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Improving the Energy Market:  
Algorithms, Market Implications, and Transmission Switching

by

Paula Ann Lipka

A dissertation submitted in partial satisfaction of the  
requirements for the degree of  
Doctor of Philosophy

in

Engineering - Industrial Engineering and Operations Research

in the  
Graduate Division

of the

University of California, Berkeley

Committee in charge:

Professor Shmuel Oren, Chair  
Professor Ilan Adler  
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Fall 2015
Improving the Energy Market:
Algorithms, Market Implications, and Transmission Switching

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Paula Ann Lipka
Abstract

Improving the Energy Market: Algorithms, Market Implications, and Transmission Switching

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Paula Ann Lipka

Doctor of Philosophy in Engineering - Industrial Engineering and Operations Research
University of California, Berkeley
Professor Shmuel Oren, Chair

This dissertation aims to improve ISO operations through a better real-time market solution algorithm that directly considers both real and reactive power, finds a feasible Alternating Current Optimal Power Flow solution, and allows for solving transmission switching problems in an AC setting. We show how to enhance the current IEEE data sets by adding appropriate thermal limits and how to convert apparent power limits to current limits, as current limits are closer to the 'actual' physical limit. We introduce a sequential linearized programming (SLP) approximation of the current-voltage formulation of the ACOPF. We show how the SLP approximation yields AC feasible generator dispatch profiles with little error in cost compared to the exact problem and that the SLP computational time is linear in the problem size. Next, we show how the SLP can be used to run a more complete real-time power market. We show how to use the SLP formulation in corrective switching and how it fixes issues with voltage and line flow. We also test out common heuristics for economic switching.

Most of the IEEE systems do not contain any thermal limits on lines, and the ones that do are often not binding. Chapter 3 modifies the thermal limits for the IEEE systems to create new, interesting test cases. Algorithms created to better solve the power flow problem often solve the IEEE cases without line limits. However, one of the factors that makes the power flow problem hard is thermal limits on the lines. The transmission networks in practice often have transmission lines that become congested, and it is unrealistic to ignore line limits. Modifying the IEEE test cases makes it possible for other researchers to be able to test their algorithms on a setup that is closer to the actual ISO setup. This thesis also examines how to convert limits given on apparent power - as is in the case in the Polish test systems - to limits on current. The main consideration in setting line limits is temperature, which linearly relates to current. Setting limits on real or apparent power is actually a proxy for using the limits on current. Therefore, Chapter 3 shows how to convert back to the best physical representation of line limits.
A sequential linearization of the current-voltage formulation of the Alternating Current Optimal Power Flow (ACOPF) problem is used to find an AC-feasible generator dispatch. In this sequential linearization, there are parameters that are set to the previous optimal solution. Additionally, to improve accuracy of the Taylor series approximations that are used, the movement of the voltage is restricted. The movement of the voltage is allowed to be very large at the first iteration and is restricted further on each subsequent iteration, with the restriction corresponding to the accuracy and AC-feasibility of the solution. This linearization was tested on the IEEE and Polish systems, which range from 14 to 3375 buses and 20 to 4161 transmission lines. It had an accuracy of 0.5% or less for all but the 30-bus system. It also solved in linear time with CPLEX, while the non-linear version solved in $O(n^{1.11})$ to $O(n^{1.39})$. The sequential linearization is slower than the nonlinear formulation for smaller problems, but faster for larger problems, and its linear computational time means it would continue solving faster for larger problems.

A major consideration to implementing algorithms to solve the optimal generator dispatch is ensuring that the resulting prices from the algorithm will support the market. Since the sequential linearization is linear, it is convex, its marginal values are well-defined, and there is no duality gap. The prices and settlements obtained from the sequential linearization therefore can be used to run a market. This market will include extra prices and settlements for reactive power and voltage, compared to the present-day market, which is based on real power. An advantage of this is that there is a very clear pool that can be used for reactive power/voltage support payments, while presently there is not a clear pool to take them out of. This method also reveals how valuable reactive power and voltage are at different locations, which can enable better planning of reactive resource construction.

Transmission switching increases the feasible region of the generator dispatch, which means there may be a better solution than without transmission switching. Power flows on transmission lines are not directly controllable; rather, the power flows according to how it is injected and the physical characteristics of the lines. Changing the network topology changes the physical characteristics, which changes the flows. This means that sets of generator dispatch that may have previously been infeasible due to the flow exceeding line constraints may be feasible, since the flows will be different and may meet line constraints. However, transmission switching is a mixed integer problem, which may have a very slow solution time. For economic switching, we examine a series of heuristics. We examine the congestion rent heuristic in detail and then examine many other heuristics at a higher level. Post-contingency corrective switching aims to fix issues in the power network after a line or generator outage. In Chapter 7, we show that using the sequential linear program with corrective switching helps solve voltage and excessive flow issues.
In memory of my grandparents:

Pauline Kowalski, Ed Kowalski, Bruno Lipka,

Anna Perry, and Richard Perry.
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5.1 AC vs. DCL Market Payments for IEEE 14-Bus Problem

5.2 AC vs. DCL Market Payments for Case 2383wp

7.1 Impact on the objective function of tightening the line limits on line 1 before any contingency occurs

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Chapter 1

Introduction

Power system operations are centralized and coordinated in many countries for better system reliability and lower cost. In the U.S. and Canada, major power system coordinators are labeled as Independent System Operators (ISOs) or Regional Transmission Operators (RTOs); this dissertation will refer to both as ISOs. These organizations are tasked with operating the grid, providing unbiased access to transmission, balancing supply and demand, and managing transmission reliability and transmission expansion for the area \[4\]. These organizations run energy markets to help meet some of their requirements.

Unlike many other markets, the electricity markets run by ISOs do not depend solely on supply and demand. In a power network, there are many physical requirements that must be satisfied. Generators have production and ramping limits, transmission lines have loading limits, voltage magnitudes have limits at each node, and voltage angle differences between buses are limited. An additional complication is that the flows on edges of the power network are not controllable; they are solely determined by the physical properties of the network and the generation. Flows cannot be directly forced down one path or another. This problem is referred to as the power flow problem; if an objective is maximized/minimized, the problem is referred to as an Optimal Power Flow (OPF) problem. There are many different objectives that can be considered depending on the decision maker; these include minimizing power production cost, minimizing transmission losses, minimizing carbon emissions, or maximizing social welfare.

The Alternating Current Optimal Power Flow (ACOPF) problem has been studied for over 50 years, starting with Carpentier \[5\]. The problem is nonlinear and nonconvex. Thousands of papers have been published on the OPF over that time proposing different solution strategies. However, no single strategy has been decided upon as the absolute best, and no ISO solves the problem to its global solution at the present time (2015). The main two methods that are in use in 2015 are to solve a modified direct current approximation or the decoupled power flow in conjunction with a power flow solver.
1.1 Power Markets

In the U.S. and Canada, power markets are run by ISOs. The ISOs in the U.S. include the following organizations:

- California ISO (CAISO)
- Electric Reliability Council of Texas (ERCOT)
- Midcontinent ISO (MISO)
- Southwest Power Pool (SPP)
- New York ISO (NYISO)
- New England ISO (ISO-NE)
- Pennsylvania-New Jersey-Maryland Interconnection (PJM).

A map of the areas that these ISOs and RTOs occupy is shown in Figure 1.1.

![Map of U.S. and Canada Major Power Markets](https://example.com/map)

Figure 1.1: Map of U.S. and Canada Major Power Markets

In all ISOs, there are several different markets to manage the power system. The longest-term market is the capacity market, which is not in place in all ISOs. All ISOs do run a day-ahead market and a real-time (5-minute ahead) market.

Capacity markets work to provide market incentives for new resources. The PJM capacity market provides locational pricing for capacity. Bids are given for resources
that increase power supply or reduce demand (generators, demand response, transmission upgrades). The bidder is then required to implement the suggested capacity three years in the future.

In the day-ahead market, power production for the next day is planned. The market is solved simultaneously for 24 hours ahead, in one hour increments. Power generators bid a cost versus power curve and bid in their startup/shutdown costs, and the ISO/RTO decides which bids to accept and the on/off schedule for all the generators that are slow to ramp up. The generator schedule is constrained by minimum up and down times, ramp rates, and economic minimum and maximum production levels. This is generally referred to as the unit commitment problem. The ISO and generator then have a contract on how much power the ISO will buy from the generator at a specific price. About 95% of the power generation is decided at this point. The ISO also runs residual unit commitment (RUC) in the day ahead. Reserves are used to deal with contingencies and discrepancies between forecasted and actual demand. The ISOs are required to meet N-1 reliability, which means that the system can still operate if any power line or generator in the network is lost. Before the market solution is decided, market power is mitigated.

Generators bid in every 15 minutes; these bids are used to clear the real time market every five minutes [7]. These shorter term markets deal with imbalances between forecasted and actual generation and load [4]. These markets also deal with production uncertainties, such as generator contingencies or renewable energy variations. The 5-minute ahead market pays different prices than the day-ahead market, and these prices typically vary across time much more than the day-ahead market. Generators are paid at the day-ahead price for production scheduled day-ahead and are paid at the real-time price for any extra production. Ancillary services may be activated as required; these may include both fast-start and slow-start generators, and reserves can be spinning and non-spinning.

To try to reduce the day-ahead and real-time price spreads, CAISO has a trading mechanism called convergence bidding. Convergence bidding allows traders to submit virtual demand and supply, which allows traders to arbitrage between the day-ahead and real-time prices. The objective of this market product is to improve price convergence between the day-ahead and real-time prices. Studies such as [8] show that this market mechanism has indeed improved price convergence.

1.2 Modeling the Power System

The Alternating Current Optimal Power Flow considers the steady state power at one time interval. It finds the best power production solution given a specific set of power demands and network requirements. It does not consider issues relating to the oscillatory nature of alternating current power such as voltage stability. However, the full ACOPF model is nonlinear and nonconvex. Generally, ISOs are dealing with a power network with thousands of buses, so the problem is also quite large for a nonlinear program. The ACOPF alone for one time interval is therefore quite hard to solve.
Ideally, the power markets would solve the Unit Commitment version of the ACOPF with reserve requirements and include the N-1 requirement to find the best generation dispatch solution. This makes the problem significantly harder. Currently, ISOs do not solve an ACOPF in the real-time or day-ahead market, although they do often incorporate AC power flow solutions to guide the market solution.

The ISOs run a modification of the direct current approximation (DC) of the OPF, detailed in Section 2.5.1. While the DCOPF ignores the role of reactive power and fixes voltage magnitudes rather than allow them to be within a range, it is a linear program that solves relatively quickly. The fact that it is a linear program means that there is no duality gap, and the dual multipliers - which give the market prices - support the market. Additionally, the linear nature of the DCOPF improves the ability of the ISO to solve larger mixed integer programs, such as the unit commitment problem or the N-1 reliability problem. ISOs often modify the DCOPF to better fit the true system; this includes altering distribution factors to try to account for losses, putting the voltage magnitude as the last or expected next operating point, and adding nomogram constraints, which usually constrain flow across cutsets [9]. The nomogram constraints are designed to try to capture voltage and reactive power constraints. Load pays for power consumed, generation gets paid for power generated, and market participants receive or pay congestion benefits/costs. Typically, the ISO alternates between the modified DCOPF and an AC feasibility check, modifying the voltage magnitude, adjusting voltage losses, and adding flow constraints as necessary to get the DC solution to be AC feasible. The ISO also runs a stability check on the solution before accepting it for the dispatch.

While the current market mechanism is fast and allows for reliable operation, it does have a number of problems. The LMPs from the DCOPF are not the true LMPs; they ignore reactive power and the losses are either ignored or inaccurate. Generally, there is not much incentive to build equipment for reactive support since reactive support with in a range is mandated and not compensated. Additionally, there may be different sets of nomogram constraints that yield the same dispatch solution but different price profiles. [10]

1.3 US Power Grid

The U.S. and Canada are divided into a set of grids, with each governed by the North American Electric Reliability Corporation (NERC). In the U.S. and Canada, there is the Western Interconnection, the Eastern Interconnection, ERCOT (Texas) Interconnection, and Quebec Interconnection, shown in Figure 1.2. The grid is primarily composed of alternating current (AC) transmission lines, although there are a few high voltage direct current lines, including the Pacific Intertie that connects Los Angeles to the northern border of Oregon.
1.4 Reliability Requirements

As there are many sources of uncertainty in the power system, and it is crucial for the power system to stay on, ISOs carefully consider measures to keep the power system in operation. Uncertainties include demand, generation and line outages, line limits, and voltage readings. Line limits may vary depending on the temperature, wind speed, and prior usage. Typically, ISOs set line limits very conservatively for normal operation, and allow for a higher limit in the case of an emergency. There are some vendors that do report what the line limits should be depending on temperature, and PJM sets the line limits according to the season. ISOs also evaluate the market solution for $N-1$ reliability, which means that if there is an outage on any one line or generator, the system should remain in operation. Typically, if they find that a market solution is not $N-1$ reliable, the system is re-solved with tighter line limits on where there was a line overload.

1.5 Transmission Switching

Due to the non-linear nature of the power system and the inability to control flow across lines, switching transmission lines on and off can be used to provide benefits to the power system. The term economic switching is used to refer to switching transmission lines in order to provide cheaper generation overall. Corrective switching is used to help restore the system in the case of contingencies, which include line and generation outages. By shutting lines on and off, we have an additional avenue to change the distribution of flow across the different lines. This possibility is very beneficial; however,
CHAPTER 1. INTRODUCTION

it does make any program that incorporates it into a mixed integer program. While some see transmission switching as being a less reliable method, it can also result in a more stable system. Also, transmission switching can be implemented while adhering to $N - 1$ requirements [11][12].

1.6 Motivation

The current method of running energy markets ignores reactive power and power losses. The interplay between reliability and the power flow in ISOs results in a less efficient dispatch than the co-optimization of both of these items. Many of the methods used to solve the ACOPF do not scale well with problem size. For example, the SDP takes 15 minutes to solve a case with 3375 buses. The method of only using real power for dispatch also means that there is often a gap in market payments for reactive power/voltage support which must be made up in other ways. The current method of using nomograms to satisfy voltage and reactive power constraints often results in the distortion of power prices. Additionally, there have not been many methods to exploit the ability to turn on and off power lines in conjunction with the ACOPF.

1.7 Scope of Work

The purpose of this dissertation is to provide a linearized AC tool in order to better solve the market as-is and to improve the possibility of solving for economic and corrective switching in real-time. In this work, we introduce a new linear approximation and solve problems up to 3375 buses. The computational time for this approximation scales linearly in problem size. We discuss how to solve the market with this new approximation and differences between the current system and the new proposed system. We also discuss the findings for using this algorithm in corrective and economic switching.

1.8 Organization

Chapter 2 introduces equations that govern power flow, the AC formulations, ways to solve the ACOPF, and the technical detail on how market prices are determined. We discuss how to create limits on current flowing across transmission lines in Chapter 3. A current limit is a convex constraint in the current-voltage formulation of the ACOPF, while a limit on real or apparent power is non-convex. This type of constraint is more appropriate for the formulation we linearize - the current-voltage formulation of the ACOPF. Additionally, the transmission line properties mean that the appropriate limit for a transmission line is on current rather than power; limits on power are used in the market. We introduce the linearized approximation to the ACOPF in Chapter 4 and show its results. In Chapter 5 we show how to run a more complete real-time market based off of the linearized formulation. In Chapter 6 we discuss economic switching
in an AC setting. We show the effectiveness of the current heuristics that are used in economic switching. Chapter 7 focuses on corrective switching in an AC setting. Chapter 8 concludes the dissertation and describes avenues for future research.
Chapter 2

Background

This chapter discusses the physical nature of power flow, power flow models, techniques to solve power flow, and commonly-used data sets. Section 2.2 gives the relationships between voltage, current, and power. Section 2.3 gives the three equivalent and most-commonly used ACOPF formulations. Section 2.4 discusses several of the approaches to finding a power flow solution. Section 2.5 explains the models the ISO markets use to solve power flow at the present time (2015). Section 2.6 summarizes and provides reference to common data sets that are used by academics to test solution techniques.

2.1 Background Notation

This section details the notation that is used commonly in the following chapters.

Superscripts:
\begin{itemize}
\item[$d$] Demand
\item[$g$] Generation
\item[$r$] Real (x-axis) component of the vector
\item[$j$] Imaginary (y-axis) components of the vector
\end{itemize}

Subscripts:
\begin{itemize}
\item[$n,m$] Buses
\item[$k(n,m)$] Line $k$ connecting buses $n$ and $m$, in the $n$ to $m$ direction
\item[$k(n)$] Property of line $k$ at bus $n$
\end{itemize}
Sets:
\( \mathcal{N} \) Set of buses \( \{1, \ldots, N\} \)
\( \mathcal{K} \) Set of lines \( \{1, \ldots, K\} \)
\( \mathcal{A}(n) \) Set of buses that are connected to node \( n \) by a transmission line (adjacent to node \( n \))
\( \mathcal{G} \) Buses with generators; \( \{1, \ldots, G\} \)
\( \mathcal{F} \) Set of flows \( \{1, \ldots, 2K\} \)
\( \mathcal{C}(n) \) Set of cycles containing node \( n \)

Indices:
\( n, m \) Bus (node) indices; \( n, m \in \mathcal{N} \)
\( k \) Three-phase transmission element; \( k \in \mathcal{K} \)
\( k(n, m) \) Flow on transmission element \( k \) from bus \( n \) to \( m \); \( k(n, m) \in \mathcal{F} \) where \( k(m, n) \) denotes the flow in the opposite direction along line \( k \), \( k(n, \cdot) \) denotes withdrawals from bus \( n \), and \( k(\cdot, n) \) denotes injections to bus \( n \)
\( k(\cdot) \) Directional flows on \( k \); \( k(\cdot) \in \mathcal{F} \)

Variables:
\( t \) Time
\( v \) Voltage
\( i \) Current
\( s \) Apparent power
\( p \) Real power
\( q \) Reactive power
\( v(t) \) Voltage at time \( t \)
\( i(t) \) Current at time \( t \)
\( s(t) \) Apparent power at time \( t \)
\( p(t) \) Real power at time \( t \)
\( q(t) \) Reactive power at time \( t \)
\( \theta_V \) Voltage angle
\( \theta_I \) Current angle
\( \theta_n \) Voltage angle at bus \( n \)
\( \psi \) Voltage phasor
\( i \) Current phasor
\( p_{k(n,m)} \) Real power on line \( k \) at the \( n \) end
\( q_{k(n,m)} \) Reactive power on line \( k \) at the \( n \) end
\( p_{n,i}^g \) Real power generation at bus \( n \) from generator \( i \)
\( q_{n,i}^g \) Reactive power generation at bus \( n \) from generator \( i \)
\( v_n \) Voltage magnitude at bus \( n \)
\( v_n^r \) Real part of voltage at bus \( n \)
\( v_n^j \) Imaginary part of voltage at bus \( n \)
\( i_n \) Current magnitude at bus \( n \)
\( i^r_n \) Real part of current injection at bus \( n \)
\( i^j_n \) Imaginary part of current injection at bus \( n \)
\( i_{k(n,m)} \) Current magnitude on \( k \) from bus \( n \) to \( m \)
\( i^r_{k(n,m)} \) Real part of current on \( k \) from bus \( n \) to \( m \)
\( i^j_{k(n,m)} \) Imaginary part of current on \( k \) from bus \( n \) to \( m \)

Parameters:

\( B^\text{MVA} \) Power base in MVA
\( R_k \) Series resistance of line \( k \)
\( X_k \) Series reactance of line \( k \)
\( G_k \) Series conductance of line \( k \)
\( B_k \) Series susceptance of line \( k \)
\( Y_k \) Series admittance of line \( k \); \( Y_k = G_k + jB_k \)
\( G_{kn}^{sh} \) Shunt conductance on line \( k \) connected to \( n \)
CHAPTER 2. BACKGROUND

$B_{kn}^{sh}$ Shunt susceptance on line $k$ connected to $n$

$Y_{kn}^{sh}$ Shunt admittance on line $k$ connected to $n$

$G_n^{sh}$ Shunt conductance at bus $n$

$B_n^{sh}$ Shunt susceptance at bus $n$

$Y_n^{sh}$ Shunt admittance at bus $n$

$\tau_{kn}$ Ideal transformer on the $n$-side of line $k$

$|\tau_{kn}|$ Transformer turns ratio on the $n$-side of line $k$

$\phi_{kn}$ Phase-shifter on the $n$-side of line $k$

$P_n^d$ Real power demand at bus $n$

$Q_n^d$ Reactive power demand at bus $n$

$P_{n,i}^{min}$ Minimum real power for generator $i$ at bus $n$

$P_{n,i}^{max}$ Maximum real power for generator $i$ at bus $n$

$Q_{n,i}^{min}$ Minimum reactive power for generator $i$ at bus $n$

$Q_{n,i}^{max}$ Maximum reactive power generator $i$ at bus $n$

$V_m$ Highest magnitude of the sinusoidal time-varying voltage

$I_m$ Highest magnitude of the sinusoidal time-varying current

$V_{n,\min}$ Minimum voltage magnitude at bus $n$

$V_{n,\max}$ Maximum voltage magnitude at bus $n$

$I_k^{max}$ Maximum current magnitude on line $k$

$C_{n,i}^q(P_{n,i}^G)$ Cost function of generator $i$ at bus $n$

$f$ Frequency in Hertz per second

$\omega$ Frequency in radians per second

$R$ Resistance

$C$ Capacitance

$L$ Inductance

$a_{nk}$ Coefficient of nomogram constraint for flow going into node $n$ from line $k$

$P_n^{input,tot}$ Maximum amount of flow allowed into node $n$

$S_{k(n,m)}^{max}$ Maximum apparent power on line $k$
CHAPTER 2. BACKGROUND

Dual Variables:

\( \delta_{k(n,m)} \) Dual variable on line power definition
\( \rho_{n,i}^{\min} \) Dual variable on minimum real power for generator \( i \) at bus \( n \)
\( \mu_{k(n,m)}^{\min} \) Dual variable on minimum real power on line \( k \) at the \( n \) end
\( \mu_{k(n,m)}^{\max} \) Dual variable on maximum real power on line \( k \) at the \( n \) end
\( \lambda_{n}^{P} \) Dual variable on the real power balance equation at bus \( n \)
\( \gamma_{nm}^{\min} \) Dual variable on the minimum angle difference
\( \gamma_{nm}^{\max} \) Dual variable on the maximum angle difference

Matrices:

\( I \) Matrix of currents
\( V \) Matrix of voltages
\( Y \) Matrix of admittances

2.2 Basic Power Equations

Most electric power systems transmit alternating voltage and current. Most systems in North America cycle at a frequency \( f \) of 60 Hz/s, or equivalently at \( \omega = 120\pi \) radians/s. Real and reactive power production is dependent on the voltage and current magnitudes and the angle between the voltage and current. Figure 2.1 shows the relationships between voltage \( v \), current \( i \), apparent power \( s \), reactive power \( q \), and real power \( p \). Voltage is defined as the energy per unit of charge and is measured in Volts, or Joules per Coulomb. Current is defined as the flow of charge, or the derivative of the charge with respect to time, and is measured in Amperes (Coulombs per second). Energy is the voltage drop times the charge, or voltage times current times time and is measured in Joules. Power is the rate of energy, or energy per unit time, and is measured in Watts (Joules per second). More details on voltage, current, energy and power can be found in [13] and [14].

Alternating voltage and current can be described by the relationship shown in equation (2.2.1). \( \theta_{V} \) is the angle of the voltage, \( \theta_{I} \) is the angle of the current, and \( t \) is time.

\[
v(t) = V^{m} \cos(\omega t + \theta_{V}) \quad i(t) = I^{m} \cos(\omega t + \theta_{I})
\]  \hspace{1cm} (2.2.1)
The time-varying real power, reactive power, and apparent power are given in (2.2.2), (2.2.3), and (2.2.4).

\[ p(t) = V^m I^m \cos(\omega t - (\theta_V - \theta_I)) \]  
(2.2.2)

\[ q(t) = V^m I^m \sin(\omega t - (\theta_V - \theta_I)) \]  
(2.2.3)

\[ s(t) = V^m I^m \cos(\omega t + \theta_V) \cos(\omega t + \theta_I) \]  
(2.2.4)

The relationship between current and voltage depends on the system elements they are flowing through. These basic elements are resistors, inductors, and capacitors. When charge flows through a resistor, the current and voltage are proportional and are in phase. Resistors have resistance \( R \), measured in units of Ohms (\( \Omega \)). Capacitors store charge between two plates and cause the current phase to lead the voltage phase by 90°. Capacitors have capacitance \( C \), measured in units of Farads \( [F = \frac{s^4 A^2}{(kg \cdot m^2)}] \), were \( s \) are seconds, \( A \) is Amperes, and \( kg \) is kilograms. Inductors are a coil where charge flowing through induces a magnetic field, which induces current. Inductors cause the current phase to lag the voltage phase by 90°. Inductors have inductance \( L \), measured in terms of Henrys \( [H = \frac{kg \cdot m^2}{(s^2 \cdot A^2)}] \). The relationships between current and voltage through different system elements are shown in equation (2.2.5).

\[ i_R(t) = \frac{v(t)}{R} = \frac{V^m}{R} \cos(\omega t + \theta_V) \]  
(2.2.5)

\[ i_C(t) = C \frac{dv(t)}{dt} = CV^m \cos(\omega t - \theta_V + 90^\circ) \]  
(2.2.6)

\[ i_L(t) = \frac{1}{L} \int v(t)dt = \frac{V^m}{L} \cos(\omega t - \theta_V - 90^\circ) \]  
(2.2.7)
To solve for voltage and current in a circuit, one can use Kirchoff’s Current Law and Kirchoff’s Voltage Law. Kirchoff’s Current Law states that the sum of currents at a node equals zero, or equivalently, total incoming current equals total outgoing current. If we let \( n \) and \( m \) designate buses and \( k(n,m) \) designate a line between bus \( n \) and bus \( m \), we can state Kirchoff’s Current Law in terms of equation (2.2.8).

\[
\sum_{m \in A(n)} i_{k(n,m)} = \sum_{m \in A(n)} i_{k(m,n)} \quad (2.2.8)
\]

Let a loop including node \( n \) be designated as \( C(n) \). Kirchoff’s Voltage Law (2.2.9) states that the sum of voltages in a loop equals zero.

\[
\sum_{n \in C(n)} v_n = 0 \quad (2.2.9)
\]

The ACOPF assumes frequency is constant and examines the steady-state rather than the instantaneous operation of the power system. In reality, if the system is unbalanced, the frequency will change in proportion to the mismatch, and there can be problems with transient stability of the voltage. In the ACOPF, the average power, voltage, current, and their phases are tracked. The average voltage and current magnitudes \( |v| \) and \( |i| \) in steady state are \( |v| = \sqrt{2}V^m \) and \( |i| = \sqrt{2}I^m \). The current and voltage vectors are typically expressed in polar or rectangular form, as shown in Table 2.1. The notation \( j \) is for an imaginary number; \( j = \sqrt{-1} \).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Polar Form</th>
<th>Conversion</th>
<th>Rectangular Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i )</td>
<td>(</td>
<td>i</td>
<td>e^{j\theta_I} )</td>
</tr>
<tr>
<td>( v )</td>
<td>(</td>
<td>v</td>
<td>e^{j\theta_V} )</td>
</tr>
</tbody>
</table>

Table 2.1: Current and Voltage in Polar and Rectangular Form

The steady state apparent, real, and reactive power can be calculated based on the voltage and current as shown in equations (2.2.10) through (2.2.12). Real power is the real component of apparent power while reactive power is the imaginary component of apparent power. Power markets run on payment for real power, but reactive power also receives compensation, generally through long term contracts.

\[
s = |v||i| \quad (2.2.10)
\]

\[
p = |v||i| \cos (\theta_V - \theta_I) \quad (2.2.11)
\]

\[
q = |v||i| \sin (\theta_V - \theta_I) \quad (2.2.12)
\]

A transmission line can be simplified from its individual components to its impedance \( Z_k \). The impedance of each transmission line, \( Z_k = R_k + jX_k \), is composed of the resistance \( R \) and reactance \( X \) of the line. The resistive component comes only from the resistors. The inductors and capacitors contribute to the reactance, with \( X = X_L + X_C \).
and $X_C = -\frac{1}{\omega C}$ and $X_L = \omega L$. Another way the lines are represented is by their admittance $Y$, which is the inverse of impedance. The admittance is $Y = 1/Z$, aka $Y = G + jB$, with $G = \frac{R}{R^2 + X^2}$ and $B = -\frac{X}{R^2 + X^2}$. Typically, the admittance for a full network is designated by a matrix of dimension $N \times N$, with the off-diagonal entries $(i,j)$ giving the admittance between node $i$ and node $j$. The on-diagonals will be the negative sum of the admittance of the row plus the shunt impedance at the node. The entire row or column should sum to zero if the shunt admittance is zero or sum to the value of the shunt admittance if the shunt admittance is nonzero.

To characterize the resistive losses and leakage flux (i.e., self-reactance), we model a practical transformer that is located on the bus $n$ side as an ideal transformer with turns ratio $|\tau_{kn}|$ in series with a series admittance $Y_k = G_k + jB_k$. Depending on if $\tau_{kn}$ is real or complex, the transformer is in-phase or phase-shifting. We can similarly represent a phase-shifter as $\tau_{kn} = |\tau_{kn}| e^{j\phi_{kn}}$. The branch admittance matrix can be represented as in (2.2.13).

$$
\begin{bmatrix}
Y_{1,1}^k & Y_{1,2}^k \\
Y_{2,1}^k & Y_{2,2}^k
\end{bmatrix}
\begin{bmatrix}
v_n \\
v_m
\end{bmatrix}
\begin{bmatrix}
\tau_{kn} \\
\tau_{km}
\end{bmatrix}
\begin{bmatrix}
Y_k + Y_{sh}
\end{bmatrix}

(2.2.13)
$$

We can model the complex current flows on line $k$ as shown in (2.2.14).

$$
\begin{bmatrix}
\hat{i}_{k(n,m)} \\
\hat{i}_{k(m,n)}
\end{bmatrix}
= \begin{bmatrix}
Y_{1,1}^k & Y_{1,2}^k \\
Y_{2,1}^k & Y_{2,2}^k
\end{bmatrix}
\begin{bmatrix}
v_n \\
v_m
\end{bmatrix}

(2.2.14)
$$

The above representation is for a two-winding transformer, and if $\tau_{kn} = \tau_{km} = 1$, then the equivalent $\pi$-model is of a transmission line. For an $N$-winding transformer, we would have a $Y$ matrix of size $N \times N$ for the unified branch model. We can represent the linear relationship between the real and imaginary parts of the complex current flows $\hat{i}_{k(n,m)}$ and $\hat{i}_{k(m,n)}$ and the complex nodal voltages $v_n$ and $v_m$ as given in (2.2.15) - (2.2.18).

$$
\begin{align*}
\hat{i}_{r_{k(n,m)}} &= \text{Re} (Y_{1,1}^k v_n + Y_{1,2}^k v_m) \\
\hat{i}_{i_{k(n,m)}} &= \text{Im} (Y_{1,1}^k v_n + Y_{1,2}^k v_m) \\
\hat{i}_{r_{k(m,n)}} &= \text{Re} (Y_{2,1}^k v_n + Y_{2,2}^k v_m) \\
\hat{i}_{i_{k(m,n)}} &= \text{Im} (Y_{2,1}^k v_n + Y_{2,2}^k v_m)
\end{align*}

(2.2.15) - (2.2.18)
$$

If we let $I$ be a matrix of the current and $V$ be a matrix of voltages, we know the following relations between current, voltage, and apparent power shown in equations (2.2.19) through (2.2.21). The symbol $*$ represents the conjugate transpose of the matrix.
\[ I = YV^* \]  
\[ V = I^* Z \]  
\[ S = VI^* = YVV^* \]

We can also rewrite these equations in terms of each individual line \( k \), as shown in equations (2.2.22) through (2.2.24).

\[ s_{k(n,m)} = p_{k(n,m)} + jq_{k(n,m)} \]  
\[ p_{k(n,m)} = v_{n}^r i_{k(n,m)}^r + v_{n}^i j_{k(n,m)}^i \]  
\[ q_{k(n,m)} = -v_{n}^r i_{k(n,m)}^i + v_{n}^i j_{k(n,m)}^r \]

2.3 The ACOPF Formulations

There are several different ways to write the ACOPF formulation. This is because current and voltage have a linear relationship and both voltage and current are vectors that can be written in either rectangular form, in terms of their x and y components, or in polar form, in terms of their magnitude and angle. The main three different formulations used for ACOPF are as follows:

PV Polar: Formulated in terms of power and voltage, in polar coordinates

PV Rectangular: Formulated in terms of power and voltage, in rectangular coordinates

IV Rectangular: Formulated in terms of current and voltage, in rectangular coordinates

The PV polar and rectangular formulations are equivalent and are equivalent to the IV formulation, depending on what flow quantity the limits are placed on. However, generally the PV formulations use apparent power for the line limits while the IV formulations use current for line limits. Even though the formulations are equivalent, the different representations result in different solution times and techniques. Castillo found that the rectangular formulations (both PV and IV) generally solved the fastest. One reason the solution times and procedures vary is that each of the formulations has different derivatives due to the different variables used.

2.3.1 Power-Voltage Polar Formulation

The PV polar formulation is detailed in (2.3.1)-(2.3.9). It has a linear voltage magnitude constraint and nonlinear real and reactive power constraints. The apparent power
CHAPTER 2. BACKGROUND

The line limit is convex, while all the other nonlinear constraints are nonconvex.

\[
\text{Min} \quad \text{Cost} = \sum_{n \in N} \sum_{i \in G} C_{n,i}^g (p_{n,i}^G) \quad (2.3.1)
\]

subject to

\[
p_{k(n,m)} = (G_{k(n)} \cos \theta_n - G_{k(m)} \cos \theta_m + B_{k(n)} \sin \theta_{nm} - B_{k(m)} \sin \theta_m) v_n v_m + G_{k(n,m)} v_n^2 \quad (2.3.2)
\]

\[
q_{k(n,m)} = (G_{k(n)} \sin \theta_n - G_{k(m)} \sin \theta_m - B_{k(n)} \cos \theta_n + B_{k(m)} \cos \theta_m) v_n v_m - B_{k(n,m)} v_n^2 \quad (2.3.3)
\]

\[
p_{k(n,m)}^2 + q_{k(n,m)}^2 \leq (S_{k(n,m)}^\text{max})^2 \quad (2.3.4)
\]

\[
\sum_{m \in A(n)} p_{k(n,m)} = -\sum_{i \in G} p_{n,i}^g + P_n^d \quad (2.3.5)
\]

\[
\sum_{m \in A(n)} q_{k(n,m)} = -\sum_{i \in G} q_{n,i}^g + Q_n^d \quad (2.3.6)
\]

\[
V_{n,m}^\text{min} \leq v_n \leq V_{n,m}^\text{max} \quad (2.3.7)
\]

\[
P_{n,i}^\text{min} \leq P_{n,i}^g \leq P_{n,i}^\text{max} \quad (2.3.8)
\]

\[
Q_{n,i}^\text{min} \leq Q_{n,i}^g \leq Q_{n,i}^\text{max} \quad (2.3.9)
\]

The direct current approximation is an approximation based off of the power-voltage polar formulation. The DCOPF is a result of fixing voltage magnitudes to 1 \((v_n = 1)\), neglecting the resistance \((R \ll X\) or \(R \approx 0\)) and assuming the small angle approximation holds \((\sin(\theta) \approx \theta\) and \(\cos(\theta) \approx 1)\).

### 2.3.2 Power Voltage Rectangular Formulation

The power voltage rectangular representation represents each complex vector in real and imaginary components rather than as a magnitude and an angle. This formulation is detailed in [2.3.10] through [2.3.19].

\[
\text{Min} \quad \text{Cost} = \sum_{n \in N} \sum_{i \in G} C_{n,i}^g (p_{n,i}^g) \quad (2.3.10)
\]

subject to

\[
p_{k(n,m)} = G_{k(n,m)} (v_n^r v_m^r + v_n^i v_m^i) + B_{k(n,m)} (v_n^i v_m^r - v_n^r v_m^i) + G_{k(n,m)} v_n^2 \quad (2.3.11)
\]

\[
q_{k(n,m)} = -G_{k(n,m)} (v_n^r v_m^r - v_n^i v_m^i) - B_{k(n,m)} (v_n^r v_m^r + v_n^i v_m^i) - B_{k(n,m)} v_n^2 \quad (2.3.12)
\]

\[
v_n^2 = (v_n^r)^2 + (v_n^i)^2 \quad (2.3.13)
\]

\[
p_{k(n,m)}^2 + q_{k(n,m)}^2 \leq (S_{k(n,m)}^\text{max})^2 \quad (2.3.14)
\]

\[
\sum_{m \in A(n)} p_{k(n,m)} = -\sum_{i \in G} p_{n,i}^g + P_n^d \quad (2.3.15)
\]
\[ \sum_{m \in A(n)} q_k(n,m) = - \sum_{i \in G} q_{n,i}^g + Q_n^d \]  

(2.3.16)

\[(V_n^{min})^2 \leq (v_r^n)^2 + (v_j^n)^2 \leq (V_n^{max})^2 \]  

(2.3.17)

\[ P_{n,i}^{min} \leq p_{n,i}^g \leq P_{n,i}^{max} \]  

(2.3.18)

\[ Q_{n,i}^{min} \leq q_{n,i}^g \leq Q_{n,i}^{max} \]  

(2.3.19)

All of the constraints in the PV rectangular formulation are nonlinear. However, only the voltage constraint is nonconvex; all the other constraints are convex.

2.3.3 Current-Voltage Rectangular Formulation

The ACOPF Model can be written in terms of current and voltage; this formulation is less commonly used. This formulation is detailed in (2.3.20) - (2.3.30):

Min Cost = \[ \sum_{n \in N} \sum_{i \in G} C_{n,i}^g \left( \sum_{G} p_{n,i}^G \right) \]  

(2.3.20)

subject to

\[ i_k(n,m) = G_k(n)v_r^n - G_k(m)v_r^m - B_k(n)v_j^n + B_k(m)v_j^m \]  

(2.3.21)

\[ i_j(n,m) = B_k(n)v_r^n - B_k(m)v_r^m + G_k(n)v_j^n - G_k(m)v_j^m \]  

(2.3.22)

\[ i_r^n = \sum_{m \in A(n)} i_k(n,m) + G_{n}v_r^n - B_{n}v_j^n \]  

(2.3.23)

\[ i_j^n = \sum_{m \in A(n)} i_j(n,m) + G_{n}v_j^n + B_{n}v_r^n \]  

(2.3.24)

\[(V_n^{min})^2 \leq (v_r^n)^2 + (v_j^n)^2 \leq (V_n^{max})^2 \]  

(2.3.25)

\[(i_k(n,m))^2 + (i_j(n,m))^2 \leq (I_k^{max})^2 \]  

(2.3.26)

\[ \sum_{i \in G} p_{n,i}^G = v_r^n i_r^n + v_j^n i_j^n + P_n^d \]  

(2.3.27)

\[ \sum_{i \in G} q_{n,i}^G = v_j^n i_r^n - v_r^n i_j^n + Q_n^d \]  

(2.3.28)

\[ P_{n,i}^{min} \leq p_{n,i}^g \leq P_{n,i}^{max} \]  

(2.3.29)

\[ Q_{n,i}^{min} \leq q_{n,i}^g \leq Q_{n,i}^{max} \]  

(2.3.30)

The IV formulation has linear constraints defining the line and nodal currents. Its line limit constraint and maximum voltage constraint are nonlinear but convex. The constraints on the minimum voltage and real and reactive power are nonconvex.
2.3.4 Solving the Exact Formulations

Many methods for solving nonlinear problems have been applied to solving these formulations. These include Lagrangian relaxation, interior point methods, Newton’s method [16], and the Davidon-Fletcher-Powell method [17], [18]. One issue with many of these algorithms is convergence. For example, Newton’s method is guaranteed to converge within a small neighborhood of the solution, but convergence may not be possible in an area outside of this neighborhood. In Figure 2.2, a starting point converges an answer within the color that it is surrounded by. This answer is not guaranteed to be a global optimum or even a feasible point. It can be difficult to recover a feasible answer from an infeasible solution found using Newton’s method.

![Figure 2.2: Convergence using Newton’s Method](image)

2.4 ACOPF Relaxations and Approximations

Due to the complexity of the ACOPF, many approximations and relaxations have been developed to try to solve it. The Semi-Definite programming and second order cone programming relaxations attempt to solve a convexified version of the ACOPF. The decoupled OPF iterates between problems solving for real power and solving for reactive power.

2.4.1 Semi-Definite Relaxation

The semidefinite programming (SDP) relaxation [19] involves using the voltage-power rectangular form of the ACOPF. The real power, reactive power, apparent power, voltage magnitudes, and voltage differences are written in terms of the voltage components. More specifically, with $X \in \mathbb{R}^{n \times 1}$ as the vector of real and imaginary voltages, all variables are written as a function of $W = XX^T$. The apparent power line limits are rewritten in terms of Schur’s complement. Then, the program is solved in terms of $W$ instead of $XX^T$. If $W$ is required to be rank one, then this formulation is the same as the original ACOPF; if the rank requirement is dropped, then the program is the SDP relaxation. If the solution to the SDP relaxation ends up having $W$ being rank one, then the SDP relaxation has zero duality gap, it has found the global optimum, and one can recover the OPF solution. If the relaxation solution has $W$ with a rank $\leq 2$, it means
the OPF has multiple solutions, and these solutions can be recovered. The IEEE 14, 30, 57, 118, and 300 bus systems, when modified to have all lines with non-zero resistance, have been shown by Lavaei and Low \[19\] to have zero duality gap.

However, the SDP relaxation is not guaranteed to have zero duality gap. Lesieutre et. al. \[20\] showed that cases with transmission constraints and negative prices will give a non-zero duality gap. If the duality gap is non-zero, it can be very difficult to recover a feasible solution. Additionally, initializing the SDP can be very difficult, as is implementing it efficiently.

### 2.4.2 Conic Relaxation

The conic relaxation is a relaxation of the SDP \[21\] \[22\]. In this formulation, the program is rewritten in terms of matrices $U + jT = XX^T$, with $X$ defined as in the SDP relaxation. Like the SDP, if rank$(U + jT)$ was required to be 1 and $W + jT$ was required to be positive semidefinite, this reformulation would be the exact formulation. However, the conic program has the relaxation first that rank$(U + jT)$ is not required to be 1, and, rather than have $U + jT \succeq 0$, it requires that all 2x2 principle minors of the matrix $U + jT$ be positive semidefinite. While conic programs in general solve to near optimality quickly, the extra feasible space of the relaxation compared to the original problem may lead to solutions that are outside the boundary of the feasible problem. Chen \[23\] has improved the conic relaxation by adding linear cuts to reduce the duality gap.

### 2.4.3 Decoupled OPF

The decoupled OPF developed by Stott and Alsac \[24\] is based on the premise that real power is much more sensitive to voltage angle than voltage magnitude and reactive power is much more sensitive to voltage magnitude than voltage angle. It also assumes a small voltage angle difference between buses such that $\cos \theta_{nm} \approx 1$ and $\sin \theta_{nm} \approx \theta_{nm}$. The real and reactive power equations (2.4.1) and (2.4.2) are only related by voltage magnitudes rather than by voltage magnitudes and the phase angle.

\[
\begin{align*}
    p_{k(n,m)} &= -B_{k(n)}v_nv_m\theta_{nm} \\
    q_{k(n,m)} &= -G_{k(n)}v_nv_m
\end{align*}
\]

### 2.4.4 Linear Approximations Incorporating Reactive Power and Voltage

There are a number of linear approximation methods to the ACOPF. Zhang \[25\] presents a Taylor series expansion of the PV Polar ACOPF by approximating voltage as $v = 1+\Delta v$ and neglecting products of the form $\Delta v_1 \Delta v_2$. The line losses are approximated by taking a piecewise linear approximation to the phase angle changes. This model
simulates very quickly and exhibits optimality gaps of less than 0.6% on 5 of the IEEE test problems of 118 buses and smaller. Coffrin [26] also presents three formulations of a linearized PV polar ACOPF formulation. These three formulations are based on a hot start, a warm start, and a flat start and solve for a feasible flow rather than the minimum cost power dispatch. The hot start model assumes the voltage magnitudes and power generation are known. It assumes that the voltage bus angles \( \theta \) are small enough such that \( \sin(\theta) \approx \theta \), but does not assume that \( \cos(\theta) \approx 1 \). Rather, it solves for the tightest linear approximation to the cosine function while satisfying load flow. The warm start model is mostly the same as the hot start model, except that the voltages are now a variable rather than an input, and the real and reactive power formulas are linearized about these target voltages. In the cold start model, the linearization point is assumed to be a voltage magnitude of 1, and the real and reactive power formulas are linearized around this point instead. The warm-start formulation appeared to capture most of the AC solution well, with the approximation and AC voltage correlated at an R-value of 0.976 or higher.

### 2.4.5 DC Approximation

The DCOPF is a linearization of the ACOPF PV polar formulation. It assumes the voltage angle differences are small, resistance is negligible compared to the reactance \( (R \ll X \text{ or } G \ll B) \), and all voltage magnitudes \( v_n \) are equal to 1. The DCOPF approximation objective is presented in (2.4.3) and its constraints are given in (2.4.4) through (2.4.11). The dual price for each constraint is listed to the right of its corresponding constraint.

\[
\begin{align*}
\text{Min} & \sum_{n \in N} \sum_{i \in G} C_{n,i}^g \left( p_{n,i}^g \right) \\
\text{subject to} & \\
p_{k(n,m)} & = B_{k(n,m)} \left( \theta_n - \theta_m \right) \\
p_{n,i}^g & \geq P_{n,i}^{\min} \\
- p_{n,i}^g & \geq -P_{n,i}^{\max} \\
p_{k(n,m)} & \geq -P_{k(n,m)}^{\max} \\
- p_{k(n,m)} & \geq -P_{k(n,m)}^{\max} \\
\sum_{m \in A(n)} p_{k(n,m)} & = - \sum_{i \in G} p_{n,i}^d + P_n^d \\
\lambda_n & \\
\theta_n - \theta_m & \geq -\theta_{nm}^{\max} \\
\theta_m - \theta_n & \geq -\theta_{nm}^{\max}
\end{align*}
\]

Many consider finding the optimal power flow to be a solved problem. However, finding an AC-feasible solution is very important. The DC solution is often not AC feasible due to its neglect of reactive power and losses [27]: therefore, lines could exceed
their thermal limits and be damaged. Switching using the DC approximation often produces a network configuration with no feasible power production schedule to satisfy network requirements. Coffrin \cite{26} finds that while the DC solution may predict active power well, it has significant errors in determining the bus angle and reactive power versus the AC solution.

2.5 Real-Time Market Operations

The ISO’s process of determining power generation for its area is very complex. It involves running a day-ahead and five-minute-ahead market as well as completing a security analysis. This thesis focuses on the real-time market. This section introduces the formulations that are used to determine prices and settlements in the market.

2.5.1 DC Dual Formulation and Prices

The dual of the DCOPF B-Theta model is given in (2.5.1) through (2.5.4).

\[
\begin{align*}
\text{Max} & \quad \sum_{n \in N} \left[ \sum_{k \in K} \mu_{k(n,m)}^\max P_{k(n,m)}^\max + \mu_{k(n,m)}^\min P_{k(n,m)}^\min \right] \\
& + \sum_{n \in N} \sum_{i \in G} \left[ \rho_{n,i} P_{n,i}^\max - \rho_{n,i} P_{n,i}^\min \right] \\
& + \sum_{n \in N} \sum_{m \in N} \theta_{nm}^\max \left( \gamma_{nm}^\max + \gamma_{nm}^\min \right) - \lambda_n P_n^d
\end{align*}
\] (2.5.1)

subject to

\[
\begin{align*}
\rho_{n,i}^\max - \rho_{n,i}^\min - \lambda_n^P &= \frac{\partial C_{n,i}^g \left( p_{n,i}^g \right)}{\partial p_{n,i}^g} \quad \hat{p}_{n,i}^g \\
\delta_{k(n,m)} - \delta_{k(m,n)} + \mu_{k(n,m)}^\max - \mu_{k(n,m)}^\min + \lambda_n^P - \lambda_m^P &= 0 \\
\sum_{m \in A(n)} B_{k(n,m)} \delta_{k(n,m)} - \sum_{m \in A(n)} B_{k(m,n)} \delta_{k(m,n)} + \gamma_{nm}^\max - \gamma_{nm}^\min &= 0 \\
\end{align*}
\] (2.5.2, 2.5.3, 2.5.4)

In the primal problem, we are trying to minimize the generator cost (maximize social welfare). In the dual problem, we are trying to maximize the network value. The system revenue is the load payment minus payments to generation and to holders of Financial Transmission rights. We can rewrite the dual objective function (2.5.16) by
using complementary slackness as detailed in (2.5.5) through (2.5.15).

\[ \mu_{k(n,m)}^{\min} \left( p_{k(n,m)} + p_{k(n,m)}^{\max} \right) = 0 \]  (2.5.5)

\[ -\mu_{k(n,m)}^{\min} p_{k(n,m)} = \mu_{k(n,m)}^{\min} p_{k(n,m)}^{\max} \]  (2.5.6)

\[ p_{k(n,m)}^{\max} \left( -p_{k(n,m)} + p_{k(n,m)}^{\max} \right) = 0 \]  (2.5.7)

\[ \mu_{k(n,m)}^{\max} p_{k(n,m)} = \mu_{k(n,m)}^{\max} p_{k(n,m)}^{\max} \]  (2.5.8)

Now, we also have from (2.5.3) that

\[ \mu_{k(n,m)}^{\max} - \mu_{k(n,m)}^{\min} = -\delta_{k(n,m)} + \delta_{k(m,n)} - \lambda_{n}^{P} + \lambda_{m}^{P} \]  (2.5.16)

The second and third terms can be written as in (2.5.9) through (2.5.15).

\[ -\mu_{k(n,m)}^{\max} p_{k(n,m)} + \mu_{k(n,m)}^{\min} \]  (2.5.9)

\[ = -p_{k(n,m)} \left( \mu_{k(n,m)}^{\max} - \mu_{k(n,m)}^{\min} \right) \]  (2.5.10)

\[ = p_{k(n,m)} \left( \lambda_{m}^{P} - \lambda_{n}^{P} - \delta_{k(n,m)} + \delta_{k(m,n)} \right) \]  (2.5.11)

\[ \sum_{k \in K} p_{k(n,m)} \left( \mu_{k(n,m)}^{\max} - \mu_{k(n,m)}^{\min} \right) \]  (2.5.12)

\[ = \sum_{k \in K} p_{k(n,m)} \left( \lambda_{m}^{P} - \lambda_{n}^{P} - \delta_{k(n,m)} + \delta_{k(m,n)} \right) \]  (2.5.13)

\[ = \sum_{k \in K} p_{k(n,m)} \left( \lambda_{m}^{P} - \lambda_{n}^{P} \right) + \sum_{k \in K} p_{k(n,m)} \left( -\delta_{k(n,m)} + \delta_{k(m,n)} \right) \]  (2.5.14)

\[ = \sum_{k \in K} p_{k(n,m)} \left( \lambda_{m}^{P} - \lambda_{n}^{P} \right) \]  (2.5.15)

So, the dual can be rewritten as in (2.5.16).

\[ \sum_{n \in N} \lambda_{n}^{P} p_{n}^{d} - \sum_{k \in K} p_{k(n,m)} \left( \lambda_{m}^{P} - \lambda_{n}^{P} \right) - \sum_{n \in N} \left( \frac{\partial C_{n,i}^{g}(p_{n,i})}{\partial p_{n,i}^{g}} \right) p_{n,i}^{g} \]  (2.5.16)

This is the load payment minus the congestion rent minus the generation rent. This is the generator’s problem: to maximize their payment subject to what the ISO is willing to pay.

### 2.5.2 Market Prices and Settlements

**Load Payment:** What each load pays for power. Each node pays the LMP times the amount of power it demands, \( \lambda_{n} p_{n}^{d} \). The total load payment is \( \sum_{n \in N} \lambda_{n} p_{n}^{d} \).

**Generation Revenue:** The total amount paid to each generator. This consists of the generation rent plus the generation cost. This is equal to \( \lambda_{n} p_{n,i}^{g} \) for generator \( i \).
CHAPTER 2. BACKGROUND

Generation Rent: Analogous to generation profit, \( \lambda g p_{n,i}^g - C_{n,i}^g (p_{n,i}^g) \).

Generation Cost: What it costs each generator to produce the amount of power \( p_{n,i}^g \): \( C_{n,i}^g (p_{n,i}^g) \).

Flowgate Marginal Price: The value of another unit of capacity on line \( k \). This is represented as \( \mu_k = \mu_{k_{\text{max}}} - \mu_{k_{\text{min}}} \).

Admittance Price: This is analogous to the value of the admittance of the line, and equals \( \mu_k - (\lambda_m^P - \lambda_n^P) \).

The Locational Marginal Price (LMP) for real power is \( \lambda_n^P \). Load pays \( \lambda_n^P \) per unit of power consumed. Generation gets paid \( \lambda_n^P \) per unit of power generated. If there is no congestion and losses are neglected, then all the nodal prices are the same, and \( \sum_{n \in N} \sum_{i \in G} \lambda_n^P p_{n,i}^g = \sum_{n \in N} \lambda_n^P p_{n}^d \). However, if there is congestion, then the difference is the congestion rent, which is \( \sum_{k \in K} (\lambda_n^P - \lambda_m^P) p_{k(n,m)} \). This is returned to the market in the form of Auction Revenue Rights or Financial Transaction Rights.

While the B-Theta model gives a more intuitive dual, ISOs more commonly use the distribution-factor model, which has fewer constraints and is faster to solve. ISOs typically only include constraints on line flows that are frequently violated.

After a solution to the DCOPF is found, a power flow is typically run to see if the solution is feasible. The ISO often adjusts voltages and incorporates losses based on the power flow solver. To get feasibility, the ISO may also reduce line limits or add more nomogram constraints. Once a feasible solution is found, generators are then instructed to produce the necessary reactive power. Typically, if the generator’s power factor is close to 1 even with the reactive power, the generator is not compensated. Reliability must run contracts with generators are in place to ensure enough reactive power is present in the network. Generators are either paid at contract or for their lost opportunity costs, depending on the contract they have with the ISO.

2.5.3 ISO Corrections

The ISO often has to adjust the DCOPF to better suit its needs. If the ISO sees that there may be a voltage problem at bus \( n \), it often imposes limits on the flow coming into \( n \), called nomogram constraints. A nomogram constraint may take the form shown in (2.5.17) where some linear combination of flows into the node have some overall limit.

\[
\sum_{k \in K} a_{nk} p_{k(n,m)} \leq P_{n}^{\text{tot}} \tag{2.5.17}
\]

These nomogram constraints may have non-zero dual prices if they are binding, and the dual of the formulation used to solve the market would have an extra term. Alvarado [28] shows that you may meet a voltage constraint via four different possible ways of adding nomogram constraints, and while the primal solution is the same in all cases,
the LMPs are very different in the different cases. Therefore, we may be distorting
the market by not using true voltage limits. Additionally, the ISOs also make sure
to incorporate losses into the formulation. One issue here is that the loss estimations
can change greatly depending on the reference bus \[29\]. The loss estimations then will
change the prices.

2.6 Common Power Network Data Sets

There are several standard test sets used to test power flow algorithms. Here, we
test on the IEEE and Polish test systems. The data for these systems was acquired from
MATPOWER \[30\].

2.6.1 IEEE Test Systems

The IEEE systems were developed based on sections of the midwestern U.S. grid in
1962. They include sizes of 14, 30, 57, 118, and 300 buses. The characteristics of these
networks are shown in Table 2.2.

<table>
<thead>
<tr>
<th># Buses</th>
<th># Lines</th>
<th># Generators</th>
<th>Gen Total Cap.</th>
<th>Total Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>20</td>
<td>5</td>
<td>7.724</td>
<td>2.590</td>
</tr>
<tr>
<td>30</td>
<td>41</td>
<td>6</td>
<td>3.350</td>
<td>1.892</td>
</tr>
<tr>
<td>57</td>
<td>80</td>
<td>7</td>
<td>19.759</td>
<td>12.508</td>
</tr>
<tr>
<td>118</td>
<td>186</td>
<td>54</td>
<td>99.66</td>
<td>42.42</td>
</tr>
<tr>
<td>300</td>
<td>411</td>
<td>69</td>
<td>326.78</td>
<td>235.26</td>
</tr>
</tbody>
</table>

Table 2.2: IEEE Network Characteristics

In these IEEE cases, the networks are fairly sparse, with the number of transmission
lines being less than twice the number of buses. Except for the 300-bus system, the
demand is significantly lower than the generation capacity in the network.

2.6.2 Polish Test Systems

The Polish systems are considerably larger than the IEEE systems. These include
sizes of 2383, 2736, 2737, 2746, 3012, 3120, and 3375 buses. Characteristics of these
systems are displayed in Table 2.3.
<table>
<thead>
<tr>
<th>Year</th>
<th>Season</th>
<th>Peak?</th>
<th># Buses</th>
<th># Lines</th>
<th># Generators</th>
<th>Gen Total Capacity</th>
<th>Total Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Winter</td>
<td>1999-2000</td>
<td>Peak</td>
<td>2383</td>
<td>2896</td>
<td>327</td>
<td>295.94</td>
<td>245.58</td>
</tr>
<tr>
<td>Summer</td>
<td>2004</td>
<td>Peak</td>
<td>2736</td>
<td>3504</td>
<td>420</td>
<td>288.80</td>
<td>180.75</td>
</tr>
<tr>
<td>Summer</td>
<td>2004</td>
<td>Off-Peak</td>
<td>2737</td>
<td>3506</td>
<td>399</td>
<td>270.75</td>
<td>112.67</td>
</tr>
<tr>
<td>Winter</td>
<td>2003-2004</td>
<td>Off-Peak</td>
<td>2746</td>
<td>3514</td>
<td>514</td>
<td>301.08</td>
<td>189.62</td>
</tr>
<tr>
<td>Winter</td>
<td>2003-2004</td>
<td>Evening Peak</td>
<td>2746</td>
<td>3514</td>
<td>520</td>
<td>321.69</td>
<td>248.73</td>
</tr>
<tr>
<td>Winter</td>
<td>2007-2008</td>
<td>Evening Peak</td>
<td>3012</td>
<td>3572</td>
<td>502</td>
<td>352.22</td>
<td>271.70</td>
</tr>
<tr>
<td>Summer</td>
<td>2008</td>
<td>Morning Peak</td>
<td>3120</td>
<td>3693</td>
<td>505</td>
<td>355.85</td>
<td>211.81</td>
</tr>
<tr>
<td>Winter</td>
<td>2007-2008</td>
<td>Evening Peak</td>
<td>3375</td>
<td>4161</td>
<td>596</td>
<td>710.95</td>
<td>483.63</td>
</tr>
</tbody>
</table>

Table 2.3: Polish System Characteristics

In the Polish system, the suffix at the end of the number of buses signifies the season, with w for winter and s for spring, and whether the case is from a peak time, p, or an off-peak time, op. For example, the 2383wp case has 2383 buses and its data is from a time of peak demand in the winter. The Polish systems generally have adequate generation capacity and have an even sparser network structure than the IEEE cases. These systems come with line limits on apparent power. We can translate these to limits on current using equation (2.2.10); this is discussed further in Chapter 3.
Chapter 3

Creating Current Limits for IEEE and Polish Systems

The amount of current that can flow through power system transmission lines is limited by thermal restrictions. The thermal ratings of the transmission lines are functions of the materials that compose them and environmental conditions. Heat loss on a transmission line is proportional to the square of the line’s current. If the line current exceeds the recommended limit for too long, the excessive heat caused by the heat loss can deform and degrade transmission lines and cause them to sag. Limiting current is the most direct measurement to limit the temperature of the line; however, this thermal limit is often converted to a limit on apparent or real power as it is easier to represent in traditional optimal power flow (OPF) formulations that consider the variables of voltage and power but not current. In the current power market, limits are placed on the real power of the line since real power is a traded quantity and current is not. Additionally, current and other line limits may be imposed to enforce stability.

The IEEE test systems and Polish systems are commonly used to test new algorithms for solving power flow problems. However, no IEEE test problems include current magnitude limits on the transmission lines, and many of the test systems include no thermal limits at all, including the 14-, 57-, 118-, and 300-bus systems. If a test problem does not include these limits, reasonable constraints may need to be created for testing purposes. In MATPOWER, the 30-bus system contains apparent power limits on lines; these constraints do not cause much congestion and therefore do not give much insight into a stressed system. The Polish systems do contain line limits, but these limits are on apparent power and need to be converted to limits on current.

In the absence of thermal constraints, such as in the 14-bus to 300-bus systems, one approach is to create constraints based on the physical characteristics and expected environmental conditions. Often, there is little information available about the lines. It takes considerable time to develop constraints based on physical characteristics, and

Some or all of the work in this chapter appeared in [31]. My co-authors provided guidance and feedback on the project. I completed the majority of the writing, testing, and coding.
CHAPTER 3. CREATING CURRENT LIMITS FOR IEEE AND POLISH SYSTEMS

it may be that these constraints rarely cause congestion and therefore are not very interesting for testing algorithms.

This chapter develops a methodology for creating constraints on the line current magnitude for the IEEE and Polish networks. It establishes standard limits that can be used in the testing of power flow algorithms. Binding constraints based on maximum current magnitude are created rather than on constraints on apparent power on the lines or on voltage angle differences.

For systems without limits, the proposed approach is to create constraints from the optimal solution (with no line limits imposed). Subsequent testing explores how modifying the line limits, created from the solution of the non-constrained problem, affects the resulting power flow solution. For the Polish systems, the apparent power limits are converted to current limits. Two types of line constraints are proposed for the test systems. The "tight" constraint level restricts current on lines to a lower amount than the "loose" constraint level.

The rest of the chapter is organized as follows: the background of how line constraints work is discussed in Section 3.1. Section 3.2 describes the approach to the formulation used and how the line constraints are constructed. Numerical results and the recommended line limits are given in Section 3.3. The chapter is summarized in Section 3.4.

3.1 Background

Chapters 1 and 2 established the use and formulation of the Alternating Current Optimal Power Flow Problem (ACOPF). Here, we discuss how the thermal limits are set for transmission lines and discuss how current and apparent power on lines are related.

3.1.1 Current Line Limits Standards

Transmission line limits used in solving power flow problems include limits on real power, voltage angle difference, apparent power, and current. The IEEE Standard 33 suggests calculating a thermal limit on current based on environmental conditions and material properties. Transmission lines generally have maximum temperature ratings. These include a normal and emergency rating, and these ratings typically depend on the characteristics of the conductor material. Resistance of a line is dependent on temperature; increasing the temperature increases the resistance. Since the IEEE test systems included only steady-state data, the current limits are also derived at steady-state. While there are many different ways to calculate maximum current, the IEEE Standard 738, shown in (3.1.1), provides one well-accepted method. In the IEEE Standard 738, \( q_c \) is the convected heat loss, \( q_r \) is the radiated heat loss, \( q_s \) is the solar heat gain, and \( R(T_C) \) is the resistance of the line per length at temperature \( T_C \).
CHAPTER 3. CREATING CURRENT LIMITS FOR IEEE AND POLISH SYSTEMS

\[ I_{\text{max}} = \sqrt{\frac{q_c + q_r - q_s}{R(T_C)}} \] (3.1.1)

The physical properties that increase the current line limit include a higher maximum temperature rating, larger line diameter, and higher line emissivity. The environmental factors that increase this limit include a higher wind speed, a lower ambient temperature, and less direct sun. The IEEE test systems present part of the Midwest electric grid. While Bočkarjova and Andersson [34] derived current line constraints on the IEEE 14-bus system using the IEEE 738 Standard, they needed to make extensive assumptions about the network characteristics and environmental conditions in order to use the IEEE standard. The data necessary to compute line limits from the IEEE standards is not readily available. One would need the diameter of the lines, their maximum temperature ratings, ambient temperature, line length, and many other parameters that are not given. Therefore, this chapter examines creating line limits based on limits that bind under nominal demand conditions rather than trying to derive them from physical properties.

3.1.2 Converting Apparent Power Limits to Current Limits

The limits on real power and voltage angle differences are most often used when solving a DC approximation of the ACOPF as reactive power is neglected in the DCOPF. Limits on apparent power, as shown in (3.1.5)-(3.1.6), or current (3.1.7) are often used when solving the AC power flow formulation. A limit on apparent power or real power (3.1.2) is often used for line limits for the power-voltage formulation of the ACOPF, and a limit on current is often used for the current-voltage formulation. The formula for reactive power on a line is shown in (3.1.3). While all of the different limits have the impact of preventing temperature on the lines from becoming too large, the different limits are not equivalent. The limits can only be equated at a fixed operating voltage magnitude and phase.

\[ p_{k(n,m)} = i_{k(n,m)}^r v_n^r + i_{k(n,m)}^j v_n^j \] (3.1.2)
\[ q_{k(n,m)} = i_{k(n,m)}^r v_n^j - i_{k(n,m)}^j v_n^r \] (3.1.3)
\[ s_{k(n,m)} = p_{k(n,m)} + jq_{k(n,m)} \] (3.1.4)
\[ |s_{k(n,m)}| = \sqrt{p_{k(n,m)}^2 + q_{k(n,m)}^2} \] (3.1.5)
\[ s_{k(n,m)}^2 = v_n^2 i_{k(n,m)}^2 \] (3.1.6)

Given a limit on the apparent power on the line, the equivalent limit on the current flow on the line can be bounded, as shown in (3.1.7).

\[ \frac{S_{k(n,m)}^{\text{max}}}{V_n^{\text{max}}} \leq I_{k(n,m)}^{\text{max}} \leq \frac{S_{k(n,m)}^{\text{max}}}{V_{n\text{min}}} \] (3.1.7)
CHAPTER 3. CREATING CURRENT LIMITS FOR IEEE AND POLISH SYSTEMS

3.2 Methodology

For the IEEE problems, the problems are solved without line constraints, and tight and loose limits are created based on restricting current below the highest current when the OPF was solved. For the Polish systems, the apparent power limits are converted to current limits. The superscripts and subscripts in this chapter are exactly the same as Chapter 2.1 so they are not included here. While many of the variables and parameters are the same as in Section 2.1, there are enough differences that a full list of variables and parameters are given in this section. One major change from Chapter 2 is that we aggregate all the different generators at a node \( n \) and average their costs; that is, 
\[
p^g_n = \sum_{i \in G(n)} p^g_{n,i}.
\]

3.2.1 Nomenclature

Variables:

- \( i^r_{k(n,m)} \): Real component of current on line \( k(n,m) \) at bus \( n \)
- \( i^i_{k(n,m)} \): Imaginary component of current on line \( k(n,m) \) at bus \( n \)
- \( i^r_n \): Real component of current injection at bus \( n \)
- \( i^i_n \): Imaginary component of current injection at bus \( n \)
- \( p_{k(n,m)} \): Real power across line \( k(n,m) \)
- \( p^g_n \): Real power generated at bus \( n \)
- \( p^g_{n,l} \): Step division of \( p^G_n \)
- \( p^\text{viol,−}_n \): Amount by which the minimum real power generation at bus \( n \) is violated
- \( p^\text{viol,+}_n \): Amount by which the maximum real power generation at bus \( n \) is violated
- \( q_{k(n,m)} \): Reactive power across line \( k(n,m) \) at bus \( n \)
- \( q^g_n \): Reactive power generated at bus \( n \)
- \( q^\text{viol,−}_n \): Amount by which the minimum reactive power generation at bus \( n \) is violated
- \( q^\text{viol,+}_n \): Amount by which the maximum reactive power generation at bus \( n \) is violated
- \( s_{k(n,m)} \): Apparent power across line \( k(n,m) \) at bus \( n \)
- \( v^\text{viol,−}_n \): Amount by which the minimum voltage requirement at bus \( n \) is violated
- \( v^\text{viol,+}_n \): Amount by which the maximum voltage requirement at bus \( n \) is violated
- \( v^r_n \): Real component of voltage at bus \( n \)
CHAPTER 3. CREATING CURRENT LIMITS FOR IEEE AND POLISH SYSTEMS

$v_n^j$ Imaginary component of voltage at bus $n$

Parameters:

$B^\text{MVA}$ Power base in MVA

$C_n^{q,2}$ Cost coefficient on the square of $p_n^G$ in MATPOWER

$C_n^{q,1}$ Cost coefficient on $p_n^G$ in MATPOWER

$C_{n,\text{pen}}$ Penalty cost of violating constraints at bus $n$

$C_n^{q,2}$ Step-cost of real power at bus $n$ at step $l$

$B_{k(n)}$ Electrical susceptance of transmission line $k$ at bus $n$

$G_{k(n)}$ Electrical conductance of transmission line $k$ at bus $n$

$I_{\text{max}}^{k(n,m)}$ Maximum allowed current on line $k(n,m)$

$P_n^d$ Real power demand at bus $n$

$Q_n^d$ Reactive power demand at bus $n$

$P_{n,\text{min}}$ Minimum required real power at bus $n$

$P_{n,\text{max}}$ Maximum allowed real power at bus $n$

$P_n^g$ Step size for real power at bus $n$; equals $\frac{P_{n,\text{max}} - P_{n,\text{min}}}{|\mathcal{L}|}$

$Q_{n,\text{min}}$ Minimum required reactive power at bus $n$

$Q_{n,\text{max}}$ Maximum allowed reactive power at bus $n$

$V_{n,\text{min}}$ Minimum required voltage magnitude at bus $n$

$V_{n,\text{max}}$ Maximum allowed voltage magnitude at bus $n$

$|\mathcal{L}|$ Total number of steps for the stepwise approximation

$l$ Index of the step; $l = 1, 2, \ldots, |\mathcal{L}|$

### 3.2.2 Formulation

The current-voltage formulation of the ACOPF, first discussed in [35] and used to solve power flow in [36], is used to consider the different current constraints. This formulation is given in [2.3.3] The current constraints could also be added to other formulations of the ACOPF, such as the polar or rectangular voltage-based formulations, given in Sections [2.3.1] and [2.3.2].
CHAPTER 3. CREATING CURRENT LIMITS FOR IEEE AND POLISH SYSTEMS

3.2.2.1 Linearizing the Cost Function

The cost function used by MATPOWER [30] is a quadratic function, shown in (3.2.1).

\[
\text{Cost} = \sum_{n \in \mathcal{N}} C_{g,2}^n \left( p_n^g \right)^2 + C_{g,1}^n p_n^g
\]  

(3.2.1)

However, most Independent System Operators and Regional Transmission Organizations accept power bids as step functions rather than a continuous cost. Therefore, the objective considered is the piecewise linear function that approximates the quadratic function. To construct the piecewise linear function, the interval \([P_{\text{min}}^n, P_{\text{max}}^n]\) is broken into \(|L|\) linear segments with length \(P_g^l = \left( P_{\text{max}}^n - P_{\text{min}}^n \right) / |L|\). There are \(|L|+1\) points where the \(l\)-th segment is associated with points in \([x_l, x_{l+1}]\), shown in (3.2.2).

\[
[x_l, x_{l+1}] := \left[ P_{\text{min}}^n + lP_g^l, P_{\text{min}}^n + (l + 1)P_g^l \right].
\]  

(3.2.2)

As long as \(|L| > 1\), the points are increasing and nonoverlapping; for \(l = 0, \ldots, |L|\), we have that \(x_0 < x_1 < \ldots < x_{|L|+1}\).

For each segment \(l \in \mathcal{L}\) and bus \(n \in \mathcal{N}\), we calculate the midpoint and apply the slope of the offer curve in order to determine the resulting cost coefficient in (3.2.3).

\[
C_{g,n,l} = C_{g,1}^n + \left( B_{\text{MVA}} \right)^2 C_{g,2}^n (x_l + x_{l+1}).
\]  

(3.2.3)

The \(B_{\text{MVA}}^2\) accounts for any per-unit scaling of the power variables. Equation (3.2.4) is the piecewise linear approximation of the aggregate offer curve in (3.2.1).

\[
\text{offers}(\cdot) = \sum_{n \in \mathcal{N}} \sum_{l \in \mathcal{L}} C_{g,n,l}^g p_{n,l}^g + C_n^0
\]  

(3.2.4)

If \(P_{\text{min}}^n > 0\), \(C_n^0 = C_{g,2}^n (P_{\text{min}}^n)^2 + C_{g,1}^n P_{\text{min}}^n\); otherwise, \(P_{\text{min}}^n = 0\) and \(C_n^0 = 0\).

Each segment \(p_{n,l}^g\) of the piecewise linear function is limited by \(P_g^l\), as shown in (3.2.5), for all \(n \in \mathcal{N}\) and \(l \in \mathcal{L}\).

\[
p_{n,l}^g \leq P_g^l = \frac{P_{\text{max}}^n - P_{\text{min}}^n}{|L|}
\]  

(3.2.5)

The aggregate of the segments at bus \(n\) must equal the total real power generation for all \(n \in \mathcal{N}\), as shown in (3.2.6).

\[
p_n^g = \sum_{l \in \mathcal{L}} p_{n,l}^g + P_{\text{min}}^n
\]  

(3.2.6)

If all cost coefficients are positive, which is true for all of the systems tested, the real power cost is convex as a function of real power, so we know that the next increment of real power generation \(p_{n,l}^g\) takes on a non-zero value only if the previous increment of real power generation is at its maximum, \(p_{n,l-1}^g = \frac{P_{\text{max}}^n - P_{\text{min}}^n}{|L|}\). Typically, generator offer curves are monotonically increasing, so all cost coefficients would be positive and the cost is convex.

Figure 3.1 displays the quadratic MATPOWER cost versus the piecewise linear approximation. The costs match perfectly at the beginning and end of each step and the estimated cost is slightly higher than the quadratic cost.
3.2.3 Current-Voltage Formulation

In addition to the cost for real power, here we put a small cost reactive power. It is priced at a fraction of real power to account for the reactive power support \[37\]. Often, better convergence results from using ‘soft’ rather than ‘hard’ constraints. Soft constraints allow deviations from the limits (for real power, reactive power, and voltage) but add the penalty cost of these violations to the objective function \[38\]. For this model, small deviations from the limits were allowed but penalized greatly in the objective function, with penalties on voltage violations \(V^\epsilon\), real power violations, \(P^\epsilon\), and reactive power violations, \(Q^\epsilon\), set to 100,000.

\[
\text{Min Cost} = \sum_{n \in N} \left[ \sum_{l \in E} C^{g,2}_{n,l} P^{g}_{n,l} + C^{g,1}_{n} p^{g}_{n} + \frac{1}{10} \left( C^{g,2}_{n} + C^{g,1}_{n} \right) q^{g}_{n} ight. \\
+ \left. V^\epsilon_n \left( v^{\text{viol},-}_n + v^{\text{viol},+}_n \right) + P^\epsilon_n \left( p^{\text{viol},-}_n + p^{\text{viol},+}_n \right) + Q^\epsilon_n \left( q^{\text{viol},-}_n + q^{\text{viol},+}_n \right) \right] 
\]  

\[(3.2.7)\]

such that:

\[
\tilde{I}^\epsilon_{k(n,m)} = G_k(n)v^\epsilon_n - G_k(m)v^\epsilon_m - B_k(n)v^j_n + B_k(m)v^j_m
\]

\[(3.2.8)\]
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\[ i_{k(n,m)}^{ij} = B_k(n)v_n^r - B_k(m)v_m^r + G_k(n)v_n^j - G_k(m)v_m^j \]  
\[ i_n^r = \sum_{m \in A(n)} i_{k(n,m)}^r + G_{sh}^n v_n^r - B_{sh}^n v_n^j \]  
\[ i_n^j = \sum_{m \in A(n)} i_{k(n,m)}^j + B_{sh}^n v_n^r + G_{sh}^n v_n^j \]  
\[ (v_n^r)^2 + (v_n^j)^2 \leq (V_{\text{max}}^n)^2 + v_{\text{viol},+}^n \]  
\[ - v_{\text{viol},-}^n + (V_{\text{min}}^n)^2 \leq (v_n^r)^2 + (v_n^j)^2 \]  
\[ (i_{k(n,m)}^r)^2 + (i_{k(n,m)}^j)^2 \leq (I_{\text{max}}^k)^2 \]  
\[ p_n^g = v_n^r i_n^r + v_n^j i_n^j + P_d^n \]  
\[ p_n^g = P_{\text{min}}^n + \sum_{l \in L} p_{n,l}^g \]  
\[ 0 \leq p_{n,l}^g \leq P_{n}^g \]  
\[ q_n^g = v_n^j i_n^r - v_n^i_i_n^j + Q_d^n \]  
\[ p_n^g \leq P_{\text{max}}^n + p_{n,\text{viol},+}^n \]  
\[ p_n^g \geq P_{\text{min}}^n - p_{n,\text{viol},-}^n \]  
\[ q_n^g \leq Q_{\text{max}}^n + q_{n,\text{viol},+}^n \]  
\[ q_n^g \geq Q_{\text{min}}^n - q_{n,\text{viol},-}^n \]

The upper bounds in (3.2.12) and (3.2.14) are nonlinear and convex; equations (3.2.15), (3.2.18), and the lower bound in (3.2.13) are nonlinear and nonconvex. The LMP is derived from the dual variable to (3.2.15). In this formulation, we use hard rather than soft constraints on current (3.2.14) to ensure we see the true impact of the current limits.

3.2.4 IEEE Systems: Create One Single Limit

The IEEE systems do not contain thermal constraints. Here, the approach to creating current constraints is to set the same limit on current on every line. This approach would be closest to the physical system if it is assumed that all lines experience similar environmental conditions and are of similar length and material.

\[ I_{\text{max}}^{k(n,m)} = I_{\text{max}}^k \quad \forall k \]  

As constraining the lines at a level greater than the highest current in the unconstrained system will not change the optimal solution, the maximum current should be set no greater than this level. Denoting the highest current in the unconstrained system as \( i^* \), the sensitivity of the system to the current limit is analyzed by studying different fractions of \( i^* \) as shown in equation (3.2.24).
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\[
I_{n,m}^{\text{max}} = \left( \frac{t}{100} \right) i^*, \quad t = 1, 2, 3, \ldots, 100 \tag{3.2.24}
\]

3.2.5 Polish Systems: Convert Apparent Power Limits to Current Limits

In these problems, limits on apparent power are given, and so these are converted to current constraints. As shown in equation (3.2.25), the maximum current on a line is bounded by the maximum apparent power.

\[
\frac{S_{n,m}^{\text{max}}}{V_{n,\text{max}}} \leq I_{n,m}^{\text{max}} \leq \frac{S_{n,m}^{\text{max}}}{V_{n,\text{min}}} \tag{3.2.25}
\]

The tight and loose limits can then be set as the lowest (3.2.26) and highest (3.2.27) bounds.

Tight Limit

\[
\frac{S_{n,m}^{\text{max}}}{V_{n,\text{max}}} \tag{3.2.26}
\]

Loose Limit

\[
\frac{S_{n,m}^{\text{max}}}{V_{n,\text{min}}} \tag{3.2.27}
\]

3.3 Numerical Results

In this section, the results for the different current constraints on both the IEEE and Polish systems are given.

3.3.1 Test Cases without Constraints on Current

The ACOPF-IV was formulated in Pyomo [39] and solved using IPOPT version 3.11 [40] with linear solver ma57. If no current constraints are added, the currents on the lines take on the values shown in Table 3.1. In Table 3.1, one sees that the average current is well below the maximum current. Additionally, some lines have near-zero current flowing through them, which appears as zero current due to the current levels being output and rounded to two decimal places.

3.3.2 IEEE System Limits on Current

In Table 3.2, the infeasibility level is defined as the solution where the system is just becomes infeasible; that is, the last current level where the penalty cost is zero. The IEEE systems do not give a feasible power flow solution using IPOPT when the current limit on the line is lowered too far. That is, there is no solution for the system to satisfy the power demand requirements given the very low line capacity.
Chapter 3. Creating Current Limits for IEEE and Polish Systems

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
</tr>
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<tbody>
<tr>
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<td>0.02</td>
<td>0.28</td>
<td>1.14</td>
</tr>
<tr>
<td>30</td>
<td>0.00</td>
<td>0.12</td>
<td>0.35</td>
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<td>57</td>
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<td>0.22</td>
<td>1.77</td>
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<td>0.44</td>
<td>4.02</td>
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<td>0.01</td>
<td>1.316</td>
<td>10.820</td>
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<td>0.36</td>
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<td>2736sp</td>
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<td>3.66</td>
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</tr>
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<td>0.00</td>
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<td>6.22</td>
</tr>
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<td>0.00</td>
<td>0.35</td>
<td>8.66</td>
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<td>0.00</td>
<td>0.31</td>
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<tr>
<td>3375wp</td>
<td>0.00</td>
<td>0.52</td>
<td>10.00</td>
</tr>
</tbody>
</table>

Table 3.1: Unconstrained Current Levels, in p.u.

<table>
<thead>
<tr>
<th># of Buses</th>
<th>Line Current Infeasibility Level (p.u.)</th>
<th>Percent of $i^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>0.227</td>
<td>20%</td>
</tr>
<tr>
<td>30</td>
<td>0.309</td>
<td>89%</td>
</tr>
<tr>
<td>57</td>
<td>1.408</td>
<td>80%</td>
</tr>
<tr>
<td>118</td>
<td>1.136</td>
<td>28%</td>
</tr>
<tr>
<td>300</td>
<td>6.780</td>
<td>63%</td>
</tr>
</tbody>
</table>

Table 3.2: Current Level Below Which the Penalty Cost is $>$ 0 for This Limit on All Lines

As the current limit is lowered, the different IEEE systems behave differently. The current limit can be lowered to less than 30% of the maximum unconstrained value on the 14- and 118-bus systems before the penalty cost is nonzero, while lowering the limit by only 11% for the 30-bus system makes the penalty cost nonzero. The limit on the 300-bus system can be lowered to 63% of its original level before there are problems finding a feasible solution.

Figure 3.2 shows the sensitivity of the systems to various current limits. On the y-axis
is the ratio of the constrained system’s objective function (including penalty costs) to the unconstrained system’s objective function; on the x-axis is the fraction of the maximum unconstrained current that is set as the current maximum (the value of $t$ in (3.2.24)). The point where the curve becomes an asymptote is where the system becomes infeasible. Reducing the current limit has different impacts on the different systems. Examining the ratio of the constrained objective function to the unconstrained objective function, the highest objective function for the 14-bus system with a zero-penalty solution is 26% higher than the unconstrained objective value; for the 30-bus system, it is only 1.7% higher. The objective function for the 57-bus system is only 0.004% higher; the 118-bus system has a larger difference with the objective function being 5.3% higher than the unconstrained function. For the 300-bus system, the objective function at the lowest current limit before the system is infeasible is 0.8% higher than with no limits on current.

Two levels of current limits are suggested - one is a strict limit ("tight") if one wants to examine a highly stressed system; the non-strict limit ("loose") aids in the analysis of a less stressed system. The tight limit is obtained when the penalty cost is close to nonzero; the loose limit is set roughly halfway between this point and the highest unconstrained current.

<table>
<thead>
<tr>
<th># of Buses</th>
<th>Tight Limit (p.u.)</th>
<th>Loose Limit (p.u.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>0.2675</td>
<td>0.7100</td>
</tr>
<tr>
<td>30</td>
<td>0.3125</td>
<td>0.3300</td>
</tr>
<tr>
<td>57</td>
<td>1.4275</td>
<td>1.6000</td>
</tr>
<tr>
<td>118</td>
<td>1.1400</td>
<td>2.3500</td>
</tr>
<tr>
<td>300</td>
<td>6.8200</td>
<td>8.8200</td>
</tr>
</tbody>
</table>

Table 3.3: Recommended Current Limits, One Level for All Lines

Figures 3.3 through 3.7 show the impacts of the different limits on each individual transmission line. If the current on the line is different at the from and to nodes, the larger value is graphed. As the current magnitude constraint is decreased, the effect on other lines is shown in Figure 3.3. In the 14-bus system when current is not constrained, one line has nearly twice the current magnitude as the line with the next highest current. The current magnitudes that change the least under restriction are the lines with comparatively lower current magnitudes. For the loose constraint, only one line is congested. For the tight constraint, four line limit constraints are binding or near binding.
CHAPTER 3. CREATING CURRENT LIMITS FOR IEEE AND POLISH SYSTEMS

Figure 3.2: One Single Limit Results

Figure 3.3: 14-Bus Line Currents at No, Tight, and Loose Limits
For the 30-bus system, the current level could not be restricted much lower than the maximum optimal current level (0.35) before the system became infeasible. In this system, there were two lines that both had high current magnitudes in the unrestricted problem, as seen in Figure 3.4. When restricting the current level, there are many lines where current actually increases on the line versus under no restriction. While the difference between the tight (0.3125) and loose (0.3300) constraints is small, these restrictions do have a large impact on the resulting current flowing through some of the lines. In both the tight and loose limit cases, two lines have binding constraints. In each of these cases, these line connects generation (either directly or indirectly) to a demand node where the other connections are not directly to generators.

For the 57-bus system, shown in Figure 3.5, the optimal solution has most of current magnitudes of half or less the highest current magnitude, similar to the pattern in the 14-bus problem. Most lines have very low current levels compared to the highest line. When the current is restricted, most of the lines exhibit minimal change, while about one third of them have currents that change significantly with the restriction. Like the 30-bus problem, the two current restrictions are close together, with 1.4275 for the tight limit and 1.6000 for the loose limit. For both limits, only two lines are congested. However, even this small difference in limits greatly impacts the current on some lines.
CHAPTER 3. CREATING CURRENT LIMITS FOR IEEE AND POLISH SYSTEMS

The 118-bus system, shown in Figure 3.6, has several lines with higher currents, similar to the 30- and 57-bus systems. The current can be restricted greatly before the problem becomes infeasible. Enforcing the tight limit has the most impact on the highest currents. Constraining the current with the loose limit impacts the currents slightly; however, constraining the current with the tight limit has a major impact on the current magnitudes. With the loose constraint, only two lines are congested. Under the tight current magnitudes, 10 of the lines have the highest permissible current under this restriction, and even lines with lower currents exhibit big differences from the unrestricted and loose current cases, both higher and lower than before.
As seen in Figure 3.7, under no restrictions, the 300-bus system has two lines with very high current magnitudes (10.82 and 10.77 p.u.), the remaining lines have current magnitudes of about 70% of that value of current or lower (all less than 7.63 p.u.). Reducing the maximum current level reduces the line current of about 2/3 of the lines and increases it on about 1/3 of the lines. The per unit value of current on lines in the 300-bus case is much higher than in the other 4 IEEE test cases.
As shown in Figure 3.8, the current magnitude constraints increase the objective value to different degrees in the different systems. The 14-bus case has the highest increase in the objective value from adding current limits, then 118, then 30 and 300, and finally 57. The 14- and 118-bus cases could have currents reduced greatly before active penalties are required to solve the problem, while 30- and 57-bus could not.
3.3.3 Polish System Limits on Current

Including the current limits only raises the objective function slightly in most cases, as given in Table 3.4. The largest impact of including line limits is on the Polish system 2383wp, where the tight limit increases system costs by 2%. There is some odd behavior; the cases with loose limits for the networks 2746wop and 3375wp find a cheaper solution than having no limits. This may be due to the interior point solver not finding the global optimum for the case with no limits, or having too large of a convergence tolerance.
CHAPTER 3. CREATING CURRENT LIMITS FOR IEEE AND POLISH SYSTEMS

<table>
<thead>
<tr>
<th>Case</th>
<th>Objective Function Ratio</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Tight Limit</td>
</tr>
<tr>
<td>2383wp</td>
<td>1.0213</td>
</tr>
<tr>
<td>2736sp</td>
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<tr>
<td>2746wop</td>
<td>1.0002</td>
</tr>
<tr>
<td>3012wp</td>
<td>1.0088</td>
</tr>
<tr>
<td>3120sp</td>
<td>1.0034</td>
</tr>
<tr>
<td>3375wp</td>
<td>1.0038</td>
</tr>
</tbody>
</table>

Table 3.4: Objective Function at Different Current Limits
CHAPTER 3. CREATING CURRENT LIMITS FOR IEEE AND POLISH SYSTEMS

3.4 Conclusion

This work has examined the impact of different current limit levels on the IEEE test systems. It has recommended standard current limit settings for testing algorithms on IEEE test systems. These limits are binding under the nominal IEEE test system settings.

For each IEEE network, one single limit is applied to all lines that is binding on at least one line in the network. The resulting problem is solved using the IV-ACOPF formulation. For each problem, a tight and a loose limit are determined. The tight limit highly constrains the system while the loose limit loosely constrains the system. For the 14-, 30-, 57-, 118-, and 300-bus problems, creating line current magnitude constraints for the ACOPF problem can result in problems that may be infeasible. As one tightens the current magnitude constraints, the objective function increases gradually at first, then increases exponentially once penalties are required to solve the problem. Different test problems exhibit different characteristics in the line current magnitude distribution and at what current magnitude level constraint the problem becomes infeasible. The Polish systems exhibit small changes between the constrained and unconstrained systems.

This work has given us appropriate line limits to incorporate into test cases to test our sequential linear programming approximation on. It also gives line limits to use to consider economic switching with.
Chapter 4

A Successive Linear Programming Approach to Solving the IV-ACOPF

The current-voltage ACOPF (IV-ACOPF) was first introduced in Section 2.3.3. This formulation was used to find appropriate current limits for the test cases in Chapter 3. This chapter uses those limits to test a new algorithm for solving the ACOPF. We propose a successive linear programming (SLP) approach to solve the IV-ACOPF, which we refer to as the SLP IV-ACOPF algorithm. The SLP IV-ACOPF leverages commercial LP solvers and can be readily extended and integrated into more complex decision processes such as unit commitment and transmission switching. We demonstrate the algorithm on the IEEE and Polish systems; it shows an acceptable quality of convergence to a best-known solution and linear scaling of computational time in proportion to network size.

The ACOPF finds the optimal power flow for a network with given real and reactive power demands, limits on a generator’s real and reactive power, and limits on the flow across lines. The problem is nonlinear and nonconvex, and finding its globally optimal solution is non-deterministic polynomial-time (NP) hard [44]. Therefore, ISOs do not solve the ACOPF exactly. Not solving the ACOPF exactly has led to inaccurate real power prices and to ignoring the role of reactive power. However, even solving the ACOPF exactly actually may create market issues. If the optimal solution is not locally convex, then we may have a duality gap, so the total market settlement would not be equal to the total power cost.

To find a solution quickly enough and avoid nonconvex pricing issues, ISOs and other grid operators use approximate solution techniques based on linear programming (LP) and mixed-integer linear programming (MILP). System operators frequently solve

Some or all of the work in this chapter appeared in [41], [42] and [43] and was done in collaboration with Anya Castillo, John-Paul Watson, Mehrdad Pirnia, Clay Campagne, Shmuel Oren, and Richard O’Neill. I worked with Clay and Mehrdad on the initial implementation and testing of the number of polygonal starting cuts as well as whether we should implement iterative constraints or not. I worked with Anya on the large-scale production level code. Anya ran the code on the test cases and compiled the results.
a modified direct current optimal power flow (DCOPF) problem or a decoupled OPF. Since a network solution that is feasible for the DC approximation is not necessarily AC feasible, ISOs use an iterative process to obtain AC feasibility. The iterative process ensures that a realistic solution is obtained and identifies constraint violations that may require preventive actions including re-dispatch, reactive power compensation, and voltage support. Some system operators solve a decoupled OPF model, described in 2.4.3 which iterates between $P-\Theta$ and $Q-V$ subproblems to achieve AC feasibility. With these approximation methods, ISOs obtain dispatches and prices within the required time limits. However, these approximations can be inaccurate when the system is stressed or when there is a strong physical coupling between real and reactive power.

The DCOPF and decoupled OPF approaches oversimplify the physical problem and require operator intervention in the day-ahead, intra-day, and real-time markets. Purchala et al. [45] demonstrate that the DCOPF approach is acceptable if the voltage profile is sufficiently flat, the R/X ratio of transmission lines is less than 0.25, and the network is not heavily loaded. Consequently, the system model which is used to clear the market may not be able to physically dispatch the resources that are required to satisfy constraints, such as voltage limits, that are not accurately reflected in the market software. Moreover, MISO intervenes at times in load pockets in order to commit units for voltage and local reliability requirements; producing voltage and reactive power schedules without compromising transmission line limits and efficiency would require ACOPF constraint modeling [46]. A report to congress prepared by the United States Department of Energy (DOE) states: “the technical quality of current economic dispatch tools–software, data, algorithms, and assumptions–deserves scrutiny. Any enhancements to these tools … will improve the reliability and affordability of the nation’s electricity supplies” [47]. Given the cost of upgrading existing ISO market software is less than $10 million dollars [48] and small increases in dispatch efficiency can yield billions of dollars per year in cost savings [49], the potential benefit-to-cost ratio of improved market software is enormous.

The computational tractability and convexity of any solution approach is critical to support market clearing strategies based on the locational marginal price (LMP). The LMP is the marginal cost of supplying the next MW of load at a particular location in time and accounts for the next incremental unit of energy, network congestion, and transmission losses. LMPs are derived from the shadow price on nodal real power balancing constraints in the optimization model. In the DCOPF and decoupled OPF models, the LMP is highly dependent on the modeling assumptions mentioned earlier. Without transparency into the exact form of the problem solved in the markets, it is difficult to assess whether price signals are ensuring efficient and reliable operations. In 2009, Liu et al. [50] reported that full derivations of the LMP are not available in ISO tariffs and other publicly available manuals. Furthermore, the clearing prices in current markets do not reflect the true marginal cost of production, which then results in uplift. Better representations of the AC power system in market software are needed to account for the physical and operational constraints [51].

Over 50 years of work has sought to find ways to solve the ACOPF better, whether...
with improved formulations, relaxations, approximations, decomposition methods, or algorithms; comprehensive surveys can be found in [49,52–59].

This chapter proposes a successive linear program (SLP) approach to solve the IV-ACOPF and presents the results of using this approach on the IEEE and Polish test systems [30]. The SLP-IV-ACOPF approximates the nonlinear, nonconvex constraints with first-order Taylor series approximations and the convex, nonlinear constraints with outer approximations. In each iteration, the evaluation points are updated to be the previous solution of the last iteration. Iterative cuts are added to bring the outer approximation of the convex constraints closer to the actual constraints. The distance the next solution is allowed to move from the previous solution is reduced in each iteration. The process completes after an iteration limit is reached or after the solution is accurate enough. This approach can be extended to include discrete controls and can be embedded within branch-and-bound algorithms to support more complex decision processes, including unit commitment and transmission switching. Since ISO market software depends on commercial LP/MILP solvers, our proposed approach is suitable for such applications, the solution is tractable and yields prices that support the market. We demonstrate an acceptable quality of convergence to the best-known solution and linear scaling of computational time in proportion to network size. The comparable nonlinear programming (NLP) approaches scale slower than linear time. The performance of the SLP, its implementation with commercial solvers, and its convex prices show that it may be implemented in the actual real-time energy market, and perhaps also for unit commitment and corrective switching.

The remainder of this chapter is organized as follows. In Section 4.1 we provide a literature review of seminal and recent linearization and convexification techniques for the ACOPF. In Section 4.3 we summarize the SLP algorithm, and in Section 4.4 we show the detailed formulation of the individual iterations of the SLP. In Section 4.6 we demonstrate the computational performance and convergence quality of our SLP IV-ACOPF algorithm. We conclude in Section 4.7 with a brief discussion of our results.

4.1 Related Work

Most LP approaches assume loose coupling between voltage angle and voltage magnitude, as initially proposed by Alsaç and Stott [60]. However, this assumption is a poor one if the system has high losses, is highly loaded, or either \( P - \theta \) or \( Q - V \) subproblems are infeasible. Kirschen and Meeteren [61] propose an improved method which reschedules real power controls in order to correct voltage magnitude violations that arise in the decoupled subproblem. However, when a high physical coupling in \( P - Q \) exists, even such reschedules can be ineffective.

Coupled models, such as the one proposed in this work, fulfill bus voltage limits and reactive power requirements during MW scheduling. Although the decoupled model has smaller subproblems in each iteration, the coupled version can take fewer iterations and be faster overall depending upon the linearization and underlying network [62]. Other
coupled models include the recent works described in [63–65]. Franco et al. [63] apply a least-squares regression to obtain a non-iterative linear approximation of the ACOPF in terms of the real and imaginary voltage components. However, the study omits numerous physical constraints (e.g., voltage magnitudes, reactive power, and line limits). Mohapatra et al. [64] formulate the ACOPF in terms of incremental variables and solve the nonlinear formulation by applying Newton’s and the primal-dual interior point methods but without line limits. Coffrin et al. [65] apply piecewise linear approximations of the cosine term and Taylor series expansions of the remaining nonlinear terms. All three studies report on limited sized networks.

There is growing interest in applying convexification techniques to the ACOPF. Like linear and quadratic programs, semidefinite programming (SDP) and second-order conic programming (SOCP) approaches can be solved in polynomial time by interior point methods. Such convex relaxations can be valuable in determining bounds on problems that are nonconvex and also to initialize local solution methods. Bai et al. [66] propose a semidefinite relaxation for which Lavaei and Low [44] derive a rank-one sufficient condition under which a globally optimal solution is guaranteed for the SDP relaxation. Lavaei and Low further prove that if the network is radial, then this sufficient condition always holds [44] and the second order conic programming relaxation (SOCP) is equivalent for these cases [67]. This can be attributed to the fact that the power flow solution of a radial network is unique [68]. A shortcoming of these convexifications is that there is no method to recover an ACOPF feasible solution when the sufficient condition is not satisfied. Therefore, Molzahn and Hiskens [69] and Josz et al. [70] propose moment relaxations to obtain tighter lower bounds. However, Lavaei et al. [71] show that even though a global optima is achieved, the solution may not satisfy the sufficient condition. Furthermore, there are still practical difficulties in efficiently implementing a SDP in a mixed integer approach, including the lack of an initialization method and limitations in scalability due to the number of linear algebraic iterations required during the solution process. Moreover, Lesieutre et al. [72] illustrate practical scenarios where the SDP fails to produce physically meaningful solutions.

There has been a number of SOCP formulations applied to the ACOPF [21, 73–76], which is considered a weaker relaxation in general. The computational effort per iteration to solve SOCP problems is greater than that required to solve linear and quadratic programs, but less than that required to solve a SDP problem of similar size and structure. Kokcuk et al. propose strong SOCP formulations, which are an order of magnitude faster than standard SDP formulations but not as tight, and are also an order of magnitude slower yet more accurate than Jabr’s original SOCP formulation [21]. The SOCP solutions are often inexact but with a finite optimality gap; closing the gap may require stronger bounds (which could guarantee a globally optimal outcome when exact) or a local solution method in order to achieve ACOPF feasibility. Although these approaches may be more suitable than SDP in a mixed integer approach, current drawbacks include the need to initialize from a strictly feasible primal-dual pair of solutions, and determining feasibility with respect to the integer variables on inexact solutions. Incorporating convex relaxations into more complex decision processes that also include mixed integer
variables is a growing research area still in early stages of development. Current state-of-the-art advancements continue to demonstrate tradeoffs between convergence quality and computational performance. Given this research context, we proceed to formulate our SLP IV-ACOPF algorithm and demonstrate scalability and performance in obtaining an ACOPF optimal solution on the full range of IEEE and Polish test networks.

4.2 Nomenclature

Sets and indices are as described in Chapter 2. There are some additional variables and parameters in the SLP-IV-ACOPF that are not in the original IV-ACOPF due to the linearization.

Variables:

- $p_n^g$: Total linearized real power generation at bus $n$
- $p_{n,l}^g$: Linear segment $l$ of generation at bus $n$
- $q_n^g$: Reactive power generation at bus $n$
- $v_n$: Voltage magnitude at bus $n$
- $v_{n}^{sq}$: Linearization of $(v_n)^2$
- $v_{n}^r$: Real part of voltage at bus $n$
- $v_{n}^i$: Imaginary part of voltage at bus $n$
- $i_{n}^r$: Real part of current injection at bus $n$
- $i_{n}^i$: Imaginary part of current injection at bus $n$
- $i_{k(n,m)}$: Current magnitude on $k$ from bus $n$ to $m$
- $i_{k(n,m)}^{sq}$: Linearization of $(i_{k(n,m)})^2$
- $i_{k(n,m)}^r$: Real part of current on $k$ from bus $n$ to $m$
- $i_{k(n,m)}^i$: Imaginary part of current on $k$ from bus $n$ to $m$
Parameters:

- \( G^s_{n} \): Shunt conductance at bus \( n \)
- \( B^s_{n} \): Shunt susceptance at bus \( n \)
- \( P^d_{n} \): Real power demand at bus \( n \)
- \( Q^d_{n} \): Reactive power demand at bus \( n \)
- \( P^\text{min}_{n} \): Minimum real power for generation at bus \( n \)
- \( P^\text{max}_{n} \): Maximum real power generation at bus \( n \)
- \( Q^\text{min}_{n} \): Minimum reactive power generation at bus \( n \)
- \( Q^\text{max}_{n} \): Maximum reactive power generation at bus \( n \)
- \( V^\text{min}_{n} \): Minimum voltage magnitude at bus \( n \)
- \( V^\text{max}_{n} \): Maximum voltage magnitude at bus \( n \)
- \( I^\text{max}_{k} \): Maximum current magnitude on line \( k \)
- \( V^h_{n} \): Step-size bound on the voltage at bus \( n \) in iter \( h \)
- \( C^2_{g,n} \): Quadratic cost coefficient for generation at bus \( n \)
- \( C^1_{g,n} \): Linear cost coefficient for generation at bus \( n \)
- \( C^g_{n,l} \): Linear segment \( l \) of the quadratic cost at bus \( n \)
- \( P^g_{n} \): Uniform piecewise segment length of the real power generation at bus \( n \)

### 4.3 Algorithm Outline

The SLP IV-ACOPF solution algorithm is shown in Figure 4.1. In the initial iteration \( (h = 0) \), we initialize variables and the evaluation points for the first order Taylor series approximations. In this chapter, we test evaluation points using a flat start, warm start, uniform start, or cold start. If the calculated voltages or line currents of this initialization are not feasible, we update them to reside within constraint bounds. We then iteratively solve the resulting LP. Following each iteration \( h \), we check whether the solution meets either of the following stopping criteria: (1) the solution is ACOPF feasible within a specified power mismatch tolerance or (2) a maximum iteration limit is reached (see Section 4.4.3). If neither of these criteria are met, we update the Taylor series evaluation points. The evaluation points are denoted by placing a caret or “hat” over the corresponding parameter; for an arbitrary evaluation point \( \hat{x}^{(h)} \), we have that \( \hat{x}^{(h)} = x^* \), which is the optimal solution from iteration \( h - 1 \).
After updating the evaluation points, we update all flowgate monitors in order to identify lines that are near or at capacities. This means we add in line limits for lines with flow above a certain percentage of the maximum flow on the line. For any evaluation point that is ACOPF infeasible according to the nodal voltage magnitude limits or the current line limits, we reset these evaluation points to be within the original bounds. Additionally, we add a tangential cutting plane for each voltage and line current infeasibility to enforce ACOPF feasibility in the following iteration. The step size bounds are also modified before re-solving the LP subproblem; these bounds limit the approximation error and control oscillations in the first order Taylor series. As in Chapter 3, we use soft constraints by adding slack variables but penalize the slack variables in the cost function.

When the process finishes, the algorithm can yield one of the four following outcomes:

1. A KKT optimal solution to the ACOPF is identified
2. The SLP solution is ACOPF feasible but not optimal
3. The SLP solution is ACOPF infeasible
4. the SLP solution is infeasible.

If outcome (1) or (2) occurs, the results are ACOPF feasible and can be implemented into a market setting. The termination of the algorithm does not show whether the outcome is (1) or (2); rather, the final point would be fed to an SDP or put into ACOPF optimality conditions to see if it is a local optima to the original ACOPF problem. If outcome (3) occurs, which happens when we terminate due to hitting the iteration limit or when there are penalty costs, the operator may still be able to use the results for dispatch if the infeasibilities are acceptable. For example, the solution may slightly violate a thermal limit on the line, but the violation may be within an acceptable range.
CHAPTER 4. A SUCCESSIVE LINEAR PROGRAMMING APPROACH TO SOLVING THE IV-ACOPF

If outcome (4) occurs, there are two possibilities: one is that a feasible and bounded ACOPF solution exists, and the other is that no feasible and bounded ACOPF solution exists. If a feasible and bounded ACOPF solution exists, then we would rerun the algorithm with different starting points and different step size limits until a feasible answer was found. If no feasible bounded ACOPF solution exists, then one would need to figure out what resources could be added or demands reduced (for example, demand response, shedding load, turning on another generator) until we can find a feasible solution. If this algorithm was implemented in an ISO context, our starting point would likely be of high quality. Also, we would likely run different starting points and step-size initializations in parallel.

A converged solution meets the mismatch tolerances as set in the stopping criteria. Since the necessary optimality conditions are related to the Taylor series approximations through first order derivatives, and first order Taylor series approximations are applied to all the nonlinear terms in the formulation, we are guaranteed an optimal solution within the specified tolerances when the SLP algorithm converges with no active penalties.

4.4 Algorithm Details

We use the model described in Section 2.2 to represent the network. In 4.4.1, we linearize and reduce the nonlinear IV-ACOPF given in Section 2.3.3 in order to construct the LP subproblem in Section 4.4.5.

We apply approximations, relaxations, penalty variables, and constraint set reduction in order to reformulate the nonlinearities in (3.2.1) – (3.2.14). We use the same piecewise linear approximation of the quadratic cost function for the objective function as used in the current constraints chapter, in Section 3.2.2.1.

4.4.1 Linearizing Voltage and Current Upper Bounds

Both the current and voltage upper bounds in the IV-ACOPF model have the form \( x^2 + y^2 \leq MAXLIM^2 \). These limits form a circle centered at the origin of a two-dimensional plane with \( MAXLIM \) as the radius. We use linear constraints to approximate the circular feasible region. The equation for any constraint placed tangent to the circle at an angle of \( \theta \) at the coordinates \( (x, y) \) is \( x \cos(\theta) + y \sin(\theta) \leq MAXLIM \). Additionally, we can represent the cosine and sine functions in rectangular coordinates.

\[
\cos(\theta) = \frac{\hat{x}}{\sqrt{(\hat{x})^2 + (\hat{y})^2}} \tag{4.4.1}
\]

\[
\sin(\theta) = \frac{\hat{y}}{\sqrt{(\hat{x})^2 + (\hat{y})^2}} \tag{4.4.2}
\]
The rectangular form a tangential cut is shown in (4.4.3).

\[
\frac{x\hat{x}}{\sqrt{(\hat{x})^2 + (\hat{y})^2}} + \frac{y\hat{y}}{\sqrt{(\hat{x})^2 + (\hat{y})^2}} \leq MAXLIM
\]  

The initial iteration of the algorithm starts with a set of constraints that form a polygon with equal lengths, containing \( N \) sides. As the iterations progress, we continue to add constraints that cut off areas that are feasible for the approximation but not the original problem. In Section 4.5, we examine the results of starting with 4, 16, 32, 128, and 360 cuts. Starting with more cuts may mean less iterations are required, but more cuts do add to the computational load.

We can examine the accuracy of the polygonal approximation with two metrics. One metric is the ratio of the truly feasible area (the circle, in blue) to the LPs approximated feasible area (the polygon). The physically infeasible area is shown in fuchsia in Figure 4.2. Another metric is the ratio of the feasible distance from the origin divided by the falsely feasible region, \( r/b \). The polygon can be divided into \( N \) identical triangles, where \( N \) is the number of outer cuts. If we examine half of one of these triangles, the width of this half triangle is \( r \), while the height is \( r \tan(\pi/N) \). The area of the whole triangle will be two of the half-triangles, which is \( r^2 \tan(\pi/N) \). The area of the entire polygon will then be \( Nr^2 \tan(\pi/N) \). So, the ratio of the circle to the polygon is \( \frac{\pi r^2}{r^2 \tan(\pi/N)} = \frac{\pi}{\tan(\pi/N)} \). If we find the ratio \( r/b \) (actual feasible distance to approximation feasible distance), it equals \( r \cos(\pi/N)/r = \cos(\pi/N) \). These two ratios (area and distance) versus number of cuts are shown in Table 4.1.

When starting with very few sides, the accuracy increases very quickly by adding more sides from only around 50% accurate for 3 cuts versus over 95% accurate for 10 cuts. After adding 20 cuts, the approximation is over 99% accurate. If even greater accuracy is required, more cuts can be added. By the time 32 cuts are added, we are already at 99.60% accuracy. Adding cuts in the early stage once again greatly increases the accuracy versus the 32 cuts, and this effect levels off the more cuts are added.
Figure 4.4: Graphs showing the ratio of actual feasible distance divided by the approximation feasible distance.

Figure 4.5: Graphs showing the ratio of actual feasible area divided by the approximation feasible area.

<table>
<thead>
<tr>
<th># Cuts</th>
<th>Angle of Cut (degrees)</th>
<th>Physically feasible/ approximated feasible area</th>
<th>Physically feasible/ approximated feasible distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>120.0</td>
<td>52.36%</td>
<td>50.00%</td>
</tr>
<tr>
<td>4</td>
<td>90.0</td>
<td>78.54%</td>
<td>70.71%</td>
</tr>
<tr>
<td>6</td>
<td>60.0</td>
<td>90.69%</td>
<td>86.60%</td>
</tr>
<tr>
<td>8</td>
<td>45.0</td>
<td>94.81%</td>
<td>92.39%</td>
</tr>
<tr>
<td>10</td>
<td>36.0</td>
<td>96.69%</td>
<td>95.11%</td>
</tr>
<tr>
<td>12</td>
<td>30.0</td>
<td>97.70%</td>
<td>96.59%</td>
</tr>
<tr>
<td>14</td>
<td>25.7</td>
<td>98.32%</td>
<td>97.49%</td>
</tr>
<tr>
<td>16</td>
<td>22.5</td>
<td>98.71%</td>
<td>98.08%</td>
</tr>
<tr>
<td>18</td>
<td>20.0</td>
<td>98.98%</td>
<td>98.48%</td>
</tr>
<tr>
<td>20</td>
<td>18.0</td>
<td>99.18%</td>
<td>98.77%</td>
</tr>
<tr>
<td>22</td>
<td>16.4</td>
<td>99.32%</td>
<td>98.98%</td>
</tr>
<tr>
<td>24</td>
<td>15.0</td>
<td>99.43%</td>
<td>99.14%</td>
</tr>
<tr>
<td>26</td>
<td>13.8</td>
<td>99.51%</td>
<td>99.27%</td>
</tr>
<tr>
<td>28</td>
<td>12.9</td>
<td>99.58%</td>
<td>99.37%</td>
</tr>
<tr>
<td>30</td>
<td>12.0</td>
<td>99.63%</td>
<td>99.45%</td>
</tr>
<tr>
<td>32</td>
<td>11.3</td>
<td>99.68%</td>
<td>99.52%</td>
</tr>
</tbody>
</table>

Table 4.1: Number of polygonal sides (cuts) approximating the feasible region versus the error.
CHAPTER 4. A SUCCESSIVE LINEAR PROGRAMMING APPROACH TO SOLVING THE IV-ACOPF

<table>
<thead>
<tr>
<th>% Accuracy</th>
<th># Cuts</th>
</tr>
</thead>
<tbody>
<tr>
<td>90.0000%</td>
<td>7</td>
</tr>
<tr>
<td>95.0000%</td>
<td>10</td>
</tr>
<tr>
<td>99.0000%</td>
<td>18</td>
</tr>
<tr>
<td>99.5000%</td>
<td>26</td>
</tr>
<tr>
<td>99.9000%</td>
<td>56</td>
</tr>
<tr>
<td>99.9500%</td>
<td>80</td>
</tr>
<tr>
<td>99.9900%</td>
<td>180</td>
</tr>
<tr>
<td>99.9950%</td>
<td>256</td>
</tr>
<tr>
<td>99.9990%</td>
<td>570</td>
</tr>
<tr>
<td>99.9995%</td>
<td>800</td>
</tr>
<tr>
<td>99.9999%</td>
<td>1800</td>
</tr>
</tbody>
</table>

Table 4.2: Number of Cuts versus Accuracy of the approximation

While not very many cuts are required to obtain 99.9% accuracy, it becomes asymptotically difficult to obtain 100% accuracy. For our choice of varying levels of cuts, the corresponding accuracy of these cuts is given in Table 4.2. The number of cuts that are tested and their approximation accuracy are shown in Table 4.3.

<table>
<thead>
<tr>
<th># Cuts</th>
<th>Angle of Cut (degrees)</th>
<th>Physically feasible/approximated feasible area</th>
<th>Physically feasible/approximated feasible distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>90.00</td>
<td>78.54%</td>
<td>70.71%</td>
</tr>
<tr>
<td>16</td>
<td>22.50</td>
<td>98.71%</td>
<td>98.08%</td>
</tr>
<tr>
<td>32</td>
<td>11.25</td>
<td>99.68%</td>
<td>99.52%</td>
</tr>
<tr>
<td>128</td>
<td>2.81</td>
<td>99.98%</td>
<td>99.97%</td>
</tr>
<tr>
<td>360</td>
<td>1.00</td>
<td>100.00%</td>
<td>100.00%</td>
</tr>
</tbody>
</table>

Table 4.3: Number of Sides of the Polygon to be Tested versus Accuracy

In the iterative method, new constraints are added on voltage and current after each iteration at the previous point if the point was AC infeasible in order to cut off ACOPF infeasible solutions. These iterative constraints are added via the following process.

1. The linear approximation model is solved.
2. The current on each line and voltage magnitude or at a bus (respectively) is computed.
3. If this magnitude is larger than the maximum allowed, a constraint is added.
4. For the constraint, the ratios of the imaginary part over the magnitude and then the real part over the magnitude are computed.
5. If the voltage magnitude at a bus exceeds the maximum, equation (18) is added. If the current magnitude on a line exceeds the maximum, equation (19) is added.

There a number of design choices that must be made for this linear approximation. These include how many initial 'cuts' to use, whether to add iterative constraints or not, and whether to restrict movement from the fixed points.

### 4.4.1.1 Taylor Series Approximations

We apply first order Taylor series approximations to linearize nonlinear terms \( p_n^q, q_n^q, (v_n)^2 \) and \((i_k)^2\) in constraints \((3.2.15), (3.2.18), (3.2.12), (3.2.13), \) and \((3.2.14)\), respectively. We do this for all buses \( n \in \mathcal{N} \) for \( p_n^q, q_n^q, \) and \( v_n^r, \) and for all flows \( k (\cdot) \in \mathcal{F} \) for \((i_k)^2\). For iteration \( h \), we use the Taylor series evaluation points (denoted with a caret or "hat") to approximate the first order linearizations.

\[
v_n^{eq} = 2\hat{v}_n^{r(h)}v_n^r + 2\hat{v}_n^{j(h)}v_n^j - (\hat{v}_n^{r(h)})^2 - (\hat{v}_n^{j(h)})^2
\]

\[
p_n^q = \hat{v}_n^{r(h)}i_n^r + \hat{v}_n^{j(h)}i_n^j + i_n^{r(h)}i_n^r + i_n^{j(h)}i_n^j\quad(4.4.4)
\]

\[
q_n^q = \hat{v}_n^{j(h)}i_n^r - \hat{v}_n^{r(h)}i_n^j + i_n^{r(h)}i_n^r - i_n^{j(h)}i_n^j + v_n^r\hat{v}_n^r + v_n^j\hat{v}_n^j + Q_n^d \quad(4.4.5)
\]

\[
\hat{i}_{k(n,m)}^{eq} = 2\hat{v}_{k(n,m)}^{r(h)}i_{k(n,m)}^r + 2\hat{v}_{k(n,m)}^{j(h)}i_{k(n,m)}^j - (\hat{v}_{k(n,m)}^{r(h)})^2 - (\hat{v}_{k(n,m)}^{j(h)})^2 \quad(4.4.6)
\]

Since a first order method is used, larger step sizes result in larger approximation error. If we designate the true nonlinear real power as \( p_n^{g,act} = v_n^r i_n^r + v_n^j i_n^j \) and reactive power as \( q_n^{g,act} = v_n^r i_n^j - v_n^j i_n^r \), the maximum errors for the first order approximation of \( p_n^q \) and \( q_n^q \) are given in \((4.4.8)\) and \((4.4.9)\).

\[
|p_n^{g,act} - p_n^q| = \left| v_n^r - \hat{v}_n^r \right| \left| i_n^r - \hat{i}_n^r \right| + \left| v_n^j - \hat{v}_n^j \right| \left| i_n^j - \hat{i}_n^j \right| \quad(4.4.8)
\]

\[
|q_n^{g,act} - q_n^q| = \left| v_n^j - \hat{v}_n^j \right| \left| i_n^r - \hat{i}_n^r \right| + \left| v_n^r - \hat{v}_n^r \right| \left| i_n^j - \hat{i}_n^j \right| \quad(4.4.9)
\]

We need step sizes that are large enough to avoid cutting off too much of the feasible region and to minimize the required number of iterations but small enough that the accuracy of the approximation is good. Depending on the penalty and real power mismatch costs, the step size is restricted at an accelerated or decelerated rate; see Section 4.5.2. At each iteration \( h \), we update the tunable parameter \( V_n^{(h)} \) and introduce the step size limits on the real and imaginary parts of the nodal voltage, \( v_n^r \) and \( v_n^j \), for all buses \( n \in \mathcal{N} \). These limits are given in \((4.4.10)\) and \((4.4.11)\).

\[
|v_n^r - \hat{v}_n^{r(h)}| \leq V_n^{(h)} \quad(4.4.10)
\]

\[
|v_n^j - \hat{v}_n^{j(h)}| \leq V_n^{(h)} \quad(4.4.11)
\]

By controlling the step size for the real and imaginary parts of the nodal voltages, we limit the approximation error in the real and reactive power as given in \((4.4.12)\) and
Figure 4.6: Outer approximation of the voltage and current phasor bounds with box constraints.

\[
|p_{n, \text{act}}^g - p_n^g| \leq 4 \left( V_n^{(h)} \right)^2 \left( |G_{nn}| + |B_{nn}| \right) \tag{4.4.12}
\]

\[
|q_{n, \text{act}}^g - q_n^g| \leq 4 \left( V_n^{(h)} \right)^2 \left( |G_{nn}| + |B_{nn}| \right) \tag{4.4.13}
\]

This also means that we limit the corresponding error in LMP as derived from the dual variable to (4.4.5).

\section*{4.4.1.2 Relaxations and Penalty Factors}

In conjunction with (4.4.4), we introduce the following box constraints on the real and imaginary parts of the nodal voltage as in (4.4.14) and (4.4.15) for all buses \( n \in \mathcal{N} \).

\[
-V_n^{\text{max}} \leq v_n^r \leq V_n^{\text{max}} \tag{4.4.14}
\]

\[
-V_n^{\text{max}} \leq v_n^j \leq V_n^{\text{max}} \tag{4.4.15}
\]

In conjunction with (4.4.7), we introduce (4.4.16) and (4.4.17) on the real and imaginary parts for all current flows \( k \in \mathcal{F} \).

\[
-I_k^{\text{max}} \leq i_{k(n,m)}^r \leq I_k^{\text{max}} \tag{4.4.16}
\]

\[
-I_k^{\text{max}} \leq i_{k(n,m)}^j \leq I_k^{\text{max}} \tag{4.4.17}
\]

These relaxed constraints bound our approximation, as illustrated in Figure 4.6.

However, this approach can result in IV-ACOPF infeasible solutions. When infeasibility occurs due to violations of the voltage (3.2.12) or current (3.2.14) upper bound, we reset the violating evaluation points of the Taylor series approximation to be within...
IV-ACOPF Feasible Region
SLP IV-ACOPF Feasible Region
Tangential Cutting Plane

Figure 4.7: The infeasible solution $x^*$ from iteration $h - 1$, the updated evaluation point $\hat{x}^{(h)}$, and the tangential cutting plane included to the constraint set for iteration $h$.

these bounds. We also include a tangential cutting plane to the constraint set for the subsequent iteration, as illustrated in Figure 4.7. We impose constraint satisfaction of the tangential cutting plane by introducing a slack variable, which is penalized in the cost function; see Appendix 4.4.2. This approach only applies for the outer approximation on the upper bounds.

When the lower bound constraint in (3.2.13) is violated, we do not introduce a tangential cutting plane, which would eliminate parts of the IV-ACOPF feasible region. Instead, we only impose constraint satisfaction by introducing slack variables that are penalized in the cost function. We treat the bounds on the first order Taylor approximations similarly. We reformulate (3.2.12), (3.2.13), (3.2.14), (3.2.20), (3.2.20), (3.2.21), and (3.2.22) as ‘soft’ constraints with (4.4.18) - (4.4.21). The slack variables $p_n^{\text{viol},+}$, $p_n^{\text{viol},-}$, $q_n^{\text{viol},+}$, $q_n^{\text{viol},-}$, $v_n^{\text{viol},-}$, $v_n^{\text{viol},+}$, and $i_k^{\text{viol},+}(n,m)$ are penalized in the objective function.

$$
P_n^{\text{min}} - p_n^{\text{viol},-} \leq p_n^g \leq P_n^{\text{max}} + p_n^{\text{viol},+} \tag{4.4.18}
$$
$$
Q_n^{\text{min}} - q_n^{\text{viol},-} \leq q_n^g \leq Q_n^{\text{max}} + q_n^{\text{viol},+} \tag{4.4.19}
$$
$$
(V_n^{\text{min}})^2 - v_n^{\text{viol},-} \leq v_n^{\text{sq}} \leq (V_n^{\text{max}})^2 + v_n^{\text{viol},+} \tag{4.4.20}
$$
$$
i_k^{\text{sq}(n,m)} \leq (I_k^{\text{max}})^2 + i_k^{\text{viol},+}(n,m) \tag{4.4.21}
$$

4.4.2 Infeasibility Handling

The following routine determines the updated evaluation point and resulting tangential cutting plane when the voltage magnitude upper bound in (3.2.12) or the thermal line limit in (3.2.14) is violated by the optimal solution of the LP subproblem in iteration...
h. Without loss of generality, we denote the real part as $x^r$, the imaginary part as $x^j$, and the upper bound as $X^{max}$.

1: if $(\hat{x}^r)^2 + (\hat{x}^j)^2 > (X^{max})^2$ then

2: $\hat{x}^r \leftarrow (\hat{x}^r) \sqrt{(X^{max})^2 / ( (\hat{x}^r)^2 + (\hat{x}^j)^2 )}$

3: $\hat{x}^j \leftarrow (\hat{x}^j) \sqrt{(X^{max})^2 / ( (\hat{x}^r)^2 + (\hat{x}^j)^2 )}$

4: add constraint: $\hat{x}^r x^r + \hat{x}^j x^j \leq (X^{max})^2 + x^{viol}$

5: end if

If we calculate the new $\hat{x}^2$ after the correction, $\hat{x}^2 = (X^{max})^2$.

4.4.3 Stopping Criteria

There are three possible scenarios where the algorithm will terminate: (1) the mismatch error on real and reactive power injections for all buses $n \in \mathcal{N}$ is less than a specified tolerance, (2) the net of these mismatches is less than a specified tolerance, or (3) the maximum iteration limit has been reached. It may be the case that multiple stopping criteria are hit at the same time. After each iteration, we first calculate what the actual real $P^*_n$ and reactive $Q^*_n$ power generation would be. Then, we calculate the difference in the actual power versus the approximated power and divide it by whichever quantity is smallest for every bus. If the quantity is 0, we set the denominator to be 0.0001. We then check if the worst mismatches for real and reactive power are acceptable, or if the total mismatch is acceptable, or if the iteration count is too high.

1: $P^*_n = P^d_n + v^{r*}_n i^{r*}_n + v^{j*}_n j^{j*}_n$

2: $Q^*_n = Q^d_n + v^{j*}_n i^{r*}_n - v^{r*}_n j^{j*}_n$

3: for all $n \in \mathcal{N}$ do

4: $\delta^p_n \leftarrow |P^*_n - p^g_n| / \min (|P^*_n|, |p^g_n|)$

5: $\delta^q_n \leftarrow |Q^*_n - q^g_n| / \min (|Q^*_n|, |q^g_n|)$

6: end for

7: if \( \max_{n \in \mathcal{N}} \delta^p_n \leq \Delta^p - tol \) and \( \max_{n \in \mathcal{N}} \delta^q_n \leq \Delta^q - tol \) or \( \frac{1}{|\mathcal{N}|} \sum_{n \in \mathcal{N}} \delta^p_n \leq \Delta^p - tol \) and \( \frac{1}{|\mathcal{N}|} \sum_{n \in \mathcal{N}} \delta^q_n \leq \Delta^q - tol \)

or $h \geq \text{LIM}$ then

8: return solution;

9: end if

4.4.4 Constraint Reduction

In large transmission networks, one major way of reducing the computational burden is to only enforce limits on some of the lines - the lines that one expects to exceed their limits. We apply the concept of flowgate monitors to solve the linearized formulation
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with a reduced constraint set. We calculate and monitor the flows for a subset of lines \(k' \in K'(h) \subset K\) where \(k'(\cdot) \in F\) are near or at \((P_{k'}^{\max})^2\). The subset of lines \(K'(h) \subset K\) is updated at each iteration \(h > 0\). The constraint set is therefore reduced to only include \(4.4.7\), \(4.4.16\), \(4.4.17\), and \(4.4.21\) for all \(k' = k(\cdot) \in F\).

4.4.5 LP Subproblem Formulation

For each iteration \(h\), the SLP IV-ACOPF solves the LP subproblem with the objective given in \(4.4.22\) subject to the constraints in \(4.4.23\) to \(4.4.53\).

\[
\text{cost} (\cdot)^{(h)} = \min \left( \text{offers} (\cdot)^{(h)} + \text{penalty} (\cdot)^{(h)} \right) \tag{4.4.22}
\]

subject to

\[
- P_n^g + \sum_{l \in L} p_{n,l}^g = -P_n^{\min} \tag{4.4.23}
\]

\[
- P_n^q + \sum_{l \in L} p_{n,l}^q \geq \left( P_n^{\max} - P_n^{\min} \right) / |L| \tag{4.4.24}
\]

\[
p_n^g + p_n^{\text{viol}-} \geq P_n^{\min} \tag{4.4.25}
\]

\[
p_n^q - p_n^{\text{viol}+} \geq P_n^{\max} \tag{4.4.26}
\]

\[
q_n^g + q_n^{\text{viol}-} \geq Q_n^{\min} \tag{4.4.27}
\]

\[
- q_n^g + q_n^{\text{viol}+} \geq -Q_n^{\max} \tag{4.4.28}
\]

\[
v_n^{sq} + v_n^{\text{viol}-} \geq (V_n^{\min})^2 \tag{4.4.29}
\]

\[
- v_n^{sq} + v_n^{\text{viol}+} \leq -(V_n^{\max})^2 \tag{4.4.30}
\]

\[
\sum_{i} v_n^{i} = 0 \tag{4.4.31}
\]

\[
\sum_{i} v_n^{i} = 0 \tag{4.4.32}
\]

\[
\sum_{i} v_n^{i} = 0 \tag{4.4.33}
\]

\[
\sum_{i} v_n^{i} = 0 \tag{4.4.34}
\]

\[
\sum_{i} v_n^{i} = 0 \tag{4.4.35}
\]

\[
\sum_{i} v_n^{i} = 0 \tag{4.4.36}
\]

\[
\sum_{i} v_n^{i} = 0 \tag{4.4.37}
\]

\[
\sum_{i} v_n^{i} = 0 \tag{4.4.38}
\]

\[
\sum_{i} v_n^{i} = 0 \tag{4.4.39}
\]

\[
\sum_{i} v_n^{i} = 0 \tag{4.4.40}
\]

\[
\sum_{i} v_n^{i} = 0 \tag{4.4.41}
\]

\[
\sum_{i} v_n^{i} = 0 \tag{4.4.42}
\]

\[
\sum_{i} v_n^{i} = 0 \tag{4.4.43}
\]
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\[ v^j_n \geq -V_n^{max} \] (4.4.44)
\[ -v^j_n \geq -V_n^{max} \] (4.4.45)
\[ -v^r_n \geq -\hat{v}^r_n + V_n^{(h)} \] (4.4.46)
\[ v^r_n \geq \hat{v}^r_n - V_n^{(h)} \] (4.4.47)
\[ -v^i_n \geq -\hat{v}^i_n + V_n^{(h)} \] (4.4.48)
\[ v^i_n \geq \hat{v}^i_n - V_n^{(h)} \] (4.4.49)
\[ -I_{k}^{max} \leq i_{k(n,m)} \leq I_{k}^{max} \] (4.4.50)
\[ -I_{k}^{max} \leq i_{k(n,m)} \leq I_{k}^{max} \] (4.4.51)
\[ i_{k(n,m)}^{eq} \leq (I_{k}^{max})^2 + i_{k(n,m)}^{viol,+} \] (4.4.52)

\[ P_n^{g}, P_n^{viol,+}, P_n^{viol,-}, q_n^{viol,+}, q_n^{viol,-}, v_n^{viol,+}, v_n^{viol,-} \geq 0 \] (4.4.53)

The term offers (\( \cdot \)) is defined in (3.2.4); here, we designate the offers for iteration \( h \) as offers (\( \cdot \))^{(h)} and penalty(\( \cdot \))^{(h)} is defined in (4.4.54).

\[
\text{penalty} (\cdot)^{(h)} = \sum_{n \in N} \left[ P_n^c (P_n^{viol,-} + P_n^{viol,+}) + Q_n^c (q_n^{viol,-} + q_n^{viol,+}) + V_n^c (v_n^{viol,-} + v_n^{viol,+}) \right] \\
+ \sum_{k' \in K^{(h)}} I_{k'}^n \left( i_{k'(n,m)}^{viol,+} + i_{k'(m,n)}^{viol,+} \right). \] (4.4.54)

4.5 Preliminary Work

There were many considerations in implementing the SLP-IV-ACOPF. We tested a number of different possible implementations on the smaller IEEE systems, up to 118 buses. We tested on these smaller systems as even poor implementations could be solved in a reasonable amount of time so the implementations on larger systems would perform well.

4.5.1 Convex Constraints Considerations

One important consideration in tuning the algorithm was how many preprocessed cuts to use to approximate the convex constraints (upper limits on voltage and current), and whether or not iterative cuts should be included. The following problems were solved on an Intel Xeon E7458 server with 64 GB of memory and 8 64-bit 2.4GHz processors. These problems were formulated using GAMS 23.6.2, and the linearization was solved with GUROBI 4.0.0; the solver for the nonlinear comparison was IPOPT 3.8.

In examining the 14 bus system, as seen in Figure 4.8 we see that if no iterative cuts are added, at least 16 polygonal cuts are needed to find an appropriate solution. The case with 4 cuts hit the maximum number of iterations without converging, in both the tight and loose limits cases. When iterative constrains are added, it appears that even
as few as four preprocessed cuts are reasonable to find a good solution. Once more than 32 preprocessed cuts are used, the linear method begins to slow down greatly and is no longer an advantage over the nonlinear version.
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Figure 4.9: 30 Bus Results for Cut Considerations

Testing the 30 bus system (Figure 4.9) shows similar results to the 14 bus system. However, in this case, more cuts are needed to produce a solution closer to the nonlinear solution, although all solutions are AC feasible except for the case of 4 cuts without iterative constraints. Here, the linear solution becomes slower than the nonlinear solution after 128 preprocessed cuts rather than 32 cuts.
In examining the case with 57 buses in Figure 4.10, we observe a similar pattern to the 30 bus system. Interestingly, the nonlinear program appears to take longer with the loosely constrained case than the tightly constrained case, but the solutions times for the linear solver seem to be consistent between the two types of constraints.
The results for the 118 bus case (Figure 4.11) with tight current constraints shows similar behavior to the other cases. The results for the 118 bus case with loose current constraints is an anomaly in this testing set - it did not converge to AC feasibility, and its CPU time reflects the maximum number of iterations for that problem.
Overall, as seen in the previous figures and in Figure 4.12, adding more than 32 cuts greatly slows the program down without improving accuracy. If iterative cuts are not used, 16 to 32 cuts are needed initially for appropriate accuracy. If iterative cuts are used, one can start with as little as four initial cuts (a box). From these results, we decided to do subsequent testing starting with four cuts and iterative constraints.
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4.5.2 Step Size Limits

Next, we examined whether the voltage step size should be reduced in subsequent iterations, with the voltage step size \( V_n^{(h)} = \frac{\alpha (V_{max})^\beta}{h^\beta} \). Here, we tested whether it should reduce quadratically or linearly (\( \beta = 1 \) or \( \beta = 2 \)) and how far the step size should be cut on the first iteration (\( \alpha = 1 \) or \( \alpha = 1/2 \)). The different step-size styles were tested in conjunction with the number of preprocessed cuts.

![Figure 4.13: Results of Different Step Sizes](image)

As seen in Figure 4.13, no single method appeared to be strictly better than the other convergence methods, although using at least some step size reduction method did often speed up the solution. From these results, we decided to pursue an adaptable method of reducing step size that accounted for the quality of the current solution rather than solely the iteration number. As the approximation nears ACOPF feasibility, we accelerate the convergence rate; if the approximation worsens, we decelerate this rate. To set this rate, the first step is to calculate the ratio \( \gamma^{(h)} \) of the penalty cost plus error divided by the total power dispatch cost plus error (4.5.1). The larger \( \gamma^{(h)} \) is, the farther we are from a feasible solution; the smaller the \( \gamma^{(h)} \) is, the closer we are to a feasible solution.

\[
\gamma^{(h)} = \frac{\text{penalty} \cdot (\cdot)^{(h)} + f \cdot (\cdot)^{(h)}}{\text{cost} \cdot (\cdot)^{(h)} + f \cdot (\cdot)^{(h)}}
\] (4.5.1)

The function \( f \cdot (\cdot)^{(h)} \) is the total error in the real power generation and is defined in (4.5.2). It is the difference in the actual power generation determined by the optimal
solution at the current iteration, \( v^r_n, v^d_n, i^r_n, i^d_n \), versus the first order Taylor series approximation of the power generation.

\[
f(h) = \sum_{n \in \mathcal{G}} \text{MISMATCH}_n |P_n^* - p^0_n|
\]

(4.5.2)

\( P_n^* \) is the actual power generation in the current iteration and is defined in (4.5.3).

\[
P_n^* = P^d_n + v^r_n i^r_n + v^d_n i^d_n
\]

(4.5.3)

As \( \gamma(h) \to 0 \), the solution becomes ACOPF feasible; the error in the real power dispatch \( f(h) \) and the penalty cost penalty \( (\cdot)^{(h)} \) approach zero. We have \( \delta = 1 - \lfloor 10 \gamma(h) \rfloor / 10 \); the smaller the \( \delta \) is, the farther away from AC feasibility we are; the larger the \( \delta \) is, the closer to AC feasibility we are. We then let the decay rate \( \beta = -a \log \gamma(h) + b \); as \( \gamma \) increases, \( \beta \) becomes smaller and reduces the rate of convergence; as \( \gamma \) decreases, we are closer to feasibility, and \( \beta \) increases. We let the constant reduction in step size \( \alpha \) be a counter-balance to the increase in convergence rate; \( \alpha = \delta / \beta \); \( \alpha \) increases as \( \gamma \) increases and as \( \beta \) decreases.

\[
V_n^r \leftarrow \alpha |V_n^{\max}| / h^\beta
\]

(4.5.4)

For faster decay, the user can increase \( a \) and \( b \). When \( \gamma(h) = 1 \), then \( \beta = b \). It is expected that these parameters would be tuned to the specific network being solved, as well as general different loading patterns, such as peak and off-peak.

### 4.6 Results

In this section, we test the computational performance and convergence quality of the SLP IV-ACOPF and compare the results to solving the nonlinear IV-ACOPF. Castillo [77] demonstrated that the mathematical formulation, solver algorithm, and initialization all contribute to the performance of ACOPF solution techniques. Here, we compare performance of the SLP versus NLP approaches for different solvers and initialization methods.

Ideally, we want to compare our solutions to the globally optimal solution. However, we are unable to obtain globally optimal rank-one solutions to the nonlinear IV-ACOPF by using a semidefinite relaxation without augmenting the formulation to include penalties on constraint violations. In many cases, these penalties were non-uniform and thus difficult to non-arbitrarily determine a global optimum. Instead, we report an upper bound on solution cost by solving the IV-ACOPF with KNITRO MS (multi-start) using the default settings [78], which we refer to as NLP/KNITRO MS. Although a global optimum is not guaranteed, the probability of finding a better local solution is higher with multi-start on the globally convergent KNITRO algorithms (Interior/CG, Interior/Direct, Active-Set). Since the KNITRO multi-start runtime is much longer than
that of our SLP IV-ACOPF algorithm, the CPU time reported is for KNITRO without
multi-start. We also solve the nonlinear IV-ACOPF with Ipopt, which uses a filter line
search to ensure global convergence [79].

Both the IV-ACOPF model and SLP IV-ACOPF algorithm are implemented in
Python 2.7 with Pyomo 3.5 [80] and executed on a workstation with four quad-core
Intel Xeon 2.7 GHz processors with hyper-threading and 512GB RAM. We solve the LP
subproblems of the SLP IV-ACOPF with either Gurobi 5.6.2 [81] or CPLEX 12.5.1 [82]
barrier method limited to two threads, and the IV-ACOPF with either Ipopt 3.11.4 con-
figured using the MA27 linear sub-solver (no multi-threading support) [79] or KNITRO
9.1, where KNITRO chooses the algorithm to apply [78]; we refer to each set of sim-
ulations as SLP/Gurobi, SLP/CPLEX, NLP/Ipopt, and NLP/KNITRO, respectively.
Since MATPOWER 5.1 [30] specifically formulates the ACOPF in polar coordinates
and augments user-specified initializations by selecting an interior point, we were un-
able to readily include the MATPOWER 5.1 solvers into our testing. A feasibility and
optimality tolerance of 1.0E−06 is applied to each solver in our study.

We execute each algorithm from multiple starting points on the IEEE networks
14, 30, 57, 118, 300 and Polish networks 2383wp, 2737sop, 2746wop, 3012wp, 3120sp,
3375wp [30]. For buses with multiple generators, we aggregate these units with an
average cost function, as in Chapter 3. We solve these networks without line limits for
a baseline case and with line limits (not including network elements) for a thermally
constrained case. We use the approach in Chapter 3 to systematically compute line
ratings in terms of current. We select one current limit for each test case from Chapter
3 and show the selection in Table 4.5.

The performance of any SLP-based algorithm is highly dependent on strategies used,
the computer implementation, and parameter settings [83]. In the proposed SLP algo-

rithm, there are parameters to control penalty factors, constraint reduction, iterative
step-size, and power mismatch tolerances. It is impossible to fine-tune these parameters
to have optimal settings for all possible problems without skewing the results, so we
demonstrate performance for the default values as reported in Table 4.4. The magni-
tude of the penalties, on average, are loosely correlated to the types of constraint most
likely to incorporate slack into the solution. For example, the penalty on the real power
is the lowest since dispatching it is already priced into the market. The iteration limit
was arbitrarily set to 20 for the reported test suite, but it could be set higher for larger
scale or more constrained networks that might require more iterations. The step-size
parameters $a$ and $b$ tune the parameters $\alpha$ and $\beta$ as described in Section 4.5.2. Most
importantly, $a$ controls the rate of decay in the allowable step-size, where the default
step-size region varies in proportion to the power of the iteration count, as determined by
$b$ when there are excessive penalties present. We apply power mismatch tolerances well
below the threshold of known state estimator precision; in practice and as we demon-
strate below, high accuracy in convergence is not required to obtain a meaningful result.
It is expected that these parameters would be tuned much more finely for the particular
network where the algorithm is applied.

We consider four types of initialization methods: a flat start, a DC warm start,
an AC warm start, and an uniform cold start. The flat start assumes unit voltage and that the output for all generators is half of their maximum output. The DC and AC warm starts are constructed from DCOPF and ACOPF locally optimal solutions, respectively, where the demand at each bus varies randomly by up to 10%; specifically, it is parameterized as $P_d^k \sim U (0.9 P_d^k, 1.1 P_d^k)$ for all $n \in \mathcal{N}$. The uniform starts assume that $v_n^r \sim U (V_n^{min}, V_n^{max})$ and $v_j^r = 0$ for all $n \in \mathcal{N}$; since that the uniform start does not incorporate any knowledge of a prior operating state, it is defined as a cold start. The sample size for the various initialization types is one sample for the flat start and 10 samples for the other start types. To reduce variance in the comparison, we use identical starting points to test SLP/Gurobi, SLP/CPLEX, NLP/Ipopt, and NLP/KNITRO.

For the nonlinear IV-ACOPF, NLP/KNITRO and NLP/Ipopt converged 99.7% and 96.9% of the time, respectively. For the SLP algorithms, a converged solution is defined as a solution that meets the mismatch tolerances defined in Table 4.4 as set in the stopping criteria. The simulation is labeled as unconverged for LP subproblem iterations.
CHAPTER 4. A SUCCESSIVE LINEAR PROGRAMMING APPROACH TO SOLVING THE IV-ACOPF

<table>
<thead>
<tr>
<th>Solver</th>
<th>Best-Case Simulations</th>
<th>Thermally Constrained Case</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$p$ $R^2$ RMSE (s)</td>
<td>$p$ $R^2$ RMSE (s)</td>
</tr>
<tr>
<td>Baseline</td>
<td>NLP/KNITRO</td>
<td>1.42 0.83 1.46</td>
</tr>
<tr>
<td>Cases</td>
<td>NLP/Ipopt</td>
<td>1.13 0.95 0.60</td>
</tr>
<tr>
<td></td>
<td>SLP/CPLEX</td>
<td>0.97 0.99 0.20</td>
</tr>
<tr>
<td></td>
<td>SLP/Gurobi</td>
<td>1.01 0.99 0.21</td>
</tr>
</tbody>
</table>

Table 4.6: The scaling factor $p$ for the experimental time complexity $\Theta$, with corresponding R-squared ($R^2$) and root mean squared error (RMSE) values, of NLP/KNITRO, NLP/Ipopt, SLP/CPLEX, and SLP/Gurobi. The exponent $p = 1$ corresponds to linear algorithmic scaling. The high R-squared values indicate that the time complexity model $n^p$ explains nearly all the variability in computational time as a function of the network size. RMSE is reported in seconds (s); a value closer to zero indicates a fit that is more useful for prediction.

<table>
<thead>
<tr>
<th>Solver</th>
<th>(CPU Time)</th>
<th>Baseline Case</th>
<th>Thermally Constrained Case</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NLP/KNITRO</td>
<td>NLP/Ipopt</td>
<td>SLP/CPLEX</td>
</tr>
<tr>
<td>IEEE-14</td>
<td>0.12</td>
<td>0.16</td>
<td>0.24 (4)</td>
</tr>
<tr>
<td>IEEE-30</td>
<td>0.19</td>
<td>0.2</td>
<td>0.84 (9)</td>
</tr>
<tr>
<td>IEEE-57</td>
<td>0.32</td>
<td>0.38</td>
<td>0.76 (5)</td>
</tr>
<tr>
<td>IEEE-118</td>
<td>0.84</td>
<td>1.18</td>
<td>2.53 (7)</td>
</tr>
<tr>
<td>IEEE-300</td>
<td>2.27</td>
<td>2.63</td>
<td>4.88 (6)</td>
</tr>
<tr>
<td>Polish-2,383</td>
<td>38.65</td>
<td>88.47</td>
<td>37.05 (6)</td>
</tr>
<tr>
<td>Polish-2,737</td>
<td>29.57</td>
<td>25.29</td>
<td>42.69 (6)</td>
</tr>
<tr>
<td>Polish-2,746</td>
<td>42.43</td>
<td>20.67</td>
<td>51.71 (7)</td>
</tr>
<tr>
<td>Polish-3,012</td>
<td>96.51</td>
<td>33.15</td>
<td>49.73 (6)</td>
</tr>
<tr>
<td>Polish-3,120</td>
<td>84.34</td>
<td>30.47</td>
<td>57.17 (6)</td>
</tr>
<tr>
<td>Polish-3,375</td>
<td>6,473.40</td>
<td>145.05</td>
<td>59.39 (6)</td>
</tr>
</tbody>
</table>

Table 4.7: The fastest recorded solver CPU time across all simulations for both baseline and thermally constrained networks. The number of SLP subproblem iterations is denoted in parentheses.

that are locally infeasible or those whose iteration count reaches the user defined limit. The SLP/CPLEX converged in 98.4% of the runs and the SLP/Gurobi in 99.1% of the runs. With this criteria, SLP with both CPLEX and Gurobi converged more often than NLP/Ipopt. Of the converged runs, 10.3% of the SLP/CPLEX and 10.3% of the SLP/Gurobi runs resulted in active penalties; these only occurred in the IEEE-118 and Polish-2,383 thermally constrained cases. The active penalty in the thermally constrained IEEE-118 is due to reactive power compensation requirements at a single
bus located in a congested area of the network. The active penalty in the Polish-2,383 is on a real power injection at a single generator bus; by increasing the real power penalty by a factor of three so that $P^e_n = 7.5 \max_n C^g_n$, SLP/CPLEX and SLP/Gurobi converge to the unpenalized optimum. The SLP parameterization ideally should be determined on a case-by-case basis, but we only examine default parameters here for the purpose of demonstrating general application and performance.

To analyze algorithm scaling properties, we determine the experimental time complexity $\Theta(T(n))$ classified by the nature of the function $T(n) = n^p$ for network size $n$ and unknown exponent $p > 0$. Instead of reporting numerous run time samples, Big-Theta ("$\Theta$") reports the exact dependence of the run time on network size for the subset of converged simulations in this study. We apply a linear regression on $\log(s) = p \log(n) + \log(b)$ to determine the power function $s = bn^p$, where $s$ is the simulation run time and the coefficient $b$ (relating to the $y$-intercept of $\log(b)$) is irrelevant when determining the order of $T(n)$. We use the MATLAB function polyfit, which minimizes the sum of the squares of the data deviations from the least-squares fit [84]. In Table 4.6, we report the experimental best-case and overall (for all the converged runs) time complexity exponent, along with the corresponding R-squared and root mean squared error (RMSE) values. The high R-squared values and low RMSE values (for NLP/Ipopt, SLP/CPLEX, and SLP/Gurobi) indicate the potential for IV-ACOPF and our SLP IV-ACOPF algorithm in practical applications. For $p$ near 1 (linear), the running time of the SLP algorithm grows nearly directly proportional to the network size $n$.

The time complexity across all the simulations, as reported in the right panel of Table 4.6, is figuratively comparable to that of the best-case, as reported in the left panel. The best-case corresponds to the fastest recorded solver CPU time for each test configuration; the run times for these simulations are reported in Table 4.7 and the number of SLP subproblem iterations (where applicable) are reported in parentheses. The best-case is of particular interest as the SLP/Gurobi, SLP/CPLEX, NLP/Ipopt, and NLP/Knitro approaches can be run in parallel from various starting points. In Table 4.8, we report the offer production costs associated with the best-known multi-start solutions, i.e., offers $\cdot^{(h)}$ in Equation (4.4.22), as determined by NLP/Knitro MS (multi-start). In Table 4.9, we report how the best-case solutions from the various algorithmic approaches compare to the multi-start solution. The results indicate that the SLP algorithm solutions are extremely close in quality to the multi-start solutions. By decreasing the mismatch tolerance and any active penalties— if present—in the SLP algorithm, we can effectively decrease the relative change in exchange for more computational time. Since NLP/Knitro and NLP/Ipopt do no worse than the NLP/Knitro MS, this indicates that the best-known optimum is most likely the only optimum found.

In Figures 4.14a and 4.14b, we report the fastest recorded solution times across the four initialization types for SLP/CPLEX and SLP/Gurobi, aggregating on both the baseline and thermally constrained cases.

The flat start does not perform well for the IV-ACOPF formulation. With the flat start ($v^r_n = 1$ and $v^z_n = 0$ for all buses), the current flows across lines are as in (4.6.1) and
## Offer Production Cost

### Table 4.8: Offer production costs for the best-known multi-start solutions found by NLP/KNITRO MS.

<table>
<thead>
<tr>
<th>Network</th>
<th>Baseline Case</th>
<th>Thermally Constrained Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>IEEE-14</td>
<td>8,091</td>
<td>9,294</td>
</tr>
<tr>
<td>IEEE-30</td>
<td>575</td>
<td>582</td>
</tr>
<tr>
<td>IEEE-57</td>
<td>41,817</td>
<td>41,978</td>
</tr>
<tr>
<td>IEEE-118</td>
<td>129,903</td>
<td>135,189</td>
</tr>
<tr>
<td>IEEE-300</td>
<td>720,149</td>
<td>726,794</td>
</tr>
<tr>
<td>Polish-2,383</td>
<td>1,858,447</td>
<td>1,863,627</td>
</tr>
<tr>
<td>Polish-2,737</td>
<td>742,679</td>
<td>742,688</td>
</tr>
<tr>
<td>Polish-2,746</td>
<td>1,185,115</td>
<td>1,185,507</td>
</tr>
<tr>
<td>Polish-3,012</td>
<td>2,581,020</td>
<td>2,597,386</td>
</tr>
<tr>
<td>Polish-3,120</td>
<td>2,137,309</td>
<td>2,140,727</td>
</tr>
<tr>
<td>Polish-3,375</td>
<td>7,402,883</td>
<td>7,415,108</td>
</tr>
</tbody>
</table>

(4.6.1) \[
\hat{r}_{k(n,m)} = G_{k(n)} - G_{k(m)}
\]

(4.6.2) \[
\hat{i}_{i(n,m)} = B_{k(n)} - B_{k(m)}
\]

(4.6.3) \[
v_{n}^{sq} = 2v_{n}^{r} - 1
\]

(4.6.4) \[
p_{n}^{q} = i_{n}^{r} + v_{n}^{r}\hat{r}_{n}^{(0)} + v_{n}^{j}\hat{j}_{n}^{(0)} - \hat{r}_{n}^{(0)} + P_{n}^{d}
\]

(4.6.5) \[
a_{n}^{q} = -i_{n}^{j} + v_{n}^{j}\hat{j}_{n}^{(0)} - v_{n}^{r}\hat{r}_{n}^{(h)} + \hat{i}_{n}^{(h)} + Q_{n}^{d}
\]

If the transmission element \( k \) has symmetric admittance, then the current across the line \( k \) is 0. If all the lines connected to node \( n \) have symmetric admittance and there are no nodal shunts, then the nodal current is zero. That means that the approximations for real and reactive power are then \( p_{n}^{q} = i_{n}^{r} + P_{n}^{d} \) and \( a_{n}^{q} = -i_{n}^{j} + Q_{n}^{d} \). The flat start gives a very poor starting point.

However, the uniform starts perform competitively compared to the DCOPF and ACOPF starts. The uniform starts do not require any knowledge of the prior operating state; by initializing \( v_{n}^{r} \) with some variation, in polar coordinates the voltage magnitudes become nonzero while the voltage angles remain zero. In the initial iteration, the current flows are nonzero; as a result, the nodal real power injections are calculated in terms of the network conductance and the nodal reactive power injections in terms of the network susceptance.
Relative Change to Baseline Case Thermally Constrained Case

<table>
<thead>
<tr>
<th>Solution</th>
<th>Table 4.8</th>
<th>NLP/ Knitro</th>
<th>NLP/ Ipopt</th>
<th>SLP/ CPLEX</th>
<th>SLP/ Gurobi</th>
<th>NLP/ Knitro</th>
<th>NLP/ Ipopt</th>
<th>SLP/ CPLEX</th>
<th>SLP/ Gurobi</th>
</tr>
</thead>
<tbody>
<tr>
<td>IEEE-14</td>
<td>0.0E+0</td>
<td>0.0E+0</td>
<td>1.2E-4</td>
<td>1.2E-4</td>
<td>0.0E+0</td>
<td>0.0E+0</td>
<td>-3.2E-4</td>
<td>-3.2E-4</td>
<td></td>
</tr>
<tr>
<td>IEEE-30</td>
<td>0.0E+0</td>
<td>0.0E+0</td>
<td>0.0E+0</td>
<td>0.0E+0</td>
<td>0.0E+0</td>
<td>0.0E+0</td>
<td>1.7E-2</td>
<td>2.4E-2</td>
<td></td>
</tr>
<tr>
<td>IEEE-57</td>
<td>0.0E+0</td>
<td>0.0E+0</td>
<td>5.3E-4</td>
<td>1.7E-4</td>
<td>0.0E+0</td>
<td>0.0E+0</td>
<td>2.4E-4</td>
<td>-4.8E-5</td>
<td></td>
</tr>
<tr>
<td>IEEE-118</td>
<td>0.0E+0</td>
<td>0.0E+0</td>
<td>1.2E-3</td>
<td>1.2E-3</td>
<td>0.0E+0</td>
<td>0.0E+0</td>
<td>3.0E-3</td>
<td>5.1E-3</td>
<td></td>
</tr>
<tr>
<td>IEEE-300</td>
<td>0.0E+0</td>
<td>0.0E+0</td>
<td>3.6E-5</td>
<td>1.9E-5</td>
<td>0.0E+0</td>
<td>0.0E+0</td>
<td>2.3E-5</td>
<td>2.2E-5</td>
<td></td>
</tr>
<tr>
<td>Polish-2,383</td>
<td>0.0E+0</td>
<td>0.0E+0</td>
<td>2.6E-4</td>
<td>4.8E-4</td>
<td>0.0E+0</td>
<td>0.0E+0</td>
<td>-1.4E-3</td>
<td>-1.8E-3</td>
<td></td>
</tr>
<tr>
<td>Polish-2,737</td>
<td>0.0E+0</td>
<td>0.0E+0</td>
<td>1.1E-4</td>
<td>1.1E-4</td>
<td>0.0E+0</td>
<td>0.0E+0</td>
<td>7.3E-5</td>
<td>5.1E-5</td>
<td></td>
</tr>
<tr>
<td>Polish-2,746</td>
<td>0.0E+0</td>
<td>0.0E+0</td>
<td>1.8E-4</td>
<td>1.8E-4</td>
<td>0.0E+0</td>
<td>0.0E+0</td>
<td>2.4E-4</td>
<td>2.3E-4</td>
<td></td>
</tr>
<tr>
<td>Polish-3,012</td>
<td>0.0E+0</td>
<td>0.0E+0</td>
<td>1.4E-4</td>
<td>3.7E-4</td>
<td>0.0E+0</td>
<td>0.0E+0</td>
<td>4.7E-4</td>
<td>4.1E-4</td>
<td></td>
</tr>
<tr>
<td>Polish-3,120</td>
<td>0.0E+0</td>
<td>0.0E+0</td>
<td>2.0E-4</td>
<td>1.4E-4</td>
<td>0.0E+0</td>
<td>0.0E+0</td>
<td>3.2E-4</td>
<td>3.0E-4</td>
<td></td>
</tr>
<tr>
<td>Polish-3,375</td>
<td>5.4E-7</td>
<td>2.7E-7</td>
<td>1.2E-3</td>
<td>1.4E-3</td>
<td>2.1E-7</td>
<td>1.5E-7</td>
<td>1.3E-3</td>
<td>1.3E-3</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.9: The relative convergence quality in the optimal solution, as compared to the best-known multi-start solution in Table 4.8 for the fastest recorded run as presented in Table 4.7. A positive (negative) metric indicates the relative increase (decrease) amount in solution value. By decreasing the mismatch tolerance in the SLP algorithm, as defined in Table 4.4, we can effectively decrease the relative change in exchange for more computational time.

![Figure 4.14](image1.png)

![Figure 4.14](image2.png)

Figure 4.14: The fastest recorded CPU time for SLP/CPLEX in (a) and SLP/Gurobi in (b) on each initialization type in both the baseline and thermally constrained Polish networks.
4.7 Discussion

This chapter introduces an SLP-based approach to solve the IV-ACOPF, which is a formulation where nonconvex constraints are isolated to each bus. The algorithm converges to an acceptable closeness to the best-known solutions and its computational time scales linearly in the network size; the algorithm requires only commercial LP solvers. The proposed approach has flexibility to focus more on optimality or on tractability and can be integrated into more complex models with integer variables that include discrete controls or discrete system states in order to represent decision-making processes in both the operational and planning levels, while still leveraging commercial solver performance. Unlike well-established methods of approximation, such as the DCOPF with loss factor modeling and AC feasibility, the SLP IV-ACOPF co-optimizes real and reactive power dispatch and enables the system operator more optimal control over system resources.

Current limitations of the proposed work is that for nonconvex problems, there is no known theoretical convergence results to the global optimum for SLP algorithms. Fletcher et al. prove global convergence (i.e., convergence to a stationary point from an arbitrary initialization point) for SLP with a filter method to solve NLP problems that contain just inequality constraints [85]. Also, given that the second order information is constant in the IV-ACOPF formulation, utilizing constant Hessian matrices can increase the efficiency of each iteration in the solution technique.

The linear nature of the SLP and direct integration of reactive power and voltage into the SLP formulation result in great potential benefits to the market. Chapter 5 describes how the SLP can be used in a real-time market setting.
Chapter 5

Running a More Complete Market with the SLP-IV-ACOPF

ISOs calculate locational marginal prices (LMPs) based on real power and do not price reactive power or voltage. This work gives a way to determine prices for reactive power and voltage in the real-time market. Previous research [41] has shown the computational and accuracy performance results of the Sequential Linear Programming approximation of the Current-Voltage formulation of the ACOPF (SLP-IV-ACOPF) for the IEEE and Polish networks [30]. This chapter bases real-time market pricing on the dual of the SLP-IV-ACOPF to provide prices for real power, reactive power, and voltage limits. We show how to distribute the complete real-time market settlement, calculating load payments, generation rents, and congestion rents and compare market outcomes to a DCOPF approach.

Nomenclature

This chapter follows much of the same notation as Chapters 3 and 4. There are enough differences that all of the nomenclature is detailed out to avoid any confusion with the previous chapters and to provide a quick reference for the reader.

Sets:

\( \mathcal{N} \) Buses \( \{1, \ldots, N\} \); \( n, m \in \mathcal{N} \)

\( \mathcal{K} \) Lines \( \{1, \ldots, K\} \); \( k \in \mathcal{K} \)

\( \mathcal{A}(n) \) Buses adjacent to node \( n \); \( m \in \mathcal{A}(n) \)

\( \mathcal{G} \) Buses with generators; \( \{1, \ldots, G\} \)

Some or all of the work in this chapter appeared in Lipka [86].
CHAPTER 5. RUNNING A MORE COMPLETE MARKET WITH THE SLP-IV-ACOPF

\( S \) Line segments tangent to voltage feasible region; \( s \in S \)

\( \mathcal{F} \) Set of flows \( \{1, \ldots, 2K\}; k(n, m) \in \mathcal{F} \)

\( \mathcal{L} \) Piecewise linear segments to approximate real power generated; \( l \in \mathcal{L} \)

\( \mathcal{H} \) Set of iterations \( \{1, \ldots, h^*\} \) to solve the problem; \( h \in \mathcal{H} \)

 Indices:

\( h^* \) Iteration when the SLP-IV-ACOPF meets its convergence criteria (detailed in [41])

\( k(n, m) \) Flow on transmission element \( k \) at bus \( n \) connecting to \( m \)

\( k(n) \) Flows on line \( k \) at the \( n \) end; \( k(\cdot) \in \mathcal{F} \)

 Variables:

For each bus \( n \):

\( p_n^g \) Total linearized real power generation

\( p_{n,l}^g \) Linear segment \( l \) of generation

\( q_n^g \) Reactive power generation

\( v_n^{eq} \) Linearization of \((v_n)^2\)

\( v_n^r, v_n^i \) Real and imaginary voltage parts

\( i_n^r, i_n^i \) Real and imaginary current injection parts

\( p_n^{viol, -} \) Violation of minimum real power limit

\( p_n^{viol, +} \) Violation of maximum real power limit

\( q_n^{viol, -} \) Violation of minimum reactive power limit

\( q_n^{viol, +} \) Violation of maximum reactive power limit

\( v_n^{viol, -} \) Violation of minimum voltage limit

\( v_n^{viol, +} \) Violation of maximum voltage limit

\( p_{k(n,m)}^{loss} \) DC piecewise linear estimation of real power loss across line \( k \)

\( \beta_{k(n),a}^+ \) Positive angle difference breakpoints

\( \beta_{k(n),a}^- \) Negative angle difference breakpoints
CHAPTER 5. RUNNING A MORE COMPLETE MARKET WITH THE SLP-IV-ACOPF

For each line $k$:

- $i_x^{k(n,m)}$: Real (x-axis) parts of current on $k$ from bus $n$ to $m$
- $i_y^{k(n,m)}$: Imaginary (y-axis) part of current on $k$ from bus $n$ to $m$
- $p_{k(n,m)}$: Real power flowing on line $k$ at node $n$

**Dual Variables: Marginal Values of the Following Constraints:**

For each bus $n$:

- $\rho_{n}\text{stepsum}$: Real power step sum
- $\rho_{n,\text{steplim}}$: Upper bound on $p_{n,l}$
- $\rho_{n}\text{low}$: Minimum real power generation at node $n$
- $p_{n}^{\text{high}}$: Maximum real power generation at node $n$
- $\gamma_{n}\text{low}$: Minimum reactive power generation at node $n$
- $\gamma_{n}\text{high}$: Maximum reactive power generation at node $n$
- $\nu_{n}\text{low}$: Minimum voltage limit at node $n$
- $\nu_{n}\text{high}$: Minimum voltage limit at node $n$
- $i_{n,\text{eq}}$: Real part of nodal current definition at node $n$
- $j_{n,\text{eq}}$: Imaginary part of nodal current node definition at node $n$
- $\lambda_{n}^{P}$: Real power nodal balance
- $\lambda_{n}^{Q}$: Reactive power nodal balance
- $\nu_{n,\text{mag}}$: Voltage squared definition
- $\nu_{n,\text{poly}}$: Polygonal constraint on voltage
- $\nu_{n,\text{iter}}(h)$: Iterative voltage cuts
- $\nu_{n}^{r,\text{LB}}$: Lower bound on the real part of voltage at node $n$
- $\nu_{n}^{r,\text{UB}}$: Upper bound on the real part of voltage at node $n$
- $\nu_{n}^{j,\text{LB}}$: Lower bound on the imaginary part of voltage at node $n$
- $\nu_{n}^{j,\text{UB}}$: Upper bound on the imaginary part of voltage at node $n$
- $\rho_{k(n)}^{\text{max}}$: Real power line limit
CHAPTER 5. RUNNING A MORE COMPLETE MARKET WITH THE SLP-IV-ACOPF

\[ \nu_{n,\text{low}} \] Real voltage box constraint lower bound at node \( n \)

\[ \nu_{n,\text{high}} \] Real voltage box constraint upper bound at node \( n \)

\[ \nu_{n,\text{low}} \] Imaginary voltage box constraint lower bound at node \( n \)

\[ \nu_{n,\text{high}} \] Imaginary voltage box constraint upper bound at node \( n \)

For each line \( k \):

\[ \iota_{k(n,m),\text{eq}} \] Real current line definition

\[ \jmath_{k(n,m),\text{eq}} \] Imaginary current line definition

\[ \delta_k \] DC with losses line real power definition

\[ \zeta_k \] DC loss breakpoints definition

\[ \phi^+_{k(n),a} \] Positive angle differences upper bounds on loss breakpoints

\[ \phi^-_{k(n),a} \] Negative angle differences upper bounds on loss breakpoints

\[ \Omega_a \] Slope of loss estimation for breakpoint \( a \)

Parameters:

For each bus \( n \):

\[ \hat{V}_{n}^{(h)} \] Real (x-axis) part of voltage at bus \( n \) at iteration \( h \)

\[ \hat{V}_{n}^{(h)} \] Imaginary (y-axis) part of voltage at bus \( n \) at iteration \( h \)

\[ \hat{I}_{n}^{(h)} \] Real (x-axis) part of current at bus \( n \) at iteration \( h \)

\[ \hat{I}_{n}^{(h)} \] Imaginary part of current at bus \( n \) at iteration \( h \)

\[ \hat{V}_{n}^{*} \] Real part of voltage at converged dispatch run \( h^* \)

\[ \hat{V}_{n}^{*} \] Imaginary part of voltage at bus \( n \) at converged dispatch run \( h^* \)

\[ \hat{I}_{n}^{*} \] Imaginary part of current at bus \( n \) at converged dispatch run \( h^* \)

\[ \hat{I}_{n}^{*} \] Imaginary part of current at bus \( n \) at converged dispatch run \( h^* \)

\[ G_n^{\text{sh}} \] Shunt conductance

\[ B_n^{\text{sh}} \] Shunt susceptance

\[ P_n^{d} \] Real power demand
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$Q^d_n$  Reactive power demand
$P^\text{min}_n$  Minimum $p^g_n$
$P^\text{max}_n$  Maximum $p^g_n$
$Q^\text{min}_n$  Minimum $q^g_n$
$Q^\text{max}_n$  Maximum $q^g_n$
$V^\text{min}_n$  Minimum voltage magnitude
$V^\text{max}_n$  Maximum voltage magnitude
$V_n^{(h)}$  Step-size bound on the voltage in iteration $h$
$C_{n}^q$  Linear cost coeff. for generation
$C_{n,l}^q$  Linear segment $l$ of the quadratic cost
$C_{n}^q$  Reactive support cost coefficient for
$P_{n,l}^q$  Maximum length of piecewise segment $l$
$P^c_n$  Dispatch run penalty costs of real power violations
$P^\text{c}_n$  Pricing run penalty costs of real power violations
$Q^c_n$  Dispatch run penalty costs of reactive power violations
$Q^\text{c}_n$  Pricing run penalty costs of reactive power violations
$V^c_n$  Dispatch run penalty costs of voltage violations
$V^\text{c}_n$  Pricing run penalty costs of voltage violations

For each line $k$:

$G_{k(n)}$  Series conductance
$B_{k(n)}$  Series susceptance
$\hat{\gamma}_{k(n,m)}^{r(h)}$  Real part of current at $h$
$\hat{\gamma}_{k(n,m)}^{i(h)}$  Imaginary part of current at $h$
$\hat{\gamma}_{k(n,m)}^{r*}$  Real part of current at $h^*$
$\hat{\gamma}_{k(n,m)}^{i*}$  Imaginary part of current at $h^*$
5.1 Introduction

Independent System Operators (ISOs) and Regional Transmission Organizations (RTOs) provide unbiased access to transmission, maintain grid reliability, and maximize social welfare. One way they fulfill their responsibilities is running energy markets, including day-ahead and real-time markets. The Optimal Power Flow (OPF) problem is the base of all these markets. The marginal values of constraints in the OPF provide market prices, and the dual objective to the OPF determines market settlements. This chapter focuses on the market prices of the basic alternating current (AC) OPF in the real-time power market. We detail the market prices and settlements of the SLP-IV-ACOPF as a possible way to run the real-time market.

5.1.1 Running the Power Market

To run the real-time power market, ISOs typically iterate between an AC power flow (ACPF) solver and a modified DCOPF. ISOs use a modified DCOPF and not the ACOPF to clear markets because the DCOPF is fast, linear, and has well-defined marginal values; the ACOPF is nonlinear and nonconvex, which means its marginal values may not support the hyperplane of the solution. The unmodified DC approximation assumes that resistances and voltage angles are relatively small, ignores reactive power, and restricts voltage magnitudes to a constant. To better suit the DC approximation to the physical system, ISOs first run the DCOPF and input the power generation solution to an ACPF solver; SOs use the ACPF solution to estimate real power losses, set voltage magnitudes, and add in nomogram constraints to approximate voltage and reactive power needs. Nomogram constraints typically limit total power flow across interfaces and may be added iteratively when the DC solution is not AC feasible. Locational marginal prices (LMPs) are the dual values on the nodal power balance constraint. Since the modified DCOPF is only based on real power, power markets do not explicitly price reactive power or compensate it within the market, although reactive power is required for normal operation of the power system and influences voltages and system stability. Reactive power can be provided by transmission, generation, or load, and is typically compensated outside of the power generation market.

5.1.2 Reactive Power Compensation

ISOs compensate reactive power production in several ways. Most ISOs pay a fixed amount per unit of reactive capacity over the year. All but MISO use one set price
for all areas; MISO pays different rates for each zone. Generators are required to be able generate reactive power over a specified power factor range at maximum real power output. The ISO sends dispatch signal to the generator for reactive power or specifies a voltage level to be maintained at the bus. CAISO, ISONE, and PJM pay units a lost opportunity cost for backing down real power if the units are forced out of the power factor range. ISONE and PJM pay startup and no load costs if a generator is committed out of merit order for reactive support purposes.

While reactive power and voltage support is a small fraction of the total power procurement cost, it can be expensive in absolute terms. PJM’s 2015 annual reactive requirement was $280M [87]; total billing for services was $50B in 2014 [88]. ISONE paid $5M in 2009 and 2010 and $5.9M for voltage in 2011; its total energy market settlement was $5.9B in 2009, $7.3B in 2010, and $6.7B in 2011 [89], [90]. This totaled $25M in 2010 and $18.6M in 2011 [91].

5.1.3 Issues with the Market

While the ISO OPF solution process produces a good approximation of the flows and prices of the power system, it has some drawbacks. Different modeling choices to represent the same item lead to different LMP profiles. The same voltage or reactive power limit can be represented by several different nomogram constraints, which can result in different nodal prices [28]. The selection of the reference bus for losses also changes the LMP profiles for the same real power dispatch [29]. The next issue is that it is unclear on how the voltage limits impact the pricing. It is unclear whether the ISO’s reactive requirement is appropriate compensation for the actual reactive power provided. Some ISOs find that the current iterative process may not satisfy all power system requirements. MISO has reported that nomograms do not work well for local voltage constraints. In CAISO, the main cause of voltage instability is when the power system cannot meet reactive power demand, and this issue is becoming worse with real power transfer increasing [92].

While transmission is compensated for static reactive power, dynamic power (from generators) is often not compensated, even if it is often more valuable. Many capacity contracts for reactive power ignore how often a unit is in service and the location of the reactive power provider. However, reactive power provides very localized services due to high reactive losses in transit over lines with high loadings. Reactive power loss is proportional to the square of the current times reactance; since reactance is typically much greater than resistance for transmission lines, reactive power losses are much larger than real power losses. A concern for market-based reactive power pricing is that it may be easier to exercise market power with reactive generation; however, the wide range of possible participants (generators, transmission, and load) in reactive markets may lessen market power. Additionally, the current system does not incentivize reactive power production or installing equipment for reactive power [93]; moving to a more comprehensive compensation of reactive power may incentivize investment.
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5.1.4 Related Work

Zhong et. al. [94] suggest making reactive power payments based on three parts: availability, operation/loss cost, and opportunity payments. In these different settlements, the real power solution is solved beforehand and used as input for the other mechanisms. Kumar et. al. examine three different ways to price reactive power and examine the corresponding real and reactive nodal prices [95]. Baughman et. al. [37] show that using power factor penalties does not accurately capture the value of reactive power and that real-time reactive power prices are greatly impacted by voltage constraints. Chattopadhyay [96] solve for prices in a formulation that includes reserve pricing and the possibility of contingencies, and shows that higher prices are often due to higher reserve prices due to contingencies. The load prices are equal to the dual on the power balance constraint plus the price on reserves divided by $1 - \kappa$, where $\kappa$ the fraction where voltage collapses, equal to the apparent power at the point of voltage collapse minus current operating point divided by the apparent power at the point of voltage collapse.

Xie et. al. [97] find LMPs and nodal prices for reactive power (LMRPs) with the interior point method. Baughman et. al. [98, 99] and Liu et. al. [100] show how to find AC LMPs using the Lagrangian; Baughman also finds reactive prices and dispatch quantities. Momoh [101] shows how to find LMPs and LMRPs using an iterative process and breaks them down into congestion and loss components. Conejo et. al. [102] shows the sensitivity of LMPs to parameters of the network. O’Neill et. al. [103] examines running a power market based on the dual of the power-voltage formulation of the ACOPF. While these papers derive prices for real and reactive power, they do not guarantee zero duality gap or discuss how to settle voltage prices.

5.1.5 Contributions

Past research on the ACOPF has largely focused on the primal formulations of the OPF, which are designed to either be faster to solve, more optimal, or more feasible [104, 106]. This research focuses on showing how to run a real-time market using the dual formulation of the SLP-IV-ACOPF algorithm. This formulation yields explicit reactive power and voltage prices in addition to more accurate and reference-independent real power prices and congestion rent. Although we do not discuss these issues in this chapter, the SLP-IV-ACOPF can also be extended to include reserves and embedded as a part of other parts of the energy market, such as the unit commitment and reliability problems.

Knowing the true real-time prices, especially over time, is beneficial for better understanding the system operation and value of different players in the market. By understanding the way voltage limits and reactive power demands affect the system, better decisions can be made for investment in reactive power equipment. By including reactive power into a market context, we can reduce uplift and adjust constraints that may be set much more tightly than necessary. In addition to its solution time, another
reason the ACOPF has not been used to run power markets is because there may be a duality gap due to the nonconvexity of the ACOPF. This would mean that the amount paid into the market would not necessarily equal the amount paid out to the market; the prices derived would not necessarily support the market. Since the SLP-IV-ACOPF is linear, there is no duality gap and the prices support the market.

5.1.6 Organization

Section 5.2 gives the formulation of the SLP-IV-ACOPF and its dual. Section 5.3 describes the DC formulation with piecewise linear losses to which we compare the SLP-IV-ACOPF. Section 5.4 defines the market prices and market settlements, from the dual of the SLP-IV-ACOPF. Section 5.5 shows how the LMPs converge. Section 5.6 displays a side-by-side comparison of market prices and settlements for running AC and DC markets. Section 5.8 concludes the chapter.

5.2 SLP-IV-ACOPF Market Solution Procedure

First, we find the optimal real and reactive power dispatch by running the SLP-IV-ACOPF until the algorithm terminates. If the algorithm terminates with an acceptable outcome, we do a pricing run where we modify the penalty values. The market prices and settlements are taken from this pricing run.

5.2.1 Dispatch Run

We use the methodology in [41] to solve for the optimal dispatch with one change: replacing the limits on maximum line current (equations 27, 32, 33, and 37 in [41]) with constraints on maximum line power transfer, shown in (5.2.1).

$$\hat{v}_{n^r}^r k(n,m) + \hat{v}_{n^r}^r k(n,m) + \hat{v}_{n^r}^r k(n,m) - p_{viol}^r k(n,m) \leq P_{max}^k + \hat{v}_{n^r}^r k(n,m) + \hat{v}_{n^r}^r k(n,m) \rho_{max}^k (n)$$  (5.2.1)

In the dispatch run, the objective is to maximize market surplus with fixed demand; this is equivalent to minimizing the total bid cost plus the penalty for violating constraints (5.2.2). The limits on real and reactive generation and voltage are considered as ‘soft’ limits to assist in solving the problem. In the dispatch run, penalties are set as described in [41].

$$\min \sum_{n \in G} \left[ \sum_{l \in L} C_{n^l}^{q^l} p_n^l + C_{n^l}^{r^l} p_n^q + C_{n^l}^{q^l} q_n + \sum_{k \in K} P_{viol}^k p_n^k (n,m) \right] + \sum_{n \in N} \left[ P_n^\epsilon \left( p_{viol,-}^n + p_{viol,+}^n \right) + Q_n^\epsilon \left( q_{viol,-}^n + q_{viol,+}^n \right) + V_n^\epsilon \left( v_{viol,-}^n + v_{viol,+}^n \right) \right]$$  (5.2.2)
As in [41], there are four potential outcomes of the SLP solution when the algorithm terminates: (1) ACOPF KKT optimal solution, (2) ACOPF feasible but not optimal, (3) SLP feasible but ACOPF infeasible, and (4) Infeasible. We accept solutions with outcome (1) or (2); the pricing run is a formality similar to executing one more iteration. With outcome (3), the operator would decide whether the 'infeasibility' was within an acceptable range; for example, it may be acceptable to slightly exceed one line limit. In outcome (3), the pricing run may have a significant impact on the prices, and the penalty costs need to be set carefully. If outcome (3) is not acceptable or outcome (4) results, we would rerun the problem with a different starting points until outcome (1) or (2) or a satisfactory outcome (3) is achieved, or investigate what additional resources are needed for feasibility.

5.2.2 Pricing Run

Similar to the ISOs, we do an extra pricing run after the problem converges to a solution. The purpose of this pricing run is to modify the penalty values to reflect opportunity costs. The different ISOs have different penalty values for the pricing run than the dispatch run for real power mismatch and transmission limit. For the purposes of this chapter, we set each penalty in the pricing run to being 20% of its value in the dispatch run, which mimics the ISO process [107].

In the dispatch run, we limit the change in voltage between iterations (5.2.27-5.2.30) to reduce the error between the estimated and actual power and to help the problem converge. These constraints are only to aid convergence and are not physical limits imposed on the system. Ideally, we would remove these constraints as they impact prices. However, without these constraints, the real and reactive power generated is different than what would result from the voltages and currents in the system. Therefore, we cannot remove these constraints during the pricing run. The objective (5.2.3) is subject to the constraints listed in (5.2.4) through (5.2.32), with the corresponding dual variable for each constraint given to the right of the constraint.

\[
\min \sum_{n \in G} \left[ \sum_{l \in \mathcal{L}} C_{n,l}^g p_{n,l}^g + C_{n}^g p_n^g + C_{n}^q q_n^q \right] + \sum_{k \in \mathcal{K}} P_{k}^k \quad \text{viol}_k(n,m) \\
+ \sum_{n \in \mathcal{N}} \left[ P_{n}^\epsilon \left( p_{n}^{\text{viol},-} + p_{n}^{\text{viol},+} \right) + Q_{n}^\epsilon \left( q_{n}^{\text{viol},-} + q_{n}^{\text{viol},+} \right) + V_{n}^\epsilon \left( v_{n}^{\text{viol},-} + v_{n}^{\text{viol},+} \right) \right] 
\]

subject to

\[
-p_n^g + \sum_{l \in \mathcal{L}} p_{n,l}^g = -P_{n}^{\text{min}} \quad \rho_n^{\text{stepsum}} (5.2.4) \\
-p_{n,l}^g \geq -\left( P_{n}^{\text{max}} - P_{n}^{\text{min}} \right) / |\mathcal{L}| \quad \rho_{n,l}^{\text{steplim}} (5.2.5)
\]
\[
p^g_n + p_{n,\text{viol},-}^n \geq p_{n,\text{min}}^n
\]
\[
- p^g_n + p_{n,\text{viol},+}^n \geq -p_{n,\text{max}}^n
\]
\[
q^g_n + q_{n,\text{viol},-}^n \geq q_{n,\text{min}}^n
\]
\[
- q^g_n + q_{n,\text{viol},+}^n \geq -q_{n,\text{max}}^n
\]
\[
v_n^{\text{eq}} + v_{n,\text{viol},-}^n \geq (V_{n,\text{min}}^n)^2
\]
\[
- v_n^{\text{eq}} + v_{n,\text{viol},+}^n \leq -(V_{n,\text{max}}^n)^2
\]
\[
i_{k(n,m)}^r - G_{k(n)} v_n^r + G_{k(m)} v_n^r + B_{k(n)} v_m^j - B_{k(m)} v_m^j = 0
\]
\[
i_{k(n,m)}^j - B_{k(n)} v_n^r + B_{k(m)} v_m^r - G_{k(n)} v_n^j + G_{k(m)} v_m^j = 0
\]
\[
i_n^r - B_{n} v_n^r + G_{n} v_n^r - \sum_{k(n,n),k(n,m)} i_{k(n,n)}^r = 0
\]
\[
i_n^j + B_{n} v_n^j + G_{n} v_n^j - \sum_{k(n,n),k(n,m)} i_{k(n,n)}^j = 0
\]
\[
p_n^g - \hat{v}_n^{r(*)} i_n^r - \hat{v}_n^{j(*)} i_n^j - \hat{v}_n^{j(*)} i_n^j = P_n^d - \hat{v}_n^{r(*)} i_n^r - \hat{v}_n^{j(*)} i_n^j
\]
\[
q_n^g + \hat{v}_n^{r(*)} i_n^r - \hat{v}_n^{j(*)} i_n^j + \hat{v}_n^{j(*)} i_n^j = Q_n^d + \hat{v}_n^{r(*)} i_n^r - \hat{v}_n^{j(*)} i_n^j
\]
\[
v_n^{\text{eq}} - 2\hat{v}_n^{r(*)} v_n^r - 2\hat{v}_n^{j(*)} v_n^j = -(\hat{v}_n^{r(*)})^2 - (\hat{v}_n^{j(*)})^2
\]
\[
- v_n^r \cos(2\pi s/S) - v_n^j \sin(2\pi s/S) + v_{n,\text{viol},+}^n \geq -(V_{n,\text{max}}^n)^2
\]
\[
- \hat{v}_n^{r(1)} v_n^r - \hat{v}_n^{j(1)} v_n^j + v_{n,\text{viol},+}^n \geq -(V_{n,\text{max}}^n)^2
\]
\[
v_n^r \geq -V_{n,\text{max}}^r
\]
\[
- v_n^r \geq -V_{n,\text{max}}^r
\]
\[
v_n^j \geq -V_{n,\text{max}}^j
\]
\[
- v_n^j \geq -V_{n,\text{max}}^j
\]
\[
- v_n^r \geq -\hat{v}_n^{r(*)} - V_{n,h}^r
\]
\[
v_n^r \geq \hat{v}_n^{r(*)} - V_{n,h}^r
\]
\[
- v_n^j \geq -\hat{v}_n^{j(*)} - V_{n,h}^j
\]
\[
v_n^j \geq \hat{v}_n^{j(*)} - V_{n,h}^j
\]
\[
- \hat{v}_n^{r(*)} i_{k(n,m)}^r - \hat{v}_n^{j(*)} i_{k(n,m)}^j - \hat{v}_n^{j(*)} i_{k(n,m)}^j + p_{k(n)}^{\text{viol}}
\]
\[
\leq -P_{k,\text{max}}^n - \hat{v}_n^{r(*)} i_{k(n,m)}^r - \hat{v}_n^{j(*)} i_{k(n,m)}^j
\]
\[
- p_{n,\text{viol},+}^n, p_{n,\text{viol},-}^n, q_{n,\text{viol},+}^n, q_{n,\text{viol},-}^n, v_{n,\text{viol},+}^n, v_{n,\text{viol},-}^n \geq 0
\]
5.2.3 Lagrangian Dual Formulation of the Pricing Run

The dual objective of the SLP-IV-ACOPF is given in (5.2.33). The constraints are given in (5.2.34) through (5.2.46) with the corresponding primal variable to the right of each constraint. The dual objective is composed of the load payment, generation rent for real and reactive power, voltage payment, congestion rent, and shunt compensation.

\[
\begin{align*}
\max \sum_{n \in N} P_n^d \lambda_n^P + Q_n^d \lambda_n^Q \\
- \sum_{n \in \mathcal{G}} \left[ \left( P_{\text{max}}^n - P_{\text{min}}^n \right) / \left| \mathcal{L} \right| \rho_{n,l}^{\text{stlim}} \right] \rho_n^{\text{high}} \\
- \left( P_{\text{max}}^n - P_{\text{min}}^n \right) \rho_n^{\text{low}} + P_{\text{min}}^n \rho_n^{\text{steps}} + Q_n^\gamma^{\text{high}} - Q_n^\gamma^{\text{low}} \right] \\
- \sum_{n \in N} \left[ \left( V_{\text{max}}^n \right)^2 \nu_n^{\text{high}} - \left( V_{\text{min}}^n \right)^2 \nu_n^{\text{low}} + \left( \nu_n^{r(*)} \right)^2 + \left( \nu_n^{j(*)} \right)^2 \right] \nu_n^{\text{mag}} + \left( V_{\text{max}}^n \right)^2 \left( \nu_n^{\text{iter(*)}} + \sum_s \nu_n^{\text{poly}} \right) \\
+ V_{\text{max}}^n \left( \nu_n^{r,\text{UB}} + \nu_n^{r,\text{LB}} + \nu_n^{j,\text{UB}} + \nu_n^{j,\text{LB}} \right) \\
+ \nu_n^{r,\text{high}} \left( \nu_n^{r(*)} + V_n^h \right) + \nu_n^{r,\text{low}} \left( -\nu_n^{r(*)} + V_n^h \right) + \nu_n^{j,\text{high}} \left( \nu_n^{j(*)} + V_n^h \right) + \nu_n^{j,\text{low}} \left( -\nu_n^{j(*)} + V_n^h \right) \\
+ \left( \nu_n^{r(*)} \nu_n^{r(*)} + \nu_n^{j(*)} \nu_n^{j(*)} \right) \lambda_n^P \\
- \nu_n^{\text{iter(*)}} + \nu_n^{\text{iter(*)}} \rho_n^{\text{high}} \right] \lambda_n^Q \left( \nu_n^{\text{iter(*)}} + \sum_s \nu_n^{\text{poly}} \right)
\end{align*}
\]

subject to

\[
\begin{align*}
\lambda_n^P + \rho_n^{\text{iter}} - \rho_n^{\text{high}} - \rho_n^{\text{steps}} &= C_n^g,1 \quad \rho_n^g (5.2.34) \\
- \rho_n^{\text{stlim}} + \rho_n^{\text{steps}} &\leq C_n^g,2 \quad \rho_n^g (5.2.35) \\
\gamma_n^{\text{high}} - \gamma_n^{\text{low}} - \chi_n^Q &= C_n^g \quad q_n^g (5.2.36) \\
- \nu_n^{\text{low}} &\leq V_n^e \quad \nu_n^{\text{viol,-}} (5.2.37) \\
- \nu_n^{\text{high}} - \nu_n^{\text{poly}} - \nu_n^{\text{iter(*)}} &\leq V_n^e \quad \nu_n^{\text{viol,+}} (5.2.38) \\
- \sum_{m \in \mathcal{A}(n)} \left[ G_{k(n,m)}^{r,eq} i_{k(n,m)}^{j,eq} + B_{k(n,m)}^{r,eq} i_{k(n,m)}^{j,eq} + G_{k(m,n)}^{r,eq} i_{k(m,n)}^{j,eq} + B_{k(m,n)}^{r,eq} i_{k(m,n)}^{j,eq} \right] \\
+ \sum_{m \in \mathcal{A}(n)} G_{k(n,m)}^{r,eq} i_{k(n,m)}^{j,eq} - \nu_n^{r(*)} \lambda_n^P + \nu_n^{j(*)} \lambda_n^Q \\
- \sum_{m \in \mathcal{A}(n)} G_{k(n,m)}^{r,eq} i_{k(n,m)}^{j,eq} + B_{k(n,m)}^{j,eq} i_{k(n,m)}^{j,eq} + G_{k(m,n)}^{j,eq} i_{k(m,n)}^{j,eq} + B_{k(m,n)}^{j,eq} i_{k(m,n)}^{j,eq} \left( \nu_n^{\text{iter(*)}} + \sum_s \nu_n^{\text{poly}} \right) \rho_n^{\text{max}} \\
- \nu_n^{r,\text{UB}} - \nu_n^{r,\text{LB}} - \nu_n^{r,\text{high(*)}} + \nu_n^{r,\text{low(*)}} - 2\nu_n^{r(*)} \nu_n^{\text{mag}} \\
- \cos(2\pi s/S) \nu_n^{\text{poly}} - \nu_n^{r(*)} \nu_n^{\text{iter(*)}} &= 0 \quad \nu_n^r (5.2.39)
\end{align*}
\]
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We represent the piecewise segments indexed by \( g \) estimating the losses 2 representing the angle difference with \( \theta_n - \theta_m \) with a convex piecewise linear function. We represent the piecewise segments indexed by \( a \) representing the angle difference with \( \beta_k^{\pm}(g) - \beta_{k(n,m)}^{\pm} = \theta_n - \theta_m \). For this implementation of \( \beta_k^{\pm}(g) - \beta_{k(n,m)}^{\pm} = \theta_n - \theta_m \), we chose breakpoints at the angle differences of 0°, 0.25°, 0.5°, 0.75°, 1°, 1.5°, 2°, 3°, 4°, 5°, 7°, 10°, 15°, 20°, 30°, and 60°. The market settlements follow from the traditional DC settlements with one extra term: a loss payment, which is the dual value of the constraint on the angle difference breakpoint (equations (9) and (10) in \[108\]) times the limit on the angle difference breakpoint. The specific details follow in the online supplement \[109\].

\[
\sum_{m \in A(n)} \left[ B_k(n) R_{k(m)}^{eq} - G_k(n) R_{k(m)}^{eq} - B_k(m) R_{k(m)}^{eq} + G_k(m) R_{k(m)}^{eq} \right] \\
- B_n R_n^{eq} + G_n R_n^{eq} - i_n^{eq} \lambda_n - i_n^{eq} \lambda_n - \sum_{m \in A(n)} i_n^{eq} \rho_k^{max} \\
- u_n^{UB} + v_n^{LB} - v_n^{high(s)} + v_n^{low(s)} - 2 i_n^{eq} v_n^{mag} \\
- \sum_{s \in S} \sin(2\pi s/S) v_n^{poly} - i_n^{eq} v_n^{iter(s)} = 0 \\
\]

\[\nu_n^{mag} - \nu_n^{high} + \nu_n^{low} = 0 \quad \nu_n^{eq} \quad (5.2.41)\]

\[i_n^{eq} = i_n^{eq} + i_n^{eq} - \rho_k^{max} i_n^{eq} = 0 \quad i_n^{eq} \quad (5.2.42)\]

\[i_n^{eq} = i_n^{eq} + i_n^{eq} - \rho_k^{max} i_n^{eq} = 0 \quad i_n^{eq} \quad (5.2.43)\]

We compare the SLP-IV-ACOPF to a DC formulation that includes losses, modifying the formulation in \[108\] to minimize a piecewise linear cost. The formulation in \[108\] estimates the losses 2\( g_k(n,m) (1 - \cos(\theta_n - \theta_m)) \) with a convex piecewise linear function. The market settlements follow from the traditional DC settlements with one extra term: a loss payment, which is the dual value of the constraint on the angle difference breakpoint (equations (9) and (10) in \[108\]) times the limit on the angle difference breakpoint. The specific details follow in the online supplement \[109\].

\[
\min \sum_{n \in G} \left[ \sum_{l \in L} C_{n,l}^{p^g} P_{n,l}^g \right] + C_n^{p^g} P_n^g \\
\text{subject to} \\
- P_n^g + \sum_{l \in L} P_{n,l}^g = - P_n^{min} \quad \rho_n^{stepsum} \quad (5.3.2) \\
- P_{n,l}^g \geq - (P_n^{max} - P_n^{min}) / |L| \quad \rho_n^{steplim} \quad (5.3.3) 
\]

5.3 DC with Losses Formulation
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\[ p_n^g + p_n^{viol,-} \geq P_n^{min} \]
\[ -p_n^g + p_n^{viol,+} \geq -P_n^{max} \]
\[ p_{k(n,m)} - B_k (\theta_n - \theta_m) = 0 \]
\[ -\sum_{m \in A(n)} p_{k(n,m)} - \sum_{m \in A(n)} p_{Loss}^k + P_n^g = P_n^d \]
\[ p_{k(n,m)} + p_{Loss}^k \leq P_{k(n,m)}^{max} \]
\[ \theta_n - \theta_m = \sum_a \beta_{k(n),a}^+ - \beta_{k(n),a}^- \]
\[ \beta_{k(n),a}^+ \leq \beta_{a}^{max} \]
\[ \beta_{k(n),a}^- \leq \beta_{a}^{max} \]
\[ p_{Loss}^k = 2g_{k(n,m)} \sum_a \Omega_a \left( \beta_{k(n),a}^+ + \beta_{k(n),a}^- \right) \]
\[ \beta_{k(n),a}^+, \beta_{k(n),a}^- \geq 0 \]

5.3.1 Dual Objective and Settlements

The dual objective is given in (5.3.14).

\[
\sum_{n \in N} \lambda_n^P P_n^d - \sum_{k \in K} \sum_a \beta_{a}^{max} \left( \phi_{k(n),a}^+ + \phi_{k(n),a}^- \right) - \sum_{k \in K} p_{k(n,m)}^{max} P_{k(n,m)}^{max} \\
- \sum_{n \in G} \left[ \sum_{l \in \mathcal{L}} \left( P_n^{max} - P_n^{min} \right) / |\mathcal{L}| P_{n,l}^{steplim} \right] + P_n^{max} \rho_n^{high} - P_n^{min} \rho_n^{low} + P_n^{min} \rho_n^{stepsum}
\]

(5.3.14)

The real power load payment \( L P_{n}^{tot} \) is \( \lambda_n^P P_n^d \). The total real power generator rent \( GR_{n}^p \) is shown in (5.3.15).

\[
\sum_{n \in G} \left[ \sum_{l \in \mathcal{L}} \left( P_n^{max} - P_n^{min} \right) / |\mathcal{L}| P_{n,l}^{steplim} \right] + P_n^{max} \rho_n^{high} - P_n^{min} \rho_n^{low} + P_n^{min} \rho_n^{stepsum}
\]

(5.3.15)

The real power congestion rent \( CR_{k}^p \) is \( \sum_{k \in K} p_{k(n)}^{max} P_k^{max} = \sum_{k \in K} p_{k(n,m)} \left( \lambda_n - \lambda_m \right) \). The DC loss payment is \( \sum_{k \in K} \sum_a \beta_{a}^{max} \left( \phi_{k(n),a}^+ + \phi_{k(n),a}^- \right) \) and would be paid to the transmission lines. The reactive power terms for load payment \( L P_{n}^q \), generation rent \( GR_{n}^q \), and congestion rent \( CR_{k}^q \) are all equal to zero.
Market Terms For the SLP-IV-ACOPF

This section describes market prices and market settlements for a real-time market running the SLP-IV-ACOPF. The prices in the SLP-IV-ACOPF include all of the prices that are presently in the market and also give additional prices on reactive power and voltage.

5.4.1 Market Prices

This section details the nodal prices for real and reactive power as well as the flowgate prices on transmission lines.

5.4.1.1 Locational Marginal Price (LMP)

The locational marginal price, $\lambda^P_n$, is the price for real power at location $n$. It is a function of generation limits and generator costs (5.4.1). The LMP for the SLP-IV-ACOPF cannot directly be broken down into reference, congestion, and loss terms due to voltage being a variable rather than a parameter. Papers that have broken the LMPs down into these components either fix voltage [11] or find the components by doing an empirical sensitivity analysis [10].

\[
\lambda^P_n = \rho^\text{high}_n - \rho^\text{low}_n + \rho^\text{stepsum}_n - C_n^g \tag{5.4.1}
\]

To further understand the components that make up the LMP, we can examine the term in (5.4.2).

\[
\frac{\partial L}{\partial \hat{v}^r_n} \hat{v}^r_n + \frac{\partial L}{\partial \hat{v}^j_n} \hat{v}^j_n = 0 \tag{5.4.2}
\]

If we let $\hat{\theta} = atan(\frac{\hat{v}^j_n}{\hat{v}^r_n})$ and solve for the LMP from (5.4.2), the LMP can be stated as shown in (5.4.5).

\[
\lambda^P_n = -\frac{1}{\bigg( -G^s_n \hat{v}_n^2 + \sum_{m \in A(n)} \hat{p}_{k(n,m)} \bigg)} \times \bigg[ -\hat{v}_n \sum_{m \in A(n)} \lambda^P_m \hat{v}_m \left[ (G_{k(n)} + G_{k(m)}) \cos \left( \hat{\theta}_n - \hat{\theta}_m \right) + (B_{k(n)} + B_{k(m)}) \sin \left( \hat{\theta}_n - \hat{\theta}_m \right) \right] 
\]

\[
+ \hat{v}_n \sum_{m \in A(n)} \lambda^Q_m \hat{v}_m \left[ (G_{k(n)} + G_{k(m)}) \sin \left( \hat{\theta}_n - \hat{\theta}_m \right) + (B_{k(n)} + B_{k(m)}) \cos \left( \hat{\theta}_n - \hat{\theta}_m \right) \right] 
\]

\[
- \lambda^Q_n \left( B^s_n \hat{v}_n^2 + \sum_{m \in A(n)} \hat{q}_{k(n,m)} \right) - \left( \nu^{r,\text{high}}_n - \nu^{r,\text{low}}_n \right) \hat{v}_n^r - \left( \nu^{j,\text{high}}_n - \nu^{j,\text{low}}_n \right) \hat{v}_n^j \tag{5.4.5}
\]
The locational marginal price has several components. The first two terms are dependent on neighboring nodes and are analogous to real and reactive power flows. The third term is the nodal reactive power price times the reactive power coming into the node. The fourth includes the box limits on voltage. The second to last term is the voltage polygonal limits. The last term is the FTR/congestion rent on the line, similar to the DC case.

From the complementary slackness conditions on $i^r_n$ and $i^j_n$, we can derive new terms for $\lambda^P_n$ and $\lambda^Q_n$.

$$\lambda^P_n = \frac{i^r_n - v^r_n}{v^r_n} = \frac{i^j_n + v^r_n}{v^j_n} \lambda^Q_n$$

(5.4.6)

### 5.4.1.2 Locational Marginal Reactive Price (LMRP)

The locational marginal price, $\lambda^Q_n$, is the price for reactive power at location $n$. It is influenced by the generator’s reactive limits and reactive cost. Even if reactive power is priced at zero, providing reactive power may have a non-zero price if a generator is at its upper or lower reactive limit (5.4.7). $C^q_n$ may be also set as the cost of the reactive equipment and its commodity costs, similar to how real power is priced. We could also extend this framework to price reactive power as a linearized quadratic cost.

$$\lambda^Q_n = \gamma^\text{high}_n - \gamma^\text{low}_n - C^q_n$$

(5.4.7)

From the complementary slackness conditions on $i^r_n$ and $i^j_n$ we can get another form of the prices for $\lambda^Q_n$.

$$\lambda^Q_n = \frac{i^r_n - v^r_n}{v^r_n} \lambda^P_n = -\frac{i^j_n + v^j_n}{v^j_n} \frac{i^j_n + v^j_n}{v^j_n} \lambda^P_n$$

(5.4.8)

### 5.4.1.3 Flowgate Prices

The flowgate price $\rho^\text{max}_k(n)$ is the value of one more unit of real power capacity on the line. It is a function of dual values on current definition constraints and the voltage fixed point.

$$\rho^\text{max}_k(n) = \frac{i^r_k(n,m) + i^r_k(n,m) - i^r_k(n,m)}{v^r_n} = \frac{j^r_k(n,m) + j^r_k(n,m) - j^r_k(n,m)}{v^r_n}$$

(5.4.9)
5.4.1.4 Voltage Prices

Due to the representation of voltage in rectangular coordinates and the linearization, there are many voltage prices. The prices $\nu_n^{mag}$, $\nu_n^{low}$, $\nu_n^{high}$, $\nu_n^{poly}$, and $\nu_n^{iter(h)}$ pertain to the voltage magnitude. The prices $\nu_n^{r,LB}$, $\nu_n^{r,UB}$, $\nu_n^{j,LB}$, and $\nu_n^{j,UB}$ pertain to the real and imaginary components of voltage; these prices can only be non-zero if the voltage component is at the maximum voltage. The prices $\nu_n^{r,high}$, $\nu_n^{r,low}$, $\nu_n^{j,high}$, $\nu_n^{j,low}$ are for voltage staying within its box convergence constraints. All the prices except those on the box convergence constraints are system requirements; the box constraints are not system requirements and are used only for convergence. Generators are incentivized to follow the market solution for all prices except for those on the box convergence constraints. For example, if $\nu_n^{r,UB} > 0$, then the generator has the incentive to push its real voltage component as high as possible; since $\nu_n^{r,UB} > 0$, by complementary slackness, that means that $v^r_n = V_{n}^{max}$, so the generator will be served by setting $v^r_n = V_{n}^{max}$ since that gives it the highest payment possible.

However, the box convergence prices $\nu_n^{r,high}$, $\nu_n^{r,low}$, $\nu_n^{j,high}$, $\nu_n^{j,low}$ should stay internal to the ISO and not be viewable to generators. For example, if $\nu_n^{r,high} > 0$ and the generator is strictly within the voltage limits, the generator has an incentive to increase voltage although the ISO’s desired behavior is for the generator to stay at its present voltage point. If the generator does not see box convergence prices, then it only sees a voltage prices of zero, so its incentive would be to stay at its present voltage setting. ISOs do not publish prices for the nomogram constraints, and not publishing the box convergence prices is analogous to this behavior.

5.4.2 Market Settlements

This section shows the equivalence between the minimization of generation cost and the total market settlement. The dual objective is comprised of the load payment, generator profit, congestion rent, voltage payment, and shunt compensation. The term $(\hat{v}_n^{r(i)} \hat{i}_n^{r(i)} + \hat{v}_n^{i(j)} \hat{i}_n^{i(j)}) \lambda_n^P + (-\hat{v}_n^{r(i)} \hat{i}_n^{i(j)} + \hat{v}_n^{i(j)} \hat{i}_n^{r(i)}) \lambda_n^Q$ is broken into congestion rent and shunt compensation later in this section.

5.4.2.1 Load Payment

As in the current market system, load pays its demand for real power times the LMP. Here, it would also pay its demand for reactive power times the LMRP. For node $n$, the real power load payment $L_{n}^{p}$ is given in (5.4.10), the reactive power load payment $L_{n}^{q}$ is given in (5.4.11), and the total load payment $L_{n}^{tot}$ is the sum of the real and reactive payments (5.4.12).

$$L_{n}^{p} = P_{n}^{d} \lambda_{n}^{P}$$
$$L_{n}^{q} = Q_{n}^{d} \lambda_{n}^{Q}$$
$$L_{n}^{tot} = L_{n}^{d} \lambda_{n}^{P} + Q_{n}^{d} \lambda_{n}^{Q}$$
5.4.3 Generation Revenue

By combining (5.4.1) and the complementary slackness conditions of (5.2.35), we can restate the LMP times generation to find the generation revenue (5.4.13).

\[
\chi_n^P p_n^g = (\rho_n^{\text{high}} - \rho_n^{\text{low}}) p_n^g + \rho_n^{\text{steps}} p_n^g - C_n^g
\] (5.4.13)

From the complementary slackness conditions on \(p_n^g\) and \(p_{n,l}^g\), we can solve for \(\rho_n^{\text{steps}}\).

\[
\rho_n^{\text{steps}} p_n^g - \rho_n^{\text{steps}} p_{n,l}^g = \sum_{l \in L} C_{n,l}^g p_{n,l}^g + \rho_n^{\text{steps}} p_{n,l}^g
\] (5.4.14)

We can also find the generation revenue for reactive power, \(\chi_n^Q q_n^g\).

\[
\chi_n^Q q_n^g = \gamma_n^{\text{high}} q_n^g - \gamma_n^{\text{low}} q_n^g - C_n^q
\] (5.4.15)

5.4.3.1 Generator Rent

The generator rent in the SLP-IV-ACOPF is the same as in the DCOPF case with an additional term for reactive power. At node \(n\), the real power component of the generation rent, \(GR_n^p\) is given in (5.4.16) and the reactive power generation rent, \(GR_n^q\) is given in (5.4.17); the total rent \(GR_n^{\text{tot}}\) paid to the generator at node \(n\) is the sum of these two terms (5.4.18).

\[
GR_n^p = P_n^{\text{max}} \rho_n^{\text{high}} - P_n^{\text{min}} \rho_n^{\text{low}} + P_n^{\text{steps}} \rho_n^{\text{steps}} - \sum_{l \in L} \left( (P_n^{\text{max}} - P_n^{\text{min}}) \rho_{n,l}^{\text{steps}} \right) / |L|
\] (5.4.16)

\[
GR_n^q = Q_n^{\text{max}} \gamma_n^{\text{high}} - Q_n^{\text{min}} \gamma_n^{\text{low}}
\] (5.4.17)

\[
GR_n^{\text{tot}} = GR_n^p + GR_n^q
\] (5.4.18)

5.4.3.2 Congestion Rent

The value of power on the line \(k\) paid to the line operator (congestion rent for line \(k\)), \(CR_k^{\text{tot}}\) (5.4.21) is composed of real power congestion rent, \(CR_k^p\), (5.4.19) and reactive power congestion rent, \(CR_k^q\) (5.4.20).

\[
CR_k^p = \hat{p}_{k(n,m)} \rho_k(n) + \hat{p}_{k(m,n)} \rho_k(m) + P_k^{\text{max}} (\rho_k^{\text{max}} + \rho_k^{\text{max}}) + \hat{p}_{k(n,m)} \chi_n^P + \hat{p}_{k(m,n)} \chi_m^P
\] (5.4.19)

\[
CR_k^q = \hat{q}_{k(n,m)} \lambda_n^Q + \hat{q}_{k(m,n)} \lambda_m^Q
\] (5.4.20)

\[
CR_k^{\text{tot}} = CR_k^p + CR_k^q
\] (5.4.21)

Since the differences between iterations decrease with each iteration, the difference in the power on lines between the last iteration \(\hat{p}_{k(n,m)}\) and current iteration \(p_{k(n,m)}\) should
be small. As will be seen in Section 5.6, in some cases, the lines pay the ISO due to line losses. If there are no line limits, we can write the congestion rent as in (5.4.22).

\[ CR_{k}^{\text{nom}} = \hat{p}_{k(n,m)} \left( \lambda_{n}^{P} - \lambda_{m}^{P} \right) + \left( \hat{q}_{k(n,m)} \right) \lambda_{m}^{Q} + \left( \hat{q}_{k(n,m)} \right) \lambda_{m}^{Q} \]  

(5.4.22)

If we assume that \( \hat{p}_{k(n)} < 0 \) and \( |\hat{p}_{k(m)}| < \hat{p}_{k(n)} \) and the same for reactive power, then \( CR_{k}^{\text{nom}} = \hat{p}_{k(n,m)} \left( \lambda_{n}^{P} - \lambda_{m}^{P} \right) - \lambda_{m}^{P} P_{k}^{\text{loss}} - \lambda_{m}^{Q} Q_{k}^{\text{loss}} \) and we could have that \( CR_{k}^{\text{nom}} < 0 \).

While operators may earn money on the transfer between the two nodes, they may have to pay the ISO for line losses.

### 5.4.3.3 Shunt Settlement

Owners of shunts (transmission equipment) receive shunt compensation at node \( n \), \( SC_{n} \), shown in (5.4.23).

\[ SC_{n} = G_{n}^{sh} \nu_{n}^{2} \lambda_{n}^{P} - B_{n}^{sh} \nu_{n}^{2} \lambda_{n}^{Q} \]  

(5.4.23)

If there is more than one shunt compensation device at the node, then the settlement would be allocated based on the proportion that each owner’s device contributed. If \( SC_{n} < 0 \), then the owner(s) will pay the ISO; if \( SC_{n} > 0 \), then the owner will get paid by the ISO. If \( \lambda_{n}^{P} > 0 \), then the conductive shunt owner gets paid as it is improving the power factor; if \( \lambda_{n}^{P} < 0 \), then the owner gets penalized as it would help increase power transfer and we would rather reduce it. For susceptive shunt equipment, if \( \lambda_{n}^{Q} < 0 \), then the network desires less reactive power; since susceptive shunt compensation tends to improve real power transfer and raise the power factor, the susceptive shunt owner gets paid more when \( \lambda_{n}^{Q} < 0 \). If \( \lambda_{n}^{Q} > 0 \), then the network desires more reactive power; since susceptive shunt compensation tends to reduce reactive power, the susceptive equipment owner is penalized.

### 5.4.3.4 Voltage Support

The value of voltage support at node \( n \), \( VV_{n} \), as shown in (5.4.25), is a function of the limits on maximum and minimum voltage.

\[ VV_{n} = V_{n}^{\text{max}} - V_{n}^{\text{min}} + \nu_{n}^{\text{high}} + \left( \hat{v}_{n}^{r(s)} \right) + \left( \hat{v}_{n}^{j(s)} \right)^{2} \nu_{n}^{\text{mag}} \]  

\[ + \left( V_{n}^{\text{max}} \right)^{2} + \left( \nu_{n}^{\text{iter(h)}} \right) + \left( \nu_{n}^{\text{polm}} \right) \]  

\[ + \nu_{n}^{\text{h,UB}} + \nu_{n}^{\text{h,LB}} + \nu_{n}^{\text{j,UB}} + \nu_{n}^{\text{j,LB}} + \nu_{n}^{\text{h,high}} + \nu_{n}^{\text{h,low}} \]  

(5.4.24)

\[ + \nu_{n}^{\text{j,low}} + \nu_{n}^{\text{j,high}} + \nu_{n}^{\text{j,low}} \]  

(5.4.25)

Voltage is a public good that results from real and reactive power. It cannot be directly attributable to marginal changes in power. Therefore, the voltage support
settlement that results from the constraints on voltage should be considered as a public good and be distributed as such. The revenue to the ISO from the voltage support settlement can be distributed in a number of different ways. It can be considered as extra revenue for the ISO to pay for reactive/voltage support, whether with fixed contracts or paying for opportunity cost. Also, knowing the voltage value over a year can help determine how to price long-term contracts. ISOs may also decide to allocate voltage support in the shorter-term, paying more to generators that provide voltage support to nodes where the support is more crucial, who provide more reactive power, and those closer to the source. This term for voltage support is analogous to the value of the interface limits times their duals or the cost of real power losses in the present energy market.

5.5 LMP Convergence

This section reports the LMP deviations between the iterations. The following box plots in Figures 5.1 through 5.2 aggregate the data for all the buses and show the mean absolute percentage difference, minimum and maximum, and error bars for the LMP differences from iteration to iteration. The outliers are represented as red pluses. Figure 5.1 shows the LMP convergence for the IEEE 14 bus problem (no line limits) when the SLP-IV-ACOPF uses a flat start. Some of the LMPs in the first iteration are negative, and they are very different from the final results. From the first to the second iteration, the LMPs change on average by over 70%. After the second iteration, the largest change the LMPs is 6%, from the second to the third iteration. After the 4th iteration, the largest LMP difference is under 1%. As seen in Figure 5.2, if a DC start is used instead of a flat start the LMP convergence improves greatly. The difference from the first to second iteration does not exceed 2.5% for any bus; after the first iteration, the difference between iterations is less than 0.5%.
Figure 5.1: LMP Differences Between Iterations for IEEE Case 14 with a Flat Start

Figure 5.2: LMP Differences Between Iterations for IEEE Case 14 with a DC Start
The same pattern is exhibited in IEEE case 14 with high and low line limits in Figures 5.3 through 5.6. If a flat start is used, the LMPs in the first iteration are very different from the final LMPs and are much closer when using a DC start.

Figure 5.3: LMP Differences Between Iterations for IEEE Case 14 with a Flat Start and 0.71 PLine Limit
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Figure 5.4: LMP Differences Between Iterations for IEEE Case 14 with a Flat Start and 0.71 PLine Limit

Figure 5.5: LMP Differences Between Iterations for IEEE Case 14 with a Flat Start and 0.2675 PLine Limit
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5.5.1 Case 2383wp

For this case, we use the 2383wp Polish system case with our real power limits equal to the given apparent power limits. With a DC start, the average LMP change from the first to second iteration is fairly small, but there are a few with up to a 60% difference in the price. The LMP difference from iteration to iteration stays under 6% after the 2nd iteration. The average LMP difference is 2% or less in all differences between iterations, even from the first to the second iteration. In all of these cases, one can see that the LMPs do converge to stable values as the algorithm converges on the primal values.
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5.6 Economic Analysis

This section discusses the differences in the market prices and payments when the OPF is solved with the DCOPF with losses (DCL) versus the SLP-IV-ACOPF. It shows the magnitudes of the new payments under the AC system: reactive power, voltage support, and reactive congestion rent.

5.6.1 Nodal Power Prices for Case 14

The LMP differences between the AC and DCL cases are shown in Figure 5.8. The y axis shows (AC LMP - DCL LMP)/(DCL LMP). With no line limits, the DCL case overestimates half of the LMPs, underestimates the LMP for bus 14, and accurately estimates the other LMPs. With a 0.71 line limit, the DCL over and under estimations by the same amount. With the 0.2675 line limit case, 5 LMPs are underestimated and the rest are fairly close. In all line limit cases, the greatest error in the DCL LMP is less than 1%. The LMRPs are shown in Figure 5.9. The range of the LMRPs tends to become smaller as the line limits are tightened. In these cases, where there is plentiful reactive power, the LMRP is small.
### Figure 5.8: Differences in AC and DCL LMPs for IEEE Case 14 with Different Line Limits

<table>
<thead>
<tr>
<th>Bus</th>
<th>% Difference in AC and DC LMPs (AC LMP-DC LMP)/DC LMP</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1.0</td>
</tr>
<tr>
<td>2</td>
<td>-0.8</td>
</tr>
<tr>
<td>4</td>
<td>-0.6</td>
</tr>
<tr>
<td>6</td>
<td>-0.4</td>
</tr>
<tr>
<td>8</td>
<td>-0.2</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>0.2</td>
</tr>
<tr>
<td>14</td>
<td>0.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bus</th>
<th>% Difference in AC and DCL LMPs (AC LMP-DC LMP)/DCL LMP</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1.0</td>
</tr>
<tr>
<td>2</td>
<td>-0.8</td>
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<tr>
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<td>6</td>
<td>-0.4</td>
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<td>10</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>0.2</td>
</tr>
<tr>
<td>14</td>
<td>0.4</td>
</tr>
</tbody>
</table>

### Figure 5.9: LMRP for Case 14

<table>
<thead>
<tr>
<th>Bus</th>
<th>Locational Marginal Reactive Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-0.2</td>
</tr>
<tr>
<td>2</td>
<td>-0.1</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0.1</td>
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<tr>
<td>8</td>
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<tr>
<td>10</td>
<td>0.3</td>
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<tr>
<td>12</td>
<td>0.4</td>
</tr>
<tr>
<td>14</td>
<td>0.5</td>
</tr>
</tbody>
</table>

- P Line Limit 0.2675
- P Line Limit 0.71
- No P Line Limit
5.6.2 Market Settlements for Case 14

Table 5.1 shows the difference in aggregate payments under a DCL and an AC system for IEEE Case 14 with three types of line limits: no line limits, a 0.71 p.u. and a 0.2675 p.u. limit on real power across lines. The tighter the line limits, the higher the congestion rent and the lower the reactive power and voltage support. This impact is likely because limiting real power across lines also limits the difference in voltages between two ends of a bus. From this case, it also appears that the AC reactive power and voltage support plus congestion rent is close to the DCL congestion rent plus loss cost for the two cases with line limits. This may mean that when ISOs use nomogram constraints, much of the voltage support and reactive power value is going to congestion rent. Since the ISO pays congestion rent, a reactive requirement, and in some cases an opportunity cost for providing reactive power, it may be paying for reactive power/voltage support multiple times. It appears that reactive power and voltage support and being incorporated into the congestion rent; that is, we may be overpaying in FTRs when part of these funds could be used to pay for reactive power and voltage support.

<table>
<thead>
<tr>
<th></th>
<th>No Line Limits</th>
<th>0.71 PLine Limit</th>
<th>0.2675 PLine Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AC DCL % Diff</td>
<td>AC DCL % Diff</td>
<td>AC DCL % Diff</td>
</tr>
<tr>
<td>Obj. Func.</td>
<td>8092.2 8085.1  0.1%</td>
<td>8488.2 8478.6  0.1%</td>
<td>9323.6 9324.1  0.0%</td>
</tr>
<tr>
<td>$L P^p$</td>
<td>10391.9 10398.4  0.1%</td>
<td>10640.0 10634.2  0.05%</td>
<td>11046.4 11020.3  0.2%</td>
</tr>
<tr>
<td>$L P^q$</td>
<td>12.3 0 100%</td>
<td>8.5 0 100%</td>
<td>7.2 0 100%</td>
</tr>
<tr>
<td>$G R^p$</td>
<td>1904.5 1940.8  1.9%</td>
<td>1101.2 1105.0  0.35%</td>
<td>881.70 875.0  0.8%</td>
</tr>
<tr>
<td>$G R^q$</td>
<td>0 0 0%</td>
<td>0 0 0%</td>
<td>0 0 0%</td>
</tr>
<tr>
<td>$V V$</td>
<td>814.9 0 100%</td>
<td>320.2 0 100%</td>
<td>188.4 0 100%</td>
</tr>
<tr>
<td>$C R^p$</td>
<td>-395.0 0 100%</td>
<td>747.4 902.9  20.80%</td>
<td>667.2 743.9 11.5%</td>
</tr>
<tr>
<td>$C R^q$</td>
<td>-6.9 0 100%</td>
<td>-5.9 0 100%</td>
<td>-6.3 0 100%</td>
</tr>
<tr>
<td>$S C$</td>
<td>-5.4 0 0%</td>
<td>-2.6 0 0%</td>
<td>-0.9 0 0%</td>
</tr>
<tr>
<td>$L o P$</td>
<td>0 372.5 100%</td>
<td>0 147.6 100%</td>
<td>0 77.3 100%</td>
</tr>
<tr>
<td>Obj. excl. $L P^p &amp; G R^p$</td>
<td>395.3 372.5 5.8%</td>
<td>1050.6 1050.5 0.0%</td>
<td>841.1 821.2 2.4%</td>
</tr>
</tbody>
</table>

Table 5.1: AC vs. DCL Market Payments for IEEE 14-Bus Problem

5.6.3 Market Settlements for Case 2383wp

The market settlements for the Polish case 2383wp are shown in Table 5.2 for both the case without line limits and with letting the limit on power across the line be the same as the limit given for apparent power on the line. In this case, the DCL load payment with line limits is higher than the AC load payment with line limits. The DCL case yields more congestion rent than the AC case; in the DCL case, the net congestion rent is paid from the ISO; in the AC case, the net congestion rent is paid to the ISO.


The power limits in this formulation are represented as box constraints, with limits on real and reactive power independently. These limits could be represented as D-curves, and generators would also receive the value of lost opportunity cost. Equation (5.7.1) represents the segments of a D-curve for a generator at bus n with segments i.

\[ -\alpha_{n,i}p^g_n - \beta_{n,i}q^g_n \geq -D_{n,i} \delta_i \]

The payment from the D curves would then be \( \sum_i -\delta_{n,i} D_{n,i} \), and it would be paid as part of the generator rent. The LMP would then be modified such that it would be the LMP not considering D curves minus \( \alpha_i \delta_i \) and the LMRP would be the LMRP not considering D curves minus \( \beta_i \delta_i \).

5.8 Conclusion and Future Work

This formulation does not discuss reserves and reserve pricing. It could be extended into a unit commitment model that would yield reserve prices and ramping products. Additionally, since reactive power requirements can be solved through investment in reactive equipment instead of market pricing [112], it would be interesting to compare these two approaches.

This work defines a more complete real-time market by pricing reactive power and voltage. It shows that the current representation of the market may be paying transmission for both congestion rent and reactive support when part of these funds should be going to reactive support. It produces LMPs based on an OPF formulation that includes
voltage and reactive power; these LMPs converge as the primal dispatch converges, and one more iteration after convergence will have a minimal impact on prices. This approach gives the explicit cost of reactive power and voltage support, which can assist in deciding where to cite more equipment and how to price long-term contracts. This representation shows how reactive power and voltage impact prices. The 14-bus example illustrates that tightening line limits as surrogates for voltage and reactive power limits likely means that voltage support is being paid twice; once via congestion rent and again as uplift or in the reactive capacity contract. With this formulation, we can explicitly separate the reactive power compensation from the congestion rent. Additionally, since we model losses, reactive power, and voltage directly, the LMPs are independent of the reference bus and nomogram constraint selections.
Chapter 6

Economic Switching

One of the major challenges in finding solutions to the OPF is that while power injections and withdrawals can be controlled, flow across lines cannot be directly controlled. This is in direct contrast to many network problems, including routing vehicles to minimize transportation time or routing messages in a wireless network. Here, there are cases where one path of power flow may be overloaded at the desired power generation while using another is very underloaded; however, one cannot simply force part of the power onto the underloaded path. One way to control the flow is to open the breakers connecting the overloaded line - effectively shutting it off, so no power or current crosses the line. The power may then flow to the underloaded one, and as long as any network constraints are not currently violated, this configuration may be better than the one with more lines in the network. This practice can also assist in finding a feasible power flow solution in a congested network. The feasible set for power flow solutions is larger with the possibility of transmission switching than with considering a static network. Transmission switching can also reduce the power production cost as one may be able to use cheaper power producers than with only the static network.

Switching lines off is common in power networks. Historically, lines are taken out of service for maintenance or switched off to reduce cost in times of low load, although these decisions are often made using experience rather than performing an optimization. The practice of transmission switching for economic benefit is starting to become more common. PJM already implements some transmission switching to reduce overloads and usually does some optimization in deciding when to do preventative maintenance on lines [114].

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The section on examining the performance of the congestion rent heuristic appeared partly in [113]. I had the idea to test the congestion rent and LMP difference heuristics on the 2383wp case and did preliminary testing in MATPOWER. Mostafa and Akshay did a more complete testing that also checked for network islanding.
6.1 Literature Review

Fisher et. al. [115] showed that transmission switching using the DCOPF solution can greatly reduce the total power cost: by switching 38 lines on the IEEE 118 bus system, the power cost was 25% less than on the original network. While 38 is a high number of lines to open on a network with 186 lines, most of the improvement was achieved by opening 3 lines, which reduced the cost by 20%. Hedman et. al. showed that transmission switching can still be beneficial even when taking into account contingencies [11] and within the context of unit commitment and reliability requirements [12].

Potluri solved the power-voltage formulation of the ACOPF [27] with an interior-point solver and found that switching lines can be beneficial in this setting. They found that opening 3 lines in the 118 bus IEEE case reduced power cost by 4.3%. The amount of savings of switching out each line was highly dependent on the problem parameters, as the incremental benefit of switching two lines instead of one was very beneficial in the case of high load but less so in the case of low load.

Many transmission switching heuristics have been proposed, with most being based on the DCOPF. Even though the DCOPF is a linear approximation of the nonlinear system, adding the binary switching variables greatly increases the size of the program and makes it nonlinear. While rarely seen in practice, the maximum number of subproblems that would need to be solved in the branch and bound tree is $2^{|K|}$, where $|K|$ is the number of lines in the network.

Since transmission switching often achieves large benefits even when only a few lines are opened, one common heuristic is to open one line at a time [116]. One cycles through solving the problem where one line is fixed as being out and the others are in, which removes the mixed integer component of the problem. Once one line is selected, it is fixed to be out and further iterations may take place. The advantage of this heuristic is that it is easily parallelized and can easily be stopped at a good answer. Additionally, to implement transmission switching, knowing that each individual switching action is feasible is important. However, this scheme may not consider combinations of lines that may be more beneficial than those found in each increment.

If it is decided that it is better to open groups of lines rather than finding the lines to switch one by one, it is common to reduce the computational effort by only allowing a small number of lines to be opened. That is, to only open up to $J$ lines, one adds the constraint $\sum_{k \in K} (1 - z_k) \leq J$. This greatly reduces the size of the branch and bound tree and eliminates nonsensical network arrangements - for example, opening all the lines - from consideration. Generally, the benefits of switching are greatest with the first few lines opened, so this method tends to come close to the cost benefits when allowing an unlimited amount of lines to open. Additionally, transmission operators prefer not to open too many lines in order to maintain reliability of the network and also to be able to implementation the solution without too much difficulty.

Another way to reduce the problem size is to assume that transmission switching cannot create islands [117, 118]. While there is an exponential number of these constraints if all of them are added from the beginning, islanding constraints can be added
in the MIP nodes to prevent branching to poor solutions. Also, including the $N - 1$ criteria to influence branching greatly reduces the size of the MIP tree.

In many of the test problems, we may have parallel lines connecting two nodes that have the same impedance and capacity. While we would get the same solution with say line A on and B off or line B on and A off, the MIP tree would go through these two cases independently. So, we could instead add a constraint to prioritize one of them, say $z_A \geq z_B$, to break the tie [119].

Liu et. al. [120] suggest ways to prescreen the lines that can be switched. It suggests adding $N - 1$ criteria by requiring that any switchable line has each end connected to two or more lines. It also suggests ranking lines according to a heuristic and only including the top lines from the heuristic in the switching problem.

Barrows et. al. [121], [122] suggest using a heuristic that limits the switchable set to those lines which increase the available branch capacity (ABC) on congested lines. That is, we want to switch lines $k(m, n)$ that increase the available capacity of line $k(i, j)$.

$$\Delta ABC_{k(i,j),k(m,n)} = 1 - \frac{LODF_{k(i,j),k(m,n)} \times p_{k(m,n)}}{P_{k(i,j)}^{max}} \quad (6.1.1)$$

Ruiz considers four different heuristics in [123]. These heuristics are congestion rent, greatest difference in locational marginal prices (LMP) when congestion rent is negative, the total cost derivative of switching the line off, and the PTDF-weighted cost derivative. All heuristics provided cost savings. The congestion rent heuristic is equivalent to another heuristic proposed by Fuller [124], who suggests ranking lines by the dual variable of the constraint that $z_k = 1$, (calculated in terms of a nonlinear program) and removing lines according to the ranking. This dual variable is equivalent to the congestion rent of the line. If power is flowing from a lower-priced node to a higher-priced node, we have a positive congestion rent, and that line is desirable to keep. On the other hand, a negative congestion rent means power is flowing from a high price to a low price, which is undesirable. This heuristic ranks lines by the most negative congestion rent and switches them first.

However, Soroush [125] shows that while the ranking lines for removal by congestion rent may work well in nominal conditions for the DCOPF, it is a poor predictor of lines to remove in the ACOPF except under low load conditions. The dual variables of the ACOPF are much better predictors of the best line to remove than the dual variables of the DCOPF. In Section 6.2, we study the congestion rent heuristic in depth on the 2383wp system, examining using dual and primal values from both the ACOPF and DCOPF. In Section 6.3 we examine over 200 heuristics on six of the Polish test systems.

6.2 Case Study: Performance of the Congestion Rent Heuristic for 2383wp

The rest of this section is organized as follows: section 6.2.1 includes the OPF formulation and a brief explanation of how the AC and DC congestion rent heuristics were
derived. Section 6.2.2 presents and discusses the results of testing the congestion rent heuristic on the 2383wp system.

6.2.1 Methodology

In this section, MATPOWER, an open source package for MATLAB that does power system simulation, is used to solve the OPF problems. The detailed formulation and solution methods for ACOPF and DCOPF problems as implemented in MATPOWER are provided in [30]. MATPOWER formulates the ACOPF problem in polar-voltage coordinates; a refresher of this formulation is shown in (6.2.1)-(6.2.10). The 2383wp system only has a linear power cost, and the objective function with this cost is presented in (6.2.1). The AC power flow equations are provided in (6.2.2) and (6.2.3). The nodal balance constraints for real and reactive power are represented by (6.2.4) and (6.2.5). As in Chapter 5, the dual variables for node balance constraints, $\lambda_P^p$ and $\lambda_Q^p$, represent the real and reactive power locational marginal prices (LMP and LMRP). The lower and upper bounds on real power, reactive power, voltage, and angle differences are given in constraints (6.2.6) through (6.2.10).

$$
\min \sum_{g \in G} C_{g,n,i} p_{g,n,i}^g$$

subject to:

$$p_{k(n,m)} = -v_n v_m (G_{k(n,m)} \cos(\theta_{nm}) + B_{k(n,m)} \sin(\theta_{nm})) + G_{k(n,m)} v_n^2$$

$$q_{k(n,m)} = -v_n v_m (-G_{k(n,m)} \sin(\theta_{nm}) + B_{k(n,m)} \cos(\theta_{nm})) - B_{k(n,m)} v_n^2$$

$$\sum_{m \in A(n)} p_{k(n,m)} + p_n^g = P_n^d$$

$$\sum_{m \in A(n)} q_{k(n,m)} + q_n^g = Q_n^d$$

$$-S_{k}^{\max} \leq s_{k(n,m)} \leq S_{k}^{\max}$$

$$P_{n,i}^{\min} \leq P_{n,i}^g \leq P_{n,i}^{\max}$$

$$Q_{n,i}^{\min} \leq Q_{n,i}^g \leq Q_{n,i}^{\max}$$

$$V_n^{\min} \leq v_n \leq V_n^{\max}$$

$$\theta_{k(n,m)}^{\min} \leq \theta_n - \theta_m \leq \theta_{k(n,m)}^{\max}$$

Using the ACOPF formulation presented, the sensitivity of the objective function to a marginal change in the status of a transmission line is calculated in [126]. This metric is used as a heuristic to estimate the benefits of switching the line. The congestion rent heuristic is shown in (6.2.11).

$$Heur_{AC} = p_{k(n,m)} \lambda_P^P - p_{k(m,n)} \lambda_P^P + q_{k(n,m)} \lambda_Q^Q - q_{k(m,n)} \lambda_Q^Q$$
In this section, we refer to the method that ranks lines based on the AC congestion rent, equation (6.2.11), as the AC Heuristic. The metric represents the economic value of the line, which equals the revenue collected from the sale of power at the importing end minus the cost of buying power at the exporting end, considering losses and reactive power. The AC heuristic considers the negative of the line value, suggesting that a line with a larger negative economic value is a potential switching candidate. A larger negative economic value would mean that the FTR holder would pay significant money to the ISO. Since we are looking at the sensitivity analysis of a binary variable, the change in the status of the line is not marginal, so it is unlikely that the heuristic estimates match the actual benefits exactly. The DC approximation, as described in Section 2.4.5, ignores reactive power and network losses. The definition of power flow across a line for the DC approximation is given in (6.2.12).

\[ p_{k(n,m)} = B_{k(n,m)}(\theta_n - \theta_m) \] (6.2.12)

The same sensitivity of the power dispatch cost to turning a line off is calculated with the DC set of assumptions in [123, 124]. The DC approximation of the congestion rent of the line is presented in (6.2.13). We refer to the method ranking lines based on this metric as the DC heuristic. The DC estimation of the lines value is the same as the AC estimation when ignoring the reactive power and losses. It is concluded in [126] that the two heuristics may produce significantly different results if the system is voltage constrained.

\[ H_{eur\ DC} = p_{k(n,m)}(\lambda_m^P - \lambda_n^P) \] (6.2.13)

### 6.2.2 Simulation Studies

We test the AC and DC congestion rent heuristics on the Polish test case 2383wp provided by MATPOWER, assuming that all of the generators are on and there are no line or generator contingencies. To study the performance of the heuristics, we compare the actual benefit from the proposed line switch with the estimated benefit calculated by the heuristics. The actual switching benefit is the total cost difference between the case in which the transmission line is in the system, and the case in which it is taken out. We simulate the performance of the heuristics under three different settings: the DC Heuristic from the DCOPF, the DC Heuristic from the ACOPF, and the AC Heuristic from the ACOPF.

1. DC Heuristic from the DCOPF: a DCOPF is solved and all the primal and dual variables are taken from the DCOPF solution. The actual benefits are calculated through the total cost comparison of the two DCOPFs: one with the line in, and one with the line out. The switching benefits are also estimated through the DC heuristic introduced in (6.2.13). A comparison between the actual and estimated benefits...
provides information on the performance of the DC heuristic with a DCOPF. The solution to the DCOPF may or may not be AC feasible, and the DC solutions here are not checked for AC feasibility.

2. DC Heuristic from the ACOPF: the dual and primal variables as well as the actual benefits are calculated through an ACOPF. The estimated switching benefits are obtained from the DC heuristic, which does not include losses or reactive power. Note that although we are using the DC heuristic, the power flow and active power LMP come from an ACOPF. A comparison between the actual and estimated benefits provides information on the performance of the DC heuristic with an ACOPF.

3. AC Heuristic from the ACOPF: the dual and primal variables are calculated through an ACOPF. The actual switching benefits are also calculated by comparing the total cost obtained from the two ACOPFs, one with the line turned on, and one with the line turned off. The benefits are estimated through the AC heuristic presented in (6.2.11). A comparison between the actual and estimated benefits provides information on the performance of the AC heuristic with an ACOPF.

Figure 6.1 compares the benefits in the DCOPF obtained by switching a line with the estimated benefits calculated by the DC heuristic (setting 1). Figure 6.2 shows the performance of the algorithm based on the DC heuristic using the DCOPF solution for the first twenty switching candidates. The first twenty switching candidates are the best twenty candidates according to the the DC congestion rent heuristic’s estimate of the benefits. The dashed line designates the maximum possible benefit from a single line switch identified by the ACOPF while the dotted line shows the maximum possible benefits of switching using a DCOPF. The results show that the algorithm is not able to find the actual best line to switch in the first twenty candidates it proposes. Five out of twenty proposed candidates are beneficial actions when tested with a DCOPF. However, there are only two candidates that provide an actual benefit in the ACOPF. In electricity markets today, all procedures are based on a modified DCOPF due to the computational complexity of ACOPF. However, operators need to make sure that the solution is AC feasible. This is often done via out of market correction mechanisms [127]. Our results suggest that switching candidates identified by the solution of a DCOPF may not be AC feasible or may not be beneficial even though DCOPF identifies them to be beneficial. This is in accordance with findings by Potluri [27].
Figure 6.1: The actual DCOPF benefits versus the estimate of benefits using the DC heuristic.

Figure 6.2: The DC Heuristic performance for the first twenty line identified by the heuristic using DCOPF. The dotted line shows the maximum possible DCOPF benefit while the dashed line represents the maximum possible ACOPF benefit.
Figure 6.3 compares the best lines to switch according to the DC heuristic from the AC solution (setting 2) versus the actual AC benefit. Figure 6.4 shows the DC estimation of the benefits of switching a line versus the actual benefit of switching the line in the ACOPF (setting 2). These two figures show similar results to Figures 6.1 and 6.2 except the DC heuristic value is taken from the AC LMPs and power flows rather than the DC LMPs and power flows. The results suggest that the algorithm is able to identify the best switching action among its first twenty proposed candidates. Six out of twenty proposed actions are beneficial. Note that the only difference between settings 1 and 2 is that the ACOPF solution is used under setting 2 for both actual and estimated benefit calculation. However, under both settings the DC heuristic presented in (6.2.13) is employed. The difference between the results comes from the fact that the dispatch and prices are different when AC power flow constraints are taken into account in the optimal power flow problem. Figures 6.5 and 6.6 show the results under setting 3 where the AC heuristic is used with ACOPF solution. The results are very similar to those of setting 2 with six beneficial solutions among the first twenty proposed actions.

The results obtained under settings 2 and 3 show that AC and DC heuristics produce very similar results when the ACOPF solution is used. Under both settings, six out of twenty proposed actions were beneficial and the algorithm was able to identify the best switching action. The only difference was a slight change in the order of the candidates. These results were expected and are in line with the conclusions of Soroush [125], which suggests the AC and DC results will be similar when the system is not heavily voltage constrained. However, the results obtained from using DCOPF solution for heuristic calculations (setting 1) are substantially different from using quantities from the ACOPF solution (settings 2 or 3). The difference appears both in the suggested switching candidates and the benefits. As was stated before, in electricity markets today, ACOPF solutions are not generally available - which would be similar to setting 1. Our results show that the studied heuristics do not provide consistent results when they are based on the DCOPF solution compared to a more realistic ACOPF. The more realistic benefits - the ACOPF based benefits, as well as the proposed candidates are different than those based on a DCOPF.
Figure 6.3: Performance of the DC heuristic for the first twenty lines identified by the heuristic using ACOPF. The dashed line shows the maximum possible benefit.
CHAPTER 6. ECONOMIC SWITCHING

Figure 6.4: The actual benefits obtained by the ACOPF versus the DC heuristic estimation of the benefits using ACOPF

Figure 6.5: The actual benefits obtained by ACOPF versus the AC heuristic estimation of the benefits using ACOPF
With high performance computing, a batch of parallel processors can be used to identify the actual benefits of the proposed candidates. Here, we show how using multiple processors would impact the performance of these algorithms. We compare the benefits obtained by the best candidate in the batch with the average estimated benefit identified by the heuristic. For this part, we only use the AC heuristic with ACOPF, similar to the conditions of setting 3, since it showed the most promise. Figures 6.7, 6.8, and 6.9 show the comparison for the batch sizes of 10, 50, and 100, respectively. The results show that by having a batch size of greater than 50, the algorithm was able to find at least one switching candidate within each batch that improved the objective function. However, even with a large batch size, there exist beneficial solutions within the batches that have negative expected benefit. These solutions correspond to the points in the fourth quadrant of Figures 6.4 and 6.5. The implication of this finding is that some lines with large congestion rents may be good switching candidates.
Figure 6.7: The maximum benefits for each batch of processors versus the average benefits identified by the AC heuristic. Each batch contains 10 CPUs.

Figure 6.8: The maximum benefits for each batch of processors versus the average benefits identified by the AC heuristic. Each batch contains 50 CPUs.
6.2.3 Results

Due to the computational complexity of the optimal transmission switching problem, different heuristics are used to obtain fast sub-optimal solutions. The heuristics are often tested on small scale systems and the scalability of their application is not well understood. We studied the performance of the congestion rent heuristic (both AC and DC formulations) on the Polish system 2383wp. The heuristics were studied under three different settings: a DC heuristic with DCOPF, a DC heuristic with ACOPF, and an AC heuristic with ACOPF. Our results suggest that the AC and DC heuristics do not perform much differently when they are based on the solution to the ACOPF. That means that the LMRP difference times reactive power on the line does not seem to have a large impact on the performance of the heuristic. However, the heuristics do produce different results if they are based on DCOPF solutions rather than the ACOPF solutions. Our results suggest that the DCOPF based solutions obtained for optimal transmission switching may not perform well under the realistic system conditions modeled by an ACOPF. Since the market procedures are based on the DCOPF, not the ACOPF, and AC feasibility is often achieved via out of market corrections, implementation of ACOPF based heuristics would not be straightforward.

Since this study examined a plain DCOPF without modifications, it is possible that running a DCOPF closer what is used by the ISO (incorporating losses, using non-unity fixed voltages, and adding nomogram constraints) may produce closer results to the ACOPF. The DC with losses formulation from Akinbode [108] used in Section 5.3 gave...
very close objective functions to the AC solution. The LMPs for the DC with losses are also not uniform for the case with no line limits, which is closer to the AC solution than the DC solution, which has the same LMP in every location. However, the market settlement with the DC with losses is very different than the AC solution to the SLP-IV-ACOPF, so it is possible that even a DC with modifications formulation may still not give LMPs and flows that are close enough to the ACOPF solution.

6.3 Metastudy on Heuristics

After performing a deep dive on the congestion rent heuristic, it is clear that what works very well in a small network may not work as well in a different and larger network. While the congestion rent heuristic performed modestly well on the 2383wp case, its benefits were not as amazing as its performance on the smaller systems that were tested on in [124] and [123]. Additionally, the congestion rent heuristic does not directly account for the structure of the network. This section aims to examine the results of many heuristics as well as how they behave on different networks. One issue with evaluating heuristics - which are evaluated empirically - is that it depends on the underlying data. What may work well on one network may not work well on another. For this study, we will examine the Polish networks 2383wp, 2736sp, 2737sop, 2746wop, 3012wp, and 3120sp and use MATPOWER for the analysis [30].

6.3.1 Heuristics Tested

We test 215 heuristics. Since many of these are sensitivity based, we only examine the impact of taking each single line out at a time. Some heuristics are based on solely the line (real power loss, percentage of line capacity used, etc.); others are based on metrics at the two ends of the line (LMP, generation rent, etc.); others are based on combinations of the line and quantities at the two ends (congestion rent). A line \( k(n,m) \) has a from end \( n \) and a to end \( m \). The heuristics are grouped into categories of heuristics involving marginal values, flow, market settlements, network requirements, network characteristics, generation, and graph structure.

Marginal value heuristics consider important prices in the program. These include the LMPs and the duals on the line limit constraints.

- Largest LMP Difference, \( \max \lambda_P^P - \lambda_P^P \)
- Smallest LMP Difference, \( \min \lambda_P^P - \lambda_P^P \)
- Largest LMRP Difference, \( \max \lambda_Q^Q - \lambda_Q^Q \)
- Smallest LMRP Difference, \( \min \lambda_Q^Q - \lambda_Q^Q \)
- Largest LMP at the from end, \( \max \lambda_P^n \)
• Smallest LMP at the from end, \( \min \lambda^P_n \)
• Largest LMP at the to end, \( \max \lambda^P_m \)
• Smallest LMP at the to end, \( \min \lambda^P_m \)
• Largest LMP (maximum of the to and from ends), \( \max \max_{n,m} \{ \lambda^P_n, \lambda^P_m \} \)
• Smallest LMP Max(either end), \( \min \max_{n,m} \{ \lambda^P_n, \lambda^P_m \} \)
• Largest LMP Min (either end), \( \max \min_{n,m} \{ \lambda^P_n, \lambda^P_m \} \)
• Smallest LMP Min (either end), \( \min \min_{n,m} \{ \lambda^P_n, \lambda^P_m \} \)
• Largest line limit dual at the from end, \( \max \mu_{k(n,m)} \)
• Largest line limit dual at the to end, \( \max \mu_{k(m,n)} \)
• Largest line limit dual difference, from end minus to end, \( \max \mu_{k(n,m)} - \mu_{k(n,m)} \)
• Largest line limit dual, maximum of from and to nodes, \( \max \max_{n,m} \{ \mu_{k(n,m)}, \mu_{k(m,n)} \} \)
• Largest admittance price at the from node, \( \lambda^P_n - \lambda^P_m - \mu_{k(n,m)} \)
• Smallest admittance price at the from node, \( \lambda^P_n - \lambda^P_m - \mu_{k(n,m)} \)
• Largest admittance price at the to node, \( \lambda^P_n - \lambda^P_m - \mu_{k(m,n)} \)
• Smallest admittance price at the to node, \( \lambda^P_n - \lambda^P_m - \mu_{k(m,n)} \)
• Largest admittance price including both nodes, \( \lambda^P_n - \lambda^P_m - \mu_{k(n,m)} - \mu_{k(m,n)} \)
• Smallest admittance price including both nodes \( \lambda^P_n - \lambda^P_m - \mu_{k(n,m)} - \mu_{k(m,n)} \)
• Largest maximum line limit dual times the line’s impedance, \( \max \mu_{k(n,m)} Z_k \)

Flow heuristics deal with the quantity of real power flow, reactive power flow, and apparent power flow on the line. The flow heuristics also consider the real, reactive, and apparent power losses. Additionally, they examine the difference in the real and reactive demands and amount of generation across the line (between the two ends).
• Largest real power on the line at the from node, \( \max p_{k(n,m)} \)
• Largest real power on the line at the to node, \( \max p_{k(m,n)} \)
• Largest real power on the line, maximum along the line, \( \max \max_{n,m} \{ p_{k(n,m)}, p_{k(m,n)} \} \)
• Smallest real power on the line, maximum along the line, \( \min \max_{n,m} \{ p_{k(n,m)}, p_{k(m,n)} \} \)
- Largest reactive power on the line at the from node, max $q_{k(n,m)}$
- Largest reactive power on the line at the to node, max $q_{k(m,n)}$
- Largest reactive power on the line, maximum along the line, max $\max_{n,m}\{q_{k(n,m)}, q_{k(m,n)}\}$
- Smallest reactive power on the line, maximum along the line, min $\max_{n,m}\{q_{k(n,m)}, q_{k(m,n)}\}$
- Largest apparent power on the line at the from node, $s_{k(n,m)}$
- Largest apparent power on the line at the to node $s_{k(m,n)}$
- Largest apparent power on the line, maximum along the line, max $\max_{n,m}\{s_{k(n,m)}, s_{k(m,n)}\}$
- Largest apparent power on the line, maximum along the line, min $\max_{n,m}\{s_{k(n,m)}, s_{k(m,n)}\}$
- Largest real power line loss, max $p_{k}^{loss} = p_{k(n,m)} + p_{k(m,n)}$
- Smallest real power line loss, min $p_{k}^{loss} = p_{k(n,m)} + p_{k(m,n)}$
- Largest reactive power line loss, max $q_{k}^{loss} = q_{k(n,m)} + q_{k(m,n)}$
- Smallest reactive power line loss, min $q_{k}^{loss} = q_{k(n,m)} + q_{k(m,n)}$
- Largest apparent power line loss, max $s_{k}^{loss} = s_{k(n,m)} + s_{k(m,n)}$
- Smallest apparent power line loss, min $s_{k}^{loss} = s_{k(n,m)} + s_{k(m,n)}$
- Largest real power demand difference between from and to nodes, max $P_{n}^{d} - P_{m}^{d}$
- Smallest real power demand difference between from and to nodes, min $P_{n}^{d} - P_{m}^{d}$
- Largest absolute real power demand difference between from and to nodes, max $|P_{n}^{d} - P_{m}^{d}|$
- Smallest absolute real power demand difference between from and to nodes, min $|P_{n}^{d} - P_{m}^{d}|$
- Largest reactive power demand difference between from and to nodes, max $Q_{n}^{d} - Q_{m}^{d}$
- Smallest reactive power demand difference between from and to nodes, min $Q_{n}^{d} - Q_{m}^{d}$
- Largest absolute reactive power demand difference between from and to nodes, max $|Q_{n}^{d} - Q_{m}^{d}|$
• Smallest absolute reactive power demand difference between from and to nodes, 
  \( \min |Q_n^d - Q_m^d| \)
• Largest real power generation difference between from and to nodes, 
  \( \max P_n^g - P_m^g \)
• Smallest real power generation difference between from and to nodes, 
  \( \min P_n^g - P_m^g \)
• Largest absolute real power generation difference between from and to nodes, 
  \( \max |P_n^g - P_m^g| \)
• Smallest absolute real power generation difference between from and to nodes, 
  \( \min |P_n^g - P_m^g| \)
• Largest reactive power generation difference between from and to nodes, 
  \( \max Q_n^g - Q_m^g \)
• Smallest reactive power generation difference between from and to nodes, 
  \( Q_n^g - Q_m^g \)
• Largest absolute reactive power generation difference between from and to nodes, 
  \( \max |Q_n^g - Q_m^g| \)
• Smallest absolute reactive power generation difference between from and to nodes, 
  \( \min |Q_n^g - Q_m^g| \)
• Largest net real power difference between from and to nodes, 
  \( \max (P_n^g - P_n^d) - (P_m^g - P_m^d) \)
• Smallest net real power difference between from and to nodes, 
  \( P_n^g - P_n^d - (P_m^g - P_m^d) \)
• Largest absolute net real power difference between from and to nodes, 
  \( \max |(P_n^g - P_n^d) - (P_m^g - P_m^d)| \)
• Smallest absolute net real power difference between from and to nodes, 
  \( \min |(P_n^g - P_n^d) - (P_m^g - P_m^d)| \)
• Largest net reactive power difference between from and to nodes, 
  \( \max (Q_n^g - Q_n^d) - (Q_m^g - Q_m^d) \)
• Smallest net reactive power difference between from and to nodes, 
  \( Q_n^g - Q_n^d - (Q_m^g - Q_m^d) \)
• Largest net reactive power difference between from and to nodes, 
  \( \max |(Q_n^g - Q_n^d) - (Q_m^g - Q_m^d)| \)
• Smallest net reactive power difference between from and to nodes, 
  \( (Q_n^g - Q_n^d) - (Q_m^g - Q_m^d) \)
Market settlement heuristics consider the congestion rent, load payments, generation payment, and generation cost. These examine both the totals of the settlements from the two nodal ends as well as the differences, sums, maximums, or minimums on the two ends.

- Largest Real Power Congestion Rent, $\max p_{k(m,n)} \lambda^P_m - p_{k(m,n)} \lambda^P_n$
- Smallest Real Power Congestion Rent, $\min p_{k(m,n)} \lambda^P_m - p_{k(m,n)} \lambda^P_n$
- Largest Reactive Power Congestion Rent, $\max q_{k(m,n)} \lambda^Q_m - q_{k(m,n)} \lambda^Q_n$
- Smallest Reactive Power Congestion Rent, $\min q_{k(m,n)} \lambda^Q_m - q_{k(m,n)} \lambda^Q_n$
- Largest AC Congestion Rent, $\max p_{k(m,n)} \lambda^P_m - p_{k(m,n)} \lambda^P_m + q_{k(m,n)} \lambda^Q_m - q_{k(m,n)} \lambda^Q_n$
- Smallest AC Congestion Rent, $\min p_{k(m,n)} \lambda^P_m - p_{k(m,n)} \lambda^P_m + q_{k(m,n)} \lambda^Q_m - q_{k(m,n)} \lambda^Q_n$
- Largest real power load payment at the from node, $\max \lambda^P_n P^d_n$
- Smallest real power load payment at the from node, $\min \lambda^P_n P^d_n$
- Largest real power load payment at the to Node, $\max \lambda^P_m P^d_m$
- Smallest real power load payment at the to Node, $\min \lambda^P_m P^d_m$
- Largest to and from nodes real power load payment, $\max \lambda^P_n P^d_n + \lambda^P_m P^d_m$
- Smallest to and from nodes real power load payment, $\min \lambda^P_n P^d_n + \lambda^P_m P^d_m$
- Largest absolute to and from nodes real power load payment, $\max |\lambda^P_n P^d_n + \lambda^P_m P^d_m|$n
- Smallest absolute to and from nodes real power load payment, $\min |\lambda^P_n P^d_n + \lambda^P_m P^d_m|$n
- Largest difference in real power load payments between the from and to nodes, $\max \lambda^P_n P^d_n - \lambda^P_m P^d_m$
- Smallest difference in real power load payments between the from and to nodes, $\min \lambda^P_n P^d_n - \lambda^P_m P^d_m$
- Largest reactive power load payment at the from node, $\max \lambda^Q_n Q^d_n$
- Smallest reactive power load payment at the from node, $\min \lambda^Q_n Q^d_n$
- Largest reactive power load payment at the to node, $\max \lambda^Q_m Q^d_m$
- Smallest reactive power load payment at the to node, $\min \lambda^Q_m Q^d_m$
- Largest reactive power load payment, summing those at the to and from nodes, $\max \lambda^Q_n Q^d_n + \lambda^Q_m Q^d_m$
• Smallest reactive power load payment, summing those at the to and from nodes, \( \min \lambda_n^Q Q_n^d + \lambda_m^Q Q_m^d \)

• Largest absolute reactive power load payment, summing those at the to and from nodes, \( \max |\lambda_n^Q Q_n^d + \lambda_m^Q Q_m^d| \)

• Smallest absolute reactive power load payment, summing those at the to and from nodes, \( \min |\lambda_n^Q Q_n^d + \lambda_m^Q Q_m^d| \)

• Largest difference in reactive power load payments between the from and to nodes, \( \max \lambda_n^Q Q_n^d - \lambda_m^Q Q_m^d \)

• Smallest difference in reactive power load payments between the from and to nodes, \( \min \lambda_n^Q Q_n^d - \lambda_m^Q Q_m^d \)

• Largest total load payment at the from node, \( \max \lambda_n^P P_n^d + \lambda_m^Q Q_n^d \)

• Smallest total load payment at the from node, \( \min \lambda_n^P P_n^d + \lambda_m^Q Q_n^d \)

• Largest total load payment at the to node, \( \max \lambda_m^P P_m^d + \lambda_m^Q Q_m^d \)

• Smallest total load payment at the to node, \( \min \lambda_m^P P_m^d + \lambda_m^Q Q_m^d \)

• Largest total load payment, summing those at the to and from nodes, \( \max \lambda_n^P P_n^d + \lambda_n^Q Q_n^d + \lambda_m^P P_m^d + \lambda_m^Q Q_m^d \)

• Smallest total load payment, summing those at the to and from nodes, \( \min \lambda_n^P P_n^d + \lambda_n^Q Q_n^d + \lambda_m^P P_m^d + \lambda_m^Q Q_m^d \)

• Largest absolute total load payment, summing those at the to and from nodes, \( \max |\lambda_n^P P_n^d| + |\lambda_n^Q Q_n^d| + |\lambda_m^P P_m^d| + |\lambda_m^Q Q_m^d| \)

• Smallest absolute total load payment, summing those at the to and from nodes, \( \min |\lambda_n^P P_n^d| + |\lambda_n^Q Q_n^d| + |\lambda_m^P P_m^d| + |\lambda_m^Q Q_m^d| \)

• Largest difference in total load payments between the from and to nodes, \( \max \lambda_n^P P_n^d + \lambda_n^Q Q_n^d - \lambda_m^P P_m^d - \lambda_m^Q Q_m^d \)

• Smallest difference in total load payments between the from and to nodes, \( \min \lambda_n^P P_n^d + \lambda_n^Q Q_n^d - \lambda_m^P P_m^d - \lambda_m^Q Q_m^d \)

• Largest real power generation payment at the from node (Generator Payment), \( \max \lambda_n^P P_n^g \)

• Smallest real power generation payment at the from node (Generator Payment), \( \min \lambda_n^P P_n^g \)

• Largest real power generation payment at the to node, \( \max \lambda_m^P P_m^g \)
• Smallest real power generation payment at the to node, min $\lambda_m^P P_m^g$

• Largest real power generation payment from adding those at the to and from nodes, max $\lambda_n^P P_n^g + \lambda_m^P P_m^g$

• Smallest real power generation payment from adding those at the to and from nodes, min $\lambda_n^P P_n^g + \lambda_m^P P_m^g$

• Largest absolute real power generation payment from adding those at the to and from nodes, max $|\lambda_n^P P_n^g + \lambda_m^P P_m^g|$

• Smallest absolute real power generation payment from adding those at the to and from nodes, min $|\lambda_n^P P_n^g + \lambda_m^P P_m^g|$

• Largest real power generation payment difference between from and to nodes, max $\lambda_n^P P_n^g - \lambda_m^P P_m^g$

• Smallest real power generation payment difference between from and to nodes, min $\lambda_n^P P_n^g - \lambda_m^P P_m^g$

• Largest reactive power generation payment at the from node, max $\lambda_n^Q Q_n^g$

• Smallest reactive power generation payment at the from node, min $\lambda_n^Q Q_n^g$

• Largest reactive power generation payment at the to node, max $\lambda_m^Q Q_m^g$

• Smallest reactive power generation payment at the at to node, min $\lambda_m^Q Q_m^g$

• Largest reactive power generation payment, adding those at from and to nodes, max $\lambda_n^Q Q_n^g + \lambda_m^Q Q_m^g$

• Smallest reactive power generation payment, adding those at from and to nodes, min $\lambda_n^Q Q_n^g + \lambda_m^Q Q_m^g$

• Largest absolute reactive power generation payment, adding those at from and to nodes, max $|\lambda_n^Q Q_n^g + \lambda_m^Q Q_m^g|$

• Smallest absolute reactive power generation payment, adding those at from and to nodes, min $|\lambda_n^Q Q_n^g + \lambda_m^Q Q_m^g|$

• Largest reactive power generation payment difference between from and to nodes, max $\lambda_n^Q Q_n^g - \lambda_m^Q Q_m^g$

• Smallest reactive power generation payment difference between from and to nodes, min $\lambda_n^Q Q_n^g - \lambda_m^Q Q_m^g$

• Largest real plus reactive power generation payment at the from node, max $\lambda_n^P P_n^g + \lambda_n^Q Q_n^g$
• Smallest real plus reactive power generation payment at the from node, \( \min \lambda_n P_n + \lambda_n Q_n \)

• Largest real plus reactive power generation payment at the to node, \( \max \lambda_m P_m + \lambda_m Q_m \)

• Smallest real plus reactive power generation payment at the to node, \( \min \lambda_m P_m + \lambda_m Q_m \)

• Largest real plus reactive power generation payments, adding those at the from and to nodes, \( \max \lambda_n P_n + \lambda_n Q_n + \lambda_m P_m + \lambda_m Q_m \)

• Smallest real plus reactive power generation payments, adding those at the from and to nodes, \( \min \lambda_n P_n + \lambda_n Q_n + \lambda_m P_m + \lambda_m Q_m \)

• Largest absolute value of the real plus absolute value of the reactive reactive power generation payments, adding those at the from and to nodes, \( \max |\lambda_n P_n + \lambda_m P_m| + |\lambda_n Q_n + \lambda_m Q_m| \)

• Smallest absolute value of the real plus absolute value of the reactive reactive power generation payments, adding those at the from and to nodes, \( \min |\lambda_n P_n + \lambda_m P_m| + |\lambda_n Q_n + \lambda_m Q_m| \)

• Largest real plus reactive power generation payments, difference between those at the from and to nodes, \( \lambda_n P_n - \lambda_m P_m + \lambda_n Q_n - \lambda_m Q_m \)

• Smallest real plus reactive power generation payments, difference between those at the from and to nodes, \( \lambda_n P_n - \lambda_m P_m + \lambda_n Q_n - \lambda_m Q_m \)

• Largest line limit dual times the power on the line at the to end plus that at the from end, \( \mu_{k(n,m)} s_{k(n,m)} + \mu_{k(m,n)} s_{k(m,n)} \)

• Largest real power generation cost at the from end, \( \max \sum_{i \in G_n} C_{n,i} P_{n,i} \)

• Smallest real power generation cost at the from end, \( \min \sum_{i \in G_n} C_{n,i} P_{n,i} \)

• Largest real power generation cost at the to end, \( \max \sum_{i \in G_m} C_{m,i} P_{m,i} \)

• Smallest real power generation cost at the to end, \( \min \sum_{i \in G_m} C_{m,i} P_{m,i} \)

• Largest real power generation cost total from the two ends, \( \max \sum_{i \in G_n} C_{n,i} P_{n,i} + \sum_{i \in G_m} C_{m,i} P_{m,i} \)

• Smallest real power generation cost total from the two ends, \( \min \sum_{i \in G_n} C_{n,i} P_{n,i} + \sum_{i \in G_m} C_{m,i} P_{m,i} \)

• Largest real power generation cost difference, from node minus to node, \( \sum_{i \in G_n} C_{n,i} P_{n,i} - \sum_{i \in G_m} C_{m,i} P_{m,i} \)
• Smallest real power generation cost difference, from node minus to node, min
\[ \sum_{i \in G_n} C_{n,i}^g P_{n,i}^g - \sum_{i \in G_m} C_{m,i}^g P_{m,i}^g \]
• Largest absolute real power generation cost difference, from node minus to node, max \[ \left| \sum_{i \in G_n} C_{n,i}^g P_{n,i}^g - \sum_{i \in G_m} C_{m,i}^g P_{m,i}^g \right| \]
• Smallest real power generation cost difference, from node minus to node, min \[ \left| \sum_{i \in G_n} C_{n,i}^g P_{n,i}^g - \sum_{i \in G_m} C_{m,i}^g P_{m,i}^g \right| \]

The network requirement heuristics examine the required line limits and demands in many different combinations.

• Largest apparent power line limit, max \[ S_{k}^{max} \]
• Smallest apparent power line limit, min \[ S_{k}^{max} \]
• Largest percentage utilized of apparent power capacity, max \[ s_{k(n,m)} / S_{k}^{max} \]
• Smallest percentage utilized of apparent power capacity, min \[ s_{k(n,m)} / S_{k}^{max} \]
• Largest real power demand at the to node, max \[ P_{m}^d \]
• Smallest real power demand at the to node, min \[ P_{m}^d \]
• Largest real power demand at the from node, max \[ P_{n}^d \]
• Smallest real power demand at the from node, min \[ P_{n}^d \]
• Largest sum of the real power demand at and from to nodes, max \[ P_{n}^d + P_{m}^d \]
• Smallest sum of the real power demand at and from to nodes, min \[ P_{n}^d + P_{m}^d \]
• Largest reactive power demand at the to node, max \[ Q_{m}^d \]
• Smallest reactive power demand at the to node, min \[ Q_{m}^d \]
• Largest reactive power demand at the from node, max \[ Q_{n}^d \]
• Smallest reactive power demand at the from node, min \[ Q_{n}^d \]
• Largest sum of the reactive power demand at the from and to nodes, max \[ Q_{n}^d + Q_{m}^d \]
• Smallest sum of the reactive power demand at the from and to nodes, min \[ Q_{n}^d + Q_{m}^d \]
• Largest apparent power demand at the to node, max \[ \sqrt{(P_{m}^d)^2 + (Q_{m}^d)^2} \]
• Smallest apparent power demand at the to node, min \[ \sqrt{(P_{m}^d)^2 + (Q_{m}^d)^2} \]
• Largest apparent power demand at the from node, max \[ \sqrt{(P_{n}^d)^2 + (Q_{n}^d)^2} \]
• Smallest apparent power demand at the from node, \( \min \sqrt{(P_d^n)^2 + (Q_d^n)^2} \)

• Largest sum of the apparent power demand at the from and to nodes, \( \max \sqrt{(P_d^m)^2 + (Q_d^m)^2 + (P_d^n)^2 + (Q_d^n)^2} \)

• Smallest sum of the apparent power demand at the from and to nodes, \( \min \sqrt{(P_d^m)^2 + (Q_d^m)^2 + (P_d^n)^2 + (Q_d^n)^2} \)

The network characteristic heuristics take into consideration the physical properties of the network in terms of the resistances and reactances on the lines.

• Largest resistance, \( R_k \)

• Smallest resistance, \( R_k \)

• Largest reactance, \( X_k \)

• Smallest reactance, \( X_k \)

• Largest impedance, \( Z_k \)

• Smallest impedance, \( Z_k \)

• Largest ratio of resistance to reactance, \( R_k / X_k \)

• Largest apparent power limit times the line’s impedance, \( \max S_{\text{max}}^k Z_k \)

The generation heuristics consider the real, reactive, and apparent power produced at the two ends of the line.

• Largest real power generation at the to node, \( \max P_g^m \)

• Smallest real power generation at the to node, \( \min P_g^m \)

• Largest real power generation at the from node, \( \max P_g^n \)

• Smallest real power generation at the from node, \( \min P_g^n \)

• Largest real power generation, totaled the from and to nodes, \( \max P_g^m + P_g^n \)

• Smallest real power generation, totaled the from and to nodes, \( \min P_g^m + P_g^n \)

• Largest reactive power generation at the to node, \( \max Q_g^m \)

• Smallest reactive power generation at the to node, \( \min Q_g^m \)

• Largest reactive power generation at the from node, \( \max Q_g^n \)

• Smallest reactive power generation at the from node, \( \min Q_g^n \)
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- Largest total reactive power generation at the to and from nodes, \( Q_m^g + Q_n^g \)
- Smallest total reactive power generation at the to and from nodes, \( \min Q_m^g + Q_n^g \)
- Largest apparent power generation at the to node, \( S_m^g = \sqrt{(P_m^g)^2 + (Q_m^g)^2} \)
- Smallest apparent power generation at the to node, \( \min S_m^g \)
- Largest apparent power generation at the from node, \( S_n^g = \sqrt{(P_n^g)^2 + (Q_n^g)^2} \)
- Smallest apparent power generation at the from node, \( \min S_n^g \)
- Largest total apparent power generation at the to and from nodes, \( S_m^g + S_n^g \)
- Smallest total apparent power generation at the to and from nodes, \( \min S_m^g + S_n^g \)

Graph heuristics examine different graph characteristics of the network. They focus on the degree of the two ends of the line, the clique size of the two ends of the line, total capacity flowing into the line, distance of the line to the network’s center, and betweenness of the line. The degree of a node is how many lines connect to the node. The clique size of the node is the size of the smallest cycle in which the node is located. Betweenness is a graph metric designed to measure how much influence a node has on the flow through the network; it is the number of shortest paths that go between all pairs of nodes in the network that pass through the node divided by the total number of shortest paths that go between all pairs of nodes in the network.

- Largest degree (number of neighboring lines) of the from node
- Smallest degree of the from node
- Largest degree of the to node
- Smallest degree of the to node
- Largest sum of the degrees at the to and from nodes
- Smallest sum of the degrees at the to and from nodes
- Largest degree examining the minimum degree of the to and from nodes
- Smallest degree examining the minimum degree of the to and from nodes
- Largest degree examining the maximum degree of the to and from nodes
- Smallest degree examining the maximum degree of the to and from nodes
- Largest clique size (size of cycle) of the from node
- Smallest clique size of the from node
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- Largest clique size of the to node
- Smallest clique size of the to node
- Largest apparent power capacity connected to the from node, $\sum_{m \in A(n)} S_{k(n,m)}^{\text{max}}$
- Smallest apparent power capacity connected to the from node, $\sum_{m \in A(n)} S_{k(n,m)}^{\text{max}}$
- Largest apparent power capacity connected to the to node, $\sum_{n \in A(m)} S_{k(n,m)}^{\text{max}}$
- Smallest apparent power capacity connected to the to node, $\sum_{n \in A(m)} S_{k(n,m)}^{\text{max}}$
- Largest total apparent power capacity connected to the to and from nodes, $\sum_{n \in A(m)} S_{k(n,m)}^{\text{max}} + \sum_{m \in A(n)} S_{k(n,m)}^{\text{max}}$
- Smallest total apparent power capacity connected to the to and from nodes, $\sum_{n \in A(m)} S_{k(n,m)}^{\text{max}} + \sum_{m \in A(n)} S_{k(n,m)}^{\text{max}}$
- Largest distance of the line to the center of the graph
- Smallest distance of the line to the center of the graph
- Largest betweenness of the lines
- Smallest betweenness of the lines

We use MATPOWER to solve the ACOPF with each line subsequently out. We remove all lines from consideration that cause islanding of the network or lines where MATPOWER cannot find a feasible solution.

6.3.2 Evaluating Heuristics

The quality of the heuristic can be measured in one form by making a priority list of the top lines according to the heuristic (according to the estimated benefit) and examining the actual benefit of taking out the line. Ideally, we would have a perfect match where the top $X$ most beneficial lines are the same as the top $X$ listed in the priority list. Here, we create a priority list for each heuristic of its top 100 lines and examine the actual top 10 most beneficial lines from the ACOPF.

The heuristics where the top 10 most AC beneficial lines were found in the priority list of 100 lines are as follows:

- Case 2383wp
  - Largest real power congestion rent
  - Largest real plus reactive power congestion rent
  - Largest LMP at the from end
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- Largest LMP at the to end
- Largest minimum LMP of the to and from ends
- Largest maximum LMP of the to and from ends

- Case 2746wop
  - Largest real power congestion rent
  - Largest real plus reactive power congestion rent

If we examine where at least 9 of the top 10 actual most beneficial lines were found in the priority list of the 100 lines, we find the same results as for all 10. If we examine where at least 8 of the top 10 actual most beneficial lines were found in the priority list of the 100 lines, the additional heuristics are as follows.

- Case 2383wp
  - Smallest reactive power load payment at the from node
  - Total smallest reactive power load payment at the to and from nodes

- Case 2736sp
  - Largest real power congestion rent
  - Largest real and reactive power congestion rent

- Case 3120sp
  - Total smallest reactive power load payment at the to and from nodes

If we examine where at least 5 of the top 10 actual most beneficial lines were found in the priority list of the 100 lines, the additional heuristics are as follows.

- Case 2383wp
  - Smallest LMRP difference
  - Largest reactive power congestion rent
  - Smallest reactive power load payment at the to node
  - Smallest reactive power load payment difference at the from and to nodes
  - Smallest admittance price at the from node
  - Smallest admittance price at the to node

- Case 2746wop
  - Largest reactive power on the line at the from node
• Case 3012wp
  – Largest LMP at the from node
  – Largest LMP at the to node
  – Largest minimum LMP of the to and from nodes
  – Largest maximum LMP of the to and from nodes

• Case 3120sp
  – Largest real power congestion rent
  – Largest reactive power congestion rent
  – Largest real plus reactive power congestion rent
  – Largest LMP at the from node
  – Largest LMP at the to node
  – Largest minimum LMP of the to and from nodes
  – Largest maximum LMP of the to and from nodes
  – Smallest reactive power load payment total of the to and from nodes

If we look at heuristics where at least five of the top 10 actual most beneficial lines are found in the top 100 of the priority list, we have the following results:

• All Cases
  – Largest real power congestion rent
  – Largest real plus reactive power congestion rent

• Three Cases
  – Largest LMP at the from node
  – Largest LMP at the to node
  – Largest minimum LMP of the to and from nodes
  – Largest maximum LMP of the to and from nodes

• Two Cases
  – Smallest reactive power load payment at the to node
  – Smallest total reactive power load payment adding those at the to and from nodes
  – Largest reactive power congestion rent
6.3.3 Conclusion

While the deep dive into the congestion rent heuristic showed that it does not give a one-to-one correspondence of the priority list to the beneficial lines, it does appear to be one of the best heuristics that were tested. Many of the heuristics did not give very good recommendations of lines to switch. Heuristics other than congestion rent that seem somewhat promising are the largest LMPs and smallest reactive power load payments at the ends of the line and the largest reactive power congestion rent.
Chapter 7
Corrective Switching

Contingencies can cause serious problems in the power network such as frequency violations, line overloads, and voltage collapse. When a generator or line is not operational, this change may have significant impacts on the ability of the system to provide power to all the load requested and may result in shedding load. While counterintuitive, one possible way to correct a contingency is to switch a transmission line out of service. Switching a transmission line out of service changes the line flows and nodal voltages, and so the switching may reduce the line overload and correct the voltages to be within their limits.

7.1 Background

There are many ways to fix a contingency, including redispatch of generation and use of FACTS devices and other voltage support, as well as phase shifters. However, these techniques alone may not be enough to fix the system, and corrective switching may be required. Additionally, corrective switching is generally regarded as a cheaper and easier option than redispatching generation.

Once a contingency has occurred, an ISO or RTO has 30 minutes to return the system back to reliability. Most utilities and ISOs use some form of post-contingency corrective switching. Post-contingency corrective switching is more commonly referred to by the term Special Protection Schemes. Special Protection Schemes suggest actions to take for different potential contingencies in the network; these actions may include switching another line out of service. These Special Protection Schemes essentially function as look-up tables, and therefore are limited - it is almost impossible to consider every different way the system could possibly have issues, especially with all of the potential distributions of load. Additionally, these Special Protection Schemes can be improved upon; by doing optimal post-contingency corrective switching, we may find a solution that sheds more load or fixes more line flow and voltage violations than by using the corresponding Special Protection Scheme.

Major challenges to implementing optimal corrective switching include its compu-
tational complexity, operator acceptance, and need to evaluate the full impact of the switch on the system. Transmission switching for contingencies is also a mixed-integer program. Operators are often reluctant to take lines out of service due to wear on the line and an unclear understanding of if switching the line out may make the system less reliable. Additionally, switching lines out can cause undesired transient behavior, such as voltage stability.

There are several approaches to using corrective switching to help resolve contingencies: real-time, planning, and robust. In the planning approach, corrective switching actions to be taken for a range of contingency scenarios are pre-computed. In robust optimization, corrective switching is used to buffer against a wide variety of potential system problems. The operator uses switching before the contingency occurs and is not expected to need to change the network solution after the contingency occurs; it is expected that the network configuration found under the robust method can deal with almost any contingency. In real time, a contingency occurs, and then a corrective action to address the contingency is found and implemented.

Corrective switching is a technique that has been explored since the 1980s [129, 130]. More recent work includes parallelization of corrective switching on large-scale problems. Using a DC model, this work on parallelization has shown that the DC corrective switching problem can be solved within the time required. There are many objectives that one can use in corrective switching. If a switch can restore full feasibility, one could even use simply the objective function of power cost. Another objective used in the literature includes minimizing total load shed, or minimizing the cost of load shed, if shedding load is considered to have different costs in different areas. Another objective is to minimize load shed plus the cost of the power generation. Some papers consider only reducing real power load shed while others consider both real and reactive load shed. Finally, if one assumes sufficient generation capacity, one could examine minimizing line flow and/or voltage violations.

Balasubramanian et. al. [131] examines the use of corrective switching for simultaneous contingencies (loss of one or two lines, generators, or the combination thereof) and shows that switching sequences for up to 13000 bus problems can be found (using parallelization) within minutes. Korad et. al. [132] examine how to configure the system better to absorb the problems contingencies may cause. Escobedo examines solving the 118 bus model with the loss of one or two lines with parallelization and using a MIP heuristic to presort what lines to look at [133]. Wrubel et. al. [134] examine an implementation of a corrective switching algorithm at PSE&G in 1995. Rolim and Machado [135] review how using corrective switching can improve reliability through reducing line overloads and voltage violations. Shao and Vittal [136] examine a corrective switching algorithm that solely uses switching and no generator redispatch to help solve network problems.

One important consideration in the literature is how to set the value of lost load (VOLL). The price can often differ due to the type of customer - i.e., often residential customers have a very low value of lost load, while industrial customers have a very high cost. Additionally, the value of lost load may vary regionally, seasonally, or generally over
CHAPTER 7. CORRECTIVE SWITCHING

In a study for ERCOT, the VOLL for residential customers is $110/MWh, while for commercial and industrial customers it is $5,679 \textsuperscript{[137]}. A review by \textsuperscript{[138]} presents a literature review of VOLL estimates. For domestic customers, the value of a one hour outage ranged from 0.32 to 22.55 pounds ($0.52 to $36.40), and the implied value of lost load ranged from 713 to 50,213 pounds ($1,151 to $81,061). For non-domestic customers, the value of lost load ranged from 9,700 to 63,140 pounds ($15,659 to $101,930).

Khorsand \textsuperscript{[139]} minimizes load shedding in the day-ahead market with a DCOPF. He uses a heuristic based MIP where one line is switched at a time. The greedy algorithm uses a line ranking based on dual variables. The gap for the greedy algorithm and heuristic MIP were 5\% and 2\%. He shows that the method improves the ability to meet load greatly.

Ayala \textsuperscript{[140]} uses a DCOPF to co-optimize reserves and energy, allowing preventative transmission switching. It minimizes the cost of energy, reserves, and load shedding. The load shedding coefficient was 10 times the highest variable cost. The switching penalty was 1\% of the lowest variable cost. Use number of cycles in a network to limit switches with lines minus buses plus one. He uses the four-bus system and the IEEE 30-bus system as examples. With the IEEE 30-bus system, he reduces the capacity of line 15 from 65 to 50 MW and divided the first three generators of bus 1, 2, and 5 into seven units.

Yi, Guo and Liang \textsuperscript{[141]} convert a network to an undirected graph. Each end of the overloaded lines are source and destination nodes. Dijkstra’s algorithm finds the shortest path. K shortest paths are also found with the K shortest path algorithm. The lines of the the K shortest paths are put into some set and are ordered from smallest to largest reactance. AC power flow is calculated after each line is dropped. It is basically a way of making a priority list. It is demonstrated on a 30-bus system.

Eickmann et. al. \textsuperscript{[142]} says that one issue of approximation is given by switching lines that have significant reactive power that are connected to voltage regulated nodes. Voltages change greatly when switching lines with the current injection approximation. It uses a similar approach to fast decoupled load flow to have voltage change states according to a Jacobian. It has one estimate the current injection from the voltages and network impedance. It basically has that $\Delta V = (Y_0 + \Delta Y)^{-1} I$; it looks at the network topology change and approximates it. Eickmann tests his method on a model of the European transmission grid, with 2769 buses and 5086 lines.

Van Acker \textsuperscript{[143]} adds a binary variable for each breaker to represent its open/closed state rather than representing all the breakers as one bus. It limits the short circuit current and circuit breaker power. Any islanding solution is ignored. It introduces a heuristic to examine substations with too high of a short circuit current. It uses a mixed integer non convex quadratic constraint problem as the model. He uses the 14-bus IEEE case as the example case.

Aazami \textsuperscript{[144]} adds in the cost of switching lines with the depreciation of switch insulators, and the cost is proportional to the number of switching operations. It uses a probabilistic MINLP to co-optimize day-ahead energy and reserve markets. It models dynamic constraints with a classical synchronous machine model. It models rotor angle
and speed. They show that switching plus stability limits are cheaper than having no option of switching, but more expensive than ignoring the stability limitations. The LMPs are significantly higher when stability constraints are included. Aazami uses the IEEE 57 bus as an example to implement switching.

Majidi-Qadikolai and Baldick [145] discuss that line switching has a cost in terms of breaker maintenance - the greater the current being cut off, the more maintenance that is required. It uses a list of monitored lines, calculates LODF for all closed lines, and uses this information to reduce the switching lines list. Lines that will overload the monitored lines are removed from the switching list. They add constraints to limit the number of times a line is switched over a number of hours as well as limiting total number of lines switched in each hour. Adding costs of breaking reduces the number of lines that are switched. This paper is more for economic switching than corrective switching. They examine this on a 13-bus system and a reduced ERCOT system (317 buses).

Dehghanian [146] introduces a framework to incorporate corrective switching into a transmission operator’s decision making. They create a switching tree, perform each switch, test for AC feasibility and stability, then do an analysis of the circuit breaker reliability and line availability, and give a final set of recommended, reliable switches. While they run a DCOPF, they check the results using an ACPF.

Majidi-Qadikolai et. al. [147] do a case study on ERCOT for contingency analysis and transmission expansion planning. They first start by solving the transmission expansion planning with ignoring contingency constraints (this is a lower bound cost). Then, they create a variable contingency list (VCL) that includes lines whose outages may result in overloads. They also modify the LODF for single circuit radial lines to be very high so the lines are always included in the VCL. They show that you get nearly the same costs by doing this analysis and doing the full model is effectively intractable; they ran it for 10 days and it did not solve.

7.1.1 Examples of Corrective Switching

This section discusses a two and a three-bus example where corrective switching has helped restore the system to meet its required network limits.

7.1.1.1 Two Bus Example

In this two bus example, one generator is attached to each bus. The generator at bus 1 costs $5 per MWh, and the generator at bus 2 costs $1.2 per MWh. The load at bus 1 is 110 MW, and 40 MVA. The generator at bus 1 is has a maximum of 10,000 MWh of production, and can provide between -1000 and 10,000 MVar of reactive power. The generator at bus 2 has a maximum of 30 MWh, with the reactive power range between -10 and 10 MVar. There are two lines connecting bus 1 and bus 2. They are identical and have essentially infinite capacity. For each line, the resistance is 0.084, the reactance is 1.8, and the susceptance is 0.3. The equivalent resistance between the two buses if
we converted the two lines to one line is 0.042 and the equivalent reactance is 0.9. The minimum voltage at both buses is 0.9 p.u., and the maximum voltage is 1.1 p.u. This example has a feasible power flow solution shown on Figure 7.1.1.1, where generator 1 produces 30MW and -10MVar and generator 2 produces 80.6MW and 1.7MVar.

\[
\begin{align*}
\text{G1} &: P_1 = 80.6, \quad Q_1 = 1.7 \\
\text{v}_1 & = 0.900 \angle 0^\circ \quad \text{R}_{eq} = 0.042 \\
\text{G2} &: P_2 = 30, \quad Q_2 = -10 \\
\text{v}_2 & = 1.092 \angle 15.3^\circ \quad \text{X}_{eq} = 0.9 \\
&D \quad \text{P}^D = 110, \quad Q^D = 40 \\
&P_{P_{loss}} = 0.55 \quad Q_{Q_{loss}} = 11.81
\end{align*}
\]

While this initial setting of the network has a feasible power flow solution, a low load situation causes problems. As we reduce the load at bus 1, we find a point where there is no longer a feasible power flow solution. Once the load is reduced to 24% of original load, Figure 7.1 shows that the voltage magnitude at bus 1 goes to 0.899 and at bus 2 to 1.100072. At the original load and for less drastic reductions in load, the two voltage magnitudes started fairly close to the minimum and maximum for the system - 0.900158 and 1.09246.

Figure 7.1: 2 Bus Network, as Load Decreases: Voltage
Figures 7.2 and 7.3 show the change in the power generation as the load is reduced, except that below a fraction of 0.24 of the original load does not have a feasible solution. As the load decreases from its original load to 30% of the original load, the generation from generator 2 stays at 30MW while that from generator 1 makes up the balance. On the other hand, the reactive power generation from generator 1 decreases as load decreases while reactive power generation from generator 1 stays the same.

The voltage angle difference was 15.28° at full load and stays here as load is reduced until real power demand drops to 28MW. At this point, the voltage angle difference drops to 13.708° at 24% of original load. After the real power demand reduces below 24% of original load, we have minimum and maximum voltage violations. However, we can help find a feasible power flow solution for this network by using corrective switching. In this case, if we remove one of the two lines, we can find a feasible power flow solution. This solution is shown in Figure 7.4.

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Figures 7.5 and 7.6 show the voltage magnitudes and angles of the system as the original load decreases before and after the corrective switch takes place. Switching
the line out reduces the voltage magnitude differences between the buses, which allows
the voltages to stay within their maximum and minimum limits even at very low load.
Switching the line out does increase the voltage angle difference at the original load, but
in this case, it only increases it to 27°, which is well within most voltage angle difference
limits.

![Figure 7.5: Voltage magnitude behavior as load reduces, with and without corrective switching](image1)

![Figure 7.6: Voltage angle behavior as load reduces, with and without corrective switching](image2)

### 7.1.2 3 Bus Network: Generator Contingency

This example shows how to use corrective switching to solve a generator contingency.
In this example, we have a 3-bus network as shown in Figure 7.7, which has a feasible power flow
solution. If generator 3 can no longer produce real power, as given in Figure 7.8, then all
of the bus voltage magnitudes drop below to 0.87 p.u., which is below the minimum bus
voltage magnitude of 0.9. Additionally, the voltage angle differences between buses 1
and 2 and between buses 2 and 3 are very large and likely exceed angle difference
limits.

If we remove one of the lines between buses 1 and 3, the new solution is shown in
Figure 7.9. Removing this line caused the voltages at all of the buses are raised and to
all be within the appropriate range. The reactive power transfer required with corrective
switching is lower than in the pre-contingency case. If we wanted to continue to remove
lines, we could remove the second line between buses 1 and 3. The solution to this
second switch is shown in Figure 7.10. This would further reduce the amount of reactive
power being transferred in the system, although it does increase the system losses and
the voltage angle differences between buses.

Removing an additional line increases the voltage angle difference between 1 and 2
and 1 and 3. Reactive power drops at nodes 1 and 3.
7.2 Methodology

While there are many ways to approach corrective switching, here we use corrective switching as a response to a contingency. The process is shown in Figure 7.11. It starts by running the SLP-IV-ACOPF on the original network, without considering any contingencies. Once the SLP-IV-ACOPF algorithm terminates, we then have the starting point for the generation schedule. The optimal generation schedule from the SLP-IV-ACOPF without contingencies is \( P^*_n \) for each generator \( n \in G \). Next, some generator or line contingency (or both) happens. After this contingency, we solve the power flow problem with real power generation changing in accordance to participation factors. We examine the solution to see if there are any voltage violations or thermal violations on the transmission lines. If there are no violations, then corrective switching is not necessary since there is not a problem. If there are violations, we then solve for
which lines to switch to reduce the total violations. The real power generation is pegged to the participation factors, which limit how the generators can be redispatched.

In the DCOPF solutions for corrective switching, total net real power generation does not change because power losses are neglected. In the ACOPF, however, switching lines on and off changes the total power losses in the system, so we may have the total net real power generation with switching out different lines. Therefore, we allow the total generation to vary from the pre-contingency case, and we notate this change as $\Delta P$. We then have that the real power generation depends on its pre-contingency dispatch plus its participation factor $\gamma_n$ times the change in total real power generation.

If we have a line contingency and not a generator contingency, we can calculate the participation factors as given in (7.2.1).

$$\gamma_n = \frac{P_{\text{max}} - P_{g^*}}{\sum_{n\in\mathcal{N}} (P_{\text{max}} - P_{g^*})}$$ \hspace{1cm} (7.2.1)

We denote the set of generators with contingencies as $\mathcal{O}$. We then ensure that generators with outages do not produce any real power by setting $P_{g^*} = 0$ and $\gamma_i = 0$ for $i \in \mathcal{O}$. The generators without outages have the participation factors as proscribed in (7.2.2).

$$\gamma_n = \frac{P_{\text{max}} - P_{g^*}}{\sum_{n\in\mathcal{N}, \text{ if } n\notin\mathcal{O}} (P_{\text{max}} - P_{g^*})}$$ \hspace{1cm} (7.2.2)

### 7.3 Formulation

The objective for corrective switching is to minimize the total voltage and real power violations, as given in (7.3.1). The constraints, given in (7.3.2)-(7.3.33) are the same as
in the dual markets chapter plus a few additional constraints. One additional constraint
is that generator redispacht depends on the participation factors \((7.3.2)\). We also remove
the penalty costs of violating the real and reactive power upper bounds and instead add
constraints that limit the violations on real and reactive power upper bounds to 0.01,
given in \((7.3.29) - (7.3.32)\).

\[
\begin{align*}
\min & \sum_{n \in N} V_n^c \left( v_n^{\text{viol},-} + v_n^{\text{viol},+} \right) + \frac{1}{2} \sum_{k \in K} P_k^e v_k^{\text{viol}} \\
\text{subject to} & \\
p_n^g - P_n^g + \gamma_n \Delta^p & = P_n^{\text{min}} \quad (7.3.2) \\
P_n^g - P_n^{\text{viol},-} & \leq P_n^{\text{min}} \quad (7.3.3) \\
p_n^g - P_n^{\text{viol},+} & \leq P_n^{\text{max}} \quad (7.3.4) \\
q_n^g - q_n^{\text{viol},-} & \leq -Q_n^{\text{min}} \quad (7.3.5) \\
q_n^g - q_n^{\text{viol},+} & \leq Q_n^{\text{max}} \quad (7.3.6) \\
v_n^{\text{viol},-} - v_n^{\text{viol},+} & \leq - \left( V_n^{\text{min}} \right)^2 \quad (7.3.7) \\
v_n^{\text{viol},-} - v_n^{\text{viol},+} & \leq \left( V_n^{\text{max}} \right)^2 \quad (7.3.8) \\
\sum_{m,n} i_{k(m,n)}^r - G_{k(n)} v_n^r + G_{k(m)} v_m^r + B_{k(n)} v_n^j - B_{k(m)} v_m^j & \leq (1 - z_k) M_k \quad (7.3.9) \\
\sum_{m,n} i_{k(m,n)}^i - G_{k(n)} v_n^i + G_{k(m)} v_m^i + B_{k(n)} v_n^j - B_{k(m)} v_m^j & \geq - (1 - z_k) M_k \quad (7.3.10) \\
\sum_{m,n} i_{k(m,n)}^j - B_{k(n)} v_n^i + B_{k(m)} v_m^i - G_{k(n)} v_n^j + G_{k(m)} v_m^j & \leq (1 - z_k) M_k \quad (7.3.11) \\
\sum_{m,n} i_{k(m,n)}^j - B_{k(n)} v_n^i + B_{k(m)} v_m^i - G_{k(n)} v_n^j + G_{k(m)} v_m^j & \geq - (1 - z_k) M_k \quad (7.3.12) \\
\sum_{m \in A(n)} i_{k(m,n)}^r & = 0 \quad (7.3.13) \\
\sum_{m \in A(n)} i_{k(m,n)}^j & = 0 \quad (7.3.14) \\
-p_n^q + \hat{v}_n^q + \hat{v}_n^r + \hat{v}_n^j + v_n^i \hat{v}_n^r + v_n^j \hat{v}_n^r + v_n^i \hat{v}_n^j & = P_n^d + \hat{v}_n^r \hat{v}_n^r + \hat{v}_n^j \hat{v}_n^j \quad (7.3.15) \\
-q_n^q - \hat{v}_n^q & \leq - Q_n^d - \hat{v}_n^r \hat{v}_n^r + \hat{v}_n^j \hat{v}_n^j \quad (7.3.16) \\
v_n^r & \leq \hat{v}_n^r + V_n^r \quad (7.3.17) \\
v_n^r & \leq - \hat{v}_n^r + V_n^r \quad (7.3.18) \\
v_n^j & \leq \hat{v}_n^j + V_n^j \quad (7.3.19) \\
v_n^j & \leq - \hat{v}_n^j + V_n^j \quad (7.3.20) \\
v_n^r \cos(2\pi s/S) + v_n^j \sin(2\pi s/S) - v_n^{\text{viol},+} & \leq \left( V_n^{\text{max}} \right)^2 \quad (7.3.21) \\
\hat{v}_n^r v_n^r + \hat{v}_n^j v_n^j - v_n^{\text{viol},+} & \leq \left( V_n^{\text{max}} \right)^2 \quad (7.3.22) \\
\hat{v}_n^r v_n^r + \hat{v}_n^j v_n^j & \leq \left( V_n^{\text{max}} \right)^2 \quad (7.3.23)
\end{align*}
\]
\[ v_r^n - v_{\text{viol}}^+ \leq V_{\text{max}}^n \]  
(7.3.24)

\[ v_r^n - v_{\text{viol}}^+ \leq V_{\text{max}}^n \]  
(7.3.25)

\[ v_j^n - v_{\text{viol}}^+ \leq V_{\text{max}}^n \]  
(7.3.26)

\[ v_j^n - v_{\text{viol}}^+ \leq V_{\text{max}}^n \]  
(7.3.27)

\[ \hat{v}_r^{i\times r} + \hat{v}_n^{i\times r} + \hat{v}_j^{j\times j} + p_{\text{viol}}^n \leq P_{\text{max}} + \hat{v}_n^{i\times r} + \hat{v}_j^{j\times j} \]  
(7.3.28)

\[ p_{\text{viol}}^+ \leq 0.01 \]  
(7.3.29)

\[ p_{\text{viol}}^- \leq 0.01 \]  
(7.3.30)

\[ q_{\text{viol}}^+ \leq 0.01 \]  
(7.3.31)

\[ q_{\text{viol}}^- \leq 0.01 \]  
(7.3.32)

\[ p_{n,l}^+, p_{n}^{\text{viol}}, p_{n}^{\text{viol}}, q_{n}^{\text{viol}}, q_{n}^{\text{viol}}, v_{n}^{\text{viol}}, v_{n}^{\text{viol}} \geq 0 \]  
(7.3.33)

### 7.4 Results

In this section, we test the post-contingency corrective switching process on different networks, considering many contingencies.

#### 7.4.1 14 Bus System with No Line Limits: Line and Outages

For this system, there is no feasible solution for outages of lines 1, 3, 6, 12, 15, 16, 17, or 19. There are no violations if there is an outage on any of the other lines in the network (2, 4, 5, 7, 8, 9, 10, 11, 13, 14, 18, 20). There are no violations with any of the possible generator contingencies.

#### 7.4.2 14 Bus System with 71MW Line Limits: Line Outages

If we examine the 14-bus system with high (71MW) line limits, the pre-contingency objective function is 84.798. The only solvable contingency is line 7*. If there is an outage on line 1, 2, or 14, there is not a feasible power flow. There are no voltage or thermal violations if lines 3, 4, 5, 6, 8, 9, 11, 13, 16, 17, 18, 19, or 20 are out. An outage of line 7 results in a flow violation of 20.1% on line 1, as shown in Figure 7.12. However, if we solve a corrective switching problem allowing up to three lines open, we see that switching line 5 off results in no violations, as seen in Figure 7.13.

If, instead of using post-contingency corrective switching to solve the problem, we pretighten the line limits on Line 1, the optimal dispatch cost is 85.6. We show how reducing the line limit impacts the objective function in 7.1.
Solving a MIP with up to 3 lines open: s.

Figure 7.12: Contingency on Line 7

Figure 7.13: Fixing flow violations due to the contingency on Line 7

<table>
<thead>
<tr>
<th>Line Limit on Line 1</th>
<th>Line Flow after Line 1 out</th>
<th>Max Flow</th>
<th>Objective Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.627</td>
<td>0.627</td>
<td>87.544</td>
</tr>
<tr>
<td>0.55</td>
<td>0.669</td>
<td>0.669</td>
<td>86.607</td>
</tr>
<tr>
<td>0.6</td>
<td>0.693</td>
<td>0.693</td>
<td>85.735</td>
</tr>
<tr>
<td>0.61</td>
<td>0.705</td>
<td>0.705</td>
<td>85.602</td>
</tr>
<tr>
<td>0.62</td>
<td>0.711</td>
<td>0.711</td>
<td>85.468</td>
</tr>
<tr>
<td>0.63</td>
<td>0.727</td>
<td>0.727</td>
<td>85.332</td>
</tr>
<tr>
<td>0.64</td>
<td>0.729</td>
<td>0.729</td>
<td>85.173</td>
</tr>
<tr>
<td>0.65</td>
<td>0.738</td>
<td>0.738</td>
<td>85.038</td>
</tr>
</tbody>
</table>

Table 7.1: Impact on the objective function of tightening the line limits on line 1 before any contingency occurs

7.4.3 14-Bus Results with 71MW Line Limits: Generator Outages

There are 5 generators in the 14 bus system. If we have an outage on generation 1, the system is feasible with just the other generators picking up the slack according to their participation factors. If generator 2 goes out, there is no feasible solution. If there is an outage of generator 3, 6, or 8, then there is a flow violation on the power traveling on line 1. Table 7.2 summarizes these results.

If generator 3 has an outage, the participation factor redispacht results in a violation on the power on line 1 of 20.8%, shown in Figure 7.14. However, we can reduce this violation to 3.2% by opening lines 5, 9, and 13, shown in Figure 7.15.

A contingency of generator 6 results in a violation on line 1 of 15.1%, shown on Figure 7.16. By switching off lines lines 4 and 6, this violation is eliminated, as shown in Figure 7.17.
Table 7.2: Generator contingencies and their impacts

<table>
<thead>
<tr>
<th>Generator #</th>
<th>Pre-Contingency Generation (MW)</th>
<th>Contingency Consequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.18</td>
<td>No violations</td>
</tr>
<tr>
<td>2</td>
<td>0.42</td>
<td>No Feasible Solution</td>
</tr>
<tr>
<td>3</td>
<td>0.67</td>
<td>Flow violation on line 1</td>
</tr>
<tr>
<td>6</td>
<td>0.10</td>
<td>Flow violation on line 1</td>
</tr>
<tr>
<td>8</td>
<td>0.26</td>
<td>Flow violation on line 1</td>
</tr>
</tbody>
</table>

Figure 7.14: Case 14 with 71MW Line Limits, Outage of Generator 3, Line 1 Violation

Figure 7.15: Case 14 with 71MW Line Limits, Outage of Generator 3, Line 1 Violation Reduction with Corrective Switching

A contingency of generator 8 results in a violation on line 1 of 16.0%, as shown in Figure 7.18. This violation can be eliminated by opening lines 4, 6, and 8, as shown in Figure 7.19.

7.4.4 14 Bus System with 26.75MW Line Limits: Line and Outages

The system is infeasible with any single line or single generator out. Full redispatch may be required in this case.
CHAPTER 7. CORRECTIVE SWITCHING

Figure 7.16: Violation on line 1 with an outage of generator 6

Figure 7.17: Fixing the violation on line 1 due to the outage of generator 6 by removing lines 4 and 6

Figure 7.18: Violation on line 1 with an outage of generator 8

Figure 7.19: Fixing the violation on line 1 due to the outage of generator 6 by removing lines 4, 6, and 8
Chapter 8

Conclusions and Future Work

8.1 Conclusions

This thesis has addressed major concerns in the marketplace about the gap between modeling the power network as an AC and DC system. It has shown an allowable market-compatible method (the SLP-IV-ACOPF) to solve the AC power flow where the prices support the market. The computational time of the SLP-IV-ACOPF scales linearly in the number of buses of the network and finds solutions very close to the best-known solutions of the original IV-ACOPF formulation. Therefore, we have a method that gives good solutions, scales up linearly so that it may be used on real power networks, and provides a valid market settlement.

The SLP-IV-ACOPF retains the payments as are used right now (real power congestion rent, real power load payment, and real power generation rent) and also gives quantities for a reactive power congestion rent, reactive power load payment, reactive power generation rent, voltage value, and shunt compensation. In comparisons between AC and DC settlements, we see that in the DC system, we may be making payments to transmission lines when they should be going to reactive power or voltage support. We also enable getting prices on reactive power and voltage to know what locations get impacted by voltage limits as well as where reactive power is most valuable.

We examine many heuristics to try to solve transmission switching faster. While no heuristics perform perfectly, we see that the congestion rent heuristic, largest LMP, and smallest reactive power heuristics do give indications of good lines to switch. While most corrective switching algorithms focus on minimizing load shed and do not consider voltage violations because they are based on DC formulations, we have introduced a method to maintain load and fix voltage and line flow violations with post-contingency corrective switching. This post-contingency corrective switching even helps out when the generators are given specified participation factors rather than allowing a new redispatch.
8.2 Future Work

There are many different possible directions for this research. This research provides one step to being able to implement an AC model in the ISO market. There are additional considerations for implementing the SLP-IV-ACOPF model in the market, and there may be future studies to understand the changes between using the SLP-IV-ACOPF versus the status quo algorithms.

8.2.1 Incorporating D-Curves

This thesis represented real and reactive power constraints as separate constraints that do not directly interact with each other. In reality, these constraints form a ‘D-curve’. At certain power factors, producing more reactive power may force less real power. These constraints were not included in the SLP-IV-ACOPF or the market solution. It would be very interesting to study the SLP-IV-ACOPF with and without D-curves and to understand the impact of D-curves on potential economic and corrective transmission switching problems.

8.2.2 Allocating Reactive Power & Voltage Support in the Real-Time market

In this thesis, we suggested putting settlements for reactive power and voltage support in a pool to be used by the ISOs. However, another interesting topic would be to allocate reactive power payments and voltage support in the real-time market. The difficulty right now with voltage support is that it is very difficult to attribute changes in voltage to individual generators. While voltage magnitude is coupled with reactive power, this is a nonlinear relationship that also depends on real power. Additionally, the voltage at one bus is influenced by voltages across the network, reactive power across the network, and physical characteristics of transmission lines. Detangling how voltage support is allocated is difficult.

8.2.3 Placement of New Reactive Capability

Another interesting topic would be to use the real-time prices of reactive power over time and figure out how to site new reactive capability, as well as the type and size of reactive capability. The impact of this new reactive capacity on old reactive equipment would also be an interesting study. It would be also very useful to see if the value of reactive capacity varied with time of day or season, and in what locations it would be more valuable.
8.2.4 Reactive Power FTR Bidding

The new sequential linear program market style allows for bidding on reactive power congestion rent. Understanding how to bid on reactive congestion rent would be interesting. Generally, the reactive power prices are much smaller than the real power prices, so the reactive congestion rent amount would generally be lower than the real power congestion rent amount. However, there may be same cases where reactive power congestion amounts to a significant payment.

8.2.5 Reactive Power Pricing and Eliminating Uplift

Generation units are often paid uplift based on startup costs for producing reactive power. It is interesting to examine whether a generator’s startup cost for reactive power could be incorporated into its bid curve, or whether the reactive power generation rent can be an alternative compensation for the uplift. Additionally, the payment for reactive power may be able to come from adding in the D-curves for the generators.

8.2.6 Lost Opportunity Cost Payments Gaming

Generators receive a lost opportunity cost if they are backed down on their real power to produce reactive power outside their power factor for the ISO. It may be possible for a generator to bid below cost with the intention of being economically dispatched at a high amount of real power, and then backed off to provide reactive power, receiving the lost opportunity cost, where perhaps otherwise they would bid in higher and not be economically be dispatched at all.

8.2.7 Other Transmission Switching Heuristics

While this thesis examined over 200 transmission switching heuristics, there are almost an unlimited number of transmission switching heuristics that can be considered as well as other reduction methods. Additionally, typically heuristics are studied by considering one line at a time. Methods that consider subsets of lines at a time to switch off would be very interesting.

8.2.8 FACTS Devices

FACTS devices change the resistance and impedance of transmission lines. Deciding how to operate and set these devices, and also where to place them, would be interesting to look at in conjunction with transmission switching. FACTS devices generally cause a discrete change in the resistance or the reactance, and since we multiply resistance and reactance by voltage, this would also be a nonlinear program.
8.2.9 N-1 SLP-IV-ACOPF

While we have demonstrated how to use post-contingency corrective switching to help contingencies and show that you can use a cheaper dispatch if you allow for post-contingency actions, the ISOs currently ensure that any solution is $N - 1$ reliable. The next step to getting the SLP-IV-ACOPF ready to implement in the market would be to be able to solve an $N - 1$ reliable version of the SLP-IV-ACOPF quickly enough for the market.
Bibliography


