer and Weil (2004) and Petrovsky-Nadeau and Wasmer (2013) incorporate matching frictions in both credit and labor markets and impose a free entry condition on both firms and banks. Since our focus is on the role of the interbank market in affecting loan supply, we assume a continuum of firms on the unit interval who are either producing or seeking finance and treat the labor market as competitive with firms taking the wage as given in deciding how much labor to employ. Beaubrun-Diant and Tripier, Wasmer and Weill, and Petrovsky-Nadeau and Wasmer all treat the opportunity cost of funds to the banks as an exogenous parameter. In contrast, we focus on the role of the interbank market
and central bank policy implementation as affecting the cost of funds for banks, and we provide a complete general equilibrium model.

3.2.1 Households

Households consume final output and supply labor to maximize

$$E_t \sum_{i=0}^{\infty} \beta^i U (C_{t+i}, 1 - N_{t+i}); \ 0 < \beta < 1.$$  

The utility function has standard properties. The household enters the period with nominal assets $A_{t-1}$ consisting of the existing stock of government debt ($B_{t-1}^h$) and holdings of high powered money ($HP_{t-1}$). These assets are allocated by the household between bank deposits ($D_t$) and bond holdings ($B_t^h$):

$$A_{t-1} \equiv B_{t-1}^h + HP_{t-1} = D_t + B_t^h.$$  

(3.1)

The household is subject to a cash-in-advance constraint that requires consumption to be purchased using initial deposit balances and current period wage receipts, or

$$D_t + w_t P_t N_t = A_{t-1} - B_t^h + w_t P_t N_t \geq P_t C_t,$$

where $w$ is the real wage and $P$ is the price level.\(^3\) In real terms the CIA constraint is,

$$d_t + w_t N_t - C_t \geq 0,$$  

(3.2)

where $d_t = D_t / P_t$ and $\xi_{cia}$ is an intercept term introduced for calibration purposes that represent all other assets that can be used to purchase consumption goods but are not

\(^3\)This constraint could, if one felt it necessary, be motivated by assuming households are anonymous to firms so firms will not sell goods on credit to be repaid in future periods. This assumption, combined with the assumption that banks cannot track future deposit activity of households would suffice.
explicitly modeled. Let \( i^d_t \) be the nominal return on bank deposits and \( i^b_t \) the nominal return on bonds. The household’s end of the period nominal wealth evolve according to

\[
A_t = A_{t-1} + i^d_t D_t + i^b_t B^h_t + w_t P_t N_t + \Pi^b_t + \Pi^f_t - P_t C_t - P_t T_t,
\]

where \( \Pi^i, i = b, f \) are bank and firm profits and \( P_t T_t \) are nominal lump-sum taxes or transfers. Define \( a_t = A_t/P_t \), and \( b^h_t = B^h_t/P_t \). In real terms, the budget constraint becomes

\[
a_t = \left( \frac{1}{1 + \pi_t} \right) a_{t-1} + i^d_t d_t + i^b_t b^h_t + w_t N_t + \left( \frac{\Pi^b_t + \Pi^f_t}{P_t} \right) - C_t - T_t. \quad (3.3)
\]

where \( a_t \equiv b^h_t + h P_t \) is such that \( a_t = d_{t+1} + b^h_{t+1} \).

The representative household maximizes the utility function subject to the CIA constraint 3.2, the budget constraint 3.3 and 3.1 expressed in real terms. The value function for the representative household is defined by

\[
V \left( \frac{a_{t-1}}{1 + \pi_t} \right) = \max_{d_t, b_t, a_t, C_t, N_t} \left[ U \left( C_t, 1 - N_t \right) + \beta E_t V \left( \frac{a_t}{1 + \pi_{t+1}} \right) \right],
\]

where \( 1 + \pi_t = P_t/P_{t-1} \) and the maximization is subject to (3.2), (3.3), and, from (3.1),

\[
\left( \frac{1}{1 + \pi_t} \right) a_{t-1} - d_t - b^h_t = 0. \quad (3.4)
\]

Let \( \mu \) and \( \lambda \) be the Lagrangean multipliers associated with the cash-in-advance and budget constraints. Let \( \varphi \) be the Lagrangean multiplier on the constraint (3.4).

\[\text{Notice that at the end of the period banks will transfer aggregate profits to the representative household, which include earnings on holdings of government bonds and cash. Thus, at the end of the period, households must hold all the supply of high powered money as well as all the supply of government bonds. Household’s nominal wealth at the beginning of period } t + 1 \text{ is } A_t = H P^h_t + B^p_t \text{ where } B^p_t = B^h_t + B^b_t \text{ are government bonds in the hand of the public and } B^b_t \text{ are bank’s holdings of government bonds. By the same token, at the end of the period, } H P^h_t = H P^h_{t+1} \text{, where } H P^h_t \text{ is the supply of high powered money.} \]
Then the first order necessary conditions for the household’s problem of maximizing utility are

\[ C_t: U_C(C_t, 1 - N_t) = (\mu + \lambda_t) \]

\[ N_t: U_N(C_t, 1 - N_t) = w_t (\mu + \lambda_t) \]

\[ a_t: -\lambda_t + E_t \beta \left( \frac{1}{1 + \pi_{t+1}} \right) V' \left( \frac{a_t}{1 + \pi_{t+1}} \right) = 0 \]

\[ d_t: \mu_t + i^d_t \lambda_t - \varphi_t = 0 \]

\[ b^b_t: i^b_t \lambda_t - \varphi_t = 0 \Rightarrow \varphi_t = i^b_t \lambda_t \]

As it is standard, the first two imply

\[ \frac{U_N(C_t, 1 - N_t)}{U_C(C_t, 1 - N_t)} = w_t. \quad (3.5) \]

while the last two imply

\[ \mu_t = \varphi_t - i^d_t \lambda_t = \left( i^b_t - i^d_t \right) \lambda_t \quad (3.6) \]

so that the excess yield of bonds over deposits measures the liquidity services provided by deposits. This in turn implies that

\[ U_C(C_t, 1 - N_t) = \mu_t + \lambda_t = \left( 1 + i^b_t - i^d_t \right) \lambda_t. \quad (3.7) \]

From the envelope theorem,

\[ V' \left( \frac{a_{t-1}}{1 + \pi_t} \right) = \lambda_t + \varphi_t = \left( 1 + i^b_t \right) \lambda_t, \]

and the first order condition for \( a_t \) can then be written as

\[ \lambda_t = \beta E_t \left( \frac{1}{1 + \pi_{t+1}} \right) V' \left( \frac{a_t}{1 + \pi_{t+1}} \right) = \beta E_t \left( \frac{1 + i^b_{t+1}}{1 + \pi_{t+1}} \right) \lambda_{t+1}. \quad (3.8) \]
or in terms of the marginal utility of consumption, the Euler equation is

\[
\frac{U_C(C_t, 1 - N_t)}{1 + \frac{i_b^t}{1 + \pi_t + 1}} = \beta E_t\left(\frac{1 + i_{t+1}^b}{1 + \pi_{t+1} + 1} \frac{U_C(C_{t+1}, 1 - N_{t+1})}{1 + i_{t+1}^d - i_{t+1}^d}\right)
\]

and the household stochastic discount factor is defined to be

\[
\frac{\beta \lambda_{t+1}}{\lambda_t} = \frac{\beta U_C(C_{t+1}, 1 - N_{t+1})}{U_C(C_t, 1 - N_t)} \frac{1 + i_t^b - i_t^d}{1 + i_{t+1}^b - i_{t+1}^d}
\]

### 3.2.2 The loan market

We assume that the process of finding a credit partner is costly in terms of time and resources, leading to the existence of sunk costs at the time of trading and a surplus to be shared between lenders (banks) and borrowers (firms in the intermediate goods sector). Search and matching frictions prevent instantaneous trading in the loan market, implying that not all market participants will end up matched at a given point in time. We allow for both exogenous and endogenous destruction of credit matches, and a matching technology that determines the aggregate flow of new credit relationships over time as a function of the relative number of lenders and borrowers searching for credit partners. Upon a successful match (i.e., a match that survives the exogenous and endogenous separation hazards), bilateral Nash bargaining between the parties determine the firm’s employment level and the loan interest rate. The latter is equivalent to choose the loan size to maximize the joint surplus to the lender and borrower, while the corresponding loan interest rate determines how the surplus is split between the two partners.

The loan market is populated by a continuum of banks and firms, with the
number of banks seeking borrowers varying endogenously and being determined by a
free entry condition to the market. We assume that banks have a constant returns
to scale technology for managing loans, so we can treat each loan as a separate match
between a bank and a firm. Each firm is endowed with one project and is either searching
for external funds or involved in an ongoing credit contract with a bank. If a firm is
matched with a bank, then the bank extends the necessary funds to allow the firm to
hire workers. There is no possibility of default, all loans are paid back at the end of the
period.

3.2.2.1 The matching process

Firms searching for external funds, $f_t$, are matched to banks seeking for bor-
rowers, $b_t^u$, according to the following constant return to scale matching function

$$m_t = \mu f_t^\varphi (b_t^u)^{1-\varphi}$$

The function $m_t$ is strictly concave with constant return to scale and determines the
flow of new credit contracts during date $t$; $0 < \mu < 1$ is a scale parameter that measures
the productivity of the matching function and $0 < \varphi < 1$ is the elasticity of match
arrival with respect to the mass of searching firms.

Matching rates The variable $\tau_t = f_t/b_t^u$ is a measure of credit market tightness,
and corresponds to the standard measure of market tightness arising in search and
matching models of the labor market. The probability that an entrepreneur with an
unfunded project is matched with a bank seeking to lend at date $t$ is denoted by $p_t^f$ and
is given by
\[ p^f_t = \mu \tau_t^{\varphi - 1} \tag{3.9} \]

Similarly, the probability that any bank seeking borrowers is matched with an unfunded entrepreneur at time \( t \) is denoted by \( p^b_t \) and is given by
\[ p^b_t = \mu \tau_t^{\varphi} \tag{3.10} \]

Since \( \tau_t = p^b_t / p^f_t \), a rise in \( \tau_t \) implies it is easier for a bank to find a borrower relative to a firm finding a lender and so corresponds to a tighter credit market. An increase (decrease) in \( \tau_t \) reduces the expected time a bank (firm) must search for a credit partner, lowering the bank’s (firm’s) expected pecuniary search costs. Since \( \tau_t = f_t / b^u_t = p^b_t / p^f_t \), at any date \( t \) the number of newly matched banks must equal the the number of newly matched firms: \( p^b_t b^u_t = p^f_t f_t \).

**Separations and the evolution of loan contracts** Loan contracts end for exogenous reasons with probability \( \delta \). Contractual parties engaged in a credit relationship that survive this exogenous separation hazard may also decide to dissolve the contract depending on the realization of the productivity of the firm’s project, taken to be \( z_t \omega_{it} \), where \( z_t \) is an aggregate component common to all firms (projects) and \( \omega_{it} \) is a firm-specific idiosyncratic component with distribution function \( G(\omega_{it}) \). As shown below, the decision to endogenously dissolve a credit relationship is characterized by an optimal reservation policy with respect \( \omega_{it} \) and denoted by \( \bar{\omega}_t \). If the realization of the idiosyncratic productivity shock \( \omega_{it} \) is above the reservation firm-specific productivity, \( \omega_{it} > \bar{\omega}_t \), both parties agree to continue the loan contract and the firm is able to produce
in the case the match survived the exogenous separation hazard. On the contrary, if the realization of $\omega_{it}$ is below $\tilde{\omega}_t$, both parties choose to dissolve the loan contract. Then, the probability of endogenous termination is defined as $\gamma_t(\tilde{\omega}_t) \equiv \text{prob}(\omega_{it} \leq \tilde{\omega}_t) = G(\tilde{\omega}_t)$.

Let $\varphi_t(\tilde{\omega}_t)$ denote the overall continuation rate, given by

$$\varphi_t(\tilde{\omega}_t) = (1 - \delta)(1 - \gamma_t(\tilde{\omega}_t))$$

while the overall separation rate is $1 - \varphi_t(\tilde{\omega}_t) = \delta + (1 - \delta) \gamma_t(\tilde{\omega}_t)$. Existence and uniqueness of the optimal reservation policy $\tilde{\omega}_t$ are shown in the appendix.

Let $f_{t-1}^m$ be the measure of intermediate good producers that enter period $t$ matched with a bank. Of those, a fraction $(1 - \delta) f_{t-1}^m$ of firms survive the exogenous hazard and a fraction $\gamma_t(\tilde{\omega}_t)$ of the survivals receive idiosyncratic productivity shocks that are less than $\tilde{\omega}_t$ and so do not produce. The mass of firms that actually produce in period $t$ is $\varphi_t(\tilde{\omega}_t) f_{t-1}^m$ and the mass of firms in a credit relationship at the end of period $t$ or beginning of period $t + 1$, denoted by $f_t^m$, is given by the number of firms actually producing during time $t$ plus all the new matches formed during the same period. Then, the evolution of $f_t^m$ is expressed as

$$f_t^m = \varphi_t(\tilde{\omega}_t) f_{t-1}^m + m_t.$$  \hspace{1cm} (3.11)

We normalize the the total number of firms in every time period to one and assume that if a credit relationship is exogenously separated at time $t$, both parties will start searching immediately during the same period of time. If the credit relationship survives the exogenous separation hazard but then endogenously separates, then both
parties must wait until next period of time in order to start searching for a credit contract again. This assumption implies that the number of firms seeking finance during period \( t \), which we have denoted by \( f_t \), is equal to the mass of searching firms at the beginning of time \( t \), \( (1 - f_{t-1}^m) \) plus the number of firms that started the period matched with a bank but were exogenously separated \( (\delta f_{t-1}^m) \). Therefore,

\[
f_t = 1 - (1 - \delta) f_{t-1}^m. \tag{3.12}
\]

Notice that there are still some firms that have been endogenously separated but cannot search in period \( t \). These firms are unmatched but waiting to start searching again next period. The number of new matches during the loan market trading session at time \( t \) can be written as

\[
m_t = \mu \tau_1 \phi - 1 \left[ 1 - (1 - \delta) f_{t-1}^m \right].
\]

Thus the evolution of \( f_t^m \) can be written as

\[
f_t^m = \varphi_1 (\tilde{\omega}_t) f_{t-1}^m + \mu \tau_1 \phi - 1 \left[ 1 - (1 - \delta) f_{t-1}^m \right].
\]

We also present a different timing assumption regarding separations and the ability to search within the same period of time that the contract separation has occurred. We can close the model by assuming that both types of separations (exogenous as well as endogenous) are able to search during the same period of time that a separation has occurred. Under this assumption, the mass of firms searching for a borrower evolve according to

\[
f_t = 1 - \varphi_1 (\tilde{\omega}_t) f_{t-1}^m \tag{3.13}
\]

with the corresponding changes in the equations for \( m_t \) and \( f_t^m \).
Credit Creation and Credit Destruction  Our timing assumption implies that the fraction $p_t f_{t-1}$, of matched firms that were exogenously separated during time $t$, are able to find a new credit relationship within the same period of time. Then, credit creation, $CC_t$, is defined to be equal to the number of newly created credit relationships at the end of time $t$ net of the number of exogenous credit separations that are successfully re-matched in a given period. That is

$$CC_t = m_t - p_t f_{t-1}. \quad (3.14)$$

The credit creation rate, $cc_t$ is

$$cc_t = \frac{m_t}{f_{t-1}} - p_t f. \quad (3.14)$$

On the other hand, credit destruction, $CD_t$, is defined as the total number of credit separations at the end of time $t$, $(1 - \varphi_t (\tilde{\omega}_t)) b_{t}^m$ net of the number of exogenous credit separations that are successfully re-matched in a given period. Thus,

$$CD_t = (1 - \varphi_t (\tilde{\omega}_t)) f_{t-1}^m - p_t f_{t-1} \Delta f_{t-1}$$

and the credit destruction rate, $cd_t$, is given by

$$cd_t = (1 - \varphi_t (\tilde{\omega}_t)) - p_t f. \quad (3.15)$$

Finally, net credit growth is defined as

$$cg_t = cc_t - cd_t$$

If instead we assume that exogenous as well as endogenous credit separations are able to search within the same period of time that separation have occurred then
credit creation and destruction rates are defined to be

\[ cc_t = \frac{m_t}{f_{t-1}} - p_t^f \varphi_t(\tilde{\omega}_t). \]  

(3.16)

and

\[ cd_t = (1 - \varphi_t(\tilde{\omega}_t)) - p_t^f \varphi_t(\tilde{\omega}_t). \]  

(3.17)

3.2.2.2 Firms and the loan market

In our setting, a credit relationship is a contract between a bank and a firm that allows the latter to operate an specific production technology, hire workers and pay their wage bill in advance of production. As long as, the credit contract prevails, the firm will receive sufficient external funds to pay workers in advance of production every period of time. After selling its output, the firm will repay its debt with the bank and transfer all remaining profits to the household. Therefore, as in De Fiore and Tristani (2012) we abstract from the endogenous evolution of net worth by assuming firms do not accumulate internal funds after repaying their debt.

Value functions Firm \( i \) is endowed with a production technology given by

\[ y_{i,t} = \xi z_t \omega_{i,t} N_{i,t}^x, \quad 0 < \alpha \leq 1, \]  

(3.18)

where \( \xi \) is a scale technology parameter that serves for calibration purposes, \( z_t \) is the aggregate productivity level with mean \( \bar{z} \), \( \omega_{i,t} \) is the firm-specific idiosyncratic productivity level drawn from a uniform distribution function \( G(\omega) \) with support \([\omega, \tilde{\omega}]\), and
$N_{i,t}$ is the firm’s employment level. Define

$$g(\omega) \equiv \frac{dG(\omega)}{d\omega} = \frac{1}{\bar{\omega} - \omega}, \text{ with } \bar{\omega} > \omega > 0$$

(3.19)

and normalize $\bar{z}$ so that the unconditional expectation of $z_t \omega_{i,t}$ is equal to one. If the firm obtains financing and produces, the firm’s instantaneous real profit flow is

$$\pi_t^f (\omega_{i,t}) = y_{i,t} - w_t R_{i,t}^l N_{i,t} - x^f$$

(3.20)

where $w_t$ is the real wage, $R_{i,t}^l$ is the firm-specific gross nominal loan interest rate bilaterally negotiated with a bank and $x^f$ is a fixed cost of production. The labor market is competitive so all firms face the same real wage. The loan principle is $w_t N_{i,t}$ and the loan contract requires the repayment of the total debt with the bank $w_t R_{i,t}^l N_{i,t}$ at the end of the same period.

Firm’s profit, $\pi_t^f$, depends on the status of the firm, that is, if the firm is searching for external funds or if it is producing. A firm searching for external funds can not produce and obtains zero real profits $\pi_t^f = 0$. Assuming no search costs for an entrepreneur, $\pi_t^f$ is defined to be

$$\pi_t^f = \begin{cases} 
\pi_t^f (\omega_{i,t}) & \text{with external funds} \\
0 & \text{without external funds}
\end{cases}$$

The state of the firm is characterized by two value functions: The value of being matched with a bank and able to produce at date $t$, denoted by $V_{t}^{FP}(\omega_{i,t})$ and the value of searching for external funds at date $t$, denoted by $V_{t}^{FN}$, both measured in terms of current consumption of the final good. Notice that if the firm is producing then its idiosyncratic productivity is common knowledge. On the contrary, if the firm is
searching for a lender, then its idiosyncratic productivity its unknown yet. Under these assumptions, the value function $V_t^{FP}(\omega_{i,t})$ is given by

$$V_t^{FP}(\omega_{i,t}) = \pi_t^f(\omega_{i,t}) + E_t \Delta_{t,t+1} \left\{ \delta V_{t+1}^{FN} + (1 - \delta) \int_{\omega} \max(V_{t+1}^{FP}(\omega_{i,t+1}), V_{t+1}^{FN})dG(\omega) \right\}$$

where $\Delta_{t,t+1} = \beta \lambda_{t+1}/\lambda_t$ is the stochastic discount factor. The value of producing is the flow value of current real profits (firm’s real cash flow) plus the expected continuation value. At the beginning of next period, the credit relationship is exogenously dissolved with probability $\delta$, and the firm must seek new financing. With probability $(1 - \delta)$, the firm survives the exogenous separation hazard and it faces the new realization of its idiosyncratic productivity level $\tilde{\omega}_{i,t+1}$. If the firm receive a realization such that $\omega_{i,t+1} \geq \tilde{\omega}_{i,t+1}$ then the loan contract will continue and the firm will obtain new external funds to produce and the corresponding value $V_{t+1}^{FP}(\omega_{i,t+1})$. In the case, the new realization is $\omega_{i,t+1} < \tilde{\omega}_{i,t+1}$, the loan contract will be dissolved and the firm will obtain the value of searching for lenders during the next period: $V_{t+1}^{FN}$. The value of searching for external funds ($V_t^{FN}$) for a firm at date $t$ expressed in terms of current consumption is

$$V_t^{FN} = p_t^f E_t \Delta_{t,t+1} \left[ \delta V_{t+1}^{FN} + (1 - \delta) \int_{\omega} \max(V_{t+1}^{FP}(\omega_{i,t+1}), V_{t+1}^{FN})dG(\omega) \right] + \left(1 - p_t^f\right) V_{t+1}^{FN},$$

where $p_t^f$ is the probability of matching with a bank. Notice that we assume matches made in period $t$ do not produce until $t + 1$. With probability $(1 - p_t^f)$, the firm does not match and must continue searching for external funds during next period.

Under Nash bargaining, the reservation productivity level $\tilde{\omega}_t$ that triggers endogenous separation is determined by the point at with the joint surplus of the match is
equal to zero. Thus, if $\omega_{i,t+1} < \tilde{\omega}_{t+1}$, both parties agree to end the credit relationship.

Notice that given existence and uniqueness of $\tilde{\omega}_{t+1}$, the integral term on the expected continuation value for both $V_t^{FP}(\omega_{i,t})$ and $V_t^{FN}$ is

$$\int_{\omega}^{\Omega} \max(V_{t+1}^{FP}(\omega_{i,t+1}), V_{t+1}^{FN}) dG(\omega)$$

$$= \gamma_{t+1}(\tilde{\omega}_{t+1}) V_{t+1}^{FN} + (1 - \gamma_{t+1}(\tilde{\omega}_{t+1})) \int_{\omega_{t+1}}^{\Omega} V_{t+1}^{FP}(\omega_{i,t+1}) \frac{dG(\omega)}{1 - \gamma_{t+1}(\tilde{\omega}_{t+1})}.$$

Therefore, the firm value functions can be written as

$$V_t^{FP}(\omega_{i,t}) = \pi_t^I(\omega_{i,t})$$

$$+ E_t \Delta_{t,t+1} \left\{ (1 - \varphi_t(\tilde{\omega}_{t+1})) V_{t+1}^{FN} + \varphi_t(\tilde{\omega}_{t+1}) \int_{\omega_{t+1}}^{\Omega} V_{t+1}^{FP}(\omega_{i,t+1}) \frac{dG(\omega)}{1 - \gamma_{t+1}(\tilde{\omega}_{t+1})} \right\}$$

and

$$V_t^{FN} = E_t \Delta_{t,t+1} \left\{ p_t^f \left[ (1 - \varphi_t(\tilde{\omega}_{t+1})) V_{t+1}^{FN} + \varphi_t(\tilde{\omega}_{t+1}) \int_{\omega_{t+1}}^{\Omega} V_{t+1}^{FP}(\omega_{i,t+1}) \frac{dG(\omega)}{1 - \gamma_{t+1}(\tilde{\omega}_{t+1})} \right] \right\}$$

$$+ \left( 1 - p_t^f \right) V_{t+1}^{FN}$$

(3.22)

Let the surplus to a producing firm be defined as

$$V_t^{FS}(\omega_{i,t}) = V_t^{FP}(\omega_{i,t}) - V_t^{FN}$$

and it is given by

$$V_t^{FS}(\omega_{i,t}) = \pi_t^I(\omega_{i,t})$$

$$+ \left( 1 - p_t^f \right) E_t \Delta_{t,t+1} \varphi_t(\tilde{\omega}_{t+1}) \int_{\omega_{t+1}}^{\Omega} V_{t+1}^{FS}(\omega_{i,t+1}) \frac{dG(\omega)}{1 - \gamma_{t+1}(\tilde{\omega}_{t+1})}.$$
Thus, the surplus to a firm for being able to produce due to a loan contract during the period, depends positively on the current flow of profits and on a fraction of the expected continuation value of the credit relationship. On the other hand, if the probability of finding a lender, $p^f_t$, is higher today, then the joint surplus will decrease since the expected continuation value of being in a credit relationship is lower because a higher $p^f_t$, induces more firms to search for lenders.

### 3.2.2.3 Banks and the loan market

There is a continuum of banks with infinite mass that are owned by the representative household. Banks operate in various centralized markets such as the interbank, bond and deposit market but also operate in the decentralized loan market. Banks activities in the centralized markets include: raise deposits from households, hold excess reserves balances with the central bank, borrow and lend reserves with other banks as part of the payments settlement system and hold government bonds. The decentralized nature of the loan market together with the existence of search and matching frictions, imply that banks have to spend time and resources searching for borrowers before being able to extend loans. Moreover, some banks may not end up with loans in their portfolio. At any point in time, banks operating in the loan market may be involved in a credit contract with a particular firm or may be seeking for a potential borrower. We assume that banks decide to enter the loan market to search for potential borrowers until the expected cost of extending a loan is equal to its expected benefit. At this point, banks will be indifferent between searching for projects or only operate in the
centralized markets of the economy.

All uncertainty is revealed before loans are extended: loans are made and paid back during the same period. Therefore, loans are not risky and there is no possibility of default. At the end of the period, the bank transfers all its profits to the representative household.

A bank can only form a credit relationship with one firm. If a bank is in a credit relationship with a firm, it can not search for a different bank until separation occurs. Bank $j$'s balance sheet expressed in nominal terms is

$$1_{\omega_{i,t}(j)} L_{i,t}(j) + B^b_t(j) + I_t(j) + H_t(j) = (1 - \rho)D_t(j)$$  \hspace{1cm} (3.23)

where $1_{\omega_{i,t}(j)}$ is an indicator function taking the value of one if bank $j$ extends a loan to a firm with idiosyncratic productivity $\omega_{i,t}$ and zero if the bank is searching for a borrower, $L_{i,t}(j)$ are loans to firm $i$, $B^b_t(j)$ equals holdings of government bonds, $I_t(j)$ is (net) lending in the interbank market, $H_t(j)$ is excess reserve holdings, and $\rho$ is the fractional reserve requirement ratio. In this section we are interested in the bank decision about $L_t(j)$ and the potential profits it may obtain by operating in the loan market and taking as given the rest of its decision variables. In the next section, we explain the decision process for the rest of the variables involved in the bank’s balance sheet.

**Value functions** Each period, when the loan market opens, a bank may be in a credit relationship with a firm or searching for potential borrowers. If a bank extends a loan
to a firm whose idiosyncratic productivity realization is $\omega_{i,t}$, bank profits are

$$\pi^b_t(\omega_{i,t}) = \left( R^i_{i,t} - R_t \right) l_{i,t}$$

where $R^i_{i,t} - R_t$ is the spread between the interest rate on the bank’s loan to a firm with idiosyncratic productivity $\omega_{i,t}$ and the bank’s opportunity cost of funds $R_t$. The determination of $R_t$ is explained below; it will be shown to be the same for all banks. Loans expressed in real terms are denoted by $l_{i,t}$ since the loan is extended to firm $i$ with idiosyncratic productivity $\omega_{i,t}$. We also assume that a bank searching for a borrower incurs a search cost of $\kappa$, measured in current consumption units and earns zero current profits in the loan market while searching for a borrower. Let $\pi^b_t(j)$ be bank $j$’s real profits. We show below that $\pi^b_t(j)$ can be written as:

$$\pi^b_t(j) = 1_{\omega_{i,t}}(j) \pi^b_t(\omega_{i,t}) - (1 - 1_{\omega_{i,t}}(j)) \kappa$$  \hspace{1cm} (3.24)

Under these assumptions the problem of a bank in the loan market can be characterized by two value functions: The value of lending to a firm with $\omega_{i,t}$ at date $t$, denoted by $V^BL_t(\omega_{i,t})$ and the value of searching for a potential borrower at date $t$, denoted by $V^BN_t$. Both value functions are measured in terms of current consumption of the final good and are given by

$$V^BL_t(\omega_{i,t}) = \pi^b_t(\omega_{i,t})$$  \hspace{1cm} (3.25)

$$+ \mathbb{E}_t \Delta_{t,t+1} \left\{ (1 - \varphi_{t+1}(\tilde{\omega}_{t+1})) V^BN_{t+1} + \varphi_{t+1}(\tilde{\omega}_{t+1}) \int_{\tilde{\omega}_{t+1}}^{\omega} V^BL_{t+1}(\omega_{i,t+1}) \frac{dG(\omega)}{1 - \gamma_{t+1}(\tilde{\omega}_{t+1})} \right\}$$
The value of extending a loan, $V_{t}^{BL}(\omega_{i,t})$, is the current value of real profits plus the expected continuation value. A bank that extends a loan to a firm with idiosyncratic productivity $\omega_{i,t}$ at date $t$ will continue funding the same firm at time $t + 1$ with probability $\varphi_{t} (\tilde{\omega}_{t+1})$. In this event, the bank obtains the future expected value of lending conditional on having $\omega_{i,t+1} \geq \tilde{\omega}_{t+1}$ given by the following conditional expectation: $\frac{\varphi_{t+1}(\tilde{\omega}_{t+1})}{\int_{\omega_{t+1}}^{\tilde{\omega}_{t+1}} V_{t+1}^{BL}(\omega_{i,t+1})(1 - \gamma_{t+1}^{-1}) dG(\omega)}$. The credit relationship will be severed at time $t + 1$ with probability $\delta + 1 - \varphi_{t} (\tilde{\omega}_{t+1})$ and the bank will obtain a future value of $V_{t+1}^{BN}$. On the other hand, the value of a bank searching for a borrower at date $t$ is given by the flow value of the search costs, $-\kappa$, plus the continuation value. A searching bank faces a probability $1 - p_{t}^{h}$ of not being matched during time $t$, obtaining a future value of $V_{t+1}^{BN}$ but a probability $p_{t}^{h}$ of being matched with a firm. If a searching bank ends up being matched with a firm at time $t$, then at the beginning of period $t + 1$ will face a probability of separation before actually extending the loan.
**Free entry condition**  In equilibrium, free entry of banks into the loan market ensures that \( V_{t}^{BN} = 0 \). Using this in (3.26), the free entry condition can be written as

\[
\frac{\kappa}{p_b^t} = \mathbb{E}_t \Delta_{t,t+1} \left\{ \varphi_t \left( \tilde{\omega}_{t+1} \right) \int_{\tilde{\omega}_{t+1}}^{\infty} V_{t+1}^{BL}(\omega_{i,t+1}) \frac{dG(\omega)}{1 - \gamma_{t+1}(\tilde{\omega}_{t+1})} \right\}
\]

(B.28)

Banks will enter the loan market until the expected cost of finding a borrower \( \kappa/p_b^t \) is equal to the expected benefit of extending a loan to a firm with idiosyncratic productivity \( \omega_{i,t+1} \geq \tilde{\omega}_{it+1} \). If the expected cost of extending a loan is lower than the corresponding expected benefits, banks will enter the loan market to search for borrowers and the probability that a searching bank finds a borrower will fall, up to the point where condition (3.28) is restored. Note that free entry of banks into the loan market modifies the value function \( V_{t}^{BL}(\omega_{it}) \) as follows

\[
V_{t}^{BL}(\omega_{it}) = \pi_{t}^b(\omega_{i,t}) + \mathbb{E}_t \Delta_{t,t+1} \left\{ \varphi_t \left( \tilde{\omega}_{t+1} \right) \int_{\tilde{\omega}_{t+1}}^{\infty} V_{t+1}^{BL}(\omega_{i,t+1}) \frac{dG(\omega)}{1 - \gamma_{t+1}(\tilde{\omega}_{t+1})} \right\}
\]

(B.29)

The net surplus for bank extending a loan is defined as \( V_{t}^{BS}(\omega_{it}) = V_{t}^{BL}(\omega_{it}) - V_{t}^{BN} \), then using (3.28), (3.30), (3.20) and (3.24), the joint surplus can be written as

\[
V_{t}^{BS}(\omega_{it}) = \pi_{t}^b(\omega_{i,t}) + \frac{\kappa}{p_b^t}
\]

(B.30)

### 3.2.2.4 Employment and the loan contract: Nash bargaining

At any point in time, a matched firm and bank that survive the exogenous and endogenous separation hazards engage in bilateral bargaining over the loan interest rate and loan size to split the joint surplus that results from the match. This joint surplus of a credit match is defined as \( V_{t}^{JS}(\omega_{i,t}) = V_{t}^{FS}(\omega_{i,t}) + V_{t}^{BS}(\omega_{i,t}) \). Using (??), (B.30), (B.20) and (B.24), the joint surplus can be written as
\[ V_t^{JS}(\omega_{i,t}) = y_{i,t} - w_t R_t N_{i,t} - x_f \]

\[ + \left( 1 - p_f \right) E_t \Delta_{t,t+1} \varphi_t (\tilde{\omega}_{t+1}) \int_{\tilde{\omega}_{t+1}}^{\omega} V_{t+1}^{FS}(\omega_{i,t+1}) \frac{dG(\omega)}{1 - \gamma_{t+1}(\tilde{\omega}_{t+1})} \]

We assume Nash bargaining with fixed bargaining shares over the loan rate, \( R_{i,t} \), and the corresponding real loan size, \( l_{i,t} \). Let \( \bar{\eta} \) be the firm’s share of the joint surplus, and \( 1 - \bar{\eta} \) the banks’. The Nash bargaining problem for an active credit relationship is

\[ \max_{\{ R_{i,t}, l_{i,t} \}} \left( V_t^{FS}(\omega_{i,t}) \right)^{\eta} \left( V_t^{BS}(\omega_{i,t}) \right)^{1-\eta} \]

where \( V_t^{FS}(\omega_{z,t}) \) and \( V_t^{BS}(\omega_{z,t}) \) are defined above and the firm’s demand for funds is given by its wage bill: \( l_{i,t} = w_t N_{i,t} \). The first order conditions imply the following optimal sharing rule:

\[ \bar{\eta} V_t^{BS}(\omega_{i,t}) = (1 - \bar{\eta}) V_t^{FS}(\omega_{i,t}) \]

and an employment condition that sets the marginal product of labor equal to the marginal cost of labor inclusive of the bank’s opportunity cost, \( R_t \), when extending a loan to a firm:

\[ \alpha z_{i,t} \omega_{i,t} \left( N_{i,t}^{*} \right)^{-1} = w_t R_t \]

for all \( \omega_{i,t} \geq \tilde{\omega}_t \). The above optimality condition can be written as the optimal loan size negotiated between credit partners as

\[ l_{i,t}^{*} = \left( \frac{\alpha z_{i,t} \omega_{i,t}}{w_t R_t} \right)^{\frac{1}{1-\alpha}}. \]

117
with the corresponding optimal negotiated loan rate, $R_{l,i,t}^t$: 

$$R_{l,i,t}^t = (1 - \eta) \left( \frac{y_{i,t}^* - x_{f}^*}{l_{i,t}^*} \right) + \eta \left( \frac{R_{tw} w_{t} N_{i,t}^* - \frac{\kappa p_f}{p_t}}{l_{i,t}^*} \right)$$

where $y_{i,t}^*$, $N_{i,t}^*$ and $l_{i,t}^*$ denote the optimal output, labor demand and loan size for a firm with idiosyncratic productivity $\omega_{i,t} \geq \tilde{\omega}_t$. Therefore, a firm with external funds in a loan contract with a bank will produce and demand labor according to the following schedules:

$$y_{i,t}^* = \left( \frac{\xi z_t \omega_{i,t}}{w_t R_t} \right)^{1/1-\alpha} \left( \frac{\alpha}{w_t R_t} \right)^{1/1-\alpha}$$

$$N_{i,t}^* = \left( \frac{\alpha \xi z_t \omega_{i,t}}{w_t R_t} \right)^{1/1-\alpha}$$

The effect of the nominal interest rate on the cost of labor is generally referred to as the cost channel of monetary policy (Ravenna and Walsh 2006). Normally, the relevant interest rate is taken to be the interest rate the firm pays on loans taken to finance wage payments. However, in this model, the loan interest rate, $R_{l,i,t}^t$, simply ensures the joint surplus generated by a credit relationship is divided optimally between the firm and the bank, with the relevant interest rate capturing the cost channel being $R_t$, the bank’s opportunity cost of funds. As shown below, $R_t$ depends on the interest rate in the interbank market and the marginal value of loans as collateral. Even though firms will face different interest rates on bank loans, since the loan rate depends on the firms idiosyncratic productivity realization, $\omega_{i,t}$, the interest cost relevant for labor demand is the same for all firms. The loan interest rate divides the joint surplus of a credit match in such a manner that a fraction $1 - \eta$, of the firm profits relative to the
loan size is obtained by the bank while a fraction $\eta$ of the bank opportunity cost of lending net of search costs and relative to the loan size is obtained by the firm.

Finally, notice that the credit contract implies that in equilibrium, there will be a size distribution of firms such that more productive firms will be able to obtain a higher amount of loans, hire more workers and become larger firms conditional on surviving.

Since both parties in a credit match have an incentive to maximize the joint surplus, the Nash bargaining protocol discussed above is equivalent to choosing the firm’s employment level to solve

$$\max_{N_{i,t}} \left[ y_{i,t} - w_t R_t N_{i,t} - x^f \right].$$

and having the loan rate to solve the Nash bargaining problem.

These results can be used to rewrite the free entry condition (equation 3.28), the net surplus for a firm (equation ??) and the joint surplus for a credit relationship (equation ??) as

$$\frac{\kappa}{p_t} = (1 - \eta) E_t \Delta_{t,t+1} \varphi_t (\tilde{\omega}_{t+1}) \int_{\tilde{\omega}_{t+1}} V_{t+1}^{BL} (\omega_{t+1}) \frac{dG(\omega)}{1 - \gamma_{t+1} (\tilde{\omega}_{t+1})}$$

(3.34)

$$V_{FS}^{t}(\omega_{i,t}) = \pi_t^f (\omega_{i,t}) + \eta \left( \frac{1 - p_t^f}{1 - \eta} \right) \frac{\kappa}{p_t^b}$$

(3.35)

$$V_{JS}^{t}(\omega_{i,t}) = \left( \pi_t^{*f} (\omega_{i,t}) + \pi_t^{*b} (\omega_{i,t}) \right) + \left( \frac{1 - \eta p_t^f}{1 - \eta} \right) \frac{\kappa}{p_t^b}$$

(3.36)

where

$$\pi_t^{*f} (\omega_{i,t}) + \pi_t^{*b} (\omega_{i,t}) = y_{i,t}^* - R_t w_t N_{i,t}^* - x^f$$
and \( \pi^f_t (\omega_{i,t}) \) is given by equation 3.20 and \( \pi^b_t (\omega_{i,t}) \) by equation 3.24 evaluated at optimal level of employment \( N^*_{t,i} \) and output \( y^*_{t,i} \) chosen by the credit match. Finally, since equation (3.32) is equivalent to \( \alpha y^*_{i,t} / N^*_{i,t} = w_t R_t \), the joint surplus of a credit relationship (equation 3.36) can be written explicitly as a function of the idiosyncratic productivity shock \( \omega_{i,t} \) in order to facilitate the characterization of the loan market equilibrium:

\[
V^JS_t(\omega_{i,t}) = (1 - \alpha) \left( \xi z_{i,t} \omega_{i,t} \right) \frac{1}{1-\alpha} \left( \frac{\alpha}{MC_t} \right)^{\frac{1}{1-\alpha}} - x^f + \left( \frac{1 - \eta p^f_t}{1 - \eta} \right) \frac{\kappa}{p^f_t^1} \quad (3.37)
\]

where \( MC_t = w_t R_t \) is the real marginal cost which is common to all firms. The joint surplus of an active credit contract between a bank and a firm is a direct function of the firm-specific productivity, \( z_{i,t} \omega_{i,t} \) and a inverse function of the marginal cost of labor. Due to the free entry condition for banks into the loan market, the term \( \kappa p_t^b \), which measures the expected search cost of extending a loan, is also the expected benefit of forming a loan contract and extending a loan to a firm. If this expected benefit is higher, while keeping constant \( p^f_t \), the joint surplus of the credit relationship increases as well. In addition, an increase in \( p^f_t \) given that \( p^b_t \) is unchanged leads to a reduction in the joint surplus. In general equilibrium, both matching rates, will change simultaneously. Notice that both matching rates can be written in terms of \( \tau_t \), thus the joint surplus is a function of the firm-specific productivity, the marginal cost of labor and the credit market tightness and it can be written as

\[
V^JS_t(\omega_{i,t}) = (1 - \alpha) \left( \xi z_{i,t} \omega_{i,t} \right) \frac{1}{1-\alpha} \left( \frac{\alpha}{MC_t} \right)^{\frac{1}{1-\alpha}} - x^f + \left( \frac{1 - \eta}{1 - \eta} \right) \frac{\kappa}{1 - \eta} \quad (3.37)
\]
Up to a first order approximation, the partial equilibrium effect of $\tau_t$ over $V_{tJS}(\omega_{i,t})$ is negative under the assumption that $\bar{\eta} = \varphi$ and that $0 < p^f < 1$, that is, if the Hosios condition holds and $p^f$ can be interpreted as a probability. Therefore, if the credit market is tighter, the joint surplus of a credit relationship falls.\footnote{The first order approximation of $V_{tJS}$ is given by}

3.2.2.5 The optimal reservation policy: Endogenous separations

The optimal reservation policy with respect to the idiosyncratic productivity shock implies the following condition:

\[
\begin{align*}
\text{if } \omega_{i,t} \leq \bar{\omega}_{i,t} & \implies V_{tJS}(\omega_{i,t}) \leq 0 \\
\text{if } \omega_{i,t} > \bar{\omega}_{i,t} & \implies V_{tJS}(\omega_{i,t}) > 0.
\end{align*}
\]

Since the joint surplus is a continuous function and it is strictly increasing in the firm’s idiosyncratic productivity level, there exists a unique threshold level, $\bar{\omega}_t$, for all firms in a credit match, defined by

\[
V_{tJS}(\bar{\omega}_t) = 0
\]

\[
\hat{V}_{tJS} = \left( \frac{\xi z \omega}{V_{JS}} \right)^{\frac{1+\pi}{1-\alpha}} \left( \bar{\omega}_{i,t} - \bar{MC}_t \right) - \left( \frac{\alpha}{1-\eta p^f} \bar{\eta} \right) \hat{\tau}_t
\]

Where a variable expressed as $\hat{x}$ denotes log-linear deviation from its steady state. If $\varphi = \bar{\eta}$ then the first order approximation of $V_{tJS}$ can be expressed as

\[
\hat{V}_{tJS} = \left( \frac{\xi z \omega}{V_{JS}} \right)^{\frac{1+\pi}{1-\alpha}} \left( \bar{\omega}_{i,t} - \bar{MC}_t \right) - \left( \frac{\alpha}{1-\eta p^f} \bar{\eta} \right) \left( 1 - p^f \right) \hat{\tau}_t
\]

thus, if $p^f < 1$, then an increase in $\hat{\tau}_t$ generates a fall in $\hat{V}_{tJS}$. Of course any value such that $\varphi - \bar{\eta} p^f > 0$ will also make the effect of $\hat{\tau}_t$ over $V_{tJS}$ negative.
such that the joint surplus is negative for any firm facing an idiosyncratic productivity $\omega_{i,t} < \bar{\omega}_t$. The optimal reservation productivity $\bar{\omega}_t$, is

$$\bar{\omega}_t = \frac{(MC_t)\alpha}{\xi z_t} H_t$$

(3.38)

where

$$H_t = \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \left[ x^f - \left( \frac{1 - \eta \mu \tau_t^-}{1 - \eta} \right) \frac{\kappa}{\mu \tau_t^p} \right]^{1-\alpha}$$

Since $\bar{\omega}_t$ is independent of $i$, the cutoff value is the same for all firms and banks. Moreover, it is decreasing in aggregate productivity $z_t$ so that a positive aggregate productivity shock means the number of credit matches that separate endogenously falls and more matched firms produce. The cutoff value is increasing in the marginal cost of labor $(MC_t = w_t R_t)$ and the firm’s fixed cost $(x^f)$.

The bank’s opportunity costs of funds $R_t$ influences the level of economy activity at both the extensive and intensive margins. From (3.38), a rise in $R_t$ increases the threshold level of the idiosyncratic productivity of firms that generate a positive joint surplus. As a consequence, fewer firms obtain financing and produce. This is the extensive margin effect. Conditional on producing, firms equate the marginal product of labor to $w_t R_t$, so a rise in $R_t$ reduces labor demand at each level of the real wage. This is the intensive margin effect. Both channels work to reduce aggregate output as $R_t$ rises. In addition, credit market conditions reflected in $\tau_t$ directly affect the extensive margin; a rise in $\tau_t$ (a credit tightening) increases $\bar{\omega}_t$ and fewer firms obtain credit.

Both, interest costs measured by $R_t$ and credit conditions measured by $\tau$ matter for employment and output.\(^6\)

\(^6\)Up to a first order approximation the effect of $\tau$ over $\bar{\omega}$ is positive if and only if $\varphi - \eta p^f > 0$, where
3.2.3 The interbank market

The interbank market involves the direct participation of banks and the central bank. Net payments between banks must be settled at the end of each period, after the interbank market has closed. The random nature of settlement payment flows from the perspective of an individual bank will generate a demand for excess reserves (reserves in excess of any required reserves). The cost of holding a level of excess reserves that, ex post, is too high or too low will depend on the opportunity costs of, in the first case, holding reserves as deposits at the central bank and, in the second case, borrowing reserves from the central bank. The central bank sets the interest paid on reserves, the rate charged on borrowed reserves, the quantity of reserves, and the haircuts applied to bank assets posted as collateral when borrowing reserves. Not all these instruments can be set independently.

3.2.3.1 Banks

Recall that the balance sheet of bank $j$ in nominal terms is

$$1_{w_{i,t}}(j) L_{i,t}(j) + B_{i}^{h}(j) + I_{t}(j) + H_{t}(j) = (1 - \rho) D_{t}(j) \quad (3.39)$$

During the period, banks make payments to and receive payments from other banks as part of the payments settlement system. Banks can trade reserve balances in a competitive interbank market at the market rate $i_{t}$. After the interbank market has closed, banks may experience a net payment shock $\phi_{t}(j) = \varepsilon_{t} D_{t}(j)$, taken to be $p'$ is the steady state value of the firm matching rate.
homogeneous of degree one in the level of the bank’s deposit liabilities. The payment shock itself is assumed to be uniformly distributed over the interval $[-\bar{\varepsilon}D_t(j), \varepsilon D_t(j)]$.

The density and cumulative distribution functions of this shock are $f(\phi) = 1/2\bar{\varepsilon}D_t(j)$ and $F(\phi) = F(\varepsilon D) = (\varepsilon + \bar{\varepsilon})/2\bar{\varepsilon}$. Since $E\phi = 0$ and $\text{var}(\phi) = \bar{\varepsilon}^2D_t^2(j)/3$, an increase in $\bar{\varepsilon}$ represents a mean preserving spread in the distribution of payment shocks. If $H_t(j) + \phi_t(j) < 0$, the bank must borrow reserves from the central bank to meet its net payment outflow. If $H_t(j) + \phi_t(j) > 0$, the bank can earn interest on its net balances by depositing them with the central bank.

Assume the central bank sets a desired interest rate (the policy rate) $i^*_t$, remunerates (required or excess) reserve balances at a rate $i^*_t - s$ and lends reserves at a penalty rate $i^*_t + s$ (see Woodford 2001, Whitesell 2003, 2006, or Walsh 2006, 2010). The rate paid on reserves places a floor on the interbank rate as no bank will lend to another at a rate less than $i^*_t - s$. And, in the absence of a collateral constraint on borrowing from the central bank, the penalty rate places a ceiling on the interbank rate as no bank will borrow in the interbank market at a rate greater than $i^*_t + s$. In this case, $s$ is the symmetric width of the channel within which the interbank rate is contained. In practice central bank lending is collateralized while interbank lending is unsecured, though the traditional analysis of a channel system (Woodford 2001, Whitesell 2003, 2006) ignores

---

7 Ashcroft, et al 2011 also models unpredictable payment flows as drawn from a uniform distribution.

8 General equilibrium models with channel systems are developed in Berensten and Monnet (2006, 2008), and Berensten, Marchesiani, and Waller (2010). See also Friedman and Kuttner (2010).

9 In fact, during 2009-2013, the federal funds rate has been below the rate the Federal Reserve pays on reserves. Beck and Klee (2011) explain that this phenomena can arise because Government Sponsored Enterprises (GSE) hold reserves but cannot earn interest on them from the Federal Reserve. As Furfine (2011) points out, there must be limits to arbitrage that prevent banks from borrowing these fed funds from GSEs and depositing them in their own interest earning reserve accounts.
collateral (but see Berentsen and Monnet 2008). We assume the central bank accepts both government bonds and commercial loans as collateral, applying a haircut to each but imposing a larger haircut on loans. If \( H_t(j) + \phi_t(j) < 0 \), the maximum a bank can borrow from the central bank is \( \xi_b B^b_t(j) + \xi_L L_{i,t}(j) \), where \( 0 < 1 - \xi_L < 1 - \xi_b < 1 \) are the haircuts on commercial loans and bonds posted as collateral. For example, the Federal Reserve currently sets \( \xi_b = 0.99 \) for U.S. bills and bonds with less than 5 years to maturity and \( \xi_L = 0.65 \) for zero coupon, normal risk-rated commercial loans of 5 years maturity. For simplicity, we assume banks hold collateralizable assets and reserves sufficient to meet all net settlement flows. This requires

\[
H_t(j) + \xi_b B^b_t(j) + \xi_L L_{i,t}(j) + \xi^{bs} \geq \bar{\epsilon} D_t(j) \tag{3.40}
\]

where \( \xi^{bs} \) is a constant that represent all different assets that can be used as collateral but are not modeled in this paper. This constant will serve for calibration purposes.

Let \( i_{i,t}^l \) be the net nominal interest rate on loans if bank \( j \) is in a loan contract with firm \( i \), and let \( x^l \) be the cost (per dollar) of servicing loans and \( x^d \) the cost of servicing deposits. Then nominal profits of bank with household deposits \( D_t(j) \) and a

\[11\] See the Fed’s Discount Window and Payment System Risk Collateral Margins Table, available at http://www.frbdiscountwindow.org/discountwindowbook.cfm?hdrID=14&dtlID=43
\[12\] This avoids needing to specify the consequences if a bank is unable to meet an extremely large unexpected outflow.
loan status $1_{\omega_{i,t}}(j)$ can be written as

$$
\Pi^h_{i,t}(j) = \left( i^*_t - i_t - x^d \right) 1_{\omega_{i,t}}(j) L_{i,t}(j) - (1 - 1_{\omega_{i,t}}(j)) P_t \kappa
$$

(3.41)

$$
+ \left[ i_t (1 - \rho) + (i^*_t - s) \rho - x^d \right] D_t(j)
$$

$$
+ \max_{B^h_t, H_t} \left\{ \left( \frac{1}{2} i^* - i_t \right) B^h_t(j) - i_t H_t(j) \right\}
$$

$$
+ \int_{-\tau D_t(j)}^{H_t(j)} (i^*_t - s) [H_t(j) - \phi_t(j)] f(\phi) d\phi
$$

$$
+ \int_{H_t(j)}^{\tau D_t(j)} (i^*_t + s) [H_t(j) - \phi_t(j)] f(\phi) d\phi \right\},
$$

where (3.39) has been used to eliminate $I_t(j)$ and the maximization is subject to (3.40). The first two terms on the right in (3.41) represent the net interest income on loans with firm $i$ and deposits where $i_t (1 - \rho) + (i^*_t - s) \rho - x^d$ is the return on an additional dollar of deposits. Notice that the net income on loans include the associated search costs of finding a borrower. The next two terms represent the interest income on bond holdings and the opportunity cost of holding excess reserves or bonds rather than lending in the interbank market. The first integral captures the outcomes where the net payment shock is such that bank ends the period with positive excess reserves. These are held in deposits with the central bank and remunerated at rate $i^*_t - s$. The second integral captures the opposite situations, where the shock is larger than $H_t(j)$, leaving the bank with a negative net position that requires it to borrow through the central bank’s lending facility at the penalty rate $i^*_t + s$.

Let $h_t(j) \equiv H_t(j)/D_t(j)$ and re-write the nominal profit function as well as the collateral constraint in terms of $h_t(j)$\textsuperscript{13}. If $\chi_t(j)$ denotes the Lagrangian multiplier

\textsuperscript{13}Given $L_{i,t}(j)$ and $D_t(j)$ the bank chooses $\{h_t(j), B^h_t(j)\}$ consistent with the following static opti-
on the collateral constraint, the first order conditions for $h_t(j)$ and $B^b_t(j)$ are

\begin{equation}
\begin{aligned}
h_t(j): & \quad -i_t + (i^*_t - s) \left[ \frac{h_t(j) + \varepsilon}{2\varepsilon} \right] + (i^*_t + s) \left\{ 1 - \left[ \frac{h_t(j) + \varepsilon}{2\varepsilon} \right] \right\} + \chi_t(j) = 0 \quad (3.42)
\end{aligned}
\end{equation}

and

\begin{equation}
\begin{aligned}
B^b_t(j): & \quad \left( i^b_t - i_t \right) + \xi_b \chi_t(j) = 0. \quad (3.43)
\end{aligned}
\end{equation}

The optimal choice of excess reserves equates the opportunity cost of holding one more unit of reserves, $i_t$, with the weighted sum of the marginal costs in expected reserve deficiency, $(i^*_t + s)[1 - (h_t(j) + \varepsilon)/2\varepsilon]$, and the marginal gains in expected interest income, $(i^*_t - s)(h + \varepsilon)/(2\varepsilon)$ from holding excess reserves and the collateral value of an extra dollar of reserve holdings $\chi_t(j)$. Equation (3.43) implies the interest rate on bonds plus their collateral value equals the interbank market rate, or

\begin{equation}
\chi_t(j) = \left( i_t - i^b_t \right) / \xi_b
\end{equation}

The minimization problem expressed in terms of $h_t(j)$:

\begin{equation}
\max_{\{h_t(j), B^b_t(j)\}} \Pi_b^t(j)
\end{equation}

s.t.

\begin{equation}
\begin{aligned}
h_t(j) D_t(j) + \xi_b B^b_t(j) + \chi_t(j) \xi_L L_{i_t}(j) + \tilde{\chi}^{bs} \geq \varepsilon D_t(j)
\end{aligned}
\end{equation}

where $\Pi_b^t(j)$ is given by

\begin{equation}
\begin{aligned}
\left( i^b_t - i_t - x^d \right) 1_{\omega_t, i_t}(j) L_{i_t}(j) - (1 - 1_{\omega_t, i_t}(j)) P_t \kappa \\
+ \left[ i_t (1 - \rho) + (i^*_t - s) \rho - \left( i^b_t + x^d \right) \right] D_t(j) \\
+ \max_{\{h_t(j), B^b_t(j)\}} \left\{ \left( i^b_t - i_t \right) B^b_t(j) - i_t h_t(j) D_t(j) \\
+ \frac{(i^*_t - s) D_t(j)}{2\varepsilon} \int_{-\varepsilon}^{h_t(j)} (h_t(j) - \varepsilon_t) d\varepsilon_t \\
+ \frac{(i^*_t + s) D_t(j)}{2\varepsilon} \int_{h_t(j)}^{\varepsilon} (h_t(j) - \varepsilon_t) d\varepsilon_t \right\}
\end{aligned}
\end{equation}

and $\tilde{\chi}^{bs}$ represents assets different from $h, B^b$ and $L$ expressed as a fraction of total deposits that can be used as collateral.
Hence, \( \chi_t \) is independent of \( j \). From (3.42), this also implies that \( h_t \) is independent of \( j \), and the demand for excess reserves is given by

\[
h_t = \left( \frac{\bar{\varepsilon}}{\delta} \right) (i_t^* - i_t + \chi_t).
\tag{3.44}
\]

Total excess reserve demand is increasing in the volatility of payment flows (measured by \( \bar{\varepsilon} \)). It is decreasing in the width of the channel \( s \) and increasing in the spread between the policy rate and the interbank rate \( i_t^* - i_t \) and the marginal value of having collateral as \( h_t \) instead of having it as \( B_t^b(j) \) or \( L_{i,t}(j) \).\(^{14}\) When \( \chi_t \) increases, the bank perceives that it is more valuable to allocate excess reserves as collateral instead of using government bonds or loans as such. In this situation, banks will increase \( h_t \) since \( B_t^b(j) \) and \( L_{i,t}(j) \) are subject to haircuts. Equation 3.44 implies that total excess reserve demand has three components: The component related to the volatility of the payment shock, thus a precautionary demand, a component related to the opportunity cost of holding excess reserves \( i_t^* - i_t \) and a component associated to the marginal benefit of having excess reserves in the form of collateral, \( \chi_t \).

Rewriting (3.42) as

\[
i_t = (i_t^* - s) \left( \frac{h_t + \bar{\varepsilon}}{2\bar{\varepsilon}} \right) + (i_t^* + s) \left[ 1 - \left( \frac{h_t + \bar{\varepsilon}}{2\bar{\varepsilon}} \right) \right] + \chi_t
\]

shows that \( i_t \) equals a weighted average of the interest rate on central bank deposits \( i_t^* - s \) and the rate of borrowing reserves \( i_t^* + s \), adjusted for the marginal value of

\(^{14}\)When the bank profit function is expressed in terms of excess reserves as a fraction of deposits \( h_t(j) \), the payment shock \( \phi_t(j) = \varepsilon_t D_t(j) \) is expressed also as a fraction of deposits. In this case, \( \varepsilon_t = \frac{\phi_t(j)}{D_t(j)} \sim \text{Unif}(-\bar{\varepsilon}, \bar{\varepsilon}) \) with density \( f(\varepsilon_t) = \frac{1}{2\bar{\varepsilon}} \) and variance \( \text{var}(\varepsilon_t) = \frac{\bar{\varepsilon}^2}{4} \). Notice that we assume that the support of the payment shock \( \bar{\varepsilon} \) evolves over time according to an exogenous autoregressive process that we specify below.
collateral $\chi_t$. Thus,
\[ i_t^* - s \leq i_t - \chi_t \leq i_t^* + s. \]

If $\chi_t = 0$ so that collateral constraints do not bind, then the standard result that the interbank rate is bounded between the rate paid on reserves ($i_t^* - s$) and the rate charged on borrowing ($i^* + s$) is obtained. When the collateral constraint binds, $\chi_t > 0$ and the rate paid on reserves provides a floor for the interbank rate, but $i_t$ could exceed the penalty rate on reserve borrowings. These bounds on $i_t$, imply
\[ i_t - i_t^* - \chi_t \]
\[ -s \leq i_t - i_t^* - \chi_t \leq s, \]
so from (3.44) reserve demand is also bounded:
\[ -\bar{\varepsilon} \leq h_t \leq \bar{\varepsilon}. \]

If the central bank sets $i = i^*$ (the interbank rate equals the central bank’s policy rate), then
\[ h_t = \left( \frac{\bar{\varepsilon}}{s} \right) \chi_t \geq 0, \]
and excess reserves are positive. In the absence of a collateral constraint, excess reserves would be zero\(^{15}\). But if the collateral constraint binds, the slope of the total reserve demand with respect to the interbank rate is positive since $\chi_t$ raises whenever $i_t$ is above $i_t^b$. Equivalently, if the central bank provides a level of total reserves equal to required

\(^{15}\)This is the case considered, for example, by Whitesell (2006). In the presence of a collateral constraint, the interest rate on unsecured borrowing can exceed the rate $i_t^* + s$ on secured borrowing by the value of the collateral necessary to access the central bank’s secured funding.
reserves so that excess reserves are zero,

\[ i_t = i_t^* + \chi_t \geq i_t^*. \]

In this case, the interbank rate exceeds the policy rate (see Berensten and Monnet 2006).

Evaluating \( \Pi_t^b(j) \) at \( h^*(j) = h_t^* \) yields:

\[
\Pi_t^b(j) = \left( i_{i,t} - i_t - x^d \right) \mathbf{1}_{\omega_i,t} (j) L_{i,t} (j) - \left( 1 - \mathbf{1}_{\omega_i,t} (j) \right) P_t \kappa 
\]

\[ + \left( i_t^* - i_t \right) B_t^b(j) + \left[ i_t (1 - \rho) - \left( i_t^d + x^d \right) - i_t h_t^* + \bar{x}_t \right] D_t(j) \]

where

\[
\bar{x}_t = (i_t^* - s) \left[ \rho + \left( \frac{(h_t^*)^2}{4\varepsilon} + \frac{h_t^*}{2} + \frac{\varepsilon}{4} \right) \right] + (i_t^* + s) \left( -\frac{(h_t^*)^2}{4\varepsilon} + \frac{h_t^*}{2} - \frac{\varepsilon}{4} \right) \]

(3.46)

where the integrals have been solved out since we already know \( h_t^* \)\(^{16}\).

When choosing deposits \( D_t(j) \) the bank takes as given its optimal decision on \( h_t^* \) and \( B_t^b \) but also takes into account the effect of \( D_t(j) \) over the collateral constraint. Taking derivatives of \( \Pi_t^b(j) \) with respect to \( D_t(j) \) yields

\[
\frac{\partial \Pi_t(j)}{\partial D_t(j)} = \zeta_t D_t(j),
\]

where

\[
\zeta_t \equiv i_t (1 - \rho) - \left( i_t^d + x^d \right) - i_t h_t^* + \bar{x}_t + \chi_t (h_t^* - \varepsilon)
\]

\(^{16}\)The integrals evaluated at the optimal demand for excess reserves, \( h_t^* (j) = h_t^* \) become:

\[
\int_{h_t^* (j)}^{\bar{x}_t} (h_t^* (j) - \varepsilon_t) d\varepsilon_t = -\frac{(h_t^*)^2}{2} + \bar{x}_t h_t^* - \frac{\varepsilon^2}{2}
\]

\[
\int_{-\bar{x}_t}^{h_t^* - \varepsilon_t} d\varepsilon_t = \frac{(h_t^*)^2}{2} + \bar{x}_t h_t^* + \frac{\varepsilon^2}{2}
\]
Competition for deposits among banks will ensure that $\zeta_t = 0$, implying an interest rate on deposits of

$$i^d_t = i_t (1 - \rho) - x^d - \bar{\rho}_t h^*_t + x_t + \chi_t (h^*_t - \bar{\chi})$$

Using the definition of $\bar{\pi}_t$ (equation 3.46) as well as equation 3.44, $i^d_t$ can be written as

$$i^d_t = i_t (1 - \rho) + (i^*_t - s) \rho - x^d + f (i^*_t - i_t, \chi_t, \bar{\chi})$$

where

$$f (i^*_t - i_t, \chi_t, \bar{\chi}) = \frac{1}{2} \bar{\pi}_t (i^*_t - i_t + \chi_t)^2 - \left( \chi_t + \frac{s}{2} \right) \bar{\chi}$$

Hence, the deposit rate is a weighted average of the interbank rate and the rate earned on required reserves adjusted for the bank’s cost of providing deposits and the effect of deposits on the need for additional collateral and excess reserves. If excess reserves are zero, (3.44) implies $i^*_t + \chi_t - i_t = 0$ and

$$i^d_t = i_t (1 - \rho) + (i^*_t - s) \rho - x^d - \left( \chi_t + \frac{s}{2} \right) \bar{\chi}.$$

In discussing the loan market earlier, the flow value (in real terms) to a bank when operating in the loan market was equal to $\pi^b_t(j) = 1_{\omega^t_t}(j) \left( R^t_{1,t} - R_t \right) l_{i,t} - (1 - 1_{\omega^t_t}(j)) \kappa$, where $1_{\omega^t_t}(j)$ indicates the status of the bank: extending a loan or searching for a borrower. Notice that the above equation is obtained by taking the following steps: 1) Expressing 3.45 in real terms:

$$\pi^b_t(j) = \left( i^d_t - i_t - x^d \right) 1_{\omega^t_t}(j) l_{i,t}(j) - (1 - 1_{\omega^t_t}(j)) \kappa + \left( i^b_t - i_t \right) b^t_i(j) + \left[ i_t (1 - \rho) - \left( i^d_t + x^d \right) - i_t h^*_t + \bar{\pi}_t \right] d_t(j)$$
where lowercase letters represent real variables. 2) Substituting the equilibrium expression for $i_t^d$ into $\pi_t^b(j)$:

$$\pi_t^b(j) = \left( i_{i,t}^d - i_t - x_t^d \right) 1_{\omega_i,t} (j) l_{i,t}(j) - (1 - 1_{\omega_i,t} (j)) \kappa$$

$$+ \left( i_t^b - i_t \right) b_t^b(j) - \chi_t (h_t^* - \tau) d_t(j)$$

and finally 3) Using the collateral constraint expressed in real terms to eliminate $b_t^b(j)$ as well as making use of $\chi_t = \frac{i_t - i_t^d}{\xi}$:

$$\pi_t^b(j) = \left( i_{i,t}^d - i_t - x_t^d + \chi_t \xi_L \right) 1_{\omega_i,t} (j) l_{i,t}(j) - (1 - 1_{\omega_i,t} (j)) \kappa$$

The term, $\chi_t \xi_L$, reflects the collateral value of extending a loan and it is a component of the net return that a bank obtains when $l_{i,t}(j) > 0$. We define the opportunity cost of extending a loan as

$$R_t = 1 + i_t - \xi_L \chi_t + x_t^d.$$  

Ceteris peribus, an increase in the haircut applied to loans used as collateral with the central bank (a fall in $\xi_L$) increases the opportunity cost of lending. As a result, the effective cost of labor increases and the demand for labor falls. This negative effect on employment holds for a given interbank rate $i_t$. In addition, an increase in the marginal value of collateral (a raise in $\chi_t$) increases the opportunity cost of lending. The gross loan rate negotiated between bank $j$ and firm $i$ in the case $l_{i,t}(j) > 0$ is defined as $R_{i,t}^d = 1 + i_{i,t}^d$. Therefore, the flow value to a bank for participating in the loan market, expressed in real terms, is given by equation 3.24.
3.2.4 The central bank

The central bank sets the required reserve ratio, the width of the channel and the haircuts on bonds and loans brought by banks as collateral against loans from the central bank. The central bank can set its policy interest rate $i^*_t$, its bond holdings and its liabilities (high-powered money) subject to its budget constraint. In nominal terms, the central bank’s budget constraint is given by

$$B^c_{tb} - B^c_{t-1} + RCB_t + (i^*_t - s) (\rho D_t + ER_t) + (i^*_t + s) BR_t = B^c_t B^c_{tb} + HP^*_t - HP^*_t-1$$

The central bank revenue is given by the interest rate payments on government debt holding $(i^*_t B^c_{tb})$, the change in high-powered money $(HP^*_t - HP^*_t-1)$ and interest payments on total reserves held with the central bank net of interest payments on total borrowed reserves, $((i^*_t - s) (\rho D_t + ER_t) + (i^*_t + s) BR_t)$. On the other hand, the central bank allocate its revenue into purchases of government debt ($B^c_{tb}$) and transfers of central bank’s receipts to the treasury ($RCB_t$). Notice that $ER_t$ and $BR_t$ denote aggregate excess reserves and aggregate borrowed reserves of the banking sector respectively. Both measures are obtained by aggregating the optimally chosen expected excess reserves and borrowed reserves of each individual bank. Recall that at the beginning of the period, each bank expected excess reserves ($ER_t(j)$) and borrowed reserves ($BR_t(j)$) are

$$ER_t(j) = \frac{D_t(j)}{2\pi} \int_{-\pi}^{h_t(j)} (h_t(j) - \varepsilon_t) d\varepsilon_t$$

$$BR_t(j) = \frac{D_t(j)}{2\pi} \int_{h_t(j)}^{\pi} (h_t(j) - \varepsilon_t) d\varepsilon_t$$

133
where the optimal level of excess reserves expressed as a fraction of deposits satisfies

the following equation

\[ h_t^* = \frac{\bar{\varepsilon}}{s} (i_t^* - \delta_t + \chi_t) \quad \text{for all } j \in [0, 1] \]

therefore

\[ ER_t(j) = \frac{D_t(j)}{2\bar{\varepsilon}} \int_{-\varepsilon}^{\varepsilon} (h_t^* - \varepsilon) d\varepsilon \]

\[ = \frac{D_t(j)}{2\bar{\varepsilon}} \left( \frac{(h_t^*)^2}{2} + \bar{\varepsilon}h_t^* + \frac{\bar{\varepsilon}^2}{2} \right) \]

and

\[ BR_t(j) = \frac{D_t(j)}{2\bar{\varepsilon}} \int_{-\varepsilon}^{\varepsilon} (h_t^* - \varepsilon) d\varepsilon \]

\[ = \frac{D_t(j)}{2\bar{\varepsilon}} \left( -\frac{(h_t^*)^2}{2} + \bar{\varepsilon}h_t^* - \frac{\bar{\varepsilon}^2}{2} \right) \]

Aggregating across all banks yields expressions for \( ER_t \) and \( BR_t \) that appear

on the central bank’s budget constraint:

\[ ER_t = \left( \frac{(h_t^*)^2}{4\bar{\varepsilon}} + \frac{h_t^*}{2} + \frac{\bar{\varepsilon}}{4} \right) D_t \]

\[ BR_t = \left( -\frac{(h_t^*)^2}{4\bar{\varepsilon}} + \frac{h_t^*}{2} - \frac{\bar{\varepsilon}}{4} \right) D_t \]

On the other hand, the treasury budget constraint is given by

\[ P_t T_t + B_t^T - B_{t-1}^T + RCB_t = P_t G_t + i_t^b B_t^T \]

where the left hand side represents treasury revenue consisting in taxes/transfers to or

from households \((T_t)\), new issues of interest-bearing government debt \((B_t^T - B_{t-1}^T)\) as
well as transfers of central bank’s receipts to the treasury \((RCB_t)\) while the right hand side represents expenditures given by government spending in goods and services \((G_t)\) as well as interest rate payments from government debt \(i^*_tB^T_t\). We assume that the total supply of government debt \(B^T_t\) is held by households \(B^h_t\), private banks \(B^b_t\) and the central bank \(B^{cb}_t\), that is

\[B^T_t = B^h_t + B^b_t + B^{cb}_t\]

therefore, the consolidated government budget constraint can be written as

\[P_tT_t + B^p_t - B^p_{t+1} + HP^s_t - HP^s_{t-1} = P_tG_t + i^*B^p_t + X_t\]  

(3.48)

where \(B^p_t = B^h_t + B^b_t\) denotes holdings of government debt by the private sector and \(X_t\) is the central bank’s interest on total reserves net of interest payments on total borrowed reserves, given by

\[X_t = (i^*_t - s)(\rho D_t + ER_t) + (i^*_t + s)BR_t\]  

(3.49)

Notice that \(BR_t\) is negative by definition.

In real terms, 3.49 is

\[b^p_t - \left(\frac{1}{1+\pi_t}\right)b^p_{t-1} + hP^s_t - \left(\frac{1}{1+\pi_t}\right)hp^s_{t-1} = G_t - T_t + i^*_t b^p_t + x_t\]  

(3.50)

where

\[x_t = (i^*_t - s)(\rho d_t + er_t) + (i^*_t + s)br_t\]  

(3.51)

\[er_t = \left(\frac{(h^*_t)^2}{4\pi} + \frac{h^*_t}{2} + \frac{\pi}{4}\right) d_t\]  

(3.52)
\[ br_t = \left( -\frac{(h_t^s)^2}{4\pi} + \frac{h_t^s}{2} - \frac{\pi}{4} \right) d_t \]  

(3.53)

To focus on the operations of the central bank, we rewrite the consolidated budget constraint in terms of the central bank holdings of government bonds as well as in terms of total government debt:

\[
\left( h_t^s - \left( \frac{1}{1+\pi_t} \right) h_{t-1}^s \right) - \left( b_t^{cb} - \left( \frac{1}{1+\pi_t} \right) b_{t-1}^{cb} \right) + i_t^b b_t^c = f_t + x_t \tag{3.54}
\]

where \( f_t \) is defined as an exogenous fiscal variable given by

\[ f_t = G_t - T_t + i_t^b b_t^T - \left( b_t^T - \left( \frac{1}{1+\pi_t} \right) b_{t-1}^T \right) \]

Notice that \( f_t \) is given by the real transfers of the central bank’s receipts to the treasury \( \left( f_t = \frac{RCB_t}{P_t} \right) \). Then, given the policy rate \( i_t^* \) and private sector decisions that determine reserve holdings \( h_t^s \), the term \( x_t \) is not controlled directly by the central bank, as emphasized by Berensten and Monet (2008). The consolidated budget constraint links changes in the supply of high powered money \( h_t^s \) with changes in the central bank’s government bond holdings \( b_t^{cb} \) via the effects of open market operations.

It can also be assumed that lump sum taxes \( T_t \) adjust in order to fund any change in \( h_t^s \) and/or changes in \( x_t \). Under this assumption the consolidated budget constraint, \( ?? \), is written as

\[ T_t + h_t^s - \left( \frac{1}{1+\pi_t} \right) h_{t-1}^s = x_t + \tilde{f}_t \]  

(3.55)
where $\tilde{f}_t$ is defined as the following exogenous fiscal shock:

$$\tilde{f}_t = G_t + \left( b_t^{cb} - \left( \frac{1}{1 + \pi_t} \right) b_{t-1}^{cb} - \phi b_t^{cb} \right) - \left( b_t^T - \left( \frac{1}{1 + \pi_t} \right) b_{t-1}^T - \phi b_t^T \right)$$

In this case, the central bank does not perform open market operations when changing the supply of high powered money.

### 3.2.5 Market Clearing and the Aggregate equilibrium

Equilibrium in the interbank market requires aggregate interbank net lending to cancel out, that is:

$$\int \frac{I_t(j)}{P_t} dj = 0$$

and to balance the reserve demand and reserve supply in real terms, so if $hp^*_t$ is the total (exogenous) real supply of high powered money set by the central bank, total reserve demand is $(\rho + h^*_t) d_t$, thus:

$$hp^*_t = (h^*_t + \rho) d_t$$

using (3.44) yields an expression for the equilibrium interbank interest rate:

$$i_t = i^*_t + \chi_t - \left( \frac{s}{\tau} \right) \left( \frac{hp^*_t}{d_t} - \rho \right), \quad (3.56)$$

Equation (3.56) illustrates that the central bank has multiple instruments for achieving a given interbank rate $i_t$. For given reserve supply relative to total deposits $\frac{hp^*_t}{d_t}$ and collateral value $\chi_t$, $i_t$ can be increased directly by raising the target policy rate $i^*_t$ or by reducing the width of the corridor $s$. Holding $i^*_t$, $\chi_t$, and $s$ constant, a decrease in the reserve supply (relative to deposit liabilities of the banking sector) increases $i_t$. 137
A further implication of (3.56) is that if $s > 0$ the equilibrium interbank rate will equal the policy rate only when $\frac{hp_t^s}{\Delta t} = \rho + (\bar{\varepsilon}/s) \chi_t \geq \rho$, that is, only when the central bank supplies a level of reserves greater than the level of required reserves and the collateral constraint binds, $\chi_t > 0$.

To summarize, in this setting the central bank has four potential policy instruments: $i^*$, $s$, $hp^s$, and $b^{cb}$, of which only three can be varied independently consistent with (??).\textsuperscript{17} If there are no open market operations and $T_t$ adjusts endogenously to any change in $hp^s$, then the central bank has three potential policy instruments: $i^*$, $s$, $hp^s$.

Aggregate output is the number of producing firms times the expected output of each firm, conditional on its $\omega_{i,t}$ realization exceeding $\tilde{\omega}_t$. Recall that the number of matched firms at the start of period $t$ is $f_{t-1}^{m}$ and that only a fraction $\varphi_t (\tilde{\omega}_t) = (1 - \delta)(1 - \gamma_t (\tilde{\omega}_t))$ of those firms survive both separation hazards and consequently end up producing. Aggregate output is then

$$Y_t = \varphi_t (\tilde{\omega}_t) f_{t-1}^{m} E \left[ y^*_t (\omega_{i,t}) \mid \omega_{i,t} \geq \tilde{\omega}_t \right],$$

where

$$E \left[ y^*_t (\omega_{i,t}) \mid \omega_{i,t} \geq \tilde{\omega}_t \right] = \int_{\tilde{\omega}_t}^{\omega} \frac{dG(\omega)}{1 - \gamma_t (\tilde{\omega}_t)}$$

and individual output for firm $i$ is written explicitly in terms of its idiosyncratic productivity level, $y_t(\omega_{i,t})$. Using the assumption that $\omega$ follows a uniform distribution with

\textsuperscript{17}If the central bank operated a channel system with an asymmetric corridor, then instead of $s$, the central bank could vary the upper and lower bounds of the corridor around $i^*$ independently. We restrict attention to a symmetric system.
density \( g(\omega) = dG(\omega) = 1/ (\bar{\omega} - \omega) \) then \( Y_t \) is

\[
Y_t = (1 - \delta)\alpha^{\frac{1}{1 - \alpha}} \left( \frac{z_t}{w_t R_t} \right)^{\frac{1}{1 - \alpha}} \left( \frac{\omega^k - (\bar{\omega}_t)^k}{k (\bar{\omega} - \omega)} \right) f_{t-1}^m \tag{3.57}
\]

where \( k \equiv (2 - a) / (1 - a) > 1 \).

Following the same steps, aggregate employment is

\[
N_t = (1 - \delta)\alpha^{\frac{1}{1 - \alpha}} \left( \frac{z_t}{w_t R_t} \right)^{\frac{1}{1 - \alpha}} \left( \frac{\omega^k - (\bar{\omega}_t)^k}{k (\bar{\omega} - \omega)} \right) f_{t-1}^m \tag{3.58}
\]

Combining (3.57) and (3.58),

\[
Y_t = \xi z_t F_t^{1-a} N_t^a \tag{3.59}
\]

where

\[
F_t \equiv (1 - \delta) \left( \frac{\bar{\omega}^k - \bar{\omega}_t^k}{k (\bar{\omega} - \omega)} \right) f_{t-1}^m \tag{3.60}
\]

Equation (3.59) is the effective aggregate production function for this economy and illustrates the way in which aggregate output depends on the aggregate productivity shock and employment but also on the number of producing firms and their average idiosyncratic productivity as reflected in \( F_t \). Credit market disruptions that lead to an exogenous rise in match breakups (a rise in \( \delta \)) acts like a negative productivity shock that it is further amplified due to endogenous changes in \( \bar{\omega}_t \) and \( f_{t-1}^m \). In addition, an increase in the cutoff productivity level \( \bar{\omega}_t \) reduces output (given \( N \)) by reducing the mass of firms that actually produce. Notice that under a perfectly competitive credit market, the term \( F_t \) does not affect the effective aggregate production function, that is \( F_t = 1 \). It is possible to think of \( F_t \) as an inefficiency wedge associated to credit market imperfections that amplifies the impact of exogenous shocks.

---

\[ ^{18} \text{See appendix.} \]
The assumption that $\omega$ follows a uniform distribution implies the following overall continuation rate:

$$\varphi_t(\tilde{\omega}_t) = (1 - \delta) \left( \frac{\tilde{\omega} - \tilde{\omega}_t}{\tilde{\omega} - \omega} \right)$$ (3.61)

Market equilibrium in the loan market requires $l_{i,t} = w_t N_{i,t}^*$ for all active firms or credit contracts, that is, for all $i$ such that $\omega_{i,t} > \tilde{\omega}_t$. Aggregating this equilibrium condition across all those active firms yields the following condition for aggregate loans and aggregate labor income

$$l_t = w_t N_t$$ (3.62)

The aggregate equilibrium takes into consideration the aggregation of the balance sheet as well as the collateral constraint for banks which are given by

$$l_t + b_t^b + h_t^{*} d_t = (1 - \rho) d_t$$ (3.63)

$$h_t^{*} d_t + \xi b_t^b + \xi_L l_t + \xi^{bs} = \bar{z} d_t$$ (3.64)

respectively. Notice that a constant term $\xi^{bs}$ has been added to the aggregate collateral constraint in order to account for the different types of assets that may serve as collateral and are not considered in the model. This new parameter will serve for calibration purposes.

As part of the aggregate equilibrium, firms and banks transfer their profits to the representative household at the end of each period. The aggregate real transfer of profits received by the household consists of the following two components:

$$\pi_t^f = Y_t - R_t^f l_t - \varphi_t (\tilde{\omega}_t) f_{t-1}^m x^f$$
\[\pi_t^b = \left( R_t^l - 1 \right) l_t + i_t^b b_t^b - i_t^d d_t + x_t - \left( x^l l_t + x^d d_t + \kappa b_t^u \right)\]

where \(\pi^f_t\) denote aggregate firm profits and \(\pi^b_t\) aggregate bank profits. In the latter case, we have used the equilibrium condition in the deposit market \(d_t = \int d_t (j) \, dj = 0\), where the integration is across all banks, the equilibrium condition in the interbank market \(\int \frac{I_t^j}{P_t} \, dj = 0\) as well as the aggregate balance sheet for the banking sector (3.39).

Since each loan contract negotiate its own loan interest rate, the average credit spread is defined as the difference between the average loan rate and the bank’s opportunity cost of lending:

\[\frac{R_t^l l_t - R_t l_t}{l_t} = (1 - \alpha) (1 - \eta) \frac{Y_t}{\varphi_t (\tilde{\omega}_t) \int_{l_{t-1}}^m l_t} - \left( \frac{(1 - \eta) x^f + \eta x^d}{l_t} \right) \] (3.65)

where

\[R_t^l l_t = E \left[ R_t^l (\omega_{it}) l_t^* (\omega_{it}) \mid \omega_{it} \geq \tilde{\omega}_t \right] \]

and

\[R_t l_t = E \left[ R_t l_t^* (\omega_{it}) \mid \omega_{it} \geq \tilde{\omega}_t \right] \]

Equilibrium in the good market requires aggregate expenditures (consumption plus government expenditures) to equalize aggregate household income net of aggregate fixed cost of production by producing firms, aggregate search costs by the banking sector as well as net of the aggregate costs of managing loans and deposits. Then the aggregate resource constraint of the economy is characterized by

\[C_t + G_t = Y_t - \left( (\varphi_t (\tilde{\omega}_t) \int_{l_{t-1}}^m x^f + x^l l_t + x^d d_t + \kappa b_t^u) \right) \] (3.66)

141
where $G_t$ is treated as exogenous and consumption must satisfy the following aggregate CIA constraint:

$$C_t = d_t + w_t N_t + \xi^{cia} \tag{3.67}$$

### 3.2.5.1 Characterization of the aggregate loan market equilibrium

In this section we characterize the loan market equilibrium in terms of two main equations. The first equation is given by equation (3.38) and relates the cutoff idiosyncratic productivity level, $\tilde{\omega}_t$, with our measure of credit market tightness $\tau_t = \frac{f_t}{b_t}$, the marginal cost of labor, $MC_t = w_t R_t$, which is common to all producing firms and the aggregate component of productivity. The second equation is an Euler equation that describes the dynamics of the credit market tightness as a function of $\tilde{\omega}_t$ and a measure of aggregate output net of fixed costs.

The equation for $\tilde{\omega}_t$ is

$$\tilde{\omega}_t = \frac{1}{\alpha} (1 - \alpha) 1^{-\alpha} \xi z_t \left[ x^f - \left( \frac{1 - \eta \mu \tau_t^q}{1 - \eta} \right) \frac{\kappa}{\mu \tau_t^q} \right]^{1-\alpha}$$

Combining the free entry condition for banks, the joint surplus of a credit relationship as well as the equation for aggregate output and the definitions for the matching rates $p_t^b$ and $p_t^f$, yields the following equation that characterizes the dynamics of $\tau$:

$$\frac{\kappa}{\mu \tau_t^q} - E_t \Delta t, t+1 \varphi_{t+1} (\tilde{\omega}_{t+1}) \left( 1 - \frac{\eta \mu \tau_{t+1}^q}{1 - \eta} \right) \frac{\kappa}{\mu \tau_{t+1}^q} \tag{3.68}$$

$$= (1 - \eta) E_t \Delta t, t+1 \left[ (1 - \alpha) \frac{Y_{t+1}}{f_t^{m}} - \varphi_{t+1} (\tilde{\omega}_{t+1}) x^f \right]$$
3.2.5.2 Monetary policy

We assume that lump sum taxes adjust whenever the real supply of high powered money (or reserve balances) changes. Therefore, the consolidated government budget constraint is consistent with equation (3.55). The model is closed by assuming the central bank sets the growth rate for the nominal reserve balances $\theta_t$, as well as the width of the corridor $s_t$ to be exogenous. In addition, the central bank sets its policy rate to be the same as the interbank interest rate:

$$i_t^* = i_t$$ (3.69)

This implies that the interbank interest rate $i_t$ is endogenous and depends on the collateral value $\chi_t = (i_t - i^b_t)/\xi^b_t$. Recall that in this case, the aggregate demand for excess reserves is $h_t^* = \frac{\bar{\varepsilon}_t}{s} \chi_t$. We assume that the volatility of the payment shock expressed as a fraction of deposits follows an exogenous process given by

$$\left(\frac{\bar{\varepsilon}_t}{\bar{\varepsilon}}\right) = \left(\frac{\bar{\varepsilon}_{t-1}}{\bar{\varepsilon}}\right)^{\rho_\varepsilon} \exp\left(\varepsilon_t^\varepsilon\right)$$ (3.70)

where $\bar{\varepsilon}$ is the steady state value of the payment shock. Real reserve balances follows the following process

$$h p^*_t = \left(\frac{1 + \theta_t}{1 + \pi_t}\right) h p^*_t-1$$ (3.71)

where $\theta_t$ is the given by

$$\left(\frac{\theta_t}{\bar{\theta}}\right) = \left(\frac{\theta_{t-1}}{\bar{\theta}}\right)^{\rho_\theta} \exp\left(\varepsilon_t^\theta\right)$$ (3.72)
3.3 Computation and simulations

The summary of the non-linear dynamic equations that characterize the aggregate equilibrium of the model is presented in appendix (??). We assume that anytime that a credit contract ends, contractual parties are able to search for a new contract within the same period of time that the credit separation has occurred. The latter holds for exogenous as well as for endogenous separations. Then, the mass of firms searching for funds follows equation (3.13) while the credit creation and credit separation rates are given by equations (3.16) and (3.17) respectively.

The model is solved by using a standard perturbation method applied to a first order approximation around the non-stochastic steady state of the model. The steady state of the model is consistent with a zero inflation rate \( \pi = 0 \) which implies an steady state value of \( \theta = 0 \) according to equation (3.71) evaluated at the steady state.

3.3.1 Calibration

The calibration strategy is as follows: First we parametrize the following 12 parameters according to the standard literature as well as based on data for the U.S great moderation period:

The value for the ratio of reserve requirements is obtained from the Federal Reserve Board’s regulation D and it is close to the 10% of liabilities requirement since the model does not take into account required reserves in the form of vault cash. On the other hand, the Federal Reserve currently sets the haircut on U.S. bills and bonds
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho )</td>
<td>Reserve requirements</td>
<td>0.094</td>
</tr>
<tr>
<td>( \xi_b )</td>
<td>Haircut on U.S bills and bonds</td>
<td>0.99</td>
</tr>
<tr>
<td>( \xi_L )</td>
<td>Haircut on loans</td>
<td>0.65</td>
</tr>
<tr>
<td>( s )</td>
<td>Width of the symmetric corridor</td>
<td>0.0025</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>Search cost for banks</td>
<td>1.58</td>
</tr>
<tr>
<td>( \bar{\eta} )</td>
<td>Bank’s Nash bargaining share</td>
<td>0.32</td>
</tr>
<tr>
<td>( \varphi )</td>
<td>Elasticity of matching function w.r.t searching firms</td>
<td>0.5</td>
</tr>
<tr>
<td>( z )</td>
<td>Aggregate technology</td>
<td>1</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Upper support of idiosyncratic productivity</td>
<td>1</td>
</tr>
<tr>
<td>( \omega )</td>
<td>Lower support of idiosyncratic productivity</td>
<td>0</td>
</tr>
<tr>
<td>( \eta )</td>
<td>Utility function parameter 1</td>
<td>1</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Utility function parameter 2</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3.1: Parameter values taken from the data

\((\xi_b)\) with less than 5 years to maturity to 99% and the haircut on zero coupon, normal risk-rated commercial loans \((\xi_l)\) of 5 year maturity to 65%.\(^{19}\) We fix the width of the symmetric corridor \(s\) to be 25 basic points as recent experience for the U.S. shows. The search cost for banks \(\kappa\) and the bank’s Nash bargaining share \(\bar{\eta}\) are taken from the baseline calibration in Petrosky-Nadeau and Wasmer (2012). Both parameters are obtained from calculating the empirical financial sector’s share of aggregate value added and matching it to their model counterpart. The data is taken from the industry value added tables provided by the Bureau of Economic Analysis over the period 1985-2002 and subtracting the share of GDP of household financial services and insurance from the National Income and Product Accounts tables. We assume that the Hosios condition holds at the steady state, implying that \(\bar{\eta} = \varphi = 0.32\) but also we consider a value of \(\bar{\eta} = 0.5\) as in Petrosky-Nadeau and Wasmer (2012).

The level of aggregate technology

\(^{19}\text{See the Fed’s Discount Window and Payment System Risk Collateral Margins Table, available at http://www.frbdiscountwindow.org/discountwindowbook.cfm?hdrID=14&dtlID=43}\)

145
in the steady state is normalized to be $z = 1$ while the support of the distribution
associated to the idiosyncratic productivity is normalized to be $[\omega, \bar{\omega}] = [0, 1]$.
Finally, the assumption of $\eta = \sigma = 1$ is consistent with a logarithmic utility function for the
representative household.

Second, we target the following 12 variables and ratios at steady state:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y$</td>
<td>GDP</td>
<td>1</td>
</tr>
<tr>
<td>$N$</td>
<td>Employment</td>
<td>1/3</td>
</tr>
<tr>
<td>$w_N$</td>
<td>Labor share</td>
<td>2/3</td>
</tr>
<tr>
<td>$\varphi(\tilde{\omega})$</td>
<td>Continuation rate</td>
<td>0.7</td>
</tr>
<tr>
<td>$cd$</td>
<td>Credit destruction rate</td>
<td>0.029</td>
</tr>
<tr>
<td>$l_d$</td>
<td>Loan deposit ratio</td>
<td>0.63</td>
</tr>
<tr>
<td>$h^*$</td>
<td>Excess reserves as a fraction of deposits</td>
<td>0.015</td>
</tr>
<tr>
<td>$i^b$</td>
<td>Bond interest rate</td>
<td>0.015</td>
</tr>
<tr>
<td>$i^d$</td>
<td>Deposit rate</td>
<td>0.0147</td>
</tr>
<tr>
<td>$i$</td>
<td>Interbank rate</td>
<td>0.016</td>
</tr>
<tr>
<td>$\varphi(\tilde{\omega})f^m_Y$</td>
<td>Fixed cost of production share of GDP</td>
<td>0.2</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Inflation rate</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3.2: Steady state targets

where $w_N$ and $\varphi(\tilde{\omega})f^m_Y$ denote the labor share on GDP and the fixed cost
of production share on GDP respectively. We assume the former to be 2/3 as it is
standard in the literature and the latter to be 20% of GDP. The steady-state value
for the continuation rate $\varphi(\tilde{\omega})$ is taken from Chowdorow-Reich (2013) who estimates
a probability between 70% and 80% that a previous loan is renovated by the same
group of banks at the U.S. syndicated loan market. The steady-state credit destruction
rate $cd$ is calculated from its empirical counterpart from Contessi and Francis (2010)
as an average during the great moderation period. We target a loan over deposit ratio
\( \frac{l}{q} = 0.63 \) using quarterly data on commercial and industrial loans and saving deposits for all commercial banks during 1985-2007. Excess reserves as a fraction of deposits \( h^* \) is set to be 1.5% by using the average of all quarterly reserve balances with federal reserve banks during the great moderation period. We target a deposit rate to be slightly lower than the bond rate in order to have the spread \( 1 + i^b - i^d \) as a tax on consumption and be consistent with the CIA framework used in the model.

Finally, we calibrate the following 12 parameters by solving the non linear steady-state of the model to be consistent with the above specified targets. The following table presents our calibration results:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \xi )</td>
<td>Scale parameter production function</td>
<td>3.8672</td>
</tr>
<tr>
<td>( \Theta )</td>
<td>Labor supply parameter</td>
<td>8.1473</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Production function elasticity</td>
<td>0.6769</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Scale parameter matching function</td>
<td>1.4108</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Exogenous separation rate at steady state</td>
<td>0.0799</td>
</tr>
<tr>
<td>( \xi_{cia} )</td>
<td>Residual parameter in CIA</td>
<td>0.9884</td>
</tr>
<tr>
<td>( \xi_{bs} )</td>
<td>Residual parameter in collateral constraint</td>
<td>-0.6458</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Subjective discount factor</td>
<td>0.9852</td>
</tr>
<tr>
<td>( \varpi )</td>
<td>Support of payment shock</td>
<td>0.0371</td>
</tr>
<tr>
<td>( x^f )</td>
<td>Fixed cost of production</td>
<td>0.4214</td>
</tr>
<tr>
<td>( x^d )</td>
<td>Cost of managing deposits</td>
<td>0.0009</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Nominal growth rate of reserve balances at steady state</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3.3: Calibrated parameter consistent with steady state targets
3.3.2 Model experiments

3.3.2.1 A payment shock on the interbank market

In this section we present the response of several variables of interest to a persistent increase in the support of the payment shock expressed as a fraction of deposits $\varepsilon_t$ (See figures 3.1-3.5). Recall that, the payment shock as a fraction of deposits is assumed to be distributed as $\varepsilon_t = \phi_t(j) \sim Unif (-\varepsilon_t, \varepsilon_t)$ with density $f(\varepsilon_t) = \frac{1}{2\varepsilon_t}$ and variance $\text{var}(\varepsilon_t) = \frac{\varepsilon_t^2}{3}$, where $\varepsilon_t$ follows (3.70). Therefore, an unexpected increase in $\varepsilon_t$ is interpreted as an unexpected higher volatility of the net payment shock.

![Graphs of model responses to a payment shock: A](image)

Figure 3.1: Model responses to a payment shock: A

Note: Source: Authors’ calculations based model simulation

An increase in $\varepsilon_t$ generates a persistent raise on the aggregate demand for excess reserves as a fraction of deposits, $h^*_t$ (See figure (3.1) and (3.2)). Given that the
central bank keeps the nominal growth of high powered money constant, the latter effect
leads to a persistent increase in the interbank rate as well as a persistent raise of two
interest rate spreads: The spread between the average loan rate and the opportunity
cost of lending and the spread between the interbank rate and the bond rate (see figure
(3.1)). The latter spread produces a significant increase in the marginal value of having
collateral in the form of excess reserves, $\chi_t$, relative to bonds and loans reinforcing the
increase in $h_t^*$. The former spread reflects a persistent tightening in credit conditions
as a consequence the negative response of the joint surplus to a loan contract. In this
new scenario, banks allocate more funds into excess reserves as a precautionary motive
since the marginal value of having additional collateral in the form of excess reserves
improves relative to other collateralizable assets. This effect is strong enough, that banks
reduce their holdings of government bonds despite the fact that the bond rate $i_t^b$ raises.
Since government bonds are assumed to be fixed in net supply and the consolidated
government budget constraint is given by (3.55), the representative household must
increase its holdings of government bonds. The increase in the deposit and bond rates
induce households to raise savings which ultimately is observed as a persistent raise in
aggregate deposits with the banking sector (see figure (3.2)).

The response of interest rates and spreads configure a higher opportunity cost
of lending, $R_t$. Moreover, banks’ aggregate balance sheet imply that the banking sector
reduces not only its government bond holdings but also its lending. This is a direct
consequence of the persistent raise in the marginal value of collateral, inducing banks
to hold more excess reserves at the expense of government bonds and loans (see figure
Figure 3.2: Model responses to a payment shock: B

Note: Source: Authors’ calculations based model simulation (3.2). The initial reduction in bank lending is further amplified in the credit market due to the presence of search and matching frictions in the form of an intensive and extensive margin effects.

Active credit matches decide to increase their reservation productivity $\tilde{\omega}_t$ as a response to the persistent increase in the real marginal cost of labor $MC_t = w_t R_t$. The Nash bargaining solution requires firms to equate their marginal product of labor to the real marginal cost of labor, a higher $MC_t$ will induce active firms to reduce employment (intensive margin effect). The increase in $\tilde{\omega}_t$ and in $MC_t$, induce a negative response on the joint surplus of a credit relationship as well as on the overall probability of continuation for credit contracts $\varphi_t (\tilde{\omega}_t)$ (see figure (3.3)).

A fraction of the mass of banks searching for borrowers will be able to exit
the loan market, therefore less banks will search for borrowers (a temporarily fall in $b_t^i$) while there will be more firms searching for lenders (a persistent raise in $f_t$). This means that the matching rate for firms falls but the corresponding matching rate for banks raises as a response to the initial shock. Therefore, the credit market becomes tighter (a persistent raise in $\tau_t$), meaning that credit market conditions worsen from the point of view of firms, reinforcing the initial effect over $\tilde{\omega}_t$, and reducing the mass of active firms at the end of each period $f_{t-1}^m$ (extensive margin effect). These effects are also reflected in a higher credit destruction rate, a lower credit creation rate and an increase in the average credit spread (see figure (3.4)).

Finally, the payment shock affect the real side of the economy as a deep and prolonged recession: aggregate output, employment and consumption persistently fall.
Figure 3.4: Model responses to a payment shock: D

Note: Source: Authors’ calculations based model simulation

together with an raise in household savings and a sharp drop in bank intermediation. The initial shock is amplified through a persistent fall in the "credit" input, that is, the term $F_t$ that appears in the effective aggregate production function (see figure (3.5)). Therefore, in our setting, any shock that affects the interbank market, produces inefficient fluctuations in the loan market that are propagated to the aggregate economy because of the existence of credit frictions.

### 3.3.2.2 A financial shock on the loan market

Figures (3.6)-(3.10) illustrate the dynamic effects of a financial shock on a number of aggregate variables. A financial shock is defined as an unexpected persistent increase in the exogenous separation rate for credit contracts $\bar{\omega}_t$. Recall, the overall
continuation rate of loan contracts (equation (3.61)) have an exogenous as well as an endogenous component. We assume that the exogenous component follows a non-linear autoregressive process given by

\[
\left( \frac{\delta_t}{\delta^{ss}} \right) = \left( \frac{\delta_{t-1}}{\delta^{ss}} \right)^{\rho_\delta} \exp \left( \epsilon_\delta \right)
\]

given this, equation (3.61) takes the following form:

\[
\varphi_t (\tilde{\omega}_t) = (1 - \delta_t) \left( \frac{\tilde{\omega}_t - \tilde{\omega}_t}{\tilde{\omega} - \omega} \right)
\]

The increase in \(\delta_t\) implies that a fraction of existing credit contracts are exogenously terminated due to the decline \(\varphi_t (\tilde{\omega}_t)\). There will be a larger mass of firms searching for funds, \(f_t\) as well as a larger mass of banks searching for profitable projects to fund \(b_t^i\) (see figure (3.6)). Free entry of banks to the loan market imply that a frac-
tion of banks that were previously engaged in a loan contract will decide to exit the market while the other fraction will stay in the loan market but searching for potential borrowers. Firms that were previously engaged in a credit contract are not able to exit the loan market, implying that all firms separated due to the initial shock will start searching for external funding. Then, at impact, the new mass of firms searching for lenders will exceed the new mass of banks searching for borrowers inducing a persistent raise in credit market tightness $\tau_t$. These new credit conditions in the loan market are exhibited by a decline in the firm’s finding rate $p^f_t$ and a raise in the bank’s finding rate $p^b_t$. The financial shock is propagated through an intensive and extensive margin effects

Figure 3.6: Model responses to a financial shock: A

Note: Source: Authors’ calculations based model simulation

with opposite effects over the employment decision of active firms. Our calibration implies a stronger employment effect of the intensive margin relative to the corresponding
extensive margin. On one hand, the mass of firms and banks that start the period in a credit contract but also survive the higher separation rate that occurs after the financial shock, will decide to raise their reservation productivity threshold $\tilde{\omega}_t$ as a response to a decline in the joint surplus to a credit relationship and a tighter credit market (see figure (3.7)). This is an extensive margin effect, associated to a selection effect that reduces the subset of firms able to obtain external funds, hire workers and produce. On the other hand, the financial shock reduces the real marginal cost of labor inducing surviving firms to hire more workers than before. Figure (3.10) shows that the net effect over employment is slightly positive since the negative response of $MC_t$ overpowers the increase in $\tau_t$ (see figure (3.7)). We suspect that the latter occurs due to the presence of wage flexibility. Despite the small increase in employment, aggregate loans fall since tighter credit conditions reduces the joint surplus to a credit relationship which in turn is reflected in less bank intermediation and a higher average loan rate spread (average credit spread). In addition, the response of gross credit flows are also in line with the tightening conditions in the loan market: The rate of credit creation falls while the credit destruction rate raises as a consequence of the financial shock.

The shock in the loan market is transmitted to the interbank market via a portfolio reconfiguration that banks perform as a response to the financial shock. Banks realize that the marginal value of collateral $\chi_t$ suddenly falls after the financial shock, inducing a higher demand for government bonds at the expense of excess reserves and loans. The lower demand for excess reserves together with a fixed nominal supply of high powered money balances reduce the interbank rate more than the consequent
reduction in the bond rate. In summary, the financial shock that started in the loan market affects also the interbank market by inducing banks switch their portfolios in favor of government bonds and less excess reserves as well as loans (see figure (3.8) and (3.9)).

Since government bonds are assumed to be in fixed net supply, households will reduce their demand for government bonds but also their bank deposits. The latter occurs because the spread between the deposit rate and the bond rate falls as well. Therefore, banks end up the period with fewer funds which in turn reduce lending even more (see figure (3.8)).

At the aggregate level, a persistent adverse financial shock generates a negative response on GDP and consumption but a slightly improvement in aggregate employ-
Figure 3.8: Model responses to a financial shock: C

Note: Source: Authors’ calculations based model simulation

The response to a financial shock is illustrated in Figure 3.8. A financial shock, modeled as an exogenous persistent increase in the separation rate of credit contracts, generates a deep and prolonged recession in terms of GDP and consumption together with a higher average credit spread and a lower marginal value of collateral. Therefore, the shock that was originated in the loan market affects the interbank market as a fall in aggregate excess reserves. As in the case of the payment shock, the transmission and propagation of a financial shock is through a persistent decline in the aggregate "credit" input, $F_t$, that affects the effective aggregate production function (see figure (3.10)).
Figure 3.9: Model responses to a financial shock: D

Note: Source: Authors’ calculations based model simulation

3.4 Conclusions

In this paper, we study the links between the central bank’s operating procedures in the interbank market, the availability of credit, and the impact of monetary policy on the real economy. To do so, we integrate two branches of the literature by incorporating a channel model of the reserve market with a credit market characterized by matching frictions and bilateral bargaining between lenders (banks) and borrowers (firms) The resulting general equilibrium framework was used to investigate the effects of alternative operating procedures on the economy’s response to a variety of shocks.

We incorporate a search-and-matching process between borrowers (firms) and lenders (banks). The financial contract and the credit interest rate are an outcome of
Figure 3.10: Model responses to a financial shock: E

Note: Source: Authors’ calculations based model simulation

a Nash bargaining, and idiosyncratic shocks to the entrepreneur’s productivity level determine the rate of endogenous match destruction. We extend the search and matching model of credit frictions to incorporating a second stage where banks operate in a centralized bond and interbank market. The nature of the interbank market – the central bank’s use of a corridor system – affects the bargaining process involving banks and firms and affects directly the equilibrium interest rate on loans. Thus, the model consists of an interbank market that involves banks and the monetary authority, and a loan market in which banks and firms participate. Banks need to meet their need for settlement balances in the interbank market and, besides interbank lending, banks can deposit excess reserves at the central bank or borrow reserves through a standing facility. The structure of the interbank market affects the lending decisions of banks in the loan
market, and the resulting spread between the average lending rate and the central bank’s policy rate depends on this matching process, the nature of Nash bargaining, and structure of the interbank market and monetary policy operating procedures. A further contribution of the present paper pertains to the cost channel of monetary policy in which, because firms must finance wage payments in advance of production, the relevant cost of labor is affected by the interest rate firms pay on loans. However, when the loan rate is the outcome of a bargaining process, its role is to split the surplus between the borrower (the firm) and the lender (the bank). It is irrelevant for the firm’s employment decision which is made to maximize the joint surplus. There is still a cost channel but it depends on the opportunity cost of funds to the bank, not the interest rate charged on the loan, and therefore it too is dependent on the structure of the interbank market. Changes in the policy interest rate, the penalty for borrowing reserves from the central bank, the interest rate paid on reserve deposits at the central bank, the supply of bank reserves by the central bank, and the volatility of settlement payment flows all influence this outside opportunity and therefore affect the equilibrium spread between the average rate on bank loans and the policy interest rate.

The transmission mechanism of two different financial shocks has been studied. The first shock is originated in the interbank market and consists of an increase in the volatility of the payment shock that each bank faces once the interbank market is closed. The second type of financial shock is originated in the loan market as an exogenous increase in the termination rate of credit contracts. The impulse responses presented show that both shocks are transmitted through the interaction of three main
variables: 1) The marginal value of having collateral in the form of excess reserves relative to other collateralizable assets that are not affected by haircuts, 2) The real marginal cost of labor and 3) The reservation productivity threshold of a loan contract. The last two variables generate intensive and extensive margin effects of financial shocks over employment decisions and are a direct consequence of the way the joint surplus to a credit relationship responds. In addition, both shocks generate deep and prolonged recessions that are characterized by sharp drops in an inefficiency wedge that appears in the aggregate effective production function of the economy (the "credit input" term: $F_t$). Financial shocks are amplified by the implied dynamics of this term, producing inefficient responses on the aggregate equilibrium of the economy.
Chapter 4

A comparative assessment of labor productivity during the Great Recession and the early 2000s recession: How choosier have employers become?

- with Christopher Limnios -

4.1 Introduction

One potential hypothesis explaining the so-called “jobless” recovery following the Great Recession is the increase in the productivity of labor in new matches formed and those which survived [McGrattan and Prescott (2012), Schaal (2011), Mulligan (2011)]. A natural place to begin the search for evidence of a jobless recovery is Okun’s

162
law, the long-standing statistical relationship between the unemployment gap and the output gap\(^1\), since according to this statistical “rule-of-thumb”, a 2% increase in output corresponds to a 1% decrease in the unemployment rate; hence, we may call a recovery in output following a recession jobless if it is not met with the corresponding fall in the unemployment rate in accordance with Okun’s law.

In order to get a sense for this empirical relationship, we regress the unemployment gap on the output gap. We construct the series for the unemployment gap by taking the difference in the quarterly civilian unemployment rate (FRED series UNEMPLOY) and the natural rate of unemployment (FRED series NROU) while we construct the series for the output gap by taking the log-deviation of quarterly real GDP (FRED series GDPC1) from its HP filtered trend component (with filtering parameter \(\lambda = 1600\)) and multiply the result by 100 to convert it to percent. Using U.S. data from 1949Q1 through 2014Q4, the resulting regression is

\[
\hat{U}_t = -0.608 \hat{y}_t
\]

\[T = 264 \quad \hat{R}^2 = 0.359 \quad F(1, 263) = 146.99 \quad \hat{\sigma} = 1.288\]

(standard errors in parentheses)

where \(\hat{y}_t\) is the output gap and \(\hat{U}_t\) is the unemployment gap defined earlier. The interpretation of this estimation is that a one percent decrease in output from trend (quarterly) would be associated with an \(\approx 0.61\) percentage point increase in the unemployment rate

\(^1\)The unemployment gap is defined as the difference between the actual unemployment rate and its natural level, while the output gap is defined as the percentage difference between the economy’s current level of output and its potential level.
above the natural rate.

Of particular interest is in observing the relative stability of the statistical relationship over time, since a break in the statistical relationship congruent with a “jobless recovery” of the Great Recession would manifest itself as a deviation of the estimation given in the above regression. Figure 1 illustrates the time series for the Okun’s Law regression coefficient resulting from the deployment of a “rolling regression” using a sliding window of 100 observations.

Figure 4.1: The time path of the regression coefficient for Okun’s law resulting by “rolling” a sliding interval of 100 observations. 95% confidence bands included.

The first data point corresponds to the regression coefficient resulting from the first 100 observations (1949Q1 - 1973Q4; this window allows for 25 years worth of data per estimation point); the window is then advanced to the next quarter and the estimation is repeated until the last observation is used. Also included is the 95% confidence interval around this estimation point (coeff ±1.96×std error). The figure demonstrates the relative stability of Okun’s Law up until 2003. What follows is a fairly
abrupt fall in the time path (low value of approximately -0.75 during 2006Q3) followed by a large rise in the coefficient (high value of approximately -0.36 during 2008Q4). The abrupt rise in the regression coefficient implies that the relationship has adjusted to one where a given increase in the output gap is now associated with a smaller decrease in the unemployment gap, providing support for the existence of a jobless recovery.

Using the estimate from the Okun’s law regression, the residual $\epsilon$ is defined by

$$\hat{\epsilon}_t = \hat{U}_t + 0.608 \hat{y}_t.$$

Figure 2 illustrates the time series of the residual\(^2\) from the regression for Okun’s Law.

Figure 4.2: The time path for the residuals from the Okun’s Law regression. Note the pattern of residual trajectories during all of the NBER recessions and how the pattern was reversed during the Great Recession.

Even though the simple Okun’s Law regression specified above actually fails the Bruesch-Godfrey autocorrelation test\(^3\), the residual series from the regression reveals

\(^2\)This series is constructed by taking the actual data for $\hat{U}$ and $\hat{y}$ and feeding it into the definition of $\hat{\epsilon}$ to get the residuals.

\(^3\)A regression of the residual upon one lag of itself (an autoregression) reveals a fairly statistically
a relatively interesting and potentially important observation; of the 10 NBER recession prior to the Great Recession, the pattern of the residual is to fall during the recession and to rise abruptly during the recovery. As figure 2 shows, this pattern is perfectly reversed (and of a relatively larger degree) during the Great Recession. The interpretation would be that before the Great Recession, the typical timing of events during an economic recovery would be a recovery in labor, closely followed by a recovery in output - a pattern which reversed itself during the Great Recession.

Figure 3 illustrates the distribution of this residual during the recovery of the Early 2000s recession.

Figure 4.3: **Source:** Authors regression residuals. Data series for the civilian unemployment rate and the long-term and short-term natural rate of unemployment come from FRED. GDP data comes from FRED, while potential GDP is the Hodrick-Prescott filtered series of the same GDP data with standard $\lambda = 1600$.

As expected, during an economic recovery from the trough of an output gap significant relationship. Authors have corrected this by incorporating lags of the independent variable into the regression; see Ball et al. (2012), for example.
through the point where $y = y^{potential}$, the mass of the residuals of the Okun regression should occupy the negative portion of the support, as during this portion of the business cycle, $\hat{y} < 0$ and $\hat{U} > 0$. Figure 4 shows the same depiction for the Great Recession.

![Okun's law residuals during the recovery of the Great Recession (2009-05 - present)](image)

Figure 4.4: **Source:** Same as figure 2.

In contrast with the Early 2000s recession, the Okun’s residuals for the (ongoing) recovery of the Great Recession are vastly positive, contradicting what we would expect from an economic “recovery”. Figure 4 shows evidence of $\hat{U}_t > -0.608 \bar{y}_t$, which offers some support to the idea that the recovery from the Great Recession has been a “jobless” one, as $\hat{U} > 0$ during this period.

We assess the validity of the theory that, consistent with a jobless recovery, existing and new employment relationships experienced a surge in productivity using a modified version of the Mortensen-Pissarides framework. Specifically, we formulate a method to quantitatively assess the productivity “cutoff”\(^4\) of firm-worker matches.

\(^4\)The “cutoff” level of productivity refers to the theoretical level where the joint surplus to the match
This allows us to compare the level of productivity of firm-employee matches during the recovery of the 2008 financial crisis in relation to the recovery of the early 2000s recession.

The two results of this chapter are:

1. During the Great Recession the productivity “spread” was 104% greater than that of the Early 2000s recession. This supports the theory of a higher level of productivity in new and existing matches contributing towards the jobless recovery.

2. Allowing the job maintenance cost to fluctuate endogenously, we show that the implied series exhibits a “pulse” during both the Great Recession and early 2000s recession; during the Great Recession the series exhibited a jump of 15% while during the early 2000s recession the jump was 5%; we interpret this difference to be largely the result of the financial turmoil during the Great Recession and subsequent increase in the difficulty of many firms to secure financing. Our implied job maintenance cost series shares a correlation coefficient of 0.52 with the St. Louis financial stress index, which we see as lending credence to our interpretation of this series as encapsulating (relative) employment financing costs.

We construct the model of the labor market in section 2 and bring the model to the data in section 3.

---

5“Spread” refers to the difference between the highest and lowest theoretical value for productivity during the specific recession.

6In the DMP model, the job maintenance cost is a cost paid by employers to “maintain a job”. We interpret this series to not only reflect the costs of recruiting a worker, but also the implicit costs facing an employer of financing their labor.
4.2 Model of the labor market

We assume that the market is populated by workers and firms. Firms endoge-
nously decide to enter the labor market by choosing to post job openings/vacancies. Both populations are either matched or not. Once a worker matches with a firm, the worker draws a productivity $\pi$ from an unspecified distribution $G$ which has direct implications for the productivity of the match. We assume that $G$ has support $[\bar{\pi}, \pi]$, where the productivity cut-off $\bar{\pi}$ marks the lower bound of a match with non-negative joint surplus$^7$. If the worker is currently employed but draws a productivity which is below the cutoff, the match terminates itself. At that point, the firm decides to endogenously enter the labor market and search again for an employee. The (now) unmatched laborer costlessly awaits its next probable match. In this framework, the cutoff productivity level $\bar{\pi}$ is endogenously determined.

4.2.1 Value functions

Workers and firms are either matched or unmatched, where each state has a corresponding value function. For each agent, their individual economic surplus is the difference in the value of being matched over unmatched. Once the agents match with each other, the joint surplus of the match is the sum of the individual surpluses. The productivity cutoff delineates a level of productivity which would result in a positive joint surplus.

$^7$Each agent's economic surplus is the difference in value between being matched and unmatched. The joint surplus to a match is the sum of the firm and laborer's individual surpluses.
4.2.1.1 The firm value functions

The value of a firm which is matched (annotated $FM$) with a worker which drew productivity above the cutoff is given by

$$
\forall_{FM}(\pi_t, \pi \geq \tilde{\pi}) = y_t - w(\pi_t) + \beta E_t \int_{\tilde{\pi}}^{\pi} [\forall_{FM}(\pi_{t+1}) + \forall_{FS}^{FS}] \, dG(\pi_{t+1}),
$$

where $w(\pi_t)$ is the wage, $\beta$ is the discount parameter, and we have assumed that the output of a match is multiplicatively linear in the product of the aggregate level of labor productivity $z_t$ and the worker’s idiosyncratic productivity level $\pi_t$ so that

$$
y_t = z_t \pi_t.
$$

This function expresses that the value of being a matched firm is the output net of costs plus the expected value of being a firm in the following period, which is made up of the expected value of continuing as a matched firm $V^{FM}$, or having the match endogenously separate and becoming a searching firm $V^{FS}$, contingent on the productivity level of the match in the following period.

The value function of a firm which has posted a job vacancy and is thus searching for a worker (annotated $FS$) is given by

$$
\forall_t^{FS} = -v + p_t^F \beta \int_{\pi}^{\infty} [\forall_{FM}(\pi_{t+1}) + \forall_{FS}^{FS}] \, dG(\pi_{t+1}) + (1 - p_t^F) \beta \forall_{t+1}^{FS},
$$

where $v$ is the cost of posting the vacancy and $p_t^F$ is the probability a firm matches with an unemployed searching worker. This function states that a firm pays $v$ to post a vacancy and with probability $p_t^F$ matches with an unemployed worker. The following
period, a productivity draw is made and if this probability is above the cutoff, a pro-
ductive match is established. If the productivity draw is below the cutoff, the match
terminates itself. With probability \(1 - p^F_t\), the firm doesn’t match with an unemployed
worker and is once again a searching firm the following period.

As the firm is endogenously entering the market for employment by choosing
whether or not to post a job vacancy, the free entry condition is the result of assuming
that enough firms would enter such that the market would be saturated to the point
that the value of being a searching firm is zero. Imposing this on the value function for
searching firms results in

\[
\frac{\text{entry condition}}{v = p^F_t \beta \int_{\bar{\pi}}^{\bar{\pi}} \mathbb{V}^{FM}(\pi_{t+1})dG(\pi_{t+1})}.
\]

The left-hand side of this expression is the per-period cost of the search and the right-
hand side is the (probabilistic) expected benefit of the search.

4.2.1.2 The worker value functions

The value function of a worker which is matched with a firm (annotated \(WM\))
is given by

\[
\mathbb{V}^{WM}(\pi_t, \pi \geq \bar{\pi}) = w(\pi_t) + \beta \int_{\bar{\pi}}^{\bar{\pi}} [\mathbb{V}^{WM}(\pi_{t+1}) + \mathbb{V}^{WU}_{t+1}] dG(\pi_{t+1}),
\]

This function expresses that the value of being an employed worker is the wage plus the
expected value of continuing on as a worker in the following period \(V^{WM}\), or having
the productivity drawn in the following period fall short of the cutoff, resulting in an
endogenous match termination, and thus unemployment, which carries value \(V^{WU}\).
The value of an unemployed worker currently searching for a firm (annotated \( WU \)) is given by

\[
\mathcal{V}_{t}^{WU} = b + p_{t}^{W} \beta \int_{\pi}^{\pi} \left[ \mathcal{V}_{t+1}^{WM}(\pi_{t+1}) + \mathcal{V}_{t+1}^{WU} \right] dG(\pi_{t+1}) + (1 - p_{t}^{W}) \beta \mathcal{V}_{t+1}^{WU},
\]

where \( b \) is the unemployment benefit a searching worker receives, and \( p_{t}^{W} \) is the probability that an unemployed worker meets a firm. This equation states that the value of being an unemployed worker is the unemployment benefit a searching worker receives, plus the probability of matching and the expected value of that match - contingent on the productivity draw - plus the expected value of not finding a match and starting the following period as an unemployed worker.

### 4.2.1.3 Firm and worker surpluses

We denote the firm’s surplus as \( \mathcal{V}_{t}^{FS}(\pi_{t}, \pi_{\geq \tilde{\pi}}) \) and it is simply the difference in the firm’s value functions, so that

\[
\mathcal{V}_{t}^{FS}(\pi_{t}, \pi_{\geq \tilde{\pi}}) = y_{t} - w(\pi_{t}) + \beta \int_{\pi}^{\pi} \mathcal{V}_{t+1}^{F}(\pi_{t+1}) dG(\pi_{t+1}) + v - \\
-p_{t}^{F} \beta \int_{\pi}^{\pi} \mathcal{V}_{t+1}^{F}(\pi_{t+1}) dG(\pi_{t+1}) - (1 - p_{t}^{F}) \beta \mathcal{V}_{t+1}^{FS}
\]

\[
= y_{t} - w(\pi_{t}) + v + (1 - p_{t}^{F}) \beta \int_{\pi}^{\pi} \mathcal{V}_{t+1}^{F}(\pi_{t+1}) dG(\pi_{t+1}) - (1 - p_{t}^{F}) \beta \mathcal{V}_{t+1}^{FS}
\]

\[
= y_{t} - w(\pi_{t}) + v + (1 - p_{t}^{F}) \beta \int_{\pi}^{\pi} [\mathcal{V}_{t+1}^{F}(\pi_{t+1}) - \mathcal{V}_{t+1}^{FS}] dG(\pi_{t+1}).
\]

Incorporating the firm’s entry condition and the formulation for output \( y_{t} \) into this last expression gives

\[
\mathcal{V}_{t}^{FS}(\pi_{t}, \pi_{\geq \tilde{\pi}}) = \pi_{t} \pi_{t} - w(\pi_{t}) + \beta \int_{\pi_{t+1}}^{\pi} \mathcal{V}_{t+1}^{FS}(\pi_{t+1}) dG(\pi_{t+1}).
\]
Finally, the entry condition can be used once again to arrive at

\[ V^{FS}(\pi_t, \pi_t, \pi_t \geq \tilde{\pi}) = z_t \pi_t - w(\pi_t) + \frac{v}{p_t}. \]  

(firm surplus)

Likewise, the worker’s surplus is denoted as \( V^{WS}(\pi_t, \pi_t, \pi_t \geq \tilde{\pi}) \) and is also defined as the difference in the worker’s value functions, so that

\[ V^{WS}(\pi_t, \pi_t, \pi_t \geq \tilde{\pi}) = w(\pi_t) - b - p_t \beta \int_{\pi_t}^{\pi_{t+1}} V^{WS}(\pi_t, \pi_t, \pi_t + 1) dG(\pi_{t+1}) - (1 - p_t) \beta V^{WS}(\pi_t + 1). \]  

(worker surplus)

4.2.1.4 The surplus sharing rule

In accordance with Nash bargaining, the joint surplus to the match is redistributed to both counterparties via the equilibrium wage in accordance with each counterparty’s relative bargaining weight. Specifically, let \( \eta \) represent the bargaining power of the firm and \( (1 - \eta) \) represent the bargaining power of the worker. The optimization problem is

\[ \max_{w^*} \left( V^{FS} \right)^{\eta} \left( V^{WS} \right)^{(1-\eta)} \]

The first order condition for this problem is

\[ \eta \left( V^{FS} \right)^{\eta-1} \frac{\partial V^{FS}}{\partial w} \left( V^{WS} \right)^{(1-\eta)} + (1 - \eta) \left( V^{FS} \right)^{\eta} \left( V^{WS} \right)^{(1-\eta)} \frac{\partial V^{WS}}{\partial w} = 0. \]
Rearranging this expression and substituting in the partial derivatives leads to the traditional sharing rule

\[(1 - \eta)V^F = \eta V^W.\] (sharing rule)

### 4.2.1.5 The productivity cutoff

The cutoff productivity level \( \tilde{\pi} \) is defined as the point where the joint surplus associated with the match is equal to zero. The joint surplus to an employment match is defined as the sum of the worker and firm surpluses and is written

\[V^J = V^W + V^F.\]

\[= z_t \pi_t - w(\pi_t) + \frac{v}{p^t} + \frac{w(\pi_t) - b + (1 - p^t_W)\beta \int_{\pi_t}^{\pi} [V^W(\pi_{t+1}) - V^W_{t+1}]dG(\pi_{t+1})}{p^t} \]

\[= z_t \pi_t - b + \frac{v}{p^t} + (1 - p^t_W)\beta \int_{\pi_t}^{\pi} [V^W(\pi_{t+1}) - V^W_{t+1}]dG(\pi_{t+1}).\]

The sharing rule can be used to write this expression as

\[V^J = z_t \pi_t - b + \frac{v}{p^t} + (1 - p^t_W)\beta \int_{\pi}^{\pi} \left( \frac{1 - \eta}{\eta} \right) [V^F(\pi_{t+1}) - V^F_{t+1}]dG(\pi_{t+1}).\]

Incorporating the entry condition for the firm results in

\[V^J = z_t \pi_t - b + \frac{v}{p^t} + (1 - p^t_W)\beta \int_{\pi_t}^{\pi} V^F(\pi_{t+1})dG(\pi_{t+1})\]

\[= z_t \pi_t - b + \frac{v}{p^t} + (1 - p^t_W)\beta \int_{\pi_t}^{\pi} V^F(\pi_{t+1})dG(\pi_{t+1})\]

\[= z_t \pi_t - b + \left[ 1 + (1 - \eta) \right] \left( \frac{v}{p^t} \right)\]

\[= z_t \pi_t - b + \left( \frac{\eta + 1 - \eta - p^t_W + \eta p^t_W}{\eta} \right) \left( \frac{v}{p^t} \right)\]

\[= z_t \pi_t - b + \left[ 1 - (1 - \eta)p^t_W \right] \left( \frac{v}{p^t} \right).\] (joint surplus)
Setting this equal to zero to isolate the cutoff productivity, we get

$$\forall J's \: = \: z_t \tilde{\pi}_t - b + \left[ \frac{1 - (1 - \eta)p_t W}{\eta} \right] \left( \frac{v}{p_t F} \right) = 0$$

$$z_t \tilde{\pi}_t = b - \left[ \frac{1 - (1 - \eta)p_t W}{\eta} \right] \left( \frac{v}{p_t F} \right)$$

or

$$\tilde{\pi}_t = \frac{b \eta p_t F - v \left[ 1 - (1 - \eta)p_t W \right]}{z_t \eta p_t F}$$  (productivity cutoff)

### 4.3 Empirical implementation

The empirical approach we take entails implementing standard parameter values used throughout the labor search literature along with the pertinent time series data into the productivity cutoff equation in order to construct a series for the idiosyncratic productivity cutoff which is consistent with the variation of the DMP model we have used. We use the resulting model and data-consistent series for the idiosyncratic productivity as a proxy for the level of employer “choosiness”. It is of particular interest to us to see how the dynamics of this implied series has evolved during both the early 2000s recession and the Great Recession in answering our research question.

The probability of a firm matching with a searching worker $p_t F$ can be written as

$$p_t F = \frac{m_t}{V_t},$$

where $m_t$ are the numbers of matches and $V_t$ are the number of open job vacancies.
(firms) searching for a worker. Assuming the same specification for $p_t^W$,

$$\frac{p_t^W}{p_t^F} = \frac{m_t}{m_t} \frac{V_t}{U_t} = \tau_t,$$

where $\tau$ represents tightness in the labor market.

Following the vast majority of the literature, we assume that the matching function takes on the Cobb-Douglas structure in vacancies and unemployed and is written

$$m_t = \psi U_t^{\alpha} V_t^{1-\alpha},$$

where $\psi$ and $\alpha$ are parameters. This functional form then implies that

$$p_t^F = \frac{m_t}{V_t} = \frac{\psi U_t^{\alpha} V_t^{1-\alpha}}{V_t} = \psi U_t^{\alpha} V_t^{1-\alpha} = \psi \left(\frac{V_t}{U_t}\right)^{-\alpha} = \psi \tau_t^{-\alpha}$$

$$p_t^W = \frac{m_t}{U_t} = \frac{\psi U_t^{\alpha} V_t^{1-\alpha}}{U_t} = \psi U_t^{\alpha-1} V_t^{1-\alpha} = \psi \left(\frac{V_t}{U_t}\right)^{1-\alpha} = \psi \tau_t^{1-\alpha}.$$

Substituting these probabilities into the productivity-cutoff equation results in

$$\bar{\pi}_t = \frac{b \eta \psi \tau_t^{-\alpha} - v \left[1 - (1 - \eta)\psi \tau_t^{1-\alpha}\right]}{\bar{s}_t \eta \psi \tau_t^{-\alpha}}.$$

### 4.3.1 Parameterization

Unemployment benefits are set to $b = 0.54 \bar{w}$, where $\bar{w}$ represents steady-state wages. The proportion of wages 0.54 is sourced from a weighted average of unemployment benefit calculators for various states in the U.S. The job posting/vacancy costs $v$ follow Petrosky-Nadeu and Wasmer (2013) and Silva and Toledo (2007) who estimate this as the cost to recruiting a worker and thus $v \sim 3.6$ percent of the wage rate; hence
we set $v = 0.036 \frac{\bar{w}}{\bar{U}}$ where $U$ and $\tau$ are the steady-state unemployment rate$^8$, and labor market tightness, respectively.

The steady-state wage $\bar{w}$ is set equal to aggregate nominal compensation as a fraction of aggregate output, or 0.438. The steady-state unemployment rate $U$ and labor market tightness $\tau$ are set to their empirical equivalents, and thus $U = 0.054$ and $\tau = 0.478$. This implies the unemployment benefit $b = 0.54(0.438) = 0.237$ and vacancy posting cost is $v = 0.036 \frac{0.443}{0.478 \cdot 0.054} = 0.618$.

We follow Walsh (2005) and Blanchard & Diamond (1989) and set $\alpha = 0.4$. Following Den Haan et al. (2000) and Petrosky-Nadeau and Wasmer (2013), based on the estimates for the U.S., we target a job filling rate $p^F = 0.4$. This then implies that the level parameter in the matching function $\psi = 0.4(0.478)^{0.4} = 0.298$. Finally, we follow the vast majority of the literature and set the bargaining weight $\eta$ symmetrically to 0.50.

Finally, for the labor market tightness, we use the ratio of total nonfarm job openings to total unemployed$^9$ and for the series for aggregate labor productivity, we use the ULC total labor productivity series$^{10}$.

---

$^8$Assuming a unit mass labor force implies that the unemployment rate is equivalent to the unemployment level.

$^9$FRED series JTSJOL/UNEMPLOY which are Job Openings: Total Nonfarm, 1000s (Level), Monthly, S.A. divided by Unemployed, 1000s, Monthly, S.A..

$^{10}$FRED series ULQELP01USQ661S which is ULC Indicators: Total Labor Productivity for the United States, Quarterly, Seasonally Adjusted.
4.3.2 Results

Incorporating the parameters and the empirical steady-state values into the productivity cutoff equation results in an expression we can use to extract the model and data-consistent time series for the idiosyncratic productivity level of the labor matches formed from December of 2000 through December of 2014

\[
\bar{\pi}_t = \frac{0.036 \tau_t^{-0.4} - 0.618 \left[ 1 - 0.149 \tau_t^{0.6} \right]}{z_t 0.149 \tau_t^{-0.4}}.
\]

The time series for \( \bar{\pi} \) is displayed in figure 5 below.

![Figure 4.5: Time series of the model-implied level of \( \bar{\pi} \).](image)

One thing to notice which is very much in line with the jobless recovery hypothesis of an increase in the productivity of existing matches following the onset of the recession. The other is the numerical degree of difference in \( \bar{\pi} \) during the latest recession and that of the late 2000s. At their most extreme (for the data available) the following
are some numbers:

\[ \tilde{\pi}_{2001:3} = 0.124 \]

\[ \tilde{\pi}_{2001:1} = 0.143, \]

where consistent with the NBER bands for the early 2000s recession, March of 2001 was commencement and November of 2001 was termination. The equivalent values for the Great recession include

\[ \tilde{\pi}_{2007:12} = 0.134 \]

\[ \tilde{\pi}_{2009:06} = 0.175 \]

The “spread” of the cutoff in the early 2000s recession is 0.124 - 0.143 = 0.019 and the spread in the latest recession is 0.175 - 0.134 = 0.041. The difference is suggestive of a more than doubling of the cutoff. More specifically,

\[
\frac{\text{late 2000s spread} - \text{early 2000s spread}}{\text{early 2000s spread}} = \frac{0.041 - 0.019}{0.019} = 1.158
\]

or a 116\% difference in the idiosyncratic productivity of existing and newly-created matches of the latest recession compared to the early 2000s recession.

### 4.3.3 Explanation of the results: What has changed?

The productivity cutoff equation is an equilibrium condition, and thus if we are to arrive at a convincing answer as to how the underlying behavior of agents changed during both recessions, we need to isolate what piece of this equation changed and to what magnitude. While the early 2000s recession had the dot-com bubble collapse in
combination with the September 11th terrorist attacks as its originating factors, the Great Recession was largely brought on by a collapse in the housing market followed by deep financial turmoil and panic in credit markets, making borrowing/seeking finance an especially difficult endeavor. The differences in their origins provides us with a clue as to what may have contributed to the more than doubling of the implied idiosyncratic productivity cutoff during the Great Recession.

Typically, small businesses finance their labor costs using some form of revolving credit. Thus, if credit markets are disrupted on par with a financial panic, we would expect to see the implicit costs of financing labor increase by a measurable degree. Turning to our variation of the DMP model, we can test this hypothesis by analyzing how the vacancy cost parameter would behave if it were allowed to vary with time.

The equilibrium wage is the solution to the Nash sharing rule and is given by

\[ w_t = \eta b + (1 - \eta) \left[ y_t + v_t \left( \frac{p^W}{p^F} \right) \right], \]

where we have allowed the job opening cost to vary with time. The intuition behind this equation is that the equilibrium wage is a weighted sum of the unemployment benefit \( b \) and the sum of output and (weighted) savings the match provides the firm; for example if \( v \) is the cost of the match, and \( p^W \) were to increase relative to \( p^F \), this implies that securing a future match would be more expensive for the firm and thus the current match the firm is in becomes more valuable, implicitly. The linear weights reflect the bargaining power of the employee and firm.
This equation can be solved for $y$ to get

$$y_t = \frac{w_t - \eta b}{1 - \eta} - v_t \left( \frac{p_t^W}{p_t^F} \right).$$

We can use this expression to eliminate $y$ from the productivity cutoff to get

$$\frac{w_t - \eta b}{1 - \eta} - v_t \left( \frac{p_t^W}{p_t^F} \right) = b - \left[ \frac{1 - (1 - \eta)p_t^W}{\eta} \right] v_t \left( \frac{p_t^W}{p_t^F} \right),$$

or, more compactly,

$$v_t = \left( \frac{w_t - b}{1 - \eta} \right) \left( \frac{\eta p_t^F}{p_t^W - 1} \right).$$

(job vacancy cost)

Finally, log-linearizing this equation about its steady-state results in

$$\hat{v}_t = \left( \frac{w}{w - b} \right) \hat{w}_t - \left[ \alpha + \left( \frac{p_t^W}{p_t^W - 1} \right) (1 - \alpha) \right] \hat{\tau}_t.$$

We can now incorporate the same parameter and steady-state values from earlier along with the appropriately transformed empirical data on the wage and labor market tightness to extract a model and data-consistent time series for the implied labor financing costs facing firms.

We assume that the data for the equilibrium wage comes from the ratio of\textsuperscript{11} the aggregate compensation of all employees (wages and salaries, in billions) to total GDP (billions). We take the constructed wage series, apply a HP filter to it with smoothing parameter\textsuperscript{12} $\lambda = 14,400$, and then take the log-difference between the actual series and the smoothed series to isolate $\hat{w}$. A similar procedure\textsuperscript{13} is used in isolating $\hat{\tau}$.

\textsuperscript{11}Total compensation comes from the FRED series A576RC1Q027SBEA Compensation of employees: Wages and salaries, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate and GDP comes from the FRED series GDP Gross Domestic Product, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate. We assume that the equilibrium wage is the ratio of the two time series.

\textsuperscript{12}While we used a smoothing parameter of $\lambda = 1600$ earlier when we regressed Okun’s law, that was because we were using quarterly data. We are now at the monthly frequency and have thus updated our smoothing parameter appropriately.

\textsuperscript{13}This is following Shimer (2005).
Incorporating the parameter and steady-state values into the job vacancy cost equation gives

\[ \hat{v}_t = (2.179)\hat{w}_t - (0.258)\hat{\tau}_t. \]

The implied series is given in figure 6 below.

![Figure 4.6: Implied series (in percentage deviation) for the firm’s implicit cost to posting a vacancy.](image)

We can draw three conclusions from observing figure 6. First, outside of recessions, the cost to posting jobs seems (relatively) stable and that during recessions, this parameter exhibits behavior which parallels a regime shift. Second, the cost facing firms to financing a job opening increased during the onset of the Great Recession to a max of 15% at its conclusion. This is a value 3 times as great as the increase in the implied vacancy posting cost during the early 2000s recession. Last, figure 6 exhibits a striking similarity with the many different “financial stress” indices provided by various Federal Reserve banks. For example, figure 7 displays the financial stress index constructed by the St. Louis Fed laid upon our series for v-hat, which may offer some credence to the
linkage between the increase in the idiosyncratic productivity level of new labor matches resulting from increased “scrutiny” brought on by (increasingly) risk-averse firms facing heightened levels of financial stress negatively impacting their ability to finance labor costs. The two series share a correlation coefficient of 0.52 ($p$-value of 0.000 under the null of no correlation). If we lag the St. Louis financial stress index by 5 months, the correlation coefficient jumps to 0.75 ($p$-value of 0.000 under the null of no correlation).

4.4 Conclusion

Using a simple variation of the canonical DMP model of the labor market, we have illustrated two primary results. The first is that consistent with the model and empirical data, the idiosyncratic threshold productivity levels for the surviving and newly-created employment matches increased to a much greater degree (slightly more than double) during the Great Recession in comparison to the early 2000s recession.
The second is that the increase in the “scrutiny” imposed by hiring firms is largely the result of the increase in risk aversion originating from the large disruptions to credit markets during the wake of the Great Recession.

This chapter can be extended in a couple of ways in order to provide more support for our hypothesis. First, the aggregate labor productivity series used can be normalized using TFP data in order to control for fluctuations in capital intensity. Second, it would be worth exploring various ways to distinguish between a shift in the vacancy posting cost $v$ and movements in the level parameter $\psi$ of the matching function. We leave these extensions for future work.
Bibliography


189


Appendix A

Appendix to chapter 2
Table A.1: Descriptive statistics for credit and employment

<table>
<thead>
<tr>
<th>Period</th>
<th>1973Q1-2012Q4</th>
<th>1984Q1-2007Q2</th>
<th>2007q3-2009q2</th>
<th>2009q3-2012q4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>SD</td>
<td>Mean</td>
<td>SD</td>
</tr>
<tr>
<td>Total loans creation</td>
<td>3.78</td>
<td>1.27</td>
<td>4.04</td>
<td>1.13</td>
</tr>
<tr>
<td>Total loans destruction</td>
<td>1.59</td>
<td>0.96</td>
<td>1.92</td>
<td>0.73</td>
</tr>
<tr>
<td>Sum total lending</td>
<td>5.37</td>
<td>1.44</td>
<td>5.95</td>
<td>1.10</td>
</tr>
<tr>
<td>Net total lending</td>
<td>2.19</td>
<td>1.74</td>
<td>2.12</td>
<td>1.55</td>
</tr>
<tr>
<td>Exc total lending</td>
<td>2.98</td>
<td>1.65</td>
<td>3.75</td>
<td>1.37</td>
</tr>
<tr>
<td>Unemployment rate</td>
<td>6.45</td>
<td>1.60</td>
<td>5.71</td>
<td>1.05</td>
</tr>
<tr>
<td>Average labor productivity</td>
<td>0.39</td>
<td>0.61</td>
<td>0.45</td>
<td>0.55</td>
</tr>
<tr>
<td>Job creation</td>
<td>4.83</td>
<td>0.84</td>
<td>4.76</td>
<td>0.57</td>
</tr>
<tr>
<td>Job destruction</td>
<td>5.24</td>
<td>1.07</td>
<td>5.14</td>
<td>0.69</td>
</tr>
<tr>
<td>Sum JC + JD</td>
<td>10.08</td>
<td>1.54</td>
<td>9.91</td>
<td>1.01</td>
</tr>
<tr>
<td>Net JC-JD</td>
<td>-0.41</td>
<td>1.17</td>
<td>-0.39</td>
<td>0.77</td>
</tr>
<tr>
<td>Exc job creation</td>
<td>9.24</td>
<td>1.42</td>
<td>9.30</td>
<td>1.01</td>
</tr>
</tbody>
</table>

Note: Lending data is based on Reports of Income and condition. Job flows data are taken from Faberman (2009) and updated with the 2011 Business Employment Dynamics Survey. The unemployment rate is seasonally adjusted and downloaded from the FRED data repository at the Federal Reserve, St Louis. Average Labor productivity is calculated from real GDP (seasonally adjusted) and hours of non-farm business employees. These data are also downloaded from FRED.
Table A.2: Standard Deviations and Correlations

<table>
<thead>
<tr>
<th>Period</th>
<th>1973q1-2012q4</th>
<th>1984Q1-2007Q2</th>
<th>2007q3-2009q2</th>
<th>2009q3-2012q4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma/\sigma^{rgdp}$</td>
<td>$\rho(., rgdp)$</td>
<td>$\sigma/\sigma^{rgdp}$</td>
<td>$\rho(., rgdp)$</td>
</tr>
<tr>
<td>Total loan creation</td>
<td>20.48</td>
<td><strong>0.311</strong></td>
<td>27.91</td>
<td>0.06</td>
</tr>
<tr>
<td>Total loan destruction</td>
<td>27.24</td>
<td>-0.46</td>
<td>32.47</td>
<td><strong>-0.28</strong></td>
</tr>
<tr>
<td>Sum total lending</td>
<td>13.50</td>
<td>0.09</td>
<td>18.32</td>
<td>-0.16</td>
</tr>
</tbody>
</table>

|                              | $\sigma/\sigma^{unemp}$ | $\rho(., unemp)$ | $\sigma/\sigma^{unemp}$ | $\rho(., unemp)$ | $\sigma/\sigma^{unemp}$ | $\rho(., unemp)$ | $\sigma/\sigma^{unemp}$ | $\rho(., unemp)$ |
| Job creation                 | 0.64           | 0.11          | 0.62           | -0.05          | 0.70           | -0.80          | 0.45           | **0.66**       |
| Job destruction              | 1.11           | **0.41**      | 0.97           | **0.32**       | 1.48           | **0.99**       | 0.67           | 0.04           |
| Sum JC + JD                  | 0.53           | **0.56**      | 0.50           | **0.32**       | 0.75           | **0.98**       | 0.32           | 0.38           |
| Real GDP                     | 0.02           | –             | 0.01           | –              | 0.022          | –              | 0.01           | –              |
| Unemployment                 | 0.12           | –             | 0.10           | –              | 0.19           | –              | 0.15           | –              |
| Number of Observations       | 160            | 94            | 8              | 14             | 20              | 14             | 20             | 14             |

Note: Lending data is compiled from the Reports of Income and Condition. Job flows data is taken from Faberman (2009) and updated with Business Employment Dynamics survey. Standard deviations and correlations are calculated using the cyclical component of the HP-filtered log-level of the variable. Boldface numbers indicate statistical significance at the 5% confidence level or better. Unemployment and RGDP are the HP filtered cyclical components of the log series of the unemployment rate and seasonally adjusted real GDP respectively.
A.0.1 Summarizing the non-linear equilibrium conditions

The system of non-linear equations that characterize the aggregate equilibrium of the model economy is:

- **Monetary policy:**

  \[ m_t = \left( \frac{1 + \theta_t}{\Pi_t} \right) m_{t-1} \]  

  where

  \[ \theta_t - \theta^{ss} = \rho (\theta_{t-1} - \theta^{ss}) + \xi_t \]

- **Euler equation:**

  \[ \frac{1}{C_t} = \beta E_t \left\{ \left( \frac{R_t}{\Pi_{t+1}} \right) \frac{1}{C_{t+1}} \right\} \]  

- **CIA constraint:**

  \[ C_t = m_t - R_t c_t \]

- **Wage setting equation:**

  \[ f_{1,t} = \left( \frac{w^*_t}{w_t} \right)^{\epsilon_w(1+\varphi)} \left( \frac{N_t}{\Delta_t} \right)^{(1+\varphi)} + (\beta \theta_w) E_t \left( \frac{1}{\Pi_{t+1}} \right) \left( \frac{w^*_t}{w^*_{t+1}} \right)^{-\epsilon_w(1+\varphi)} \]  

  \[ f_{2,t} = \chi_t \left( \frac{w^*_t}{w_t} \right)^{-\epsilon_w (1+\varphi)} \left( \frac{N_t}{\Delta_t} \right)^{(1+\varphi)} \]  

  \[ \beta \theta_w E_t \left( \frac{1}{\Pi_{t+1}} \right) \left( \frac{w^*_t}{w^*_{t+1}} \right)^{-\epsilon_w (1+\varphi)} \]  

  \[ f_{2,t+1} \]
• Aggregate wage index in real terms:

\[ 1 = \theta_w \left( \frac{w_{t-1}}{w_t} \frac{1}{\Pi_t} \right)^{1-\epsilon_w} + (1 - \theta_w) \left( \frac{w_t^*}{w_t} \right)^{1-\epsilon_w} \]  

(D7)

• Wage dispersion:

\[ \Delta^w_t = \theta_w \left( \frac{w_{t-1}}{w_t} \frac{1}{\Pi_t} \right)^{-\epsilon_w} \Delta^w_{t-1} + (1 - \theta_w) \left( \frac{w_t^*}{w_t} \right)^{-\epsilon_w} \]  

(D8)

• Price setting equation:

\[ g_{1,t} = \beta C_t \Pi_t^* + \theta_p E_t \left( \frac{\Pi_t^*}{\Pi_{t+1}^*} \right) g_{1,t+1} \]  

(D9)

\[ g_{2,t} = \beta \frac{1}{\mu_t} C_t + \theta_p E_t g_{2,t+1} \]  

(D10)

\[ g_{1,t} = \left( \frac{\epsilon_p}{\epsilon_p - 1} \right) g_{2,t} \]  

(D11)

• Aggregate price index in terms of inflation rates:

\[ 1 = \theta_p \left( \frac{1}{\Pi_t} \right)^{1-\epsilon_p} + (1 - \theta_p) (\Pi_t^*)^{1-\epsilon_p} \]  

(D12)

• Price dispersion:

\[ \Delta^p_t = \theta_p \left( \frac{1}{\Pi_t} \right)^{-\epsilon_p} \Delta^p_{t-1} + (1 - \theta_p) (\Pi_t^*)^{-\epsilon_p} \]  

(D13)

• Unemployment:

\[ U_t = 1 - \frac{N_t}{L_t} \]  

(D14)

• Aggregate labor supply:

\[ w_t = R_t C_t \chi_t (L_t)^\psi \]  

(D15)
• Resource constraint:
\[ \Delta_t^p C_t = Y_t^f \] (D16)

• Aggregate final good:
\[ Y_t^f = Y_t^I - \left( b_t^p \kappa + \varphi_t (\tilde{\omega}_t) f_{t-1}^m x_t^f \right) \] (D17)

• Aggregate bank’s balance sheet:
\[ l_t + er_t + \xi^{bs} = d_t \] (D18)

• Consistency of aggregate loans and deposits:
\[ l_t = \varphi_t (\tilde{\omega}_t) f_{t-1}^m d_t \] (D19)

• Aggregate equilibrium in the loan market:
\[ l_t = \frac{w_t N_t}{\Delta_t^w} \] (D20)

• Aggregate production function for the intermediate good sector:
\[ Y_t^I = A_t \xi^p (F_t)^{1-\alpha} \left( \frac{N_t}{\Delta_t^w} \right)^\alpha \] (D21)

• Aggregate employment:
\[ N_t = \left( \frac{\alpha A_t \xi^p}{\mu_t^p w_t R_t} \right)^{\frac{1}{1-\alpha}} F_t \Delta_t^w \] (D22)

• Credit friction input (credit miss-allocation "input") \( F_t \):
\[ F_t = (1 - \delta) \left( \frac{(\tilde{\omega})^k - (\tilde{\omega}_t)^k}{k (\tilde{\omega} - \omega)} \right) f_{t-1}^m \] (D23)
• Credit market tightness:

\[ \tau_t = \frac{f_t}{b_t} \]  

(D24)

• Measure of firms in a credit relationship:

\[ f_t^m = \varphi_t (\tilde{\omega}_t) f_{t-1}^m + p^f_t f_t \]  

(D25)

• Measure of firms searching for credit:

\[ f_t = 1 - (1 - \delta) f_{t-1}^m \]  

(D26)

• Overall continuation rate for a credit contract:

\[ \varphi_t (\tilde{\omega}_t) = (1 - \delta) \left( \frac{\omega - \tilde{\omega}_t}{\omega - \omega} \right) \]  

(D27)

• Reservation productivity:

\[ \left[ \alpha^\alpha (1 - \alpha)^{1-\alpha} A t^\xi w^\rho_m \gamma_t \right]^{1/\alpha} = (\mu_t^p w_t R_t)^{\alpha/\alpha} \left[ \left( \frac{1 - \eta p^f_t}{1 - \eta} \right) \frac{\kappa}{p_t^b} \right] \]  

(D28)

• Evolution of credit market tightness:

\[ \frac{\kappa}{\mu_t^p p_t^b} - \beta_t \Delta_{t,t+1} \frac{1}{\mu_t^p} \left( 1 - \theta p_t^f \right) \frac{\kappa}{\mu_{t+1}^p p_{t+1}^b} \]  

\[ (1 - \eta) \beta_t \Delta_{t,t+1} \frac{1}{\mu_t^p} \left( 1 - \alpha \frac{Y_{t+1}^f}{f_t^m} - \varphi_{t+1} (\tilde{\omega}_{t+1}) x^f \right) \]  

(D29)

• Stochastic discount factor:

\[ \Delta_{t,t+1} = \beta \left( \frac{R_t}{R_{t+1}} \frac{C_t}{C_{t+1}} \right) \]  

(D30)
• Matching rate for firms:

\[ p^f_t = \mu \tau_t^{\nu-1} \]  \hspace{1cm} (D31)

• Matching rate for banks:

\[ p^b_t = \mu \tau_t^\nu \]  \hspace{1cm} (D32)

• Gross real interest rate:

\[ 1 + r_t = \frac{R_t}{E_t \Pi_{t+1}} \]  \hspace{1cm} (D33)

• Credit destruction rate:

\[ cd_t = 1 - \varphi_t (\tilde{\psi}_t) - p^b_t \delta \]  \hspace{1cm} (D34)

• Credit creation rate:

\[ cc_t = \frac{m_t}{f^m_{t-1}} - p^f_t \delta \]  \hspace{1cm} (D35)

• Labor share of GDP:

\[ LS_t = \frac{w_t N_t}{Y^f_t} \]  \hspace{1cm} (D36)

• Fixed cost of production share of GDP:

\[ FCS_t = \varphi_t (\tilde{\psi}_t) \frac{f^m_{t-1} x^f}{Y^f_t} \]  \hspace{1cm} (D37)

• Definition of aggregate loans as a fraction of deposits:

\[ \hat{l}_t = \frac{l}{d} \]  \hspace{1cm} (D38)

• Definition of aggregate excess reserves as a fraction of deposits:

\[ \hat{er}_t = \frac{er}{d} \]  \hspace{1cm} (D39)
A.0.2 Introducing absconding and credit rationing into the loan contract

In this section we consider the possibility that borrowers may abscond with the funds obtained from banks. Given this, banks introduce an incentive compatibility constraint in the loan contract such that the value of the net surplus for an intermediate good producer in a credit contract is greater or equal to the value of absconding. We assume, that in the case of absconding the firm is able to produce and generate profits but does not repay the bank. The bank is able to recover an exogenous fraction $\theta_t$ of the profits made by the intermediate good producer whom is not allowed to participate in the loan market anymore. Therefore, if the producer decide to abscond, then it will obtain

$$1 - \theta_t \left( \frac{y_t(\omega_{z,t}) - x_f}{\mu_t} - \tilde{l}_t(j,\omega_{z,t}) \right)$$

where $\tilde{l}_t(j,\omega_{z,t}) = w_t N_t(\omega_{z,t})$ denotes the loan extended by bank $j$ to producer $z$ in this new context. The optimal credit contract in this scenario is characterized by credit rationing. A bank will prefer to loan only a fraction $q_t(j,\omega_{z,t})$ of its deposits and park the rest of it as excess reserves at the central bank.

Banks’ balance sheet

Under the possibility that the borrower may abscond with the funds obtained from a loan, the balance sheet for bank $j$ is

$$\chi_t(j) \left( \tilde{l}_t(j,\omega_{z,t}) + \frac{ER_t(j)}{P_t} \right) + (1 - \chi_t(j)) \frac{ER_t(j)}{P_t} = \frac{D_t(j)}{P_t}$$
such that if $\chi_t(j) = 1$ the bank extends a loan to an intermediate good producer characterized by $\omega_{z,t}$ and

$$\tilde{l}_t(j,\omega_{z,t}) = q_t(j,\omega_{z,t}) \frac{D_t(j)}{P_t}$$

and

$$\frac{ER_t(j)}{P_t} = (1 - q_t(j,\omega_{z,t})) \frac{D_t(j)}{P_t}$$

with $q_t(j,\omega_{z,t}) \leq 1$. On the other hand, if $\chi_t(j) = 0$, the bank sets $\frac{ER_t(j)}{P_t} = \frac{D_t(j)}{P_t}$.

Notice that if $q_t(j,\omega_{z,t}) = 1$ then we are back to the standard case since whenever $\chi_t(j) = 1$ then $l_t(j,\omega_{z,t}) = \frac{D_t(j)}{P_t}$ and $\frac{ER_t(j)}{P_t} = 0$. The possibility of credit rationing means that if the bank extends a loan, then it will not use all its available funds, that is $\tilde{l}_t(j,\omega_{z,t}) \leq \frac{D_t(j)}{P_t}$. Under this circumstances, individual bank profits expressed in real terms are given by

$$\pi^b_t(j) = \chi_t(j) \left( R_t^l(j,\omega_{z,t}) \tilde{l}_t(j,\omega_{z,t}) + R_t^e \frac{ER_t(j)}{P_t} \right) + (1 - \chi_t(j)) \left( R_t^e \frac{ER_t(j)}{P_t} - \frac{\kappa}{\mu_t^p} \right) - R_t^d \frac{D_t(j)}{P_t}$$

Substituting out the term $(1 - \chi_t(j)) \frac{ER_t(j)}{P_t}$ from bank $j$ balance sheet into $\pi^b_t(j)$ yields

$$\pi^b_t(j) = \left( R_t^l(j,\omega_{z,t}) - R_t^e \right) \chi_t(j) \tilde{l}_t(j,\omega_{z,t}) - (1 - \chi_t(j)) \frac{\kappa}{\mu_t^p} + \left( R_t^e - R_t^d \right) \frac{D_t(j)}{P_t}$$

As before, optimality with respect to deposits requires $(R_t^e - R_t^d) D_t(j) = 0$ every period. Since household deposits are always positive in equilibrium, the bank will choose...
to collect deposits until the gross interest rate on excess reserves is equal to the gross interest rate on deposits, that is $R^r_t = R^d_t = R_t$. Substituting the optimality condition with respect to $D_t(j)$ into the profit function yields the flow value of a bank

$$\pi^b_t(j) = \left( R^d_t(j, \omega_{z,t}) - R_t \right) \chi_t(j) \tilde{l}_t(j, \omega_{z,t}) - (1 - \chi_t(j)) \frac{\kappa}{\mu^p_t}$$

Bank profits can be written as

$$\pi^b_t(j) = \begin{cases} \pi^b_t(j, \omega_i) = \left( R^d_t(j, \omega_{z,t}) - R_t \right) \chi_t(j) \tilde{l}_t(j, \omega_{z,t}) & \text{if extends a loan to firm } \omega_z \\ -\frac{\kappa}{\mu^p_t} & \text{otherwise} \end{cases}$$

The net surplus to a bank of being in an active credit contract is given by

$$V_{tBS}(\omega_{z,t}) = \pi^b_t(j, \omega_i) + \frac{\kappa}{\mu^p_t \mu^d_t}$$

where $\frac{\kappa}{\mu^p_t \mu^d_t}$ is equal to the continuation value due to free entry of banks. The surplus to a firm is

$$V_{tFS}(\omega_{z,t}) = \pi^f_t(\omega_{z,t}) + E_t \Delta_{t,t+1} \left( 1 - p^f_t \right) \varphi_t(\tilde{\omega}_{t+1}) \int_{\tilde{\omega}_{t+1}} V_{t+1FS}(\omega_{z,t+1}) \frac{dG(\omega)}{1 - \gamma_{t+1}}$$

where

$$\pi^f_t(\omega_{z,t}) = \frac{y_t(\omega_{z,t}) - x^f_t}{\mu^p_t} - R^d_t(j, \omega_{z,t}) \tilde{l}_t(j, \omega_{z,t})$$

A.0.3 The loan contract

If the borrower has the possibility of absconding, the lender will modify the loan contract to ensure that absconding does not arise in equilibrium. The loan contract will be such that the borrower have incentives to abide the terms of the contract and repay the principal and interest rate to the lender at the end of the period. We assume
the borrower is able to produce in the case of absconding, the contract must offer the household at least a fraction of the profits it obtains when not repaying to the bank. The loan contract solves the following Nash bargaining problem:

$$\max \{ R_l^t(j, \omega, z, t), \tilde{l}_t(j, \omega, z, t) \} (V_t^{FS}(\omega, z, t))^\eta (V_t^{BS}(\omega, z, t))^{1-\eta}$$

subject to the following incentive compatibility constraint

$$V_t^{FS}(\omega, z, t) \geq (1 - \theta_t) \left( \frac{yt(\omega, z, t) - x^f}{\mu_t^P} - \tilde{l}_t(j, \omega, z, t) \right)$$

where

$$\tilde{l}_t(j, \omega, z, t) = q_t(j, \omega, z, t) \frac{D_t(j)}{P_t}$$

The first order condition with respect to $R_l^t(j, \omega, z, t)$ yields the following sharing rule for the surplus generated in a credit relationship:

$$(1 - \eta) \left( \frac{V_t^{FS}(\omega, z, t)}{V_t^{BS}(\omega, z, t)} \right)^\eta \left( \frac{V_t^{BS}(\omega, z, t)}{V_t^{FS}(\omega, z, t)} \right)^{1-\eta} + \lambda_t(\omega, z, t)$$

where $\lambda_t(\omega, z, t)$ is the Lagrange multiplier associated with the incentive compatibility constraint. If the constraint is not binding, $\lambda_t(\omega, z, t) = 0$, we obtain the standard sharing rule for a Nash bargaining problem: $(1 - \eta) V_t^{FS}(\omega, z, t) = \eta V_t^{BS}(\omega, z, t)$ as in the case with no possibility of absconding. Let $MPL_t(\omega, z, t)$ denote the marginal product of labor, $MPL_t(\omega, z, t) = \alpha A_t w_t N_t^*(\omega, z, t)^{\alpha-1}$, and recall the loan extended by bank $j$ is used by the intermediate producer to cover its labor costs $\tilde{l}_t(j, \omega, z, t) = w_t N_t^*(\omega, z, t)$. Then, the first order condition with respect to $\tilde{l}_t(j, \omega, z, t)$ implies:

$$\eta \left( \frac{V_t^{BS}(\omega, z, t)}{V_t^{FS}(\omega, z, t)} \right)^{1-\eta} \left( \frac{MPL_t(\omega, z, t)}{\mu_t^P w_t} - R_l^t(j, \omega, z, t) \right) + (1 - \eta) \left( \frac{V_t^{FS}(\omega, z, t)}{V_t^{BS}(\omega, z, t)} \right)^\eta \left( (R_l^t(j, \omega, z, t) - R_t) \right)$$

$$= \lambda_t(\omega, z, t) \left[ (1 - \theta_t) \left( \frac{MPL_t(\omega, z, t)}{\mu_t^P w_t} - 1 \right) - \left( \frac{MPL_t(\omega, z, t)}{\mu_t^P w_t} - R_l^t(j, \omega, z, t) \right) \right]$$
Notice that the term \( \frac{MPL_t(z,w,t)}{\mu^p_t w_t} - R_t(j,\omega,z,t) \) is the marginal profit for an intermediate producer that decide to finish the project and not abscond. By the same token, the term \( R_t(j,\omega,z,t) - R_t \) is the marginal benefit for a bank that extends a loan to a non absconding producer. Those two marginal profits are weighted by \( \eta \) \((\frac{V^BS_t(\omega,z,t)}{V^{FS}_t(\omega,z,t)})^{1-\eta} \) and \((1-\eta) \) \((\frac{V^FS_t(\omega,z,t)}{V^{BS}_t(\omega,z,t)})^{\eta} \). If the bank knows with certainty that the borrower will always pay back the loan, then \( \lambda_t(j,\omega,z,t) = 0 \) and the first order condition with respect to \( \tilde{I}_t(j,\omega,z,t) \) would imply \( MPL_t(j,\omega,z,t) = \mu^p_t w_t R_t \). If the bank knows that there is a possibility that the borrower will abscond, the weighted average of marginal profits do not cancel out as in the standard case but are equal to the marginal opportunity cost of absconding which is given by the term \((1-\theta_t) \left( \frac{MPL_t(\omega,z,t)}{\mu^p_t w_t} - 1 \right) - \left( \frac{MPL_t(\omega,z,t)}{\mu^p_t w_t} - R_t(j,\omega,z,t) \right) \).

Combining the first order conditions yields an equation for \( \lambda_t(\omega,z,t) \) given by:

\[
\lambda_t(\omega,z,t) = (1-\eta) \left( \frac{V^{FS}_t(\omega,z,t)}{V^{BS}_t(\omega,z,t)} \right)^{\eta} \left( \frac{MPL_t(\omega,z,t)}{\mu^p_t w_t} - R_t \right) \frac{MPL_t(\omega,z,t)}{\mu^p_t w_t} - 1 \)
\]

Notice that since \( \lambda_t(\omega,z,t) \geq 0 \) then it must be the case that \( \frac{MPL_t(\omega,z,t)}{\mu^p_t w_t} \geq R_t \) and \( \frac{MPL_t(\omega,z,t)}{\mu^p_t w_t} \geq 1 \). If \( \frac{MPL_t(\omega,z,t)}{\mu^p_t w_t} \leq R_t \) the firm and the bank are not willing to participate in the credit relationship. Substituting the solution for \( \lambda_t(\omega,z,t) \) into the sharing rule yields the optimal hiring condition for an intermediate good producer in a credit relationship:

\[
(1-\eta) V^{FS}_t(\omega,z,t) \left( R_t - 1 - \theta_t \left( \frac{MPL_t(\omega,z,t)}{\mu^p_t w_t} - 1 \right) \right) = \eta V^{BS}_t(\omega,z,t) (1-\theta_t) \left( \frac{MPL_t(\omega,z,t)}{\mu^p_t w_t} - 1 \right)
\]

The above equation clearly shows that when \( (1-\eta) V^{FS}_t(\omega,z,t) = \eta V^{BS}_t(\omega,z,t) \) which is equivalent to having \( \lambda_t(\omega,z,t) = 0 \), the intermediate good producer will hire workers up to the point where \( MPL_t(\omega,z,t) = \mu^p_t w_t R_t \). When \( \lambda_t(\omega,z,t) > 0 \) then it must be the case that \( MPL_t(\omega,z,t) > \mu^p_t w_t R_t \) and the intermediate good producer will hire an inefficiently
low number of workers due to the existence of equilibrium credit rationing. In order to focus on the implications of credit rationing, we assume that the incentive compatibility constraint is always binding, that is

\[ V_{FS}^t(\omega_{z,t}) = (1 - \theta_t) \left( \frac{y_t(\omega_{z,t}) - x^f}{\mu_t^p} - w_t N_t(\omega_{z,t}) \right) \]

since \( V_{BS}^t(\omega_{z,t}) = \pi_b^t(j,\omega_i) + \kappa \mu_t^p p_t^b \) and \( \pi_b^t(j,\omega_i) = (R_l^t(j,\omega_{z,t}) - R_t) w_t N_t(j,\omega_{z,t}) \) then the ratio \( \frac{V_{FS}^t(\omega_{z,t})}{V_{BS}^t(\omega_{z,t})} \) is given by

\[ \frac{V_{FS}^t(\omega_{z,t})}{V_{BS}^t(\omega_{z,t})} = (1 - \theta_t) \left( \frac{y_t(\omega_{z,t}) - x^f}{\mu_t^p} - w_t N_t(j,\omega_{z,t}) \right) \frac{R_t}{R_l^t(j,\omega_{z,t}) - R_t} w_t N_t(j,\omega_{z,t}) + \frac{\kappa}{\mu_t^p p_t^b} \]

substituting \( \frac{V_{FS}^t(\omega_{z,t})}{V_{BS}^t(\omega_{z,t})} \) into the optimal sharing rule yields the equilibrium loan interest rate equation:

\[ \left( R_l^t(j,\omega_{z,t}) - R_t \right) \tilde{t}_t(j,\omega_{z,t}) = \left( \frac{1 - \eta}{\eta} \right) \left( \frac{y_t(\omega_{z,t}) - x^f}{\mu_t^p} - w_t N_t(\omega_{z,t}) \right) \left( \frac{R_l - 1 - \theta_t \left( \frac{MPL_t(\omega_{z,t})}{\mu_t^p w_t} - 1 \right)}{ MPL_t(\omega_{z,t}) \mu_t^p w_t - 1 \right) - \kappa \frac{\mu_t^p p_t^b}{\mu_t^p p_t^b} \]

The joint surplus of a credit relationship under the possibility of absconding is

\[ V_{JS}^t(\omega_{z,t}) = (1 - \theta_t) \left( \frac{y_t(\omega_{z,t}) - x^f}{\mu_t^p} - w_t N_t(\omega_{z,t}) \right) + \left( R_l^t(j,\omega_{z,t}) - R_t \right) w_t N_t(\omega_{z,t}) + \frac{\kappa}{\mu_t^p p_t^b} \]

The optimal level of employment is obtained by maximizing the joint surplus of a credit relationship with respect to \( N_t(\omega_{z,t}) : \)

\[ MPL_t(\omega_{z,t}) = \mu_t^p w_t + \mu_t^p w_t R_t \frac{R_l^t(j,\omega_{z,t})}{1 - \theta_t} - \mu_t^p w_t R_t \frac{R_l^t(j,\omega_{z,t})}{1 - \theta_t} \]

and the cut-off productivity level solves \( V_{JS}^t(\tilde{\omega}_{z,t}) = 0 \) which implies:

\[ 0 = (1 - \theta_t) \left( \frac{y_t(\tilde{\omega}_{z,t}) - x^f}{\mu_t^p} - w_t N_t(\tilde{\omega}_{z,t}) \right) + \left( R_l^t(j,\tilde{\omega}_{z,t}) - R_t \right) w_t N_t(\tilde{\omega}_{z,t}) + \frac{\kappa}{\mu_t^p p_t^b} \]
This extension of the credit contract has several implications. First, the loan interest rate $R^t_l (j, \tilde{\omega}_{z,t})$ affects the joint surplus of a credit relationship and the cut-off $\tilde{\omega}_{z,t}$. This occurs only when the incentive compatibility constraint is binding since the net surplus that a borrower obtains from an active credit match has to be equal to its absconding value. By definition, the absconding value for an intermediate good producer does not take into account $R^t_l (j, \tilde{\omega}_{z,t})$ which implies that the joint surplus will depend on it. Under this assumptions is more difficult to compute the optimal equilibrium values $y^*_t (\omega_{z,t})$, $N^*_t (\omega_{z,t})$ and $\tilde{l}_t (j, \omega_{z,t})$ since we need to solve a non linear system of equations.
Appendix B

Appendix to chapter 3

B.0.1 Characterization of the aggregate non-linear equilibrium

Lower case letters denote real variables $x = \frac{X}{P}$. Variables written as $\tilde{X}$ are expressed as fraction of deposits, that is $\tilde{X} = \frac{X}{D} = \frac{x}{d}$

We assume that household’s period utility function is

$$U(C_t, 1 - N_t) = \frac{C_{t}^{1-\sigma}}{1 - \sigma} - \Theta \frac{N_{t}^{1+\eta}}{1 + \eta}$$

The aggregate equilibrium of the model is characterized by the following dynamical system of non-linear equations:

1. **Nominal supply of reserves:**

   $$HP_t^s = (1 + \theta_t) HP_{t-1}^s$$  \hspace{1cm} (D1.1)

   where $\theta_t$ is given by

   $$\left( \frac{\theta_t}{\theta_{ss}} \right) = \left( \frac{\theta_{t-1}}{\theta_{ss}} \right)^{\rho \theta} \exp \left( \epsilon_t^\theta \right)$$  \hspace{1cm} (D1.2)
2. Policy rate:

\[ i_t = i_t^* \quad \text{(D2)} \]

3. Gross inflation:

\[ 1 + \pi_t = \frac{P_t}{P_{t-1}} \quad \text{(D3)} \]

4. Euler equation:

\[ \lambda_t = \beta E_t \left( \frac{1 + i_{t+1}^b}{1 + \pi_{t+1}} \lambda_{t+1} \right) \quad \text{(D4)} \]

5. Marginal utility of income:

\[ \lambda_t = \frac{C_t^{-\sigma}}{1 + i_t^b - i_t^d} \quad \text{(D5)} \]

6. The marginal value of collateral:

\[ \chi_t = \frac{i_t - i_t^b}{\xi_b} \quad \text{(D6)} \]

7. Interest rate on deposits:

\[ i_t^d = i_t (1 - \rho) + (i_t^* - s_t) \rho - x^d - \chi_t \bar{\varepsilon} + \frac{1}{2} s_t \left( i_t^* - i_t + \chi_t \right)^2 - \frac{s_t \bar{\varepsilon}_t^2}{2} \]

using the definition of \( h^* \) we have

\[ i_t^d = i_t (1 - \rho) + (i_t^* - s_t) \rho - x^d - \chi_t \bar{\varepsilon} + \frac{1}{2} s_t \left( h^* \right)^2 - \frac{s_t \bar{\varepsilon}_t^2}{2} \quad \text{(D7)} \]

8. Interbank market equilibrium: In real terms

\[ hp_t^* = (h_t^* + \rho) d_t \quad \text{(D8)} \]
where

\[ hp_t^s = \frac{HP_t^s}{P_t} \]  

(D8.1)

as a fraction of deposits:

\[ \hat{hp}_t = h_t^* + \rho \]

where

\[ \hat{hp}_t = \frac{hp_t^s}{d_t} \]

or in nominal terms:

\[ HP_t^s = h_t^* D_t + \rho D_t \]

9. The demand for excess reserves:

\[ h_t^* = \frac{\bar{\varepsilon}_t}{s} (i_t^* - i_t + \chi_t) \]  

(D9)

notice that \( h_t^* \) is expressed as a fraction of deposits and that \( \bar{\varepsilon}_t \) follows the following process:

\[ \left( \frac{\bar{\varepsilon}_t}{\bar{\varepsilon}} \right) = \left( \frac{\bar{\varepsilon}_{t-1}}{\bar{\varepsilon}} \right)^\rho \exp \left( \varepsilon_t^\delta \right) \]

10. Consolidated gov. budget constraint: In real terms

\[ T_t + \left( hp_t^s - \left( \frac{1}{1 + \pi_t} \right) hp_{t-1}^s \right) = x_t + f_t \]  

(D10)

where \( f_t^p \) is treated as exogenous and given by

\[ f_t = G_t + \left( b_t^c - \left( \frac{1}{1 + \pi_t} \right) b_{t-1}^c - i_t b_t^c \right) - \left( b_t^T - \left( \frac{1}{1 + \pi_t} \right) b_{t-1}^T - i_t b_t^T \right) \]

As a fraction of deposits:

\[ \frac{T_t}{d_t} + \left( \frac{hp_t^s}{d_t} - \left( \frac{1}{1 + \pi_t} \right) \frac{hp_{t-1}^s d_{t-1}}{d_t} \right) = \frac{x_t}{d_t} + \frac{f_t}{d_t} \]
or

\[ \hat{T}_t + \left( \hat{h}_t - \frac{1}{1 + \pi_t} \right) \hat{h}_t \frac{d_{t-1}}{d_t} = \hat{x}_t + \hat{f}_t^g \]

11. Central bank’s net payment of interest on reserves:

\[ x_t = (i_t^s - s_t) (r_d + \epsilon r_t) + (i_t^s + s_t) br_t \quad (D11) \]

as a fraction of deposits

\[ \hat{x}_t = (i_t^s - s) (r + \epsilon r_t) + (i_t^s + s) \hat{br}_t \]

12. Aggregate excess reserves in the banking system:

\[ er_t = \left( \frac{(h_t^s)^2}{4\bar{\epsilon}_t} + \frac{h_t^s}{2} + \frac{\bar{\epsilon}_t}{4} \right) d_t \quad (D12) \]

as a fraction of deposits

\[ \hat{er}_t = \frac{(h_t^s)^2}{4\bar{\epsilon}} + \frac{h_t^s}{2} + \frac{\bar{\epsilon}}{4} \]

13. Aggregate borrowed reserves in the banking system:

\[ br_t = \left( \frac{-(h_t^s)^2}{4\bar{\epsilon}_t} + \frac{h_t^s}{2} - \frac{\bar{\epsilon}_t}{4} \right) d_t \quad (D13) \]

as a fraction of deposits

\[ \hat{br}_t = \frac{-(h_t^s)^2}{4\bar{\epsilon}} + \frac{h_t^s}{2} - \frac{\bar{\epsilon}}{4} \]

14. Aggregate collateral constraint:

\[ h_t^s d_t + \xi b_t^b + \xi_L l_t = \bar{\epsilon}_t d_t + \xi^{bs} \quad (D14) \]

as a fraction of deposits

\[ h_t^s + \xi b_t^b + \xi_L \hat{l}_t = \bar{\epsilon} + \hat{\xi}^{bs} \]
15. **Aggregate banks balance sheet:**

\[ l_t + b_t^b + h_t^* d_t = (1 - \rho) d_t \]  

(D15)

as a fraction of deposits

\[ \hat{l}_t + \hat{b}_t^b + h_t^* = (1 - \rho) \]

16. **The CIA constraint:**

\[ C_t = d_t + w_t N_t + \xi_{cia} \]  

(D16)

17. **Labor supply:**

\[ \Theta N_t^\eta C_t^\sigma = w_t \]  

(D17)

18. **Aggregate loans:**

\[ l_t = w_t N_t \]  

(D18.1)

if interbank market is written as a fraction of deposits then we need the following equation

\[ l_t = \hat{l}_t d_t \]  

(D18.2)

19. **Aggregate resource constraint of the economy:**

\[ Y_t = C_t + G_t + \varphi_t (\bar{\omega}_t) f^m_{t-1} x_f + \kappa b_t^h + x^d d_t \]  

(D19)

where \( G_t \) is exogenous.

20. **Opportunity cost of lending:**

\[ R_t = 1 + i_t - \chi_t \xi_L \]  

(D20)
21. Aggregate employment:

\[ N_t = \frac{\alpha z_t \xi}{w_t R_t} F_t^{\frac{1}{1-\alpha}} \]  
(D21)

22. Aggregate output:

\[ Y_t = z_t \xi (F_t)^{1-\alpha} (N_t)^\alpha \]  
(D22)

23. Credit market tightness:

\[ \tau_t = \frac{f_t}{b_t} \]  
(D23)

24. Number of firms in a credit relationship:

\[ f^m_t = \varphi_t (\bar{o}_t) f^m_{t-1} + p^f f_t \]  
(D24)

25. Number of firms searching for workers:

\[ f_t = 1 - \varphi_t (\bar{o}_t) f^m_{t-1} \]  
(D25)

26. Continuation rate:

\[ \varphi_t (\bar{o}_t) = (1 - \delta_t) \left( \frac{\bar{o} - \bar{o}_t}{\bar{o} - \bar{\omega}} \right) \]  
(D26)

where \( \delta_t \) follows an AR(1) process given by

\[
\begin{pmatrix}
\delta_t \\
\delta_{ss}
\end{pmatrix} = \begin{pmatrix}
\delta_{t-1} \\
\delta_{ss}
\end{pmatrix} \rho_\delta \exp (\epsilon_t)
\]

27. Credit input: \( F_t \):

\[ F_t = (1 - \delta_t) \left( \frac{(\bar{o})^k - (\bar{o}_t)^k}{k (\bar{o} - \bar{\omega})} \right) f^m_{t-1} \]  
(D27)
28. Cut-off productivity level:

\[
\left[ \alpha (1 - \alpha)^{1-\alpha} \xi z_t \tilde{\omega}_t \right]^{1-\alpha} = (MC_t)^{1-\alpha} \left[ x^f - \left( \frac{1 - \eta p_t^f}{1 - \eta} \right) \frac{\kappa}{p_t^f} \right] \tag{D28}
\]

29. Credit market tightness: In terms of \( \tau_t \):

\[
\frac{\kappa}{\mu(\tau_t)\varphi} - E_t \Delta_{t,t+1} \varphi (\tilde{\omega}_{t+1}) \left( 1 - \eta \mu (\tau_{t+1})^{\varphi-1} \right) \frac{\kappa}{\mu(\tau_{t+1})\varphi} = (1 - \eta) E_t \Delta_{t,t+1} \left( (1 - \alpha) \frac{Y_{t+1}}{f_{t+1}^m} - \varphi_{t+1} (\tilde{\omega}_{t+1}) x^f \right)
\]

30. Stochastic discount factor:

\[
\Delta_{t,t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t} \tag{D30}
\]

31. Credit destruction rate:

\[
cd_t = 1 - \varphi_t (\tilde{\omega}_t) - p_t^f \varphi_t (\tilde{\omega}_t) \tag{D31}
\]

32. Credit creation rate:

\[
cc_t = \frac{m_t}{f_{t-1}^m} - p_t^f \varphi_t (\tilde{\omega}_t) \tag{D32}
\]

where the flow of new matches \( m_t \) is

\[
m_t = \mu_t f_t^{\varphi} \left( b_t^{\mu} \right)^{1-\varphi}
\]

then \( cc_t \) can be written as

\[
cc_t = \frac{\mu_t \tau_t^{\varphi} b_t^{\mu}}{f_{t-1}^m} - p_t^f \varphi_t (\tilde{\omega}_t)
\]

33. Matching rate for firms:

\[
p_t^f = \mu (\tau_t)^{\varphi-1} \tag{D33}
\]
34. Matching rate for banks:

\[ p^b_t = \mu (\tau_t)^{\phi} \quad \text{(D34)} \]

35. Gross real interest rate:

\[ 1 + r_t = \frac{1 + i^b_{t+1}}{1 + \pi_{t+1}} \quad \text{(D35)} \]

36. Average credit spread:

\[ \frac{R^l_t l_t - R^l l_t}{l_t} = (1 - \alpha) (1 - \eta) \frac{Y_t}{\varphi_t (\omega_t) f^m_{t-1} l_t} - \left( \frac{(1 - \eta) x^f + \eta \eta^m}{l_t} \right) \quad \text{(D36)} \]

37. Real marginal cost:

\[ MC_t = w_t R_t \quad \text{(D37)} \]