Investigation of Magnetic Field and Current Topology in Z-pinch Plasmas

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Investigation of Magnetic Field and Current Topology in Z-pinches

A dissertation submitted in partial satisfaction of the requirements for the degree Doctor of Philosophy in Engineering Sciences (Engineering Physics)

by

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2015
The dissertation of Derek Alexander Mariscal is approved, and it is acceptable in quality and form for publication on microfilm and electronically:

Chair

University of California, San Diego

2015
DEDICATION

To Tom L. Egger, thank you for instilling the skepticism and obdurate determination necessary for the pursuit of a career in science.
Modern science has been a voyage into the unknown, with a lesson in humility waiting at every stop. Many passengers would rather have stayed home.

—Carl Sagan
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ABSTRACT OF THE DISSERTATION

Investigation of Magnetic Field and Current Topology in Z-pinches

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Z-pinches plasma dynamics are largely determined by the current and magnetic field topology of the system. Measurement of the magnetic fields allows for the inference of the current distribution, but is in practice difficult to measure in experiments. This leads to a high dependence on numerical simulations for extracting this information, but without quality experimental data, may not be entirely reliable for this purpose.

Proton deflectometry, or proton radiography, is a relatively new diagnostic developed for investigating electromagnetic fields in high energy-density plasmas since it provides data with high spatial and temporal resolution compared to traditional field diagnostics. It was developed in the laser-plasma-interaction commu-
nity as a means of determining electric and magnetic field strength and orientation during laser-driven plasma experiments. In this work, the method was developed for use on Mega-Amp-scale pulsed-power-driven plasma experiments.

In one configuration, a proton beam was directed radially, with respect to the z-axis of a pulsed-power-driven short-circuit load. In this setup, an azimuthally symmetric magnetic field is generated around the short-circuit load, as a current pulse, 0.6 MA in 0-100% rise-time 200 ns with an approximately sine-squared waveform.

Scaled laboratory astrophysics experiments modeling the dynamics of universal astrophysical phenomena such as plasma jets have recently become an area of great interest. Such experiments are vital to resolving long-standing questions about the roles of various physical processes in the dynamics of such objects. The application of proton deflectometry to scaled laboratory astrophysics experiments revealed details of the current and magnetic field topology which was previously accessible only in numerical codes. One such load is the radial foil load, designed to replicate the propagation of a jet during the formation stages of a star. The data from this work was used to benchmark a resistive MHD code, Gorgon, designed to reproduce the Z-pinch experiments as well as astrophysical phenomena. The simulation results were found to agree with the experimental data, meaning that the current and magnetic field topology could be recovered from the code. The demonstration of this diagnostic technique opens up many possibilities for examining the current and magnetic field topology in other Z-pinch experiments.
Chapter 1

Introduction to Z-pinch Physics

1.1 Introduction

This chapter is meant to give a brief overview of the broad range of experiments known as Z-pinches, or as also commonly known, zed pinches. The “Z” in Z-pinch refers to the typical direction of current driven by a pulsed power driver, while the pinch refers to the phenomenon of plasma acceleration toward the z-axis of the load, which is typically azimuthally symmetric about this axis, by the Lorentz, or $\mathbf{J} \times \mathbf{B}$, force, as will be outlined below. Here, the intention is to elucidate the purpose of Z-pinches, as well as some of the physical processes that control the behavior of these complex systems.

Much of the research in Z-pinches over the course of the 20th century has been driven by experiments aimed at initiating and sustaining thermonuclear reactions, and therefore some of the examples given will be closely related to this motivating scheme, such as the equilibrium Z-pinch, and wire array Z-pinches. Through the explanation some of the basic physics involved in these experiments, it will be seen that the Z-pinch has evolved into an extremely useful tool in several other aspects of research, often falling under the category of High Energy Density Physics (HEDP), which is most simply defined as a system in which a pressure (or energy density) of one MegaBar is reached, $P \geq 1 \text{ MB}$ [11]. These systems have many applications including, but not limited to, Inertial Confinement Fusion (ICF) schemes [12,13] and scaled laboratory astrophysics experiments.
This brief examination will also present some of the challenges associated with precisely controlling the physical processes, especially instability-forming mechanisms, which are often detrimental to their desired use. Once the reader is familiarized with the Z-pinch, it will then be clear as to why recovering the details of the magnetic field topology, and consequently the current configuration which supports it, is of importance in this class of experiments.

The latter class, scaled laboratory astrophysics, will be the primary focus of this work, specifically in examining experiments in the laboratory which are representative of the dynamics involved in jet launching and collimation in young stellar objects. For example, although wire array Z-pinch experiments have been largely used to create powerful X-ray sources, when the configuration is changed they may be used to create plasma jets where the magnetic field may be dynamically significant. Astrophysics has historically been confined primarily to observation, from which theoretical models are constructed. Numerical simulations utilizing these theoretical models and constrained by observational data, have been very useful tools for investigating astrophysical phenomena, however, there is often still disconsensus in the validity of the models, which are necessarily incomplete due to the relatively limited observational data [14,15].

Recent advances in technology, especially in the development of high power drivers, both laser and pulsed-power, have enabled the study of astrophysical phenomena experimentally [16]. Information gathered from these types of experiments is vital to improving theory, and consequently the numerical tools used to investigate not only astrophysics, but also a wide range of plasma physics. Again, in these types of experiments, the magnetic field is often dynamically relevant, and thus must be properly characterized in order to make any sort of meaningful connection to astrophysical systems.

1.2 Laboratory Astrophysics with Z-pinches

Although Z-pinch research was initially, and still is, intended as a potential means of achieving fusion energy (itself also an astrophysical phenomenon),
it has also proven to be a valuable tool in astrophysics research [16–18]. This is in large part due to the advancements in technology which have enabled access to the High Energy Density Physics, HEDP, regime, in which pressures $\geq$ 1 MBar are achieved [11]. Laboratory astrophysics experiments have been motivated by a disconnect in astrophysics between observational data and models [14, 15]. Both Z-pinches and laser-driven experiments have seen recent progress in modeling astrophysical phenomena in the laboratory [16]. One area in which Z-pinches may be of particular value is in the modeling of astrophysical outflows, often manifesting in the form of jets, and especially in cases where the magnetic field is thought to be dynamically significant [16].

Jets and outflows are ubiquitous in the universe, observed to emanate from Young Stellar Objects, YSO’s, with accretion disks [19], as well as from the center of galaxies [20]. These objects may evolve on vastly different spatial scales. For example, those from Herbig-Haro objects (HH), are observed to span up to $\sim$1 parsec, while outflows from the centers of galaxies may be on the order of $\sim$100’s of parsecs. Through observations, it is difficult, if not impossible, to observe the launching region of jets in HH objects due to the presence of the accretion disk and surrounding envelope, which typically obscure this region. Thus, the only way to determine the launching mechanism is by observing the jet far from the source, which has emerged from this region, and use it as a history of the ejection to work backwards from.

These, as with most observational data, are complicated by the time-scale on which astrophysical phenomena typically evolve, often much greater than human lifespans, and of course, the spatial scales, which are many orders of magnitude greater than human experience. The advancement of technology in pulsed-power driven experiments has enabled the study of these processes, to varying degrees of relevancy, due to the advantageous use of scaling. High power laser experiments have demonstrated the ability to perform experiments relevant to supernova explosions and hypersonic, radiatively cooled, plasma jets.

Initially, much of the astrophysical research with Z-pinches focused on the acquisition of data concerning plasma properties. These include opacity measure-
ments, spectral data, and equation-of-state data [16]. More recently, it was realized that astrophysical dynamics could be modeled in the laboratory, with a good degree of similarity, by using special Z-pinch configurations [21, 22]. This effort was initialized by Imperial College, in London, UK, at the Mega-Ampere Generator for Implosion Experiments, MAGPIE, facility, and promptly followed by various other Z-pinch facilities in the United States. Early plasma jet experiments utilizing a Z-pinch driver were carried out via conical wire arrays on the MAGPIE, by Lebedev and Ampleford [21, 23]. In this configuration, experiments were able to drive high Mach number, hydrodynamically collimated jets, which are similar to many astrophysical jets. The conical wire array jets were primarily collimated by hydrodynamic means, but a more accurate picture of jets from astrophysical objects requires the inclusion of a dynamically significant magnetic field during the launching and collimation phases.

To this end, radial wire arrays [22], and radial foil loads [24], have been used to produce hypersonic, radiatively cooled, and \textit{magnetically driven} plasma jets. These jets are similar to those in the so-called “magnetic tower” model proposed by [25].

There are two sets of equations which are useful for describing astrophysical outflows. One is the equations of hydrodynamics, or Euler equations, in which the plasma is not magnetized. The other is is the ideal MHD equations, which are relevant when magnetic fields are dynamically significant. Both of these sets of equations allow for the comparison of experimental systems to astrophysical systems so long as care is taken in defining and obtaining relevant similarity parameters. In this way, Z-pinch plasma systems may be used to aid in the understanding of the dynamics of these large, slowly evolving, systems.

While the ultimate goal of scaled laboratory astrophysics experiments is to simulate an astrophysical process from start to finish, in general, the similarity between the systems is often short-lived, and thus only a piece of the astrophysical picture is typically obtained in any single experiment [11]. Further, the conditions in the laboratory systems are often not entirely satisfactory in terms of directly scaling to astrophysical outflows. These types experiments are, however,
very important, since they can challenge numerical codes, e.g. hydrodynamic, radiation-hydrodynamics, and MHD codes. If the codes are able to accurately reproduce experimental systems which are similar to astrophysical phenomena, then the results of numerical models of astrophysical systems can be taken with more confidence. This is the realm of this work, where the data from experiments is used to benchmark a MHD code, with experiments which create conditions similar to astrophysical outflows.

1.2.1 Observations and Models of Plasma Jets

Although plasma jets are observed to be associated with many phenomena in the universe, this discussion will be restricted to YSO’s. High velocity, $v_{\text{jet}} \sim 100-1,000$ km/s, collimated outflows, often $\leq 10^\circ$, or jets, from protostellar objects may be the most commonly observed outflows [26]. A YSO is essentially a protostellar object with an associated accretion disk, whereas an HH object is a system with an associated/observable outflow. In observations, a bow shock at the leading edge of the jet, due to the interaction of the outflow with the interstellar medium (ISM), as well as knotty structure along the length of the jet, indicating that rather than a continuous outflow, these jets often have multiple ejection events. These outflows are important in star formation for several reasons. One of which is that they remove significant mass and angular momentum from the star-disk system. This helps to avoid spin-up due the contraction during the accretion phase. Secondly, on larger scales, they can help disrupt star-forming clouds, by injecting this momentum into neighboring regions. Examples of these types of outflows are seen in Figures 1.1 and 1.1.

Along with these long, highly collimated outflows, are less collimated “molecular outflows” and disk winds. These outflows are typically nearer to the source, and expand radially with respect to the source object. The typical velocities for these outflows are slower by comparison with jets, at $\sim 1-30$ km/s, and the eventually form a large cavity in the surrounding cloud/envelope environment.
Figure 1.1: Three Harbig-Haro objects, showing jets from young stellar objects, and the jets extending from the source. Top-A long clumpy jet. Bottom-Left-The complex bow structure of the jet interacting with the interstellar medium (ISM) Bottom-Right-Complex shock structure from bow shock piling up (Credit-NASA, ESA, and P. Hartigan).
Figure 1.2: Harbig-Haro object 111, showing the remarkable degree of collimation in the jet. The jet extends 12 light years, or $10^{19}$ cm, from the source. Infrared observations are used to better view the source object which is surrounded by an envelope of gas and dust, while the jet, which has exited the envelope, is viewed in the visible spectrum. (Credit-NASA, Hubble Space Telescope, and B. Reipurth (CASA, University of Colorado).

The Magnetic Tower Model

In young stellar objects, the primary jet launching mechanism is believed to be due to a dynamically significant magnetic field \cite{14,15,25} in a model known as the magnetic tower configuration. In this model, initially poloidal (or axial) magnetic field lines are threaded through the source object as well as the accretion disk. Differential rotation between the star and the disk allows for the “winding up” of the magnetic field, resulting in a primarily toroidal magnetic field configuration, which extends vertically with respect to the axis of rotation.

Due to the frozen in condition, the lines are not able to splay out to infinity, as would be the case in the absence of a medium surrounding the accretion disk, but rather must wind up and increase in their extent in the vertical direction, with respect to the cylindrical geometry of the system \cite{25}. This is due to the fact that the pressure of the ambient medium would require a large amount of work in order to move the surrounding medium. Thus, rather than field lines increasing in radius after each turn, the vertical extent must increase with each turn. The wound magnetic field will become very long and thin after many turns, as seen in Figure 1.4.
Figure 1.3: Schematic of basic properties of a outflows from YSO’s on two different length scales. a) Jet launching and collimation originates at the disk scale. b) At the envelope scale, the bow shock, knotty structure of the collimated can be seen propagating far from the source. Figure from Frank et al. Protostars and Planets VI (2014) [14,15]

Figure 1.4: The developmental stages of the magnetic tower jet collimating configuration. The flux lines are frozen in to the disk, and are wound due to the differential rotation of the disk. Field lines must extend vertically as they are wound, resulting in a toroidal magnetic field which may collimate the outflow.
Due to magnetic hoop stresses, a pinching effect could then be responsible for collimating the very long and narrow outflows observed from YSO’s. A simple schematic of the winding of the magnetic field, along with the resulting magnetic tower, is shown in Figure 1.4. It is this poloidal/toroidal field that is thought to be responsible for the collimation of the plasma jet. This mechanism has been studied numerically for both YSO’s and black hole situations. Examples of these studies on magnetic tower jet formation from black holes are shown in Figure 1.5, and have shown that the general field configurations are similar for both types of source objects with accretion disks, and that the magnetic pressure of the tower could be responsible for the acceleration of the jet [27].

Figure 1.5: (a) Side view of the magnetic tower field winding up for an accretion disk surrounding a black hole. Figure from Uzdensky and MacFayden The Astrophysical Journal (2006) [28] (b) The magnetic tower field configuration from a 3D MHD simulation of an accretion flow surrounding a black hole. From Kato et al. The Astrophysical Journal (2004) [29]

1.3 Basic Z-pincho Theory: The MHD Equations

In order to examine how Z-pincho experiments may be used for laboratory astrophysics experiments, it is first necessary to examine the general dynamics
of Z-pinch systems. This begins with familiarizing the reader with a relevant set of equations often used to describe a Z-pinch plasma system. In most cases, Z-pinch plasma dynamics are well described by a set of equations known as the equations of magneto-hydrodynamics. These equations describe plasmas which can be described as a fluid where magnetic fields are important in the dynamics. In order to define the regime in which MHD is valid we need to examine some properties of the plasma of interest. For a plasma to be considered in a fluid state, the rate of macroscopic changes must be large, i.e. changes occur slowly, compared to the collision rate between the particle species [30]. Secondly, compared to the scale length of the system we wish to describe, the collisional mean free path of the particles must be small.

\[ \tau_{\text{changes}} \gg \tau_{\text{collisions}} \quad \text{or} \quad \nu_{\text{changes}} \ll \nu_{\text{collisions}} \]  

(1.1)

and

\[ L_{\text{system}} \gg L_{mfp} \]  

(1.2)

In this case, each element of the fluid can be specified by its own density, \( \rho \), temperature, \( T \), and mean velocity, \( V \). The plasma state is assumed, that is, the particles exhibit collective behavior due to electromagnetic forces, rather than purely hydrodynamic forces, as in the case of neutral particles. If these conditions are met, then the MHD equations may be used to describe the system.

### 1.3.1 The Resistive MHD Equations

The resistive MHD equations in Gaussian units are [11]: The continuity equation:

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{V} = 0 \]  

(1.3)

The equation of motion, or momentum equation:

\[ \rho \left( \frac{\partial \mathbf{V}}{\partial t} \right) = -\nabla P + \frac{\mathbf{J} \times \mathbf{B}}{c} + \rho \mathbf{G} \]  

(1.4)

Ohm’s law:

\[ \nabla \times \mathbf{E} + \frac{\mathbf{V} \times \mathbf{B}}{c} = \eta \mathbf{J} \]  

(1.5)
Faraday’s law:
\[ \nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} \]  
(1.6)
and Ampere’s law:
\[ \nabla \times \mathbf{B} = 4\pi \mathbf{J} \]  
(1.7)

Equation 1.4 is the equation of motion, where we have neglected gravity for our purposes. In astrophysical systems, this is generally invalid, however, on the scale of Z-pinches, the effects of gravity are negligible. Lastly, the MHD equations are not a closed set, and thus an equation of state, a means of relating the pressure to the density is necessary [11].

In fact there two different sets of MHD equations which may be used to describe the plasma, depending on what conditions are present. By manipulating the equations one can find:
\[ \frac{\partial \mathbf{B}}{\partial t} = \eta c^2 \frac{4\pi}{\nabla^2} \mathbf{B} + \nabla \times (\mathbf{V} \times \mathbf{B}) \]  
(1.8)
If the fluid is at rest, or \( \mathbf{V} = 0 \), then this simplifies to a pure diffusion equation for the magnetic field. This may then be rearranged to find a dimensionless parameter, the magnetic Reynolds number, which determines a key difference between the two sets of MHD equations, known as the ideal and resistive MHD equations. In hydrodynamics, the Reynolds number describes the ratio of the thermal forces to the viscous forces. Analogously, in MHD, the magnetic Reynold’s number describes the ratio of the effects of magnetic forces to resistive forces, or magnetic advection to magnetic diffusion.
\[ Re_M = \frac{4\pi V L}{\eta c^2} \]  
(1.9)
Where \( V \) and \( L \) are the characteristic velocity and length scale of the system, \( c \) is the speed of light, and \( \eta \) is the plasma resistivity. If the magnetic Reynolds number is large, then the resistivity is small or negligible. This implies that the magnetic diffusion is small, and the magnetic field lines are said to be “frozen in” to the plasma. This situation corresponds to the regime of ideal MHD. The resistive MHD equations then provide a means of describing the general dynamics of Z-pinch plasma systems, and thus are used throughout this work. Of course, it is simple to show that in the limit that plasma resistivity is negligible, the resistive
MHD equations reduce to the ideal case, by dropping terms with resistivity, \( \eta \). Due to this flexibility, the resistive MHD equations generally represent a more accurate description of the system than the ideal case. However, the ideal MHD equations are often applicable in astrophysical systems, and will be discussed further in Section 1.5.2.

1.3.2 The Basic Z-pincho

![Z-pincho image]

**Figure 1.6:** A cylindrically symmetric column of plasma carrying a current.

Early in the 20th century, William Bennett investigated a simple, and very special case of what would become known as a Z-pincho [31]. This case was that of the self focusing of fast electrons, which could theoretically reach a static equilibrium, where thermal forces are exactly balanced by magnetic forces. To begin, we consider an infinitely long, cylindrically axisymmetric column of plasma, with a current density \( \mathbf{J} \). The general schematic for this situation is seen in Figure 1.6. Of course, we seek a system which is at equilibrium, where \( v = \partial r / \partial t \) is equal to zero.

\[
\rho \frac{\partial \mathbf{v}}{\partial t} = -\nabla P + \mathbf{J} \times \mathbf{B}
\]  

(1.10)

In this simplified case, we will consider only these two forces. Thus, the equation
of motion is just:

$$\nabla P = \mathbf{J} \times \mathbf{B} \quad (1.11)$$

Here, \( \rho \) is the density, \( \mathbf{v} \) is the velocity, \( \mathbf{J} \) is the current density, \( B \) is the magnetic field vector, and \( P \) is the pressure. This equation simply says that the thermal pressure force must be equal to the \( \mathbf{J} \times \mathbf{B} \) force.

Since the magnetic field is purely azimuthal in this case, e.g. \( B_\theta = \frac{\mu_0 I}{2\pi a} \), the magnetic pressure must balance the thermal pressure. It is also assumed that we are dealing with an ideal gas, such that

$$P = (Z + 1)n_i k_B T \quad (1.12)$$

Where \( Z \) is the ionization number, \( n_i \) is the ion density, \( k_B \) is the Boltzmann constant, and \( T \) is the temperature averaged over the volume. It is also assumed that the plasma is quasi-neutral, such that \( n_e = Z n_i \), and that the plasma is in thermal equilibrium, with \( T_i = T_e = T \). After some manipulation and integrations, we find the Bennett relation:

$$\mu_0 I^2 = 8\pi (Z + 1) N k_B T \quad (1.13)$$

where \( N \) is the ion line density. The Bennett relation shows that the current required to confine a given ion line density and temperature. Note that this has no dependence on the internal structure of the plasma, and that modest currents could then confine a plasma at significantly high temperatures. As an example, if we take \( N \) to be \( 10^{20} \text{m}^{-1} \) and \( T \) to be 1 eV, 10,000 K, it would take a mere 25 kA to maintain equilibrium. Further, maintaining plasma conditions which would be suitable for thermonuclear burn would be relatively easy. Considering a hydrogen plasma with a temperature of 10 keV, and a density of \( 10^{20} \text{m}^{-1} \), and noting that a fully ionized hydrogen plasma has an ionization of \( Z=1 \), it would only take a current of 1 MA to maintain the state. The parameters here are chosen as a reasonable example, since this state would allow thermonuclear reactions to take place, and mega-Amp currents are readily produced by university-scale machines. Further, from this well known result, it is clearly seen that by increasing the current, the pressure, and therefore the density and temperature, are increased. This model,
however, has neglected quite a lot of important physics, not the least of which is that the plasma has the freedom to radiate away energy.

The incorporation of this fact was analyzed by Pease and Braginskii [32,33]. The Pease-Braginskii current, improves on the Bennett relation model by including the effects of both Ohmic heating and radiation loss effects. In this case, the ratio of radiation losses to Ohmic heating is determined only by the current, and the two energy sinks/sources will balance at the Pease-Braginskii current. For a hydrogen plasma with a parabolic density profile, this critical current is:

\[ I_{PB} = 0.433(\ln \Lambda)^{1/2} MA \]  

Where \( \ln \Lambda \) is the Coulomb logarithm, and \( \Lambda \) is the plasma parameter. This parameter is the ratio of the distance of closest approach for a Coulomb collision in a plasma, to the the classical distance of closest approach for a Coulomb collision. This is just \( \Lambda = 4\pi n_e \lambda_D^3 \), where \( \lambda_D \) is the Debye length, and \( n_e \) is the electron density. The Debye length is simply a measure of the effectiveness of charge screening effects in a plasma, which is proportional to the number of electrons in a Debye sphere.

### 1.3.3 Instability Development in Z-pinch

Of course, in reality, there are even more physical processes which should be accounted for in this analysis. The results presented in the previous section are not necessarily incorrect, however, the biggest problem encountered in real experiments is that the equilibrium Z-pinch is in fact, highly unstable. There are, of course many instability modes for Z-pinch plasmas, however, two are especially prevalent in the case of a cylindrical plasma confined by a magnetic field [35]. These instabilities are the \( m=0 \), or sausage mode, and the \( m=1 \), or kink mode, as seen in Figure 1.7. These modes may be examined by considering appropriate boundary conditions for the plasma, and applying perturbation theory to the ideal MHD equations, where small displacements to the plasma-vacuum interface are considered [36]. This analysis results in solutions of the form \( \exp(im\theta + ikz + \gamma t) \), meaning that the oscillations are in the \( \theta \) and \( z \) directions, while the growth rate of
Figure 1.7: Drawings showing two common instability modes in real Z-pinches, the a) m=0, or sausage instability, and the b) m=1, or kink instability. Figure from J. Chittenden lecture on Z-pinch physics. [34]

the perturbation amplitude is equal to $\gamma$. The relevant terms are seen by expanding the $J \times B$ term in the momentum balance of the ideal MHD equations [37].

$$
\nabla P = \frac{1}{c} (B \cdot \nabla) B + \frac{1}{c} \frac{B^2}{8\pi} 
$$

(1.15)

The first term on the right hand side represents the “tension” along magnetic field lines, while the second term is the magnetic pressure term.

The Sausage Instability

The m=0, or sausage instability, is seen in Figure 1.7 a), and is due to the second term on the right side of equation 1.15. Here, when a small displacement in the $-\hat{r}$ direction surface of the equilibrium results in the magnetic pressure being stronger than the thermal pressure. If the plasma is incompressible, this will result in excess plasma being “squeezed” like a tube of toothpaste, forcing plasma into the adjacent $\pm\hat{z}$ regions. Here, the plasma “bulges” outward, resulting in the thermal pressure being greater than the magnetic pressure, which serves to further enhance
the instability. This is then seen schematically in the figure, where a sausage like plasma is formed.

**The Kink Instability**

The m=1, or kink instability, is seen in Figure 1.7 b), and is also due to the second term on the right side of equation 1.15. Here, when a small displacement of the column results in the magnetic pressure being stronger than the thermal pressure due to the bunching of magnetic field lines in this displaced region. This time, rather than squeezing the column, the column is correspondingly displaced on the opposite side, leading to a spreading out of the field lines here, and thus a decreased magnetic pressure. This again serves to enhance the instability.

### 1.4 Physics of Wire Z-pinches

![Figure 1.8](image.png)

**Figure 1.8:** a)-b) Schematics of the core-corona structure in a single wire explosion Z-pinch experiment. c) Experimental laser Schlieren images, which identify density gradients in the plasma, showing development of m=0 instabilities and their growth, during a single wire experiment. Figure c) is adapted from Chitten-den et al. Physical Review E (2000) [38].

It is now useful to look at an example of a Z-pinch experiment. One of the simplest experiments, which is similar to the example of a cylindrical plasma column configuration, is the so-called single wire explosion. In this scheme, a single thin, \( \sim 5 - 50\mu m \) diameter, wire is connected to two electrodes, and driven by a current, typically a few hundred kA. When the current pulse begins, the
wire is quickly Ohmically heated due to the large, high-frequency, current. This drives ablation of the wire, and the formation of a coronal plasma surrounding the wire [38]. This plasma is low density, and at a much higher temperature than the cold and dense wire core. Thus, current preferentially flows in this highly conductive plasma, rather than the resistive wire core. This is seen in a) and b) of Figure 1.8. As the current continues to rise, the magnetic field generated around the wire also increases in strength, until it becomes large enough to confine the coronal plasma, and pinch it back toward the wire axis. This further aids in ablation by forcing the hot plasma back against the cold wire core. Further, the coronal plasma is observed to become $m=0$ unstable, and necking is observed in the coronal plasma. The cold and dense wire core then persists until all of the wire material has been ablated, typically occurring in the necking regions of the instability first. So, even in this simple scheme, instability is observed due to the distribution of current and magnetic field, which prevents reaching of the desired equilibrium plasma pinch.

1.4.1 The Dynamic Z-pinch

![Figure 1.9](image)

Figure 1.9: Two views of a cylindrically symmetric thin plasma shell carrying a current.
The dynamic Z-pinches were later examined as a potential means of bypassing the instabilities observed in single wire experiments. To first approximation, it is a thin, cylindrical plasma shell implosion [39]. This scheme relies upon a cascade of energy conversion [40], seen schematically in Figure 1.10. First, the magnetic energy is converted to kinetic energy as it accelerates a thin plasma shell toward the z-axis. When the plasma collides on axis, it stagnates, and the kinetic energy is converted to thermal energy, i.e. the kinetic energy goes into heating the plasma. Finally, the thermal energy is converted to radiated away.

**Figure 1.10:** The various stages of energy conversion in the ideal dynamic Z-pinched implosion.

Seen in Figure 1.9 a cylindrically symmetric thin plasma shell carries a current. The thickness of the shell $\Delta r$ is small compared to the radius of the cylinder, and carries a current. Due to the symmetry, this problem may be further simplified, and the shell mass per unit length is summed, to $m_o$. Since the shell is thin, the thermal pressure is negligible, so that the force per unit length is solely due to the magnetic pressure, or more precisely the magnetic pressure times the surface area [39]. Recalling $B$ in terms of $I$, the force per unit length is:

$$F_{\text{pul}} = J \times B = -\frac{B^2}{2\mu_o}2\pi r \hat{r}$$

where the magnetic field is the value at the surface of the pinch, and is a function of time, such that $B = B(t)$. Similarly, $I$ is also a function of time, $I = I(t)$, and recalling that

$$B = \frac{\mu_o I}{2\pi r}$$

equation 1.16 becomes:

$$F_{\text{pul}} = m_o \ddot{r} = -\frac{\mu_o I^2}{4\pi r}$$

(1.18)
In order to view obtain a general form of this action, we now switch to dimensionless variables.

\[ \tilde{r} = \frac{r}{r_o}, \quad \tilde{t} = \frac{t}{\tau}, \quad \tilde{I} = \frac{I}{I_{max}} \]  

(1.19)

Here \( r_o \) is the initial radius of the cylinder, \( I_{max} \) is the maximum value of the current, while \( \tau \) is the time at which the current maximum is reached. When these values are substituted into equation 1.18, we arrive at the so-called 0-D implosion trajectory expression [39].

\[ \ddot{\tilde{r}} = -\frac{\mu_0 I_{max}^2 \tau^2}{4\pi m_o r_o^2} \tilde{I}^2 = -\Pi \tilde{I}^2 \]  

(1.20)

The dimensionless scaling parameter for the problem, \( \Pi \) is introduced here. This parameter describes the implosion trajectories for the parameters involved. If implosions have the same function of \( \tilde{I} \), i.e. the same current driver is used, the timing of the pinch arrival on axis will be the same, though their trajectories may differ slightly, if the value of \( \Pi \) is matched. It also implies that for a given current function, or current driver, there exists an optimal \( \Pi \), since it is assumed the the largest possible amount of electrical energy is imparted to the shell’s kinetic energy if the time at which the shell is fully imploded coincides with \( \tau \), or the time at which the current has reached its peak value.

1.4.2 Cylindrical Wire Array Implosions

One of the primary means of approximating the dynamic pinch in experiments is known as a cylindrical wire array [39, 41–43]. Since wires were shown to rapidly expand upon current initiation, it seems that if enough wires are used, one could readily produce a thin plasma shell, and thus implode it in a fashion which is similar to the 0-D trajectory. Obtaining a more realistic implosion trajectory in the case of a wire array depends primarily on three factors; the shape of the driving current pulse, the initial mass per unit length of the array, that is, the sum of the mass contained in all the wires in the array, divided by the array length, and the mass ablation rate, given by the rocket model [42].

\[ \frac{dm}{dt} = \frac{\mu_0 I^2}{4\pi V_{abl} r} \]  

(1.21)
where $V_{abl}$ is a characteristic, and empirically determined constant. This ablation rate is also proportional to the square of the drive current, and the strength of the magnetic field, determined by both the radius of the array, and the magnitude of the drive current. Using this information, one can find the timing of a given implosion, and match this to the current driver being used. The optimum timing for any implosion, at least in this vastly simplified model, is at or just before maximum current in order maximize the kinetic energy imparted to the plasma before it arrives on axis. For example, one can consider the MAGPIE facility at Imperial College, in London, UK. This driver delivers a maximum of 1.4 MA in an approximately sine squared waveform, with a 10-90% rise-time of 240 ns. Since we would like to implode at, or just before the maximum current in this configuration, to optimize the density achievable, it is desirable to match the relevant parameters so that the implosion arrives on axis at \( \sim 240 \) ns.

\[
m_{pul} \frac{\partial v}{\partial t} = J \times B - \nabla P = J_z B \hat{r} - \nabla P = \frac{I(t)}{(\Delta r)^2} - \frac{2I(t)}{r} - \nabla P \tag{1.22}
\]

where $m_{pul}$ is the array mass per unit length, $J_z$ is the current density in the vertical direction, $I(t)$ is the current as a function of time, and $\Delta r$ is the shell thickness, and $r$ is the position of the shell at a given time. Since we are only considering motion of the “shell”, and $\Delta r \ll r$, $v(t)$ may be written in terms of $r$ [43].

\[
\frac{\partial^2 r}{\partial t^2} = \frac{2I^2}{m_{pul}(\Delta r)^2 r} - \frac{1}{m_{pul}} \nabla P \tag{1.23}
\]

This equation may then be integrated to obtain a function describing the position of the imploding array as a function of time, similar to the 0-D implosion considered in the imploding thin-shell approximation considered previously. This, at first glance, may seem to be a far more unstable configuration to begin with, however, through optimization of the array radius and number/diameter of the wires, it has been shown that this configuration is a powerful and efficient source of soft X-rays created to date [44–46].

The cylindrical wire array implosion is characterized by four distinct, but closely related, stages. These phases are initiation, ablation, implosion, and stagnation. Each stage may present its own instabilities, which may or may not con-
Figure 1.11: a) A schematic of a cylindrical wire array. b) A slice through the array during the ablation phase. c) The global magnetic field redirects the coronal plasma expansion and accelerates it toward the axis of the array.

Contribute to instabilities in the subsequent stages [42, 47]. In the initiation stage, each wire initially behaves as it would on its own, quickly expanding, forming a corona, and developing m=0 instabilities. In this case, each wire will carry its own private magnetic flux, $B_{wire}$. During the ablation phase, the current carried by the corona of each wire contributes to a global magnetic field, seen in Figure 1.11. The interaction of the current with this B field, through the JxB force, sweeps plasma away from its initial location about the wire cores, and toward the common axis of the array, redistributing the mass within the array [43]. This action is responsible for mitigating the growth of the m=0, and what is observed is what is known as the modified m=0 instability [48]. During the ablation stage, mass is redistributed within the array, forming a precursor column of plasma, surrounded by a standing shock. The end of this phase begins at $\sim 80\%$ of the total implosion time, and the main implosion begins [42], seen in Figure 1.13. It is important to note that some areas along the wire length which have not run out of mass at this time, and thus this mass does not participate in the main implosion. When this mass finally implodes, it is known as the “trailing mass of the implosion. This in itself causes a degradation in the power of the final emitted X-ray pulse.

In the ideal case of the plasma shell, all the plasma kinetic energy arrives on axis at the same time, thermalizes, and releases the energy as radiation in a
Figure 1.12: a) A low wire number cylindrical array is a poor approximation of a b) thin plasma shell, leading to a departure from the ideal 0-D implosion trajectory. Increasing the wire number, however, increases symmetry, and the power of the final emitted X-ray pulse [49].

Figure 1.13: A 0-D implosion trajectory calculated for imploding arrays on MAG-PIE at Imperial College. Data from implosions of 16 and 32 wire Al and W arrays is shown, where optical streak photography is used to track the implosions. Unlike the 0-D implosion trajectory, the implosion of the arrays do not start until \( \sim 80\% \) of the total implosion time is reached. Figure from Lebedev et al. PoP (2002) [43]
short pulse. As might be expected, in low wire number cases, more instability is introduced in the final implosion, which ultimately determines the radiated power, as seen in Figure 1.14 b). Symmetry is improved by increasing the wire number, which decreases the instability of the implosion, and thereby increases the final radiated X-ray pulse power. In the case of real cylindrical wire implosions, for appropriately chosen initial conditions, a pulse width of \(\sim 4\) ns nanoseconds, powers \(\geq 280\) TW, and efficiency \(\sim 17\%\) from stored energy to final emitted energy, have been achieved on the 20 MA Z accelerator at Sandia National Laboratory [50]. This process has been found to be quite powerful and efficient, however, it is practicality is limited by the ability to form a very light and uniform plasma shell.

**Figure 1.14**: The stages of implosion for a cylindrical wire array shown with a 0-D implosion trajectory calculated for a 16 wire array on MAGPIE. Each stage has unique instabilities associated with it, and the early modes cascade to the later instabilities leading to degradation of the final X-ray pulse. b) X-ray diode signals from the implosion of two cylindrical wire arrays. The wire material, array radius, and mass per unit length are similar for the two arrays, but the wire number is different. Significant non-uniformity in the implosion of the lower wire number array leads to a degraded power in the X-ray emission. Figure s) adapted from Chittenden summer school, and b) from Bland et al PoP (2007) [51].

Of course, there is still significant instability even in rather “uniform” wire array implosions. As seen in Figure 1.14, the magneto-Raleigh-Taylor (M)RT instability is one of the most prevalent during the implosion. This instability is due to the “light” magnetic field accelerating the “heavy” plasma during the course of the implosion. Further, this instability may be enhanced due to the
incomplete ablation of wire mass prior to the start of the main implosion. This means that some sections of the array will begin to implode before others, resulting in magnetic “bubbles” formed in the implosion, which contribute to non-uniformity of the implosion [52]. The result is that rather than a shell implosion, the implosion looks like those in Figure 1.15.

![Figure 1.15: a)-d) Experimental and e)-h) simulated, XUV self-emission images of an imploding cylindrical wire array on MAGPIE. The implosion is non-uniform along the axial direction, and significant trailing mass does not participate in the main implosion, due to several instabilities. a-d) from Bland et al PoP (2007) [51] and e-h) from Chittenden Lecture on Z-pinches (2009) [34]](image)

Thus, the magnetic field topology and current distribution have a very large impact on the dynamics of Z-pinch experiments. As will be seen in Chapter 3, this information often goes undiagnosed due to the difficulty in making direct measurements of these parameters in plasmas. Knowing this topology is very important, for all Z-pinch experiments, and is of particular importance in the field of laboratory astrophysics with Z-pinches.
1.5 Similarity Conditions for Scaled Laboratory Astrophysics Experiments

1.5.1 Hydrodynamic Similarity

Again, attempts to model astrophysical phenomena in the laboratory relies upon the ability to directly two systems which evolve on vastly different temporal and spatial scales. To this end we must develop the theory behind scaling, the simplest of which is for hydrodynamic systems. The Euler equations describe the behavior of ideal hydrodynamics at any scale where a fluid approximation is valid, and dissipation is negligible, as discussed by Ryutov et al. The Astrophysical Journal (1999) [53]. The Euler equations for a compressible, polytropic, gas are:

the continuity equation,

$$\frac{\partial p}{\partial t} + \nabla \cdot \rho \mathbf{v} = 0$$ (1.24)

the momentum equation,

$$\rho \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla P$$ (1.25)

and the energy equation.

$$\frac{\partial P}{\partial t} - \gamma \frac{P}{\rho} \frac{\partial \rho}{\partial t} + \mathbf{v} \cdot \nabla P - \gamma \frac{P}{\rho} \mathbf{v} \cdot \nabla \rho = 0$$ (1.26)

Here the assumption of a polytropic gas means that the energy per unit volume, or pressure, is proportional to the density, $\epsilon = const. \times \rho$. For adiabatic processes, the pressure is related to the density, $P \propto \rho^\gamma$, where $\gamma$ is the adiabatic exponent.

In order to maintain the validity of the Euler equations, the system must be absent of dissipation, e.g. viscosity and heat transport must be negligible. Similar to the assumptions under which MHD is valid, the system must remain localized, $L_{\text{system}} \gg L_{\text{mfp}}$, i.e. collisionality is high., This may be expressed in dimensionless form, and is known as the localization parameter.

$$\delta = \frac{L_{\text{mfp}}}{L_{\text{system}}} \ll 1$$ (1.27)
The importance of viscous effects are described by the Reynolds number, which is the ratio of the inertial forces to the viscous forces.

\[ Re = \frac{VL}{\nu} \]  

(1.28)

Here \( V \) is the characteristic velocity of the system, often the sound’s speed, \( L \) is the characteristic length, and \( \nu \) is the viscosity. If this number is large, then the viscous forces are small compared to the inertial forces. The Peclet number describes the heat transport for a hydrodynamics system, and is the ratio of the convective heat transport to the conductive heat transport.

\[ Pe = \frac{VL}{\chi} \]  

(1.29)

This number describes the ratio of advective transport vs collisional transport of heat, or thermal conduction, where \( \chi \) is the thermal diffusivity. Of course, if the thermal conduction is small, then the Peclet number is large, and is typically in the range of \( Pe \approx 10^5 \) to \( 10^{10} \) for astrophysical outflows [54]. In order to describe two different systems via the Euler equations, equations 1.24-1.26 must remain invariant under a transformation of variables from \( r, \rho, P, t, \) and \( V \) to [53,54]:

\[
\tilde{r} = \frac{r}{L^*}, \quad \tilde{\rho} = \frac{\rho}{\rho^*}, \quad \tilde{P} = \frac{P}{P^*}, \quad \tilde{t} = \frac{t}{L^* \sqrt{\frac{P^*}{\rho^*}}}, \quad \tilde{V} = V \sqrt{\frac{\rho^*}{P^*}}
\]  

(1.30)

The values with a tilde denote the dimensionless variables, while the values with an asterisk refer to the value of the parameter at some characteristic point. Thus, there is correspondence between any two systems which satisfy 1.30, known as the Euler similarity. Now we consider a system where the initial values are given by [53,54]:

\[
V_{(t=0)} = V^* F\left( \frac{r}{L^*} \right), \quad P_{(t=0)} = P^* G\left( \frac{r}{L^*} \right), \quad \rho_{(t=0)} = \rho^* H\left( \frac{r}{L^*} \right)
\]  

(1.31)

where \( F, G, \) and \( H \), are dimensionless functions. Then we consider another system with different characteristic parameters, \( L^*_1, \rho^*_1, P^*_1, \) and \( V^*_1 \), where the dimensionless functions remain the same, i.e. the initial state of the second system is geometrically similar to the first system. Then the two systems will behave similarly
if the last condition of 1.30 is satisfied, or the Euler number, \( Eu = V \sqrt{\frac{\rho}{P}} \), remains invariant between the two systems [11,53].

\[
Eu = V^* \sqrt{\frac{\rho^*}{P^*}} = V^*_1 \sqrt{\frac{\rho^*_1}{P^*_1}} = \text{invariant} \quad (1.32)
\]

This is a very broad similarity condition, having only two constraints which depend on only the three parameters which describe the initial state of the systems, e.g. \( \rho^*, P^*, \) and \( v^* \). The Euler number is also similar to the Mach number, \( M = \frac{V}{C_s} \), the ratio of the fluid velocity to the sound speed. For a polytropic gas, the sound speed involves the adiabatic exponent, \( \gamma = 1 + 1/\text{constant} \), and may be expressed as, \( C_s = \sqrt{\frac{\gamma P}{\rho}} \) [11]. Using this information, the \( Eu \) number may be expressed as:

\[
Eu = \frac{V}{C_s} \sqrt{\gamma} = M \sqrt{\gamma} \quad (1.33)
\]

To summarize, in order for the Euler equations to hold; the localization must remain small, \( \delta = \frac{l_{mfp}}{l_{system}} \ll 1 \), the viscous effects and heat conduction must also remain small, i.e. \( Re, Pe \gg 1 \) [53], and lastly, the effect of radiative losses from the plasma should be unimportant. The conditions for two hydrodynamic systems to evolve similarly are as follows. The initial distributions must be geometrically similar, i.e. \( F, G, \) and \( H \) are the same, and the distributions of the density, pressure, and velocity satisfy the transformation as in Equation 1.32. The Euler number and adiabatic index must also remain invariant between the two systems. As will be discussed in the next section, these conditions are also necessary in ideal magneto-hydrodynamic (MHD) situations.

### 1.5.2 Ideal MHD Similarity

When a dynamically significant magnetic field is present, the ideal magneto-hydrodynamics may be used to describe the system. Thus, we now look at the similarity conditions for an ideal MHD system. The ideal MHD equations may be written as [55]: the continuity equation

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{V} = 0 \quad (1.34)
\]
The momentum equation:

\[
\rho \left( \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = -\nabla P - \frac{1}{4\pi} \mathbf{B} \times \nabla \times \mathbf{B}
\] (1.35)

Ohm's law:

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{V} \times \mathbf{V}
\] (1.36)

As before, there are several conditions which must be satisfied in order for these equations to hold, and correspond to a well-behaved ideal MHD system with the absence of dissipation [55]. The first condition is that the particles must again be localized with respect to the size of the MHD system, a condition set by any system which we wish to describe with hydrodynamics. This is generally true, except in the presence of shocks, which will not be covered here. Second, since we wish to describe a system in the absence of dissipative processes, viscosity, thermal conductivity, and Ohmic dissipation must be negligible. The relevant dimensionless parameters which describe these conditions are the Reynolds number, and the Peclet number as in the hydrodynamic scaling, however, in ideal MHD a new dimensionless parameter is introduced, the magnetic Reynolds number. This number, \(Re_M\), describes the role of Ohmic dissipation, and must be large for these equations to remain valid.

\[
Re_M = \frac{VL}{D_M}
\] (1.37)

Here, \(V\) is the velocity, \(L\) is the characteristic scale length of the system, and \(D_M\) is the magnetic diffusivity. The magnetic diffusivity is

\[
D_M = \frac{\eta c^2}{4\pi}
\] (1.38)

where \(\eta\) is the resistivity, and \(c\) is the speed of light. The large value of \(L\) in astrophysical systems typically makes the magnetic Reynolds number quite large, often \(Re_M \sim 10^{10} - 10^{18}\) [30]. Additionally, small \(\eta\) corresponds to high electrical conductivity, and thus an ideal MHD situation, while also aiding in the increase in \(Re_M\). In experiments, this number is typically much smaller, but high plasma temperatures imply low resistivity (Spitzer), and thus \(Re_M \geq 1\) in many cases.
The similarity parameters in Equation 1.32, and the dimensionless functions remain the same for this case. However, a new term must be introduced. The ideal MHD equations must similarly remain invariant upon the transformation to this new variable [54]. This term is:

\[
\tilde{B} = \frac{B}{\sqrt{P^*}} \tag{1.39}
\]

where \(B^*\) is the magnetic field, and the corresponding initial state is given by:

\[
B_{(t=0)} = B^* K\left(\frac{r}{L^*}\right) \tag{1.40}
\]

where \(K\) is another dimensionless function describing the initial distribution. The condition set by Equation 1.40 must now also remain invariant between the two systems. This term is similar to the plasma beta, \(\beta \propto \frac{8\pi P^*}{B^*} \), which is the ratio of the thermal pressure to the magnetic pressure. Using this, the additional condition that must be satisfied in order for both systems to evolve similarly may be expressed as:

\[
\frac{B^*}{\sqrt{P^*}} = \sqrt{\frac{8\pi}{\beta^*}} = \text{invariant} \tag{1.41}
\]

To summarize, in order for the ideal MHD equations to hold; the same conditions set forth in the previous section must hold, however, now the additional constraint is that Ohmic dissipation must be small, or \(Re_M \gg 1\) [55]. Further, the localization parameter may be modified to read as \(\delta = \frac{r_{Li}}{L_{\text{system}}} \ll 1\), where \(r_{Li}\) is the ion Larmor radius, which essentially means that particles should be localized along the magnetic field in the plasma [55]. Again, the effect of radiative losses from the plasma should be unimportant. The conditions for two ideal MHD systems to evolve similarly are as follows. Again, the initial distributions must be geometrically similar, i.e. \(F, G, H,\) and \(K\) are the same, and the distributions of the density, pressure, velocity, and the magnetic field satisfy the transformation as in Equation 1.39. The Euler number, adiabatic index, and beta parameter must also remain invariant between the two systems.

Thus, if these conditions are satisfied, it is expected that a laboratory plasma system may evolve similarly to an astrophysical system despite the large difference in spatial and temporal scales. Again, in general the shear size of astrophysical jets, \(\sim 10^{15}m\) generally leads to very large values of \(Re, Pe,\) and \(Re_M\) [56].
These values can be $Re > 10^8$, $Pe > 10^7$, and $Re_M > 10^{15}$. Typical experiments generally cannot reach these values, due in part to the small spatial scales of these systems, nor can these large values be obtained even in numerical simulations. However, by satisfying the general conditions that these numbers are $\gg 1$ implies that the large scale dynamics may at least be somewhat similar, and that the differences may primarily appear on smaller spatial scales [55].

1.5.3 Similarity for Plasma Jets

There are several other parameters which are important when comparing astrophysical and laboratory plasma jets [57]. Some of these are simply familiar parameters describing the jets such as velocity, density, and temperature, e.g. $V$, $\rho$, and $T$. Others are those which describe the system with dimensionless parameters, which add to the confidence in comparisons between the two systems. For instance, many laboratory experiments examine jet propagation into vacuum, or very low background density. This is not necessarily relevant to the astrophysical case, where jets are often propagating into the interstellar medium, or ISM, which is comparable to the jet density. This parameter is the jet-to-ambient density ratio, $\eta = \rho_{jet}/\rho_{ambient}$.

Another important parameter is the sonic Mach number, which is simply the velocity of the jet divided by the sound speed of the plasma, or $M = v_{jet}/C_s$. Last, is the jet cooling parameter, which quantifies the effects of radiative cooling on the jet dynamics [58]. This is the length over which the jet is radiatively cooled over the radius of the jet, or $\chi = d_{cool}/r_{jet}$, where $\chi < 1$ implies that the jet is radiatively cooled, and $\chi > 1$ implies that the jet is adiabatic.

Comparing all of these parameters ensures not only the validity of the equations being used to describe the systems of interest, but also the degree to which they are similar. In this way, laboratory experiments can be used to further the understanding of the physics of astrophysical outflows.
1.6 History of Laboratory Astrophysics with Z-pinchess

There are three different Z-pinch configurations which have been used to model astrophysical outflows which will be discussed here. These configurations are the conical wire array, the radial wire array, and the radial foil load. Each presents a unique set of conditions, which allow for the study of similar systems, where the aim is to model astrophysical jet dynamics away from the source, where the particular launching mechanism is not of concern [59]. Their applicability is typically limited in that not all dimensionless scaling criteria are met, however, the data from these experiments has led to improvements in the numerical models used to study both experimental and astrophysical outflows, and provides insight into the dynamics of astrophysical outflows. A fourth configuration, the x-pinch, is also useful in modeling astrophysical jets, but due to the similarity with conical wire arrays, will not be discussed here. The interested reader may find more information on this class of experiments, in which the author was involved, in reference [9].

1.6.1 High Mach Number Hydrodynamic Jets

High Mach number jets have been produced through the use of conical arrays [23, 57]. These arrays are similar to cylindrical wire arrays as discussed in section 1.4.2, however, and inclination angle is now introduced to the array. Plasma is ablated in a direction which is normal to the wires, and accelerated toward the common axis as before, however, due to the inclination angle of the array, a precursor is not formed in the usual manner. Instead, the arrival of plasma on the axis results in the formation of a conical shock, which redirects the velocity of the plasma in the axial direction. This plasma then travels axially upward in a collimated jet. This jet may be considered as purely hydrodynamic, with a Mach number in the range of \( M \geq 20 - 30 \), velocity \( v_{\text{jet}} \sim 200\text{km/s} \), [57]. Further, using wire materials with higher Z, radiative effects become increasingly more important. For example, jets from Al, Cu, and W are shown in Figure 1.17. The higher Z materials allow for radiative collapse, due to the large availability of tran-
Figure 1.16: a) Schematic of a conical wire array, and how the conical shock results in a high Mach number jet propagating in the $\hat{z}$ direction. b) An experimental laser interferometry image of a conical array with a jet emerging above the anode. Figure from Lebedev et al. Applied Physics Journal (2002) [57]

sition states, of the jet, thereby increasing the jet collimation and Mach number. Although these experiments have relevance to the dynamics of jets propagating far from their source, there is likely little connection with the launching region of the jet. As discussed, a magnetic field is thought to be dynamically important in the launching and collimation of astrophysical jets, and thus a laboratory model which includes this is desirable.
Figure 1.17: Laser shadow images of jets from conical wire arrays of three different wire materials, aluminum, iron, and tungsten. The jet is seen to be more collimated with increasing Z of the material. Figure from Lebedev et al. Applied Physics Journal (2002) [57]

1.6.2 Z-pincho Models of Magnetic Tower Jets

There are two relevant Z-pinch configurations which may have some connection with the picture of the magnetic tower jets in YSO’s. These configurations are the radial wire load, and the radial foil load, which are discussed here. Although small modifications to these setups can significantly alter the dynamics, the common features of the load dynamics will be discussed here.

The Radial Wire Configuration

This radial wire configuration brought several interesting aspects which are relevant to astrophysical jets, the important of which was as a model for the proposed magnetic tower jet [22]. In this configuration, seen in Figure 1.18, Wires are placed radially about two concentric electrodes. The current flows radially along the wires, and vertically through the central cathode. The current in the wires, $J = -I_{\text{wire}} \hat{r}$, then interacts with the large toroidal magnetic field around the cathode, $B = -\frac{\mu_0 I_{\text{tot}}}{2\pi r} \hat{\phi}$, to produce a force, $F \propto J \times B \propto \frac{I^2}{r} \hat{z}$, which redirects ablated plasma in the vertical direction. Thus, the $J_rxB\phi$ force increases with
Figure 1.18: A schematic of the radial wire setup. Wires connect the central cathode to the anode. When current flows through the array, a magnetic field is generated around the cathode, as well as the wires. The JxB force then accelerates the ablated plasma vertically.

decreasing radius. This action results in plasma being distributed above the plane formed by the wires, as seen in Figure 1.20 a).

Recalling the rocket model for describing the mass ablation rate, in equation 1.21, it is seen that the ablation rate increases with increasing global magnetic field, so the mass ablation rate is also higher nearer to the cathode. The radial density profile as a function of time above the array of wires can then be solved for, giving [22]:

$$\rho(r,t) = \frac{\mu_0}{8\pi^2 r^2 V_{abl}^2} I^2 \left( t - \frac{z}{V_{abl}} \right)^2$$

(1.42)

where $V_{abl}$ is an empirically determined ablation velocity, which is often $\sim 100 \text{ km/s}$. An example of this profile is seen in Figure 1.19, which is seen to be in agreement with experimentally measured values. This distribution of mass means that there is a radial density gradient present, which is responsible for the collimation of a plasma jet on axis. To a smaller extent, this high density region is collimated by the magnetic field which is advected with the plasma flow [22]. This situation is similar to the conical wire array jets, and again, is affected by the wire properties, whereby materials such as W allow the higher density plasma column to collapse to higher densities through the action of radiative cooling. This in turn
Figure 1.19: The density profile of ablated plasma above the plane of the radial wire array. The dots and triangles correspond to values obtained on the left and right hand side of an interferometry image used to measure the density. Figure from Lebedev et al. Monthly Notices of the Royal Astronomical Society (2005) [22]

Figure 1.20: A cartoon of the general radial wire dynamics. Plasma is ablated above the array. When the wire mass is completely ablated near the cathode a magnetic cavity is formed. Current flowing along the cathode and downward through the jet causes the jet to be collimated via the magnetic field. Figure from Lebedev et al. Monthly Notices of the Royal Astronomical Society (2005) [22]
means that at locations nearest to the cathode, the wires will be depleted of mass sooner than at larger radii. When this happens, the last remaining plasma will be accelerated in the z-direction, creating a gap in the wire mass. The plasma, which has been accelerated upward, will create a “cavity”. this forces the current to, flow along the boundaries set by the cavity, and then vertically downward through the previously collimated jet, seen in Figure 1.20 b). The new current configuration changes the direction of the jxb force, which drives the cavity to expand both radially and vertically, while simultaneously pinching the jet, and causing it to become unstable, Figure 1.20 c).

This evolution is supported by the experimental evidence seen in Figures 1.21 and 1.22 for an experiment utilizing MAGPIE, 1 MA in 240 ns, with 16x13µm W wires [22]. In the shadow case, the plasma column/jet is seen above the forming magnetic cavity, followed by pinching and disruption of the plasma jet inside the column, however, the jet continues to propagate after this cavity has broken up. In the time-gated extreme ultraviolet (XUV) self emission images, filtered to show photon energies $\sim$150-280 eV, the strongest emission begins after the cavity began formation. The emitting regions near the boundaries of the cavities indicate that current is flowing along the walls. Numerical simulations of these loads were

![Figure 1.21: Three laser shadow images at different times showing the evolution of a radial wire experiment on MAGPIE. Figure from Lebedev et al. Monthly Notices of the Royal Astronomical Society (2005) [22]](image)
used to characterized some of the parameters such as magnetic Reynolds number and plasma beta, which are difficult to accurately measure during experiments, and are summarized in Figure 1.23. Some of the parameters which determine the applicability of this load as a model for astrophysical outflows as measured in the experiments, and combined with the results of numerical simulations are summarized in 1.1. Although some of the dimensionless scaling parameters are similar, such as plasma beta, Mach number, and localization parameter are similar, other important parameters are different by more than several orders of magnitude. In particular, the Reynolds, magnetic Reynolds, and Peclet numbers for radial wire experiments are quite different from that of a YSO.
Figure 1.23: 2D RZ simulation of a radial wire array is used to summarize the plasma parameters. In particular, the plasma beta is much less than one, while the magnetic Reynolds number is much greater than one, inside the magnetic cavity. Figure from Lebedev et al. Monthly Notices of the Royal Astronomical Society (2005) [22]
Table 1.1: Summary of the relevant parameters for both YSO and radial wire array jets. Table adapted from Ciardi et al. Monthly Notices of the Royal Astronomical Society (2005) [22]

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<th>Flow Variables</th>
<th>Units</th>
<th>YSO</th>
<th>Radial Wire Array</th>
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<tr>
<td>Length</td>
<td>cm</td>
<td>$10^{17}$</td>
<td>$2 - 4$</td>
</tr>
<tr>
<td>Radius</td>
<td>cm</td>
<td>$10^{17}$</td>
<td>0.5</td>
</tr>
<tr>
<td>Dynamical age</td>
<td>ns</td>
<td>$10^{22}$</td>
<td>$200 - 400$</td>
</tr>
<tr>
<td>Temperature</td>
<td>eV</td>
<td>$0.5 - 100$</td>
<td>$5 - 200$</td>
</tr>
<tr>
<td>Fluid velocity</td>
<td>km/s</td>
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</tr>
<tr>
<td>Density</td>
<td>g/cm$^3$</td>
<td>$10^{-18} - 10^{-20}$</td>
<td>$10^{-4} - 10^{-6}$</td>
</tr>
<tr>
<td>Magnetic Field</td>
<td>G</td>
<td>$10^{-3} - 10^{3}$</td>
<td>$10^{4} - 10^{6}$</td>
</tr>
<tr>
<td>Radiative cooling time</td>
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</tr>
<tr>
<td>Mean free path</td>
<td>cm</td>
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<td>$10^{-5}$</td>
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<table>
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<th>Symbol</th>
<th>YSO</th>
<th>Radial Wire Array</th>
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<td>Mach number</td>
<td>M</td>
<td>$5 - 40$</td>
<td>$\gg 5$</td>
</tr>
<tr>
<td>Density contrast</td>
<td>$\eta$</td>
<td>1-2</td>
<td>$1 - &gt; 10^3$</td>
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<tr>
<td>Cooling parameter</td>
<td>$\chi$</td>
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<td>$&lt; 1$</td>
</tr>
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<td>Localization parameter</td>
<td>$\delta$</td>
<td>$&lt; 10^{-6}$</td>
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<td>$&gt; 10^7$</td>
<td>$50 - 10^4$</td>
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<tr>
<td>Reynolds number</td>
<td>$Re$</td>
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<td>$&gt; 10^4$</td>
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<tr>
<td>Magnetic Reynolds number</td>
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<td>$10 - 10^3$</td>
</tr>
<tr>
<td>Beta parameter</td>
<td>$\beta$</td>
<td>0.01-100</td>
<td>0.01-100</td>
</tr>
</tbody>
</table>
1.6.3 The Radial Foil Configuration

![Diagram of radial foil configuration]

**Figure 1.24:** A schematic of the radial foil setup as viewed a) from the top, and b) a cutaway view of this setup as seen from the side. The current and magnetic field orientation determine the direction of the JxB force.

The radial foil load is similar to the radial wire load in configuration, however, as implied by the name, the wires are now replaced by a foil which is stretched across the outer electrodes, and contacted underneath by the cathode. There are several differences between the radial wire and radial foil load, however the dynamics are very similar. The first is that the current density will increase with decreasing radius, since $J \propto \frac{I}{2\pi rh}$, where $h$ is the thickness of the foil, which is contrary to the radial wire array where the current density is constant along the radius. This means that in the radial foil case, the JxB force is proportional to $1/r^2$, while in the radial wire case, it is proportional to $1/r$. In both cases, the mechanism for the initial jet launching mechanism is similar. Again, the current flows radially inward along the foil and down through the cathode, ablating mass and forming an ambient plasma above the foil. Measurements with interferometry show that this plasma expands axially, and decays exponentially with height, indicating that the expansion is isothermal [60], and this ambient plasma has a sound speed $C_s \sim 9$ km/s.

At the cathode, most of the current flows along the outer surface, which
Figure 1.25: a) Early time foil ablation as seen from side-on laser interferometry. b) Axial line-outs of the line-integrated electron density above the foil at different radial positions showing the decrease in mass density as a function of increasing radius. Figure from Suzuki-Vidal Proc. APS DPP (2007) [24]

means that little current flows in the foil directly above the cathode. This means that there is less mass ablated just above the axis of the cathode. This sets up a void in the radial density profile, which sets up a density gradient in the -\( \hat{r} \) direction. This density gradient is then responsible for initially accelerating/collimating a jet, which propagates in the +\( z \) direction, as seen in Figure 1.25.

If the foil is sufficiently thin, \( \sim \leq 5\mu m \), then the foil mass may be completely depleted due to ablation near the cathode [61]. When this happens, then an “episode” is formed. The mass depletion suddenly allows the magnetic flux initially confined below the surface of the foil to move above the initial foil height. The \( J_r x B_\phi \) force component drives expansion of a magnetic cavity, where the new current path is then along the inner surface of this bubble. As in the radial wire case, this new current path returns through the jet, which drives instabilities here. Although this jet becomes unstable, it is not observed to be destroyed, but rather continues to propagate as a collection of knots. Eventually, the plasma near the cathode will fill the original gap, allowing current to once again flow there. The plasma which has filled this gap is then accelerated by the \( J_x B_z \) force, forming yet another magnetic cavity, or a second “episode”. Due to the lower amount of mass in this plasma, it is accelerated faster than the previous episode, and is observed
to expand with a higher velocity than the original cavity. This whole system is driven by the Poynting flux, a schematic of which is seen in Figure 1.26.

**Figure 1.26**: XUV self-emission image of radial foil system after formation of second episode with a superimposed schematic of the Poynting flux orientation. Figure from Suzuki-Vidal Proc. APS DPP (2007) [24]

Instabilities in the jet ultimately lead to increased impedance here, which contributes to cavity/jet breakup, and the current then re-strikes near the cathode, forming yet another episode, which will follow a similar evolution [62]. Using thin, 5μm, foils, the process of cavity formation and jet disruption occurs multiple times. This process can lead to 3-4 episodes per pulse. The episodic nature found in such experiments is at least qualitatively similar to the YSO situation observed in nature in Figures 1.1 and 1.1 where long jets with knotty internal structure propagate far from their sources.

**Applicability as Scaled Models of Astrophysical Outflows**

The implications of radial wire and foil loads as models of astrophysical outflows, and their applicability as models of such outflows were discussed by Lebedev [22] and Ciardi [56], which are briefly summarized here.

1) These experiments show that magnetic pressure gradients are able to
drive outflows, and that magnetic hoop stresses can collimate these outflows, which is similar to astrophysical models for outflows.

2) There are both magnetic and hydrodynamic mechanisms for collimating the outflows in the experiments. First, a jet is formed which is hydrodynamically collimated by pressure gradients, before the jet becomes magnetically confined within the cavity. After current again flows near the base of the jet, trapped magnetic flux in the previous bubble may continue to assist in collimation of the ballistically propagating clumpy jet.

3) In astrophysical models, a current in the outflow is expected to support the toroidal magnetic field, however where the return current would flow is unclear. These experiments show that the return current can flow along the magnetic cavity.

4) The time-dependent nature of the experiments is similar to astrophysical outflows, which are not necessarily continuous, and reproduce the knotty structure often observed in astrophysical outflows.

1.6.4 Current and Magnetic Field Measurements in Radial Foil Loads

Attempts to measure the current in the plasma above the radial foil load have been very limited in scope, and the problems associated with the technique used to obtain these measurements will be elucidated in Chapter 3. Thus, numerical simulations of radial foil systems have provided most of the current and magnetic field topology for this situation. Of course, this information is vital if a connection to astrophysical observations is to be made.

Using a magnetic pickup coil, described later in Chapter 3 and in which the component of the magnetic field measured is set by the plane formed by the coil in the probe, the toroidal magnetic field was measured in a magnetic cavity from a radial foil load, as seen in Figure 1.27. This probe was positioned 10 mm above the surface of the foil, and \( \sim 13 \) mm from the axis of the load, so that the probe would be within an expanding magnetic cavity at some time during the experiment. The voltage induced in the probe was seen to remain near zero until this magnetic cavity reached the probe. By using the expansion velocity of the cavity wall, and
assuming a temporal profile for the area of the probe coil which is inside the cavity, a magnetic field $\sim 0.3$ T was present within the magnetic cavity formed during the experiment. By assuming that the total current through the load is flowing along the cavity, and through the central jet, the magnetic field magnitude at this location is estimated to be $\sim 1.5$ T. Of course, this measurement is taken to be a mere estimate, especially since the probe is seen to disrupt the dynamics of the system at the location where the measurement is taken. Further, the author of that work suggests that spatially resolved measurements of the magnetic field are needed in order to have full confidence in comparisons with 3D modeling of the problem.

Figure 1.27: a) Early time foil ablation as seen from side-on laser interferometry. b) XUV self-emission image of radial foil system after formation of second episode. Figure from Suzuki-Vidal, et al. PoP (2010) [61]

Similar measurements of the magnetic field above the surface of radial foil loads on COBRA at Cornell University, 1 MA in 100 ns, by Gourdain et al. with two probes, instead of one, have measured the field magnitude in two, toroidal and vertical, directions [63]. In that work, the probes were placed $\sim 3$ mm from the
axis of the load, and $\sim 3$ mm above the surface of the foil. The probe measuring the toroidal component of the magnetic field did not remain at zero, but instead measured a small field, indicating that a “secondary” current flowed in the ambient plasma above the foil, and through the forming jet. Additionally, a jump in the signal was observed when the cavity wall crossed the location of the toroidal probe, but not in the vertical (or poloidal) probe. This indicates that current was indeed flowing through the plasma sheet in the vertical direction, supporting the toroidal magnetic field. As the jet was pinched, a jump in signal in the vertical channel was observed, most probably due to the development of the kink instability in the plasma jet. The probe signal corresponded to $\sim 3$ T, indicating that $\sim 50$ kA, or 10% of the total current was carried in the jet at that time.

From these measurements, it is clear that the magnetic field plays an important role in the system dynamics. However, the limited amount of measurements, and poor spatial resolution, makes it difficult to fully characterize the role of the magnetic field in the dynamics of the system. Spatially resolved measurements of the magnetic field in the plasma would greatly improve the understanding of the dynamics of these systems, and improve the confidence in their applicability as scaled laboratory astrophysics experiments. As previously mentioned, these measurements are at present difficult to make, and specifically with the use of magnetic pickup coils, or B-dot probes, often disrupt the plasma dynamics. In this work, a method of making such measurements, namely proton deflectometry, is developed, and applied to radial foil loads in an attempt to better understand the current and magnetic field topology. Such measurements are crucial not only to understanding these systems, but in validating the numerical codes used to simulate them.
Chapter 2

The Nevada Terawatt Facility

2.1 Introduction

Figure 2.1: The layout of the Zebra bay at the Nevada Terawatt facility, showing the location of Zebra and its load chamber. Also shown is the Leopard laser and the Phoenix target chamber for laser-only experiments. The beam transport is used to deliver the Leopard laser to the Zebra load chamber for coupled shots.

The experiments presented in this work were performed at the Nevada Terawatt Facility, located in Stead, Nevada. It is operated in conjunction with
support from the U.S. Department of Energy, and the University of Nevada, Reno. The facility currently operates two devices which are capable of delivering powers in the Terawatt regime. One driver is a pulsed-power $Z$-pinch machine, Zebra. The other driver is a laser, Leopard. Nominal operation modes give a peak power delivered at the load or target of 2 TW and 50 TW respectively, although both offer a long-pulse mode of operation, which deliver lower power. A general layout of the facility is shown in Figure 2.1 The Leopard laser can either be delivered to the Phoenix target chamber, for laser only experiments, or optionally delivered to the Zebra load chamber through a beam transport, for Leopard and Zebra coupled shots. This ability to couple a high-power laser to a MA-scale $Z$-pinch machine makes the Nevada Terawatt Facility a unique facility. At the time of writing, it is the only facility in the world which is capable of coupling a short-pulse high-intensity laser to a mega-amp-scale $Z$-pinch driver. Here the drivers will be outlined in detail, giving the reader a fundamental tutorial on the operation and specifications of each device.

2.2 The Zebra Driver

The Zebra driver, as it is known today, was originally a pulsed-power device operating at Los Alamos National Laboratory. At that time, it was known as HDZP-II, which is described in detail by Schlater et al. Plasma Physics and Controlled Fusion (1990) [65]. It was relocated to the Nevada Terawatt Facility, where it was refurbished, installed, and demonstrated on a $Z$-pinch load in 1999 [66]. $Z$-pinch drivers share general features, in that they are all predicated on the charging of a Marx bank in parallel, before discharging in series in order to provide a large voltage, and subsequently large current, to a load. This process occurs in several distinct stages, where the characteristics of the electrical pulse is modified in order to deliver the desired waveform to the load. Most importantly the pulse is compressed temporally, leading to higher delivered power. The first process is the Marx discharge, occurring over $\sim 1\mu s$, this discharge then is delivered to the pulse forming line, PFL, which compresses the pulse to $\sim < 1\mu s$. After this,
it is delivered to the vertical transmission line, through a magnetically insulated transmission line, MITL, and finally delivered to the load in a pulse which rises from zero to maximum in $\sim 100$ ns.

### 2.2.1 The Marx Bank Discharge

A simple picture of the Marx bank is a collection of capacitors charged in parallel through charging resistors, with the capacitor pairs connected by spark gap switches. The Marx bank on Zebra consists of 32 1.3 $\mu$F capacitors, which may be charged to a maximum of 100 kV per capacitor. For the Zebra generator, the Marx banks are submerged in $\sim 6,000$ gallons of transformer oil in order to maintain electric isolation. A diagram of portion of the Marx bank is shown in Figure 2.3. Here, it is seen that the capacitors are charged with opposite polarity, with a spark gap switch between each pair, resulting in 16 total switches. This allows for the eventual series discharge when the spark gaps are closed. The spark gaps themselves consist of a cell containing two electrodes and filled with $SF_6$ gas. $SF_6$ acts as a dielectric until sufficient voltage causes electrical breakdown, at which
Figure 2.3: A diagram of a portion of the Marx bank on Zebra. a) The capacitors are charged in parallel through resistors in alternating polarities, and each pair of capacitors are connected via a spark gap switch. b) When the switches are closed by the trigger Marx voltage, the capacitors are connected in series.
point it becomes a conductor, and the switch is said to be closed. The voltage breakdown of at different pressures is well characterized, which gives additional control over the timing of the switches [67]. When the capacitors are charged to the desired voltage, these spark gaps are triggered to induce electrical breakdown in the switch by a separate Marx bank, known as the trigger Marx, and the total voltage in the Marx is increased via a process known as erection. Since the capacitors are arranged in series, the maximum total voltage capable from the Marx bank is then:

\[ V_{\text{Marx}} = 32 \times V_{\text{charge}} = 3.2 \text{ MV} \]  

where \( V_{\text{charge}} \) is the potential to which the capacitors are charged. This large voltage is a requirement for a “fast” Z-pinch driver, and more precisely for efficient transfer of stored electricity to a load, as was outlined by Turchi [40]. The maximum energy stored in the Marx bank is calculated as:

\[ W_{\text{stored}} = N \times \frac{C \times V_{\text{charge}}^2}{2} = \frac{(32 \times 1.3\mu F) \times (100\text{kV})^2}{2} = 208\text{kJ} \]  

Where \( C \) is the capacitance value of the capacitors in the bank, \( N \) is the number of capacitors. Typically, however, the Marx bank is charged to \( \sim 80 \text{ kV} \), which then results in a maximum voltage of \( V_{\text{Marx}} = 2.56\text{MV} \) and a total stored energy of \( W_{\text{stored}} = 133\text{kJ} \).

### 2.2.2 Intermediate Electrical Storage

The Marx bank discharges the capacitor energy in \( 1 \mu s \), which is too long for efficient transfer of energy to an imploding system [40], thus an intermediate electrical transfer system is needed to compress this pulse temporally. The next stage is the intermediate electrical storage section, or pulse forming line. It is comprised of two large, \( \sim 1-2 \text{ m} \), concentric, stainless steel cylinders, immersed in a large pool of deionized water. Thus this section acts as a capacitor. This intermediate capacitor has an effective capacitance of 27 nF. A schematic of this layout is shown in Figure 2.2. This section is then connected to the pulse forming line, PFL, discussed in the following section, via a rimfire switch. This switch is, as with the switches within the Marx bank, filled with SF\(_6\) gas. Contrary
to the Marx switches, which use a triggering voltage to aid in the breakdown of the SF$_6$, the rimfire switch operates in self-break mode, relying on the buildup of sufficient potential to break down the gas in the switch. The SF$_6$ pressure then determines at what voltage the switch breaks down, and it is adjusted such that the intermediate capacitor is nearly fully charged by the time the SF$_6$ breaks down. The combination of the intermediate electrical storage and the rimfire switch serves as an added control measure of the final delivered current pulse. The intermediate electrical storage is then connected to the vertical transmission. The vertical transmission line is comprised of another pair of large several concentric metal cylinders, immersed in the same pool of deionized water, as seen in Figure 2.2. The charging time for the transmission line is about 1/4 that of the Marx discharge, at 250 ns. This transmission line then discharges through the MITL to the load, with a transit time of 100 ns. Thus the initial $\sim 1\mu$s discharge pulse from the Marx has been compressed temporally by a factor of 0.1.

### 2.2.3 The Magnetically Insulated Transmission Line

In order to deliver the pulse to the load, it must first be spatially decreased from the size of the transmission line, $r_{TL} \sim 1$ m, down to a size of a typical load, $r_{load} \sim 1$ cm. In order to accomplish this, a carefully designed device known as a magnetically insulated transmission line (MITL) is used, and seen in Figure 2.4. Here, a “stack” consisting of concentric metallic rings, and dielectric rings formed with an angle on the inner surface of 45 degrees, are used to transmit the initially spatially large, $r_{TL} \gg r_{load}$, down to the desired size within the vacuum load chamber. The choice of this design is used to prevent electrons emitted from the transmission line from crossing the vacuum gap, where it could potentially provide a short-cut for the current, and thereby degrade the total amount of current delivered to the load. When the rising pulse is being delivered, there is a vertically upward oriented electric field, $E_z$, while there is simultaneously an azimuthally positive magnetic field about the line, $B_\theta$. For an electron emitted from the MITL, the $E \times B$ drift is radially inward. This action serves to keep “flashover” across the dielectric surfaces. Similarly, for the interior transmission line, emitted electrons
are confined to the electrode, for the same reason. It is important to note, however, that when $\frac{dt}{dt}$ changes sign, after the maximum of the current pulse, the $\text{ExB}$ direction will also flip. This means that the MITL is no longer effective, and the current is often observed to “crowbar”, or short circuit, making measurements of the current very difficult, if not impossible, to determine since the magnetic insulation is no longer effective.

### 2.2.4 Final Output to Load

As previously stated, Zebra may be operated in two modes, either short or long pulse. The inner conductor of the vertical transmission line is connected to the outer connector via 8 pin to pin water switches. The gap between these switches is set to a distance which allows the transmission to fully charge before these gaps close. By leaving the water gaps open, or optionally closing them, the pulse is drastically altered. When the gaps are left open, the transmission line is
Figure 2.5: Examples of typical current pulses delivered to the load by Zebra for both short and long pulse mode. Figure from UNR NTF website. [64]

allowed to charge to its maximum voltage before the pulse is transferred to the load section, while simultaneously decreasing the pulse width. Conversely, closing these gaps corresponds to the operation of Zebra in long-pulse mode. The closed gaps prevent the transmission line from fully charging before delivering the pulse to the load section, i.e. the voltage is lower and the pulse is longer than in short-pulse mode.

Combining all of these machine characteristics, the nominal pulse modes and their characteristics can be summarized as follows. In short pulse mode, the Zebra machine nominally delivers a maximum current of $\sim 1$ MA, with a rise-time of $\sim 100$ ns, a voltage of $\sim 2$ MV, and at an impedance of 1.9 $\Omega$ [66]. This translates to a peak power delivered of $\sim 2$ TW. Long pulse operation delivers a maximum current of about 0.6 MA, with a rise time of $\sim 200$ ns, and a voltage of $\sim 1$ MV. The impedance of this machine means that it is relatively insensitive to the characteristics of the load selected, i.e. virtually any practical diameter load, $\sim \leq$ a few cm, can be driven without any worry over the consequences of the changing impedance of the load.

2.2.5 The Load Chamber and Diagnostic Access

The Zebra chamber was, of course, designed to allow for diagnostic access. Unlike laser-driven experiments, however, the discharge of such large energies must be coupled directly to the load environment. This means that vibrations from the
Figure 2.6: a) An overhead view of the load chamber. 16 ports allow for diagnostic access at center-line separations of 22.5°. b) A 3D cutaway view of the Zebra load chamber, with the typical location of the load shown.

discharge are directly transferred to the chamber. In short-pulse operation, the machine discharge can cause accelerations of up to 300 g. The electromagnetic pulse, a short burst of electromagnetic energy, is also very strong at distances within and near the chamber, which is often problematic when fielding diagnostics within the chamber. Both of these characteristics of a MA-scale driver can be detrimental to sensitive diagnostics placed within the chamber. To this end, most diagnostics are located outside of the chamber, or placed in tubes extending several meters from the load axis. In order to facilitate this, 8 3” diameter and 8 2” diameter ports are alternately arranged azimuthally about the chamber circumference, angularly spaced by 22.5 degrees, as seen in Figure 2.6. This allows for variety in the suite of diagnostics employed. A scaffolding of large steel beams constructed ~ 2 feet above the surface of the pool which allows for suspension of diagnostics, as well as support for the beam transport. Output from electrical diagnostics are fed to Faraday cage boxes, in order to protect the equipment inside from the electromagnetic pulse produced by the current discharge.
Figure 2.7: Layout of the Leopard laser front-end. After amplification, the beam is imaged to the CPA compressor chamber for temporal pulse compression before being sent to the target chamber, either Phoenix or Zebra.

2.3 The Leopard Laser

High-power, ultrafast, lasers are relatively new systems. These lasers produce powers on the order of 0.001-100 J, with pulse durations on the order of 10-1000 fs, resulting in powers in the terawatt, $\geq 10^{12}$ W, and petawatt, $\geq 10^{15}$ W, range. Such high power lasers have been enabled through the use of the chirped-pulse amplification (CPA) technique [68] [69] [70] [71] and broad-bandwidth lasing materials such as Ti:sapphire, 800 nm, and ND:glass, 1.06 $\mu$m [72]. When focused to small spots, 1-100 $\mu$m diameter FWHM, irradiances can reach $> 10^{20}$ W·cm$^{-2}$. The Leopard laser is one such system, and was at least in part designed and built to enhance the diagnostic capability of Z-pincha experiments on Zebra. A high-power, ultrafast, laser enables a multitude of diagnostics which are often not available, especially at University-scale Z-pincha laboratories. Given the correct parameters and setup, such a laser can be used to enable unique diagnostics such as X-ray backlighting, optical/X-ray Thomson scattering, X-ray plasma absorption spectroscopy, and proton deflectometry. The following section briefly describes
the components of the Leopard laser, along with the output beam specifications at outlined by Wiewior et al. [73] [74].

As with high current Z-pinch devices, these high-power ultra-fast lasers also share many common features. The component stages of such lasers include a front-end, where an initial or seed pulse is generated, an amplification stage, where the low energy seed pulse is amplified to the desired energy, and finally a compressor stage, where the beam pulse width is reduced temporally via the CPA technique. After these stages, the light is then delivered to a target via a beam transport line. While the two initial stages may be carried out in air, from the compressor stage onward, the light is transported in vacuum.

![Figure 2.8: A general schematic for a laser pulse amplified and compressed via the CPA technique. Figure adapted from Backus et al. Review of Scientific Instruments (1998) [72]](image)

### 2.3.1 Seed Pulse Generation

The Leopard laser utilizes commercially available products to generate the seed pulses at the front end. A femtosecond oscillator is pumped via a frequency-doubled diode-pumped solid-state Nd:YVO$_3$ laser, both manufactured by SpectraPhysics. From this a 130 fs seed pulse is sent through an Öffner-type pulse stretcher, which results in a pulse which is approximately four orders of magnitude
longer. A stretched pulse is then selected and amplified by a Ti:sapphire regenerative amplifier at 500 Hz. Before this pulse is sent on, it must first pass through a fast Pockels cell. Pockels cells are devices which utilize an electro-optic medium, which is usually a crystal. When a voltage is applied, the refractive index of the medium is slightly altered in proportion to the applied voltage/electric field, which is the well-known Pockels effect. This effect may be utilized to alter the phase delay within the medium, and in effect, remove undesired portions of the laser pulse before they are amplified. These undesired portions are known as pre-pulses, and are due to a phenomenon known as amplified spontaneous emission, or ASE. The utilization of fast Pockels cells improves the power contrast ratio of the final laser pulse, which we define as the ratio of the power in the pre-pulse to the power in the main laser pulse.

\[ r_c = \frac{P_{\text{pre-pulse}}}{P_{\text{main-pulse}}} \]  

(2.3)

High contrast ratio is extremely important in the generation of high-energy laser-accelerated proton beams, and will be discussed in further detail in Chapter 3. Lastly, after the selected pulse has passed through the Pockels cell, it is passed through a serrated aperture. This changes the spatial profile of the laser pulse such that is optimal for energy extraction from the amplifiers.

### 2.3.2 Pulse Amplification

Although the amplification will increase the energy of the seed pulse to the final energy, the pulse stretching in the previous step is important in order to keep the power of the pulse below the threshold for self-focusing in the amplifiers. There are five amplifiers that the pulse from the oscillator passes through. The beam spatial profile is progressively spatially expanded as it moves through the amplifier chain. The first amplifier is a 6 mm diameter phosphate glass amplifier, after which the pulse is sent through two 19 mm diameter amplifiers utilizing silicate glass and phosphate glass respectively. After this, the pulse will be passed through a 45 mm diameter phosphate glass amplifier, before final amplification by a 94 mm diameter disk-type amplifier comprised of a phosphate/silicate glass. In addition to the amplifiers in the chain, there are also two additional fast Pockels cells, in
order to maintain high contrast ratio in the amplified pulse, while also preventing damage from ASE to the amplifiers. As an added protection for the amplifiers, Faraday isolators are also employed. These isolators are simply advanced polarization devices, which minimize the transmission of reflected pulses due to their polarization misalignment with respect to the polarization of the isolator. This becomes more important as higher powers are achieved, and even small fractions of reflected light energy can become significantly amplified, leading to damage in the amplifier chain. After the pulse has been amplified, it is finally transported to the pulse compression chamber for temporal compression to the final desired pulse length.

2.3.3 Pulse Compression

The Leopard laser utilizes a double pass, dual diffraction grating configuration. The gratings are 1740 lines/mm, with a $\lambda/8$ surface figure at 1 $\mu$m wavelength light. As previously mentioned, the beam compressor is performed under a vacuum of $10^{-6}$ Torr. This serves to maintain the cleanliness of the optical elements. Were dust or other contaminants to settle on the surface of the gratings, severe damage could occur when the high-power light pulse is incident. Although the amplifiers on the front-end of Leopard are capable of amplifying pulses up to 100 J, they are limited to a maximum of 30 J due to the damage threshold of the gratings used in the CPA system. In practice, the pulses are typically limited to just half of this, and so the pulse energies generally average around 15 J when Leopard is operated in short-pulse mode.

2.3.4 Adaptive Optics, Beam Transport, and Focusing

Due to the many elements present in the Leopard laser from seed pulse to compression, significant distortion to the wavefront often occurs. This distortion is manifested as intensity variations within the laser beam profile. These variations may cause inefficient amplification, or worse, damage to optical components. It may also be present in the final focused beam profile, which can cause undesired
irradiance variations, which may ultimately lead to harmful beam-plasma interactions, such as self-focusing, which are often detrimental to the desired interaction. Additionally, the variations in the laser wavefront may lead to a divergent beam profile, which degrades the ability of the off-axis parabola (OAP) to focus the beam. Adaptive optics were originally developed in the astrophysics community as a means of compensating for atmospheric distortions when using ground-based telescopes, and are quite simply, deformable mirrors. Using small actuators attached to the rear surface of a mirror, and controlling software, the mirror may be deformed in such a way that it compensates for distortions in the incident light. This technology was later adapted for use in lasers to compensate for the intensity variations found in all amplified laser wavefronts, and has subsequently become a standard component in high-power lasers.

![Schematic of adaptive optics system](image)

**Figure 2.9**: Schematic of adaptive optics system. Part of the beam is picked off by a beam splitter so that the wavefront can be monitored and fed to the control system for the deformable mirror. Figure adapted from The LYOT Project website [75]

When the adaptive optics system is employed, the final focused laser spot is a factor of $1/8$ the size of the spot without the use of adaptive optics. This becomes very important when laser irradiance is important. Adaptive optics systems offer one more important advantage. After a single shot at full energy, the amplifiers and amplifying media will be thermally “hot”. The expansion in the materials introduces significant waveform abberations. Typical cool-down times
are 1-2 hours before the amplifiers have returned to their original state. Since these distortions may be compensated for by the adaptive optics, the cooling period becomes a lesser concern in the repetition rate. Instead, the minimum cooling time is set by the flashlamp for generating the seed pulse, at about 15 minutes. Since the Phoenix and Zebra chambers typically take around 45 minutes to reach vacuum pressures which are acceptable for firing either the laser, the pulsed-power machine, or both, these requirements become the limiting factor for the amount of shots that can be taken in an operational day.

In order to focus the Leopard beam in this work, an off-axis parabolic, or OAP, mirror was used. Experiments performed in the Phoenix target chamber utilized a 30° dielectric OAP, as seen in Figure 2.10 a). During coupled shots, a 90° metallic off-axis parabola, as seen in Figure 2.10 b), was chosen over a dielectric type, which has more ideal optical properties, due to financial considerations. This is because debris typical of a Z-pinch experiment is likely to destroy the surface of any optical component in the chamber, and a dielectric parabola costs about 15-20 times more than the metallic type. The risk to components in the Zebra chamber will be discussed further in Chapter 6.

![Figure 2.10: a) The dielectric OAP used during laser-only experiments in the Phoenix chamber, and b) the 90° off-axis parabola used for focusing the Leopard laser beam during coupled shots in the Zebra chamber.](image-url)

The parabola used in these experiments was 101.6 mm diameter, with a
76.2 mm effective focal length, giving a $f/#$ number of 1.5. The diameter of the parabola ensured that the full Leopard beam diameter, $\sim$80 mm, to be focused, while the focal length was chosen due to target chamber dimensions and commercially available options. The parabolic mirror shape is first machined from aluminum, then coated with gold, and a proprietary coating for protection of the gold surface. Due to the small $f$ number, the focus of the laser was very sensitive to changes in position between the optical axis of the laser and the OAP.

![Figure 2.11](image)

Figure 2.11: a) An image of the minimum Leopard focal spot focused using a dielectric OAP. b) A 3D plot of the minimum focal spot. c) A line-out across the focal spot plotted along with a Gaussian curve with 5 $\mu$m full-width half-maximum for reference.

Final beam output $\sim$15 J in 350 fs, with a contrast ratio $\sim 10^{-8}$. When using the dielectric parabola, the final focal spot diameter was $\sim$5 $\mu$m, as seen in Figure 2.11. Assuming that 50% of the laser energy is contained in this spot gives a calculated on-target intensity of $\approx 1 \times 10^{20}$ W cm$^{-2}$. The diameter of the focal spot was slightly larger when the metallic OAP was used, at $\sim$7.5 $\mu$m, translating to a calculated on-target intensity of $\approx 5 \times 10^{19}$ W cm$^{-2}$.
Chapter 3

Z-Pinch Diagnostics

3.1 Introduction

Z-pinch experiments are an extremely hostile environment for diagnostics. An example of the destructive force of the enormous magnetic and radiation pressures exerted by a Z-pinch shot on the Z machine at Sandia National Laboratory is shown in Figure 3.1.

![Pre-shot, Shot, Post-shot images of Z machine chamber](image)

**Figure 3.1:** The experimental chamber before and after a shot on the Z machine at Sandia National Laboratory as an extreme example of the level of debris present in Z-pinch experiments at the MA current scale.

Diagnostics are of paramount importance in order to study the myriad of physical processes occurring during the course of an experiment. Although there are a multitude of diagnostics available, there are some which are standard within the Z-pinch field, and several of the most important of these will be described here. One of the most important parameters in any Z-pinch experiment is the shape of
the current pulse, i.e. the timing and value, delivered to the load, and thus differential B-dots are placed at various points in the machine to measure this. The aim of the other major diagnostics are to make quantitative estimates/measurements of the plasma properties, including density, temperature, and velocity, throughout the evolution of a Z-pinch experiment. One of the most common tools for measuring density and velocity is through the use of lasers. Additionally, there has been much progress in methods for recovering electromagnetic field information from within the plasma. Most of these diagnostics operate on a timescale between 0.1-10 ns, in order to take snapshots of the dynamic system as it evolves over the typical current drive time of \( \approx 100-200 \) ns. Measurements of these parameters are often challenging, especially given the violent nature of such experiments. Mega-Volt potentials, intense X-rays, and high-velocity ballistic debris are just some of the issues to contend with in order to make a high-quality measurement in a Z-pinch environment.

3.2 Machine Current Measurement

Perhaps the most important diagnostics, are those that monitor the driving current pulse. Knowing the details of the current pulse throughout the experiment is crucial, since without this information, it is not possible to estimate how much energy is delivered to the plasma. In order to accomplish this, the current is measured at multiple points within the machine. This is done with devices known as a B-dots and V-dots. Capacitive probes, V-dots, or resistive voltage dividers are often employed within the machine to monitor the load voltage throughout the machine pulse. Differential B-dot probes and Rogowski coils, however, are readily installed near the load, where current measurements are most important for the experiment.
3.3 Differential B-dot Monitors

Differential B-dot probes are used to infer the current through an element located somewhere in the machine [76–78]. The basic design of differential B-dot probes is shown in Figure 3.2. The two magnetic pickup coils are embedded in a conductor, and detect the changes in magnetic flux in a hole placed within the current delivery/return structure. Embedding the probes within the structure of the machine helps to avoid perturbation of the effectiveness of the current-transmitting components, while simultaneously protecting the probes from electric fields. Two separate signals are gathered from the two coils within each B-dot probe. The coils are arranged such that they have opposite polarity and are at different azimuthal locations with respect to the load at the center of the machine. The resulting signals are processed to remove common-mode noise and then integrated to produce the current waveform. These differential B-dots are embedded in the anode mesa, in three azimuthally symmetric locations about 15 cm from the axis of the load. Each B-dot is calibrated, and the resulting signals, voltage traces throughout the current drive, are then easily integrated to give the total current through each region. This data is then combined and mean-averaged, resulting in the current waveform of the machine, and monitor any potential asymmetries in the current transmission. This allows for detection of signals in the electromagnetically noisy environment of a driver such as Zebra. Each B-dot probe in one of these devices is itself a diagnostic which may be used independently to investigate magnetic fields.
Figure 3.3: The positions of three differential B-dot probes placed in the return current mesa of Zebra. Another differential B-dot is embedded in the load hardware to monitor current near the load.
within plasma loads, a simple version of which is discussed further in Section 3.6.1.

### 3.3.1 Rogowski Coils

Rogowski coils are another common method of measuring the machine current. As with differential B-dot probes, these probes utilize coils to detect magnetic flux. The simplest design is shown in Figure 3.4. Here a helical coil is arranged concentrically about a current carrying conductor. The helical coil detects magnetic flux inside the area of the loops via an induced EMF, which is proportional to the time derivative of the magnetic flux.

![Figure 3.4: a) A basic Rogowski coil design. A helical coil of wire is placed around a conductor carrying current, I. b) A cross section of the coil showing the magnetic flux generated by the current carrying conductor and the induced EMF.](image)

In Figure 3.4, $r_0$ is the major radius of the coil, while $r_1$ is the minor radius.

$$V = -M \times \frac{dI}{dt}$$  \hspace{1cm} (3.1)

where $M$ is the mutual inductance, a geometric factor, proportional to both the size of the cross-sectional area of the coil and the number of wire turns. The coil is thus sensitive to changes in current. When constructing such a probe, tight and
uniform spacing of the coils is important in order to shield from unwanted electromagnetic interference. By placing them near the load, a better measurement of the current delivered to the load than inferring from remote current monitors, such as those embedded in the mesa, can be obtained. It may be dangerous to assume that all of the current measured by the differential B-dot probes embedded in the machine hardware has been delivered to the load, since it is possible that some current is lost somewhere along the MITL or in the load hardware. Current loss in these areas may not be detected in the B-dot probes embedded in the machine, leading to inaccurate estimates of the total energy delivered to the plasma. Rogowski coils are often placed around current-return posts, which are themselves placed concentrically around the load at a radius of several centimeters in order to measure the current as close to the load as possible, and thus ensuring that an accurate current reading is obtained. In this case, it is important that the coil is placed accurately about the axis of the current carrying post, so that a reliable and reproducible measurement is made. As discussed in the previous section, differential B-dot probes embedded in the top of the load hardware allow for current measurements even closer to the load, however, due to the ease of fielding such diagnostics, they are often deployed redundantly in the system. Rogowski coils may also be fielded in a differential configuration by placing an identical coil on another return post, but oriented in such a way as to produce a voltage of opposite polarity.

3.4 Laser Diagnostics

Laser-based diagnostics are an extremely convenient means of measuring density distributions of a plasma. They are also advantageous because of the wide availability of nanosecond and sub-nanosecond range of laser pulses, which makes them ideal for taking snapshots of Z-pinch plasma systems, which evolves on a time-scale which is 3→4 orders of magnitude larger. At the NTF, the laser pulse width is 150 ps, providing more than adequate temporal resolution for the work in this thesis. All three laser diagnostics operate similarly, typically by expanding a
laser beam’s initial diameter, to a size which is similar to the plasma system size, typically about 2 cm diameter, through a system of mirrors and lenses. It is then directed at the Z-pinch, which allows for measurements over the entire region of interest.

After the laser pulse has passed the plasma system, it is collected again through a system of lenses and mirrors, and directed at a detector. Just before the detector, the light is typically filtered and attenuated to avoid saturation of the detector. Neutral density attenuators are placed before the detector, typically a charged-coupled device (CCD) sensor, in order to cut the intensity of the light, which not only can easily saturate the CCD pixels, but potentially damage it due to the power of the laser pulse. After the ND filters, a band-pass filter, which passes light of the same wavelength as the laser light, in most cases 532 nm, and attenuates other frequencies. The exact combination of filters is determined by the ability to attenuate as much optical wavelength radiation from the Z-pinch load as possible, while retaining an acceptable signal levels from the laser pulse. The CCD cameras are triggered before the start of the current pulse, and remain on throughout the experiment, thus the timing of the images is set simply by the timing of the laser pulse firing relative to the Z-pinch current, while the temporal resolution is set by the pulse width of the laser.

While the laser typically fires a single pulse during the course of a Z-pinch shot, multiple images from a single experiment may be gathered in several ways. The simplest way of achieving this is to split the beam after it has exited the plasma system, and directing the separate beams through the system along slightly different paths, to different detectors, allowing multiple images at similar times. The second method is to split the beam into the desired number of pulses before it is directed through the system of interest, and direct each pulse through a different path length, called a delay arm, which introduces a timing difference in the images due to the difference in path lengths. Through this method, typically achieved pulse separations of a few tens of nanoseconds are possible, practically limited by the size of the laboratory. Additionally, the beams must either be separated spatially, i.e. directed through different ports, or the polarization must be altered,
so that only the desired light reaches the detector. For NTF experiments, the CCD was a 16-bit MicroLine ML8300, with 3448x2574 5.4 µm width pixels. Finally, slight modifications to the setup can provide different types of measurements, as will be discussed here.

### 3.5 Laser Propagation in Plasmas

All laser diagnostics, however, have an important limitation in their usefulness. This is due to the interaction between laser light and plasmas. The propagation of an electromagnetic wave, or laser light, in a plasma is highly dependent on the distribution of the free electrons [79]. In order to see this more clearly, however, one must consider the natural oscillations of a plasma.

A simplified picture of a plasma is two unmagnetized fluids comprised of electrons and ions, respectively. If the electrons are displaced relative to the ions, they will be pulled back to their original positions due to the Coulomb force. Of course, due to the finite momentum of the electrons, they will not return to their original positions, and the Coulomb force will again attempt to restore the original position of the electrons. This natural oscillation is known as the plasma frequency. To obtain an expression for this frequency, we need only to consider the electron motion due to the restoring force, while ignoring the electron thermal motions, i.e. the electron plasma is “cold”. If we consider a one-dimensional case of a large slab of electron and ion fluid, and displace the electron species by a small amount, Δx, an electric field develops due to the charge separation.

\[
E = \frac{-en_e \Delta x}{\epsilon_0}
\]  

(3.2)

The equation of motion for the electrons is then:

\[
m_e \frac{d^2 \Delta x}{dt^2} = eE = e \left( \frac{-en_e \Delta x}{\epsilon_0} \right)
\]  

(3.3)

Assuming harmonic motion, the fundamental frequency describing the natural oscillations of the electrons in the plasma is:

\[
\omega_{pe}^2 = \frac{e^2n_e}{\epsilon_0m_e}
\]  

(3.4)
To see the effect of the electron density on a light wave propagating through a plasma under simplified assumptions, we examine the equations describing an electromagnetic wave in a medium [80]. First, we consider the sinusoidally varying electric field in a planar light wave traveling in the $\hat{z}$ direction with the electric field in the $\hat{x}$ direction, which is $E_{\text{laser}} = E_x e^{(kz - \omega t)} \hat{x}$. Then, the combination of Ampere’s law along with Faraday’s law will produce the desired equation.

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (3.5)$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} \quad (3.6)$$

By taking the time derivative of equation 3.6 and the curl of 3.5, these two equations may be combined, resulting in:

$$\nabla \times \left( \frac{\partial \mathbf{B}}{\partial t} \right) = \nabla \times (-\nabla \times \mathbf{E}) = \mu_0 \frac{\partial \mathbf{J}}{\partial t} + \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} \quad (3.7)$$

$$\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} \quad (3.8)$$

Using the vector identity 3.8, equation 3.7 becomes:

$$\nabla^2 \mathbf{E} - \nabla(\nabla \cdot \mathbf{E}) = \mu_0 \frac{\partial \mathbf{J}}{\partial t} + \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} \quad (3.9)$$

which the equation for an electromagnetic wave. Of course, the second term on the RHS must be dealt with. This is done here by assuming again that it is only the electron inertia which is important, i.e. the electrons are “cold” and the ion motion is negligible, so that the current is proportional to the electron motion induced by the electric field, thus:

$$\mathbf{J} = -e n_e v_x \mathbf{\hat{x}} \quad (3.10)$$

while the electron equation of motion is:

$$m \frac{dv_x}{dt} = -e E_x \quad (3.11)$$

Substituting these equations 3.10 3.11 into equation 3.9, and carrying out the differentiation, yields the dispersion relation:
\[
\frac{k_L^2 c^2}{\omega_L^2} = 1 - \frac{\omega_p^2}{\omega_L^2}
\]  
(3.12)

where \(k_L^2\) is the laser wavenumber, \(\omega_L^2\) is the frequency of the laser, and \(\omega_p^2\) is the plasma frequency. When the RHS of eqn 3.12 \(\to 0\), then the wavenumber becomes imaginary, and thus the laser cannot propagate. The RHS goes to zero when \(\omega_p \approx \omega_L\), which occurs when the electron density reaches the so-called critical density.

\[
n_e \approx \frac{1.1 \times 10^{21}}{\lambda_{L(\mu m)}^2} [cm^{-3}]
\]  
(3.13)

Here, \(\lambda_{L(\mu m)}\) is the wavelength of the laser light in microns, and in the case of 532 nm laser light, the wavelength used in this work, the critical density is \(n_e = 3.9 \times 10^{21}cm^{-3}\). Thus, laser refraction and absorption processes occur only in the underdense plasma, where the electron densities are below \(n_e\). This criteria means that laser probing techniques are sensitive to electron densities, and have an upper cutoff to the density of plasma which they can reach. Given this information, and that the index of refraction is defined as the ratio of the phase speed of light in vacuum to the phase speed of light in a medium, we may rearrange to find the expression for the index of refraction in a plasma. This is found to be:

\[
N = \frac{c}{v_{ph}} = \frac{c k_L}{\omega_L} = \left[1 - \frac{\omega_p^2}{\omega_L^2}\right]^{1/2}
\]  
(3.14)

It is also useful to have this expressed in terms of the free electron densities, which is:

\[
N = \left[1 - \frac{n_e}{n_c}\right]^{1/2}
\]  
(3.15)

Although we have just examined the effect of electron density in an unmagnetized plasma, there are in fact, usually non-negligible magnetic fields in Z-pinch plasmas, which may change this simplified solution depending on both the strength and orientation of the magnetic fields present. Further, the index of refraction depends on both ion density and atomic processes, so the full solution is more complicated than presented here. A more complete description may be found in Alpher et al. Physics of Fluids (1959) [81].
3.5.1 Laser Shadow Imaging

Laser shadow imaging, or refractive index imaging, is the simplest of the laser diagnostics options. No change from the setup described above is needed, and so snapshots can be taken with this method by simply varying the timing of the laser pulse relative to the current pulse. This diagnostic is useful for monitoring motions of dense, above the critical density, regions of plasma, but little else in the way of quantitative data can be obtained. Of course, as with the other laser based diagnostics, it is sensitive to density gradients, or variations in the index of refraction within the plasma, which will manifest as distortions in the resulting laser image. These may manifest as bright or dark spots in the image, depending on the strength of the density gradient. Too large of a density gradient will refract rays outside of the acceptance angle for the optics system, which may appear as dark spots in the image. Although many lenses may be employed in the optics setup, in general, the acceptance angle of the system is determined by the radius of the collecting lens, $r_{\text{lens}}$, and the source-lens distance, $d_{SL}$ as seen in Figure 3.5.

![Figure 3.5: The acceptance angle of a source-lens system.](image)

$$\Theta_{acc} = \arctan\left(\frac{r_{\text{lens}}}{d_{SL}}\right)$$ (3.16)

Dark regions, of course, may also be due to overdense plasma. An example of the usefulness of this configuration of laser probing is seen in Figure 3.6, where it has been employed to monitor the bulk motions of an imploding cylindrical wire array. The instabilities and resulting disordered implosion can easily be seen.
3.5.2 Density Gradient Imaging

Density gradient imaging, or schlieren imaging, is a diagnostic which is capable of identifying gradients in plasma density. This configuration is similar to the laser shadow setup, but has one key difference. After the load region, the light is recollected by a focusing optic, and rather than simply allowing this light to propagate to the detector, a small object is placed near the focus of the optic. This blocks most, although ideally all, of the laser light when there is no load or plasma present, thus the “background” picture taken just prior to firing the machine is completely dark. When a medium is in the beam path before the focusing optic and beam block, light rays will have their paths deviated due to the variations in the index of refraction. Assuming again that the laser is traveling in the \( \hat{z} \) direction, this deviation creates a pattern of light and dark regions, which can be used to infer the density gradient of the plasma. This technique is particularly useful for studying the dynamics of plasma expansion and compression, as it provides a visual representation of the density variations in real-time.
direction, the angle by which a ray will be deflected is:

$$\Theta_y = \int \frac{d}{dy} N(x, y) dl$$  \hspace{1cm} (3.17)

where $N(x, y)$ is the index of refraction as a function of $x$ and $y$, paths due to the presence of the medium are allowed to bypass the block. This setup is shown schematically in Figure 3.7. Provided this deviated angle is less than the acceptance angle of the optical setup, $\theta_{refraction} \leq \theta_{acceptance}$, it is then imaged on the detector. The resulting images show the location of gradients in the plasma density, and thus this configuration is known as dark-field schlieren imaging. By varying the focal length of the lens and the size of the beam stop in the setup, the sensitivity to gradients can be varied.

**Figure 3.7**: The basic setup for dark-field laser schlieren imaging of plasma density gradients. The sensitivity range for the magnitude of the gradients is determined by the optics used, and the size of the beam block.

There are, of course variations on this setup, which include changing the type of beam block used. For example, if a flat edge is used to block the beam, then only gradients perpendicular to the orientation of the edge will be imaged. In light-field schlieren imaging, a small aperture is placed at the focus of the lens. This produces an image similar to the shadowgraphy setup, however, when light rays are refracted due to the density variations in the free electron population, they are then blocked by the aperture, leading to dark regions in the image. All configurations give data which is rather difficult to quantify reliably. The information, as in shadowgraphy, is primarily used to identify qualitative plasma features, such as coronal plasma structures and shocks.
Figure 3.8: An example of a laser schlieren image. Two wires are placed adjacent to each other, separated by 8 mm, and driven by a 200 kA current pulse. The plasma gradients are easily identified as the light regions in the image. The corona structure surrounding the wires is visible, as well as the formation of a precursor column of plasma near $x = 0$ mm.
3.5.3 Laser Interferometry

Laser interferometry is the most quantitative of the three laser diagnostics described in this section. There are multiple variations on laser interferometry scheme, however they all operate on the same principle, and produce similar results. All setups require that there are two beams split from the same pulse, which travel equal distances before being recombined to form an interference pattern. Changes to the optical path lengths introduced by the plasma will manifest as a shift in an otherwise uniform interference pattern. Of course, as previously discussed, it is the variations in the electron density that produce the strongest changes to the index of refraction, and thus the shift in fringes corresponds to the changes in electron density over the path taken by the photons.

When the two beams recombine on another beam splitter, and the electric fields of the two beams interfere, an interference pattern is formed. Phase differences introduced by the plasma shift the interference fringes in the resulting image, and the total phase change in the probe beam is \[ \phi = \int kdl = \int N \frac{\omega}{c} dl \] (3.18)

In any of the various interferometry schemes, it is vital that the path lengths taken by each of the two beams remain the same. Otherwise, they will not combine before entering the detector, and hence, will not interfere. The maximum error in path length difference is then set by the pulse width. For example, a pulse width of 150 ps corresponds to a \( \delta x \) of \( \sim 3 \) cm. Obviously, this is a crude estimate, which assumes a “top hat”, or square, waveform, but it does represent the maximum error, which would result in the two beams missing each other entirely. Ideally, both images should arrive at the interfering medium at the same time, in order to maximize the contrast of the interference fringes, or the interference between the electric fields. A better approximation of the nature of the pulse width, is well represented by a Gaussian, rather than a square waveform. This simply means that in practice, the interference contrast is typically more sensitive to differences in path length than this crude estimate suggests. The majority of the path length taken by the probe beam, however, is not inside the plasma, and so can be compared with the reference
beam, which is completely outside of the plasma. Including both portions, the only
significant changes are then assumed to be due to the plasma in the probe beam,
so that the integrals need only be taken over the path length through the plasma,
e.g. $x_p^- \rightarrow x_p^+$:

$$\Delta \phi = \int_{x_p^-}^{x_p^+} (k_{\text{plasma}} - k_0) \, dl = \int_{x_p^-}^{x_p^+} (N - 1) \frac{\omega}{c} \, dl = \frac{\omega}{c} \int_{x_p^-}^{x_p^+} \left[ \left( 1 - \frac{n_e}{n_c} \right)^{1/2} - 1 \right] \, dl \quad (3.19)$$

Where $k_0$ is the wavenumber in the absence of plasma. Since the plasma density
must be quite small in order for the beam to propagate, i.e. $n_e \ll n_c$, we can
expand the index of refraction of the plasma.

$$N \approx 1 - \frac{1}{2} \frac{n_e}{n_c} \quad (3.20)$$

Substituting eqn 3.20 into eqn 3.19, we can simplify the expression for the phase
change to

$$\Delta \phi = \frac{-\omega}{2cn_c} \int n_e \, dl \quad (3.21)$$

When the “background” image is taken, the fringes in the image represent contours
of constant phase, and when a plasma is introduced, the fringes may shift due the
difference in phase introduced in the probe arm by the plasma. The number of
fringe shifts, $F$, at any point on the detector relative to the background image
then represent the line averaged refractive index along the path taken by a ray,
according to:

$$F = \frac{\Delta \phi}{2\pi} \approx -4.46 \times 10^{-18} (\lambda_{\mu m}) \int n_e [\text{cm}^{-3}] \, dl \quad (3.22)$$

where $\lambda_{\mu m}$ is the laser wavelength in microns, in this case 0.532 $\mu m$, and the
electron density is in units of $\text{cm}^{-3}$. A 2D map of the line-integrated electron density
can then be obtained by comparing the shift in fringes between the background
image and the shot image. An example of interferometry data is shown in Figure
3.9. The information is still limited, as in other types of laser diagnostics, by
the critical density. Additionally, if large gradients in plasma density are present,
rays may be deflected out of the optical system entirely. This information may
only be extracted in areas where $n_e \ll n_c$, and where fringes have not become too
distorted to track, as in the case of the precursor column in the center of the experimental image, seen in Figure 3.9. Fringe shifts smaller than a few camera pixels
Figure 3.9: An example of electron density measurements from laser interferometry in a Mach-Zehnder setup. The left-hand image is the “background” image taken just prior to the shot. The experimental image is the image taken at 224 ns after current start, and finally, the “unfolded” image gives quantitative integrated electron density information from the difference in interferometric fringe shifts from the experimental and background image.

also cannot be resolved, and thus information is again lost. In order to obtain the line-integrated electron density, the fringe shifts must be followed, and if done manually, the minimum detectable electron density is a $\frac{1}{4}$ fringe shift, or about $10^{-17} \text{cm}^{-2}$. This is improved by using a program designed for calculating such shifts, Interferometric Data Evaluation Algorithms, or IDEA [83]. The upper limit on density detection is absolutely limited by the critical density, while practically limited by the existence of large gradients in the density. IDEA improves the data unwrapping through a series of calculations. First, areas which do not contain data are removed from the data set. The remaining portion of the interferogram is forward transformed with a 2D-Fast-Fourier-Transform algorithm. The carrier frequency is then isolated, and the image is back-transformed, yielding modulo $2\pi$ phase data, or values between $\pm \pi$. This creates a continuous phase plot of the data. This procedure is repeated for the background image. The resulting phase plot is subtracted from the data from the “shot”, and becomes a 2D plot of the phase shift in the probe data introduced by the plasma. When multiplied by the appropriate constants, as in equation 3.22, the final data representing the line integrated electron density, or areal electron density, is obtained. An example of this is seen in Figure 3.9.
Mach-Zehnder Interferometry

Figure 3.10: The basic schematic of Mach-Zender interferometry. The beam is split into probe and reference beams. The probe beam is sent through the plasma, while the reference beam is diverted around experimental chamber. Finally they are recombined at the second beam splitter before the CCD detector.

The basic layout for Mach-Zehnder, MZ, interferometry is shown in Figure 3.10. The beam is split into two equal intensity beams via a partially reflecting mirror. One beam is sent through the Z-pinch system, the probe beam, while the other is sent around the experimental chamber, the reference beam. The two beams are then recombined on another beam splitter before being imaged onto the detector. Typically, a system of lenses are used to recollimate the beam after it has exited the experimental chamber.
Shearing Interferometry

Due to the size constraints set by many Z-pinch target chambers, the MZ scheme for interferometry may be impractical, and thus a more compact interferometry scheme is needed. One such scheme is the so-called shearing interferometry setup. In this case, rather than splitting the laser beam before entering the target plasma, the beam is split after it has passed through the Z-pinch load, and the two beams are recombined shortly after. This scheme, however, requires that a portion of the beam has not passed through the plasma, and thus this portion of the image is considered to be a reference beam. Although shearing interferometry is sometimes more practical to implement, especially when laser safety is a prohibitive factor for setting up MZ interferometry, it is significantly limited by comparison. It is often difficult to probe the load region while still having enough beam “left over” for interference purposes. In regions of overlap where the reference beam has also been perturbed by the target plasma, the electron density information is lost, since it is not practical to attempt to account for both distortions.

![Figure 3.11](image.png)

**Figure 3.11:** The basic schematic of shearing interferometry. The probe beam is split, sent around the optical path, and recombined at the beam splitter.

As with the machine current measurements, lasers are also capable of making measurements of the magnetic field distribution within a plasma, and will also be discussed in the following section.
3.6 Magnetic Field Measurements in Z-Pinch Plasmas

As outlined in Chapter 1, the current topology, and the corresponding magnetic fields associated with them, in Z-pinch plasmas strongly dictates the dynamics of the system. Thus, it is of paramount importance to be able to diagnose the magnetic fields in Z-pinches in order to improve the understanding of the dynamics of these complex systems, and subsequently improve theory and numerical codes describing them. There are several techniques which are occasionally used to this end, including but not limited to; electrical probes, laser Faraday polarimetry, and Zeeman-splitting spectroscopy. As expected, measuring magnetic fields is a rather difficult task in this environment, and thus these diagnostics are not fielded in most experiments, but rather are used intermittently. Often, the measurements are only possible with specific and limited Z-pinch loads, and even then, only at early times in the experiment when plasma densities and temperatures are low. Much of the work in this thesis was to develop a reliable way to examine these fields in any Z-pinch experiment. The usual techniques for making these measurements are presented here, followed by a discussion of a promising new technique for determining current and magnetic field topology in Z-pinch experiments.

3.6.1 B-dot Probes

Electrical B-dot probes, or magnetic pickup coils, generally consist of an insulated conductor configured in such a way as to measure magnetic fields either globally, or at a point within the system of interest. The most commonly used electrical probes for measuring magnetic fields in plasmas are B-dot probes [61, 84–87]. The advantage of B-dot probes is that they are relatively simple probes to construct and field in an experiment, and operate on a very simple principle. The basic design of such a probe is shown in Figure 3.12. The simplest design of such a probe is a coaxial cable, which has a small loop formed at the end and is connected to the return conductor. This design allows for the straight-forward exploitation of Faraday’s Law, equation 3.23, whereby the magnetic flux normal to the plane
of a conducting loop will induce a voltage with is proportional to the change in magnetic flux with time:

$$EMF = V = \frac{\partial \Phi}{\partial t} = \frac{\partial}{\partial t} (\mathbf{B} \cdot \mathbf{A}) = \frac{\partial}{\partial t} (B_{\text{normal}} \ast A)$$  \hspace{1cm} (3.23)$$

where $V$ is the voltage induced, $\mathbf{B}$ is the magnetic flux, and $\mathbf{A}$ is the sampling area inside the loop formed by the conductor. This probe may be constructed out of commercially available semi-rigid coaxial cable, to form a loop which is of a suitable size, diameter $\sim 1$ mm, to be inserted within a Z-pinch experiment, where a typical size is of the order of 1 cubic centimeter. To protect the probe from directly conducting current from the plasma load, it is typically sealed by coating it in a high-temperature epoxy. The probe is then connected to an oscilloscope which will measure the voltage induced on the probe by magnetic fields produced during the course of the experiment.

These probes are typically calibrated by placing them near a cylindrical short-circuit load, where a simple field is generated. In this way, they are essentially cross-calibrated with machine current B-dots with known calibration factors so that the current can be used to calculate the field from the short-circuit load. The voltage trace is combined with the measured value of the probe area, and through a simple numerical integration, quantitative magnetic field magnitudes normal to the probe as a function of time are obtained. One of the most common probe failures is physical destruction of the probe during the course of an experiment.
This is due to the frequently very-high, MV, potentials present, high temperatures several eV, and copious amounts of electromagnetic radiation. During the course of a Z-pinch discharge, conditions often cause a breakdown in the probe insulation, leading to unreliable data. This is readily observed on many oscilloscope traces, when the signal suddenly increases to voltages which correspond to unrealistic field values. One of the most significant drawbacks of these probes is that they interfere with the flow of plasma when placed within a wire array, which leads to the obvious conclusion that the dynamics of the system have been significantly altered. An example of this can be seen in Figure 3.13, where two small, loop area $\sim 2.5 \text{mm}^2$, B-dot probes inserted into a four-wire Z-pinch load are disrupting the flow of plasma toward the common axis. Since these probes measure the

Figure 3.13: Two B-dot probes are inserted near the axis of a wire array. The laser interferogram shows strong plasma flow disruption due to the presence of the probes.
magnetic flux at just location throughout the experiment, the spatial resolution is severely limited with this diagnostic. Further, it is clear that the addition of more probes, in order to increase the spatial resolution, would lead to further alteration of the system dynamics.

### 3.6.2 Faraday Rotation

A much less intrusive diagnostic for measuring magnetic fields in Z-pinch plasmas is Faraday rotation, or Faraday polarimetry. This technique utilizes the interaction of polarized laser light with magnetic fields in media, and is dependent on both the magnetic field in the plasma, as well as the electron density along the path of a ray passing through the plasma. This technique has been used to make magnetic field measurements in Z-pinch plasma experiments [52, 88–90].

In order to see how the Faraday effect can be used to make field measurements, we follow the method of Hutchinson [80]. As before, it is necessary to look at an electromagnetic wave traversing a medium, however, this time, the magnetic field is non-zero, thus equation of motion for the electrons in the “cold” plasma approximation now includes an extra term:

$$\frac{d\mathbf{v}}{dt} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B}_0)$$ (3.24)

When waves propagate perpendicular to the magnetic field, the refractive index is the same as in equation 3.15, however, when the waves propagate parallel to the magnetic field, it is modified to [80]:

$$N = \left[ 1 - \left( \frac{\omega_p^2}{\omega^2} \right) \left( 1 - \frac{\omega_p^2}{\omega^2} \right) \right]^{1/2}$$ (3.25)

where $\Omega$ is the electron gyrofrequency, $\Omega = \frac{eB_0}{mc}$. Utilizing the Appleton-Hartree dispersion relation to first order, equation 3.25 is approximately [80]:

$$N = \left[ 1 - \left( \frac{\omega_p^2}{\omega^2} \right) \pm \left( \frac{\omega_p^2}{\omega^2} \right) \left( \frac{\Omega}{\omega} \right) \cos \theta \right]^{1/2}$$ (3.26)

where $\theta$ refers to the angle of wave propagation relative to the magnetic field. This correction to the previously derived index of refraction, equation 3.15, is usually
insignificant if $\Omega/\omega \ll 1$. However, this correction may be exploited in order to obtain information about the magnetic field. Examining the last term in equation 3.26, it is seen that waves which propagate parallel to the magnetic field will be most affected by this change in index of refraction due to the magnetic field, i.e. the largest phase changes, since:

$$\Omega \cos \theta = \frac{e}{m_e} \mathbf{B} \cdot \frac{\mathbf{k}}{k}$$  \hspace{1cm} (3.27)

Thus, in the presence of a significant magnetic field, a linearly polarized wave will have its electric field vector rotated as it propagates due to the difference in the index of refraction. Of course, we still have the usual dependence on electron density, so the angle $\phi$, in radians, by which it is rotated may be expressed as [80]:

$$\phi = \frac{e^3 \lambda^2}{8\pi^2 \epsilon_0 m_e^2 c^3} \int n_e \mathbf{B} \cdot d\mathbf{l}$$  \hspace{1cm} (3.28)

where $e$ is the electron charge, $\lambda$ is the wavelength of the probe beam, $m_e$ is the electron mass, $c$ is the speed of light, $n_e$ is the electron density, $\mathbf{B}$ is the magnetic field, and $d\mathbf{l}$ is the optical path length element.

In order to design a Faraday rotation diagnostic, one must first estimate the characteristics of the plasma system. This is accomplished by making some simplifying assumptions about a $Z$-pinch load, as outlined by Tatarakis et al. [88]. Using typical values for $Z$-pinch experiments, and changing to a dimensionless form, equation 3.28 may be rewritten as [88]:

$$\phi_{\text{deg}} = 15 \left( \frac{\lambda}{1\mu m} \right)^2 \left( \frac{n_0}{10^{26} \text{m}^{-3}} \right) \left( \frac{B_0}{100 \text{T}} \right) \int y \xi_n \xi_B b \cdot \left( \frac{dy}{100 \mu m} \right)$$  \hspace{1cm} (3.29)

where $n_0$ is the peak electron density, $B_0$ is the peak magnetic field magnitude, and $b$ is the magnetic field direction. $\xi_n$ and $\xi_B$ represent the distribution functions of the electron density and magnetic field. The general geometry of a simple $Z$-pinch load is assumed to be an azimuthally symmetric cylinder of plasma. With an axially flowing current, and the magnetic field will be in the azimuthal direction about the column. The distribution functions may then be assumed, in an oversimplified manner. A uniform current distribution throughout the plasma column means that the magnetic field distribution is proportional to the radius [88].

$$\xi_n = \frac{r}{R_0}$$  \hspace{1cm} (3.30)
where $R_0$ is the outer radius of the column. Similarly, a parabolic electron density profile is assumed [88]

$$\xi_n = 1 - \left( \frac{r}{R_0} \right)^2$$

(3.31)

The integral in equation 3.29 is then [88]

$$\phi_{\text{deg}} = \int_y \left( 1 - \left( \frac{r}{R_0} \right)^2 \right) \left( \frac{r}{R_0} \right) b \cdot \left( \frac{dy}{100\mu m} \right)$$

(3.32)

By assuming that refraction is negligible, this may be solved analytically, and manipulating for dimensionless variables again, we arrive at (an approximate) function describing the rotation angle for this specialized case [88].

$$\phi_{\text{deg}} = \left[ \frac{2x}{3R_0} \left( 1 - \frac{x^2}{R_0^2} \right)^{3/2} \right] \left[ 192 \left( \frac{\lambda}{1\mu m} \right)^2 \left( \frac{N_0}{10^{19} m^{-1}} \right) \left( \frac{I}{100 kA} \right) \left( \frac{100 \mu m}{R_0} \right)^2 \right]$$

(3.33)

where $N_0$ is the line density along the optical path and $I$ is the current. The left hand term is the dimensionless distribution for the rotation angle in a parabolic electron density profile, while the term on the right describes the plasma parameters. From this it is seen that, at least in this specialized case, the rotation angle is directly proportional to both the line electron density and the current in the plasma. For example, if a 100 kA current flows through a 2 mm radius plasma with a line density of $6.7 \times 10^{20} m^{-1}$, and 532 nm wavelength light is used, a maximum rotation angle of about 2 degrees is expected. This layout for performing such a measurement is shown in Figure 3.6.2. A laser, which is linearly polarized, is directed through the plasma column. After it exits the plasma, the laser passes through an “analyzing” polarizer. Prior to the experiment, the analyzing polarizer is set to an angle relative to laser polarization angle which allows adequate light to pass, and a “background” image is recorded. For the moment, we consider two positions within the plasma column. Since the magnetic field at these two locations are in opposite directions, we expect that the polarization rotation will be positive on one side and negative in the other. Relative to the brightness on the background image, when no current and thus no field was present, the resulting image should be brighter on the +x edge of the column, and darker on the -x side.
Faraday polarimetry, while extremely useful in situations where it can be reliably fielded, is essentially limited to systems with low wire numbers \[52,88,89\] so that a relatively clear line of sight is maintained throughout the experiment. Although not overly difficult, one must also simultaneously field interferometry in order to make accurate measurements of the electron density along the same path as the polarimetry, else no quantitative data may be extracted. This has proven to be a difficult task in the \(Z\)-pinch environment. The most obvious limitation, however, is that the measurements are limited by the critical density for the laser wavelength used. This means that high-density regions are essentially inaccessible. The use of shorter wavelength laser light, for example UV at 355 nm, enables access to slightly higher plasma densities, since the corresponding critical density is \(n_c = 9 \times 10^{21} \text{cm}^{-3}\).

### 3.6.3 Zeeman Splitting Spectroscopy

The least intrusive method for diagnosing magnetic fields in \(Z\)-pinch plasmas is through the exploitation of the Zeeman effect [91–94]. The Zeeman effect
is an observed splitting of atomic spectral lines which is due to the interaction of the magnetic dipole moment of an orbital electron, due to its orbital angular momentum, with an externally applied magnetic field.

Figure 3.15: The magnetic dipole moment of an orbital electron and an external B field. The torque exerted depends on the angle between the dipole moment and the external field.

The magnetic dipole of an electron will experience a torque in the presence of an externally applied magnetic field, and the resulting change in orbital energy is proportional to the product of the magnetic moment and the external magnetic field according to:

$$\Delta E_B = -\mu \cdot B$$

(3.34)

For now, however, only a magnetic field in the \( \hat{z} \) direction is considered. The magnetic moment of an electron is associated with the orbital angular momentum, \( \mu \), which is classically:

$$\mu = -\frac{e}{2m_e} L$$

(3.35)

Where \( e \) is the electron charge, \( m_e \) is the electron mass, and \( L \) is the orbital angular momentum, which is \( m_e \ell \hbar \). Assuming the magnetic field is in the \( z \)-direction, the classical shift in energy due to the field is

$$\Delta E_B(\theta) = \frac{e}{2m_e} L_z B = \frac{m_e e \hbar}{2m_e} B \cos(\theta)$$

(3.36)

where \( \hbar \) is Planck’s constant. In general, however, the electron spin is non-zero, i.e. has its own moment, and thus often cannot be ignored, so more completely, the
magnetic moment depends on the total angular momentum, which is the sum of the orbital angular momentum and the electron spin, $S$.

$$\mu = \frac{-e}{2m_e}(L + gS) \quad (3.37)$$

where $g$ is a factor which describes the magnitude of the spin contribution, which is predicted from quantum electrodynamics, and is approximately twice the contribution from the orbital angular momentum. We then define the total angular momentum as $J$, so again with the B field in the $\hat{z}$ direction we have:

$$\mu_{J_z} = \frac{g_L e}{2m_e} J_z \quad (3.38)$$

where $g_L$ is the so-called Lande’ g-factor, which is a geometric factor accounting for the vector of each orbital and spin angular momentum as they relate to the total angular momentum vector, $J$, since they will continually change during the interaction with the magnetic field. The derivation of this factor can be found in [95] pp.191. This factor is:

$$g_L = 1 + \frac{J(J + 1) + S(S + 1) - L(L + 1)}{2J(J + 1)} \quad (3.39)$$

Furthermore, the angular momentum is quantized according to $J_z = m_j \hbar$, where $m_j = 0, 1, 2, \ldots$, and we also introduce the Bohr magneton, $\mu_B = \frac{e\hbar}{2m_e}$, which is the unit expressing the electron magnetic dipole moment. Including this, the general expression for the shift in orbital energy is then:

$$\Delta E = g_L \mu_B m_j B \quad (3.40)$$

where $m_j$ is the magnetic quantum number. The shift in transition energy, or photon energy, between the upper and lower sub-levels of the shifted energies is then:

$$\Delta E_B = \Delta \hbar \nu = \mu_B B(g_L' m_j' - g_L m_j) \quad (3.41)$$

Where the primed quantities denote the initial state, i.e. before the transition occurs. Photons are emitted when electrons relax from the excited levels to the lower levels, and transitions occur only between states where $\Delta m_j = m_j - m_j' = -1, 0, +1$. Further, the transitions which will be shifted by the magnetic field
are only those with $\Delta m_j = \pm 1$, which are known as the $\sigma$ components, while the unshifted components, e.g. $\Delta m_j = 0$, are known as the $\pi$ components [94]. If the spin is 0, then the Lande’ $g$-factor is the same for the upper and lower levels, $g_L = g'_L$, and the splitting is said to be due to the “normal” Zeeman effect. When the spin is non-zero, more complex splitting patterns will emerge, i.e. when $g_L \neq g'_L$. These more complex patterns are commonly known as the “anomalous” Zeeman effect. Some examples of Zeeman splitting are seen in Figure 3.16. Finally, the wavelength shift due to the magnetic field from equation 3.41 is:

$$\Delta \lambda_B \approx -\frac{\lambda_0^2}{\hbar c} \Delta E_B$$

Polarization of the components of the Zeeman spectra is important, and depend on the viewing angle relative to the orientation of the magnetic field [93]. When viewing the spectra parallel to $B$, only the $\sigma$ components are observable, with the light being circularly polarized, e.g. left handed when $\Delta m = +1$ and opposite for $\Delta m = -1$. When viewing perpendicular to $B$, both $\pi$ and $\sigma$ components are observable, however in this orientation the $\pi$ components are linearly polarized and parallel to $B$, while the $\sigma$ components are linearly polarized, but perpendicular to $B$.

In order to exploit this effect for measuring magnetic fields in a plasma, one must choose suitable lines to “follow” and monitor for splitting, while also being
aware of the magnitudes of fields that are expected to be observed. By isolating the relevant lines in a plasma and looking for the strength of the Zeeman splitting effect, which will be proportional to the magnetic field in the plasma, information about the magnetic field can be deduced. For example, the \( ^2S_{1/2} - ^2P_{1/2} \) transition of the carbon IV doublet, \( \lambda_0 = 581.2 \text{ nm} \) has been used in ZaP Z-pinch experiments for field measurements [93], and when viewed parallel to \( \mathbf{B} \), gives the \( \sigma \pm \) lines.

This technique is quite difficult in practice, however. There are several major issues to contend with in this method, which have to do with other contributions to line shifts. High velocities, especially in a plasma which is imploding, can shift the lines due to the Doppler effect. For the preceding example, the FWHM Doppler broadening in a 100 eV temperature C IV plasma is about 0.13 nm, which is about an order of magnitude larger than the Zeeman shift of the \( \sigma \) components in a magnetic field of 1 T. Additionally, the presence of the Stark effect is appreciable in Z-pinch plasmas. The Stark effect is the broadening of spectral lines due to the presence of a large electric field [96], and can often obscure the Zeeman effect. Similarly, high temperatures may also broaden spectral lines. Another disadvantage is that it is quite difficult to temporally resolve such measurements. Lastly, this technique is practically limited to low spatial resolution, since one must choose one spatial location in which to make the measurement. Due to these issues, this technique has only minimally been applied to Z-pinch plasmas, and at the time of writing, only to plasma focus type Z-pinches.

### 3.7 Proton Beams as an Electromagnetic Field Diagnostic

The limitations of the aforementioned diagnostics were the motivation for developing proton deflectometry in Z-pinch plasma experiments. The information from these other diagnostics essentially limits our understanding of Z-pinch plasmas and thus the quality of the theory describing the dynamics of these systems, since they provide limited field information with which to challenge theory and numerical tools. Proton deflectometry, or proton radiography, is a relatively
new diagnostic which was developed for diagnosing electromagnetic fields in laser-plasma interaction experiments [97–99]. The need for such a diagnostic arose due to the difficulty of measuring field strength and orientation in such a challenging environment. Several other methods have been devised to accomplish this task include electrical probes, Faraday rotation, Stark-effect spectroscopy, and Zeeman-splitting spectroscopy, as discussed above. These alternate methods, however, are all limited in their ability to diagnose electromagnetic fields in plasmas as completely as with proton deflectometry. Proton probing has been performed with two types of proton production. The first type are those from the interaction of a short-pulse high-intensity laser with a thin foil [100], and the second is through the implosion of capsules filled with $D - ^3 He$, which produce mono-energetic protons as a byproduct of fusion burn [101]. The former method was used in this work, and thus will be the focus of this discussion. Proton deflectometry is an

![Figure 3.17: The basic layout for proton radiography/deflectometry. A proton beam traverses the plasma of interest, and is collected on a detector. The resulting image shows distortions in proton beam due to the electromagnetic fields in the plasma.](image)

extremely valuable diagnostic which has been developed in, and primarily used by, the laser-plasma interaction community. After the advances in laser technology, specifically in the improvements in decreased pulse widths, enabled access to a new regime of laser driven experiments. This special regime concerning the interaction of high-energy, ultrafast, laser pulse interactions with matter led to the discovery
of many new physical processes, including the production of multi-energetic, 1-50 MeV, high-fluence, low divergence, proton beams. Shortly after the physical description of the processes involved in the production of such beams, it was realized that such beams could be useful as a diagnostic. The basic schematic for proton probing of electromagnetic fields is shown in Figure 3.17. Here, a proton beam is directed through a plasma of interest, and the resulting proton beam is collected after it has exited the plasma. The resulting image contains an image which will change from its “natural” shape depending on the specific configuration of electromagnetic fields it has encountered in the plasma. High-energy protons have several properties which make them advantageous as a potential diagnostic tool. The first of which, is that high energy protons have a large mean free path compared to the scale length of many laser driven experiments, making them relatively insensitive to density. Secondly, the charged particles, protons, respond to electromagnetic fields. Third, due to their high velocities and the typical scale length of the system of interest, the integration times are small, resulting in highly temporally-resolved snapshots. For example, a proton with 5 MeV kinetic energy has a velocity of about $3 \times 10^7$ m/s, and for a system size of 1 cm, this corresponds to a probe duration of about 300 ps. Lastly, because the proton beam is driven on a time scale which is comparable to the driving laser pulse width, 0.01-1 ps, the probe duration is small compared to the timescale of plasma system evolution, which are often orders of magnitude longer. These factors mean that proton beams, of suitably high energy, are ideal for measurements of electromagnetic fields in dense, up to solid density, plasmas. As discussed above, it is extremely difficult to make field measurements in such plasmas, and the advent of high-energy proton beams allow for investigation of plasma regions which were previously inaccessible. These advantages will be discussed in detail here.

### 3.7.1 History of Laser-Accelerated Proton Beams

Proton deflectometry has been used frequently as a diagnostic for high energy-density plasma experiments in recent years, especially in laser-plasma interaction experiments [97–99,102–108]. Some of the first observations of fast ions
emitted from laser-plasma interactions were in the 1960’s, with the use of relatively low power, MegaWatt, laser pulses [109–111]. These early observations showed the production of ions with a kinetic energy of \( \sim 1 \) keV per particle. Henceforth, when an energy is given for a particle or ion beam, it will similarly be in units of eV, and in the case of a proton beam it is representative of the particle kinetic energy per particle in the beam, unless otherwise specified. In the 1980’s higher intensity, \( \sim 10^{14} W \text{cm}^{-2} \), laser pulses from the Helios CO\(_2\) laser at Los Alamos National Laboratory interacting with solid targets were found to generate up to 1 MeV energy ions, and that the ion species were primarily protons [111]. Progress in laser technology, largely enabled by the implementation of Chirped Pulse Amplification (CPA) technology discussed in Chapter 2, enabled lasers capable of delivering, \( \sim 1-1000 \) Joules, in \( \sim 0.01-1 \) picoseconds. The interaction of these TeraWatt-PetaWatt, ultrafast, high-contrast, lasers with solid targets revealed a new regime of ion acceleration [100, 112–123]. These experiments routinely recorded high-energy, 1-50 MeV, ions in a highly directional beam, when such laser pulses were incident on solid foil targets. For example, in one such experiment at Lawrence Livermore National Laboratory with the Nova laser, \( \lambda_L \sim 1 \mu m \), \( E_L \sim 150 - 750 \) J, pulse width \( \tau_L \approx 0.5 - 5 \) ps, and focused to a spot with a diameter, \( d \approx 8 \mu m \) FWHM, delivered peak intensities (irradiances) of \( \sim 3 \times 10^{20} \) W cm\(^{-2}\) [113,115]. The interaction of this pulse with flat Al or CH targets having dimensions of \( \sim 1 \) mm \( \times 1 \) mm and thicknesses of \( \sim 50-125 \) \( \mu m \), produced protons with energies greater than 30 MeV. Additionally, the beams were shown to be very well collimated and highly directional, in some cases having a beam half-opening angle of \( \sim 15 \) degrees, which decreased with increasing energy [124], and a smooth spatial profile [125]. These high-energy ions were determined to be protons by utilizing proton-induced nuclear reactions in titanium, specifically Ti\(^{48}(p,n)V^{48}\). By using wedge type targets as seen in Figure 3.18, it was realized that these protons were accelerated approximately normal to the rear surface, the non-laser facing surface, and thus that the highest energy protons originated from this side of the target [121]. Based on this information, a model known as the target-normal sheath-acceleration mechanism, or TNSA mechanism, [100] was developed to explain the origin of such beams.
Figure 3.18: When the laser pulse is incident on the front surface, the proton beam is observed to be normal to the rear surface, indicating the origin of the proton beam is from the rear. Lower energy protons are also observed normal to the front surface.

Shortly after the discovery of high-energy proton beams, it was realized that they could be used as a diagnostic in high energy-density plasmas [98, 103]. Because any particle with a net electric charge will experience accelerations due to external electromagnetic fields, they can potentially be used to map the topology of fields in plasmas. Additionally, due to the proton’s relatively large mass compared to an electron’s, when it is accelerated to high velocity, or MeV energies, it will have a long mean free path compared to the typical system which is being probed, meaning that protons may penetrate high plasma densities. This is, of course, only possible if one has reasonable confidence in the characteristics of the initial state of the charged particle beam, and can determine the trajectories of the particles. This particular requirement is a difficult task, which typically necessitates extensive numerical modeling.

3.8 Production of High-Energy Laser-Accelerated Proton Beams

3.8.1 The Target-Normal Sheath-Acceleration Mechanism

The mechanism for producing these high-energy proton beams is known as the Target Normal Sheath Acceleration mechanism [100]. The basic layout
Figure 3.19: General schematic of the TNSA mechanism for proton beam acceleration. a) The CPA laser pulse is incident on the thin metallic target. A pre-plasma is formed due to the interaction of the laser pre-pulse with the target. The expansion of the pre-plasma moves the critical density surface away from the target surface, where hot electrons are accelerated when the main laser pulse arrives, and these electrons pass through the target until sufficient electric potential prevents them from escaping. b) A magnified view of the box in a) showing the hot electrons escaping from the rear surface of the target, creating the sheath field and ionizing contaminants on the rear surface of the target. c) Protons from a thin contamination layer at the rear surface of the target are accelerated by the sheath field, from a virtual point source.
for producing a proton beam via this method is shown in Figure 3.19. First, a CPA pulse of laser light, typically \( \sim 1 \) micron wavelength, is incident on a flat metallic target, typically a 1-50 micron thickness. The laser pre-pulse, inherent to all laser pulses and discussed further in the following section, creates a pre-plasma on the laser facing side of the target, henceforth referred to as the front surface. This pre-plasma will move the critical density away from the surface of the target, and thus the region where laser absorption occurs, away from the target surface. When the main pulse arrives, the electrons are accelerated to relativistic energies in the radial/forward direction. These high-energy electrons propagate through the pre-plasma, and due to the low stopping power of solids at such energies, pass through the target to exit the rear surface. The charge imbalance induced in the target by the escaped electrons causes a large electric field to develop at the rear surface. Hydrocarbons are inherently adsorbed by all targets due to ever present water vapor and vacuum pump oils in the experimental chamber [111]. Unless the targets are purposely cleaned to remove them, these atoms are then ionized, and accelerated by this electric field. The fact that the ions which are accelerated are found to be primarily protons comes from the fact that protons have much smaller mass than C, O, and target atoms, which are also present. This means that the protons are preferentially accelerated from the ionized contaminant layer at the rear surface. The strength of the accelerating electric field, and consequently the number and energy of protons accelerated by it depends on several parameters, including laser energy, focal spot size, target thickness, and the ratio of the power contained in the main laser pulse to the power in the laser pre-pulse. The construction of the TNSA model will be explained in more detail below after a brief discussion of the mechanisms by which intense laser light is coupled to a plasma.
3.8.2 The Motion of an Electron in an Electromagnetic Field

The equation of motion for an electron in an electromagnetic field is:

\[
\frac{dp}{dt} = -e(E + v \times B) \tag{3.43}
\]

Where \( \mathbf{v} \) is the velocity, \( \mathbf{E} \) is the electric field, \( \mathbf{B} \) is the magnetic field, and for generality, \( p = \gamma m_e v \). Then we define a linearly electromagnetic wave traveling in \( \hat{z} \), with the real components of \( \mathbf{E} \) and \( \mathbf{B} \) oscillating in \( \hat{x} \) and \( \hat{y} \) as \( \propto \cos(kz - \omega t) \), e.g. \( \mathbf{E}_L = E_L \cos(kz - \omega t)\hat{x} \). We can then separate the three components of the electron motion.

\[
\frac{dp_x}{dt} = -e(E_x - v_z B_y) \hat{x} \tag{3.44}
\]

\[
\frac{dp_y}{dt} = 0 \hat{y} \tag{3.45}
\]

\[
\frac{dp_z}{dt} = -e(v_x B_y) \hat{z} \tag{3.46}
\]

For the moment, consider only a weak electromagnetic field, such that the \( \mathbf{v} \times \mathbf{B} \) components are negligible, i.e. only the first term in the RHS of equation 3.44 is important.

\[
\frac{dp_x}{dt} = -eE \tag{3.47}
\]

Which using the fact that the velocity will also be oscillatory as \( \propto \cos(\omega t) \), it is easily shown that the quiver velocity of the electron in the presence of an electric field is, more generally:

\[
v_{\text{quiver}} = -\frac{eE_L}{m_e \omega_L} \tag{3.48}
\]

Where \( \omega_L \) is the laser frequency, and \( E_L \) is the amplitude of the laser electric field. From this we can find the normalized vector potential, or pump strength, \( a_0 \), which is defined as the quiver velocity divided by the speed of light \( c \) [126].

\[
a_0 = \frac{v_{\text{quiver}}}{c} = \frac{eE_L}{mc_e \omega_L} = 8.5 \times 10^{-10} \lambda_L [\mu m] \left( I_L [W cm^{-2}] \right)^{1/2} \tag{3.49}
\]

Here \( \lambda_L \) is the laser wavelength, and \( I_L \) is the intensity of the laser pulse. When \( a_0 \sim 1 \), the interaction is said to be relativistic. If the laser irradiance is sufficiently
high, around $10^{18}$ W · cm$^{-2}$, the relativistic regime of laser-plasma interactions is accessed, due to the energy gained by the electrons during the interaction. Since this work uses $\lambda_L = 1.064\mu m$ wavelength light, the threshold for relativistic interactions, i.e. when $a_0 \sim 1$, occurs when the intensity,

$$I_L = \left( \frac{1}{(8.5 \times 10^{-10})(1.064)} \right)^2 \approx 1.2 \times 10^{18} W cm^{-2} \quad (3.50)$$

If we then average the kinetic energy of the electron over one laser period, $KE_{avg} = \langle \frac{1}{2} m_e v_{quiver}^2 \rangle$, we find what is known as the ponderomotive potential:

$$U_p[eV] = \langle \frac{1}{2} m_e v_{quiver}^2 \rangle = \frac{e^2 E_L^2}{4 m_e \omega_L^2} = 9.3 \times 10^{-14} (\lambda_L[\mu m])^2 I_L[W cm^{-2}] \quad (3.51)$$

$$\gamma m_e > m_e - \text{rest} \quad (3.52)$$

A force arises from the gradient of the ponderomotive potential, which may accelerate electrons out of the laser interaction region. Applying the gradient to the ponderomotive potential in equation 3.51, this force may then be expressed as:

$$F_p = -\nabla U_p = -\frac{e^2 E_L^2}{4 m_e \omega_L^2} \nabla E_L^2 \quad (3.53)$$

From this it is immediately obvious that an electron in the presence of an oscillating electric field of a laser will be pushed away from the interaction region. In the above derivation, a weak electromagnetic field was considered. When the field is much larger, such that $a_0 \sim 1$, the magnetic field components can no longer be disregarded, and the electron has a component of motion along the wave vector $k$, or in this case $\hat{z}$ which is explored here [127]. A more general version of equation 3.43 is:

$$\frac{\partial \mathbf{p}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{p} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad (3.54)$$

We now include $\gamma$, which is by definition:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{\mathbf{p}^2}{\gamma^2}}} \quad (3.55)$$

Given that $p^2 = p_{quiver}^2 / m_e^2 c^2$, we may solve again for $\gamma$, which is:

$$\gamma = \sqrt{1 + p^2} \quad (3.56)$$
for linearly polarized light, as considered here. It is now useful to define the
electric and magnetic fields in terms of the vector potential, $A = \mathbf{p}/m_e c$, such that
$\mathbf{E} = -\nabla \phi - \frac{1}{c} \frac{\partial \mathbf{A}}{\partial t}$ and $\mathbf{B} = \nabla \times \mathbf{A}$. Equation 3.54 may now be written as:

$$\frac{\partial \mathbf{p}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{p} = e \left[ \nabla \phi + \frac{1}{c} \frac{\partial \mathbf{A}}{\partial t} - (\mathbf{v} \times (\nabla \times \mathbf{A})) \right]$$

(3.57)

substituting for $\mathbf{v}$ and $\mathbf{A}$, this is:

$$\frac{\partial \mathbf{p}}{\partial t} + \frac{1}{\gamma m_e} \mathbf{p} \cdot \nabla \mathbf{p} = e \left[ \nabla \phi + \frac{1}{\gamma m_e c} \frac{\partial \mathbf{p}}{\partial t} - \left( \frac{1}{\gamma m_e} \mathbf{p} \right) \times \nabla \times \left( \frac{1}{m_e c} \mathbf{p} \right) \right]$$

(3.58)

With some simplification via algebra and vector identities, the longitudinal component of the momentum becomes

$$\frac{\partial \mathbf{p}}{\partial t} = e \nabla \phi - m_e c^2 \nabla (\gamma - 1)$$

(3.59)

The first term on the RHS of equation 3.59 is the usual force due to an electrostatic potential, while the second term represents the force due to the gradient of the relativistic ponderomotive potential. Thus the relativistic ponderomotive potential is just $U_p = m_e c^2 (\gamma - 1)$, which in the non-relativistic limit reduces to the previously recovered result in equation 3.51. This potential energy may be thought of as an effective temperature to make an estimate of the electron “temperature” using $\text{Energy} \approx k_B T_h$, where $k_B$ is the Boltzmann constant, and $T_h$ is the hot electron temperature. In terms of practical units, this is:

$$T_h [\text{keV}] = 511 \left[ \sqrt{1 + \frac{I [\text{W cm}^{-2}]}{2.8 \times 10^{18} (\lambda [\mu \text{m}])^2}} - 1 \right]$$

(3.60)

The absorption of laser energy into the target plasma is responsible for accelerating electrons to very high energies in the TNSA model, i.e. creating the “hot” electron population, and is assumed to be via the ponderomotive force as will be shown below. There are, of course, a multitude of other mechanisms and processes determining the coupling of the laser energy to the plasma electrons. Some of the other mechanisms include inverse Bremsstrahlung [128], vacuum heating [129], the jxb mechanism [127, 130], and resonance absorption [128, 131] however, they are not be discussed here. There are a couple of points which are worth reinforcing
here. One is that the laser absorption processes occur in the underdense plasma, densities below $n_c$. Second, although not predicted by the relativistic ponderomotive potential, the hot electron spectrum predicted by numerical simulations and measured in experiments resembles that of a relativistic Maxwellian distribution with a finite cut-off energy [127,132].

### 3.8.3 Construction of the TNSA model

The emission of ions, especially protons, was observed as far back as 1986, by Gitomer. At that time, several models were proposed for accelerating protons to the energies observed [111]. By inferring the energy transferred from the laser to the electrons via observed X-ray yield, they posited that the ions were accelerated by a population of hot electrons, and thus that the bulk ion energy observed was roughly proportional to the electron energy:

$$E_{\text{ion}} = \alpha T_{\text{hotelectron}}$$

(3.61)

where, the proportionality constant, $\alpha$, was somewhere $\sim 2-12$, depending on the specific model used. Experiments with the Petawatt laser [113] necessitated modification of this model to a more physical picture in order to explain the resulting proton energy spectrum. Wilks [100], placed physical limits on the model presented by Gitomer [111], which resulted in the proposition of the target-normal sheath-acceleration mechanism as the process for accelerating high-energy proton beams from the interaction of ultra-intense, short-pulse, laser light with solid targets.

In order to construct a more physical picture of the ion acceleration, we first seek a mechanism which could produce an electric field for accelerating the protons. A simplified picture of this begins with an examination of a model proposed by Denavit [133] for collisionless plasma expansion into a vacuum. For simplicity, the expansion is assumed to be one-dimensional, or planar, which in our assumption of a flat foil target, is a reasonable approximation. The ion fluid continuity equation is:

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_i) = 0$$

(3.62)
while the equation of motion for the ion fluid expansion is:
\[
\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} = -\frac{C_s^2 \partial n_i}{n_i \partial x}
\]  
(3.63)

where \( n_i \) and is the ion density, \( v_i \) is the ion velocity, and the sound speed is \( C_s = \sqrt{Z T_e / m_i} \). Using the \( \pi \) theorem, we may reduce the number of variables by switching to \( \xi = x / C_s t \), and find the self-similar solutions:

\[
\frac{v_i}{C_s} = \xi + 1 = \frac{x}{C_s t} + 1
\]  
(3.64)

and

\[
\frac{n_i}{n_0} = \exp(-\xi - 1) = \exp\left(-\frac{x}{C_s t} + 1\right)
\]  
(3.65)

In order to find the electric field near the ion front, we look at the electron fluid equation of motion. Assuming a dynamic equilibrium, where the electron inertia is negligible, and recalling that the expansion is isothermal, this is just

\[
\frac{\partial p_e}{\partial x} = -n_e e E
\]  
(3.66)

Noting that the plasma is quasineutral, \( n_e \approx Z n_i \), and solving for the electric field we obtain:

\[
E = \frac{T_e}{e C_s t}
\]  
(3.67)

By setting the characteristic scale length of the problem to \( L_n = C_s t \), this may then be written as

\[
E = \frac{T_e}{e L_n}
\]  
(3.68)

A simple schematic of the development of this field due to plasma expansion is shown in Figure 3.20. This is then a simple picture of the electric field which will accelerate the ions, which still has some limitations which need to be addressed. The most significant problem is that as the scale length goes to zero, the field becomes infinite, implying that ions would be accelerated to infinite energy. Denavit [133] notes that there are several physical mechanisms which prevent this from occurring. The first is that the ions will eventually “catch up” to the electrons due to the finite inertia of the electrons, which we neglected here. Additionally, the
expansion is not truly isothermal, since the plasma is cooling near the front. An effective way to limit the ion acceleration energy was proposed by Kishimoto [134]. It was noted that the electron Maxwellian was truncated at a finite energy, and that by setting the local scale length near the front to the electron Debye length, \(\lambda_{\text{Debye}} = \sqrt{k_B T_{e\text{-hot}}/4\pi n_{e\text{-hot}}e^2}\), where quantities are now denoted as “hot” since is is the energetic species we are interested in. Equation 3.68 then becomes

\[
E = \frac{T_{e\text{-hot}}}{e\lambda_{\text{Debye}}}
\]  

(3.69)

Experimentally, the energy spectrum of the electrons has been observed to be close to a relativistic Maxwellian distribution. Thus, \(T_{e\text{-hot}}\) is then approximately given by

\[
T_{e\text{-hot}} \approx mc^2 \left(1 + \frac{2U_p}{mc^2}\right)^{1/2}
\]  

(3.70)

where \(U_p\) is the ponderomotive potential [100].

Of course, this model is quite a simplistic picture, since in reality, there are two distinct electron populations and temperatures. Additionally, the hot electron temperature, and ion scale length evolve quite rapidly throughout the experiment. It is of note that this model is a decent approximation only when the main laser pulse is interacting with the plasma/target. Modifications to the
plasma expansion model were presented by Mora [135] which further constrains the fluid expansion model, and extend the usefulness of the TNSA model. In this work, it was noted that when \( \lambda_{\text{Debye}} > L_n \), the self-similar solution is not relevant. When the opposite is true, \( \lambda_{\text{Debye}} \ll L_n \), the self-similar solution again predicts acceleration of protons to infinite energy. Through the use of an expansion of the electric field, or the sheath field, it was found that the field strength evolution is asymptotic in time,

\[
E_{\text{ion-front}} \approx 2E_0 \left(2 \exp(1) + \frac{\omega_{pi}^2 t^2}{2\pi^2}\right)^{-1/2}
\]

where \( E_0 = \sqrt{n_{e0} k_B T_e / \epsilon_0} \) is the initial electric field, and \( \omega_{pi} = \sqrt{n_{e0} e^2 / M_i \epsilon_0} \) is the ion plasma frequency. This changes the previous assumption that was assumed in the construction of the TNSA model. In reality, the hot electrons that escape the rear surface of the target are eventually eventually held back as a correspondingly large potential due to charge imbalance is built up on the target. This more realistic picture of the electric field evolution was then used in order to find the maximum energy of accelerated ions, or the cutoff energy.

\[
\varepsilon_{\text{max}} \approx 2\varepsilon_0 (\ln(2\tau))^2
\]

Where \( \varepsilon \) refers to the ion energy, and \( \tau = \omega_{pi} t / \sqrt{2 \exp(1)} \) is the characteristic time, or an effective acceleration time. Finally, Mora was also able to calculate the number of accelerated protons per unit energy via the fluid model as

\[
\frac{dN}{d\varepsilon} \approx \frac{n_{i0} C_s t}{\sqrt{2\varepsilon \varepsilon_0}} \exp\left(-\sqrt{\frac{2\varepsilon}{\varepsilon_0}}\right)
\]

The accuracy of these models has been tested both experimentally and numerically, and have done a remarkable job of predicting the characteristics of the resulting proton beams. Of course, improvements in numerical tools, allowing for the inclusion of more realistic physics, have improved the understanding of the underlying influences, but the simple picture brought about by the consideration of a plasma expanding into a vacuum is nonetheless quite sufficient for explaining the basic processes leading to the acceleration of high-energy proton beams from the interaction of high-intensity laser pulses with thin foils.
3.8.4 Scaling of Proton Beam Energy with Driver Properties

By now, it is clear that there are quite a few parameters which may influence the characteristics of the final proton beam which is accelerated. Until now, the specifics of the laser driver used for proton acceleration have not been discussed, so this section will briefly do so. Several studies have examined this scaling [136–138]. Fuchs et al. summarized the scaling of laser-accelerated proton beam characteristics with the driver parameters in 2006 [136]. In this work, the proton beam maximum energy and the conversion efficiency of laser energy into energetic protons were measured by varying laser parameters, while keeping the target material, aluminum, consistent. First the target thickness was varied, as in the preceding section, until the optimal target thickness was found. This thickness, 25 μm in this case, corresponds to the greatest maximum energy, and the highest conversion efficiency. This target thickness was then held while the laser intensity was varied, by changing the amount of energy in the pulse, ~ 1-10 J, and keeping the pulse width and focal spot diameter constant at ~ 320 fs and ~ 5 μm, respectively. They found that the maximum proton energy and laser-proton conversion efficiency increased as the laser energy was increased, as seen in Figure 3.21 and from this, constructed a simple scaling by modifying the scaling presented by Mora in equation 3.72 to include experimental parameters. The maximum energy then becomes:

\[ E_{\text{max}} \approx 2T_{\text{hot}} \left[ \ln(t_p + \sqrt{t^2_p + 1}) \right]^2 \]  \hspace{1cm} (3.74)

where \( E_{\text{max}} \) is the maximum ion energy, and \( t_p = \omega_p t_{\text{acc}}/\sqrt{2 \exp(1)} \) is the acceleration time normalized to the ion plasma frequency, while \( t_{\text{acc}} \) is a limiting acceleration time, which is proportional to the laser pulse time, \( t_{\text{acc}} \sim \alpha \tau_{\text{laser}} \). In numerical calculations performed using this scaling the best match to experimental data was found when \( \alpha = 1.3 \). Adding more realistic values to the expression for the number of ions with a specific energy per ion is done by further consideration of the mechanisms which led to the model, especially in the coupling of laser energy to the accelerating field. Of course, \( T_{\text{hot}} \) is still given by equation 3.70, but now
a conversion efficiency from laser energy into hot electrons sent into the target is considered. The number of electrons propagating into the target is then given by:

\[ N_{e-hot} = \frac{f E_L}{T_{hot}} \]  \hspace{1cm} (3.75)

where \( f \) is the fraction of the laser energy, \( E_L \) that is converted into fast electrons. From [139, 140], this fraction scales with the laser energy approximately as \( f = 1.2 \times 10^{-15} I_0^{0.74} [W/cm^2] \). These electrons are then spread over the surface of the sheath-accelerating, and using the assumed spherical expansion of the hot electrons intersecting with the planar target, the area of this sheath is:

\[ S_{sheath} = \pi (r_0 + d \tan \theta)^2 \]  \hspace{1cm} (3.76)

where \( r_0 \) is the radius of the sphere initiated at the location of the laser spot, \( d \) is the target thickness, and \( \theta \) is the half angle divergence of the hot electrons inside the target. Since the hot electrons travel very fast, nearly \( c \), and are accelerated during the laser pulse, \( \tau_L \), we may then estimate \( n_{e0} \), which is:

\[ n_{e0} = \frac{N_{e-hot}}{c \tau_L S_{sheath}} \]  \hspace{1cm} (3.77)

This information may then be applied to equation 3.73, which becomes:
\[
\frac{dN}{dE} \approx n_e 0 C_s t_{acc} S_{sheath} \exp \left( -\frac{\sqrt{2E}}{T_{hot}} \right)
\]

(3.78)

where \( C_s \) is the usual ion sound speed.

Figure 3.22: a) The scaling of maximum proton energy as a function of laser pulse-width. The maximum observed proton energy is shown for various drivers, and is seen as circles or squares, depending on the intensity of the driver. b) Conversion efficiency as a function of laser intensity times the laser wavelength are shown. Here, the scattered data represent the number of protons in a 1 MeV bin centered at 10 MeV proton energy are plotted for various laser drivers. Figure from Fuchs et al. Nature Physics 2006 [136].

The model presented by Fuchs [136], may then be applied to the Leopard laser at NTF in order to make some quantitative estimates of the proton beam characteristics from this driver. Two cases are plotted, with only the hot electron temperature varied in order to get an upper and lower estimates on the proton beam energy characteristics. The first case should be considered as an upper bound, and uses a Leopard energy of 15 J, 350 fs pulse-width, and focal spot diameter of 5 \( \mu \)m, and assuming that 50\% of the laser energy is contained in the focal spot, the laser intensity is \( \sim 1 \times 10^{20} \text{ W cm}^{-2} \). The target thickness is fixed at 2 \( \mu \)m for both cases. The resulting hot electron temperature is calculated to be \( \sim 4.3 \text{ MeV} \), with a maximum proton energy, \( E_{max} \sim 16 \text{ MeV} \). The total number of protons in an energy bin between 1.2 MeV, the experimental lower detection limit, and 16 MeV, the theoretical maximum, is \( \sim 1.8 \times 10^{11} \). The second case
uses a measured hot electron temperature from Leopard experiments with 2 \( \mu \)m Ti targets, which is \( \sim 2 \) MeV [141]. The model then predicts that the proton cutoff energy is \( \sim 8 \) MeV, with a similar number of total protons, though now within a smaller energy bin of 1.2 to 8 MeV, and should be considered a reasonable lower bound on the proton energy characteristics. The number of protons for a given energy are plotted in Figure 3.23.

![Figure 3.23](image)

**Figure 3.23**: Predicted number of protons as a function of proton energy as calculated from the scaling models in Fuchs et al. Nature Physics 2006 [136]. The two lines are for the purely theoretical case and for a case using an experimentally measured hot electron temperature.

Although there are many factors which influence the final proton beam characteristics, two are of particular importance, the laser pre-pulse and the target thickness, which will be discussed in the following sections.

### 3.8.5 The Influence of Amplified Spontaneous Emission

An unperturbed rear surface is important for several reasons, and the characteristics of the prepulse may strongly influence the state of this at the time of
the main pulse arrival [142–145]. The strongest effect of this region being perturbed, is of course the degradation of the sheath field due to the increased ion scale length. The shape of this region has also been shown to strongly influence the directionality of the proton beam [142,146].

An ideal, laser pulse would be a pulse which has a temporally Gaussian profile with a pulse length between $1/e$ times the maximum intensity, from rising edge to maximum value, which is $\frac{1}{\Delta \omega} = \frac{1}{\text{laserbandwidth}}$ [11, p. 461-464]. As discussed in Chapter 2, amplified spontaneous emission, or laser pre-pulse, is inherent to all real laser pulses. Although the pre-pulse on Leopard has not been fully characterized, the power contrast ratio $r_c$, has been measured at $\sim 10^{-8}$, and since Leopard nominally delivers a $\sim 15$ J $\sim 350$ fs pulse in a $\sim 7\mu m$ diameter spot, the power in the pre-pulse is $P_{\text{pre-pulse}} \approx r_c \times \frac{15}{350} J = \approx 4 \times 10^5$ W. ASE in this class of laser typically occurs over the period of a few nanosecond, but as an upper limit, it is assumed that the pre-pulse duration is of the order of $t_{pp} = 0.5$ ns, from consideration of the fastest Pockels cell with a voltage rise time $\sim 250$ ps. This may then be used to estimate the amount of energy contained in the pre-pulse, $E_{pp} \approx 0.2$ mJ. If this light is focused on to the target in a spot size with a diameter $\sim 7\mu m$, this gives an intensity of $\sim 10^{12}$ W cm$^{-2}$. This is important in the formation of the pre-plasma, a requisite component of proton acceleration via the TNSA model, which requires that the bulk of the laser energy be deposited in a plasma, rather than at the target surface, in order to most efficiently couple the laser energy into hot electrons. When this pre-pulse laser light is incident on the surface of the target, ablation is quickly initiated, forming the pre-plasma, which expands away from the surface of the target. The scale length of the pre-plasma, essentially the distance of the plasma from the target surface, determines where the interaction of the main pulse with the critical density surface will occur.

### 3.8.6 The Influence of Target Thickness

The target thickness is an important parameter in this acceleration scheme. It can influence several parameters, including the maximum proton energy. Several of these influences are discussed here. The key to accelerating ions to the
energies typically observed, > 50MeV is the ion density scale length. As discussed above, when the characteristic scale length becomes small, the accelerating field, or \( E_{\text{sheath}} \), becomes large, \( \sim MV/\mu m \) or \( TV/m \) [100].

One way of keeping this scale length short, is through the particular experimental geometry using ultra-short laser pulses and thin targets. This allows the hot electrons generated at the critical density surface on the front side of the target to penetrate beyond the rear surface before ion expansion has occurred at the rear surface, i.e. a sharp density surface. As will be discussed later, when this surface is significantly perturbed, the ion acceleration is significantly degraded. Ions are also observed to be emitted from the front surface of the target, although with significantly less energy. The characteristic scale length on this side is on the order of 100’s of microns due to the nature of the laser pre-pulse interaction, which often arrives \( \sim \) ns before the main pulse and allows for significantly more plasma expansion. If, for example, the scale length of the preplasma is 500 \( \mu m \) at the front surface and 50 \( \mu m \) at the rear surface, this longer scale length would result in an accelerating field which is approximately an order of magnitude lower than that of the rear surface.

For the scaling presented in equation 3.68, the accelerating field is much

Figure 3.24: Data showing the maximum proton energy vs the target thickness. Three data sets are shown, each for a different pre-pulse duration, \( \tau_{\text{ASE}} \). Figure from Kaluza et al. Physical Review Letters (2004) [142]
smaller than for the rear surface where little or no expansion has occurred on the
time-scale of the main laser pulse, e.g. $\sim<10\mu m$ in $<1$ ps. Numerical simulations
have shown that the target influence can enhance the hot electron density at the
rear surface for appropriately chosen thicknesses. Mackinnon and Sentoku et al.
[147, 148] found that the electron bunch length was approximately equal to the
laser pulse length, and since they are relativistic in this case, they travel at nearly
the speed of light. When the electrons reach the rear edge of the conducting target,
they are reflected by the development of a sheath field. The thickness of the target
then determines the circulation length, and therefore whether the hot electron
bunching is enhanced. For sufficiently intense/energetic pre-pulses, a shock may

![Figure 3.25](image)

**Figure 3.25:** a) A schematic of hot electron recirculation and bunching enhance-
ment. b) If the target is thin, electrons may reflect from both boundaries while
the laser pulse is still “on” allowing for for enhancement of the sheath field. c)
The predicted scaling of maximum proton energy as a function of target thickness
from modeling. Figure from Sentoku et al. Physics of Plasmas (2003)

be launched into the target. If the shock is significantly strong, it will “break
out” at the rear surface, thus destroying the sharp density gradient required for
large sheath fields and subsequent proton acceleration. Whether or not a shock
launched by the pre-pulse will be influential places a dependence on the thickness
of the target, i.e. $\tau_{bo} = d_t/c_{shock} < \tau_{pp}$. Here, $\tau_{bo}$ is the time it takes for the shock
to break out, $d_t$ is the target thickness, $c_{shock}$ is the shock speed, and $\tau_{pp}$ is the
interval between the start of the ASE and the main pulse arrival [142,149].
3.9 Advantages of Proton Radiography as a Magnetic Field Diagnostic

3.9.1 Insensitivity to Density

Proton probing operates in two regimes. One in which the plasma density effects are negligible, and the resulting images are purely due to electromagnetic field effects on the proton test particles, known as proton deflectometry, and the other in which the plasma density is not negligible, and the resulting images are due to both plasma density variations and electromagnetic fields, known as proton radiography. In this work, we focus on the former mode of operation, the reasons for which will be elucidated here.

In a low order approximation from kinetic gas theory, a target consists of hard spheres, representing the target particles. An incident particle, the colliding species, then sees the spheres as a projected circular area. Consider a particle traveling with velocity $v$ and incident on an infinitesimally thin slab. If they are assumed to be stationary, and the slab has a thickness $\Delta x$, the probability of a proton passing through the target is dependent only on the size of the particles, or the projected cross-sectional area normal to the proton direction.

The probability will also then have a dependence on the number of particles per unit area, or the areal density of the particles.

$$C_{\text{probability}} \propto \frac{A_{\text{slab particles}}}{A_{\text{slab}}}$$  \hspace{1cm} (3.79)

Where $C_{\text{probability}}$ is the collision probability, $A_{\text{slab particles}}$ is the area of the particles in the slab, and $A_{\text{slab}}$ is the area of the slab. Thus, if $A_{\text{slab particles}}$ is equal to $A_{\text{slab}}$, the the proton will experience a collision. This is an extreme limit, which is analogous to a dense region of potential particles with which the proton may encounter. The mean free path for an incident particle is then approximately:

$$l_{mfp} \sim \frac{1}{\sigma n}$$  \hspace{1cm} (3.80)

Where $\sigma$ represents the “area” of the particle in the thin slab, and $n$ is the number of particles in the slab per unit area. Collision theory is, of course, much more
Figure 3.26: A particle is incident on a slab target consisting of the collision species. The effective cross-sectional area of the collision species is a 2D projection of their spherical area.

complex than represented here, having dependencies on the thermal motion of the target species, the angle of incidence, the impact parameter, charge screening in a plasma, the particular charge distribution for ionized atoms, and the velocity of the colliding species. A detailed description of collisions involving ions in matter may be found in [150].

When calculating the range of protons in matter, two primary types of collisions are important; Coulombic collisions with electrons and nuclei. In dense regions where the likelihood of collisions is higher, a high-energy proton has another advantage. Since the likelihood of encountering electrons as collision particles, is much higher in elements with higher atomic numbers than hydrogen, the proton still has a long mean free path due to its large momentum, compared to that of an electron. When these factors are combined in a calculation, it results in the total stopping power, where stopping powers are given in units of energy times area per mass, usually MeV cm\(^2\) g\(^{-1}\). The continuous slowing down approximation range, or CSDA range, approximates the average path length traveled by a charged particle in a material as it slows down to rest [151], where the rate of energy loss at every point along the track is assumed to be equal to the total stopping power. This
range can be calculated by:

\[ CSDA_{\text{range}} = \int_{E_{\text{lower}}}^{E_{\text{upper}}} \left( \frac{1}{P_{\text{stopping}}} \right) dE \] (3.81)

Where \( P_{\text{stopping}} \) is the stopping power, either electronic, nuclear, or total, and \( E \) is the energy.

Such information can be used to predict the penetration depth of particles with respect to a medium under normal conditions. This is known as the collisional stopping power of a material, and it is tabulated for most elements, and a limited set of materials which are commonly used in experiments, in the National Institute of Standards and Technology, or NIST, database [152]. An example of this information is shown in Figure 3.27. The collisional stopping power may then be used in tandem with a simplified power loss model in a widely used code known as SRIM [150], or Stopping and Range of Ions in Materials. This fact is important not only in calculating the range of protons as they traverse a target plasma, but is also important in the design of proton detectors, as will be discussed further in section 3.10.

Figure 3.27: The continuous slowing down approximation range and projected range calculated for protons in aluminum. The penetration depth increases with increasing proton energy. Figure generated from NIST PSTAR [153]
3.9.2 Space-Charge Effects

In order to consider the proton beam as a collection of individual test particles which probe the electromagnetic field structure, the space-charge effects must also be negligible [154]. If the proton beam is comprised solely of protons, i.e. no electrons are present, then a simple estimate of the electric potential of this beam can be made. This is done by assuming that the beam is a spherical “cloud” of protons using a characteristic size, \( \text{diameter} = a_0 \), and the total number of protons, \( N \), as seen in Figure 3.28.

\[
\Phi \approx \frac{eN}{a_0}
\]

The potential is then \( \Phi \approx \frac{eN}{a_0} \), where \( e \) is the familiar unit charge. The space-charge induced change in energy for the protons is then calculated using \( \frac{q\Phi}{E_p} \), where \( E_p \) is the proton energy. Thus, this expression becomes

\[
\Delta E_{\text{protoncloudenergy}} \approx \frac{e^2N}{a_oE_p}
\]  

(3.82)

Using typical experimental parameters of \( E_p \sim 5 \text{MeV} \), \( N \sim 10^{12} \), and \( a_o \sim 6 \text{mm} \), this change in energy due to space-charge effects is estimated to be \( \sim 0.1 \text{ MeV} \). Of course, this effect is decreased at higher proton energies, and due to the divergent nature of laser-accelerated proton beams, \( a_o \) also increases in time, therefore the space-charge effects are further minimized. Lastly, despite these estimates, which should be taken as an estimate of a theoretical proton beam which has no co-moving electrons, it has been found experimentally that laser-accelerated proton beams drag a population of electrons with it, beam making it charge neutral [155].

Figure 3.28: A spherical “cloud” of protons, which is expanding with time. Initially cloud has diameter \( a_0 \) at \( t_0 \), and later \( a_1 \) at \( t_1 \).
3.9.3 Beam-Plasma Instabilities

Proton radiography would be of little use if it excited instabilities in the plasma of interest, since this would perturb the dynamics of the system. As shown in the previous chapter, this would be analogous to the B-dot probe interfering with plasma flow. It can be shown that the proton beam is unlikely to perturb the system through simple estimates. In the limit where a beam is divergent with respect to direction and thus energy, the growth rate for the beam-plasma instability, $\gamma_{\text{beam-plasma}}$, may be estimated as [156]:

$$\gamma_{\text{beam-plasma}} \approx \omega_{\text{plasma}} \left( \frac{n_{\text{proton-beam}}}{n_{\text{plasma-ions}}} \right) = \sqrt{\frac{4\pi Z^2 e^2 n_{\text{plasma-ions}}}{m_{\text{plasma-ion}}} \left( \frac{n_{\text{proton-beam}}}{n_{\text{plasma-ions}}} \right)}$$

(3.83)

Where $\omega_{\text{plasma}}$ is the familiar plasma frequency. The strongest dependence here is on the number density of the proton beam, since $\gamma$ scales with $n_{\text{proton-beam}}$ and $n_{\text{plasma-ions}}^{-1/2}$. This implies that the e-folding times for instability growth are smallest when plasma density is lowest. To estimate the growth time for such an instability, we may use a typical plasma density of $10^{18} \text{ cm}^{-3}$ and a proton beam density of $10^{12} \text{ cm}^{-3}$, we find that the growth time is on the order of 100 ns. This time is several orders of magnitude larger than the probing duration, which is around $\sim$ few ns for the lowest energy portion of the beam, and around 300 ps for the highest energies of the proton beam used in this work. The fact that the growth times are so much larger means that beam-plasma excitations can be largely neglected over the probing interval, again, ensuring that the distortions in the final beam profile are due to the interaction of the protons with the electromagnetic fields in the plasma, and that the system has not been perturbed.

3.9.4 High Spatial Resolution

Another key advantage of using proton beams as a diagnostic arises from the fact that the source size is quite small, while the fluence is quite high, which leads to very high spatial resolution [102]. The source size has been investigated both experimentally and numerically, and is typically 100-200 $\mu m$ in the transverse direction, with respect to the direction of the beam acceleration [122,157,158]. The
size and position of the virtual source, however, are determined experimentally and can be as small as 2-3 µm [102]. Due to the high fluence, the spatial resolution is primarily determined by the size of the virtual proton source. This means that it may be implemented in a point-projection scheme, despite the fact that the source position and size are often virtual [121], and thus the magnification of resulting images is $M = 1 + L_{s-o}/L_{o-d}$ where $L_{s-o}$ is the distance from the source to the object, and $L_{o-d}$ is the distance from the object to the detector, as seen in Figure 3.29.

![Figure 3.29](image.png)

**Figure 3.29**: The point projection layout and relevant distances for calculating the magnification of the system for proton probing. The magnification is set by the distances between the proton source and object, and the distance from the object to the detector.

### 3.9.5 Collisional Blurring

When proton radiography is implemented on plasma systems with relatively low number densities, the effects of collisional blurring can be largely ignored, and thus the resulting images are purely due to the effect of the electromagnetic fields on otherwise unperturbed proton trajectories [154]. The collisional blurring may be calculated using a combination of number densities and proton energies in the
Transport Range of Ions in Materials code, TRIM [150]. As an example, consider a volume of aluminum gas with density $\rho \approx 1 \times 10^{-3} \text{g/cc}$, and a scale length $\sim 5 \text{ mm}$ along the direction of proton propagation. TRIM predicts that 1 MeV protons will penetrate this density, however, the collisional blurring of the beam corresponds to $\sim 170 \mu \text{m}$ over 5 mm, or about $2^\circ$, or 0.03 mrad, half-opening angle. This example roughly corresponds to an upper limiting case as will be discussed later. For higher energy protons the blurring is less of a concern. The impact of collisional blurring on the results of this work will be discussed further in Chapter 6.

3.9.6 Effect of Proton Beam Energy Spectrum

As previously discussed, proton beams generated from this scheme are proportional to the hot electron energy, which is itself similar to a relativistic Maxwellian. Regardless of the exact spectrum, the importance of this is that the energy spectrum is finite, and with sufficient bandwidth, that if one is able to resolve the energies, then one can utilize this property. By properly utilizing this property of TNSA proton beams, the amount of information that can be gathered in a single experiment is greatly enhanced, including a high dynamic range for the magnitude of fields probed, and the ability to gather multiple temporal snapshots of the evolution of the $Z$-pinch plasma evolution. In this work, the typical energy range examined was $\sim$1-10 MeV. The first advantage that is immediately obvious arises from the proportionality of the force on a proton traveling with velocity, $v_p$, in the presence of a magnetic field with strength $B$. From the Lorentz force, $F = v_p \times B$, it is obvious that protons with different energies, and therefore different $v_p$, the force is proportionally different. Due to this, the different energy components of a proton beam will respond to the magnetic fields accordingly. This essentially implies that the dynamic range of the diagnostic is large, with lower energy particles being sensitive to lower magnitude fields, while the higher energy particles allow for detection of larger magnitude fields.

As previously covered in section 3.8.4, beam production occurs roughly over the time-scale of the driving laser pulse, which in this work is 350 femtoseconds. This means that excellent temporal resolution is obtained, since $Z$-pinch experi-
Figure 3.30: a) The relevant distances for determining the timing of proton images of a plasma system. b) The frame start time relative to the time at which the proton beam is accelerated and the frame integration time is calculated and plotted for several proton energies. The energies are 1.2, 4.4, 6.6, 8.3, and 9.7 MeV, and the integration time for a frame corresponding to each energy decreases with increasing energy while the frame start time is increased with decreasing energy.

ments typically evolve over 100’s of nanoseconds. However, since the different energy components of the proton beam move with different velocities, two important temporal characteristics of the diagnostic arise. The first temporal characteristic is the time at which each snapshot of the field is taken, which is set by two relevant spatial dimensions. These dimensions are defined by the experimental geometry, as seen in Figure 3.30. The first is the length between the target/source and the plasma system of interest, $L_{t-s}$, defining the time at which probing is initiated relative to the CPA laser pulse arrival. The second is the plasma system size, $L_{ps}$, defined as the length of the region where magnetic fields of sufficient strength to affect the trajectory of a proton in the detected spectrum exist. In this work, $L_{t-s}$ is typically 1 cm, while $L_{ps}$ is typically 3 cm, which are both set by the size constraints of the experimental hardware. The second of these is the temporal integration time, or probe duration. For a 1 MeV proton, the time to traverse this region is $\sim 2$ ns, while for a 10 MeV proton, the integration time is only $\sim 0.7$ ns. After the protons have exited the system, they propagate ballistically to the detector, and thus this length is of little importance to the integration time. This means that the temporal integration time is smaller as the proton energy is in-
creased, while both integration times are small compared to the Z-pinch evolution time. Another consequence of these characteristics is the ability to resolve multiple temporally separated frames with one proton beam. The probe initiation and integration time are shown in Figure 3.30, where calculations for several proton energies are shown.

3.10 Diagnosing Proton Beams

In order to utilize the features of multi-energy proton beams described in Section 3.9.6, the detector must be able to resolve the features of the different energy bins. To this end, radiochromic film, or RCF, was utilized in this work. Radiochromic film is a special film which was designed for use in the medical field for radiology medicine, but is frequently used in experimental physics as laser-accelerated proton detector [159]. The films used in this work were manufactured by Gafchromic\textsuperscript{TM}, and are designed for applications in radiotherapy [160].

There are several types of RCF, however they are all composed of a polyester as a substrate, and an organic monomer suspended in a gel, as seen in Figure 3.31 a). This monomer polymerizes and changes from transparent to a blue color when struck with ionizing radiation, thus it does not require development. The sensitivity, response/dose, is then just determined by the thickness of the active layer. The thickness of the active layer can vary by 10% from batch to batch, and up to 5% within each batch. This results in an uncertainty in dose of $\sim 20\%$ [161]. Since the film is sensitive to ionizing radiation of almost any type, including high-energy photons, electrons, or ions, it is shielded with various materials which remove electrons and heavy ions, while reducing the amount of X-rays which can potentially induce unwanted signal on the film. In this way, for appropriately chosen materials, the signal on the RCF may be considered to be almost entirely due to protons.

RCF is typically deployed in a package known as a film stack, in which multiple layers of RCF and filter materials are placed. Using information from the NIST pstar [153, 162], estar [163], as well as tabulated data from the Lawrence
Berkeley Laboratory Center for X-Ray Optics [164], the energies of particles and photons which will be allowed to penetrate the shielding material, and the subsequent layers of RCF can be estimated. The thickness and material chosen to wrap the film pack then determines the minimum energy of a proton which may penetrate into the first layer of film.

In this work, the shielding material was typically 16 \( \mu \text{m} \) aluminum, which was wrapped around the entire film stack. This sets the minimum energy of protons which may penetrate beyond this filter layer at \( \sim 1.2 \text{ MeV} \). Due to the Bragg peak absorption in the materials, each layer of film will collect a signal which is primarily due to protons within a rather well defined energy band. In order to calculate the energy of protons which will be deposited within each layer, a Matlab routine was utilized [165]. This calculation takes an energy bin and resolution, film type, and filter material and thickness as input parameters. It contains penetration depth information from Stopping and Range of Ions in Matter [150], SRIM, calculations for the filter materials, as well as the film material compositions. SRIM calculates the total stopping power of a material, e.g. polycarbonate, as a sum of all of the stopping powers from each individual material, with a correction for the field

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**Figure 3.31:** The film structure of the two most commonly employed RCF types in this work. (Left) The Gafchromic® HD-V2, and (Right) the EBT3 type. Figure adapted from Gafchromic® film specifications. [160]
structure due to complex electron orbits in specific atoms. From this information, one can construct a virtual stack of radiochromic film, including the substrate and active layers, and find the approximate energy deposition within each layer. This information is then represented as a curve, which shows the amount of energy deposited a single proton as a function of its initial energy in the active layer of a particular film in the stack.

Figure 3.32: Sample Bragg peak absorption calculation for a stack of RCF used in the experiments. Each curve corresponds to a layer of film within the pack, showing the amount of energy deposited vs the initial proton energy.

An example of this data is shown in Figure 3.32, where the calculation has been performed for the proton energies expected in the experiment from characterization, and a film pack composed of one layer of HD-V2 film, followed by 4 layers of EBT3 film, all wrapped in 16 μm aluminum. In this case, we can see that film type HD-V2, high definition, has a smaller bin width than the film type EBT3, about 0.5 MeV and 2 MeV respectively, due to the difference in the active layer thickness. This is due to the difference in film construction. The structure of two of the different film types are seen in Figure 3.31 While HD-V2 is composed of a substrate and the reactive gel, the EBT film is a sandwich of polyester, reac-
tive gel, and polyester. For the HD-V2, this means that the energies detected are
dependent on the orientation of the film with respect to the direction of incoming
protons, e.g. when the gel faces the proton beam it will detect lower energy protons,
and when the orientation is flipped, it is sensitive to higher energy protons due to
the filtering provided by the plastic layer. In the case of EBT3, it is symmetric,
so the sensitivity is the same irrespective of the orientation.

Occasionally a third film type was used, MD-55-v2. This film is similar
to the EBT3 film, in that it is composed of an active layer sandwiched between
two layers of polyester, however, the thickness of the active layer is 15 µm. This
film then provided an intermediate sensitivity between the two primary types used.
Ultimately, the use of this additional film deemed unnecessary during deflectometry
experiments, and thus was primarily used during characterization experiments.

3.11 History of Proton Deflectometry as an EM
Field Diagnostic

The use of proton beams as a diagnostic has been developed and demon-
strated extensively for laser-plasma interaction experiments [97–99, 102–108, 166].
While there are numerous examples, it is useful to examine one particular case
where magnetic fields were examined in a laser plasma experiment, since it is the
magnetic field, and not the electric field that we seek to investigate in this work.
The electric field is generally negligible in this work, and will be discussed further
in Chapter 6. In this experiment, seen in Figure 3.33 a), a foil is driven by a
long pulse, nanosecond pulse width, laser. This drives ablation from the target
surface, and due to the $\nabla T_e \times \nabla n_e$ mechanism, generates a magnetic field with a
maximum strength of $\sim 50$ T [166]. The proton beam used for probing is driven
by a short-pulse laser, and passes through a metallic mesh, which is imprinted
on the profile of the proton beam. This mesh imprint serves to make distortions
due to the encountered electromagnetic fields easier to detect and quantify. The
beam is incident normal to the plane of the interaction target foil, and from the
opposite side where the interaction target is being driven by the long pulse laser.
Due to the orientation of the magnetic field, protons are deflected radially outward with respect to the beam axis of symmetry. This results in an image of the mesh which appears magnified. Numerical simulations were then iterated to find a best match according to the strength and orientation of the magnetic field driven by the aforementioned mechanism.

**Figure 3.33:** a) The experimental setup for an experiment measuring magnetic fields in a laser-plasma interaction experiment. The proton and interaction targets are planes extending in and out of the page. b) The accelerations experienced by protons passing through the magnetic field region, as seen from the proton source. c) The resulting experimental deflectogram recorded on radiochromic film showing a magnified grid in the central region. d) The simulated deflectogram of the experiment in this configuration also showing a magnified grid. Figure adapted from Romagnani et al. Laser and Particle Beams (2008) [166]

In fact, the proton deflectometry technique has also recently been applied to a Z-pinch in order to map the return currents [167]. This differs significantly from this work in the following ways. The primary difference is that the Z-pinch in the recent example was driven by a laser [168]. In this scheme, a laser is incident on the end of a small wire, and thus a current is driven through it. This current is much smaller than the currents typically used in pulsed-power-driven Z-pinches, with a detected maximum current of \( \sim 7 \text{ kA} \), and the corresponding magnetic fields generated were relatively small, \( \sim 1 \text{ T} \). This is in contrast with currents in the present work, which have a maximum of 600 kA, and magnetic fields up to 100’s of Tesla, depending on the specific load used. Also, the protons used to produce a deflectogram of the laser-driven Z-pinch were not generated via the TNSA scheme, but rather from fusion generated protons from an imploded capsule of D³He fuel. This deflectometry scheme is quite different than the scheme used for the present work, since the source size is larger, \( \sim 45 \mu \text{m} \), resulting in
decreased spatial resolution as compared to the TNSA generated proton source. Fusion generated protons are also monoenergetic, and for the DD reaction are \( \sim 14.7 \) MeV, with a narrow bandwidth. Although this scheme is both reliable and useful, the use of TNSA generated proton beams offer significant opportunities to obtain more information when used as a diagnostic as compared to fusion generated proton sources, while also being significantly simpler to implement.
Chapter 4

Numerical Simulations

4.1 Introduction

Numerical tools were vitally important in this work. This is primarily due to the fact that interpretation of proton deflectometry data is not possible without them. Furthermore, it is simple to see that the problems cannot be reduced to two dimensional approximations, and likewise analytic solutions, due to the inherently three dimensional nature of the problems. This can be seen through the use of a vastly simplified model of proton deflectometry for a Z-pinch experiment. It is known that Z-pinch systems can have very complicated magnetic field associated with them, but it is instructive to first consider the simplest case imaginable, to determine how simple or difficult it will be to interpret the resulting data from such an experiment. The simplest example is of course for a single proton approaching a wire carrying a current in the -z direction from the radial direction with respect to the axis of the wire, as seen in Figure 4.1. The direction of the current is, of course, arbitrary, but by choosing this direction, which corresponds to the experimental situation, it will be easier to understand the experimental data when it is examined in detail in the following chapter. In this case there is a symmetric azimuthal B field, increasing in magnitude with decreasing radius.

\[ B_\theta = \mu_0 \frac{I}{2\pi r} \]  \hspace{1cm} (4.1)
For the moment, it is not necessary to consider the change in magnitude, but rather assume that the magnitude is strong enough to impact the proton trajectory over the relevant scale lengths, while not large enough to trap the proton within the field. Additionally, it is assumed that the proton momentum is initially solely in the $-r$ direction. Then, the right-hand rule may be used to intuitively examine this simplified scenario, which reveals the complexity of this simple deflection. Looking at Figure 4.1, it is seen that there are three primary deflections in this geometry. Firstly, a proton traveling in the minus $r$ direction with respect to the $z$-axis of the load would first experience an axially upward acceleration. With a component of the proton momentum now in the axial direction, it next experiences a radially outward acceleration. Finally, on the other side of the wire, with respect to the side initially approached by the proton, the magnetic field vector is flipped. This means that the proton will then experience an axially downward acceleration. This means that tracking even a single proton deflection is an inherently three-dimensional problem for this probing configuration. Due to the complex, nature of a proton beam interacting with the electromagnetic fields of a $Z$-pinch plasma experiment, extensive numerical modeling was necessary to interpret experimental data, which this simplified model quickly reveals. In addition to these three deflections, it is important to consider that proton beams are inherently divergent, which is the first complication introduced. Secondly, there are many protons in a beam, typically

![Figure 4.1](image-url): The three primary deflections experienced by a proton approaching radially with respect to the axis of a pulsed-power-driven short-circuit magnetic field.
with $\sim 10^{12}$, each with its own velocity vector, which must be accounted for. Lastly, the magnetic field increases in magnitude nearer to the short-circuit load, leading to variations in the strength of the accelerations experienced by a proton in the real experiments. From this, one quickly realizes the complexity of the deflected proton trajectories in even the simplest scenario. In order to accurately interpret the results of any real deflection scenario, accurate 3D simulations of both the $Z$-pinch load, and subsequent proton trajectories were required. Although one could calculate the trajectory for a single proton given appropriate initial conditions, it would need to be iterated for every proton, including its energy and initial vector, while also including the effect of having real materials present in the short-circuit load as boundary conditions.

Of course, numerical tools for carrying out such calculations exist, and so they were employed in this work. Two codes were used for the primary simulations, and were both equally important to the interpretation of the experimental data. Gorgon [169], is a code which was designed to model $Z$-pinch experiments, and is widely regarded as one of the most reliable tools for simulations within the $Z$-pinch community. Large Scale Plasma [170], or LSP, is a code which is often used in the laser-plasma interaction community, and is better suited to model the physics of proton beams than the MHD code. Both of these codes, along with their use in this work, will be discussed in this chapter. Due to the three dimensional requirement for simulation of the problem in this work, both codes utilized 3D Cartesian space for calculations. To enable the large number of calculations for the problem, typically in excess of $10^6$ cells in a single run, the parallel features of both codes were used. These implementations utilize a version of a message-passing interface, or MPI, in order to split the calculations among an array of processors. Without this load sharing feature, the calculations performed in this work would not be possible in a reasonable time-frame. This is especially important due to the many simulation iterations required to match the experimental results. Below, the framework for the Gorgon code will be discussed, as well as some of the relevant features of the LSP code. Some of the numerical tools used to support the running of both codes, as well as to analyze resulting data, will also be discussed.
4.2 The Gorgon Code

The Gorgon code was developed by Dr. Jeremy Chittenden at Imperial College in London, UK [171]. Early versions were based off of several other MHD codes, namely those of Bell’s MH2D [172] and Stone’s ZEUS-2D [173, 174, 174], developed for ideal MHD simulations and astrophysical flows respectively. Since many Z-pinch phenomena are described well by the magneto-hydrodynamic equations, the resistive MHD equations were used as the basis for Gorgon [171]. The code essentially has a hydrodynamic framework, with the extra physics required by MHD built in. In order to define the regime in which MHD is valid we need to examine some properties of the plasma of interest.

4.2.1 The Resistive MHD Equations in Gorgon

Although the ideal MHD equations can be employed to get a general picture of the plasma dynamics, closer inspection of a real system generally requires adjustments to this set of equations. Some of these corrections include terms for resistivity, viscosity, thermal conduction, dissipation, and radiation. This is the regime which Gorgon was built to handle, as it essentially solves the discretized resistive MHD equations on an Eularian grid. Although it would be preferable to include all calculations, considerations of available computational power limit how much can be realistically modeled in a single calculation. The Gorgon version employed in this work is a single fluid model, which also allows the ion and electron populations to be out of thermal equilibrium, while also implementing a Van-Leer advection algorithm. In general, the resistive MHD equations provide a much better representation of the dynamics of a laboratory MHD system. For a typical Z-pinch, the magnetic Reynold’s number can vary dramatically within the system, but in general the number is ≤ 1 in the bulk of the plasma. Additionally, for a real system, we will also consider the fluid elements to be compressible. Of course, these equations are not a closed set, so it is necessary to employ several different plasma models to truncate the set. The resistive MHD equations for Gorgon
are then [171,175]:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0 \tag{4.2}
\]

\[
\left( \frac{\partial (\rho \mathbf{V})}{\partial t} + \nabla \cdot (\rho \mathbf{VV}) \right) = -\nabla P + \mathbf{J} \times \mathbf{B} - \nabla \cdot \pi \tag{4.3}
\]

\[
\frac{\partial U}{\partial t} = \nabla \cdot (U \mathbf{V}) - (PI + \pi) \cdot \mathbf{V} - \nabla \cdot (-\kappa \nabla T) + \eta |\mathbf{J}|^2 + \sum_{i=0}^{Z_{upper}} \left( \frac{\partial n_z}{\partial t} \sum_{i=1}^{Z_{upper}} \mathbf{E}_{ionization}(i) \right) \tag{4.4}
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) - \nabla \times (\eta \mathbf{J}) \tag{4.5}
\]

While the continuity equation remains unchanged, there are several new terms introduced. The last term on the right hand side of eqn 4.3 is the divergence of \( \pi \). Here \( \pi \) is the viscous stress tensor, which is a linear approximation of the stresses introduced by viscosity, which in this case is limited by an artificial viscosity in order to provide numerical stability [171].

4.2.2 The Energy Equation

A means of including the effects due to the aforementioned corrections such as resistivity, etc., is also required. These terms are in equation 4.4. This equation describes the change in internal energy for the system, thus \( U \) on the left hand side is the energy density. In order from left to right, the terms on the right hand side are the energy advection, the compressional heating, the thermal diffusion, Ohmic heating, and an ionization energy sink. This sink is dependent on the time rate change of \( n_z \), the number of ions with ionization, \( Z \), and the energy required to reach the ionization state. A Thomas-Fermi ionization model is used to calculate the electron temperature and average ionization state for the plasma, while the Saha equation is then iteratively solved to determine the various ionization states available [171].

To account for radiative losses in the plasma, an “optically thin” radiation loss model is used as a first-order approximation. This is necessary to ensure that the plasma does not artificially retain too much energy, causing it to heat
up rapidly. This parameter is calculated as an approximate emissivity parameter, which may later be used to synthesize diagnostic images, as discussed in section 4.3.1 below.

4.2.3 Current Driver Models

Gorgon also offers flexibility in how the simulated load is driven, through three options. All work on the same principle, by setting potentials on the upper and lower z boundaries of the simulation domain. The first current-drive option is set by specifying a current amplitude and waveform, which is well approximated for many pulsed-power machines by:

\[ I(t) = I_{\text{max}} \times \sin^2 \left( \frac{\pi}{2} \times \frac{t}{\tau_{\text{max}}} \right) \]  (4.6)

The peak current is specified by $I_{\text{max}}$, while the time that the current reaches this value is set by $\tau_{\text{max}}$. While this approximation is often quite accurate, it does not take into account the machine’s circuit response to fast changes in load inductance, such as in an imploding array or a x-pinch load. As outlined in Chapter 2, it also does not account for MITL failure, in which case the current may be diverted somewhere along the power feed. However, given this information, this approximation is at least valid for well-behaved loads up to the time of peak current. The second option allowed by Gorgon is to use an experimentally measured waveform as the current drive. This is a relatively straightforward approach, and will be able to more accurately model an experimental drive current than through the previous option. Again, however, it does not necessarily account for the machine’s circuit response, since there is no feedback mechanism between the simulated load and the read-in current waveform. The last option is the most accurate, though it introduces more calculations, and will therefore come at an added computational cost. This third option is to use a circuit model of the machine discharge in order to calculate the circuit response of the machine to a change in the load inductance. Of the three options, the first option was typically preferred in this work, since the loads chosen in the experiment are of the non-imploding type. This is especially true of the radial foil loads used. Choices of “thick” foils and wires ensured
that electrical contact between the anode and cathode were retained throughout the main pulse, which means that the load inductance is approximately constant throughout the current drive.

### 4.2.4 Modeling Wire Initiation

Although the framework for making a MHD calculation is now in place, if we would like to model a real Z-pinch experiment, then a means of first initiating a plasma is required. Since many Z-pinch experiments utilize solid wires as the material a means of transitioning from solid $\rightarrow$ plasma is needed, without expending too much computational effort. That is, to save on computational power, a simplified model of a wire which is reasonably able to reproduce the experimental situation is necessary in order to completely model the experiment from start to finish. As discussed in Chapter 1, even single wire experiments are quite complex in structure, and X-ray radiography of wires during the ablation stage of cylindrical wire array experiments show that the wires are in a complex mixed state between all material phases; solid, liquid, gas, and plasma [176]. The problem of modeling a single wire with Gorgon has been extensively examined by Chitten-den [177]. The first issue here is that typical wire diameters range from a few to 50 $\mu$m, while typical spatial resolution, or cell sizes, are 50-200 $\mu$m for 3D simulations.

![Figure 4.2](image)

**Figure 4.2:** a) A slice through a 16 x 25 $\mu$m tungsten cylindrical wire array. The white box shows the zoomed in region for b) a wire as initiated by Gorgon before current start. b) 25 ns after current start, the wire has expanded and is ablating material toward the axis of the array.
This means that the resolution typically prohibits the resolution of any single wire. This is not necessarily a large problem since experimental data has shown that the wire diameter typically expands 10-100 fold during the first few nanoseconds of the current pulse. While the wire corona quickly expands, the cold, dense, wire core is observed to persist until the entirety of the wire mass has been ablated much later during the experiment. Of course, this is dependent on the wire diameter, i.e. the amount of mass available to ablate, and the current drive. Recalling the rocket model in Chapter 1, the rate of mass ablation for a wire depends primarily on the square of the current, and has only a weak dependence on the wire material, with an ablation velocity which is assumed constant, and determined empirically. This is important because it implies that if the modeled wire similarly quickly expands to a much larger diameter, it may be decently approximated by the code with somewhat large cell sizes. This in fact, has been shown to be the case with carefully chosen initial conditions for the simulated wire. The wire is initiated as a neutral, cold, dense, gas, having a temperature which is typically set to 0.01 eV, or 100 K. This itself avoids the calculation of the phase transition from solid to gas. With the choice of the low temperature, the sound speed is sufficiently low that the motion of the gas is negligible, especially on the time scale of the simulation. The mass of the wire is also spread across multiple cells. This is done by placing 99% of the wire pass per unit length in the wire location, while randomly redistributing 1% of the total wire mass per unit length in radially adjacent cells surrounding the cell containing the majority of the mass, as seen in Figure 4.2 b).

It is thought that the initial inhomogeneity in the coronal structure is due to variations in the mass ablation rate along the core, which is plausibly due to axial differences in the properties along the wire core [177]. In order to simulate the properties of real wires, small 0.1% variations in the initial temperature are seeded throughout the wire core. This small variation in temperature leads to small changes in the resistivity of the wire core, which correspondingly changes the early-time ablation rate, and has been shown to reliably reproduce the features of wire ablation observed in experiments, including the flaring structure in the coronal plasma surrounding the wire.
4.2.5 Modeling of Experimental Hardware

Another useful feature of the Gorgon code is the ability to incorporate models of the experimental hardware. This not only gives a more accurate representation of the system, but allows for more complex setups, where free expansion of plasma into a vacuum is useful. An example of this is for a x-pinch driven jet. In this case, it would be useful to allow the jet to propagate above the hardware, where it may be used for an interaction experiment. Shown in Figure 4.3 we can see a jet freely expanding above the x-pinch hardware. In fact, in this setup, a low density and temperature cylindrical gas has been placed above the jet, so that a simulated interaction between a jet and an ambient medium simulation can be performed. Additionally, the foam has been set with a linearly increasing mass density in the vertical direction. When the jet interacts with the gas, a bow shock is seen to form, corresponding to the astrophysical picture of a YSO jet interacting with the interstellar medium.

Although this experiment was not realized in the laboratory, Gorgon provided a means of studying the feasibility of such and experiment, again demonstrating the power of such a numerical tool. This is, of course, made possible only by the ability to include a simple model of the hardware, so that the x-pinch and jet may be driven, and allowed to propagate beyond what would otherwise be the conducting boundary. In order to model the experimental hardware in Gorgon, desired cells are defined as electrode material, and can be specified in the code to match the experimental hardware. This material is defined to be highly conductive, while also being thermally insulated. Further, the position of electrode material is fixed throughout the experiment, ensuring that it does not interact with the plasma.

There is also an option in Gorgon to model the magnetically insulated transmission line. In most cases, this practically translates to more flexibility in the ability to more completely model the experiments. This becomes especially important for the modeling the loads examined in this work. Without this capability, the only option to model such loads would be in a two-dimensional R-Z geometry. Simulating in two-dimensions offers the advantage of using more cells,
Figure 4.3: a) X-Z Slice from a 3D Gorgon simulation of a W wire x-pinch jet interacting with a low density gas. b) Synthetic laser interferogram unfold showing areal electron density and structure of bow shock in low density gas.

i.e. better problem resolution, however, it cannot reproduce the three-dimensional effects of the experiment, and thus is a poor approximation.

The option to simulate a transmission line is even important to the interpretation of short-circuit load data, as will be covered in the following chapter. Although calculations of the short-circuit loads will be presented in more detail in Chapter 5, an example of this setup is shown here to demonstrate how the code solves the field. Rather than setting the potentials at the upper and lower z boundaries, the potentials are now set by the dimensions of the transmission line connected to the lower z boundary of the 3D grid. Starting from the axis of the load and working radially outward, there are three important zones at the bottom surface of the computational grid. Current is allowed to flow only through the central electrode, through the electrode cells in region I. Region II, is filled with vacuum cells, providing insulation between the inner and outer electrodes. Finally, current is allowed to flow through the outer electrode, or the return current “can” in region III. As previously mentioned, this is a more accurate representation of the experiment, and allows for extremely valuable versatility. This advantage does, however, come at yet another increase in computational cost, as all three spatial dimensions must be increased to accommodate the additional cells representing the hardware.
Figure 4.4: A 3D view of a short-circuit load in Gorgon simulation space. An expanded view shows the three regions where current enters and exits the simulation space through the lower z boundary.

4.3 Gorgon Output

Gorgon outputs a variety of useful parameters during the course of a simulation. These parameters include both scalar data, such as temperature, mass density, and electron density, as well as vector data, such as magnetic field and instantaneous velocity. An example of some of these output parameters are shown in Figure 4.5. Here a 3D simulation of a 4 wire x-pinch is sliced through a plane defined by two of the crossing wires in order to show typical output.

Figure 4.5: Slices through a simulation of a x-pinch performed with Gorgon showing a) mass density, or rho, b) electron density, or \( n_e \), c) magnetic field magnitude, and d) temperature.
4.3.1 Simulated Diagnostics

This output may then be further processed in order to make qualitative comparisons between experiment and simulation in addition to the quantitative output. Gorgon includes some powerful post-processing tools which enable straightforward comparison of simulated data with experimental data. While some of the diagnostics are crude approximations that provide only qualitative comparison, others are more precise calculations, which allow for quantitative comparison between experiment and simulation. These comparisons allow for quick sanity checks, and can give valuable insight when experimental design is under consideration. The quantitative data enable direct comparison to experiments and more importantly, offer the ability to validate the code against experimentally measured values.

Laser Shadow

One of the most common diagnostics in the Z-pinch community, as covered in the previous chapter, is the laser shadow diagnostic. This diagnostic is sensitive primarily to variations in the electron density along the laser path, and the wavelength of the photons used in the experiment. The laser shadow calculation first uses characteristics of the laser beam and optics used in the experiment to calculate the divergence/convergence of the laser beam when it approaches the simulation domain. This information is used as the initial state of the simulated laser photons. The position of the beam axis relative to the detector, or virtual camera, may optionally be adjusted to best match the experiment. The laser beam is then modeled as an area of virtual photons, each with it’s individual wave vector $\hat{k}$, which are advanced through the simulated grid. As the photons enter the simulation, they are subject to refractions due to the electron density along their path. Photons may also be removed from the simulation if they reach a cell which has a density above the critical density for the laser wavelength. Further, the laser wavelength may be selected, but is typically set to 532 nm, which is a standard laser probing wavelength used among Z-pinch laboratories.
Figure 4.6: An example of a synthetic laser-shadow diagnostic image from a Gorgon simulation of a x-pinch. Changes in the index of refraction throughout the plasma cause brightening and darkening in the synthetic image.

Line Integrated Electron Density

Although the laser shadow uses optics equations to calculate deviations in photon paths due to plasma, such calculations are not necessary to reproduce the data gathered from laser interferometry. As outlined in the previous chapter, the laser interferometry diagnostic is primarily sensitive to variations in line electron density. Since the code outputs the electron number density, it is trivial to integrate along a single direction to obtain the line-integrated electron density. It is then only necessary to set the limits on the dynamic range of the experimental diagnostic during analysis of the post-processing integration to make high quality comparisons to the simulated diagnostic. An example of this is seen in Figure 4.7.

Self Emission Imaging

The synthetic self-emission diagnostic is meant to reproduce images similar to those produced by a time-gated XUV framing camera [178]. In order to produce these images, two parameters are used, the temperature and the emissivity of the plasma elements. By integrating through the simulation volume along a chosen
Figure 4.7: The line-integrated electron density from a Gorgon simulation of a x-pinch load at 50 ns after current start. The limits are set to typical values for interferometry with a 532 nm probe laser at $1 \times 10^{17} \text{cm}^{-2}$ and $1 \times 10^{19} \text{cm}^{-2}$. Values above the maximum are typically not accessible due to the difficulty in resolving interferometry fringes.

axis, a value for the emitted radiation that would escape is approximated. These values are then gathered on a detector plane, which results in a pseudo self-emission image. The version of this synthetic diagnostic does not discern between the energy of the photons, since these are not calculated during the course of the simulation, but despite the crude approximation it is still a powerful tool for making comparisons to experimental data.

Proton Deflectometry

Since the aim of this work is to investigate magnetic field topology in order to determine current configuration, simulated proton deflectometry is by far the most important simulated diagnostic afforded by Gorgon. This simulated diagnostic is a simplistic model, which calculates the force on a virtual particle in each cell it passes through, and thus the resulting change in path due to the Z-pinch produced magnetic fields. Essentially,

$$\mathbf{F}_{p^+} = q_{p^+}(\mathbf{v}_{p^+} \times \mathbf{B}) \quad (4.7)$$
Figure 4.8: A synthetic XUV self-emission diagnostic image of a x-pinch at 150 ns after current start. Some flaring in the coronal plasma surrounding the wires is seen, as well as the jets formed above and below the cross-point of the wires.

The virtual particle’s momentum is then carried over to the next cell that it enters. Additionally, since laser-accelerated proton beams are at relativistic energies, a correction to the particle momentum at large velocities is included. Although the virtual particles do not interact with the Z-pinch plasma, interactions with cells defined as electrode material are included. This routine is accomplished by simply checking whether the virtual particle has entered a cell defined as electrode. If it has, then the virtual particle is simply removed from the simulation space. Although this may seem to be an insignificant portion of the deflectometry calculation, it was found to be very important in reproducing experimental deflectometry data from short-circuit loads. The direction of probing is always normal to the XZ boundary, while the user is allowed to set the location of the proton point source in XYZ coordinates. The minimum cell size used in this work was typically limited to 200 µm, meaning that the difference in source position is not resolved by the code. It can be seen that this change in source position is relatively insignificant in this work. Since it is a point source, or virtual point source, the magnification of the system is set by the distances between the source, object, and image plane. Then, the magnification is easily calculated.
Figure 4.9: a) A point source, object, and image plane for the calculation of the magnification of the projected image.

\[
\tan \alpha = \frac{l_o}{d_{so}} = \frac{l_i}{d_{si}}, \quad \Rightarrow M = 1 + \frac{d_{oi}}{d_{so}} \quad (4.8)
\]

If we then introduce the error in position \(\Delta x\):

\[
\tan \alpha = \frac{l_o}{d_{so} \pm \Delta x} = \frac{l_i}{d_{si} \pm \Delta x}, \quad \Rightarrow M = 1 + \frac{d_{oi} \pm \Delta x}{d_{so} \pm \Delta x} \quad (4.9)
\]

So, in the limit where \(|\Delta x| \ll d_{so}, d_{oi}\) the magnifications remains approximately unchanged. For example, the error in the position of the source relative to the load and image plane is estimated to be \(\pm 500\mu m\) in the experiments while \(d_{so}, d_{oi} \sim 2.5 cm\). Therefore the difference between the virtual source position and the simulated point source position are not significant. The sensitivity of the final proton deflectogram to small changes in source position has been investigated with the Gorgon code, and the results were nearly indistinguishable.

In fact, the final proton deflectometry result is more sensitive to other parameters, such as the proton beam flux and energy. These parameters are, of course, chosen in the simulations, by setting the opening angle for the proton beam, as well as the number and energy of protons in the simulation. The current version of this synthetic diagnostic allows only for a square beam profile, however, this fact is not overly important in reproducing the final proton distribution, as will be seen in the following chapter. As always, increases in the number of protons in a given calculation, increases the time required for a calculation, so it
was important to balance between obtaining a realistic result, and using the experimentally measured characteristics of the proton beam. The topic concerning the reproduction of realistic proton beams for deflectometry will be examined in detail in the following chapter. In general, this tool has been extremely valuable in the design of experiments, as well as in the interpretation of experimental proton deflectometry results.

4.4 The Large Scale Plasma Code

Due to the limitations of Gorgon’s synthetic proton deflectometry diagnostic, the Large Scale Plasma code was employed. The LSP code comes with a wide variety of physics packages, which can optionally be included or excluded during the compilation of the executable. This allows for flexibility in the examination of the critical physics in proton deflectometry, as well as in the ability to debug and optimize the code.

Of particular interest in this work was the impact of magnetic fields generated by the proton beams, as well as the interaction of protons with the background Z-pinch produced plasma. The code was originally designed by Welch [170, 179–181] for the purpose of examining heavy ion beam physics, and has most widely been used within the laser-plasma interaction community. Although it is capable of calculations in a wide variety of simulation geometries, such as one, two, and three-dimensional space in Cartesian, cylindrical, or spherical geometry, it was used only in 3-D Cartesian space for this work for reasons outlined at the beginning of this chapter. Because of the wide variety of physics models which may be used in the simulation of a problem, only the models which were typically enabled will be discussed.

LSP employs a direct implicit algorithm which relaxes the typical particle-in-cell requirements, which usually require that the plasma and cyclotron frequencies both be adequately resolved during each time step. In this scheme, either may be under-resolved during a time step, though not simultaneously. The algorithm
describing the particle momentum is [181]:

\[ p_{n+1/2} = p_{n-1/2} + \Delta t \left[ a_n + \left( p_{n-1/2} + p_{n+1/2} \right) \frac{q B_n x_n}{2 \gamma_n m c} \right] \quad (4.10) \]

Here \( n \) is one full time step, \( \gamma \) is the relativistic factor, \( m \) is the mass of the particle, \( c \) is the speed of light, and \( a_n \) is

\[ a_n = \frac{1}{2} \left( a_{n-1} + \frac{q}{m} E_{n+1} (x_{n+1}) \right) \quad (4.11) \]

which is the mean of the fields from the previous and current time step. Thus equation 4.11 shows that the momenta are advanced by using half of the electric field value at the old position, and half of the electric field value at the new position. The most relevant discussion of the LSP physics, e.g. advancement of fields, momenta, and energies, for three dimensional modeling of proton beams may be found in Welch et al. PoP (2006) [181].

Simulations by both Gorgon and LSP required many iterations in order to find the optimal matches to experimental data. To this end, when synthetic proton deflectometry was performed in LSP, only one half of the problem was examined. This is justified because all of the Z-pin on loads in this work were azimuthally symmetric. With the proton beams incident from a single radial direction, this means that only one half of the Z-pin load, and thus one half of the beam, was necessary to examine the deflection trajectories. This approximation is of course invalid if protons are deflected toward the missing half of the simulated loads, however, this was not the case in the loads examined in this work.

### 4.5 Gorgon Input for LSP Simulations

In order to use LSP for simulated proton deflectometry, data from Gorgon needed to be converted for input in LSP. Typically, data from a single output time selected in a Gorgon simulation was used for this purpose. LSP accepts input for field and particle or fluid density, as well as allowing for the specification of these two types of parameters through the use of selected functions. The three-dimensional magnetic field data is only accepted in the format output by an old
code, MAG3D. However, several routines for converting the Gorgon data to the required format were developed and optimized in this work. Slices of 3D data in Gorgon data were converted cell-by-cell into the required order and units. An example of this is shown in Figure 4.10, where the magnetic fields and mass density have been imported into LSP for a simulated deflectometry experiment.

Figure 4.10: Comparison of mass density contour, and magnetic field slice through, a simulated x-pinch load. The top two images are from Gorgon, while the bottom images show the data as imported and viewed in LSP.

4.5.1 Proton Beam Input

For the simulation of a proton beam, the following simulation parameters were used. Dynamic fields are allowed so that self-generated fields from the particles entering the simulation are accounted for, however, the Z-pinich produced fields are held static throughout the simulation. LSP includes several models for
describing the energy and velocity of the input proton beam, however most were designed for one and two-dimensional space.

The LSP code does, however, include an option to extract particles crossing a specified plane within the simulation space, which contains the particle charge, position, and momentum at a specified time. These files are written in a binary format which is readable by LSP, and is meant to provide a means of restarting LSP simulations. Additionally, there is a tool which allows for converting these output files to ASCII text, and vice-versa. This option was exploited in order to construct a realistic model of the experimental proton beam in the calculations. A separate calculation utilizing the Octave math program was developed, which allowed for the definition of every particle’s charge, position, velocity, and time of insertion. This enabled the injection of particle beams with nearly any spatial and temporal profile desired. The resulting ASCII file was then converted to the XDR binary format and read as input by LSP.

4.5.2 The Effect of Cell Size

The cell size is an important parameter in all numerical calculations. In general, smaller cell size results in better problem resolution, and therefore more accurate results. In fluid calculations, however, this parameter cannot be too small, else the fluid approximation breaks down. A smaller cell size means that the computational effort is vastly increased. This is especially true in 3D calculations. For example, consider a typical Gorgon simulation domain of 5 cm x 5 cm x 5 cm, divided into cells with 200 µm length in x, y, and z. This means that \(250^3 = 1.5625 \times 10^7\) calculations are performed over each time step. If 100 µm cell size is desired, or double the resolution, then the increase in calculations is cubic, or \(2^3 = 8\times\) the previous number. Assuming that the codes scales linearly with the number of processors and cells, if the previous resolution requires 100 processors to complete a calculation in a reasonable amount of time, \(~7\) days, then \(~2\) months would be required to complete a calculation at 100 µm with 100 processors. Alternatively, the calculation could be completed in a similar time, but would require 800 processors. The cell size in both codes was generally limited to
200 µm. This limitation was ultimately set by the availability of computational resources, however, it was found that this resolution was adequate for reproducing the experimental data, at least in short-circuit loads. This resolution, however, may be a limiting factor in the ability to reproduce the experimental data from plasma loads, as will be discussed further in Chapter 6.
Chapter 5

Development of Proton
Deflectometry for MA-Scale
Z-Pinches

5.1 Introduction

This chapter will outline the mechanisms responsible for the creation of laser-accelerated high-energy proton beams, as well as the specific advantages of proton probing over the previously discussed methods for making electromagnetic field measurements in plasma experiments. After developing the theory behind the use of proton beams as a diagnostic, along with an example of such use in laser-plasma-interaction experiments, the development of the diagnostic for use on z-pinch experiments will be discussed. This process includes the characterization of proton beams produced at the Nevada Terawatt Facility, and testing of the method on Zebra driven short-circuit loads. These experimental results are then used as a benchmark test for the two codes, Gorgon and LSP, the results of which are discussed in detail.
5.2 Proton Beam Characterization on Leopard

The first step in the implementation of proton deflectometry for z-pinch experiments was to characterize and optimize the proton beams produced by the Leopard laser at the Nevada Terawatt Facility. This included a parametric scan to optimize the maximum energy and flux of the proton beam to be used as a diagnostic. Proton beam generation and optimization were performed first offline in the Phoenix target chamber, where the optical table was decoupled from the vacuum chamber, minimizing any motions associated with evacuation of the chamber, which may adversely affect the focus of the laser beam on target and thus degrade proton production.

In order to produce the highest energy proton beam possible with the given laser specifications, extensive parameter space was examined. Various target thicknesses, ranging from 2-50 µm, and materials, Au, Cu, and Ti were tested as candidates. The targets were mounted by gluing the foils to 50 µm wires, which were in turn glued to thin glass support-stalks, seen in Figure 5.3 b). Several smaller diameter wires were then mounted on the top edge of the target to aid in finding the minimum focal spot diameter. This was done by viewing the laser spot with a microscope and a CCD while moving the small diameter wires in the laser light until the laser light was not visible on the microscope, assuring that the focal spot was at least approximately equal to the diameter of the wire. This was confirmed by calibrating the image taken with the microscope, and through this procedure the minimum focal spot diameter was found to be $\sim 5 \, \mu m$ FWHM, as seen in Figure 2.11.

The Leopard short pulse laser, 1064 nm wavelength and 350 fs pulse duration, delivered an average of 13.4±1.3 J at normal incidence to the target. Assuming that $\sim 50\%$ of the laser energy is in this focal spot, the calculated peak laser intensity is then $\sim 9 \times 10^{19} \, W \, cm^{-2}$, ensuring that the laser is of sufficient intensity to reach the relativisitc regime, where the TNSA mechanism is responsible for accelerating the protons for use as probing particles. The detector, radiochromic film, was placed 2 cm behind the target in order to diagnose the proton beam. Ultimately, it was determined that the optimal targets were 1.5 x 1.5 mm, 2 µm
thick Titanium foils by utilizing radiochromic film signals.

The results of one such shot are displayed in Figure 5.1, with each layer’s labeled energy corresponding to the calculation shown previously in Figure 3.32. Figure 2 displays a sample proton spectrum produced by Leopard with the optimal target, and observed with RCF. The RCF detector stack is located 20 mm behind the target, and is completely wrapped in light-tight 16 $\mu$m thick Al to reduce signals from other high-energy sources, e.g. photons, electrons, and ions from carbon, oxygen and titanium. Low energy protons will deposit their energy within the front layers of the film stack, and higher energy protons will deposit their energy deeper within the film stack.

Figure 5.1: Sample RCF spectrum from Leopard only proton characterization shot. The film pack was wrapped in 16 $\mu$m aluminum, and two additional strips form a cross pattern. The outer regions are then effectively shielded with 32 $\mu$m while the region where the strips overlap is shielded with 48 $\mu$m, which increases the minimum energy of detected protons to $\sim$1.75 MeV and $\sim$2.25 MeV, respectively. The decrease in beam divergence is clearly seen within the additional foil strips on the first layer of film. Higher energy components of the proton beam are only weakly affected by the additional foil strips, and thus the image of the cross is only faintly visible on the higher sensitivity films.

5.2.1 Typical Proton Beam Characteristics

The typical characteristics of proton beams accelerated by the Leopard laser are summarized in this section. Multiple target types were tested in order to find the best conversion efficiency into protons, which is proportional to the signals obtained in the films and can be seen in Figure 5.2. The optimal targets were found to be 2 $\mu$m titanium foil. This target type produced reliable proton flux at the relatively large distances used in this work. The beam divergence was found to be $\sim$65 degrees full-opening angle at the lowest energies detected $\sim$1.1 MeV,
decreasing to $\sim 40$ degrees full-opening angle at $\sim 10$ MeV. The energy deposited in each film by the particles has a rather large uncertainty due to both the $\sim 20\%$ batch-to-batch variation in the RCF [159] and the use of a non-calibrated scanner. Thus, the uncertainty in the dose, or energy deposited in each film, is estimated to be approximately one order of magnitude.

**Figure 5.2**: Approximate energy deposited in each layer for some of the target types examined. Although the energy deposited in the first film was similar for several targets, the use of 2 $\mu$m titanium foil targets deposited the most energy in each layer of film at higher proton energies compared to the other target types.

### 5.3 Deflectometry Benchmarking Tests with Short-Circuit Loads

A short-circuit load is beneficial as a test load for several reasons. One reason was that for appropriately chosen rod diameters, little or no X-ray background was created, which could have ablated the side of the proton target facing the load, and therefore potentially interfering with the production of a high-energy
proton beam due to the creation of a preformed long scale-length plasma on the accelerating surface of the target. Secondly, there was no debris launched from the load, which can severely damage the RCF package. Lastly, they produced uniform electromagnetic fields, which were simple to analyze and model. Combining these three advantages over plasma loads, it was reasoned that the data collected from these tests would be prime data for benchmarking the two codes used to model the experiments, Gorgon and LSP. Benchmarking the codes was of paramount importance before applying proton deflectometry to plasma loads, which can have far more complicated electromagnetic geometries. For quantification of proton beam distortions due to the observed magnetic field, a 127 µm thick copper mesh with 400 lines per inch, Figure 5.3 b), was placed in the path of the proton beam, near the laser target. The copper mesh was then imprinted on the beam, so that distortions were more apparent, and more easily compared to simulations of the experiment.

5.3.1 Experimental Setup

Figure 5.3: a) The experimental setup for short-circuit deflectometry tests of the proton deflectometry diagnostic. b) Drawing of the grid-target setup for imprinting a periodic pattern on the proton beam profile. c) Picture of the experimental hardware, where 6 return-current posts have been removed in order to allow the proton beam to travel through the load region with minimal clipping due to the electrodes.

The experimental geometry is shown in Figure 5.3. A proton beam was produced by the short-pulse high-intensity laser solid target interaction, and directed in the radial direction with respect to the z-axis of a short circuit load driven by
Zebra. Zebra is capable of delivering 1 MA peak current with a 100 ns, 0-100% current rise time. In these experiments it was operated in long pulse mode, delivering 0.6 MA peak current with a 200 ns, 0-100% rise time. The initial field measurements were done with short-circuit loads comprised of either a 6 mm or 3 mm diameter stainless steel rod, giving peak calculated magnetic fields at the surface of the conductor of 40 T and 80 T, respectively, at 0.6 MA.

A reduced diameter current return was designed in order to minimized the distance between the target and detector, which is ultimately limited by the diameter of this hardware. This setup consisted of 16 return-current posts, each with a diameter of 3.175 mm, placed on a 38 mm diameter circle surrounding the load. The number and placement of the return-current posts could be varied by removing some of the return-current posts. This enabled access for proton deflectometry, as well as opening a path for laser diagnostics when plasma loads were used.

Operation of Zebra in long-pulse mode allowed for not only a slightly less hostile environment in terms of the vibrations experienced by the laser beam transport and optics within the Zebra chamber, but also allowed for a slower variation in the fields, making the quasi-static magnetic fields approximation used in simulations more valid. In a test shot where only the Leopard laser was fired, the first layer of film, as seen in Figure 5.4 a), the contrast of the grid was very good, however, the contrast is degraded as the proton beam energy is increased, i.e. when the film layer depth increases. This is expected from CSDA data obtained from NIST pstar [162].

Optimization of the film pack combination and filtering was crucial in this work. Although protons with energy ~10 MeV were observed, the population of protons with energies >3 MeV was much smaller than the population at the lower energies. This is evident in layer two of Figure 5.1, where second layer of HD-V2 type film shows very little signal, and is supported by the TNSA model, which predicts that the number of protons will decrease as a function of increasing proton energy. This is a rather small energy window with which to work, and thus the filtering and films were optimized to maximize the amount of information obtained
Figure 5.4: Sample RCF spectrum from Leopard only shot using the film pack type used during deflectometry shots. A copper mesh was placed two millimeters behind the laser target, allowing the mesh to be imprinted on the proton beam profile. The mesh contrast decreases as proton energy decreases due to the difference in proton stopping power at higher energies.

during the course of each shot, while keeping the background as low as possible.

In order to detect the higher energy protons, the EBT3 type film was used, which, due to the thickness of the active layer being $\sim 4 \times$ thicker than the active layer in HD-V2 is $\approx 10,000$ times more responsive than HD-V2. While this is advantageous for detecting the higher energy protons at a lower flux, it also makes it more sensitive to doses of X-rays and high-energy electrons, which may penetrate to these deeper layers. This is, however, offset by the lower stopping powers of X-rays and high-energy electrons in the film layers. Additionally, as the source of X-rays is considered a point source in this situation, the X-rays generated by the laser-target interaction should only produce a uniform background on the films, which is distinct from the proton signal.

To improve the signal to noise ratio, where noise is taken to be any signal which is not due to the laser-accelerated proton beam, film packs were typically wrapped in 16 $\mu$m aluminum foil. This foil alone serves to cut out proton energies below $\approx 1.2$ MeV, X-rays $\sim 3$ keV, and electrons below $\approx 4.4$ keV. This filtering was appropriate for short-circuit, discussed in this chapter, and radial foil loads, discussed in Chapter 6. When more violent loads were examined, such as single wires, x-pinches, hybrid x-inches, and radial wire arrays, this filtering was inadequate, primarily due to the direction of ballistic debris, which destroyed the aluminum foil, as well as the first 1-2 layers of RCF, an example of which is shown
In Figure 6.2.

In such loads, it was necessary to mechanically reinforce the foil with 50 µm Kapton tape. When placed over the aluminum foil, this insulating tape was found to be sufficient to withstand the debris from these loads, however, it came at a cost of about half of the proton energy spectrum which could be detected with the aluminum filtering alone. This filtering was chosen after an experiment examined the effectiveness of multiple combinations of filters during a single wire shot (show the setup pic, and hopefully there is a picture of the background). The combination of 16 µm aluminum with the ≈ 50 µm Kapton tape and the first layer of substrate in the EBT3 type RCF, set the minimum detected proton energy at ≈ 4.4 MeV. This is clearly seen in a test shot in Figure 5.5. Here, only Leopard was fired, while half of the film pack was shielded with Al only, and the other was shielded with the combination filter of Al and Kapton.

5.3.2 General Results

Next, coupled Leopard and Zebra shots gave the first field measurements via proton deflectometry. The setup given above remained the same for the coupled shots, with two subtle differences. One is that the film sizes were increased in order to capture the desired features. The size increase was deemed necessary from initial predictions made by the simulation work completed prior to the experimental series. The other difference is that rather than keeping all 16 return-current posts in place, 6 were selectively removed in order to allow the proton beam to enter and exit the field region, as seen in Figure 5.3 c). For the experimental geometry used, protons were deflected away from the short circuit in the outward-radial direction. During the course of the deflection, protons also experience an axial displacement. Combining the two displacements, a radially and vertically offset deflection pattern was observed. This is seen in Figure 5.6, where increasing the current in the 6 mm diameter short-circuit proportionally increased the strength of the magnetic field generated, and the deflected pattern is also seen to increasingly diverge. This effect is displayed in Figure 5.6 c), where results from the 6 mm short-circuit carrying 565 kA current produced a maximum surface field of 37.6
Figure 5.5: The first layer of a RCF stack from a Leopard only test shot with different filtering for each side. The left side uses only the standard 16 µm Al filter, while the right side uses 50 µm Kapton tape to reinforce the Al filter. The addition of the Kapton filter shifts the minimum energy of the protons which can reach this film layer from 1.2 → 4.7 MeV.

Figure 5.6: a) Only Leopard is fired, and so the result is a proton radiograph of the short-circuit hardware. b) When the current in the load is 446 kA, the calculated magnetic field magnitude at the surface of the load is 29.7 T, and two diverging “wings” are observed. c) Zebra current is increased to 565 kA, giving a calculated maximum magnetic field strength of 37.6 T, and the divergence between the features increases. Shadows due to the return-current posts are visible in each image.
T which split the initially round probing beam into two symmetrically diverging features. At the outer edges of the deflection features, the shadows of two outer return-current posts can be seen. As the current in the load was increased, or the diameter was reduced, the magnitude of the magnetic field at the surface of the conductor was increased, and a larger angle between the deflection features was observed as expected.

5.4 Modeling of Experimental Results

In order to better interpret the experimental results, numerical modeling of the results were necessary. This modeling helps to elucidate the physical picture of the deflections, as well as enable quantitative measurements of field magnitudes and topology. As discussed in Chapter 4, the Gorgon and LSP codes were used to this end. Sample deflection trajectory calculations for short-circuit loads are shown in Figure 5.7. For these calculations, Gorgon was first used to simulate the short-circuit loads, as seen in Figure 5.8. Here the dimensions of the hardware match the experimental conditions, including the removal of the return-current posts, as in the experiment. It was then run up to maximum current, with a

Figure 5.7: a) Simulated proton trajectories when probing a short-circuit load from the radial direction, and b) a LSP simulation of proton trajectories when probing a short-circuit load, where only probing of one side of the load is shown, and spatial dimensions are in cm.
specified $\sin^2$ current waveform. The exact waveform is not necessarily important in this calculation, since it is a vacuum calculated magnetic field. This means that there is no real dynamic dependence on the current waveform, and only the current magnitude is important. Thus, currents measured by differential b-dots in the experiment were used to select the corresponding times from the simulations where simulated proton deflectometry was performed, which were typically within $\pm$ 5 ns of the experimental times. Similarly, since the vacuum calculated fields were spatially uniform, small adjustments to the magnetic field data, could be made by including a proportionality constant to adjust the exact current in the load in simulations. After the short-circuit calculations were complete in Gorgon, proton probing was carried out with the Gorgon proton probing post-processor, which calculated the integrated proton paths from the source, through the magnetic field generated by the load, and finally on to a detector. Proton probing in LSP was accomplished by using the Gorgon magnetic field output as 3D magnetic field information for LSP, before probe particles were injected and tracked through the simulation space and eventually being gathered at an extraction plane. Using the same short circuit magnetic field data from the Gorgon modeling as initial conditions for both codes, provided a means of benchmarking two codes against the experimental data, as well as against each other. Since proton probing was carried out in the radial-probing configuration in all cases, the problem is symmetric. In order to minimize the computational time required for a single run, only half of the problem was simulated in LSP. The magnetic field is held static throughout the simulation in both Gorgon particle tracking post-processing and the LSP modeling, which is a reasonable approximation based on the proton integration times, as discussed in section 3.9.6. As a quick approximation it is useful to examine the fastest changing component of the current waveform, seen in equation 5.2.

$$I_{zebra} = I_{max} \sin^2 \left( \frac{\pi}{2} \frac{t}{\tau} \right)$$

(5.1)

Where $I_{max}$ is 600 kA for Zebra, and $\tau$ is the time at which the current is maximum, or 200 ns. The maximum rate of change in I occurs at $\sin^2 \left( \frac{\pi}{4} \right)$, or at $t = \tau/2 = \frac{200}{2} = 100$ ns.
100 ns, thus

$$\left(\frac{I_{\text{zebra}}}{dt}\right)_{\text{max}} = \frac{I_{\text{max}} \pi}{2\tau} \sin^2\left(\frac{\pi}{4}\right) \approx \frac{5}{5} \text{kA}$$

(5.2)

If the longest integration time is considered, or the slowest protons, then there is a total current change of $\sim 10 \text{kA}$. This means that the change in magnetic field magnitude during the $\text{maximum}$ rate of change in the current is

$$\Delta |B| = \frac{B_{\text{probe-start}} - B_{\text{probe-end}}}{B_{\text{probe-start}}} \approx 3\%$$

(5.3)

This change is negligible, and is further minimized as the proton energy increases, and at times away from this maximum. Thus holding the magnetic fields constant during the simulated proton deflectometry is valid.

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**Figure 5.8**: The simulated experimental hardware in Gorgon as viewed from a) the proton beam source, and b) orthogonal to the direction of proton beam propagation. Load diameter is 6 mm, while the 10 return-current posts are 3.175 mm diameter. Magnetic field topology is represented by the colored lines.

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### 5.5 RCF modeling

Experimental proton deflectometry data is of little use if it cannot be reproduced numerically. However, if it can be reproduced, the current and magnetic
field topology may be recovered from the Gorgon simulations. To this end, it was necessary to reproduce the radiochromic film signals gathered from experiment, in the two codes. This not only allowed benchmarking between the experiment and simulation, but also between the two codes. Reproducing RCF from the experiment is a rather complex task in itself, and presented unique challenges in each code.

5.5.1 Synthetic RCF in Gorgon

As discussed in Chapter 4, Gorgon’s synthetic proton diagnostic uses a single energy for the protons in each calculation. This is different than LSP which requires that all beam parameters are set prior to the simulation start, and allows for some more flexibility than allowed by LSP, since each deflectometry calculation for a given energy is independent. This fact allows for a better representation of radiochromic film, since the subsequent output for each energy can be summed in whichever way is specified, e.g. different weighting for different energies based on the Bragg peak calculations for a given layer of RCF. It also allows for the examination of the dependence of a given deflectometry feature on the proton energy. This is because of the ability to specify each discrete energy, which also sets the minimum energy resolution for each bin. An example of this is seen below in Figure

Using the information from the RCF energy bin calculator described in Chapter 3, the energies were linearly summed over the predicted bin. A custom curve matching algorithm performed in Octave was used to find the best match between the experiment and simulation. This was accomplished by first taking an averaged line-out of the experimental RCF data, which is used as the reference curve. Octave then was used to incrementally sum the output data from the individual deflectometry runs in Gorgon, i.e. \( RCF_1 = E_1 + E_2, RCF_2 = E_1 + E_2 + E_3, \) etc. After each summation, a line-out with the same dimensions and location used in the experiment was taken across the resulting synthetic RCF. This curve was then compared against the reference curve until a “best match” was found. After this best fit was found, the information on the energy bin was then used to
define the proton beam for LSP.

**Figure 5.9:** An example of how synthetic RCF images are recovered from Gorgon simulations. The images from each proton energy used in the calculation are progressively added until the best match between experiment and simulation is found. a) Deflectograms with energies 1.2, 1.5, and 1.8 MeV are summed. The signals from each energy are clearly seen, with the higher energies closer to the center of the image. b) Smaller energy increments, 0.02 MeV, are used to reconstruct the same image as in a). The different energy bins are indistinguishable, and thus the resulting synthetic RCF deflectogram is a more accurate representation of the experiment.

### 5.5.2 Synthetic RCF in LSP

As previously mentioned, LSP relies upon the magnetic field information from Gorgon simulations, thus the comparison of the synthetic RCF signals from both codes was used to cross benchmark the codes. LSP can include far more particle physics than the Gorgon proton probing post-processor, however, a stripped down model was used in order to get a one to one comparison. For this reason, LSP was run with the same cell resolution, 200 $\mu$ m cubic cells, as the Gorgon simulations. An image of the simulated proton beam as it passes through the short-circuit load region is displayed in Figure 5.10.

This resolution was found to be adequate for reproducing the experimental results reported here. LSP allows for particle extraction at specified planes within the simulation space. This feature was used to reproduce simulated deflectograms along the proton propagation path, which is seen in Figure 5.11. This helps to elucidate how the initially round beam is affected by the magnetic field produced by the short-circuit load. Reconstruction of synthetic RCF also required its own special math routine, and is outlined here. LSP records the position and velocity
Figure 5.10: An example of a proton beam probing the 6 mm diameter short-circuit carrying a 446 kA current in LSP. a) An XY plane slice of the magnetic field through the short-circuit load. b) The different proton energy components of the beam propagating through the field region, with the highest energies at the front of the beam. The simulation domain has been trimmed in this view for clarity.

of each proton that passes through the specified extraction plane at each time step. The resulting data is a list of the protons which have passed through the plane at a specified simulation time.

The kinetic energy of each proton is first calculated from its velocity, and protons with the desired energy for a layer of synthetic RCF are separated. The size and resolution of the synthetic RCF is then specified, usually 5 cm square with 100 µm resolution. Once this has been defined, the program searches through the positions of the protons to find any which have landed within a pixel, and counts the total number of protons in this position. This is performed sequentially over each pixel in the image, after which the counts are deployed to an array having the specified dimensions of the synthetic RCF. Again, due to the constraints on simulation space, only one half of any particular simulated deflectometry calculation was performed. In order to reproduce a full synthetic film layer, the data was duplicated, flipped horizontally, and placed adjacent to the original half.
Figure 5.11: By simulating the RCF at locations along the proton beam propagation the distortions due to the magnetic field are seen. The image locations are at a) 1, b) 2, c) 3, and d) 4 cm relative to the proton injection plane, which is itself 1 cm from the proton source. The proton energy bin for the synthetic RCF images is similar to the experiment, containing energies between 1.2 and 1.7 MeV, and only one half of the image is simulated for reasons outlined in the text.

5.6 Deflection Dependence on Proton Beam Divergence

The laser produced proton beam has an inherent divergence. The divergence of the proton beam means that different regions of the beam were affected differently, with the initially upward traveling protons experiencing a larger initial deflection, which then contributed to a larger radial deflection than those with an initially downward trajectory. This in turn results in a characteristic angle between the deflection features, which cannot be recovered when no divergence in the beam is included. An example of how this divergence results in the diverging features is shown in Figure 5.12, where a weakly divergent proton beam comprised of many small beamlets passes near a short-circuit load. When a more realistic proton beam based on experimental characterization of the proton beams produced by Leopard is injected, the experimental features are recovered. These calculations also included the experimental hardware, so that even the shadows of the return current posts were present. Injection of a realistic beam in three dimensional simulation space required the development of a custom routine. This calculation approximates the proton source as a point source. Each proton, defined by its own velocity vector is then “warped” to the edge of the simulation space,
Figure 5.12: LSP simulation of proton beamlets, where the full beam divergence has a small opening-angle of 20 degrees, passing near the short-circuit load carrying a 400 kA current as viewed from the source of the protons. a)-d) are increasing times, and the location of the extraction plane is shown for each image on an XY plane in e). The portion of the beam passing nearest to the short-circuit, the left hand side of a)-d), are deflected more strongly than those further from the load.

along with the injection time, which is dependent on the proton energy, with the highest energy protons entering first. This results in a vast savings in computational effort due to a large reduction in simulation space, and is valid because the magnetic field outside the simulation space is too weak to have affect the proton trajectories.

The proton probing post-processor in Gorgon employs a similar technique, however, the custom routine for LSP allows for customization of the beam profile. The most significant difference between the two beam profiles is that it is rectangular in Gorgon, whereas the beam profile in LSP is round, as in the experiment. As will be shown, this is of little consequence when viewing the regions of interest, and results in only minor differences in the details of the deflectograms. For both codes, the same scalings for producing an approximately realistic proton beam were used. This includes a calculation which decreases the full opening-angle of the proton beam from 60° at 1 MeV energy to 20° at 10 MeV. These calculation also included adjustments for the number of protons in each energy bin, the energy resolution of the energy bin, as well as adjustments for the source and detector
positions in the simulations.

Figure 5.13: A comparison of two Gorgon short-circuit proton probing simulations. The only difference between the two simulations is that one beam has a full opening angle of 45 degrees (red) while the other is 60 degrees (green), which are superimposed. The energy of the protons is 1.2 MeV, and the overall shape of the “wings” remains the same, with only the far regions differing.

The angle between the features is primarily dependent on the magnitude of the azimuthal magnetic field and the energy of the protons. This effect was reproduced in both Gorgon and LSP simulations. Over the divergence angle range explored in simulations, typically between 45 to 90 degrees full opening angle, the divergence angle of the beam merely determined the vertical and horizontal extent of the deflection feature, while the angle between the deflection features was constant for the given field strength. An example of this is shown in Figure 5.13. In reality, the true divergence angle of the laser produced proton beams in this experiment is $\sim 60$ degrees full-opening angle at the lowest detected proton energies, $\sim 1.2$ MeV, where the field measurement is the most important factor in determining the angle between the features.

5.7 Comparison with Experimental Results

The results of these benchmark tests may found in Mariscal et al. Applied Physics Letters (2014) [1]. The deflection trajectories were similar in both Gorgon
and LSP simulations, and both similarly reproduced the final beam distribution as observed in the experiment. Lineouts taken at the proton source height from the experiment and simulations, in Figure 5.14 (d), display the detailed profiles in the final proton distribution from the experiment and LSP when the current in the load is 446 kA, where the current in the experiment was measured by a differential B-dot probe embedded near the short-circuit load. Incongruities between the experimental and simulated lineouts are due to the relatively low resolution, 200 µm cell size, used in simulations, as well as variations in the proton beam spatial profile, which are not present in simulations. Gorgon simulations significantly reduce computational cost compared to LSP simulations, and are adequate for reproducing the experiment in this configuration.

Gorgon was then used to examine the predicted lineout peaks for a range of currents at roughly 50 kA intervals for this configuration shown in Figure 3(a). The distance from the center of the RCF to the peak of the lineout profile increases at 0.045 mm/kA, as seen in Figure 3(b), and similarly displays the agreement between simulations and experiment at the two currents probed, 446 and 565 kA, which correlate to magnetic field magnitudes at the surface of the short-circuit conductor of 29.7 and 37.7 T respectively. These configurations show that the minimum magnetic field detectable is 7 T, however, taking line-outs at a location
Figure 5.15: (a) Lineouts from Gorgon-simulated RCF at ∼50 kA intervals showing the increase in distance from the axis with increasing Zebra current. (b) Simulated and experimental RCF lineout peak locations and corresponding maximum magnetic field magnitudes. Figure from Mariscal et al. Applied Physics Letters (2014) [1]

above the proton source height enable measurements below this limit since the inherent beam divergence deflects these protons further from the load. However, this is at the cost of wider peaks in the profile, which can decrease the accuracy of the measurement. Conversely, taking lineouts at locations below the proton source height enables the measurement of larger magnetic fields.

Several small differences were present in the comparison of the experiment and simulation, however, they did not invalidate the predictions. For one the mesh imprint on the beam was not present in these simulations. Another reason for the slight variation in the results between experiment and simulation was that the beam profile generated by Gorgon was square, rather than circular, which makes very little difference in the final result. Simulations with LSP were shown to produce the same results, though at a significantly higher computational cost, leading to the increased reliance on Gorgons post-processor for the calculations. LSP calculations were primarily reserved for the exploration of interaction phenomena to assess the importance of these factors in the calculations, which become important when examining plasma loads.
5.8 Acknowledgements

Chapter 5 contains material that is partially a reprint of the material as it appears in D. Mariscal, C. McGuffey, J. Valenzuela, M. Wei, J. Chittenden, N. Niasse, R. Presura, S. Haque, M. Wallace, A. Arias, A. Covington, H. Sawada, P. Wiewior, and F. Beg, “Measurement of pulsed-power-driven magnetic fields via proton deflectometry,” *Applied Physics Letters*, vol. 105, no. 22, p. 224103, 2014. The dissertation author was the primary investigator and author of the paper, and would like to thank all of the co-authors at UC San Diego and the UN Reno who have given written permission for the use of the material in this dissertation.
Chapter 6

Proton Deflectometry of Scaled Laboratory Astrophysics Experiments

6.1 Introduction

In this chapter, the application of the proton deflectometry diagnostic to a Z-pinch plasma load will be covered. The loads examined in this work, radial foil loads, were designed to replicate the dynamics of astrophysical outflows. The aim of applying deflectometry to such a load is not only important to determining the magnetic field and current topology as a demonstration of the utility of the diagnostic, but also presents a rigorous challenge to the Gorgon code which is used to model these (and many other) loads. The importance of such tests cannot be understated, since codes are typically validated against more easily measured quantities such as density, temperature, and velocity, which does not necessarily extend their applicability for determining the magnetic field and current distributions. These experiments provide data which is directly representative of these distributions. Thus, if the data is reproduced numerically, the applicability of the code to recover the current and magnetic field topology can be taken with significantly greater confidence.
6.2 Challenges of Proton Deflectometry in a Z-Pinch Environment

Applying the proton probing method to plasma systems presents several challenges. For many z-pinch loads, significant ionizing radiation is generated during the current pulse. Toward the end of the current pulse, ballistic debris is produced from the load. Further, on a driver like Zebra, which has a high impedance, very large voltages are present. Each of these can have detrimental effects on any equipment placed inside the load chamber, such as radiochromic film and laser optics, especially if the items are sensitive to any of these sources.

The ionizing radiation often produced in z-pinch loads (X-ray, XUV, gamma) has the potential to initiate ablation on the laser target. As discussed in Chapter 3, this could form a pre-plasma at the rear surface, which is detrimental to the formation of the short scale-length plasma sheath necessary for acceleration of protons to high energies. A significant radiation background may also inhibit the detection of protons in the RCF by inducing a background signal which is large enough to make the proton signal indiscernible. As seen in Figure 6.1, the charged particles and radiation emitted from the current feed of the Zebra driver, during a shot where only Zebra was fired, induced such a large signal on all film layers that signal from proton deflectometry would be obscured. This particular issue necessitated that the film pack be moved further from the load region. Additionally, the ballistic debris produced by most loads is especially harmful to the radiochromic film stack, and is not as easily avoided. An example of the effects of ballistic debris on the RCF stack is shown in Figure 6.2.

Of course, as previously discussed, it is possible to shield the film stack more heavily, giving better immunity to radiation background, as well as mechanical reinforcement against ballistic debris. However, given that there is a finite cutoff energy in the proton beam spectrum, this comes at the cost of a reduced usable proton energy bandwidth, as well as a reduced proton beam flux.

Although the off-axis parabola used for focusing the Leopard laser light is not adversely affected by the radiation emitted by z-pinch loads, the reflecting
Figure 6.1: a)-f) All layers of film in the RCF package had a saturated signal when the film pack was placed near the current return during a Zebra short-circuit shot. Additionally, the first film pack was mechanically damaged, splitting the film into two pieces.

surface is sensitive to debris. The parabolas used in this work were originally coated with Au, giving a high $\geq 97\%$ reflectance. Even with significant shielding between the load and the parabola, it was often coated in whichever material was used in the load, as the load material was vaporized during Zebra shots, resulting in an unknown reflectance and surface roughness. An example of this is seen in Figure 6.3, when a single wire load was used during a Zebra only shot. Attempts to reuse load-coated parabolas showed a decreased ability to focus the laser light to small diameter focal spots, and resulted in a decrease in proton beam energy and flux.
Figure 6.2: a) The first layer of the RCF package, which was shielded with 16 μm aluminum, from a single wire Zebra shot, with the approximate position of the main ballistic debris impact shown. b) The first two layers of the same RCF stack, where mechanical damage is easily visible on the first two layers of film. The deflectometry signal is only barely discernible due to the difference in its angle relative to the blast streaks.

Figure 6.3: The off-axis parabola used for focusing the laser is shown both a) pre-shot and b) post-shot. The Zebra load was a single 1 mm diameter stainless steel wire, and the parabola is coated in this material post-shot.
6.3 Deflectometry of Radial Foil Systems

The radial foil load was chosen due to the interest in determining the effect of current and magnetic fields on the collimation of the jet produced by this load. These loads, as previously discussed in Chapter 1, have been extensively studied for their exotic properties, complex magnetic field structures, and their similarities to astrophysical outflows, especially jets produced in the magnetic tower model. Compared to many other load choices, this also happened to be an ideal candidate for proton deflectometry, since it inherently aids in overcoming the previously mentioned challenges. The primary benefit of using such a system was that the debris was mostly directed in the vertical direction, keeping the RCF detector and the OAP clear of danger. To further benefit the success of the proton deflectometry data, low Z, aluminum, relatively thick, 16 and 12.5 µm, foils were used. These foil parameters were found to induce only a minor radiation background on the RCF stack, which when present, was readily identifiable, and distinct from the signal due to protons. Using these loads, no additional filtering of the RCF stack was necessary, other than the usual 16 µm aluminum foil used in short-circuit proton deflectometry tests.

![Figure 6.4](image)

**Figure 6.4:** a) Plasma is ablated above the surface of the radial foil. The gradient in density is responsible for the formation of a hydrodynamic jet. b) If the mass becomes depleted near the cathode, a magnetic bubble, or episode will begin to form. c) The current path in the magnetic cavity causes pinching of the jet in this region, driving it unstable, until the current re-strikes near the cathode and another episode begins. Figure from Suzuki-Vidal Physics of Plasmas 2010 [61]

As previously discussed in Chapter 1, the general dynamics of these systems
can be qualitatively illustrated with an examination of the JxB force present in
the system. Figure 6.4 displays a schematic of the stages of radial foil evolution.
A circular foil is stretched across concentric electrodes, while the central cathode
supports the middle of the foil. Current flows radially through the foil and down
through the cathode, where a large magnetic field is created around the electrode
just below the surface. This directs the JxB force in the upward z-direction.
The foil begins to ablate, and the rate of mass ablation is proportional to the
current density, which increases as the radius decreases, creating a radial gradient
in the density above the foil. Just above the electrode, little/no current flows, so
no ablation occurs here. This creates a void in the ablated plasma distribution
above the foil, which in turn sets up a density gradient which is responsible for
the acceleration of a hydrodynamically collimated jet in the center of this halo
plasma, as seen in Figure 6.4. As more plasma is ablated, the region where current
is initially confined to flow begins to move further away in the upward z direction.

Due to the thick foils used in this work, it is expected that multiple episodic
ejections will not be formed, as in the case discussed in Chapter 1. Rather, by
using both thicker foils, and the long-rise, lower-peak-current, Zebra pulse, the
radial foil evolution should be similar to the reported episodic foil experiments,
with a significantly slower time evolution. Previous experiments on MAGPIE, 1
MA in 240 ns, showed that at least one magnetic “episode”, or magnetic cavity, is
formed with the use of thicker, 15 µm, Al foils [61]. Thus, it was expected that the
deflectometry would be performed in a region of evolution similar to a) and b) in
Figure 6.4. In effect, rather than forming multiple episodes as in the experiments
discussed in Chapter 1, the formation of the halo plasma and hydrodynamic jet is
expected, while the formation of a single magnetic cavity during the course of the
current drive may be possible.

6.3.1 Experimental Setup

The experimental setup is shown in Figure 6.5, which was similar to the
radial proton probing configuration used in the short-circuit loads. A 12.5 µm thick
Al foil was stretched across a 38 mm diameter outer cylindrical electrode, while
Figure 6.5: a) The setup for proton probing of radial foil loads, including the approximate positions of the target, mesh, RCF detector, and foil load. The proton beam direction and laser probing direction are also shown for reference. b) A cutaway of the load hardware showing the position of the 3 mm diameter cathode contacting the bottom surface of the foil. c) The experimental setup as viewed from the top of the Zebra load chamber, and d) a side-on view of the load.

The diameter of the foil which was subjected to the current pulse was 25 mm, i.e. the inner radius of the outer cylindrical electrode was 25 mm. A 3 mm diameter cathode contacts the bottom surface, on the central axis. As with the short-circuit loads, a Cu mesh was placed ≈ 2 mm behind the target, so that it was imprinted on the proton beam profile, making distortions due to the encountered magnetic fields more apparent. In these experiments, a thicker copper mesh, 127 µm, was placed in the path of the proton beam in order to improve grid contrast at higher energies compared to the short-circuit case. This can be seen in a Leopard only shot in Figure 6.9. The distance between the target and the axis of the load was 3.2 cm, while the distance between the RCF detector and the axis of the load was 2.8 cm.

The target height was 1.5 cm above the surface of the foil. Given the distance between the target and RCF, and the divergence of the proton beam, the target height was chosen to maximize the amount of information gathered from each shot. Assuming a 30° proton beam half-opening angle, the proton beam would be nearly 3 cm in diameter at the load axis, thereby maximizing the beam area at the axis of the load. However, the beam is expanding as it approaches the axis of the load, and so does not probe close to the surface of the foil during the approach to the axis of the load.

The vertical extent of the beam was also important, but was deemed to
be of less importance for the following argument, which is seen schematically in Figure 6.6. Here, it is seen that a proton in the region before the cathode, relative to the -r proton approach direction, would be deflected downward toward the foil if significant magnetic field exists above the surface, and therefore would likely not contribute to any signal collected at the RCF, making proton flux in this region essentially unnecessary. The proton beam does, however, reach the surface of the foil before the cathode region at the largest beam-divergence angles. Due to the current flowing anti-parallel with respect to the beam incidence past the cathode region, the protons are deflected in the upward direction where it can be collected by the detector, meaning that the usage of the beam as a probe is maximized.

![Figure 6.6](image)

**Figure 6.6**: a) A r-z slice through the experimental hardware, showing the direction that a proton is accelerated toward during its approach toward the cathode. b) On the other side of the cathode, the direction of current, and thus magnetic field are flipped, so that proton trajectories are deflected vertically upward.

The film packs were also sized in order to maximize the amount of data collected during each shot, while taking into consideration the practical size limitations set by the size of the film sheets as delivered, 8" x 10". Again assuming that the beam has a 30° half-opening angle, the beam should be approximately 6 cm at the RCF detector, with no deflection. Thus, the RCF film size was 2.5" x 5", or 64 mm x 127 mm for deflectometry shots.

The film pack and holder are seen in Figure 6.7, where the holder was used to maintain a consistent position of the film while also aiding in keeping the film layers flat. The two wire fiducials seen on the surface of the film pack, were used during every shot. These fiducials are used to align films for analysis. Lastly, as with the short-circuit loads, Zebra was operated in long-pulse mode,
Figure 6.7: The RCF film pack within the holder as set up for deflectometry shots. This particular filtering wrapping the film pack was used for Leopard-only proton production and characterization and had two different types of filtering. The different filtering on each side shifts the minimum energy of protons which can reach each film layer in the stack. The resulting proton signal is shown in Figure 6.9.

delivering a maximum current of 0.6 MA in approximately 200 ns. This further aided in maintaining a relatively quiescent environment, conducive to making the first successful magnetic field measurements in z-pinch plasma loads via proton deflectometry.

6.3.2 Load Background and Proton Flux Assessment

Prior to attempting proton deflectometry shots on radial foil loads, the background levels on the RCF were assessed. This was accomplished by taking radial foil shots with everything in place as if a coupled shot was to be taken, i.e. the RCF and laser target were in place. It is worth noting that energetic protons have been observed to be ejected from magnetic-tower jets in experiments [182], however, this is unlikely to be the source of the observed proton signal in these experiments due to the lack of observed signal on RCF stacks during Zebra-only shots. Radial foil shots taken without the Leopard laser, and thus without proton production, showed a small, but discernible background, seen in Figure 6.8. The uniformity of this background suggests that it is likely due to X-rays from the load.

In order to test the ability to detect the proton signal at this distance from the target, i.e. at a lower flux, the same setup was used during laser-only shots.
Figure 6.8: A sample of the typical background produced on RCF stack films during a Zebra only 16µm radial foil shot. The background is visible on some deflectometry shots, and is typically more apparent in the more sensitive film layers, EBT3 type, which are deeper in the film stack.

Results from a test shot using a special set of film pack filters is shown in Figure 6.9, which shows that a usable proton signal is available up to energies $\sim 10$ MeV.

The signals on these films are then compared against each other by taking a lineout across the first film in the stack at similar locations. This comparison is shown in Figure 6.10, where it is seen that the proton flux is above the background level produced by the radial foil load for at least half of the film. Further, the background produced by the radial foil loads is seen to be quite uniform, therefore, the proton signal should still be detectable in the regions where it is lower than the background.
Figure 6.9: RCF signals from a Leopard only shot with the RCF at the same location as during the deflection shots. Half of the film pack is filtered differently as seen in Figure 6.7., showing different energy components, i.e. the minimum proton to reach each side is different in each side. The minimum proton energy to reach each side of each layer of film is labeled for reference.

Figure 6.10: (a) Location of lineouts across similar regions of RCF for Leopard only and Zebra shots. The filtering for the film packs is the same for both lineouts, e.g. 16 µm aluminum foil. (b) Although not the brightest region of the proton signal, the proton signal is discernible from the uniform background produced by the radial foil load.
6.3.3 General Experimental Deflectometry Results

![Figure 6.11](image)

**Figure 6.11**: RCF signals from deflectometry of a 12.5 µm radial aluminum foil load. The film pack is comprised of, A, one layer of HD film, followed by, B-D, 3 EBT3 films.

Experimental proton deflectometry of 12.5 µm thick radial aluminum foil loads yielded rather unexpected results. The deflection pattern was primarily a dome shape, as seen in Figure 6.11. In addition to this, significant “flaring” features were observed just above and surrounding the dome shape. The reason for the dome shape is fairly easy to visualize using the simplified schematic shown in Figure 6.6. Protons are deflected initially downward, and thus away from the detector, as they approach the cathode. This action serves to clear the region below the dome shape of protons. Conversely, beyond the cathode protons are deflected upward. On the lateral sides of the cathode, with respect to the direction of the beam, protons see a degraded component of the upward accelerating magnetic field. Thus, they are decreasingly deflected as a function of increasing lateral distance from the cathode, which results in a semicircle, or dome, deflection pattern.

Although much of the original grid pattern is difficult to discern, there are areas within the RCF signals which are clearly distorted mesh patterns as seen in Figure 6.12, meaning that the signals are due to protons, and not from radial-foil accelerated particles or X-rays. The presence of the plasma may lead to collisional
Figure 6.12: A magnified portion of the deflectometry images shows the distorted grid pattern within the RCF signals, meaning that the signal is due to charged particle deflections, and not X-rays.

blurring, which can affect the grid contrast as the beam passes through the long scale-length plasma and will be discussed further in Section 6.5.

The increased momentum of higher energy protons means that they will be deflected less strongly than the lower energy components of the proton beam. This was observed in the experiments as the deflections were observed to decrease, i.e. the features were more localized, as a function of increasing film depth in the detector, i.e. as the proton energy increased. An example of this is seen in Figure 6.13, where two layers in the same RCF film stack are compared. The central signal on the RCF is likely a blurred proton signal, which will be neglected for the moment, but will be discussed in Section 6.7.
Figure 6.13: The energy dependence of the deflected protons is seen when two layers of RCF from the same shot are superimposed. Films A and C are those from Figure 6.11. Film A is represented in turquoise and the signal from film C in magenta. The lower energy particles detected in Film A are deflected further from the high field region than the higher energy particles in Film B.

6.3.4 Evolution of Radial Foil Systems

A time sequence of deflectometry results is shown in Figure 6.14, where the resulting RCF images are representative of the evolution of the current topology within the radial foil system. Early on, at 56 ns before peak Zebra current, a smooth dome shape is observed. This dome shape is due to particles being deflected both upward and outward by the magnetic field above the surface of the foil. The upper portion of the beam, however, remains relatively unperturbed, thus the dome shape is superimposed on this undeflected portion of the beam. At this time, the shadowgraphy shows the expansion of the foil near the cathode, with no evidence of jet formation. While the dome feature increases in size, the signal begins to weaken at the apex of the dome feature by 36 ns after peak Zebra current. The shape of this feature distorts to complex shapes later in time, with the signal nearly disappearing at the apex.
Figure 6.14: A time sequence of deflectometry results with (left) the signal from the third layer in the RCF stack, corresponding to a minimum proton energy of $\sim 7.1$ MeV and (right) the simultaneous laser shadowgrams of the cathode region.
The laser shadowgraphy shows the continued expansion of the foil near the cathode, as well as evidence of a jet feature emerging. As in the short-circuit case, it may then be expected that the dome feature is diverging due to current flowing vertically along the jet.

![Proton Deflectometry vs Interferometry]

**Figure 6.15**: Comparison between deflectometry and interferometry of radial foil loads at -56 and +36 ns relative to peak Zebra current.

For a more detailed comparison, the interferometry data is unfolded to give a 2D map of the areal electron density, as seen in a comparison between two times in Figure 6.15. This comparison confirms the shadowgraphy data, showing that the jet is only just beginning to emerge from the cathode region at 56 ns before peak Zebra current, and is well formed by 36 ns after peak current. Again, the distorted mesh pattern visible in the deflectometry means that these features are not due to X-ray or particle emission from the load itself.

### 6.4 Modeling of Radial Foil Loads

As previously mentioned, modeling of the experimental results is crucial in order to interpret the data. The primary tools used, Gorgon and LSP, proved to
be reliable for short-circuit data, but plasma loads serve to provide a rigorous test for Gorgon. Again, the Zebra load was modeled with Gorgon, while both codes were used to examine the problem of proton deflectometry.

6.4.1 Radial Foil Loads in Gorgon

For the modeling of the radial foil load in Gorgon, a new method of setting the initial mass distribution was used. As covered in Chapter 4, the mass for wire arrays is set by placing the majority of the wire mass in one or several cells, and distributing the remaining mass in radially adjacent cells. To better model the mass distribution of a foil, however, the mass contained in the surface area of one computational cell is now distributed between two vertically adjacent vertical cells. Additionally, a small percentage of the mass in these cells is used to smooth out any “stepping” in the mass distribution, leading to a better representation of the foil, and avoiding complications which may arise due to sharp jumps in density.

As with the short-circuit loads, the current drive is modeled as an idealized pulse with sin$^2$ waveform peaking at a maximum current of 600 kA, 200 ns after current start. The cells were cubic, and 100 $\mu$m in length per edge, run on a 400x400x400 grid. One immediately obvious fact is that the foil thickness cannot be resolved. The inability to resolve the foil thickness in this work is important because of the effect is has on the magnetic field diffusion. For a static plasma, the magnetic diffusion time scales as $L^2$. Thus, starting with a cell size of 100 $\mu$m, and the foil thickness of 12.5 $\mu$m, and assuming constant parameters, except for the scale length, it is seen that the field diffusion time is greatly affected. It is estimated that the true magnetic field diffusion time is $\sim 1/256$ that of the diffusion time for this cell size. Further, the initial mass distribution is spread over at least two computational cells, and thus must diffuse through at least 200 $\mu$m. This in turn contributes to a decrease in the field diffusion/current transport far from the foil.

An example of this can be seen in Figure 6.16, where two simulations of the radial foil load were run. One was run with 200 $\mu$m cell size, while the other was run with 100 $\mu$m cell size. Line-outs of the magnetic field magnitude were taken at
Figure 6.16: Two r-z slices through the center of Gorgon simulations of radial foil loads with 12.5 μm aluminum foils. The two calculations are run with a cell size of a) 200 μm, and b) 100 μm. The approximate line-out locations are shown for c)-e). There is \( \sim 10\% \) greater magnetic field magnitude for the 100 μm cell size compared to the 200 μm simulation.
a height of 3, 7, and 9 mm above the foil for both cases. The field magnitude at this location is seen to be larger by $\sim 10\%$ when the cell resolution is halved. Due to the limited availability of computational resources, it was not possible to examine at what cell resolution this solution converged, however, simulations on a suitably sized machine could be used to determine the importance of this parameter. This difference in field magnitude was found to have no significant impact on the proton deflectometry results, and the impact of the field magnitude is discussed further in section 6.6.2.

Figure 6.17: A time sequence of the sliced mass density from a simulated 12.5 $\mu$m Al radial foil load. The formation of the jet is clearly seen, as well as the evolution of the acceleration region just above the cathode.

With this limitation know, simulations were run with the practical cell size of 100 $\mu$m. One such simulation of a 12.5 $\mu$m thick radial foil load is shown in Figure 6.17. In the figure, the mass density is sliced through the diameter of the radial foil load. The electrodes are not shown for clarity, but are discernible from sharp boundaries where plasma is not present. The general evolution of the radial foils is seen to be in rough agreement with data from laser diagnostics during the experiments, with the formation of a high density region, or jet, clearly visible by peak current, or 0 ns. Of course, the region which is of particular interest is the region near the cathode, where no clear magnetic bubble has been formed by 200 ns after peak current. More specifically, the current and magnetic field distribution
above and near this region was of importance to the deflectometry results, and will be discussed further below.

**Areal Electron Density Comparison**

The only other available diagnostics for proton probing were laser shadowgraphy and interferometry. Thus, a comparison of interferometry results are a reasonable parameter to begin with when checking the validity of the code results. A comparison between the density profiles at \(\sim56\) ns before peak Zebra current is seen in Figure 6.18. Here, both experiment and simulation show the jet as a high density region with a width \(\sim1\) mm. The simulated profile is slightly more collimated, and higher density, while the experimental lineout shows a slightly asymmetric profile. In general, however, Gorgon appears to replicate the density data quite well.

![Figure 6.18: A comparison of experimental and simulated areal electron density.](image)
a) The experimental interferometry unfold at 56 ns, and simulated line integrated electron density at 55 ns, before peak Zebra current. The maximum and minimum values for areal electron density are the same for both images. b) Lineouts across each image at 3.8 mm above the original foil position are plotted as a function of distance.
6.5 Collisional Energy Loss and Blurring in a Plasma

As previously discussed, significant plasma density may impact the results of the proton deflectometry. To this end, several methods were used to examine the importance of the plasma density on the final RCF signal.

6.5.1 Theory of Proton Energy Loss in a Plasma

Although the higher energy component of the proton spectrum has a sufficiently long mean free path that stopping can largely be negleted, with these systems, a significant amount of mass is ablated, which produces a long scale-length plasma which is capable of stopping the lowest energy component of the proton beam. For example, the energy loss of protons in plasmas can be estimated using the following general formula [183–186]:

$$-\frac{dE}{dx} = \left(\frac{\omega_p e}{v_p}\right)^2 \left[ G\left(\frac{v_p^2}{v_{th}^2}\right) \ln \Lambda + H\left(\frac{v_p}{v_{th}}\right) \right]$$  \hspace{1cm} (6.1)

Here, $\omega_p$ is the electron plasma frequency, $e$ is the electron charge, $m_e$ is the electron mass, $v_p$ is the velocity of the proton, $v_{th} = \sqrt{2T/m_e}$ is the thermal velocity of the electrons in the plasma, and $\ln \Lambda$ is the Coulomb logarithm. Last, $G(x) = erf\left(x^{1/2}\right) - 2\left(\frac{x}{\pi}\right)^{1/2}e^{-x}$ is the Chandrasekhar function, where large-angle scattering terms are neglected [183,187] and $H$ is a function which varies among the different stopping models in the literature, but generally is not important unless $v_p < v_{th}$, which is not the case here, and thus is taken to be $\sim 0$. Using appropriate values for the plasma of interest and an initial proton energy of 7 MeV, a conservative estimate of the energy loss is $dE/dx \sim 0.5$ MeV/cm. For the radial foils systems, where the plasma scale length, at least of significant density, is $\sim 1$ cm, this corresponds to an overall energy loss of $\sim 1$ MeV. These types of losses are evident in the first layer of RCF, where the energy loss is greater, and the minimum detectable energy was typically $\sim 1.2$ MeV. For the higher energy protons, corresponding to the deeper layers of RCF, stopping is much less of a concern since energy losses become less than the energy windows for the RCF layers, however the
mesh imprinted on the beam was degraded significantly, indicating that collisional blurring of the proton beam was significant.

### 6.5.2 Effect of Proton Losses on RCF Signals

**Figure 6.19**: a) $\rho R$ calculated by integrating through the mass density profile from a Gorgon simulation at 100 ns after peak Zebra current. Each of the values in this image is then compared against b) CSDA range data for aluminum from NIST pstar in order to find c) the minimum energy of a proton which would be required to penetrate the integrated density value.

Another way to examine the impact of density on proton transport is to use a combination of Gorgon calculations and stopping power data from NIST pstar. This approximation is done by taking line integrated mass density profiles, or rhoR, from Gorgon, since we have reasonable agreement with the experiment in terms of line integrated electron density. Each value on this image is then compared against data from NIST pstar using the Continuous Slowing Down Approximation range in order to calculate the minimum energy of a proton which could penetrate this density, as shown in Figure 6.19. Of course, the real proton trajectories are not all aligned with respect to the direction of integration, so these numbers are a lower estimate. Using this approximation, it can be seen that proton losses due to plasma density should not be significant enough to reproduce the experimentally
observed deflections for even the lowest energy protons.

6.5.3 SRIM Calculations for Examining Density Effects

The TRIM code [150] was also used to calculate collisional stopping of protons in the plasma in order to further examine the effects of plasma density on the deflectometry results. For these calculations, an ion species, protons, is selected, as well as an atomic, or target, species. To approximate the radial foil plasma, mass density profiles from Gorgon simulations at +100 ns relative to peak current, which are in reasonable agreement with experimental laser interferometry, were used to set up an approximate target in TRIM. This is taken as an upper limit to blurring effects in the experiments, since this is the latest time probed, and the mass density profile above the foil increases with time. There are then five regions consisting of the halo, jet boundary, jet, jet boundary, and finally the halo again. The density is assumed to be constant within each of these regions at $1 \times 10^{-4}$ g/cc for the halo, $1 \times 10^{-3}$ g/cc for the jet boundary, and $1 \times 10^{-2}$ g/cc for the jet. The width of the regions are 10 mm, 2 mm, and 1 mm, for the halo, jet boundary, and jet, respectively. Again, this is only an approximation of the actual mass density, and even more so, since the target layers are planar with respect to the incident protons.

The transport calculations for three proton energies are shown in Figure 6.20. These calculations show that the lowest energy protons detected, 1.2 MeV, are likely significantly affected by the highest density region, the jet. In fact, for this density profile, TRIM predicts that although nearly all protons with an initial kinetic energy of 1.2 MeV will escape the plasma, the average proton energy after passing through this density is $\sim 0.9$ MeV. This means that these protons are unlikely to make it to the first layer of RCF. Instead, signals on this RCF are either from protons which have not passed through this region of significant density, or protons which were initially of higher energy which were down-shifted as they traversed the plasma region. These same calculations also predict that the beam is blurred by $\sim 300 \ \mu m$ over 25 mm for this energy, or $\sim 12$ mrad. Together, these two factors account for the typically low and blurry signal in the first layer.
Figure 6.20: Example TRIM calculations for three different proton energies, through a mass density profile similar to the experiment at +100 ns after peak current. The longitudinal and transverse distributions are shown for 1.2 MeV, 4.4 MeV, and 10 MeV protons. The collisional blurring is most significant for the lower energy protons.

Of course, the collisional effects due to the plasma density are also higher than predicted here, since the TRIM calculation does not include the effect of increased temperature. It is clear that the plasma density has an impact on the deflectometry results, and at lower energies it is more appropriate to use the term proton radiography because of this.

6.6 LSP Modeling

The results in section 6.8 prompted the use of LSP to verify the lack of significant proton deflection for radial foil loads. It was first used to check the previous result, which was confirmed. It was then used to examine the effect of several other parameters which could cause the disagreement between experiment and simulation. The possible issue with Gorgon being unable to properly calculated the field and current transport was examined. An electric field was included in
the calculations as another potential explanation. Lastly, the electrons which are always injected along with the proton beam in order to keep the beam initially charge neutral were included in the synthetic RCF to assess whether this could be a potential cause for the discrepancy. Each issue is covered in detail below.

6.6.1 Deflectometry of Radial Foil Loads in LSP

As previously covered, the magnetic field from Gorgon is used as input for LSP in order to perform deflectometry calculations. When the same magnetic field data from the upper-limiting case shown in Figure 6.32, no significant changes were found. The results of this calculation are seen in Figure 6.21. In this simulation, the proton beam energy bin is 1.2-1.9 MeV. This was used since it should be representative of the lower detection limit of the diagnostic, while the probing time is again at 100 ns after peak Zebra current. Again, only a slight distortion to the beam profile near the bottom edge of the synthetic RCF is observed, confirming that the field topology predicted by Gorgon is too weak to have a significant impact on proton trajectories, even at the lower proton energies.

Figure 6.21: Synthetic RCF signal from LSP-simulated proton deflectometry of a radial foil load at 100 ns after peak Zebra current. Differing from Figure 6.32, the beam is comprised of an energy bin of 1.2-1.9 MeV, and only a slight distortion to the beam profile can be seen at the lower portion of the image.
6.6.2 Artificial Enhancement of Magnetic Field Transport

A simple, though not physical, method of examining the dependence of the deflectometry signals on the magnetic field magnitude above the foil is by artificially enhancing this field in simulations. As discussed in the previous chapter, since the magnetic field data in LSP simulations is imported from Gorgon, the magnitude of the field may be adjusted by simply multiplying by a specified constant.

This of course does not account for the change in load dynamics if this were the case, and is only intended to explore the effect of a stronger magnetic field above the surface of the radial foil load. Three cases were examined; the original magnetic field magnitude, double the magnetic field magnitude, and ten times the magnetic field magnitude, where an order of magnitude increase in magnetic field is similar to the case when a magnetic episode has formed, since this would correspond to nearly all of the load current flowing above the surface of the foil. The results of these calculations for a 4.4-5 MeV proton beam can be seen in Figure 6.22. Although none of the simulations show particularly good agreement with the experiment, they do show that around $10 \times$ the magnetic field magnitude must be present above the foil in order to reproduce a similar horizontal size of the semicircle deflection feature, while the vertical extent of the deflection is still significantly lower than observed in the experiments. Further, if the field were enhanced by this factor, it is expected that the dynamics of the system would be significantly altered. Again, the only way to properly determine whether this is possible would be to run Gorgon simulations at adequate resolution to determine how much the current and field transport is affected by smaller computational cell size.
6.6.3 Effect of an Electric Field

In order to include electric field data in LSP calculations the electric field due to the z-pinch load was approximated from Gorgon data. Since Gorgon does not output electric field data directly, this was accomplished by first using 3D magnetic field data to determine the current density in the system via Ampere’s Law in steady state. e.g. $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$. The simple Ohm’s law, $\mathbf{E} = \eta \mathbf{J}$ was then used in combination with the Spitzer resistivity to work out an approximate magnitude for the electric field. The Spitzer resistivity model is based on electron-ion collisions and is [188]

$$\eta_{\text{Spitzer}} = \frac{\pi Z e^2 m_e^{1/2} \ln \Lambda}{16 \pi^2 \epsilon_o^2 (k_B T_e)^{3/2}}$$  \hspace{1cm} (6.2)$$

where $Z$ is the ion charge, $e$ is the electron charge, $m_e$ is the electron mass, $\ln \Lambda$ is the Coulomb logarithm, $\epsilon_o$ is the vacuum permittivity, $k_B$ is the Boltzmann constant, and $T_e$ is the electron temperature. In order to calculate the electric field, the resistivity was calculated at each point using the electron temperature given by Gorgon, while $Z$ was taken to be 3, and a value of 10 was taken for the Coulomb logarithm. This rather large value for the Coulomb Logarithm was used in order to obtain an upper estimate of the electric field magnitude so that the maximum contribution of electric fields to the deflection trajectories could be examined. Once this data was calculated, it was imported into LSP for simulations, but was found to have very little effect on the deflectometry results, as expected. Of course, this
is to be taken only as a reasonable estimate the electric field magnitude.

![LSP with E and B fields](image)

**Figure 6.23**: Synthetic RCF signal from LSP-simulated proton deflectometry of a radial foil load where an approximate electric field has been included. The inclusion of the electric field slightly modifies the structure of the beam profile compared to the case when only the magnetic field is present.

### 6.6.4 Inclusion of Electrons in RCF Analysis

The electrons which are at least initially co-moving with the proton beam have a low kinetic energy compared to the fast electrons which may be generated by the interaction of the laser pulse with the target. The kinetic energy of an electron moving with the same velocity of a 10 MeV proton corresponds is $\sim 5.5$ keV, and $\sim 540$ eV at the velocity of a 1 MeV proton. As seen in Figure 6.24, these electrons can penetrate the 16 $\mu$m Al foil used to shield the film packs, however, the low stopping power of the electrons in the films should mean that the signals due to these electrons is low. Further, LSP simulations have shown that even the relatively small magnetic field magnitudes predicted by Gorgon, $\sim 7$ T, quickly remove the majority of co-moving electrons from the proton beam, and direct them away from the RCF detector.

Figure 6.25 shows the protons and electrons as they propagate through the magnetic field produced by the radial foil load at 1.7 ns after they are injected. The proton energy spans 1.2-1.9 MeV, while the electron energy spans 0.653-1.04
Figure 6.24: Electron CSDA ranges for several materials from NIST estar. (a) The CSDA range predicted for Cu, Al, and Kapton, the three most commonly used filters. (b) The CSDA range for the same materials, but energies closer to those used in simulations for the co-moving electrons.

Figure 6.25: An equal number of electrons with the same velocity are injected with the protons in order to keep the beam charge neutral at the time of injection. (a) A 3D view of an LSP simulation showing the electron and proton positions at 1.7 ns after injection. (b) The YZ view, or the view from the injection plane, and (c) the YX, or overhead view, of the particles at the same time as the 3D view.
keV. The electron trajectories are much more affected by the magnetic field than the protons. Although the distribution seen in Figure 6.25 (b) resembles the distribution in the experiment, the majority of the electrons are actually directed out of the simulation space, away from the detector. Further, the energy of these electrons is too low to penetrate the 16 um Al filter in front of the film stack. This does, however, mean that higher energy electrons could be responsible for the deflection features observed in the experiments.

6.7 High Energy Electrons

As previously mentioned, the appearance of the central signal is most likely due to protons, since it falls off quickly with increasing film depth. Again, X-rays generated by the load or the interaction of the laser with the target may also be ruled out as the origin, since these sources are not collimated, and thus should produce a rather uniform background on the RCF. Further, X-rays with sufficient energy to penetrate to these layers of film have a low stopping power, and thus the signal should not drop off quickly with increasing film depth, as the central signal does. Experiments with radial wire arrays have shown that ions are accelerated out of the load [182], it is highly unlikely that this signal is due to the load, however, since the background signal observed on the RCF was quite low and uniform when only Zebra was fired. Further, the radial wire arrays are more similar to thin foil loads which produce magnetic episodes, and therefore strong pinches which are capable of accelerating ions to energies that would be required to be detected in these experiments.

Another possibility then, is that the deflected signals are actually due to high energy electrons, rather than protons. Hot electrons are produced during high-intensity laser-matter interactions, and in fact are a prerequisite for accelerating the proton beam in the first place. In this case, due to the opposite charge, electrons would be deflected up and out of the field region before the cathode as shown in Figure 6.26.
Figure 6.26: A simple schematic showing how protons would be deflected by the magnetic field of the radial foil load above the surface of the foil. This schematic shows that most of the upward deflection of electrons would occur in this region.

6.7.1 High Energy Electron Measurements

The electron spectrum was not measured during these experiments, but may be estimated to be similar to the data with thicker Cu targets shown in Figure 6.27. In these characterization experiments, electrons up to $\sim 5$ MeV were recorded with an electron spectrometer, also seen in Figure 6.27.

Further, recent data using Leopard interacting with $2 \mu$m Ti foils has shown that the hot electron slope temperature is $\sim 1.2-1.9$ MeV. An example of this data is shown in Figure 6.28, where data from an electron spectrometer shows a good fit with a hot electron spectrum having $T_{\text{hot}} = 1.3$ MeV.
Figure 6.27: a) An electron spectrometer signal for a 10 µm Cu foil driven by the Leopard laser. b) Line-outs of the electron spectrometer signal for four different target thicknesses showing maximum electron energies of ∼5 MeV, with the highest numbers of the distribution ∼2 MeV. Figure from H. Sawada private communication (2015) [141]

Figure 6.28: An electron spectrometer signal for a 2 µm Ti foil driven by the Leopard laser. Figure from H. Sawada private communication (2015) [141]
6.7.2 RCF Response to Electrons

One of the types of film used, EBT3, is known to be sensitive to high energy electrons. The preceding version, EBT2, has been shown to be sensitive to electron doses [189]. These two film types differ only in the thickness of the plastic and active layers, while the organic polymer which is sensitive to ionizing radiation remains the same. More recent results also show that EBT3 is also sensitive to electrons with 6 MeV energy [190], as shown in Figure 6.29 a).

The continuous slowing down approximated range for electrons in polyethylene is shown in Figure 6.29 b). The thick polyethylene layers sandwiching the active layers is the most significant component of the film when considering stopping powers. This calculations shows that electrons with energies above ∼250 keV are able to penetrate through all layers of EBT3 film. The first layer of film, HDV2, is also included in this calculation, but not plotted for clarity. In combination with the previously shown CSDA range for aluminum, Figure 6.24, it is reasonable to expect that the high energy electrons produced during the laser-target interaction could contribute to the signal on the RCF.

Figure 6.29: a) A dose response curve for 6 MeV electrons detected by EBT3 film from Sorriaux et al. Medica Physica (2013) [190]. b) Electron CSDA range in polyethylene, the most significant component of the film composition for stopping power calculations.
6.8 Proton Deflectometry Comparison

Prior to simulating the deflectometry the magnetic field magnitude was extracted from Gorgon by taking radial lineouts of the magnetic field magnitude from a simulation at 50 ns intervals, shown in Figure 6.30 c). The magnetic field profile peaks at $\sim 7$ T at 100 ns after peak Zebra current, after which it begins to decrease. The magnetic field magnitude decreases exponentially with increasing lineout height, as is shown in Figure 6.31. In fact, the fall off is well approximated with a single exponential of the form $B(z) = B_o e^{Sx}$. For example, $B_o$ is $\sim 61$ T, and $S = -1/6$ when the radial location of the lineout is 5 mm. This indicates that any significant proton deflections should occur near the region proximal to the cathode and near the foil surface.

![Image of mass density and magnetic field magnitude](image)

**Figure 6.30**: a) A r-z slice through the mass density at 100 ns after peak Zebra current showing no evidence of magnetic episode formation. b) A r-z slice through the magnetic field magnitude with lines outlining the current path and magnitude at 100 ns after peak Zebra current. The large dashed line shows the location of the lineouts in c). c) The radial magnetic field profile at 5 mm above the original foil height at several times throughout the current drive.

Deflectometry was then tested at several different times throughout the current drive. The result of one such calculation shows the deflectometry result at 100 ns after peak Zebra current, when Gorgon indicates that the magnetic field distribution above the foil is highest. For the energies used in the calculation,
4.7-5.7 MeV, no significant deflection of the proton beam was found. Rather, only a slight distortion to the beam profile is observed near the bottom of the synthetic RCF, as shown in Figure 6.32.

![Figure 6.31](image)

**Figure 6.31:** a) Vertical lineouts of the magnetic field magnitude at several radii. b) The vertical lineout of magnetic field magnitude at a radius of 5 mm from the axis of the load is plotted along with an exponential fit.

This should be expected from the calibration tests of the diagnostic, where Gorgon predicted that the minimum detectable field with lower energy protons was \( \sim 7 \) T. Of course, this is a very different field configuration, however, higher energy protons are deflected less than the lower energy protons. When lower energy protons were used, similar results were found. That is, deflection features were not similar to those observed in the experiment.
Figure 6.32: Synthetic RCF signal from Gorgon proton deflectometry of a radial foil load at 100 ns after peak Zebra current. The beam is comprised of an energy bin of 4.7-5.7 MeV, similar to the second layer of RCF in the experimental film stack. Only a slight distortion to the beam profile can be seen at the lower portion of the image.

6.8.1 Simulated Deflectometry Including High Energy Electrons

To investigate this potential additional component of the deflectometry signals the Gorgon proton probing post-processor was modified to include an electron deflectometry capability. LSP calculations would seem to be the logical choice, since it is already capable of including electrons in the simulation, however, with the available resources and injection schemes it is not well suited for this task. In fact, when this was attempted, large sheath fields due to a sudden introduction of a large number of high energy electrons prohibited the electrons from propagating further.

Using this new tool, the radial foil load was again probed, but this time with electrons. The energy bin of the electrons was 1.2-2 MeV, with a large beam opening angle of $\sim$75 degrees. The time selected for this was 35 ns after peak Zebra current, similar to one of the experimental shots. The electron positions for a single run using electrons with energy 1.2 MeV are seen in Figure 6.33. In this 3D view, the upper and lower half of the electron beam have been colored differently, green and black, respectively. In Figure 6.33 c) most of the electrons
Figure 6.33: Gorgon high energy electron probing at several times including the radial foil plasma load at 35 ns after peak Zebra current. A side-on, 45 degree rotated, and view from the RCF plane are shown for two times, a) 250 ps and b) 500 ps after particle injection. c) The view from the RCF plane at 750 ps.
have reached the detector, showing that the majority of the signal on the RCF from high energy electrons is in fact due to the upper half of the electron beam. This means that the data is representative of the magnetic field topology above the surface of the radial foil load, which is also shown in the 3D figure.

**Figure 6.34:** The synthetic RCF resulting from a Gorgon calculation using electrons with energy 1.2-2 MeV. This is compared to the signal observed on the third layer of RCF in the film stack, where the minimum proton energy is 7.1 MeV and the minimum electron energy is >∼0.6 MeV.

The synthetic RCF resulting from this calculation are shown in Figure 6.34, and is compared with the experimental result at 36 ns after peak Zebra current. While this result immediately reproduces both the general size and shape of the experimental result, there is still the problem of the earlier films in the stack, which contain a large circular/ellipsoid signal near the center of the film. This signal, as earlier mentioned, is then the proton signal which has not been deflected by the relatively weak magnetic field produced by the radial foil load. The lack of modulation in the signal, i.e. little or no grid pattern from the Cu mesh, is due to the blurring of the beam structure caused by the long scale-length plasma. This blurring was predicted by the TRIM collisions discussed in section 6.5.

In fact, the first film shown in Figure 6.11 displays this central signal. The
lower portion of the signal is blurred by the presence of significant plasma, while the upper portion still shows some of the Cu grid since it has not passed through the denser region. The reason that this does not appear in the following layers is due to the narrower beam opening angle, meaning that the majority of the beam has passed through the plasma. Further, the disappearance of the central signal by the fourth layer of RCF implies that the proton beam cutoff energy in this case was $\sim 7$ MeV.

![Image of Figure 6.35](image)

**Figure 6.35:** The synthetic RCF signal from Gorgon with both the proton and high energy electron signals included. This result is compared to the second layer of RCF from the experiment at two similar times, -55 ns and +35 ns, respectively. The minimum proton energy to reach the experimental film is 4.7 MeV while the proton energy range for the simulations is 4.7-5.7 MeV. The high energy electron energies span 1.2-2 MeV. A grid with 10 mm spacing is superimposed in order to aid comparison.

With this, it is then concluded that the experimental deflectometry signals are actually a superposition of the signals from both the protons and high energy electrons. The persistence of the deflected feature from electrons is expected since their lower stopping power compared to protons means that a very similar size and shape of the signal *should* be imprinted on all of the higher sensitivity films, as is observed. When protons and electrons are included in the calculations, the general experimental result is recovered, and is shown in Figure 6.35.
Figure 6.35 shows this superposition with the electrons being responsible for the dome shape and the central signal being due to the protons, as highlighted by the dashed circle in the synthetic images. The distance between the bottom of the film and the apex of the experimentally observed semi-circle, as well as the horizontal extent of this feature are both reproduced by the simulations. The simulations show more structure in both proton and electron signals due to the fact that Gorgon does not have the ability to include density or collisional effects in the calculations. In both experiment and simulation, only some slight distortion to the lower portion of the proton signal is observed. The proton beam profile structure seen at -55 ns in the simulations is not seen in experiment due to the blurring effect of the plasma density. Recalling that the plasma density is highest near the lower-central region of the radial foil load, the lower-central region of the experimental deflectometry shows no significant modulation due to the copper mesh, while the mesh is faintly visible in the upper portion of the image. For this same reason, the apex of the dome signal appears much brighter in simulations at 35 ns after peak Zebra current.

6.9 Conclusions

The inclusion of high-energy electrons is apparently quite important to the interpretation of these results, and indeed provides the correct result without any need for modifying the underlying physics of Gorgon. It is also important to note that there are some details of the experiment which are not fully reproduced, such as the diminished signal near the apex of the dome feature. The examination of density effects showed that this component in calculations cannot be neglected in this case. In order to model this, calculations utilizing LSP should be performed including both the high energy electrons and the background plasma density from Gorgon. This task is not trivial, and will require more resources than available at UCSD. Even without these effects, however, the modeling results appear to be adequate for explaining the experimental data.
6.9.1 Dynamic Range of Proton Probing Diagnostic

While the short-circuit work showed that the minimum magnetic field magnitude for significantly impacting proton trajectories was \( \sim 7 \) T, the fact that high energy electron signals were recovered essentially increases the lower end of the dynamic range for field sensitivity of the proton probing technique. In the short-circuit case, the field magnitudes probed in experiments were large enough such that these electrons were almost entirely removed from the load region due to deflection by the magnetic field. This means that they did not reach the detector, and thus did not contribute to the final signal on the RCF.

This increase in the dynamic range, however, should only be useful in cases where the signal from the electrons is large enough, i.e. they are bunched such that the dose is larger, and that the signal is distinct from that from protons. In either case, the high energy electrons should be included in the modeling of any such proton probing experiment in order to determine whether or not they will contribute to the final signal.

6.9.2 Current Topology in Radial Foil Loads

As with all Z-pinch loads, the current topology in radial foil loads is important to the dynamics. One of the biggest questions was how much of the drive current was flowing vertically downward through the jet. Since Gorgon appears to reproduce the bulk deflectometry results, which depend on the magnetic field topology being correct, the current topology may be inferred from the Gorgon simulations. Using the instantaneous magnetic field vectors, it is trivial to use post-processing tools to determine the current density via Ampere’s Law. This is performed for each cell in the computational grid, resulting in the current density vector. An area is then chosen to integrate these values which results in the total current.

This method was used to find the total current flowing downward through the jet as a function of height relative to the foil height. The results of this are shown in Figure 6.36. Similar to the results reported by Gourdain [63] the fraction of current flowing through the jet is between \( \sim 5-18\% \) at \( \sim 5 \) mm above the cathode.
Figure 6.36: a) Horizontal slices through the vertical current density, $J_z$, at 5 mm intervals above the cathode along with a r-z slice through the mass density from a Gorgon simulation at +35 ns. b) The fraction of total current flowing along the jet, inside a circle with radius 1.5 mm as a function of height relative to the foil height.

In this work, the maximum fraction of current flowing through the jet occurs at $\sim 100$ ns after peak drive current, at 50 kA, or $\sim 18\%$ of the total drive current at this time. Were a magnetic episode to form, this should be closer to 100%, which is not predicted by Gorgon. Similarly, looking at Figure 6.30 b), it is seen that the current flow remains close to the surface of the foil, even up to 100 ns after peak current. With the current confined within several millimeters above of the foil, no significant pinching of the jet can occur above this height, and thus the jet collimation is aided by the magnetic pressure only below about 5 mm above the cathode.

6.9.3 Implications for Radial Foil Loads as Lab Astro Loads

Of course, as previously discussed, the region of interest for this work is the region near the cathode, since this is the launching and collimation region for the jet. Gorgon simulations indicate that this is the region which has the largest impact on deflectometry results, and also happens to be the region where the transition from a hydrodynamically confined jet to a magnetically confined jet begins. In other words, this is the region where the transition to an astrophysical-like system occurs. Further, this is also the region which is typically obscured by standard lab
diagnostics, as well as in observations.

In the case that the first episode has begun to form in this region we expect that the magnetic pressure would begin to dominate the dynamics near the base of the jet. This can be represented by the plasma beta, the ratio of thermal to magnetic pressure, a 2D r-z map of which is calculated and shown in Figure 6.37.

![Log(\beta) - XZ @ +35 ns](image)

**Figure 6.37**: The log of the plasma beta calculated from Gorgon using a r-z slice at 35 ns after peak Zebra current. a) The plasma beta for the full diameter of the foil and height of the jet. b) A narrowed view of the jet region showing that the jet plasma is thermally dominated.

This calculation shows that the jet is thermally dominated until beyond the radius of the cathode. After this, the log of the plasma beta is weakly below 1, indicating that this is the region where the magnetic pressure is significant. The strongest magnetic field magnitude is confined below the surface of the foil, around the cathode, throughout the simulation time, which stops at 200 ns after peak Zebra current. This result means that the jets launched from these 12.5 \( \mu \text{m} \) thick foils with the Zebra current pulse are most similar to conical wire array experiments. These jets do not transition to a magnetically dominated system, similar to the episodic jets reported by Suzuki-Vidal *et al.* [60] and Ciardi *et al.* [59], and remain a primarily hydrodynamic system, as in conical jet experiments [191].
Chapter 7

Conclusions and Future Outlook

The successful demonstration of this diagnostic is important not only from an experimental standpoint, but also from a simulation standpoint. Although Gorgon is a code which was originally designed to model Z-pinch experiments, it has reached into other realms in recent years. Continuous tests to codes used to model experiments is crucial to improving the understanding and theory of plasma physics. This in turn should extend the applicability of not only this code, but also the understanding of the laboratory astrophysics experiments. More specifically, these results lead to increased confidence in simulations aimed at modeling Z-pinch experiments that attempt to replicate the dynamics of astrophysical outflows.

Since the technique was used to examine a Z-pinch plasma load which is commonly used as a model for astrophysical jets, this validation is all the more important. The physics of the experiments and simulations of laboratory astrophysics experiments must be well understood in order to perform a properly scaled experiment which is relevant to astrophysical dynamics.

The agreement found between experiment and simulation allows for the interpretation of the experimental data through Gorgon. The experiments were indeed a rigorous test for Gorgon, which is often only compared against more easily measured experimental parameters such as density and temperature. This means that the results from Gorgon are valid for at least the two cases examined, namely short-circuits and thick radial foil loads. More importantly, this means that Gorgon’s field and current topology are correct, although there may be some small-
scale features that differ. Due to the geometry of the setup, and the limitations set by the proton beam characteristics driven by the Leopard laser, it is unlikely that these features could be resolved with the current energy spectrum of protons produced.

It is also promising for potential use on world-class facilities such as Z-R, 26 MA in $\sim 100$ ns, at Sandia National Laboratory. This diagnostic has been planned at this facility for some time, with the potential coupling of the Z-Petawatt laser to Z-R, which is capable of generating proton beams with maximum energies of $\sim 40$ MeV [192]. Unfortunately, due to the extremely large magnetic fields produced, $\sim 4000$ T for 20 MA and a radius of 1 mm, proton energies approaching 4.5 GeV energies would be required to probe an imploded load at such high currents [192]. However, proton probing could still potentially be useful in early-time scenarios, where the load has not yet imploded, and the current is much lower. As a current topic of interest, the compression of magnetic flux may also be examined by probing the fringe fields, above and below the imploded load, in order to assess the degree to which the seeded field in the axial direction has been compressed. Examining this type of load in this configuration drops the energy requirement for protons considerably. Further, with only modest improvements to the proton conversion efficiency, this diagnostic is well suited for investigating current and magnetic field topology in pulsed-power experiments $\sim 1$ MA.

### 7.1 Improvements to the Experimental Setup

Experimentally, there are some slight modifications to the setup which are proposed. Due to increased radiation from more violent loads such as imploding wire arrays, compared with radial foil experiments, it may be necessary to include a “shine shield” between the load and the thin foil used to produce the proton beams. A thin, several micron thick, aluminum foil would likely be sufficient for blocking much of the XUV emission, and would then prevent the thin metallic target from ablating on the rear surface. As previously discussed, ablation of the rear surface is detrimental to the formation of the electrostatic sheath field at the
rear surface, which is responsible for the acceleration of the proton contaminants. Additionally, by moving the target closer to the interaction region, higher flux, and higher spatial resolution is possible. This, again, makes the shine shielding even more crucial, with the corresponding increase in self-emission flux from the conical arrays. From experimental deflectometry trials with single wires and cylindrical arrays, the debris produced by these loads is also likely significant, and thus the RCF film pack shielding, typically 16 µm Al, should be mechanically reinforced with Kapton tape, as previously discussed. This again comes at a cost of shrinking the usable proton energy bandwidth, but is necessary.

Although proton deflectometry is an established method of measuring magnetic fields in laser-plasma interaction experiments, the data in this work suggests that there are significant challenges which have yet to be fully overcome in order to make reliable measurements in Z-pinch experiments at the Nevada Terawatt Facility. Much of the difficulty in this work stemmed from a low conversion efficiency, and correspondingly low maximum proton energy. These challenges should be rather easily circumvented if a higher power driver is used, since a higher conversion efficiency may be achieved. With a higher proton conversion efficiency and higher maximum proton energy, probing of higher magnetic fields is possible.

The work presented in this thesis has demonstrated that it is possible to measure magnetic fields in Z-pinch plasmas via charged particle deflectometry, and that it is suitable for use on university-scale drivers. Proton probing for Z-pinch plasmas opens up many possibilities. As previously mentioned, this diagnostic allows for electromagnetic field detection in regions of dense plasma which were previously inaccessible. This fact means that with adjustments to the setup, and optimization of the proton source, that higher quality data may be obtained, which will ultimately lead to improvements in the numerical tools used to model Z-pinches.
Bibliography


