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THE HIGH SENSITIVITY OF THE MAXIMUM LIKELIHOOD ESTIMATOR
METHOD OF TOMOGRAPHIC IMAGE RECONSTRUCTION

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Summary
In recent work we have shown that PET images obtained by the MLE iterative
method of image reconstruction converge towards strongly deteriorated ver­
sions of the original source image. In the present work we show that the
image deterioration is caused by an excessive attempt by the algorithm to
match the projection data with high counts and that we can modulate this
effect. We compare a source image with reconstructions by filtered back­
projection and by the MLE algorithm and show that the MLE images can have
similar noise to the filtered backprojection images at regions of high ac­
tivity and very low noise, comparable to the source image, in regions of
low activity, if the iterative procedure is stopped at an appropriate
point.

Introduction
After the Maximum Likelihood Estimator (MLE) method of image reconstruc­
tion was proposed by Shepp and Vardi\textsuperscript{1} for emission tomography, it has of­
ten been observed that continuation of the iterative process beyond a cer­
tain point results in strong image deterioration. Starting from the origi­
nal activity distribution shown in Fig. 1, we recently calculated images
in a 128 x 128 pixel image plane by a random process\textsuperscript{2}. These images are
called "source images". Using a matrix of detection probabilities calcu­
lated for one 512-detector ring of the ECAT-III instrument at UCLA\textsuperscript{3}, pro­
jection data were obtained again by a random process. The data were then
used as input to the MLE algorithm. By using the same matrix of detection
probabilities in the reconstruction as in the source image generation we
avoided questions regarding accuracy of a particular matrix to define an
instrument accurately. We looked ex­
clusively at the behavior of the MLE
algorithm.

Fig. 1 Initial source distribution
for the images shown in this paper.
The relative intensities are indi­
cated.
In the work of Ref. 2 we were able to confirm the process of image deterioration and study the convergence characteristics of the MLE algorithm. Figure 2 shows the log likelihood for the images obtained at different numbers of iterations for a source image with 2 million counts (2M). The horizontal line indicates the likelihood for the true source image. The conclusions from the previous work can be summarized as: 1) the MLE algorithm actually converges towards an image that maximizes the likelihood that the initial projection data would have come from a source distribution corresponding to the obtained image, 2) the asymptotic maximum likelihood image can be a very deteriorated version of the original source distribution, although the quality of the reconstruction increases as the number of counts in the projection data increases, and 3) the original source image is not a maximum likelihood image for the projection data. Further data, images and discussion are given in Ref. 2.

From the above findings one could conclude that the MLE algorithm does not have much future in medical tomography, since reconstructions converge towards images that could be sufficiently different from the source image to lead to false diagnosis. It is clear, however, that images obtained with a moderate number of iterations appear to be good representations of the source image. We have now quantified the noise in different parts of the reconstructed images and compared them with the Filtered Backprojection (FBP) method. We have understood the causes of image deterioration and controlled it within a certain range. The analysis leads us to the conclusion that the MLE algorithm can yield excellent images with very low noise in regions of low counts if used properly.

Reconstruction Procedure
Based on our understanding of the properties of the MLE algorithm, we felt it would be important to incorporate a method of changing the weight given
to tubes d in the process of maximization. The notation of Shepp and Var-
di1 is used throughout this paper. We call this modified method the
Weighted Likelihood Estimator (WLE). We seek to maximize the function:

\[ WL(\lambda) = WP(n^*|\lambda) = \prod_{d=1}^{D} \left[ e^{-\lambda^*(d) n^*(d) / n^*(d)!} \right]^{s \cdot n^*(d) + t} \]

where \( n^*(d) \) is the number of counts detected in a tube d, and \( \lambda^*(d) \) is the
projection into tube d of the reconstructed image. With \( s = 0 \) and \( t = 1 \),
the function WL is identical to the likelihood function L of Ref. 1. Keeping \( t = 1 \), \( s > 0 \) will give higher weight to those tubes that have
higher number of counts, while making \( s < 0 \) will decrease their weight.
Unlike L of Ref. 1, WL(\lambda) does not have the meaning of the probability to
obtain the projection data \( n^* \) from the image \( \lambda^* \). However, if \( s \) is small,
both L and WL increase monotonically by iteration. The iterative formula
for the maximization of Eq. 1, obtained by a method similar to that of
Ref. 1 is the following:

\[ \lambda_{\text{new}}(b) = \lambda_{\text{old}}(b) \left[ 1 + \sum_{d=1}^{D} \left[ s \cdot n^*(d) + t \right] p(b^d, d) \frac{n^*(d) - 1}{\sum_{b'^d=1}^{B} \lambda_{\text{old}}(b'^d)p(b'^d, d)} \right] \] (2)

Results of Reconstructions
We have used a source image with 2M counts based on the activity distri-
bution shown in Fig. 1. Figure 3a shows a cut through the source image.
Figure 3b shows the FBP results with the Shepp–Logan filter and Figs. 3c,
d and e show the results from the unmodified MLE at 9, 32 and 200 itera-
tions, respectively. We have also carried out reconstructions with the
WLE for values of \( s = 0.0025 \) and \(-0.0015\). It is observed that the onset
of image deterioration in regions of high activity comes early in the
first case and is delayed in the second case. No substantial differences
are observed in regions of low activity.

Evaluation of Results
We have defined two regions, 1 and 2, in the source image representing
high (1.0) and low (0.05) activity regions, respectively, as shown in
Fig. 4. The mean values of the reconstructions and the standard error
from the mean have been calculated in each zone. Figures 5a and b show
error plots for the two regions. In region 1 we observe that the error
for the FBP is a factor of 2 higher than the source image error, normal for that method of reconstruction. For the WLE method, we see a substantial influence of the parameter s on the iteration number at which the std. error is equal to that of the FBP method. In region 2, the error of the FBP is ~ 0.05, of the same magnitude as the signal, while the WLE results remain under 0.01 (near the source noise) up to iterations 40 to 60, depending weakly on parameter s. Even at iteration 200, with \( s = 0 \) or -0.0015 the std. error remains under 0.02, with a marked superiority over the FBP method.

Discussions and Conclusions
The behavior of the WLE reconstructions when the parameter s is changed indicates that the progressive deterioration of the images is due to an attempt by the algorithm to match excessively well projection data with high number of counts. We have made the observation earlier\(^2\) that the MLE algorithm gains more likelihood by matching projection data with low number of counts than with high counts. We are now finding that, due to the imperfect nature of the count limited projection data, the MLE method still tries too hard to match regions of high activity and yields unacceptable images if allowed to iterate without limits. Considering that it is possible to obtain images

Fig. 3 Cuts through source and reconstructed images. a) source image with 2 million counts. b) reconstruction by filtered backprojection, Shepp-Logan filter. c) reconstruction by maximum likelihood estimate, 9 iterations. d) ditto, 32 iterations. e) ditto, 200 iterations.
with the MLE that have similar noise as the FBP in regions of high activity, and much lower noise in regions of low activity when the iterations are stopped at an appropriate point, it appears that it would be fruitful to define a criterion for iteration stopping based on statistical considerations. We are continuing work in that direction.

Fig. 4 Source distribution showing Regions 1 and 2 for noise evaluation.

Fig. 5 Plots of standard error from the mean as a function of iteration number for different values of parameters in Eqs. 1 and 2. a) for region 1, with high counts (1.0). b) for region 2, with low counts (0.05).

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