Title
Nutrition and Risk Sharing within the Household

Permalink
https://escholarship.org/uc/item/2td8z1g3

Authors
Dubois, Pierre
Ligon, Ethan A.

Publication Date
2010-01-25
NUTRITION AND RISK SHARING WITHIN THE HOUSEHOLD

PIERRE DUBOIS AND ETHAN LIGON
NUTRITION AND RISK SHARING WITHIN THE HOUSEHOLD

PIERRE DUBOIS AND ETHAN LIGON

ABSTRACT. Using data on individual consumption from farm households in the Philippines, we construct a direct test of risk-sharing within the household. We contrast the efficient outcomes predicted by the unitary household model with the outcomes we might expect if food consumption delivers not only utils, but also nutrients affecting future productivity.

The efficiency conditions which characterize the within-household allocation of food under the unitary model are violated, as consumption responds to earnings shocks. If productivity depends on nutrition, this explains some but not all of the response, as earnings “surprises” have some effect on the cost and composition of diet.

1. INTRODUCTION

In recent years, a variety of authors have sought to test the hypothesis that intra-household allocations are efficient. Often these have been construed as tests of the “unitary household” model, associated with Samuelson (1956) and Becker (1974). In Becker’s formulation, an altruistic household head dictates allocations of goods such as food and leisure, giving more to some (favored) dependents, and less to others. A celebrated prediction of this model is the so-called “Rotten Kid Theorem”; given the ability to structure incentives within the household, the head can induce even entirely selfish children to act in the interests of the altruistic head (and by extension, in the interests of the entire household).

Full intra-household efficiency implies both productive efficiency, as well as allocational efficiency. Other authors who have conducted tests of intra-household efficiency have tested only one or another of these. Udry (1996),
for example, focuses on productive efficiency, while a much larger num-
ber of authors have focused on allocational efficiency (e.g., Thomas, 1990;
Lundberg et al., 1997; Browning and Chiappori, 1998; Bobonis, 2009).
One important difficulty (which the previous authors each address in dis-
tinct ingenious but indirect ways) involved in testing intra-household alloca-
tional efficiency is that intra-household allocations are seldom observed—
ordinarily the best an econometrician can hope for is carefully recorded
data on household-level consumption. In this paper we exploit a carefully
collected dataset which records expenditures for each individual within a
household, and thus are able to conduct the first direct test of intra-household
allocational efficiency of which we are aware.

By allocational efficiency we mean, in effect, that the marginal rate of
substitution between any two commodities will be equated across house-
hold members. Importantly, we follow the Arrow-Debreu convention of
indexing commodities not only by their physical characteristics, but also by
the date and state in which the commodity is delivered. Thus, allocational
efficiency implies not only that people within a household consume apples
and oranges in the correct proportion, but also that within the household
there is full insurance. The tests we conduct here are really a joint test of
these two sorts of allocational efficiency (allocation of ordinary commodi-
ties, and allocation of state-date contingent commodities).

Without pretending any sort of exhaustive comparison of our paper with
existing literature, we will briefly describe two papers, each of which shares
(different) points of similarity with the present paper. Dercon and Krishnan
(2000) test the hypothesis of full intra-household risk-sharing in Ethiopia
by looking at the response of individual nutritional status to illness shocks.
In order to deal with limitations of their data, they assume that utility de-
pends on food consumption only via anthropometric status. So, for ex-
ample, children are implicitly assumed to be indifferent between consum-
ing a varied diet with fruit, meat, and vegetables and a subsistence diet
of beans, provided that either diet results in similar weight-for-height out-
comes. With this assumption, Dercon and Krishnan reject intra-household
efficiency, at least for poorer households, but their results are also consist-
tent with efficient intra-household allocation if people derive utility directly
from food consumption. Our data allow us to distinguish between these
possibilities, and so we allow individual utility functions to depend on con-
sumption both directly and via the influence of consumption on nutritional
outcomes. Using the same dataset as we do, Foster and Rosenzweig (1994)
doesn’t address the question of intra-household allocation at all, but rather
asks whether or not individual anthropometric measures depend on the na-
ture of the contract governing compensation for off-farm work, interpreting
this as a test for the importance of incentives. As in Dercon and Krishnan
(2000), Foster and Rosenzweig assume that food only influences utility to the extent that it influences measures of weight for height, but find that indeed incentives provided in the workplace outside the household influence consumption and physical status. In contrast to Foster and Rosenzweig, our focus is on the allocation of goods within the household, and on the role that food consumption may play in providing incentives above and beyond the determination of weight and height.

We proceed as follows. First, we provide an extended description of the data in Section 2. We describe some patterns observed in the sharing rules of Philippino households, including expected levels of consumption, and both individual and household-level measures of risk in both consumption and income.

Second, in Section 3 we formulate a simple ‘naive unitary’ dynamic model, in which utility depends on consumption, but productivity does not. An altruistic head allocates consumption goods and assigns activities to other household members. From this model we derive a simple restriction on household members’ marginal rates of substitution. Working with a parametric representation of individuals’ utility functions, we estimate a vector of preference parameters, which allows us to characterize changes in intrahousehold sharing rules as a function of individual characteristics such as age and sex.

Third, in Section 4, we consider the possibility that food consumption influences future productivity. In particular, while food consumption produces both direct utility (which depends on the quantity and quality of different kinds of foodstuffs), and also represents a sort of human capital investment which influences labor productivity (but this investment depends only on the quantity and nutritional content of foodstuffs, and not food quality). This leads us to consider a model of nutritional investments, which reproduces some of the features of models formulated by, e.g., Pitt et al. (1990); Pitt and Rosenzweig (1985). In this model the head takes into account the effect that consumption will have on both utility and productivity. This model also implies a set of restrictions on household members’ marginal rates of substitution which distinguish it from the ‘naive unitary’ model. In particular, one prediction of this model is that if there’s an anticipated increase in the marginal product of labor for household member $i$, then nutritional investment in this member will increase at the same time that the quality of food consumed by $i$ decreases.

Fourth, Section 5 presents our main results. In brief, maintaining the hypothesis that the unitary household model we specify is correct, we estimate a collection of preference parameters. We permit heterogeneous risk preferences within the household, along the lines of Mazzocco (2007), and take
A key result involves estimates of the ratio of relative risk aversions of the female head to the relative risk aversion of (i) males in the household; and (ii) other females in the household. Our estimates of these ratios indicate that risk aversion doesn’t vary greatly across females, but that female risk aversion is roughly fifty per cent greater than the risk aversion of males. We also estimate how the ages of individual household members affects the rate of growth of nutritional intakes, and find particularly large effects for boys. We find evidence that sickness and pregnancy both have a negative effect on nutritional intakes for women. Taken together, these estimates of the effects of individual characteristics could be used to construct much richer models of household-level nutritional demand than currently prevail in the literature.

The version of the unitary household model we estimate usefully accounts for much of the variation we observe in consumption growth rates within the household, but relies on the hypothesis that allocations within the household are efficient, both within and across different date-states. The hypothesis of efficiency across different date-states amounts to assuming that there is full risk sharing within the household. We test this hypothesis using an approach similar in spirit to the inter-household tests of full risk sharing devised by Townsend (1994), and ask whether unexpected changes in individual earnings have any effect on the allocation of consumption, and reject the hypothesis of full risk-sharing. Section 6 concludes.

2. The Data

The main data used in this paper are drawn from a survey conducted by the International Food Policy Research Institute and the Research Institute for Mindanao Culture in the Southern region of the Bukidnon Province of Mindanao Island in the Philippines during 1984–1985. These data are described in greater detail by Bouis and Haddad (1990) and in the references contained therein. Additional data on weather used in this paper were collected by the first author from the weather station of Malay-Balay in Bukidnon.

Bukidnon is a poor rural and mainly agricultural area of the Philippines. Early in 1984, a random sample of 2039 households was drawn from 18 villages in the area of interest. A preliminary survey was administered to

---

1 It's more usual in the economics literature to imagine that adult males play the role of household head. Our assumption that females are responsible for the kinds of allocational decisions we model in this paper is motivated by our reading of a small literature in anthropology and sociology which describes the division of household responsibilities in the rural Philippines (e.g., Illo, 1995; Eder, 2006).
<table>
<thead>
<tr>
<th>Age Group</th>
<th>Expend.</th>
<th>Rice (kg)</th>
<th>Corn (kg)</th>
<th>Staples (lbs)</th>
<th>Meat (lbs)</th>
<th>Veg. (lbs)</th>
<th>Snacks (lbs)</th>
<th>Calories</th>
<th>Protein</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample</td>
<td>5.914</td>
<td>0.724</td>
<td>1.203</td>
<td>0.3</td>
<td>1.876</td>
<td>0.477</td>
<td>0.809</td>
<td>1823.626</td>
<td>54.664</td>
</tr>
<tr>
<td>Male</td>
<td>6.428</td>
<td>0.802</td>
<td>1.271</td>
<td>0.295</td>
<td>1.984</td>
<td>0.484</td>
<td>1.068</td>
<td>1926.152</td>
<td>57.508</td>
</tr>
<tr>
<td>Female</td>
<td>5.361</td>
<td>0.64</td>
<td>1.13</td>
<td>0.306</td>
<td>1.76</td>
<td>0.47</td>
<td>0.53</td>
<td>1713.438</td>
<td>51.606</td>
</tr>
<tr>
<td>≤ 5 years</td>
<td>3.802</td>
<td>0.398</td>
<td>0.727</td>
<td>0.226</td>
<td>1.407</td>
<td>0.241</td>
<td>0.386</td>
<td>1137.078</td>
<td>34.886</td>
</tr>
<tr>
<td>6–10 years</td>
<td>4.792</td>
<td>0.607</td>
<td>1.044</td>
<td>0.276</td>
<td>1.629</td>
<td>0.362</td>
<td>0.422</td>
<td>1603.433</td>
<td>48.102</td>
</tr>
<tr>
<td>11–15 years</td>
<td>6.878</td>
<td>0.872</td>
<td>1.51</td>
<td>0.37</td>
<td>2.239</td>
<td>0.62</td>
<td>0.632</td>
<td>2232.415</td>
<td>66.056</td>
</tr>
<tr>
<td>16–25 years</td>
<td>7.929</td>
<td>1.061</td>
<td>1.606</td>
<td>0.362</td>
<td>2.271</td>
<td>0.758</td>
<td>1.238</td>
<td>2412.472</td>
<td>71.975</td>
</tr>
<tr>
<td>26–50 years</td>
<td>8.879</td>
<td>1.009</td>
<td>1.612</td>
<td>0.359</td>
<td>2.491</td>
<td>0.693</td>
<td>2.074</td>
<td>2416.091</td>
<td>72.495</td>
</tr>
<tr>
<td>&gt; 50 years</td>
<td>7.119</td>
<td>0.877</td>
<td>1.44</td>
<td>0.247</td>
<td>1.875</td>
<td>0.567</td>
<td>1.363</td>
<td>2136.464</td>
<td>61.653</td>
</tr>
<tr>
<td>≤ 5 years (Male)</td>
<td>3.719</td>
<td>0.419</td>
<td>0.733</td>
<td>0.23</td>
<td>1.405</td>
<td>0.216</td>
<td>0.293</td>
<td>1166.963</td>
<td>35.476</td>
</tr>
<tr>
<td>6–10 years (Male)</td>
<td>4.943</td>
<td>0.671</td>
<td>1.04</td>
<td>0.269</td>
<td>1.712</td>
<td>0.366</td>
<td>0.44</td>
<td>1638.98</td>
<td>48.926</td>
</tr>
<tr>
<td>11–15 years (Male)</td>
<td>7.107</td>
<td>0.924</td>
<td>1.669</td>
<td>0.379</td>
<td>2.279</td>
<td>0.582</td>
<td>0.719</td>
<td>2360.029</td>
<td>69.036</td>
</tr>
<tr>
<td>16–25 years (Male)</td>
<td>9.465</td>
<td>1.21</td>
<td>1.828</td>
<td>0.355</td>
<td>2.623</td>
<td>0.837</td>
<td>1.956</td>
<td>2688.411</td>
<td>80.574</td>
</tr>
<tr>
<td>26–50 years (Male)</td>
<td>10.411</td>
<td>1.18</td>
<td>1.769</td>
<td>0.348</td>
<td>2.69</td>
<td>0.74</td>
<td>3.007</td>
<td>2653.379</td>
<td>79.364</td>
</tr>
<tr>
<td>&gt; 50 years (Male)</td>
<td>7.96</td>
<td>1.039</td>
<td>1.497</td>
<td>0.229</td>
<td>1.944</td>
<td>0.626</td>
<td>1.732</td>
<td>2300.727</td>
<td>66.907</td>
</tr>
<tr>
<td>≤ 5 years (Female)</td>
<td>3.901</td>
<td>0.372</td>
<td>0.72</td>
<td>0.221</td>
<td>1.409</td>
<td>0.272</td>
<td>0.497</td>
<td>1101.531</td>
<td>34.184</td>
</tr>
<tr>
<td>6–10 years (Female)</td>
<td>4.638</td>
<td>0.54</td>
<td>1.048</td>
<td>0.283</td>
<td>1.544</td>
<td>0.357</td>
<td>0.399</td>
<td>1566.915</td>
<td>47.256</td>
</tr>
<tr>
<td>11–15 years (Female)</td>
<td>6.657</td>
<td>0.822</td>
<td>1.357</td>
<td>0.362</td>
<td>2.201</td>
<td>0.657</td>
<td>0.549</td>
<td>2109.172</td>
<td>63.179</td>
</tr>
<tr>
<td>16–25 years (Female)</td>
<td>6.614</td>
<td>0.934</td>
<td>1.415</td>
<td>0.367</td>
<td>1.97</td>
<td>0.69</td>
<td>0.623</td>
<td>2176.366</td>
<td>64.617</td>
</tr>
<tr>
<td>26–50 years (Female)</td>
<td>6.858</td>
<td>0.783</td>
<td>1.406</td>
<td>0.374</td>
<td>2.228</td>
<td>0.63</td>
<td>0.844</td>
<td>2102.985</td>
<td>63.43</td>
</tr>
<tr>
<td>&gt; 50 years (Female)</td>
<td>4.573</td>
<td>0.39</td>
<td>1.269</td>
<td>0.302</td>
<td>1.667</td>
<td>0.391</td>
<td>0.248</td>
<td>1639.464</td>
<td>45.756</td>
</tr>
</tbody>
</table>

**TABLE 1.** Mean Daily Food Consumption. The first column reports mean total food expenditures per person (in constant Philippine pesos). The next six columns report means for particular sorts of food expenditures (differences between total food expenditures and the sum of its constituents is accounted for by “other non-staple” foods). The final two columns report individual calories and protein derived from individual-level food consumption.
each household to elicit information used to develop criteria for a stratified random sample later selected for more detailed study. The preliminary survey indicated that farms larger than 15 hectares amounted to less than 3 percent of all households, a figure corresponding closely to the 1980 agricultural census. Only households farming less than 15 hectares and having at least one child under five years old were eligible for selection. Based on this preliminary survey, a stratified random sample of 510 households from ten villages was chosen. Some attrition (mostly because of outmigration) occurred during the study and a total of 448 households from ten villages finally participated in the four surveys conducted at four month intervals beginning in July 1984 and ending in August 1985. The total number of persons in the survey is 3294.

The nutritional component of the survey interviewed respondents to elicit a 24-hour recall of individual food intakes, as well as one month and four month interviews to measure household level food and non-food expenditures. Food intakes include quantity information for a highly disaggregated set of food items. Individual food expenditures can be computed using direct information on the prices and quantities of foods purchased, and on quantities consumed out of own-production and in-kind transactions.

Later in the paper we will concern ourselves with changes in individuals’ shares of consumption, intentionally neglecting to explain differences in levels of consumption, where theory has less to say. However, some of these differences are interesting, and so some information on levels of individual expenditures along with caloric and protein intakes are given in Table 1. Turning to the final columns of the table, we first note that the average individual in our sample is not terribly well-fed. Comparing the figures in Table 1 to standard guidelines for energy-protein requirements (WHO, 1985) reveals that even the average person in our sample faces something of an energy deficit.

When we consider the average consumption of different age-sex groups, it becomes clear that particular groups are particularly malnourished. Also, these figures show clearly that the relationship between consumptions and age follows consistently an inverse U shaped pattern which is quite reassuring about the reliability of these measures.

The picture of inequality drawn by our attention to energy and protein intakes is, if anything, exacerbated by closer attention to the sources of nutrition. While all of the foods considered here are sources of calories and protein, it also seems likely that food consumption is valued not just for its nutritive content, but that individuals also derive some direct utility from certain kinds of consumption. This point receives some striking support from Table 1. Consider, for example, average daily expenditures by males aged 26–50, compared with the same category of expenditures by women of
the same age. The value of expenditures on male consumption of all staples is 28 per cent greater than that of females of the same age. This difference seems small enough that it could easily be attributed to differences in activity or metabolic rate. However, compare expenditures on what are presumably superior goods: expenditures on male consumption of meat (and fish), vegetables, snacks (including fruit) is 424 per cent greater than the corresponding expenditures by women in the same age group. Since nothing like a difference of this size shows up in calories or protein, this seems like very strong evidence that intra-household allocation mechanisms are designed to put a particularly high weight on the utility of prime-age males relative to other household members, quite independent of those prime-age males’ greater energy-protein requirements. Note that although these differences in consumption seem to point to an inegalitarian allocation, these differences provide no evidence to suggest that household allocations are inefficient.

3. The Basic Model

Consider a household having \( n \) members, indexed by \( i = 1, 2, \ldots, n \), where an index of 1 is understood to refer to the household head. Time is indexed by \( t = 0, 1, \ldots, T \), where \( T \) may be infinite. During each period, member \( i \) consumes a \( K \)-vector of goods \( c_{it} = (c_{it1}, \ldots, c_{itK}) \). At the same time, \( i \) undertakes \( m \) additional activities \( a_{it} \), which may include things from which she derives pleasure (say dancing, playing games, or dressing up), and others which she finds unpleasant (e.g., plowing a field, watching a child, or cleaning the stables).

Household member \( i \) derives direct utility from consumption and activities. Further, at time \( t \) person \( i \) possesses a set of characteristics (e.g., gender, weight, age) which we denote by the vector \( b_{it} \). These characteristics may have an influence on the utility she derives from both consumption and activities. Thus, we write her momentary utility at \( t \) as some \( U(c_{it}, b_{it}) + Z_i(a_{it}, b_{it}) \), where the function \( U \) is assumed to be increasing, concave, and continuously differentiable in each of the consumption goods.

Of course, unpleasant activities aren’t undertaken for their own sake; rather, they may be useful in production. Let \( y \) be a vector of goods (e.g., corn, sugarcane, household services). In general, there will be uncertainty in production; we regard \( y \) as a random variable with joint p.d.f. \( f(y|a) \).

We’re interested in characterizing the set of efficient allocations for the household. Following Becker (1974), we imagine that the altruistic household head is responsible for allocating consumption and assigning activities within the household; however, it’s important to note that this is simply a device for characterizing the set of efficient allocations. As forcefully argued
by Chiappori (1988, 1992), in a static model the restriction of efficiency tells us nothing about the shares of consumption we expect to observe in the household (in our setting, the hypothesis of efficiency tells us nothing about the altruism of the head). However, in a dynamic setting, the hypothesis of efficiency puts very strong restrictions on the evolution of these shares, and it is these restrictions which we exploit in this paper.

In any event, the household head associates an altruism weight with the utility of each household member (with the normalization that the weight for the head is equal to one). We generalize the usual problem by permitting this weight to vary over time. In particular, let the altruism weight associated with member $i$’s utility at time $t$ be given by $\alpha_{it} \in (0, 1]$, and suppose that the evolution of altruism over time is given by

$$ \log \alpha_{it+1} = \log \alpha_{it} + \varepsilon_{it+1} $$

where $E_t(\alpha_{it+1}) = \alpha_{it}$. Note that having a constant weight over time (the usual case) is a special instance of (1).\(^2\) In the general case this specification implies that future changes in altruism parameters are unpredictable, and specifically that the sequence $\{\varepsilon_{it} - 1\}$ is a martingale difference sequence.

We formulate the problem facing the head recursively. At the beginning of a period, the head takes as given an $n$-vector reflecting her current sentiments toward other household members ($\alpha$), a list of the characteristics of household members ($b$), prices ($p$), and the total of household expenditures for the period $x$. Given her preferences, she then chooses consumptions and allocations subject to the constraints implied by these prices and resources. In particular, let $H(\alpha, p, x)$ denote the discounted, expected utility of the head given the current state, and let this function satisfy

$$ H(\alpha, p, x) = \max_{\{(c_i, a_i)\}_{i=1}^n} \sum_{i=1}^n \alpha_i \left( U(c_i, b_i) + Z_i(a_i, b_i) \right) + \beta \int H(\hat{\alpha}, \hat{p}, \hat{p}' \sum_{i=1}^n \hat{y}_i, \hat{b}) \, dG(\hat{\alpha}, \hat{p}, \hat{y}_1, \ldots, \hat{y}_n, \hat{b}|\alpha, p, a_1, \ldots, a_n, b) $$

subject to the household budget constraint

$$ p' \sum_{i=1}^n c_i \leq x. $$

Here variables with ‘hats’ denote future realizations of the variable, and the distribution function $G$ denotes the joint distribution of next period’s

\(^2\)Alternatively, if one wished to avoid invoking paternal altruism, one could interpret the evolution of these coefficients as multiplicative preference shocks.
prices and output for each of the $n$ household members given this period’s activities and prices.

It’s very important to notice that in the present model consumption assignments yield utility, but do not affect future characteristics $b$. For some sorts of physical characteristics (e.g., weight) this is obviously unrealistic, and in Section 4 we relax this assumption. One of our aims is to test whether or not consumption is allocated so as to take into account the benefits of “nutritional investment;” if so, this is a factor influencing intra-household allocation which is inappropriately neglected in the standard unitary model.

First order conditions from this problem imply that

$$\frac{U_k(c_{1t}, b_{1t})}{U_k(c_{it}, b_{it})} = \alpha_{it}$$

$k = 1, \ldots, K$, and $i = 1, \ldots, n$, where $U_k(c, b)$ denotes the marginal utility of the $k$th consumption good. From this, it’s easy to see that the head will allocate consumption so that members’ marginal rates of substitution are all equated. As a consequence, the unitary household model implies that

$$\frac{U_k(c_{1t+1}, b_{1t+1})}{U_k(c_{it+1}, b_{it+1})} / \frac{U_k(c_{1t}, b_{1t})}{U_k(c_{it}, b_{it})} = \frac{\alpha_{it+1}}{\alpha_{it}} = e^{e_{it+1}}$$

Accordingly, we interpret the unitary household model as implying that ratios of marginal rates of substitution between the head and any household member will vary over time only in unpredictable ways.

A solution to the sharing problem facing the household head is a set of functions which indicate the expenditures assigned to each household member $i$, $x_i = \tilde{e}_i(\alpha, x, p, b)$, $i = 1, \ldots, n$, and individual demand functions $c_i = c(x_i, p, b_i)$. We can use these demands to define indirect period-specific utilities from consumption,

$$v(x_i, p, b_i) \equiv U(c(x_i, p, b_i), b_i)$$

It’s also convenient to define a corresponding individual expenditure function mapping momentary utility $w$ from consumption (given prices and characteristics) into expenditures on consumption for $i$, so that $x_i = e(w, p, b_i)$, satisfies

$$x_i \equiv e(v(x_i, p, b_i), p, b_i)$$

so that $e$ is a sort of inverse of the indirect utility function $v$.

We can substitute the indirect utility functions $v$ into the head’s problem,
yielding

\[ H(\alpha, p, x, b) = \max_{\{a_i, (x_i, a_i)\}_{i=2}^n} v(x - \sum_{i=2}^n x_i, p, b) + Z_1(a_1, b_1) \]
\[ + \sum_{i=2}^n \alpha_i(v(x_i, p, b_i) + Z_i(a_i, b_i)) \]
\[ + \beta \int H(\hat{\alpha}, \hat{p}, \hat{\beta} \sum_{i=1}^n y_i, \hat{b}) dG(\hat{\alpha}, \hat{p}, y_1, \ldots, y_n, \hat{b}|\alpha, p, a_1, \ldots, a_n, b) \]

First order conditions for this reformulation of the problem imply that \( \alpha_{it} = \frac{\nu'(x_{1t}, p_{1t}, b_{1t})}{\nu'(x_{it}, p_{it}, b_{it})} \) for \( i = 1, \ldots, n \) and \( t = 1, \ldots, T \). As a consequence,

\[ \frac{\nu'(x_{1t+1}, p_{1t+1}, b_{1t+1})}{\nu'(x_{it+1}, p_{it+1}, b_{it+1})} = \frac{\alpha_{it+1}}{\alpha_{it}} = e^{\varepsilon_{it+1}}, \]

where the notation \( \nu'(x, p, b) \) denotes the partial derivative of \( v \) with respect to expenditures \( x \). Note the similarity of restrictions on consumptions (3) to restrictions on indirect utilities (4); we will exploit this similarity to use both expenditures and quantities of goods consumed in our empirical work.

To conduct estimation and inference, we need to specify at least some components of agents’ preferences over food consumption. The within-period allocation of total consumption expenditures \( x \) to goods with prices \( p \) can be completely characterized by an indirect period-specific utility function \( v(x, p, b) \). We’re interested in characterizing food expenditures at different levels of aggregation (across goods); accordingly, for any partition of foodstuffs into \( S \) different categories, we let \( x^s \) denote expenditures on the \( s^{th} \) category. Then following e.g., Blundell et al. (1994), we represent these momentary preferences by the conditional indirect utility function \( \nu^S(x^1, \ldots, x^S, p, b) \), when the household head is constrained to spend \( x^s \) on the \( s \)th expenditure category. As above, when the head is not so constrained, we represent his conditional indirect utility by \( \nu(x, p, b) \). The restrictions we then place on these different representations of the household head’s indirect utility are given by:

**Assumption 1.** The \( S \) categories of expenditures are aggregable in the sense that \( \nu^S(x^1, \ldots, x^S, p, b) = \nu(x, p, b) \), where \( x = \sum_{r=1}^S x_r \). Further, there exist household-specific, possibly time-varying ‘price indices’ \( \pi^S_{nt}(p) \) and a set of functions \( \nu^S(x^1, \ldots, x^S, b) \) such that the indirect utility functions satisfy

\[ \frac{\partial}{\partial x^S} \nu^S(p, x^1, \ldots, x^S, b) = \pi^S_{nt}(p) \frac{\partial}{\partial x^S} \nu^S(x^1, \ldots, x^S, b) \]

for all \( s = 1, \ldots, S \).
Note that this condition is satisfied by the class of indirect utility functions having the PIGL or PIGLOG property (Muellbauer, 1975), described by Deaton and Muellbauer (1980) and widely used in the empirical literature (e.g., Blundell et al., 1993, 1994).

A consequence of Assumption 1 is that the ratio of marginal utilities of expenditures \( s \) of any two members of household \( h \) does not depend on the unknown price index \( \pi_{ht}(p) \). Specifying the function \( v^s \) will then allow us to work with within-household ratios of marginal utilities of consumption.

We want to permit a great deal of heterogeneity in preferences over different consumption goods. Accordingly, we partition the vector of personal characteristics \( b_{it} \) into three distinct parts (this follows the line of Dubois, 2000). Let \( \upsilon_i \) denote time invariant characteristics of person \( i \) (such as sex), and let \( \zeta_{it} \) denote time-varying characteristics of the same person (such as age and health). Both \( \upsilon_i \) and \( \zeta_{it} \) are assumed to be observed by the econometrician. In contrast, let \( \xi_{it} \) denote time-varying characteristics of person \( i \) at time \( t \) which aren’t observed in the data.

Recalling that consumption consists of \( K \) elements \((c^1, ..., c^K)\), we parameterize the utility \( U \) of person \( i \) at date \( t \) by

\[
U(c_{it}, b_{it}) = \sum_{k=1}^{K} \exp \left( \upsilon_i' \gamma_k + \zeta_{it}' \delta_k + \xi_{it} \right) A^k_i B^k_i (c^k_{it}) \frac{1 - \theta^k_i \upsilon_i}{1 - \theta^k_i \upsilon_i}
\]

Here \((\gamma, \delta, \theta_1, ..., \theta_K)\) are each vectors of unknown parameters. Thus, the factor \( \exp(\upsilon_i' \gamma_k + \zeta_{it}' \delta_k + \xi_{it}) \) allows the utility (and marginal utility) of all consumption to vary according to both observed and unobserved characteristics (as in, e.g., Blundell et al., 1994). Note in particular that one can model differences in the utility derived from consumption foodstuffs according to features such as age and sex. The (possibly unobserved) factors \( \{A^k_i\}_{k=1}^{K} \) govern the relative, idiosyncratic utility a given person derives from different consumption goods: think of invariant differences in preferences over vegetables and sweets, for example. In contrast, the factors \( \{B^k_{it}\} \) govern time-varying differences in preferences over different commodities; think of seasonal differences in preferences for starchy foods. Finally, the linear functions \( \theta^k_i \upsilon_i \) can be regarded as the relative risk aversion person \( i \) has over variation in the consumption of good \( k \), so that risk attitudes can vary according to sex, ethnicity, or other time-invariant characteristics. Given our previous remarks, an almost identical parameterization will serve for modeling the indirect utility of expenditures \((x^1, ..., x^S)\).

Now, under the unitary model, and with the specification of preferences given above, the ratio of the intertemporal marginal rate of substitution of consumption of the household head 1 over that of person \( i \) is equal to the
proportional change in the altruism parameter for person $i$, and can be written as

$$\exp\left[(\Delta\zeta_{1t+1} - \Delta\zeta_{it+1})\delta + (\Delta\xi_{1t+1} - \Delta\xi_{it+1})\right] \left(\frac{x_{1t+1}^k}{x_{1t}^k}\right) - \theta'_i u_i = e^{e_{it+1}}$$

where $\Delta$ is the first difference operator. This specification of preferences is a straightforward generalization of the commonly used CRRA preferences, but it’s worth noting that these preferences are not generally Gorman-aggregable. As a consequence, an efficient allocation will not generally give household members fixed shares; rather the shares will vary with total household expenditures and with changes in the time-varying characteristics of household members.

4. NUTRITIONAL INVESTMENT

We now extend the model of Section 3 in another direction, and take into account the possibility that current consumption provides some sort of nutrition to household members, which in turn may affect the future (dis)utility associated with some particular activities. This new model is very much in the spirit of, say, Stiglitz (1976), or Dasgupta and Ray (1986).

Notation is as in Section 3. Recall that at date $t$, member $i$ is described by some set of physical characteristics $b_{it}$, which may include things like gender, height, weight, health, and so on. Earlier, $b_{it}$ evolved according to some unspecified stochastic process, but this evolution was assumed to be independent of current activities and consumption. In this extended version of the model, member $i$ consumes a $K$-vector of foods $c_{it}$, as before, but now she derives not only direct utility from this consumption, but may also derive $l$ consumption services $s_{it}$, related to consumption by

$$s_{it} = \varphi c_{it}$$

where $\varphi$ is an $l \times K$ matrix which determines the mapping from food into consumption services which generate utility. At the same time, $i$ undertakes $m$ additional activities $a_{it}$, as before. Momentary utility for person $i$ at $t$ is given by $U(c_{it}, b_{it}) + Z_i(a_{it}, b_{it})$.

The key difference between this model and the model of Section 3 is that the physical characteristics of household members evolve in response to a law of motion $M$, so that

$$b_{it+1} = M(b_{it}, c_{it}).$$

Note that this law of motion permits consumption at time $t$ to influence subsequent characteristics. Though this law of motion is a first-order Markov process, one could allow more complicated temporal dependence through
clever specification of the vector $b_{it}$, permitting it, for example, to include
lagged variables.

As before, let $y$ be a vector of goods (e.g., corn, sugar, household services). In general, there will be uncertainty in production; we regard $y$ as a
random variable with joint p.d.f. $f(y|a)$. Note the implicit restriction: the
probability of corn yields being high depends on the field being properly
plowed, but it doesn’t depend on the physical characteristics of the person
who actually performed the plowing.

The new problem facing the household head requires him to take into
account the influence of current consumption on future productivity:

$$H(\alpha, p, x, b_1, \ldots, b_n) = \max \left\{ \sum_{i=1}^{n} \alpha_i (U(c_i, b_i) + Z_i(a_i, b_i)) \right\}$$

subject to the budget constraint

$$p' \sum_{i=1}^{n} c_i \leq x$$

and the law of motion for physical characteristics

$$\hat{b}_i = M(b_i, c_i).$$

The distribution function $G$ denotes the joint distribution of next period’s
prices and output for each of the $n$ household members given this period’s
activities and prices. The value $\hat{p}' \sum_{i=1}^{n} y_i$ represents the next period budget
of the household. Note that $G$ no longer governs the evolution of $b_i$; rather,
this evolution proceeds according to (8).

5. Empirical Tests

We’ve presented two distinct models of intra-household allocation. Each
of these models can be characterized by positing a different rule govern-
ing the evolution of the household head’s altruism parameters $\{\alpha_i^t\}$. The
first model is a simple version of the unitary household model, in which
food consumption is allocated to different household members in order to
produce utility; the weight of each members’ utility depends on the time-
varying altruism of the head toward that member. In this model, changes in
a member’s share of consumption (allowing for age-sex specific mappings
from consumption to utility) are due only to unpredictable changes in the
altruism of the head.

The second model (nutritional investment) is one in which the allocation
of food affects not only the utility of different household members, but
also the production possibility set of the household. In this model, the allocation of energy and protein in the household may respond not only to unpredictable changes in the head’s altruism, but may also vary because the productivity of particular household members may depend on the consumption assignment in a way which varies over time. The most obvious example might have to do with the additional energy required by some household members during different seasons: household members who engage in heavy agricultural labor may be assigned a disproportionate share of calories during the harvest season, for example, or these same people may receive a greater share of protein in advance of a period of hard labor.

5.1. Estimating the Unitary Household Model. Equation (6) gives a relationship between the growth rate of consumption and expenditures for the household head and that of each household member if preferences are as assumed in (5) and if intra-household allocations are efficient. Recall that while shares of consumption and expenditures depend on total household expenditures and individual characteristics, they should not depend on the realization of any idiosyncratic shock unless that shock directly influences preferences. As a first pass at testing this restriction, we take logs of (6) and rearrange, yielding the estimating equation

\[ \Delta \log(x^k_{it}) = \Delta \log(x^k_{1t}) \frac{\theta'_k \upsilon_i}{\theta'_k \upsilon_1} + (\Delta \xi_{it} - \Delta \xi_{1t})' \frac{\delta}{\theta'_k \upsilon_i} + \frac{\epsilon_{it} + \Delta \xi_{it} - \Delta \xi_{1t}}{\theta'_k \upsilon_i} \]

subject to the restriction that the unobserved time-varying characteristics are mean independent of the observed characteristics \((\upsilon_i, \xi_{it})\) (i.e., that \(E(\xi_{it} | \xi_{1t}, \upsilon_i) = E(\xi_{1t})\)). It’s important to note that this restriction does not directly bear on changes in a member’s share of total household resources—the expression for such a share depends on the preferences of every household member. Rather, we characterize only the changes in the growth rate of expenditures and consumption relative to the household head. To reiterate, if the unitary household model is correct, the disturbances in (9) will be unrelated to individual-specific outcomes, such as off-farm labor income or changes in the composition of household income. This can be tested by introducing overidentifying variables in equation (9).

It may be worth dwelling on the interpretation of (9). Note that there’s no prediction regarding the level of a member’s share; only a prediction about what produces changes in that share. Thus, this equation is of no use in trying to understand inequality in the allocation of household resources; only in understanding changes in the way in which those resources are shared. One feature of the environment which may help to explain changes in household shares has to do with heterogenous risk preferences: if household member \(i\) is more risk averse than the household head, then changes
in total household resources will produce smaller percentage changes in \( i \)'s consumption than it will in the consumption of the head (and conversely). Changes of this sort will be captured by our estimates of \( \theta'_{k} \), which enter the first term on the right-hand-side of equation (9). Alternatively, changes in the relative needs of different household members may result in changes in shares of food expenditures and nutrition. For example, as a small boy matures into a grown man, one would expect that person’s share of household resources to increase, basically as a consequence of changes in the utility that person derives from food consumption. Changes of this sort are captured by changes in \( \zeta_{it} \), and depend on the vector of parameters \( \delta \).

Our first attempts to estimate (9) are reported in Table 2. Here we exploit the relationship between ratios of direct and indirect utility given by (3) and (4) to estimate a system of three equations, each of the form of (9), but with different measures of consumption.

Our first measure of consumption is individual food expenditures; our second is individual caloric intake; and our third is total protein intake. For time-varying individual characteristics \( \zeta_{it} \), we use a set of (per round) time effects; interactions between sex and the logarithm of age in years, and between sex and the number of days sick in the most recent period; an indicator with the value of one if person \( i \) is in the second or third trimester of pregnancy; and a measure of lactation (the number of minutes spent nursing per day). For the fixed individual characteristics \( \upsilon_{i} \) governing relative risk aversion \( (\theta'_{k}\upsilon_{i}) \), we’ve simply used gender. Since residuals from these three different equations are \textit{a priori} related, we’ve used a three-stage least squares procedure to estimate this system of seemingly unrelated regressions.

In the first stage, we use data on changes in log household-level food expenditures (collected via a different survey instrument than our data on individual-level consumption) to instrument for changes in the log of the household heads’ consumption. In the second stage we use these first stage results to estimate each equation separately, and then use estimated residuals from this stage to construct estimates of the covariance matrix of residuals across equations. The third stage uses this estimated covariance matrix to compute more efficient point estimates and consistent estimates of the standard errors of the estimated coefficients (details may be found in Appendix A).

Table 2 shows the results of the base nutrient instrumental variables regressions (that is, the regressions for individual food expenditures, calories intakes and protein intakes). The results show a large difference in elasticities for males and females: for males in the household, the results show that individual shares of food expenditures increase at a rate 50–60% more than the female household head (60% more for total food expenditures,
<table>
<thead>
<tr>
<th></th>
<th>Expend.</th>
<th>Calories</th>
<th>Protein</th>
<th>$F$ (p-value)</th>
<th>Expend.</th>
<th>Calories</th>
<th>Protein</th>
<th>$F$ (p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{u_{1}}$: Male</td>
<td>1.6038*</td>
<td>1.4944*</td>
<td>1.6179*</td>
<td>1428.5493</td>
<td>1.6148*</td>
<td>1.5000*</td>
<td>1.6180*</td>
<td>1413.2552</td>
</tr>
<tr>
<td></td>
<td>(0.0306)</td>
<td>(0.0408)</td>
<td>(0.0296)</td>
<td>(0.0000)</td>
<td>(0.0310)</td>
<td>(0.0411)</td>
<td>(0.0296)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>$\theta_{u_{1}}$: Female</td>
<td>1.0152*</td>
<td>1.1512*</td>
<td>1.0625*</td>
<td>781.9572</td>
<td>1.0031*</td>
<td>1.1568*</td>
<td>1.0670*</td>
<td>766.2346</td>
</tr>
<tr>
<td></td>
<td>(0.0258)</td>
<td>(0.0391)</td>
<td>(0.0246)</td>
<td>(0.0000)</td>
<td>(0.0260)</td>
<td>(0.0392)</td>
<td>(0.0247)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>$\delta_{v_{1}}$: Age male</td>
<td>0.4148*</td>
<td>0.3330</td>
<td>0.4452*</td>
<td>4.8577</td>
<td>0.4653*</td>
<td>0.3719*</td>
<td>0.4549*</td>
<td>4.9419</td>
</tr>
<tr>
<td></td>
<td>(0.1346)</td>
<td>(0.1819)</td>
<td>(0.1446)</td>
<td>(0.0022)</td>
<td>(0.1358)</td>
<td>(0.1835)</td>
<td>(0.1455)</td>
<td>(0.0020)</td>
</tr>
<tr>
<td>$\delta_{v_{1}}$: Age female</td>
<td>0.1814</td>
<td>0.0398</td>
<td>0.0670</td>
<td>0.6868</td>
<td>0.2010</td>
<td>0.0555</td>
<td>0.0684</td>
<td>0.7474</td>
</tr>
<tr>
<td></td>
<td>(0.1559)</td>
<td>(0.2108)</td>
<td>(0.1675)</td>
<td>(0.6093)</td>
<td>(0.1563)</td>
<td>(0.2116)</td>
<td>(0.1677)</td>
<td>(0.5237)</td>
</tr>
<tr>
<td>Days sick, male</td>
<td>0.0048</td>
<td>0.0078</td>
<td>0.0052</td>
<td>1.2738</td>
<td>0.0047</td>
<td>0.0077</td>
<td>0.0051</td>
<td>1.2421</td>
</tr>
<tr>
<td></td>
<td>(0.0032)</td>
<td>(0.0043)</td>
<td>(0.0034)</td>
<td>(0.2814)</td>
<td>(0.0032)</td>
<td>(0.0043)</td>
<td>(0.0034)</td>
<td>(0.2926)</td>
</tr>
<tr>
<td>Days sick, female</td>
<td>-0.0038</td>
<td>-0.0058</td>
<td>-0.0096</td>
<td>3.6258</td>
<td>-0.0042</td>
<td>-0.0060</td>
<td>-0.0096</td>
<td>3.3356</td>
</tr>
<tr>
<td></td>
<td>(0.0040)</td>
<td>(0.0054)</td>
<td>(0.0043)</td>
<td>(0.0124)</td>
<td>(0.0040)</td>
<td>(0.0054)</td>
<td>(0.0043)</td>
<td>(0.0185)</td>
</tr>
<tr>
<td>Pregnant</td>
<td>-0.1676*</td>
<td>-0.2411*</td>
<td>-0.1847*</td>
<td>1.7809</td>
<td>-0.1668*</td>
<td>-0.2488*</td>
<td>-0.1833*</td>
<td>1.8456</td>
</tr>
<tr>
<td></td>
<td>(0.0835)</td>
<td>(0.1130)</td>
<td>(0.0898)</td>
<td>(1.4844)</td>
<td>(0.0838)</td>
<td>(0.1135)</td>
<td>(0.0899)</td>
<td>(1.3659)</td>
</tr>
<tr>
<td>Nursing</td>
<td>-0.0098</td>
<td>-0.0132</td>
<td>-0.0092</td>
<td>0.4182</td>
<td>-0.0095</td>
<td>-0.0129</td>
<td>-0.0094</td>
<td>0.3747</td>
</tr>
<tr>
<td></td>
<td>(0.0102)</td>
<td>(0.0138)</td>
<td>(0.0109)</td>
<td>(0.7399)</td>
<td>(0.0102)</td>
<td>(0.0138)</td>
<td>(0.0109)</td>
<td>(0.7713)</td>
</tr>
<tr>
<td>Second quarter</td>
<td>0.0749*</td>
<td>0.1409*</td>
<td>0.1071*</td>
<td>8.7307</td>
<td>0.0648*</td>
<td>0.1305*</td>
<td>0.0911*</td>
<td>6.1360</td>
</tr>
<tr>
<td></td>
<td>(0.0208)</td>
<td>(0.0279)</td>
<td>(0.0221)</td>
<td>(0.0000)</td>
<td>(0.0227)</td>
<td>(0.0305)</td>
<td>(0.0242)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>Third quarter</td>
<td>0.0128</td>
<td>0.1056*</td>
<td>0.1086*</td>
<td>14.0427</td>
<td>0.0283</td>
<td>0.1162*</td>
<td>0.1045*</td>
<td>7.7453</td>
</tr>
<tr>
<td></td>
<td>(0.0209)</td>
<td>(0.0283)</td>
<td>(0.0225)</td>
<td>(0.0000)</td>
<td>(0.0237)</td>
<td>(0.0321)</td>
<td>(0.0254)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>Fourth quarter</td>
<td>0.0836*</td>
<td>0.0125</td>
<td>-0.0162</td>
<td>15.6535</td>
<td>0.0722*</td>
<td>0.0096</td>
<td>-0.0030</td>
<td>5.0024</td>
</tr>
<tr>
<td></td>
<td>(0.0213)</td>
<td>(0.0281)</td>
<td>(0.0224)</td>
<td>(0.0000)</td>
<td>(0.0285)</td>
<td>(0.0381)</td>
<td>(0.0303)</td>
<td>(0.0018)</td>
</tr>
<tr>
<td>$y_{t_{r+1}}^{P}$</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0.1632</td>
<td>0.0619</td>
<td>-0.2186</td>
<td>2.4632</td>
</tr>
<tr>
<td></td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>(0.2757)</td>
<td>(0.3709)</td>
<td>(0.2940)</td>
<td>(0.0605)</td>
</tr>
<tr>
<td>$y_{t_{r+1}}^{u}$</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>-0.0089</td>
<td>-0.0140</td>
<td>0.0309</td>
<td>4.9312</td>
</tr>
<tr>
<td></td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>(0.0260)</td>
<td>(0.0352)</td>
<td>(0.0279)</td>
<td>(0.0020)</td>
</tr>
<tr>
<td>$y_{t+1}^{P}$</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>-0.7757*</td>
<td>-0.6427</td>
<td>-0.2094</td>
<td>6.9830</td>
</tr>
<tr>
<td></td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>(0.2654)</td>
<td>(0.3563)</td>
<td>(0.2823)</td>
<td>(0.0001)</td>
</tr>
<tr>
<td>$y_{t+1}^{u}$</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0.0404</td>
<td>-0.0640</td>
<td>-0.0340</td>
<td>3.9608</td>
</tr>
<tr>
<td></td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>(0.0297)</td>
<td>(0.0402)</td>
<td>(0.0319)</td>
<td>(0.0078)</td>
</tr>
</tbody>
</table>

Table 2: Expenditure & nutritional intakes within the household. Point estimates may be interpreted as changes in person $i$'s share of household food expenditures/calories/protein relative to the share of the female household head. Figures in parentheses are standard errors.
49% more for Calories, and 62% more for protein). In sharp contrast, for other females in the household, the elasticities are not much greater than one (1.5% more for expenditures, 15.1% for Calories, and 6.25% more for protein).

To interpret these changes, consider that one measure of household sharing is given by the coefficients associated with the household heads’ consumption growth. If all household members had homogeneous risk attitudes, then these coefficients would be equal to one under the null hypothesis of perfect risk-sharing. Since in fact these coefficients are all much greater than one for males, on a strict interpretation of (9) this implies that males are considerably less averse to risk than are other household members, and bear a disproportionate amount of the aggregate risk faced by the household. Further, males’ tolerance of variation in the consumption of Calories is less than their tolerance of variation in either expenditures or protein (relative to the household head), suggesting that when the household faces an adverse shock, males substitute toward less expensive sources of calories to a greater extent than do females.

As with risk attitudes, maturity has a very different influence on consumption shares across sexes. In particular, on average a one percent increase in the ratio of a male’s age to head’s age results in a 0.41 per cent increase in the value of food consumed by that male, a corresponding 0.33 per cent increase in calories and a 0.45 per cent increase in protein intake, increases that are jointly highly significant. For females the point estimates suggest that age increases consumption shares, but none of the point estimates are either individually or jointly significantly different from zero. Accordingly, males not only bear the largest share of risk in the household, but also assume additional risk as they age at much a greater rate than do females.

Interestingly, neither males nor females experience much of a reduction in calories and protein when ill, despite one’s presumption that ill household members are apt to be less active. Sickness has no significant effect on male’s consumption shares relative to the household head. Sickness causes females to have a (jointly) significant decrease in consumption shares, but of apparently small magnitude. Surprisingly, being pregnant appears to result in a larger fall in women’s share of food expenditures, calories, and protein than does being sick but these effects, though large and individually significant, are not jointly statistically significant.\(^3\)

\(^3\)WHO (1985) estimates that the energy needs of well-nourished women amount to 350 Calories more per day, or roughly a 15 per cent increase, when in the second and third trimesters of pregnancy, though there’s evidence that at least part of this energy cost is made up via reduced activity. Most strikingly, pregnancy seems to lead to a 16.9 per cent
5.2. **Testing the Unitary Household Model.** The estimates presented in Table 2 shed light on the intra-household allocation of consumption given the validity of our specification of preferences and given the hypothesis that intra-household allocations are Pareto optimal, governed by (3). In this case, the residuals from (9) will be orthogonal to all other information, shocks, and other outcomes which might affect the household or the individuals in it. In particular, surprises in individual labor earnings ought not to have any effect on the sharing rule.

Our next order of business, then, is to construct predictions of labor earnings for different individuals, and then to use these to construct measures of unpredicted earnings shocks. Wages in this agricultural region have considerable seasonal variation, and vary also with weather shocks. Accordingly, we use two sorts of information to predict wages. First are a variety of fixed (or slowly varying) individual characteristics, such as sex, education, age, weight, and height (and squares of these last three quantities); next are month and village specific observations and predictions of weather.

Our construction of these weather predictions is worthy of some note. From a single weather station in Malay-Balay, Bukidnon, we have monthly information about the weather in this region over the period 1961 to 1994. These data include information on maximum rainfall, humidity, the number of rainy days per month and a measure of cloudiness. We assume that the weather at time \( t+1 \) is unknown at time \( t \), but that the weather history is known, and can be used to predict future weather outcomes. We use these relatively long time series on weather variables to estimate a prediction rule for these variables (after some experimentation, we settled on regressing each of these variables on lags of six, twelve, and twenty-four months). We then interact these weather variables with a complete set of village dummy variables. By themselves, these predicted weather variables explain eleven per cent of observed variation in log earnings.

When we include these weather variables interacted with municipality along with individual characteristics, we’re able to account for 22 per cent of observed variation in log earnings. Education, age, and sex all are important for determining earnings; physical characteristics less so (none is individually significant in the predicted earnings regression).

In any event, we use the predicted earnings regression to construct predicted earnings \( y_{it+1}^p \) and ‘unpredicted’ earnings \( y_{it+1}^u \), computed as the forecast error in the predicted earnings regression. We then add the change in the log of these earnings variables for both person \( i \) and the household head reduction in woman’s share of household protein relative to the head, while WHO guidelines suggest that such women ought to receive an increase of roughly similar magnitude. Reductions in activity will presumably have no direct effect on a pregnant woman’s need for protein.
to the base regression (9). Results are reported in the right-hand panel of Table 2.

By introducing overidentifying individual earnings variables in these equations, one can test for perfect risk sharing within the household. The results show clearly a rejection of full risk sharing since unpredicted earnings shocks for the head, and both predicted and unpredicted individual earnings shocks have a (jointly) significant effect on shares.

Our results amount to a firm rejection of the null hypothesis that changes in earnings are orthogonal to changes in consumption shares. However, the pattern of results suggests another puzzle, as the patterns of signs associated with the earnings variables vary in surprising ways. In particular, an unpredicted one per cent increase in person i’s earnings leads to an estimated 0.04 increase in i’s share of food expenditures relative to the head, but if anything appears to have a negative effect on nutrition. Related, the effect of surprises in the heads’ earnings, though jointly significantly different from zero also have disparate signs, with apparent decreases in expenditures and Calories, and an apparent increase in protein.

However, more surprising is that predictable increases in earnings lead to a quite large and significant decrease in one’s share of expenditures, as well as large (but not individually significant) decreases in Calories and protein. In particular, we estimated that a one percent increase in predicted earnings leads to a 0.78 per cent decrease in the share of food expenditures, a 0.64 per cent decrease in the share of calories, and a 0.21 per cent decrease in the share of protein. This is inconsistent not only with the strong predictions of our model of the unitary household, but is also inconsistent with much less restrictive models, a point we shall return to later.

5.3. Tests of the Nutritional Investment Model. Our second model has the property that the household may make investments in the nutrition of members where the marginal return to those investments may be particularly high. Without much better data on production, this is hard to test directly. However, once again we can marshal some evidence which is at least extremely suggestive.

In particular, as discussed in Section 3, we can also use the consumption expenditures by food categories to implement the same tests. In particular, we can look to see how shocks to earnings affect different of these food categories. The key to our test is to note that if nutritional investment is driving changes in shares, then predicted or realized changes in earnings ought to affect nutritional intakes; e.g., a family member who is expected to spend

---

4Because consumption of some food items is sometimes zero, we replace the logarithmic transformation of food expenditures by the inverse hyperbolic sine (Robb et al., 1992; Browning et al., 1994).
<table>
<thead>
<tr>
<th></th>
<th>Rice</th>
<th>Corn</th>
<th>Staples</th>
<th>Meat</th>
<th>Veg.</th>
<th>Snacks</th>
<th>Other</th>
<th>$F$ ($p$-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\theta}{\psi}: \text{Male}$</td>
<td>1.2287*</td>
<td>1.2498*</td>
<td>1.3484*</td>
<td>1.4999*</td>
<td>1.2036*</td>
<td>1.3690*</td>
<td>1.4421*</td>
<td>1960.8081</td>
</tr>
<tr>
<td></td>
<td>(0.0200)</td>
<td>(0.0210)</td>
<td>(0.0319)</td>
<td>(0.0335)</td>
<td>(0.0283)</td>
<td>(0.0327)</td>
<td>(0.0310)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>$\frac{\theta}{\psi}: \text{Female}$</td>
<td>0.7688*</td>
<td>0.7578*</td>
<td>0.9235*</td>
<td>0.9143*</td>
<td>0.6536*</td>
<td>0.7779*</td>
<td>0.8984*</td>
<td>860.7171</td>
</tr>
<tr>
<td></td>
<td>(0.0208)</td>
<td>(0.0204)</td>
<td>(0.0317)</td>
<td>(0.0291)</td>
<td>(0.0238)</td>
<td>(0.0289)</td>
<td>(0.0260)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>$\frac{\delta}{\psi}: \text{Age male}$</td>
<td>0.0672</td>
<td>0.0693</td>
<td>−0.0224</td>
<td>0.3243</td>
<td>0.3240*</td>
<td>0.0464</td>
<td>0.1557</td>
<td>2.4428</td>
</tr>
<tr>
<td></td>
<td>(0.1161)</td>
<td>(0.1138)</td>
<td>(0.1042)</td>
<td>(0.1748)</td>
<td>(0.1013)</td>
<td>(0.1261)</td>
<td>(0.1187)</td>
<td>(0.0168)</td>
</tr>
<tr>
<td>$\frac{\delta}{\psi}: \text{Age female}$</td>
<td>0.1045</td>
<td>0.0608</td>
<td>0.0104</td>
<td>0.0686</td>
<td>0.0707</td>
<td>0.0265</td>
<td>0.0389</td>
<td>0.3343</td>
</tr>
<tr>
<td></td>
<td>(0.1346)</td>
<td>(0.1318)</td>
<td>(0.1207)</td>
<td>(0.2025)</td>
<td>(0.1175)</td>
<td>(0.1464)</td>
<td>(0.1373)</td>
<td>(0.9386)</td>
</tr>
<tr>
<td>Days sick, male</td>
<td>0.0048</td>
<td>−0.0022</td>
<td>−0.0064*</td>
<td>0.0101*</td>
<td>−0.0081*</td>
<td>0.0039</td>
<td>−0.0041</td>
<td>3.6502</td>
</tr>
<tr>
<td></td>
<td>(0.0027)</td>
<td>(0.0027)</td>
<td>(0.0025)</td>
<td>(0.0041)</td>
<td>(0.0024)</td>
<td>(0.0030)</td>
<td>(0.0028)</td>
<td>(0.0006)</td>
</tr>
<tr>
<td>Days sick, female</td>
<td>0.0008</td>
<td>−0.0033</td>
<td>−0.0033</td>
<td>−0.0082</td>
<td>−0.0041</td>
<td>0.0042</td>
<td>0.0046</td>
<td>1.4864</td>
</tr>
<tr>
<td></td>
<td>(0.0034)</td>
<td>(0.0034)</td>
<td>(0.0031)</td>
<td>(0.0052)</td>
<td>(0.0030)</td>
<td>(0.0037)</td>
<td>(0.0035)</td>
<td>(0.1668)</td>
</tr>
<tr>
<td>Pregnant</td>
<td>0.1394</td>
<td>0.0378</td>
<td>0.0011</td>
<td>−0.1611</td>
<td>−0.0179</td>
<td>−0.0254</td>
<td>−0.0336</td>
<td>0.8679</td>
</tr>
<tr>
<td></td>
<td>(0.0722)</td>
<td>(0.0707)</td>
<td>(0.0647)</td>
<td>(0.1085)</td>
<td>(0.0627)</td>
<td>(0.0783)</td>
<td>(0.0736)</td>
<td>(0.5310)</td>
</tr>
<tr>
<td>Nursing</td>
<td>0.0068</td>
<td>−0.0126</td>
<td>−0.0042</td>
<td>0.0025</td>
<td>0.0057</td>
<td>−0.0058</td>
<td>−0.0056</td>
<td>0.6336</td>
</tr>
<tr>
<td></td>
<td>(0.0088)</td>
<td>(0.0086)</td>
<td>(0.0079)</td>
<td>(0.0132)</td>
<td>(0.0076)</td>
<td>(0.0095)</td>
<td>(0.0090)</td>
<td>(0.7285)</td>
</tr>
<tr>
<td>Second quarter</td>
<td>−0.0461*</td>
<td>0.0794*</td>
<td>0.0185</td>
<td>−0.0254</td>
<td>0.1337*</td>
<td>0.0185</td>
<td>0.0256</td>
<td>11.9649</td>
</tr>
<tr>
<td></td>
<td>(0.0176)</td>
<td>(0.0173)</td>
<td>(0.0159)</td>
<td>(0.0266)</td>
<td>(0.0167)</td>
<td>(0.0194)</td>
<td>(0.0181)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>Third quarter</td>
<td>−0.0029</td>
<td>−0.0074</td>
<td>0.0267</td>
<td>0.0502</td>
<td>0.0089</td>
<td>−0.0380</td>
<td>−0.0101</td>
<td>1.7347</td>
</tr>
<tr>
<td></td>
<td>(0.0179)</td>
<td>(0.0176)</td>
<td>(0.0161)</td>
<td>(0.0270)</td>
<td>(0.0156)</td>
<td>(0.0195)</td>
<td>(0.0184)</td>
<td>(0.0960)</td>
</tr>
<tr>
<td>Fourth quarter</td>
<td>0.0720*</td>
<td>−0.0073</td>
<td>−0.0031</td>
<td>0.0141</td>
<td>0.0096</td>
<td>0.0522*</td>
<td>0.0025</td>
<td>4.1328</td>
</tr>
<tr>
<td></td>
<td>(0.0182)</td>
<td>(0.0175)</td>
<td>(0.0161)</td>
<td>(0.0270)</td>
<td>(0.0156)</td>
<td>(0.0196)</td>
<td>(0.0184)</td>
<td>(0.0001)</td>
</tr>
</tbody>
</table>

**Table 3.** Expenditure shares for different food groups within the household. Point estimates may be interpreted as changes in person $i$'s share of expenditures on each of the various food groups relative to the share of the household head. Figures in parentheses are standard errors.
<table>
<thead>
<tr>
<th></th>
<th>Rice</th>
<th>Corn</th>
<th>Staples</th>
<th>Meat</th>
<th>Veg.</th>
<th>Snacks</th>
<th>Other</th>
<th>F (p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{11}^*:\text{Male}</td>
<td>1.2273*</td>
<td>1.2488*</td>
<td>1.3494*</td>
<td>1.4975*</td>
<td>1.1985*</td>
<td>1.3687*</td>
<td>1.4465*</td>
<td>1958.7733</td>
</tr>
<tr>
<td></td>
<td>(0.0199)</td>
<td>(0.0210)</td>
<td>(0.0319)</td>
<td>(0.0334)</td>
<td>(0.0282)</td>
<td>(0.0327)</td>
<td>(0.0311)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>$\theta_{11}^*:\text{Female}$</td>
<td>0.7685*</td>
<td>0.7589*</td>
<td>0.9222*</td>
<td>0.9133*</td>
<td>0.6547*</td>
<td>0.7776*</td>
<td>0.8940*</td>
<td>859.1147</td>
</tr>
<tr>
<td></td>
<td>(0.0207)</td>
<td>(0.0203)</td>
<td>(0.0317)</td>
<td>(0.0290)</td>
<td>(0.0237)</td>
<td>(0.0289)</td>
<td>(0.0261)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>$\delta_{10}^*:\text{Age male}$</td>
<td>0.0563</td>
<td>0.0910</td>
<td>-0.0112</td>
<td>0.3182</td>
<td>0.3066*</td>
<td>0.0499</td>
<td>0.2041</td>
<td>2.3978</td>
</tr>
<tr>
<td></td>
<td>(0.1166)</td>
<td>(0.1143)</td>
<td>(0.1047)</td>
<td>(0.1755)</td>
<td>(0.1016)</td>
<td>(0.1269)</td>
<td>(0.1196)</td>
<td>(0.0189)</td>
</tr>
<tr>
<td>$\delta_{10}^*:\text{Age female}$</td>
<td>0.0989</td>
<td>0.0697</td>
<td>0.0187</td>
<td>0.0624</td>
<td>0.0670</td>
<td>0.0284</td>
<td>0.0634</td>
<td>0.3544</td>
</tr>
<tr>
<td></td>
<td>(0.1345)</td>
<td>(0.1318)</td>
<td>(0.1208)</td>
<td>(0.2024)</td>
<td>(0.1172)</td>
<td>(0.1465)</td>
<td>(0.1375)</td>
<td>(0.9285)</td>
</tr>
<tr>
<td>Days sick, male</td>
<td>0.0047</td>
<td>-0.0022</td>
<td>-0.0063*</td>
<td>0.0099*</td>
<td>-0.0080*</td>
<td>0.0039</td>
<td>-0.0041</td>
<td>3.5381</td>
</tr>
<tr>
<td></td>
<td>(0.0027)</td>
<td>(0.0027)</td>
<td>(0.0025)</td>
<td>(0.0041)</td>
<td>(0.0024)</td>
<td>(0.0030)</td>
<td>(0.0028)</td>
<td>(0.0008)</td>
</tr>
<tr>
<td>Days sick, female</td>
<td>0.0008</td>
<td>-0.0033</td>
<td>-0.0035</td>
<td>-0.0081</td>
<td>-0.0040</td>
<td>0.0041</td>
<td>0.0041</td>
<td>1.4179</td>
</tr>
<tr>
<td></td>
<td>(0.0034)</td>
<td>(0.0034)</td>
<td>(0.0031)</td>
<td>(0.0052)</td>
<td>(0.0030)</td>
<td>(0.0037)</td>
<td>(0.0035)</td>
<td>(0.1929)</td>
</tr>
<tr>
<td>Pregnant</td>
<td>-0.1340</td>
<td>0.0350</td>
<td>-0.0053</td>
<td>-0.1490</td>
<td>-0.0262</td>
<td>-0.0214</td>
<td>-0.0384</td>
<td>0.8171</td>
</tr>
<tr>
<td></td>
<td>(0.0721)</td>
<td>(0.0707)</td>
<td>(0.0648)</td>
<td>(0.1085)</td>
<td>(0.0626)</td>
<td>(0.0784)</td>
<td>(0.0737)</td>
<td>(0.5729)</td>
</tr>
<tr>
<td>Nursing</td>
<td>0.0064</td>
<td>-0.0124</td>
<td>-0.0036</td>
<td>0.0019</td>
<td>0.0058</td>
<td>-0.0057</td>
<td>0.0047</td>
<td>0.5960</td>
</tr>
<tr>
<td></td>
<td>(0.0088)</td>
<td>(0.0086)</td>
<td>(0.0079)</td>
<td>(0.0132)</td>
<td>(0.0076)</td>
<td>(0.0095)</td>
<td>(0.0090)</td>
<td>(0.7598)</td>
</tr>
<tr>
<td>Second quarter</td>
<td>-0.0632*</td>
<td>0.0820*</td>
<td>0.0381*</td>
<td>-0.0452</td>
<td>0.1396*</td>
<td>0.0191</td>
<td>0.0344</td>
<td>11.8320</td>
</tr>
<tr>
<td></td>
<td>(0.0192)</td>
<td>(0.0189)</td>
<td>(0.0147)</td>
<td>(0.0290)</td>
<td>(0.0179)</td>
<td>(0.0212)</td>
<td>(0.0198)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>Third quarter</td>
<td>-0.0193</td>
<td>0.0054</td>
<td>0.0425*</td>
<td>0.0387</td>
<td>0.0027</td>
<td>-0.0351</td>
<td>0.0160</td>
<td>1.6831</td>
</tr>
<tr>
<td></td>
<td>(0.0203)</td>
<td>(0.0199)</td>
<td>(0.0182)</td>
<td>(0.0305)</td>
<td>(0.0176)</td>
<td>(0.0221)</td>
<td>(0.0208)</td>
<td>(0.1080)</td>
</tr>
<tr>
<td>Fourth quarter</td>
<td>0.1022*</td>
<td>-0.0253</td>
<td>-0.0313</td>
<td>0.0346</td>
<td>0.0168</td>
<td>0.0471</td>
<td>-0.0314</td>
<td>3.9598</td>
</tr>
<tr>
<td></td>
<td>(0.0244)</td>
<td>(0.0238)</td>
<td>(0.0218)</td>
<td>(0.0365)</td>
<td>(0.0211)</td>
<td>(0.0264)</td>
<td>(0.0248)</td>
<td>(0.0002)</td>
</tr>
<tr>
<td>$y_{I_{r+1}}^p$</td>
<td>-0.4689*</td>
<td>0.2663</td>
<td>0.4538*</td>
<td>-0.3725</td>
<td>-0.0590</td>
<td>0.0565</td>
<td>0.5270*</td>
<td>1.9934</td>
</tr>
<tr>
<td></td>
<td>(0.2359)</td>
<td>(0.2314)</td>
<td>(0.2117)</td>
<td>(0.3550)</td>
<td>(0.2047)</td>
<td>(0.2565)</td>
<td>(0.2414)</td>
<td>(0.0520)</td>
</tr>
<tr>
<td>$y_{U_{r+1}}^u$</td>
<td>0.0096</td>
<td>0.0061</td>
<td>-0.0450*</td>
<td>0.0640</td>
<td>-0.0371</td>
<td>0.0061</td>
<td>-0.0456*</td>
<td>2.0214</td>
</tr>
<tr>
<td></td>
<td>(0.0224)</td>
<td>(0.0219)</td>
<td>(0.0201)</td>
<td>(0.0337)</td>
<td>(0.0194)</td>
<td>(0.0243)</td>
<td>(0.0229)</td>
<td>(0.0486)</td>
</tr>
<tr>
<td>$y_{U_{r+1}}^u$</td>
<td>0.2376</td>
<td>-0.4189</td>
<td>-0.1532</td>
<td>0.3310</td>
<td>0.3302</td>
<td>-0.0640</td>
<td>-0.7497*</td>
<td>2.7966</td>
</tr>
<tr>
<td></td>
<td>(0.2264)</td>
<td>(0.2221)</td>
<td>(0.2031)</td>
<td>(0.3406)</td>
<td>(0.1965)</td>
<td>(0.2461)</td>
<td>(0.2321)</td>
<td>(0.0066)</td>
</tr>
<tr>
<td>$y_{U_{r+1}}^u$</td>
<td>0.0708*</td>
<td>-0.0592*</td>
<td>-0.0052</td>
<td>0.0399</td>
<td>-0.0319</td>
<td>0.0376</td>
<td>0.0152</td>
<td>1.7887</td>
</tr>
<tr>
<td></td>
<td>(0.0256)</td>
<td>(0.0251)</td>
<td>(0.0229)</td>
<td>(0.0385)</td>
<td>(0.0222)</td>
<td>(0.0278)</td>
<td>(0.0262)</td>
<td>(0.0847)</td>
</tr>
</tbody>
</table>

**Table 4.** Expenditure shares for different food groups within the household. Point estimates may be interpreted as changes in person $i$'s share of expenditures on each of the various food groups relative to the share of the household head. Figures in parentheses are standard errors.
long hours behind a plow might plausibly receive extra protein in advance of plowing, and extra calories during the same period as the plowing occurs. However, if two different sorts of food both have the same nutritional value, but consumption of one sort gives higher levels of utility (and hence is presumably more costly), then our model of nutritional investment would predict increases in calories and protein in response to increases in earnings, but not necessarily in categories of food which are superior in terms of utility.

Following this logic, we reorganize food expenditures into groups according to type, rather than nutrients. These groups include rice, corn, other staples, meat and fish, vegetables, snacks and fruit, and a residual “other” category. Basic results from our specification for the unitary model appear in the left-hand panel of Table 3.

The expenditure elasticity of individual demand for these food groups shows the same division by gender that we observed for total expenditures and nutrition. The expenditure elasticities for males with respect to the head’s expenditures range from 1.20 for vegetables to 1.50 for meat (all are significantly greater than one), while expenditure elasticities for other females in the household range from 0.65 for vegetables to 0.92 for “other staples” (all are significantly less than one). However, unlike total expenditures, shares of expenditures for most food groups do not increase sharply with age for males. Only for vegetables do we see large and statistically significant increases in expenditure shares, a result consistent with a mounting body of evidence that children don’t like to eat their vegetables (Blanchette and Brug, 2005).

Sickness had no significant effect on total expenditure or nutrition for males, but perhaps these were masked by compositional changes in diet as we do see significant effects across different food groups for males. In particular, there’s some evidence of substitution away from corn (the main staple), “other staples”, and vegetables toward rice, meat, and snacks, a finding which may suggest some “coddling” of sick males. In contrast, though sickness leads females to consume a smaller share of protein, it has no significant effect on expenditures for any given food group. Neither do pregnancy or nursing lead to significant changes in shares of any food expenditure group.

In Table 4 we add earnings changes to the base specification (as with Table 2), and find additional evidence against the unitary household model. Increases in the head’s predicted earnings or a decrease in person i’s predicted earnings both lead to a decrease in person i’s share of rice (the preferred staple) relative to the head, but an increase in i’s share of other less desirable staples and “Other”. This strongly suggests that changes in one person’s expected
earnings leads to rather large compositional changes in diet, even when the overall effects on nutrition are more modest.

The magnitude of the estimated effects of unpredicted changes to earnings are generally much smaller than are the effects associated with predicted changes, but are also often statistically significant. Unpredicted changes to head’s earnings result in significant decreases in $i$’s share of expenditures on “Other Staples” or “Other”, while unpredicted changes to $i$’s own earnings lead to a significant shift in expenditure shares away from corn and toward rice.

Overall, one can see that changes in earnings lead to changes in the composition of diet, perhaps particularly between less-desirable (corn and “other staples”) and more-desirable (rice and meat). These changes pose a challenge to our formulation of the unitary model. Though our formulation of preferences allows demand for different sorts of food to vary with various time-varying observables, risk sharing within the unitary household should rule out variation in diet (either in quantities or composition) in response to earnings shocks.

6. Conclusion

In this paper we’ve constructed a direct test of the hypothesis that food is efficiently allocated within households in part of the rural Philippines. Conditional on our specification of preferences (a generalization of CES utility), we’re able to reject this hypothesis, as the allocation of food expenditures, calories, and protein seems to depend on the realization of each individual’s off-farm earnings.

We then turn to an alternative explanation of this feature of the data. We consider a model in which food consumption produces not only utils, but also functions as a form of nutritional investment, which may be used to directly influence workers’ productivity. Perfectly predictable variation in individual earnings turns out to significantly effect expenditures and nutrition, consistent with the hypothesis of nutritional investment. At the same time, unpredicted shocks to individual earnings tend to lead to increases in total food expenditure shares, but decreases in shares of Calories and protein. Earnings shocks also lead to changes in the composition of diet, in what we interpret as shifts between more and less desirable types of food. We’re left with strong evidence against a unitary household model without nutritional investment, and hints that the allocation of food within the household may serve both to nourish as well as to provide some kind of incentives.
Appendix A. Estimator

In this appendix we devise an estimator with which to estimate the system of equations

$$\Delta \log (x^k_{it}) = \Delta \log (x^k_{1t}) + (\Delta \zeta_{it} - \Delta \zeta_{1t}) \frac{\delta}{\theta_k' \upsilon_i} + \nu^k_{it}$$

for $k = 1, 2, 3$ and with $\nu^k_{it} = \frac{v^k_{it}}{\theta_k' \upsilon_i}$.

$$cov\left(\nu^k_{it}, \nu'^{k'}_{i't}\right) = cov\left(\frac{v^k_{it}}{\theta_k' \upsilon_i}, \frac{v'^{k'}_{i't}}{\theta_k' \upsilon_{i'}}\right) = \frac{1}{\theta_k' \upsilon_i \theta_k' \upsilon_{i'}} cov(\epsilon^k_{it}, \epsilon'^{k'}_{i't})$$

(1) One can assume that

$$cov\left(\nu^k_{it}, \nu'^{k'}_{i't}\right) = \sigma_{kk'} \text{ if } i = i'$$

$$= 0 \text{ if } i \neq i'$$

and

$$E\left(\nu^k_{it} | \Delta \log (x^k_{1t}), (\Delta \zeta_{it} - \Delta \zeta_{1t})\right) = 0$$

In this case, FGLS allows to estimated consistently all parameters, for all $k \frac{\theta_k' \upsilon_i}{\theta_k' \upsilon_{i'}}$, $\frac{\delta}{\theta_k' \upsilon_i}$

(2) One can still have

$$cov\left(\nu^k_{it}, \nu'^{k'}_{i't}\right) = \sigma_{kk'} \text{ if } i = i'$$

$$= 0 \text{ if } i \neq i'$$

but that

$$E\left(\nu^k_{it} | \Delta \log (x^k_{1t}), (\Delta \zeta_{it} - \Delta \zeta_{1t})\right) \neq 0$$

because of an endogeneity problem, that is that shocks $\nu^k_{it}$ are correlated with the head’s shock and thus with the head’s changes of log-consumption (or log-protein, or log-calories).

Then, taking advantage of the availability of household level data on consumption measures, coming from a different module an measurement method of the survey, we use these data as instrumental variables for household head’s changes in consumption or food intakes.

Denoting by $c^k_{it}$ the household level consumption of good $k$ by household of individual $i$ at period $t$, we assume that

$$E\left(\nu^k_{it} | c^k_{it}, (\Delta \zeta_{it} - \Delta \zeta_{1t})\right) = 0$$
Then we can estimate parameters, for all $k$, $\frac{\theta'_{k}v_{i}}{\theta'_{k}v_{i}'}$, $\delta$ by 3SLS.

(3) Assuming that

$$\text{cov}\left(\epsilon_{it}^{k}, \epsilon_{i't}^{k'}\right) = \sigma_{kk} \text{ if } i = i'$$

$$= 0 \text{ if } i \neq i'$$

then

$$\text{cov}\left(\nu_{it}^{k}, \nu_{i't}^{k'}\right) = \text{cov}\left(\frac{\epsilon_{it}^{k}}{\theta'_{k}v_{i}}, \frac{\epsilon_{i't}^{k'}}{\theta'_{k}v_{i'}}\right) = \frac{\sigma_{kk'}}{(\theta'_{k}v_{i})(\theta'_{k'}v_{i'})} \text{ if } i = i' \text{ and } 0 \text{ if } i \neq i'$$

Notations:

$$\epsilon^{k} = \begin{bmatrix} \epsilon_{11}^{k} \\ \vdots \\ \epsilon_{NT}^{k} \end{bmatrix}, X^{k'} = \begin{bmatrix} X_{11}^{k} \\ \vdots \\ X_{NT}^{k} \end{bmatrix} = \begin{bmatrix} (X_{11}^{k}(1), \ldots, X_{11}^{k}(p)) \\ \vdots \\ (X_{NT}^{k}(1), \ldots, X_{NT}^{k}(p)) \end{bmatrix} \text{ (dim: } NT \times p)$$

$$Z^{k} = \begin{bmatrix} Z_{11}^{k} \\ \vdots \\ Z_{NT}^{k} \end{bmatrix} = \begin{bmatrix} (Z_{11}^{k}(1), \ldots, Z_{11}^{k}(k)) \\ \vdots \\ (Z_{NT}^{k}(1), \ldots, Z_{NT}^{k}(k)) \end{bmatrix} \text{ (dim: } NT \times k)$$

$$\varepsilon = \begin{bmatrix} \varepsilon_{1} \\ \varepsilon_{2} \\ \varepsilon_{3} \end{bmatrix} \text{ (dim: } 3NT \times 1), X = \begin{bmatrix} X^{1} & 0 & 0 \\ 0 & X^{2} & 0 \\ 0 & 0 & X^{3} \end{bmatrix} \text{ (dim: } 3p \times 3NT), \beta = \begin{bmatrix} \beta_{1} \\ \beta_{2} \\ \beta_{3} \end{bmatrix} \text{ (dim: } 3p \times 1)$$

$$Z = \begin{bmatrix} Z^{1} \\ Z^{2} \\ Z^{3} \end{bmatrix} \text{ (dim: } 3NT \times k), Y = \begin{bmatrix} Y^{1} \\ Y^{2} \\ Y^{3} \end{bmatrix} \text{ (dim: } 3NT \times 1)$$

Then

$$Y = X'\beta + \varepsilon$$

dim : $(3NT \times 1) = (3NT \times 3p) \times (3p \times 1) + (3NT \times 1)$
Denoting
\[ P_Z = Z(Z'Z)^{-1}Z' \]
\[ \text{dim } : (3NT \times 3NT) \]
we have
\[ \beta_{OLS} = (X'X)^{-1}X'Y \]
\[ \beta_{IV} = (X'P_ZX)^{-1}X'P_ZY \]
\[ \beta_{GLS} = (X'\Omega^{-1}X)^{-1}X'\Omega^{-1}Y \]
\[ \beta_{IVGLS} = (X'P_Z\Omega^{-1}P_ZX)^{-1}X'P_Z\Omega^{-1}Y \]

\[ \text{var} (\beta_{OLS}) = (X'X)^{-1} \]
\[ \text{var} (\beta_{IV}) = (X'P_ZX)^{-1} \]
\[ \text{var} (\beta_{GLS}) = (X'\Omega^{-1}X)^{-1} \]
\[ \text{var} (\beta_{IVGLS}) = (P_ZX\Omega^{-1}X'P_Z)^{-1} \]

where \( \Omega = E(\varepsilon \varepsilon') \)
\[ \Omega = E(\varepsilon \varepsilon') = E \begin{pmatrix} e_1^1 & e_2^1 & e_3^1 \\ e_2^2 & e_2^2 & e_2^3 \\ e_3^3 & e_3^3 & e_3^3 \end{pmatrix}' = \begin{pmatrix} \sigma_{11} e_{11}' & \sigma_{12} e_{12}' & \sigma_{13} e_{13}' \\ \sigma_{22} e_{22}' & \sigma_{22} e_{22}' & \sigma_{23} e_{23}' \\ \sigma_{33} e_{33}' & \sigma_{33} e_{33}' & \sigma_{33} e_{33}' \end{pmatrix} \] (dim : \(3NT \times 3NT\))

Using the matrix notations:
- When
  \[ \text{cov} \left( \varepsilon_{i,i'}, \varepsilon_{i,i'}' \right) = \sigma_{kk} \text{ if } i = i' \]
  \[ = 0 \text{ if } i \neq i' \]
then
\[ \Omega = \Sigma_3 \otimes I_{NT} \]
with
\[ \Sigma_3 = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{pmatrix} \]
\[ \Omega^{-1} = \Sigma_3^{-1} \otimes I_{NT} \]
Thus
\[ \beta_{GLS} = \left( X' \left( \Sigma_3^{-1} \otimes I_{NT} \right) X \right)^{-1} X' \left( \Sigma_3^{-1} \otimes I_{NT} \right) Y \]
\[ \beta_{IVGLS} = \left( X'P_Z \left( \Sigma_3^{-1} \otimes I_{NT} \right) P_ZX \right)^{-1} X'P_Z \left( \Sigma_3^{-1} \otimes I_{NT} \right) Y \]
• When 
\[ \text{cov} \left( \varepsilon_{it}^k, \varepsilon_{i't}^{k'} \right) = \sigma_{kk'} \lambda (z_{it} - z_{i't'}) \]
then
\[ \Omega = \Sigma_3 \otimes \Sigma(z) \]
with
\[ \Sigma(z) = I_{NT} + \lambda \left[ (z \ast J_{1,NT}) - (z \ast J_{1,NT}') \right] \]
\[ \Omega^{-1} = \Sigma_3^{-1} \otimes \Sigma(z)^{-1} \]

• When 
\[ \text{cov} \left( \varepsilon_{it}^k, \varepsilon_{i't}^{k'} \right) = \sigma_{kk'} \lambda (z_{it} - z_{i't'}) \]
\[ \text{cov} \left( \frac{\varepsilon_{it}^k}{\theta_k^i v_i}, \frac{\varepsilon_{i't}^{k'}}{\theta_k^{i'} v_{i'}} \right) = \sigma_{kk'} \frac{\lambda (z_{it} - z_{i't'})}{(\theta_k^i v_i) (\theta_k^{i'} v_{i'})} \]
then
\[ \Omega = \Sigma_3 \otimes \Sigma(z) \]
with
\[ \Sigma(z) = \left( I_{NT} + \lambda \left[ (z \ast J_{1,NT}) - (z \ast J_{1,NT}') \right] \right) \cdot (A) \]
where ·/ is for the element by element division and
\[ A \left[ it, i't' \right] = (\theta_k^i v_i) (\theta_k^{i'} v_{i'}) \]
\[ \Omega^{-1} = \Sigma_3^{-1} \otimes \Sigma(z)^{-1} \]

REFERENCES


TOULOUSE SCHOOL OF ECONOMICS (IDEI, INRA, GREMAQ)

UNIVERSITY OF CALIFORNIA, BERKELEY