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Journal
Astrophysics and Space Science, 104(1)

ISSN
0004-640X

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Publication Date
1984-09-01

DOI
10.1007/bf00653998

THE ANGULAR MOMENTUM-VS-MASS RELATION FOR SPECTROSCOPIC BINARIES

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(Received 24 January, 1984)

Abstract. The spectroscopic binaries, considered as a single data point, fall roughly on the 'universal' power-law of index 1.8 for angular momentum vs total mass, as defined by planets, spiral galaxies, and numerous other objects. But the individual systems in the *Seventh Catalogue of the Orbital Elements of Spectroscopic Binary Systems* define a curve of rather shallower slope, 1.63 $\pm$ 0.07, over more than two orders of magnitude in mass and four in angular momentum. Various subsets (long and short periods; single and double line systems; known and unknown orbital orientations) all yield slopes from 1.48 to 1.77. These values, as well as the slightly larger one found for eclipsing systems by Sisteró and Marton, are very much what one would expect, given the form of Kepler's Third Law and the stellar mass-radius relation.

If only these well-known pieces of physics are at work, then the still-wider visual binaries should yield a slope near $\frac{5}{3}$. Catalogues currently in press will permit easy testing of this prediction. It seems unlikely that deep clues to the origin of either binary systems or angular momentum are to be found from considerations of this nature.

1. Introduction

The history of radio astronomy reveals that, when you do not understand a phenomenon very well, it is sometimes useful to plot it on log-log paper (Scheuer, 1968). Formation processes – for planetary systems, stars, galaxies, and whatever else – are arguably the least-understood phenomena of astronomy today. Brosche (1963) appears to have been the first to suggest that one might gain insight into them by plotting total angular momentum vs total mass in logarithmic coordinates for as many kinds of objects and systems as could be measured.

On scales from asteroids and planets to whole galaxies, he found an approximate universal relation, with $J \propto M^b$ and $b \approx 2$, the scatter being typically 1–2 orders of magnitude over a range of nearly 20 orders in M and 40 in J. Other authors have made similar plots on scales from the whole universe (Sisteró, 1983) down to single galaxies (Carrasco *et al.*, 1982) and binary stars (Sisteró and Marton, 1983). The values of b found range from 1.7 to 2.0; and the conclusions drawn range from prosaic reaffirmations of Kepler's laws and known properties of spiral galaxies (Carrasco *et al.*, 1982) to the almost-mystical (Muradian, 1980; Sisteró, 1983).

My attention was drawn to the problem by the work of Sisteró and Marton (1983) on the 1048 eclipsing binary systems for which orbit parameters are tabulated by Brancewicz and Dworak (1980). They found exponents of 1.88 $\pm$ 0.07 for the total population and various subsets, with both the $\log J$ vs $\log M$ plot and a single point for

Astrophysics and Space Science **104** (1984) 133–143. 0004–640X/84/1041–0133$01.65.

the average of all systems falling close to previous 'universal' relations. It seemed (and was) straightforward, though tedious, to make similar plots for the 978 spectroscopic systems whose orbital elements are given in the *Seventh Catalogue* (Batten *et al.*, 1978). Section 3 describes this process and its results.

2. Predictions

Kepler's third law and the definition of orbital angular momentum can be combined in several ways. One yields

$$J \sim M^{5/3} P^{1/3} . \tag{1}$$

Thus, a population of binaries with a restricted range of period, but otherwise randomly selected, should show $b = 1.67$. The other combination yields

$$J \sim M^{3/2} a^{1/2} . \tag{2}$$

For systems which are or have been in contact, the semi-major axis, a, must be roughly the sum of the stellar radii. The mass-radius relation for Main-Sequence stars changes slope at 1.5–2.0$M_{\odot}$ as the envelopes go from convective to radiative, but a crude average for spectral types B–K is $R \propto M^{0.8}$. Thus, contact or near-contact systems should show

$$J \sim M^{1.5} M^{0.4} = M^{1.9} . \tag{3}$$

This is very close to what Sisteró and Marton (1983) found for eclipsing systems, a large fraction of which either are contact systems or are likely to have passed through a contact phase during pre-Main-Sequence contraction.

3. Analysis and Results for Spectroscopic Binaries

The *Seventh Catalogue of the Orbital Elements of Spectroscopc Binaries* (Batten *et al.*, 1978) lists periods, P, velocity amplitudes, K_1 or K_1 and K_2, and eccentricities for 978 systems with published orbits. P and K immediately yield $a \sin i$, where a is the semi-major axis for the star whose lines are measured and i is the angle of inclination of the orbit plane to the plane of the sky. Auxiliary information tabulated includes spectral types and i, where it is known from a light curve or a visual/astrometric orbit.

To turn these into masses and angular momenta, it is necessary (a) to estimate the mass of the primary star from its spectral type, (b) to deduce the most probable value of the mass ratio $q = M_2/M_1$ for the two-thirds of the systems where only one set of lines was measured, and (c) to choose a likely value of $\sin i$ for the 700 systems where it is not known.

The procedures used for (a), (b), and (c) are described by Trimble (1974, 1978) and are very similar to those used by Kraitcheva *et al.* (1979) and others. The mean expected value of $\sin i$ for a population randomly oriented in space, but with discovery probability proportional to velocity amplitude is 0.879; and q can be calculated unambiguously from

M_1, i, and a so-called mass function, which depends only on P, K_1, and (weakly) eccentricity. Then, for circular orbits,

$$J = M_1(1 + 1/q)\left(\frac{a_1 \sin i}{\sin i}\right)^2 \frac{2\pi}{P} \,. \tag{4}$$

Treated in this way, the systems yield total masses from 9×10^{32} to 2×10^{35} g and orbital angular momenta from 8×10^{51} to 3×10^{55} g cm^2 s^{-1}. The rotational angular momenta are not necessarily negligible for close systems in corotation, being of order $(R/a)^2$ times the orbital ones. But very few systems have published rotational velocities; thus the rotational moments have been ignored.

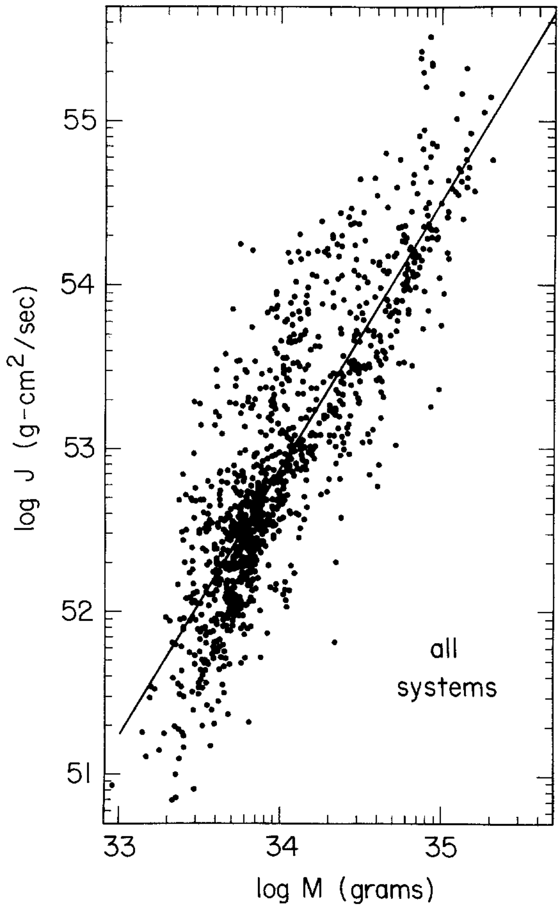

Fig. 1. Plot of angular momentum, J, vs mass, M, for all spectroscopic binaries in the Batten *et al.* (1978) catalogue (cgs units, logarithmic coordinates). Each of the 978 systems is represented by a single point, and the straight line is a formal least-squares fit to $\log J = \log J_0 + b \log M$, with all points given equal weight.

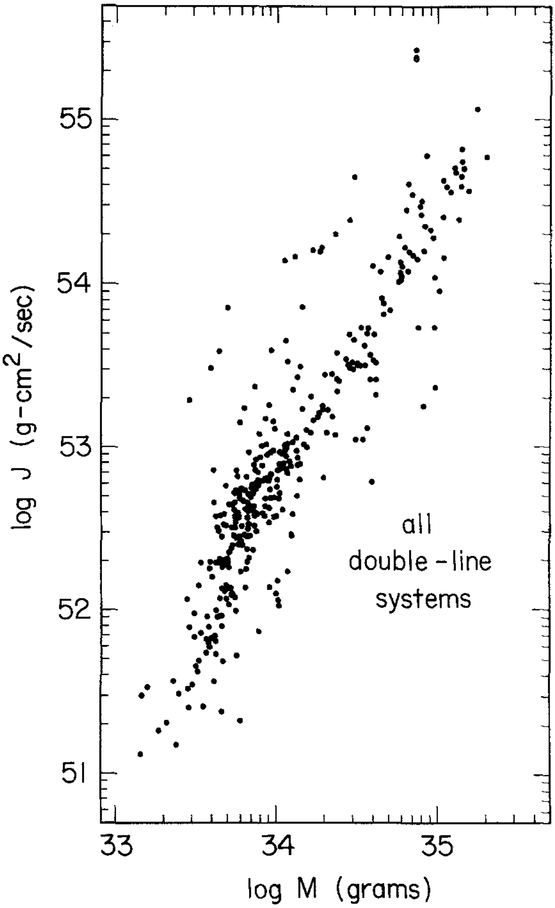

Fig. 2. Same as Figure 1, but for the 333 double-line systems, separately.

Figures 1–8 show J vs M for the 978 SB's and subsets of known and assumed orbital orientation; of measured and calculated mass ratio; and of long and short period. The straight lines, where shown, are formal least-squares fits to $\log J = \log J_0 + b \log M$. Because the eye tends to perceive the regions of maximum point density and to ignore outliers completely, a free hand 'best fit' would look rather different for some of the samples. The break to a steeper slope for masses less than about 5×10^{33} g seems to be a real feature of the total population and most subsamples. It is also apparent in the plots of Sisteró and Marton (1983) for eclipsing systems.

Table I lists $\log J_0$ and b for the various samples. All slopes are in the range 1.48 to 1.77, smaller than those found for eclipsing systems by Sisteró and Marton (1983), and smaller than typical 'universal' slopes of 1.8 to 2.0.

A single point representative of the spectroscopic binaries as a whole does, however, fall reasonably close to the universal relations found by Brosche (1963) and later

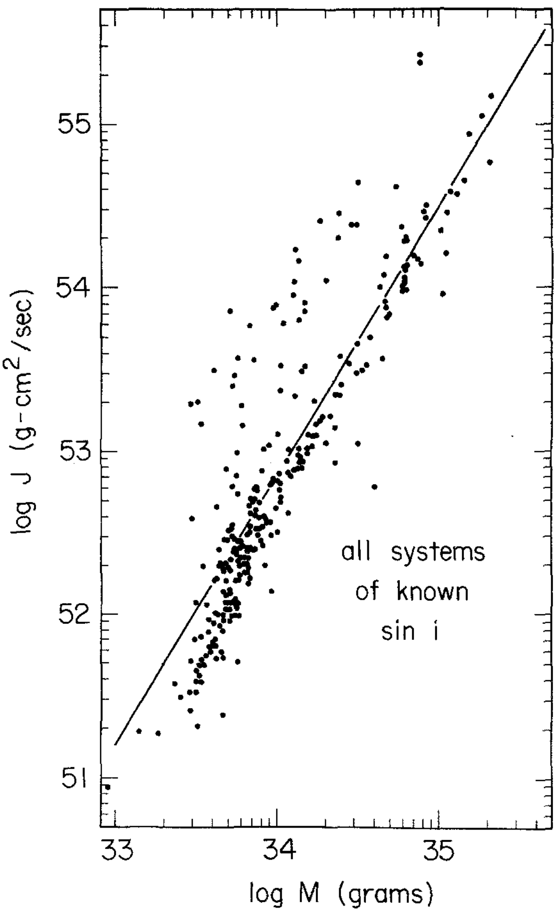

Fig. 3. Same as Figure 1, but for the 272 systems of known orbital orientation i, separately.

authors. The centroid of the points shown in Figure 1 occurs at $M = 1.07 \times 10^{34}$ g and $J = 9.02 \times 10^{52}$ g cm^2 s^{-1}. The universal J–M relation can be approximated by a curve through the data for the Earth ($M = 5.976 \times 10^{27}$ g; $J = 5.885 \times 10^{40}$ g cm^2 s^{-1}; Weast and Astle, 1982) and the metagalaxy ($M = 2.7 \times 10^{55}$ g; $J = 1.2 \times 10^{91}$ g cm^2 s^{-1}; Sisteró, 1983). This yields $\log J_0 = -9.75$ and $b = 1.82$ and a prediction of $J = 1.4 \times 10^{55}$ g cm^2 s^{-1} for $M = 1.07 \times 10^{34}$ g. Thus, an averge point representing the spectroscopic binaries falls somewhat above the universal relation, but within the order-of-magnitude scatter representative of other kinds of systems. Eclipsing binaries have shorter periods, on average, than spectroscopic ones and thus lower J's for a given M. The least squares fit of Sisteró and Marton (1983) gives $J = 5.5 \times 10^{52}$ g cm^2 s^{-1} at $M = 1.07 \times 10^{34}$ g, also well within the general scatter.

Comparison of the various subsamples shown in the figures and in Table I does not produce any surprises. The most discordant b's belong to the smaller samples, and

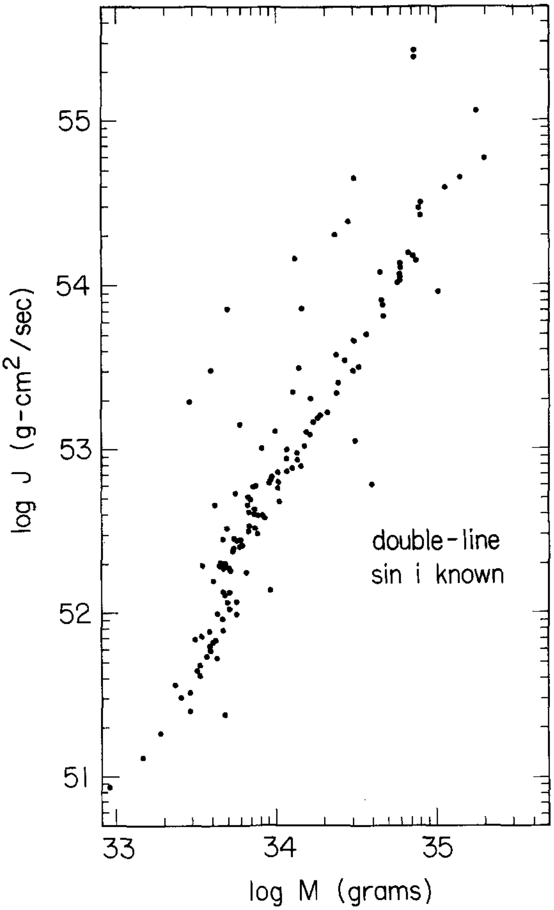

Fig. 4. Same as Figure 1, but for the 134 double-line systems of known i.

systems grouped in hundreds by right ascension yielded b's from 1.45 to 1.78, suggesting that the differences in slope among subsamples are purely statistical. The short period ($P < 2^{\mathrm{d}}$) and long period ($P > 1000^{\mathrm{d}}$) groups have nearly the average slope, and, not surprisingly, it is very nearly the one predicted by Equation (1) for a sample with restricted period range obeying Kepler's laws. The difference between $\log J_0$ values for the two (-2.73 for short P's; -1.51 for long ones) is that expected from Equation (1) for two groups differing by a factor of 5000 in mean period, which is very nearly the case for these two samples.

4. Implications and Conclusions

Features of the present results that seem to require some explanation include (a) the value of the slope, b, and its deviation from the one for eclipsing binaries, (b) the break

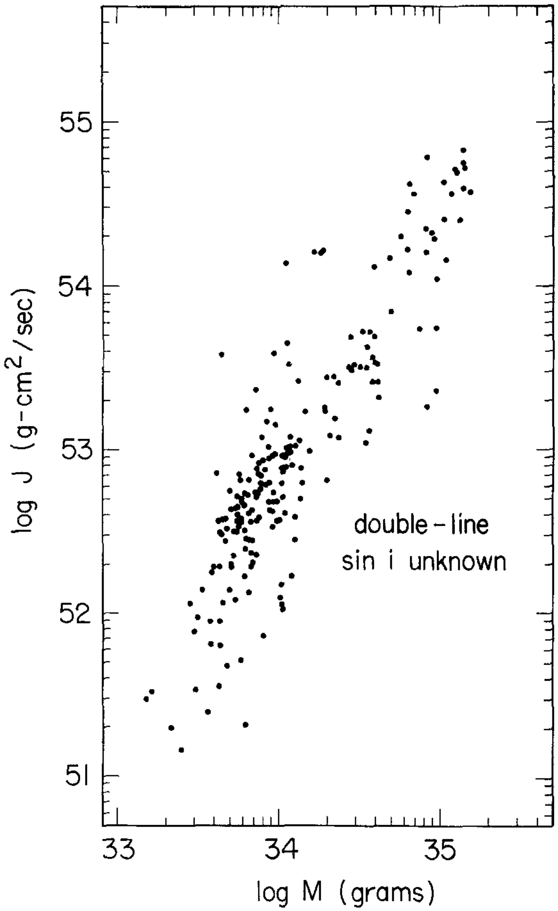

Fig. 5. Same as Figure 1, but for the 199 double-line systems whose orbital inclination i, is not known.

in slope for both kinds of systems near $M = 5 \times 10^{33}$ g (readily noticeable in Figures 1–4), and (c) the near-agreement between an average point representing all systems and the prediction of the universal relation.

The slope, $b = 1.93$, found for eclipsing binaries by Sisteró and Marton (1983) is very nearly what Equation (3) leads us to expect for systems that are or have been in contact. The shallower slope, $b = 1.62$, found here for spectroscopic systems is, on the other hand, quite close to that expected from Equation (1) for a sample dominated simply by Kepler's laws. If there is no more physics involved than this, then a sample of visual binaries, which have surely not passed through a contact phase, should also yield a b near $\frac{5}{3}$. I intend to test this prediction when the new catalogues of visual binary orbits (Worley and Hinds, 1984) and parallaxes (van Altena, 1984) become available.

The break in slope occurs at a system mass where, for average values of mass ratio, the primary is switching from a convective to a radiative envelope. Since upper Main-

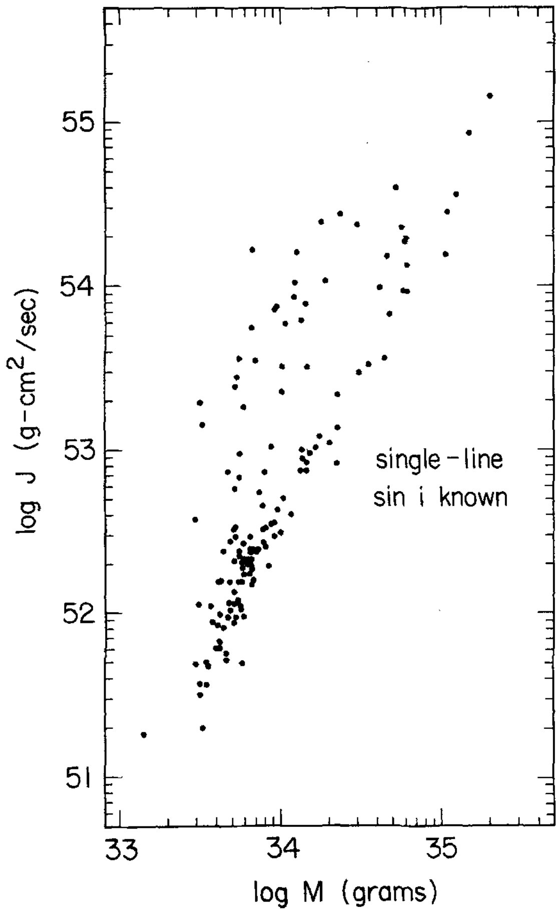

Fig. 6. Same as Figure 1, but for the 138 single-line systems of known i.

Sequence, radiative stars have $R \propto M^{0.6}$ and lower Main Sequence, convective ones have $R \propto M^{1.0}$, Equation (2) leads us to expect a steepening of the slope at low masses of the sort seen should occur for contact systems. That spectroscopic as well as eclipsing systems show the effect suggests that binary systems not currently in contact may acquire their final angular momenta quite early in the pre-Main-Sequence contraction phase, though other arguments based on statistics of systems near contact (Rucinski, 1983) suggest rapid loss of angular momentum during late Hayashi and early Main-Sequence phases. Visual binaries have surely never been in contact. Thus, if a pre-Main-Sequence contact phase is responsible for the slope change, they will surely not show the effect. If they do, another explanation must be sought.

Prosaic remarks can also be made about point (c). The constraints imposed by the finite sizes of stars and by the short lifetimes of systems whose separations are not small compared to the average interstellar distance guarantee that a system of

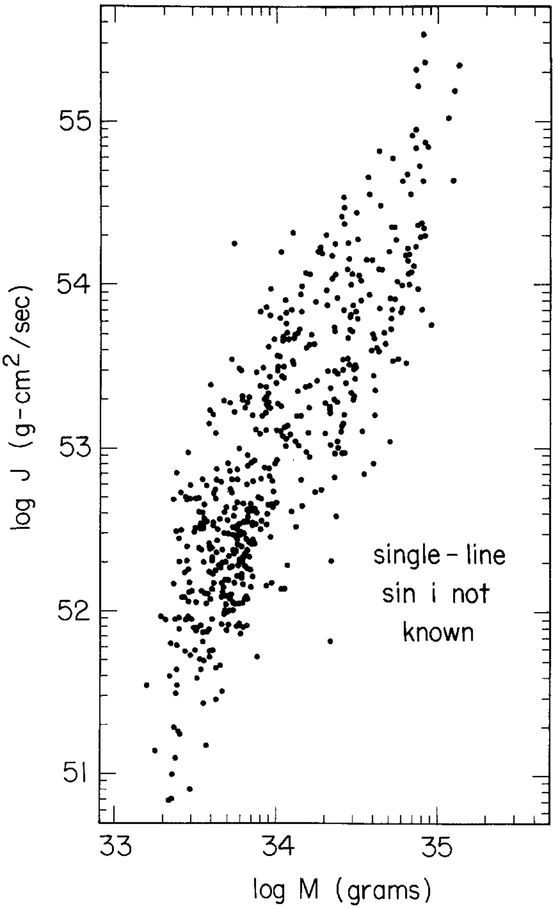

Fig. 7. Same as Figure 1, but for the 507 single-line systems whose orbital inclination i, is not known. Predictably, this sample shows the widest scatter.

$M = 1.07 \times 10^{34}$ g must have J between about 6×10^{51} and 5×10^{55} g cm^2 s^{-1}. Addition of visual binaries and common proper motion pairs to the available SB's and EB's will produce a sample covering the whole possible range and having an average J somewhere in the middle, at a value dependent largely on the relative numbers of the three kinds of systems included.

An equivalent statement is that $J(M)$ for the total binary population is largely a consequence of the narrow mass range possible for stars and the actual distribution of semi-major-axes. This distribution is rather flat over the whole possible range for the few nearly-complete samples available, leading to $\langle J \rangle \approx 10^{54}$ g cm^2 s^{-1} at our reference momentum of 1.07×10^{34} g. Admittedly, the cause for the flat distribution in a is not well understood, but the problem now sounds like it belongs to the details of star formation theory rather than to fundamental physics. This result and those of Carrasco

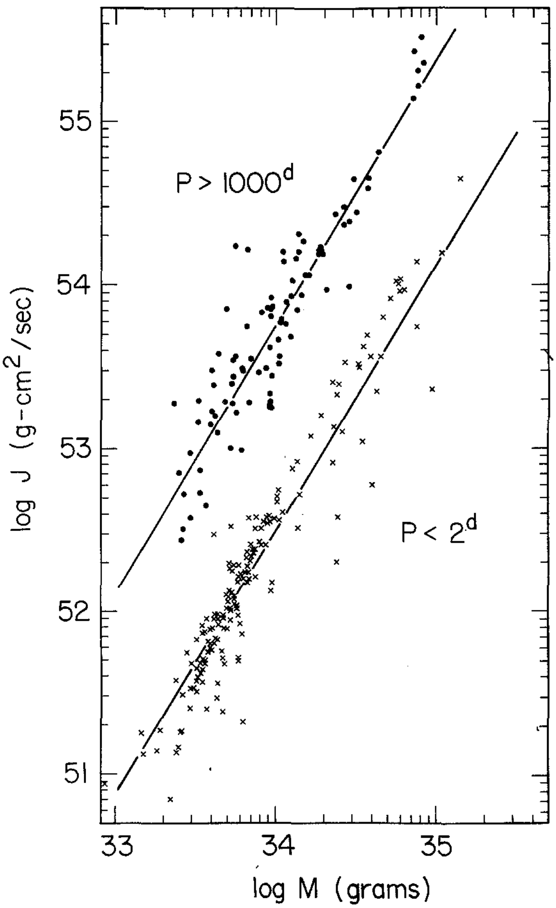

Fig. 8. Same as Figure 1, but for the longest period ($P > 1000^d$) and shortest period ($P < 2^d$) systems, separately.

et al. (1982) for spiral galaxies raise the suspicion that the other portions of the universal $J(M)$ relation may be susceptible of equally prosaic explanations and that there is less to it than meets the eye.

Acknowledgements

I am grateful to Robert S. Harrington, USNO, for acting as a guide to the literature of visual binaries, and to Tom Hartquist for a penetrating remark on their use to distinguish alternative hypotheses.

TABLE I

Least squares fits to $\log J = \log J_0 + b \log M$ (cgs units) for the spectroscopic binaries and various subsamples thereof. Small differences in b make large differences in $\log J_0$ simply because the systems all have masses very much larger than 1 g. The values found for eclipsing binaries by Sisteró and Marton (1983) and a 'universal' relation, drawn through data points for the Earth and the metagalaxy, are given for comparison

Sample	N	$\log J_0$	b
All systems	978	−2.17	1.62
RA = 0–12 hr	451	−4.39	1.68
RA = 13–24 hr	527	−0.13	1.56
Known i	272	−5.62	1.72
Unknown i	706	−0.78	1.58
Double line	333	−1.59	1.60
Single line	645	−2.82	1.64
Known i, double line	134	−7.36	1.77
Known i, single line	138	−3.89	1.67
Unknown i, double line	199	+2.51	1.48
Unknown i, single line	507	−2.46	1.63
$P < 2$ days	163	−2.73	1.62
$P > 1000$ days	96	−1.51	1.63
Eclipsing binaries	1084	−12.81	1.928
Universal		−9.75	1.819

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