Title
Reputation Effects of Information Sharing

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Reputation Effects of Information Sharing*,**

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Abstract

This paper analyzes a model of investment and return in an economy characterized by information asymmetry between an investor and a manager. The realized value of the uncertain state of nature is the manager’s private information. The paper first considers an economy where the manager cannot share her private information with the investor. Therefore, dividend payment is the only reputation building tool available to the manager. If the investor’s prior beliefs about the manager’s trustworthiness are sufficiently high, then the manager will return a dividend consistent with the lower possible state of nature having occurred and the investor will revise such beliefs downwards. However, if the beliefs are not so high, then the equilibrium will be mixed strategies.

The paper then compares such a dividend-only economy with one where information sharing is an additional tool available for building reputation. Information sharing disciplines the potential opportunism accruing to a manager out of her informational advantage. It provides ex post verifiability of the state of nature and thereby, obviates downward revision of an investor’s prior beliefs about a manager’s trustworthiness. This results in a greater region of pure strategy play in the dividend-and-information-sharing economy. Since such pure strategy play implies investment with certainty, information sharing leads to higher investment. Further, pure strategy play implies returns consistent with the actual state of nature in the dividend-and-information-sharing economy while it implies dividend consistent with the lower state of nature in the dividend-only economy and these lead to higher return in information sharing.

Keywords. Information, Reputation, Investment, Return, Trust.

JEL codes: M41, D82, C11, C73.
1. Introduction

“It takes 20 years to build a reputation and five minutes to ruin it. If you think about that, you’ll do things differently.”

- Warren Buffett

At one end of the spectrum are the Warren Buffetts and Cynthia Cooper (Cynthia Cooper was the whistle-blower at WorldCom) while at the other end are Enrons and WorldComs. Interestingly enough, the final disintegration of Arthur Andersen (one of Big Five Accounting firms) was not caused directly by Enron audit deficiencies, but by a decision to shred Enron audit documents, and the conviction on the charge of obstruction of justice that resulted. The ensuing loss of reputation meant Arthur Andersen’s clients walked away, and even though on May 31st, 2005, the U.S. supreme Court overturned the conviction on the grounds that the “jury instructions failed to convey the requisite consciousness of wrong-doing”, it was too late – a firm of 85,000 people worldwide, including 24,000 in the US had disintegrated. This example underscores the importance of reputation in a world characterized by information asymmetry.

The role information plays in facilitating reputation for trustworthiness is the focus of this paper. This paper first analyzes an economy where there is information asymmetry between an investor and a manager and the only reputation-building tool available to the manager is the dividend she pays out. The information asymmetry arises from an uncertain state of nature, the realization of which is the manager’s private information. In the equilibrium of a finitely repeated game, a rational manager will pay dividends consistent with the lower possible state of nature having occurred, if the investor’s prior beliefs about the manager’s trustworthiness are sufficiently high. Lack of ex post verifiability of the state of nature implies that the manager is able to get away with pretending that the lower possible state of nature has occurred; however, this leads to a downward revision of the investor’s ex ante beliefs about the manager’s trustworthiness. Given that the investor’s beliefs about the manager’s trustworthiness are high, therefore, she will invest with certainty. That is, the equilibrium will be in pure strategies.
If the investor’s prior beliefs about the manager’s trustworthiness are not sufficiently high, then the equilibrium will be in mixed strategies. The investor will invest with a certain probability strictly less than one while the manager will pay dividend with a certain probability strictly less than one. The choice of respective probabilities will be such that it will make the other agent indifferent between the choices available to her in her action space. While pure strategy play leads to downward revision of the investor’s beliefs about the manager’s trustworthiness, mixed strategy play ensures that the investor’s prior / ex ante beliefs about the manager’s trustworthiness are updated upwards to a point at which the investor then invests with a probability strictly less than one.

The paper then compares such a dividend-only economy with one where sharing of private information is an additional reputation-building tool available to the manager. It finds that in equilibrium, the dividend-only economy will have a smaller region of pure strategy play compared to the dividend-and-information-sharing economy. Pure strategy region in both the economies has the investor investing with certainty while the mixed strategy region has the investor investing with a certain probability strictly less than one. Greater region of pure strategy play in the dividend-and-information-sharing economy and the concomitant investment with certainty will lead to higher investment in information sharing.

The pure strategy region in the dividend-only economy has the manager paying out a dividend consistent with the lower state of nature having occurred. On the other hand, the pure strategy region of the dividend-and-information-sharing economy introduces the possibility of sharing manager’s private information about the realized state of nature. It, thereby, obviates such obfuscation by the manager as is possible in the dividend-only economy. Further, as noted earlier, there is greater region of pure strategy play in the dividend-and-information-sharing economy. Taken together, the greater region of mixed strategy play combined with pure strategy play of payment of dividend consistent with the lower state of nature in the dividend-only economy leads to higher return in information sharing.
Fundamental to the above analysis is the existence of trustworthy manager. In the dividend-only economy, the trustworthy manager is defined as one that always pays a fair dividend to the investor. In the dividend-and-information-sharing economy, the corresponding definition is of a manager that pays a fair dividend and chooses to share her private information. The finitely repeated nature of the game allows for opportunities for reputation building in both the economies. While the payoff maximizing nature of the rational manager pushes her towards paying out as little as possible in dividends, the possible existence of the trustworthy manager forces egalitarianism. In equilibrium, the rational manager is forced to mimic the trustworthy type in an attempt to build reputation for being trustworthy. The nature of such mimicking is different across the two economies with the resultant differences in investment and return.

This paper is closely related to Einhorn and Ziv (2008) and Beyer and Dye (2012). Einhorn and Ziv (2008) shows that current period’s disclosure impacts the firm’s reputation for being informationally endowed by increasing the market’s expectation of information endowment of the firm and thus, creates an implicit disclosure cost for future. Beyer and Dye (2012) studies managerial efforts at building reputation for being ‘forthcoming’ by disclosing all the earnings forecast received in the classic Dye (1985) and Jung and Kwon (1988) voluntary disclosure framework.

In the current study, the counterpart of Beyer and Dye’s ‘forthcoming’ manager is the ‘trustworthy’ manager. In the literature following Berg, Dickhaut and McCabe (1995), ‘trust’ is widely perceived as making the risky choice of passing the endowment to the second player in a sequential game for an expanded pie, some of which may be possibly returned. This paper follows such notion of trust in studying the managerial efforts at building reputation for being trustworthy. While trust has mostly been examined using behavioral theories and human subjects experiments, at a theoretical level, trust has been conceptualized as a strategic attempt to build reputation in a repeated game (e.g. Kreps and Wilson (1982); Kreps, Milgrom, Roberts and Wilson (1982)). This paper allows for information asymmetry in a reputation-building model to examine the role of information sharing in building reputation for being trustworthy. Another paper that allows for
information asymmetry in a reputation-building model is Lunawat (2013a). While Lunawat (2013a) establishes information sharing as a reputation building tool, the current paper focuses on differences in reputation building opportunities, in investment and in return across dividend-only and dividend-and-information-sharing economies.

This paper is also related to the repeated cheap-talk models that address reputation concerns pertaining to disclosure of private information. In these models, there is value to truthful disclosures because there is the possibility of untruthful disclosures and hence, a rational agent tries to develop reputation for making truthful disclosures. Sobel (1985) introduces a repeated cheap-talk model to analyze the effect of reputation on strategic transmission of information. He shows that a rational agent will develop a reputation for credible communication by mimicking the behavior of the honest type and identifies situations where a rational agent cashes in on her reputation. Benabou and Laroque (1992) and Morris (2001) extend Sobel’s model by incorporating noisy information. Stocken (2000) uses a repeated cheap-talk game to derive conditions where voluntary disclosure of non-verifiable information can be credibly made in the presence of verifiable mandatory disclosures. He shows improvement in communication flowing from managerial efforts at building a reputation for truthful disclosure. While the cheap-talk models allow for the possibility of untruthful disclosure, I follow the Grossman (1981) and Milgrom (1981) approach of requiring disclosure / information sharing to be truthful. Dispensation of the possibility of untruthful disclosures is consistent with existence of a fairly strong legal and regulatory framework and also allows veering the focus from managerial efforts at building a reputation for truthful disclosures to managerial efforts at building a reputation for being trustworthy.

This paper contributes to the extant literature on investor protection and dividend policies by formalizing managerial reputation-building as the force that drives dividend payment. Holding a firm’s investment policy constant, dividend payouts reduce the firm’s retained earnings, thereby leaving shareholder wealth unchanged (Modigliani and Miller (1958)). Therefore, it is puzzling that firms still choose to pay dividends. A possible explanation is that firms signal future growth by paying dividends so that firms that pay dividends
experience higher future growth while those that do not pay dividends experience lower future growth (e.g. Bhattacharya (1979)). However, this explanation has mixed empirical support (e.g. Aharony and Swary (1980), DeAngelo, DeAngelo and Skinner (1996) and Benartzi, Michaely and Thaler (1997)). Another explanation is rooted in the agency theory (e.g. Jensen (1986), Hart and Moore (1974)). According to this, investor preference for dividends over retained earnings arises from the potential of managerial / insider expropriation of retained earnings. This explanation has found support in both empirical (LaPorta, Lopez-de-Silanes, Shleifer and Vishny (2000)) and experimental (LaRiviere, McMahon and Neilsen (2016)) work. This paper puts forth managerial reputation building as an alternative explanation for dividend payment. Lunawat (2013b) provides experimental evidence supporting the equilibrium behavior derived in the current paper.

The rest of the paper proceeds as follows. Section 2 introduces the model of the dividend-only economy. Section 3 defines and characterizes the equilibrium of the model. Section 4 discusses the dividend-and-informing-sharing economy. Section 5 compares the investment and return in the two economies. Section 6 summarizes and concludes.

2. Model

This section develops the model of the dividend-only economy. There are two players, a sender / investor and a receiver / manager\(^1\). Nature moves first and selects the manager’s type to be either trustworthy or untrustworthy. I will define momentarily what I mean by each type. The manager knows her type but the investor does not. The game then proceeds through 2 periods in each of which the investor and the manager make a sequence of choices. In what follows, the subscript \(t\) \((t = 1, 2)\) will be used to denote a period.

\(^{1}\) The sender-receiver game I define derives from the “investment game” first studied experimentally in Berg, Dickhaut and McCabe, 1995. In this investment game a sender is endowed with 10 units of wealth and decides how much of this endowment to send to a receiver. The amount sent by the sender is tripled before it reaches the receiver. The receiver decides how much of this tripled amount to keep and how much to send back.
The investor is endowed with $e > 0$ units of wealth and chooses whether to invest in the
manager. Regardless of whether the investor chooses to invest, $e$ is common knowledge.
That is, both the investor and the manager know the amount of wealth the investor is
endowed with. The investor’s decision to invest is denoted by $m_{t}^{ND} = e$ and the investor’s
decision not to invest is denoted by $m_{t}^{ND} = 0$. If the investor chooses to invest, then the
amount $e$ is multiplied by a multiplier $\lambda_{t}$ before the manager receives it. $\lambda_{t} \in \{l, h\}$ and is
equally likely to be either $l$ or $h$ in every period. Assume $1 \leq l < 2 < h, l + h > 4$. It is as if
the manager has some production technology because of which she is able to grow the
investment of $e$ to $e\lambda_{t}$. The multiplied amount ($e\lambda_{t}$) may be thought of as the gross income
of the firm comprising the investor and the manager. I will explain momentarily the
parameter restriction of $1 \leq l < 2 < h, l + h > 4$.

Now the manager receives $e\lambda_{t}$ and learns $\lambda_{t}$. However, the investor does not learn $\lambda_{t}$. That
is, $\lambda_{t}$ is the manager’s private information. After the manager receives $e\lambda_{t}$, she chooses to
send back $k_{t}^{ND}$ to the manager. If $\lambda_{t} = l$, $k_{t}^{ND} \in \{0, el / 2\}$. If $\lambda_{t} = h$, $k_{t}^{ND} \in \{0, el / 2, eh / 2\}$. The idea is that if the low state of the world (namely, $l$) occurs, then the manager either
can choose to send back half or chose to send back nothing to the investor. However, if the
high state of the world occurs (namely, $h$), then the manager has the additional option of
choosing to send back an amount (namely, $el / 2$) consistent with the low state of the world
having occurred.\footnote{Note that this idea can also be captured by expanding the set of $k_{t}$ to include more elements than the ones
specified here. The equilibrium and other results derived will be qualitatively similar. The set of $k_{t}$ used here
is the most parsimonious one possible.} The investor receives $k_{t}^{ND}$ and the manager keeps the residual
$e\lambda_{t} - k_{t}^{ND}$.

The amount sent back by the manager ($k_{t}^{ND}$) may be thought of as the dividend the manager
pays to the investor. A trustworthy manager is defined as one that always chooses to return
returns half of what she receives (That is, if $\lambda_{t} = l$, she chooses $k_{t}^{ND} = el / 2$ and if $\lambda_{t} = h$,
she chooses $k_{t}^{ND} = eh / 2$). An untrustworthy manager is defined as a manager that is not
trustworthy. I have $l / 2 < 1$ so that a realization $\lambda_{t} = l$ implies a negative net return for
investor, $h / 2 > 1$ so that a realization $\lambda_{t} = h$ implies a positive net return for investor, and
\[(l + h) / 4 > 1\] so that the expected net return for investor, if manager is trustworthy, is positive. Risk neutrality, additively separable utility and no time discounting are assumed. Figure 1 describes the timeline of the dividend-only economy.

\[
l + h \\
\]
3. Equilibrium

A perfect Bayesian equilibrium of this game is defined as follows. An equilibrium comprises a strategy for each player and for each period $t$ a function $P_t^{ND}$ that takes the history of moves up to period $t$ into numbers in $[0, 1]$ such that:

(i) Starting from any point in the game where it is the manager’s move, the manager’s strategy is a best response to the investor’s strategy.

(ii) Starting from any point in the game where it is the investor’s move, the investor’s strategy is a best response to the manager’s strategy given that the investor believes with probability $P_t^{ND}(h_t)$ that the manager is trustworthy.

(iii) The game begins with $P_1^{ND} = \theta$.

(iv) Each $P_t^{ND}$ is computed from $P_{t-1}^{ND}$ and the manager’s strategy, using Bayes rule whenever possible.

I will first define the function $P_t^{ND}$ as:

B(i) Set $P_1^{ND} = \theta$.

B(ii) If the investor does not invest in period 1 ($m_1^{ND} = 0$), then $P_{2}^{ND} = P_1^{ND}$.

B(iii) If the investor invests in period 1 ($m_1^{ND} = e$) and the manager returns non-zero amount in period 1 ($k_1^{ND} > 0$) and $P_1^{ND} > 0$, then $P_2^{ND} = \max(4/(l+h), \theta/(2-\theta))$.

B(iv) If the investor invests in period 1 ($m_1^{ND} = e$) but the manager does not return anything in period 1 ($k_1^{ND} = 0$), then $P_{2}^{ND} = 0$.

B(v) If $P_1^{ND} = 0$, then $P_{2}^{ND} = 0$.

$P_t^{ND}$ is the probability with which the investor believes the manager is trustworthy. In the beginning of period 1, the investor believes with probability $\theta$ that the manager is trustworthy (Point B(i) above). If the investor does not invest in period 1, then no Bayesian updating occurs and the investor’s posterior beliefs about the manager’s trustworthiness are the same as her prior beliefs about the manager’s trustworthiness (Point B(ii) above).
If the investor invests but the manager does not return anything in period 1, then the investor knows that the manager is untrustworthy and therefore sets $P_{2}^{ND} = 0$ (Point B(iv) above).

If the investor invests in period 1 and the manager returns a non-zero dividend in period 1 and $\theta > 0$, then the posterior belief $P_{2}^{ND} = \max \left( 4/(l + h), \theta / (2 - \theta) \right)$ (Point B(iii) above). To elaborate on this point, note first that a trustworthy manager, by definition always chooses to return half of what she receives. I will show later that if $P_{1}^{ND}$ is greater than a certain threshold, then the rational manager returns a dividend consistent with the lower state of nature having occurred. That is, $k_{1}^{ND} = el/2$. The investor rationally anticipates this and revises her posterior beliefs downwards to $\theta l (2 - \theta)$. However, if $P_{1}^{ND}$ is not greater than the said certain threshold, then the rational manager mimics the trustworthy type with a certain probability strictly less than one. This probability is such that the investor’s updated beliefs about a manager’s trustworthiness are given by $P_{2}^{ND} = 4/(l + h)$. In the proof in the Appendix, I show that this is consistent with the Bayesian updating criterion specified in the definition of the equilibrium.

Now, I will describe the strategies of the investor and the untrustworthy manager in terms of $P_{i}^{ND}$. The investor’s strategy may be outlined as:

I(i) If $P_{1}^{ND} > (4/(l + h))^2$, the investor invests in period 1 ($m_{1}^{ND} = e$), otherwise she does not ($m_{1}^{ND} = 0$).

I(ii) If $P_{2}^{ND} > 4/(l + h)$, the investor invests in period 2 ($m_{2}^{ND} = e$) and if $P_{2}^{ND} < 4/(l + h)$, the investor does not invest in period 2 ($m_{2}^{ND} = 0$).

I(iii) If $P_{2}^{ND} = 4/(l + h)$ and $k_{1}^{ND} = eh/2$, with probability $V_{1}^{ND} = h/(l + h)$, the investor chooses to invest ($m_{2}^{ND} = e$) and with probability $1 - V_{1}^{ND}$, the investor chooses not to invest ($m_{2}^{ND} = 0$).
I(iv) If $P_2^{ND} = 4/(l + h)$ and $k_1^{ND} = e/2$, with a probability $V_2^{ND} = l/(l + h)$, the investor chooses to invest ($m_2^{ND} = e$) and with a probability $1 - V_2^{ND}$, the investor chooses not to invest ($m_2^{ND} = 0$).

The investor follows a threshold strategy where she invests if her beliefs about the manager’s trustworthiness ($P_t^{ND}$) are above the threshold for the period and does not invest if her beliefs are below the threshold. Points I(i) and I(ii) state this for periods 1 and 2 respectively. It follows from these two points that the investment thresholds for periods 1 and 2 are respectively $(4/(l + h))^2$ and $4/(l + h)$.

In period 2, if the investor’s beliefs about the manager’s trustworthiness are on the investment threshold, she is indifferent between investing and not investing. She chooses to invest with a probability $V_0^{ND}$ that makes the rational manager indifferent between mimicking (to be a trustworthy type) and not mimicking (Points I(iii) and I(iv) above). If period 1 dividend is consistent with the high state of nature having occurred, then $V_1^{ND} = V_1^{ND}$ and is given by $V_1^{ND} = h/(l + h)$ (Point I(iii) above). However, if period 1 dividend is consistent with the low state of nature having occurred, then $V_0^{ND} = V_2^{ND}$ and is given by $V_2^{ND} = l/(l + h)$ (Points I(iv) above).

By definition, a trustworthy manager always chooses to return half of what she receives. An untrustworthy manager’s strategy depends on $t$ and $P_t^{ND}$. The strategy may be outlined as:

- **M(i)** The manager does not return anything in period 2 ($k_2^{ND} = 0$).
- **M(ii)** If $P_1^{ND} \geq 8/(l + h + 4)$, the manager chooses $k_1^{ND} = e/2$.
- **M(iii)** If $P_1^{ND} < 8/(l + h + 4)$ and $\lambda_1 = l$, the manager chooses $k_1^{ND} = e/2$ with some probability $S_3^{ND}$ ($0 \leq S_3^{ND} < 1$) and chooses $k_1^{ND} = 0$ with probability $1 - S_3^{ND}$. 


M(iv) If $P_{1}^{ND} < 8/(l + h + 4)$ and $\lambda_1 = h$, the manager chooses $k_1^{ND} = eh/2$ with some probability $S_1^{ND} (0 < S_1^{ND} < 1)$, chooses $k_1^{ND} = el/2$ with some probability $S_2^{ND} (0 \leq S_2^{ND} < 1, 0 < S_1^{ND} + S_2^{ND} < 1)$ and chooses $k_1^{ND} = 0$ with probability $1 - S_1^{ND} - S_2^{ND}$ such that $S_1^{ND} = S_2^{ND} + S_3^{ND}$. If $P_1^{ND} \leq 4/(l + h)$ then $S_1^{ND} = S_2^{ND} + S_3^{ND} = P_1^{ND} (l + h - 4)/4(1 - P_1^{ND})$.

The untrustworthy / rational manager chooses to return nothing as dividend in the last period (Point M(i) above) because the alternative of paying a non-zero dividend is personally costly to her and since it is the last period, there are no associated expected future benefits. Consider the case where $P_1^{ND} \geq 8/(l + h + 4)$. The investor’s beliefs about a manager’s trustworthiness are so high that the latter can get away with pretending that the lower possible state of nature has occurred. That is, the manager pays $el/2$ as dividend, thereby implying that the lower state of nature occurred (Point M(ii) above).

The intuition behind why the manager is able to get away with such pretense is the following. Even though the investor revises her posterior beliefs downwards, the prior is so high that the posterior remains at or above the investment threshold for period 2. To see this more clearly, note that since $P_1^{ND} \geq 8/(l + h + 4) > 4/(l + h) > (4/(l + h))^2$, therefore by I(i), the investor invests in period 1. By M(ii), the manager returns a non-zero amount of $el/2$. Then, by P(iii), $P_2^{ND} = \max \left(4/(l + h), \theta/(2 - \theta)\right)$. Since $\theta = P_1^{ND} \geq 8/(l + h + 4)$, therefore $P_2^{ND} = \max \left(4/(l + h), \theta/(2 - \theta)\right) = \theta/(2 - \theta) \geq 4/(l + h)$ and by I(ii) – I(iv), the investor invests with non-zero probability in period 2.

Points M(iii) and M(iv) talk about the case where the investor’s beliefs about the manager’s trustworthiness are not so high. Then, the manager’s best response is in mixed strategies. If the lower state of nature occurs (that is, $\lambda_1 = l$), then with probability $S_3^{ND}$, the manager chooses to pay a dividend consistent with the lower state of nature having occurred and with complementary probability pays a dividend of zero. That is, with probability $S_3^{ND}$, she mimics to be a trustworthy type (Point M(iii)).
Similarly, if the higher state of nature occurs (that is, \( \lambda_1 = h \)), then with probability \( S_1^{ND} \), the manager chooses to pay a dividend consistent with the higher state of nature having occurred. That is, with probability \( S_1^{ND} \), she mimics to be a trustworthy type. Note that this case admits the additional possibility of the manager paying a dividend consistent with the lower state of nature having occurred. Therefore, Point M(iv) states that the manager may pay such a dividend with a probability \( S_2^{ND} \) and then may pay a zero dividend with the complementary probability of \( 1 - S_1^{ND} - S_2^{ND} \). In the proof in the Appendix, I show that \( S_0^{ND} \) is so chosen that the investor’s updated period 2 beliefs about the manager’s trustworthiness (namely, \( P_2^{ND} \)) are exactly on the investment threshold for period 2 (namely, \( 4/(l+h) \)). Note that if \( P_1^{ND} < 8/(l+h+4) \), \( \lambda_1 = h \) and \( k_1^{ND} = eh/2 \), then \( S_0^{ND} = S_1^{ND} \). If \( P_1^{ND} < 8/(l+h+4) \), \( \lambda_1 = h \) and \( k_1^{ND} = el/2 \), then \( S_0^{ND} = S_2^{ND} \). If \( P_1^{ND} < 8/(l+h+4) \) and \( \lambda_1 = l \), then \( S_0^{ND} = S_3^{ND} \).

**Proposition 1.** The strategies (I(i) – I(iii), M(i) – M(iv)) and beliefs (B(i) – B(v)) described above constitute a perfect Bayesian equilibrium.

Proof in Appendix.

4. **Dividend-and-Information Sharing Economy**

4.1 **Model.** Consider the earlier game with the following modification. The manager makes an information sharing decision before the investor makes an investment decision. The manager chooses whether to share private information she will learn in the course of the game. A decision to share private information is denoted by \( d_t = 1 \) and a decision not to share private information is denoted by \( d_t = 0 \). Note that the manager is not privy to the private information at the time she makes the choice of whether to share it. It is information she will learn in the course of the game.
The investor sees the manager’s information sharing decision, is endowed with $e > 0$ units of wealth and then makes the investment decision. The investor’s decision to invest is denoted by $m_i^D = e$ and the investor’s decision not to invest is denoted by $m_i^D = 0$. As before, if the investor chooses to invest, then the amount $e$ is multiplied by a multiplier $\lambda_t$ before the manager receives it. $\lambda_t \in \{l, h\}$ and is equally likely to be either $l$ or $h$ in every period. Assume $1 \leq l < 2 < h$, $l + h > 4$. Now the manager receives $e\lambda_t$ and learns $\lambda_t$. However, the investor learns $\lambda_t$ only if the manager had chosen to share her private information. In this sense, $\lambda_t$ is the manager’s private information – she always learns the realized value of $\lambda_t$, but the investor’s knowledge of $\lambda_t$ is dependent on the manager’s information sharing decision.

After the manager receives $e\lambda_t$, she chooses to send back $k_i^D$ to manager. If $\lambda_t = l$, $k_i^D \in \{0, el / 2\}$. If $\lambda_t = h$, $k_i^D \in \{0, el / 2, eh / 2\}$. In this economy, a trustworthy manager is defined as one that always chooses to share her private information ($d_t = 1$) and always chooses to return returns half of what she receives (That is, if $\lambda_t = l$, she chooses $k_i^D = el / 2$ and if $\lambda_t = h$, she chooses $k_i^D = eh / 2$). An untrustworthy manager is defined as a manager that is not trustworthy. As before, risk neutrality, additively separable utility and no time discounting are assumed. The modified timeline may be described as in Figures 2.
Figure 2 – Timeline of the Dividend-and-Information-Sharing Economy

<table>
<thead>
<tr>
<th>Untrustworthy manager</th>
<th>The manager chooses whether to share private information that she will learn in course of the game.</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>The investor sees the manager’s disclosure decision, is endowed with $e$ units of wealth and chooses whether to invest.</td>
</tr>
<tr>
<td></td>
<td>If the investor chooses to invest, then the manager receives $e\lambda_t$ where $\lambda_t \in {l, h}$, chooses to return $k^{D}_t$ to the investor and keeps the residual $e\lambda_t - k^{D}_t$. If $\lambda_t = l$, $k^{D}_t \in {0, e\ell / 2}$. If $\lambda_t = h$, $k^{D}_t \in {0, e\ell / 2, e\ell / 2}$.</td>
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<tr>
<td></td>
<td>The investor receives $k^{D}_t$ and learns $\lambda_t$ if the manager had earlier chosen to share her private information.</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Nature chooses the manager’s type to be trustworthy or untrustworthy.</th>
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<tbody>
<tr>
<td>Repeat for 2 periods</td>
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<table>
<thead>
<tr>
<th>Trustworthy manager</th>
<th>The manager chooses to share private information that she will learn in course of the game.</th>
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</thead>
<tbody>
<tr>
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<td></td>
<td>If the investor chooses to invest, then the manager receives $e\lambda_t$ where $\lambda_t \in {l, h}$, chooses to return $k^{D}_t$ to the investor and keeps the residual $e\lambda_t - k^{D}_t$. If $\lambda_t = l$, $k^{D}_t = e\lambda_t / 2$ to the investor and keeps the residual $e\lambda_t - k^{D}_t$.</td>
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<tr>
<td></td>
<td>The investor receives $k^{D}_t$ and learns $\lambda_t$.</td>
</tr>
</tbody>
</table>

| Repeat for 2 periods |

Nature chooses the manager’s type to be trustworthy or untrustworthy.
4.2 Equilibrium. Analogous to the equilibrium of the dividend-only economy, a perfect Bayesian equilibrium of this dividend-and-information-sharing economy comprises a strategy for each player and for each period \( t \) a function \( P_t^D \) that takes the history of moves up to period \( t \) into numbers in \([0, 1]\). Before describing the definition of \( P_t^D \), note that the set of subscripts for \( P_t^D \) will be different from the set of subscripts for each of \( d_t, m_t^D, \lambda_t \), and \( k_t^D \). While each of \( d_t, m_t^D, \lambda_t \), and \( k_t^D \) is subscripted using the set \( \{1, 2\} \), \( P_t^D \) is subscripted using the set \( \{0, 1, 2\} \). The investor enters the game with \( P_0^D = \delta \). She sees \( d_1 \) and updates to \( P_1^D \) before choosing \( m_1^D \). Since the investor’s updating from \( P_0^D \) to \( P_1^D \) occurs before her choice of investment for the first period \( m_1^D \), this necessitates the introduction of an additional element (namely, ‘0’) in the set of subscripts for \( P_t^D \).

Set \( P_0^D = \delta \). Then, the definition of \( P_t^D \) follows analogously from the definition of \( P_t^{ND} \) and may be stated as:

\[ B'(i) \quad \text{If the manager does not share her private information in period 1 (} d_1 = 0, \text{ then } P_1^D = 0 \text{ and if the manager her shares private information in period 1 (} d_1 = 1, \text{ then } P_1^D = P_0^D = \delta.} \]

\[ B'(ii) \quad \text{If the investor does not invest in period 1 (} m_1^D = 0, \text{ then } P_2^D = P_1^D.} \]

\[ B'(iii) \quad \text{If the investor invests in period 1 (} m_1^D = e, \text{ and the manager shares her private information in period 2 and the manager returns half of what she receives in period 1 (} d_2 = 1, k_1^D = e\lambda_1 / 2) \text{ and } P_1^D > 0, \text{ then } P_2^D = \max(4/(l + h), \delta).} \]

\[ B'(iv) \quad \text{If the investor invests in period 1 (} m_1 = e, \text{ but the manager either does not share her private information in period 2 or the manager does not return half of what she receives in period 1, (} d_2 = 0 \text{ or } k_1^D = 0) \text{ then } P_2^D = 0. \]

\[ B'(v) \quad \text{If } P_{t-1}^D = 0, \text{ then } P_t^D = 0. \]
Note that the updating rule (Point B’(iii) above) here is different from the corresponding rule in the dividend-only economy because no downward revision of beliefs occurs in this dividend-and-information-sharing economy.

Now, I will describe the strategies of the investor and the untrustworthy manager in terms of $P_t^D$. The investor’s strategy may be outlined as:

I(i) If $P_1^D > (4/(l + h))^2$, the investor invests in period 1 ($m_1^D = e$), otherwise she does not ($m_1^D = 0$).

I(ii) If $P_2^D > 4/(l + h)$, the investor invests in period 2 ($m_2^D = e$) and if $P_2^D < 4/(l + h)$, the investor does not invest in period 2 ($m_2^D = 0$).

I(iii) If $P_2^D = 4/(l + h)$, with a probability $V_2^D = \lambda l(l + h)$, the investor invests ($m_2^D = e$) and with a probability $1 - V_2^D$, the investor does not invest ($m_2^D = 0$).

The investor continues to follow the threshold strategy. Further, the investment threshold for periods 1 and 2 in this dividend-and-information-sharing economy (Points I'(i) and I'(ii) above) are identical to the respective thresholds in the dividend-only economy. However, unlike the dividend-only economy, the investor’s best response in the mixed strategy region is not dependent on the dividend $k_t^D$ anymore (Point I'(iii) above) because at the discretion of the manager, the investor is now able to see the realized value of the state of nature.

By definition, a trustworthy manager always chooses to share her private information and always chooses to return half of what she receives. An untrustworthy manager’s strategy depends on $t$ and $P_t^D$. The strategy may be outlined as:

M'(i) If $\delta > (4/(l + h))^2$, then $d_1 = 1$.

M'(ii) The manager does not return anything in period 2 ($k_2^D = 0$).
M'(iii) If \( P^D_1 \geq 4/(l + h) \), the manager chooses to share her private information in period 2 and chooses to return half of what she receives in period 1 (\( d_2 = 1 \) and \( k^D_1 = e\lambda / 2 \)).

M'(iv) If \( P^D_1 < 4/(l + h) \), then with a probability \( S^D_1 = P^D_1 (1 - 4/(l + h)) / (4/(l + h))(1 - P^D_1) \), the manager returns half of what she receives in period 1. With probability \( 1 - S^D_1 \), the manager does not return anything in period 1. In the instance where the manager returns half of what she receives in period 1, she chooses to share her private information in period 2. That is, with a probability \( S^D_1 \), the manager chooses \( d_2 = 1 \) and \( k^D_1 = e\lambda / 2 \) and with a probability \( 1 - S^D_1 \), the manager chooses \( k^D_1 = 0 \). Note that if \( P^D_1 = 0 \), then \( S^D_1 = 0 \) and if \( P^D_1 = 4/(l + h) \), then \( S^D_1 = 1 \).

The manager always shares her private information in period 1 because not doing so gives out her type in the beginning of the game itself and is therefore, a dominated strategy (Point (M'(i) above). If the investor’s period 1 beliefs about the manager’s trustworthiness are at or above the investment threshold for period 2, then with certainty the manager mimics to be trustworthy (Point M'(iii) above). However, if the investor’s period 1 beliefs are below the threshold, then the manager’s best response is in mixed strategies. That is, with a certain probability (\( S^D_1 \)) strictly less than one, she mimics to be trustworthy (Point M'(iv) above).

The strategies and beliefs described above constitute a perfect Bayesian equilibrium. This follows from the equilibrium of the \( n \) period version of this game, which is derived in Lunawat (2013a). The investor’s and the manager’s best responses across the dividend-only and the dividend-and-information-sharing economies are summarized in Table 1.
<table>
<thead>
<tr>
<th>Case 1: $8/(l + h + 4) &lt; P_0^D = P_1^{ND} &lt; 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_1^D = m_1^{ND} = e$</td>
</tr>
<tr>
<td>$m_2^D = m_2^{ND} = e$</td>
</tr>
<tr>
<td>$k_1^D = e \lambda_i / 2$, $k_1^{ND} = e / 2$</td>
</tr>
<tr>
<td>$k_2^D = k_2^{ND} = 0$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 2: $4/(l + h) &lt; P_0^D = P_1^{ND} &lt; 8/(l + h + 4)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_1^D = m_1^{ND} = e$</td>
</tr>
<tr>
<td>$m_2^D = e$, Investor invests with probability $V_0^{ND}$</td>
</tr>
<tr>
<td>$k_1^D = e \lambda_i / 2$, Manager returns with probability $S_0^{ND}$</td>
</tr>
<tr>
<td>$k_2^D = k_2^{ND} = 0$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 3: $(4/(l + h))^2 &lt; P_0^D = P_1^{ND} &lt; 4/(l + h)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_1^D = m_1^{ND} = e$</td>
</tr>
<tr>
<td>Investor invests with probability $V_2^D$, Investor invests with probability $V_0^{ND}$</td>
</tr>
<tr>
<td>Manager returns with probability $S_1^D$, Manager returns with probability $S_0^{ND}$</td>
</tr>
<tr>
<td>$k_2^D = k_2^{ND} = 0$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 4: $0 &lt; P_0^D = P_1^{ND} &lt; (4/(l + h))^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_1^D = m_1^{ND} = 0$</td>
</tr>
<tr>
<td>$m_2^D = m_2^{ND} = 0$</td>
</tr>
<tr>
<td>$k_1^D = k_1^{ND} = 0$</td>
</tr>
<tr>
<td>$k_2^D = k_2^{ND} = 0$</td>
</tr>
</tbody>
</table>

Table 1 – The Investor’s and the Manager’s Best Responses across the Dividend-Only and the Dividend-and-Information-Sharing Economies
4.3 Return Probabilities Across the Economies. Consider the case where 

\[(4/(l+h))^2 < P_0^D = P_1^{ND} < 4/(l+h).\] 

Then, \[P_2^D = \max(4/(l+h), P_0^D) = 4/(l+h)\] and \[P_2^{ND} = \max(4/(l+h), P_1^{ND}/(2-P_1^{ND})) = 4/(l+h).\] This implies (using Bayesian updating) that \[S_1^D = S_1^{ND} = S_2^{ND} + S_3^{ND}.\] Therefore, \[S_1^D = S_1^{ND}\] and \[S_1^D \geq S_2^{ND}, S_1^D \geq S_3^{ND}.\] In other words, when the low multiplier \((\lambda_1 = l)\) occurs, then under the conditions specified above, the probability \(^3\) that \(k_1^0 = el/2\) is at least as high in the dividend-and-information-sharing economy as in the dividend-only economy \((S_1^D \geq S_3^{ND})\). When the high multiplier \((\lambda_1 = h)\) occurs, then under the conditions specified above, the probability that \(k_1^0 = eh/2\) is the same across the two economies \((S_1^D = S_1^{ND})\). But the probability that \(k_1^0 = el/2\) is higher in the dividend-only economy since \(k_1^D = el/2\) given \(\lambda_1 = h\) is a zero probability event in the dividend-and-information-sharing economy. Further, the probability that \(k_1^0 = eh/2\) in the dividend-only economy is not higher than the probability that \(k_1^0 = eh/2\) in the dividend-and-information-sharing economy \((S_1^D \geq S_2^{ND})\).

Note that while in the dividend-and-information-sharing economy, the probability of return \((S_1^D)\) is independent of the multiplier / state of the world, in the dividend-only economy, it \((S_0^{ND})\) depends on the multiplier. Specifically, in the dividend-only economy, the probability of return in a low-multiplier case may be lower than in the dividend-and-information-sharing economy \((S_1^D \geq S_3^{ND})\) while in a high multiplier case, it may be higher than in the dividend-and-information-sharing economy \((S_1^D \leq S_1^{ND} + S_2^{ND})\).

5. Investment and Return

5.1 Threshold for Maximum Investment. Consider the case where 

\[4/(l+h) < P_0^D = P_1^{ND} < 8/(l+h+4).\] 

That is, in the beginning of the game, the investor’s beliefs about the manager’s trustworthiness are above \(4/(l+h)\) but below \(8/(l+h+4)\).

---

^3 Note that \(k_1^0 = k_1^D\) in the dividend-and-information-sharing economy while \(k_1^0 = k_1^{ND}\) in the dividend-only economy.
Then, the investor will invest probability 1 in both periods 1 and 2 in the dividend-and-information-sharing economy since $P_2^D = P_1^D = P_0^D > 4/(l + h) > (4/(l + h))^2$. In the dividend-only economy, the investor will invest with probability 1 in period 1 since $P_1^{ND} > 4/(l + h) > (4/(l + h))^2$. But in period 2, if the investor invests, she will invest with probability $V_0^{ND} < 1$. Note that this possible investment in period 2 will occur only if the period 1 return is strictly positive so that $P_2^{ND} = \max\left(P_1^{ND}/(2 - P_1^{ND}), 4/(l + h)\right) = 4/(l + h)$.

Now consider the case where $P_0^D = P_1^{ND} > 8/(l + h + 4)$. Since $8/(l + h + 4) > 4/(l + h)$ and $P_2^{ND} > 4/(l + h)$, the investor will invest in periods 1 and 2 in both the dividend-and-information-sharing and the dividend-only economies. Note that in the dividend-and-information-sharing economy, the threshold for maximum investment to occur is $4/(l + h)$ while in the dividend-only economy, the threshold for maximum investment to occur is $8/(l + h + 4)$. Similarly, the thresholds for maximum return to occur are different. This difference in thresholds will lead to higher expected investment and expected return in dividend-and-information-sharing economy. This is derived formally next.

### 5.2 Investment Across the Two Economies.

Proposition 2 below states the conditions under which the expected investment in the dividend-and-information-sharing economy is at least as much as in the dividend-only economy and the conditions under which the expected investment in the dividend-and-information-sharing economy is greater than that in the dividend-only economy.

**Proposition 2.** In general, $E_i(m_1^D + m_2^D) \geq E_i(m_1^{ND} + m_2^{ND})$. Further $E_i(m_1^D + m_2^D) > E_i(m_1^{ND} + m_2^{ND})$ if and only if $4/(l + h) < P_0^D = P_1^{ND} \leq 8/(l + h + 4)$.

**Proof.** There are four cases to consider. First, $P_0^D = P_1^{ND} > 8/(l + h + 4)$. Then, $P_2^D = \max(4/(l + h), P_0^D) = P_0^D > 8/(l + h + 4) > 4/(l + h)$. And, $P_2^{ND} =$
max\left( \frac{4}{l+h}, P_{1}^{ND} \right) = P_{1}^{ND} > \frac{4}{l+h}. That is, in both the dividend-and-information-sharing and the dividend-only economies, the investor’s beliefs about the manager’s trustworthiness are above the investment threshold for period 2 and therefore, the investor invests with probability 1 in period 2. Further, 
P_{0}^{D} = P_{1}^{ND} > \frac{8}{l+h+4} > (\frac{4}{l+h})^{2}. That is, in both the economies, the investor’s beliefs about the manager’s trustworthiness are above the investment threshold for period 1 and therefore, the investor invests in period 1, too. This implies \( E_{r}(m_{1}^{D} + m_{2}^{D}) = E_{r}(m_{1}^{ND} + m_{2}^{ND}) = 2e. \)

Second, \( 4/(l+h) < P_{0}^{D} = P_{1}^{ND} \leq 8/(l+h+4). \) Then, \( P_{2}^{D} = \max(4/(l+h), P_{0}^{D}) \)
= \( P_{0}^{D} > 4/(l+h). \) But, \( P_{2}^{ND} = \max\left(4/(l+h), P_{1}^{ND} \left(2 - P_{1}^{ND}\right)\right) = 4/(l+h). \) Therefore, in the dividend-and-information-sharing economy, the investor invests with probability 1 in period 2, but in the dividend-only economy, investor invests with probability \( V_{0}^{ND} \) in period 2. Since \( P_{0}^{D} = P_{1}^{ND} > \frac{4}{l+h} > (\frac{4}{l+h})^{2}, \) investor invests in period 1 in both economies. This implies \( E_{r}(m_{1}^{D} + m_{2}^{D}) = 2e > E_{r}(m_{1}^{ND} + m_{2}^{ND}) = e + S_{0}^{ND} V_{0}^{ND} e. \)

Third, \( (4/(l+h))^{2} < P_{0}^{D} = P_{1}^{ND} \leq 4/(l+h). \) Then, \( P_{2}^{D} = \max(4/(l+h), P_{0}^{D}) = 4/(l+h)\)
and \( P_{2}^{ND} = \max\left(4/(l+h), P_{1}^{ND} \left(2 - P_{1}^{ND}\right)\right) = 4/(l+h). \) Therefore, in the dividend-and-information-sharing economy, the investor invests with probability \( V_{2}^{D} \) in period 2 while in the dividend-only economy, investor invests with probability \( V_{1}^{ND} \) if \( k_{1}^{ND} = eh/2 \) and with probability \( V_{2}^{ND} \) if \( k_{1}^{ND} = el/2. \) Further, since \( P_{0}^{D} = P_{1}^{ND} > (\frac{4}{l+h})^{2}, \) the investor invests in period 1 in both the economies. This implies \( E_{r}(m_{1}^{D} + m_{2}^{D}) = E(e + S_{1}^{D} V_{2}^{D} e) = E(e + eS_{1}^{D} \lambda_{1} / (l+h)) = e + eS_{1}^{D} / 2. \) And, \( E_{r}(m_{1}^{ND} + m_{2}^{ND}) = E(e + V_{0}^{ND} e) \)
= \( e + eV_{1}^{ND} \text{Prob}(k_{1}^{ND} = eh/2) + eV_{2}^{ND} \text{Prob}(k_{1}^{ND} = el/2), \) inserting the definition of \( V_{0}^{ND} \)
= \( e + eV_{1}^{ND} \text{Prob}(\lambda_{1} = h \text{ and } k_{1}^{ND} = eh/2) + eV_{2}^{ND} \text{Prob}(\lambda_{1} = h \text{ and } k_{1}^{ND} = el/2) \)
+ \( eV_{2}^{ND} \text{Prob}(\lambda_{1} = l \text{ and } k_{1}^{ND} = el/2), \) further expanding on the definition of \( V_{0}^{ND} \)
\[ e + eV_{1,ND}S_{1,ND}/2 + eV_{2,ND}S_{2,ND}/2 + eV_{3,ND}S_{3,ND}/2, \] from the definition of \( S_{1,ND}, S_{2,ND} \) and \( S_{3,ND} \)

\[ = e + (e/2)(V_{1,ND}S_{1,ND} + V_{2,ND}S_{2,ND} + V_{3,ND}S_{3,ND}) \]

\[ = e + (eS_{1,ND}/2)(V_{1,ND} + V_{2,ND}), \] since \( S_{1,ND} = S_{2,ND} + S_{3,ND} \)

\[ = e + eS_{1,ND}/2, \] since \( V_{1,ND} + V_{2,ND} = 1 \)

\[ = e + eS_{1,D}/2, \] since \( P_{1,D} = P_{1,ND} \) and \( P_{2,D} = P_{2,ND} \) implies \( S_{1,D} = S_{1,ND} \)

That is, \( E_1(m_{1,D} + m_{2,D}) = E_1(m_{1,ND} + m_{2,ND}) \).

Fourth, \( P_{0,D} = P_{1,ND} < (4/(l + h))^2 \). Then, \( P_{1,D} = P_{0,D} < (4/(l + h))^2 \). Therefore, the investor does not invest in period 1 in either economy. Since \( P_{2,D} = P_{1,D} < (4/(l + h))^2 < 4/(l + h) \) and \( P_{2,ND} = P_{1,ND} < (4/(l + h))^2 < 4/(l + h) \), therefore, the investor does not invest in period 2 also in either economy. This implies \( E_1(m_{1,D} + m_{2,D}) = E_1(m_{1,ND} + m_{2,ND}) = 0 \).

Q.E.D.

The proof of Proposition 2 hinges on the differences in period 2 investment across the two economies. Figure 3 summarizes this period 2 investment across the two economies under different priors. The four different regions or intervals in which the priors fall are shown on the number line. Then, for each of the four intervals, the table compares the expected total investment, the probability of investing in period 2 and the posterior probability across the two economies. The first column of the table states the case where investment does not occur in either economy. The second column states the case where investment occurs with a certain probability strictly less than one in both the economies. In the third column, the priors are at a point where investment occurs with certainty in the dividend-and-information-sharing economy but occurs with a probability strictly less than one in the dividend-only economy. The fourth column states the case where the priors are so high that investment occurs with certainty in both the economies.
Figure 3 – Comparing Investment Across the Dividend-only and the Dividend-and-Information-Sharing Economies

Notes to Figure 3: In the second row titled ‘Probability of Investing in Period 2’, the first number in each of the cells shows the probability with which the investor invests in period 2 in the dividend-and-information-sharing economy while the second number in each of the cells shows the probability with which the investor invests in period 2 in the dividend-only economy.

5.3 Return Across the Two Economies. The untrustworthy manager does not return anything in period 2 ($k_2^D = k_2^{ND} = 0$) and therefore, in comparing the return across the two economies, I will focus on period 1 return only. Proposition 3 below states the conditions under which the period 1 expected return from an untrustworthy manager in the dividend-and-information-sharing economy is at least as much as in the dividend-only economy and the conditions under which the period 1 expected return from an untrustworthy in the dividend-and-information-sharing economy is greater than that in the dividend-only economy.

Proposition 3. In general, $E_1(k_1^D) \geq E_1(k_1^{ND})$. Further, $E_1(k_1^D) > E_1(k_1^{ND})$ if and only if $P_0^D = P_1^{ND} \geq 4/(l+h)$.
Proof. There are three cases to consider. First, \( P_0^D = P_1^{ND} \geq 8/(l + h + 4) \). Since
\( P_1^D = P_0^D \geq 8/(l + h + 4) > 4/(l + h) \), therefore \( k_1^D = e\lambda_1 / 2 \) and since \( P_1^{ND} \geq 8/(l + h + 4) \), therefore \( k_1^{ND} = e\lambda / 2 \). This implies \( E_i(k_1^D) = E(e\lambda_1 / 2) = e(l + h) / 4 > E_i(k_1^{ND}) = e\lambda / 2 \).

Second, \( 4/(l + h) \leq P_0^D = P_1^{ND} < 8/(l + h + 4) \). Since \( P_1^D = P_0^D \geq 4/(l + h) \), therefore \( k_1^D = e\lambda_1 / 2 \). Since \( P_1^{ND} < 8/(l + h + 4) \), therefore manager returns with probability \( S_0^{ND} \). This implies \( E_i(k_1^D) = E(e\lambda_1 / 2) = e(l + h) / 4 \). And \( E_i(k_1^{ND}) = (e\lambda / 2)S_3^{ND} / 2 + (e\lambda / 2)S_2^{ND} / 2 = eS_1^{ND} (l + h) / 4 \). That is, \( E_i(k_1^D) > E_i(k_1^{ND}) \).

Third, \( P_0^D = P_1^{ND} < 4/(l + h) \). Since \( P_1^D = P_0^D < 4/(l + h) \), therefore manager returns with probability \( S_1^D \) and since \( P_1^{ND} < 4/(l + h) < 8/(l + h + 4) \), therefore manager returns with probability \( S_0^{ND} \). Further, \( P_1^D = P_1^{ND} \) and \( P_2^D = P_2^{ND} \) implies \( S_1^D = S_1^{ND} = S_2^{ND} + S_3^{ND} \). Then, \( E_i(k_1^D) = E(eS_1^D \lambda_1 / 2) = eS_1^D (l + h) / 4 \). And \( E_i(k_1^{ND}) = E(S_0^{ND} k_1^{ND}) = S_1^{ND} / 2 (eh / 2) + S_2^{ND} / 2 (e\lambda / 2) + S_3^{ND} / 2 (e\lambda / 2) = eS_1^{ND} (l + h) / 4 \). That is, \( E_i(k_1^D) = E_i(k_1^{ND}) \).

Q.E.D.

Figure 4 summarizes period 1 return across the two economies under different priors. The three different regions or intervals in which the priors fall are shown on the number line. Then, for each of the three intervals, the table compares the total expected return, the amount of return and the probability of return across the two economies. The first column describes the case where mixed strategy occurs in both the economies. The second column describes the case where pure strategy play occurs in the dividend-and-information-sharing economy but mixed strategy play occurs in the dividend-only economy. The third column states the case where pure strategy play occurs in both the economies. Note that in the dividend-and-information-sharing economy, pure strategy play implies that the rational / untrustworthy manager returns half of what she receives as dividend to the investor. But in the dividend-only economy, pure strategy play implies that the rational / untrustworthy manager returns a dividend consistent with the lower state of nature having occurred.
### Table 1

<table>
<thead>
<tr>
<th>Amount and Probability of Return</th>
<th>$\lambda_i = l$</th>
<th>$\lambda_i = h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k^D = \frac{e \lambda_i}{2} = \frac{el}{2}$</td>
<td>with probability $S^D_1$, $k^{ND} = \frac{el}{2}$ with probability $S^{ND}_1$</td>
<td>$k^D = \frac{e \lambda_i}{2} = \frac{eh}{2}$</td>
</tr>
<tr>
<td>$k^D = \frac{e \lambda_i}{2} = \frac{el}{2}$</td>
<td>with probability $S^D_1$, $k^{ND} = \frac{el}{2}$ with probability $S^{ND}_2$</td>
<td>$k^D = \frac{e \lambda_i}{2} = \frac{eh}{2}$</td>
</tr>
</tbody>
</table>

### Figure 4

**Comparing Period 1 Return from an Untrustworthy Manager Across the Dividend-only and the Dividend-and-Information-Sharing Economies**

#### 5.4 Example

Consider $l = 1$, $h = 5$ and $e = 10$. Then, the differences across the dividend-only and the dividend-and-information-sharing economies will occur in the region where prior probability is in the interval $(0.67, 1)$. That is, the differences across the two economies will occur when $0.67 < P^D_0 = P^{ND}_1 < 1$. Figure 5 summarizes these differences across the two economies. The number line divides the interval $(0.67, 1)$ into two sub-intervals, namely, $(0.67, 0.8)$ and $(0.8, 1)$. Then, for each of the two sub-intervals into which
the prior probability falls, the table compares the posterior probability, the expected total investment and the period 1 expected return from an untrustworthy manager across the two economies.

The first column of the table describes the case where the prior probability is in the sub-interval (0.67, 0.8). In the dividend-and-information-sharing economy, the posterior probability is equal to the prior probability. This probability is above the investment threshold for both periods 1 and 2. Since the investor’s beliefs about the manager’s trustworthiness are above the investment threshold for both periods 1 and 2, therefore, the investor invests with certainty in both periods. This makes the expected total investment equal 20. The untrustworthy manager returns half of what she receives in period 1 and this implies that the expected total return from an untrustworthy manager is 15.

Now, consider the dividend-only economy. The posterior belief gets revised downward to 0.67. This figure of 0.67 is exactly the investment threshold for period 2. Now, the investor’s period 1 beliefs about the manager’s trustworthiness are above the investment threshold for period 1 but the period 2 beliefs are exactly on the investment threshold. Therefore, the investor invests with certainty in period 1 but invests with a certain probability strictly less than 1 in period 2. This makes the expected total investment less than 20. The untrustworthy manager mimics to be a trustworthy type but such mimicking occurs with a certain probability strictly less than 1. This implies that the expected total return from an untrustworthy manager is less than 15. Note that while the equilibrium in the dividend-and-information-sharing economy is in pure strategies, the equilibrium in the dividend-only economy is in mixed strategies.

The second column of the table describes the case where the prior probability is in the sub-interval (0.8, 0.1). In the dividend-and-information-sharing economy, the posterior probability is equal to the prior probability. This probability is above the investment threshold for both periods 1 and 2; and therefore, the investor invests in both periods. This makes the expected total investment equal 20. The untrustworthy manager returns half of what she receives in period 1 and this implies that the expected total return from an untrustworthy manager is equal to 15. Now, consider the dividend-only economy. The
posterior belief gets revised downward. However, even with such downward revision, the posterior belief remains above the period 2 investment threshold of 0.67. Since the investor’s beliefs about the manager’s trustworthiness are above the investment threshold for both periods 1 and 2, therefore, the investor invests in both periods. This makes the expected total investment equal 20. In period 1, the untrustworthy manager returns a dividend consistent with the lower possible state of nature having occurred. This implies that the expected total return from an untrustworthy manager is 5. Note that while the equilibrium in both economies is in pure strategies, the period 1 expected return from an untrustworthy manager is lower in the dividend-only economy.

<table>
<thead>
<tr>
<th>Posterior Probability</th>
<th>( P^D_2 = P^D_1 = P^D_0 &gt; 0.67 )</th>
<th>( P^ND_2 = 0.67 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P^D_1 )</td>
<td>( P^D_0 &gt; 0.8 )</td>
<td>( P^ND_1 &gt; P^ND_2 &gt; 0.67 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Expected Total Investment</th>
<th>( E_i(m^D_1 + m^D_2) = 20 )</th>
<th>( E_i(m^ND_1 + m^ND_2) &lt; 20 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( E_i(m^D_1 + m^D_2) = 20 )</td>
<td>( E_i(m^ND_1 + m^ND_2) = 20 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Expected Total Return from an Untrustworthy Manager</th>
<th>( E_i(k^D_1) = 15 )</th>
<th>( E_i(k^ND_1) = 15 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( E_i(k^ND_1) &lt; 15 )</td>
<td>( E_i(k^ND_1) = 5 )</td>
</tr>
</tbody>
</table>

Figure 5 – Comparing Posterior Probability, Expected Total Investment and Period 1 Expected Return from an Untrustworthy Manager Across the Dividend-only and the Dividend-and-Information-Sharing Economies for the case where \( l = 1, h = 5, e = 10 \)
The key here is that when prior probability is high but not very high (more specifically, it is between 0.67 and 0.8), the posterior probability in the dividend-and-information-sharing economy remains above the investment threshold for period 2 while the posterior probability in the dividend-only economy is exactly at investment threshold for period 2. Therefore, the equilibrium is in pure strategies in the dividend-and information-sharing economy while it is in mixed strategies in the dividend-only economy. Pure strategy play implies investment with certainty while mixed strategy implies investment with a certain probability strictly less than one. This leads to maximum possible expected investment in the dividend-and-information-sharing economy while the expected investment in the dividend-only economy is strictly less than the maximum possible of 20. Similarly, pure strategy play in the dividend-and-information-sharing economy implies that first-period return consistent with the actual state of nature occurs with certainty while mixed strategy play in the dividend-and-information-sharing economy implies that first-period return consistent with the actual state of nature occurs with a certain probability strictly less than one. This leads to higher return in information-sharing.

When the probability is very high (more specifically, it is higher than 0.8), the posterior probability in both the economies remains above the investment threshold for period 2. Therefore, equilibrium is in pure strategies in both the economies. As such pure strategy play implies investment with certainty, expected investment in both economies is equal to the maximum possible of 20. Now, pure strategy play implies a first-period return consistent with the actual state of nature in the dividend-and-information-sharing economy while it implies a first-period return consistent with the lower state of nature in the dividend-only economy. This leads to lower expected return in the dividend-only economy. Figure 6 summarizes this reasoning.
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<table>
<thead>
<tr>
<th>Dividend-and-Information-Sharing Economy</th>
<th>Dividend-only Economy</th>
<th>Dividend-and-Information-Sharing Economy</th>
<th>Dividend-only Economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>• posterior probability &gt; investment threshold for period 2</td>
<td>• posterior probability = investment threshold for period 2</td>
<td>• posterior probability &gt; investment threshold for period 2</td>
<td>• posterior probability &gt; investment threshold for period 2</td>
</tr>
<tr>
<td>• equilibrium is in pure strategies</td>
<td>• equilibrium is in mixed strategies</td>
<td>• equilibrium is in pure strategies</td>
<td>• equilibrium is in pure strategies</td>
</tr>
<tr>
<td>• investment with certainty</td>
<td>• investment with certainty</td>
<td>• investment with certainty</td>
<td>• investment with certainty</td>
</tr>
<tr>
<td>• first-period return consistent with the actual state of nature occurs with certainty</td>
<td>• first-period return consistent with the actual state of nature occurs with certain probability strictly less than one</td>
<td>• first-period return consistent with the actual state of nature occurs with certainty</td>
<td>• first-period return consistent with the lower state of nature occurs with certainty</td>
</tr>
</tbody>
</table>

\[ P_0^D = P_1^{ND} \]

Figure 6 – Explaining Higher (or Equal) Expected Total Investment and Higher Period 1 Expected Return from an Untrustworthy Manager in the Dividend-and-Information-Sharing Economy for the case where \( l = 1, h = 5, e = 10 \)

6. Conclusion

This paper shows that information-sharing and the concomitant credibility allow for a greater region of indiscriminate mimicking of the trustworthy managers by the untrustworthy / rational ones. In this sense, the institution of information sharing provides reputation building opportunities over and above those provided by the institution of dividend payment. Characteristic features of the region of perfect mimicking are higher probability of investment and higher probability of high returns on investment. By allowing
for a greater region of perfect mimicking, information sharing results in greater investor-manager trust, higher investment, and higher return.

In accounting practice, there are several instances of managers providing information voluntarily with the evident purport of building a reputation amongst investors. This paper, while confirming such anecdotal behavior, provides the strategic basis for it. In this paper, the implications for practice emerge from how such voluntary information sharing provides greater opportunities at the building of trust and trustworthiness and how greater trust translates into greater investment and return.

There is increasing demand for enhancing the granularity of public disclosure. For example, there is debate about whether the results of the annual stress tests conducted by the Federal Reserve should be disclosed at an individual bank-level or at an aggregate industry-level. In a similar vein, there is debate about whether PCAOB (Public Company Accounting Oversight Board) inspection reports delineating quality control deficiencies should be disclosed at a firm-specific level or an industry level. Taken to the limit, firm-specific disclosures and granular information are analogous to the dividend-and-information-sharing economy while industry-level disclosures and coarse information are analogous to the dividend-only economy. This paper suggests that reputation and trust are important components that need to be considered in such debates.

Given the complicated nature of the equilibrium described in this paper, it is natural to ask whether people actually behave as predicted and whether such behavior results in higher investments and returns in the dividend-and-information-sharing economy compared to the dividend-only economy. I ran a human subjects experiment and found support for the equilibrium behavior predicted here (Lunawat (2013b)). After controlling for prior beliefs, I also found support for higher investment in information sharing.

This paper analyzes a setting where the manager always learns the true state of nature. An interesting extension could modify the setting to one where the manager does not learn the state of nature perfectly and instead receives only a noisy signal of the state of nature. This will allow an examination of how the reputation effects change as the precision of the
manager’s private information increases. Another possible extension would be one where the manager receives a perfect signal about the state of nature but the probability that she receives such a signal is strictly less than one so that the investor is unsure about the manager’s endowment of private information. This will allow an analysis of how strategic considerations about managerial reputation building interact with investor uncertainty about manager’s endowment of private information.

Future work could also look at modifying this setting to incorporate differences in managerial talent. One way to model differences in managerial talent could be to define a manager with higher ability as one that has a relatively higher probability of getting a higher state of nature. Another way to model differences in managerial talent could be the following: the manager receives a perfect signal about the state of nature but the probability that she receives such a signal is strictly less than one. Then, a manager with higher ability could be defined as one that has a relatively higher probability of being informed. Regardless of whether differences in managerial talent is modeled as differences in access to better technology or as differences in access to better information, it will shed light on interaction between managerial efforts at reputation building and differences in managerial talent.

References


Appendix

Proof of Proposition 1. There are two things to verify. First, the investor’s beliefs must be consistent with the manager’s strategy, in the sense that Bayes Rule holds wherever applicable. Second, starting from any information set in the game no player has a profitable deviation, that is, no player has an incentive to deviate.

I will first verify the criterion of Bayesian consistency. If the investor does not invest in period 1, then she does not learn anything about the manager’s type and therefore $P_{2}^{ND} = P_{1}^{ND}$. If $P_{1}^{ND} = 0$, the untrustworthy manager chooses $k_{1}^{ND} = 0$ and Bayes rule implies $P_{2}^{ND} = P_{1}^{ND}$.

If $P_{1}^{ND} \geq 8/(l + h + 4)$, the untrustworthy manager chooses $k_{1}^{ND} = el/2$ and Bayes Rule requires:

$$P_{2}^{ND} = \frac{\text{Prob (manager is trustworthy | manager chooses } k_{1}^{ND} = el/2)}{\text{Prob (manager chooses } k_{1}^{ND} = el/2)}$$

$$= \frac{\text{Prob (manager is trustworthy and chooses } k_{1}^{ND} = el/2)}{\text{Prob (manager chooses } k_{1}^{ND} = el/2)}$$

$$= \frac{1/2.P_{1}^{ND}}{1/2.P_{1}^{ND} + (1 - P_{1}^{ND})} = \frac{P_{1}^{ND}}{2 - P_{1}^{ND}}$$

$$= \frac{\theta}{2 - \theta}, \text{ since } P_{1}^{ND} = \theta.$$
Now, if $P_{1}^{ND} < 8/(l + h + 4)$ and $k_{1}^{ND} = el/2$, it could be $\lambda_{1} = l$ and the manager chose $k_{1}^{ND} = el/2$ with probability $S_{3}^{ND}$ or it could be $\lambda_{1} = h$ and the manager chose $k_{1}^{ND} = el/2$ with probability $S_{2}^{ND}$. Recall that if $P_{1}^{ND} \leq 4/(l + h)$ then $S_{2}^{ND} + S_{3}^{ND} = P_{1}^{ND} (l + h - 4)/(4(1 - P_{1}^{ND})).$ By Bayes Rule:

$$P_{2}^{ND} = \frac{Prob(\text{manager is trustworthy} \mid \text{manager chooses} \ k_{1}^{ND} = el/2)}{Prob(\text{manager chooses} \ k_{1}^{ND} = el/2 \mid \text{manager is trustworthy}) \cdot Prob(\text{manager is trustworthy}) + \frac{Prob(\text{manager chooses} \ k_{1}^{ND} = el/2 \mid \text{manager is untrustworthy}) \cdot Prob(\text{manager is untrustworthy})}{Prob(\text{manager is untrustworthy} \mid \text{manager chooses} \ k_{1}^{ND} = el/2) \cdot Prob(\text{manager is untrustworthy})} + \frac{Prob(\text{manager chooses} \ k_{1}^{ND} = el/2 \mid \text{manager is trustworthy}) \cdot Prob(\text{manager is trustworthy})}{Prob(\text{manager chooses} \ k_{1}^{ND} = el/2 \mid \text{manager is trustworthy}) \cdot Prob(\text{manager is trustworthy})} + \frac{Prob(\text{manager chooses} \ k_{1}^{ND} = el/2 \mid \text{manager is untrustworthy}) \cdot Prob(\text{manager is untrustworthy})}{Prob(\text{manager chooses} \ k_{1}^{ND} = el/2 \mid \text{manager is untrustworthy}) \cdot Prob(\text{manager is untrustworthy})}}$$

$$= \frac{1/2. P_{1}^{ND}}{1/2. P_{1}^{ND} + 1/2(1 - P_{1}^{ND}) S_{3}^{ND} + 1/2(1 - P_{1}^{ND}) S_{2}^{ND}}$$

$$= \frac{P_{1}^{ND}}{P_{1}^{ND} + (1 - P_{1}^{ND})(S_{3}^{ND} + S_{2}^{ND})} = 4(l + h)$$

Finally, if $P_{1}^{ND} < 8/(l + h + 4)$ and $\lambda_{1} = h$, the manager chooses $k_{1}^{ND} = eh/2$ with probability $S_{1}^{ND}$ and if $P_{1}^{ND} \leq 4/(l + h)$ then $S_{1}^{ND} = P_{1}^{ND} (l + h - 4)/(4(1 - P_{1}^{ND})).$ By Bayes Rule:

$$P_{2}^{ND} = \frac{Prob(\text{manager is trustworthy} \mid \text{manager chooses} \ k_{1}^{ND} = eh/2)}{Prob(\text{manager chooses} \ k_{1}^{ND} = eh/2 \mid \text{manager is trustworthy}) \cdot Prob(\text{manager is trustworthy}) + \frac{Prob(\text{manager chooses} \ k_{1}^{ND} = eh/2 \mid \text{manager is untrustworthy}) \cdot Prob(\text{manager is untrustworthy})}{Prob(\text{manager chooses} \ k_{1}^{ND} = eh/2 \mid \text{manager is untrustworthy}) \cdot Prob(\text{manager is untrustworthy})} + \frac{Prob(\text{manager chooses} \ k_{1}^{ND} = eh/2 \mid \text{manager is trustworthy}) \cdot Prob(\text{manager is trustworthy})}{Prob(\text{manager chooses} \ k_{1}^{ND} = eh/2 \mid \text{manager is trustworthy}) \cdot Prob(\text{manager is trustworthy})} + \frac{Prob(\text{manager chooses} \ k_{1}^{ND} = eh/2 \mid \text{manager is untrustworthy}) \cdot Prob(\text{manager is untrustworthy})}{Prob(\text{manager chooses} \ k_{1}^{ND} = eh/2 \mid \text{manager is untrustworthy}) \cdot Prob(\text{manager is untrustworthy})}}$$

$$= \frac{Prob(\text{manager chooses} \ k_{1}^{ND} = eh/2 \mid \text{manager is trustworthy}) \cdot Prob(\text{manager is trustworthy})}{Prob(\text{manager chooses} \ k_{1}^{ND} = eh/2 \mid \text{manager is trustworthy}) \cdot Prob(\text{manager is trustworthy}) + \frac{Prob(\text{manager chooses} \ k_{1}^{ND} = eh/2 \mid \text{manager is untrustworthy}) \cdot Prob(\text{manager is untrustworthy})}{Prob(\text{manager chooses} \ k_{1}^{ND} = eh/2 \mid \text{manager is untrustworthy}) \cdot Prob(\text{manager is untrustworthy})} + \frac{Prob(\text{manager chooses} \ k_{1}^{ND} = eh/2 \mid \text{manager is trustworthy}) \cdot Prob(\text{manager is trustworthy})}{Prob(\text{manager chooses} \ k_{1}^{ND} = eh/2 \mid \text{manager is trustworthy}) \cdot Prob(\text{manager is trustworthy})} + \frac{Prob(\text{manager chooses} \ k_{1}^{ND} = eh/2 \mid \text{manager is untrustworthy}) \cdot Prob(\text{manager is untrustworthy})}{Prob(\text{manager chooses} \ k_{1}^{ND} = eh/2 \mid \text{manager is untrustworthy}) \cdot Prob(\text{manager is untrustworthy})}}$$
\[
\frac{1/2.P_{1}^{ND}}{1/2.P_{1}^{ND} + 1/2.(1 - P_{1}^{ND})S_{1}^{ND}} = 4/(l + h)
\]

This satisfies the criterion of Bayesian consistency\(^4\).

I will now verify that the investor’s strategy is optimal. The investor’s payoff from not investing (choosing \(m_{i}^{ND} = 0\)) = \(e\). If \(P_{1}^{ND} \geq 8/(l + h + 4)\), the manager chooses \(k_{i}^{ND} = e/l / 2\). The investor’s expected payoff from investing (choosing \(m_{i}^{ND} = e\)) = \(eP_{1}^{ND}((l + h)/4) + e(1 - P_{1}^{ND})(l/2)\) is greater than the investor’s payoff from not investing (choosing \(m_{i}^{ND} = 0\)) = \(e\) if \(eP_{1}^{ND}((l + h)/4) + e(1 - P_{1}^{ND})(l/2) > e\). Simplifying yields the investor’s expected payoff from choosing \(m_{i}^{ND} = e\) is greater if \(P_{1}^{ND} > 2.(2 - l)/(h - l)\).

Since \(P_{1}^{ND} \geq 8/(l + h + 4)\), I will have shown the investor’s expected payoff from choosing \(m_{i}^{ND} = e\) is higher if I can show \(2.(2 - l)/(h - l) < 8/(l + h + 4)\). Simplifying, \(2l + 8 < l^2 + lh + 2h\). \(\text{RHS} = l(l + h) + 2h > 4l + 2h\), \(5 = 2l + 2(l + h) > 8 + 2l = \text{LHS}\). Note that \(P_{1}^{ND} \geq 8/(l + h + 4)\) implies \(P_{1}^{ND} > (4/(l + h))^2\) since \((4/(l + h))^2 < 8/(l + h + 4)\).

If \(P_{1}^{ND} < 8/(l + h + 4)\) and \(\lambda_{i} = l\), the manager chooses \(k_{i}^{ND} = e/l / 2\) with probability \(S_{3}^{ND}\) and chooses \(k_{i}^{ND} = 0\) with probability \(1 - S_{3}^{ND}\). If \(P_{1}^{ND} < 8/(l + h + 4)\) and \(\lambda_{i} = h\), the manager chooses \(k_{i}^{ND} = eh/2\) with probability \(S_{1}^{ND}\), chooses \(k_{i}^{ND} = e/l / 2\) with probability \(S_{2}^{ND}\) and chooses \(k_{i}^{ND} = 0\) with probability \(1 - S_{1}^{ND} - S_{2}^{ND}\). This implies the investor’s expected payoff from investing (\(m_{i}^{ND} = e\)) = \(eP_{1}^{ND}((l + h)/4) + e(1 - P_{1}^{ND})(l/4)(S_{2}^{ND} + S_{3}^{ND}) + e(1 - P_{1}^{ND})(h/4)S_{1}^{ND}\). Consider \(P_{1}^{ND} > 4/(l + h)\). Since \(S_{1}^{ND} = S_{2}^{ND} + S_{3}^{ND}\) therefore the expression for the investor’s expected payoff from investing simplifies to \(eP_{1}^{ND}((l + h)/4) + e(1 - P_{1}^{ND})(l + h)/4)S_{1}^{ND}\). The investor’s payoff from investing is higher if \(eP_{1}^{ND}((l + h)/4) + e(1 - P_{1}^{ND})(l + h)/4)S_{1}^{ND} > e\) or equivalently, \((l + h)/4)(P_{1}^{ND} + (1 - P_{1}^{ND})S_{1}^{ND}) > 1\). Since \(l + h > 4\) therefore LHS >

\(^4\)Note that there is some arbitrariness involved in belief updating when \(4/(l + h) < P_{1}^{ND}\).

\(^5\)Since \(l + h > 4\).
\[ 1 + \left(\frac{(l + h)/4}{(1 - P_1^{ND}) S_1^{ND}}\right) > 1 > \text{RHS}. \] That is, if \( P_1^{ND} > 4/(l + h) \) it is optimal for the investor to invest. Now, consider \( P_1^{ND} \leq 4/(l + h) \). Recall that \( S_1^{ND} = S_2^{ND} + S_3^{ND} = P_1^{ND} (l + h - 4) / 4(1 - P_1^{ND}) \). Inserting the expressions for \( S_1^{ND} \) and \( (S_2^{ND} + S_3^{ND}) \) and simplifying yields the investor’s expected payoff from investing = \( e P_1^{ND} \left(\frac{(l + h)/4}{2}\right) \). Now \( e P_1^{ND} \left(\frac{(l + h)/4}{2}\right) > e \) if \( P_1^{ND} > (4/(l + h))^2 \). That is, when \( P_1^{ND} \leq 4/(l + h) \), it is optimal for the investor to invest if \( P_1^{ND} > (4/(l + h))^2 \) and not to invest if \( P_1^{ND} < (4/(l + h))^2 \).

In period 2, the untrustworthy manager does not return anything \( (k_2^{ND} = 0) \). The investor’s expected payoff from investing \( (m_2^{ND} = e) = (e/2)(l + h)/2 P_{2^{ND}} \). Since \( (l + h) > 4 \), the investor’s expected payoff from choosing \( m_2^{ND} = e \) is greater if \( P_{2^{ND}} > 4/(l + h) \) and the investor’s expected payoff from choosing \( m_2^{ND} = 0 \) is greater if \( P_{2^{ND}} < 4/(l + h) \). At \( P_{2^{ND}} = 4/(l + h) \), the investor’s payoff from investing equals the investor’s payoff from not investing. That is, the investor is indifferent between choosing \( m_2^{ND} = e \) and choosing \( m_2^{ND} = 0 \). Therefore, the investor chooses to invest with a probability that makes the manager indifferent between the options available to her. That is when, \( k_1^{ND} = el/2 \), with probability \( V_2^{ND} = l/(l + h) \), the investor chooses to invest and with a probability \( 1 - V_2^{ND} \), the investor chooses not to invest. To verify the manager’s indifference at this point, note that if \( \lambda_1 = l \), the manager’s payoff from choosing \( k_1^{ND} = 0 \) is \( el \) and the manager’s expected payoff from choosing \( k_1^{ND} = el/2 \) is \( el/2 + ev_2^{ND} (l + h)/2 \). Since \( V_2^{ND} = l/(l + h) \), the manager’s expected payoff from choosing \( k_1^{ND} = 0 \) equals the manager’s payoff from choosing \( k_1^{ND} = el/2 \). If \( \lambda_1 = h \), the manager’s payoff from choosing \( k_1^{ND} = 0 \) is \( eh \) and the manager’s expected payoff from choosing \( k_1^{ND} = el/2 \) is \( eh - el/2 + ev_2^{ND} (l + h)/2 \). Since \( V_2^{ND} = l/(l + h) \), the manager’s expected payoff from choosing \( k_1^{ND} = 0 \) equals the manager’s payoff from choosing \( k_1^{ND} = el/2 \). Now when, \( k_1^{ND} = eh/2 \), with probability \( V_1^{ND} = h/(l + h) \), the investor chooses to invest and with a probability \( 1 - V_1^{ND} \), the investor chooses not to invest. To verify the manager’s indifference at this point, note that the manager’s payoff from choosing \( k_1^{ND} = 0 \) is \( eh \) and
the manager’s expected payoff from choosing $k_1^{ND} = eh/2$ is $eh/2 + eV_1^{ND}(l+h)/2$. Since $V_1^{ND} = h/(l+h)$, the manager’s expected payoff from choosing $k_1^{ND} = 0$ equals the manager’s payoff from choosing $k_1^{ND} = eh/2$.

Finally, I need to verify that the untrustworthy manager’s strategy is optimal. In period 2, the manager’s payoff from choosing $k_2^{ND} = 0$ is $e\lambda_2$ while the manager’s payoff from not choosing $k_2^{ND} = 0$ is strictly lower making it optimal for the manager to choose $k_2^{ND} = 0$.

If $P_1^{ND} \geq 8/(l+h+4)$, then the investor chooses $m_2^{ND} = e$ since $P_1^{ND} \geq 8/(l+h+4)$ implies $P_2^{ND} > 4/(l+h)$. Thus, the manager’s payoff from choosing $k_1^{ND} = 0$ is $e\lambda_1$ while the manager’s expected payoff from choosing $k_1^{ND} = el/2$ is $e\lambda_1 - el/2 + eE(\lambda_2) = e\lambda_1 - el/2 + e(l+h)/2$, making it optimal for the manager to choose $k_1^{ND} = el/2$.

If $P_1^{ND} < 8/(l+h+4)$, then by definition of $P_1^{ND}$, it must be that $P_2^{ND} = 4/(l+h)$. If $\lambda_1 = l$ and $k_1^{ND} = el/2$, then with probability $V_2^{ND} = l/(l+h)$, the investor chooses $m_2^{ND} = e$ and with probability $1-V_2^{ND}$, the investor chooses $m_2^{ND} = 0$. Thus, while the manager’s payoff from choosing $k_1^{ND} = 0$ is $el$, the manager’s expected payoff from choosing $k_1^{ND} = el/2$ is $el/2 + eE(\lambda_2)V_2^{ND} = el/2 + e((l+h)/2)l/(l+h)$. At this point, the manager is indifferent between choosing $k_1^{ND} = 0$ and choosing $k_1^{ND} = el/2$. Therefore, the manager chooses $k_1^{ND} = el/2$ with probability $S_3^{ND}$ and $k_1^{ND} = 0$ with complementary probability. If $P_1^{ND} < 8/(l+h+4)$, $\lambda_1 = h$ and $k_1^{ND} = el/2$, then also with probability $V_2^{ND} = l/(l+h)$, the investor chooses $m_2^{ND} = e$ and with probability $1-V_2^{ND}$, the investor chooses $m_2^{ND} = 0$. Again, while the manager’s payoff from choosing $k_1^{ND} = 0$ is $eh$, the manager’s expected payoff from choosing $k_1^{ND} = el/2$ is $eh - el/2 + eE(\lambda_2)V_2^{ND} = eh - el/2 + e((l+h)/2)l/(l+h)$. Therefore at this point too, the manager is indifferent between choosing $k_1^{ND} = 0$ and choosing $k_1^{ND} = el/2$. If $P_1^{ND} < 8/(l+h+4)$, $\lambda_1 = h$ and
\[ k_1^{ND} = eh/2, \] then with a probability \( V_1^{ND} = h/(l + h), \) the investor chooses \( m_2^{ND} = e \) and with a probability \( 1 - V_1^{ND}, \) the investor chooses \( m_2^{ND} = 0. \) The manager’s payoff from choosing \( k_1^{ND} = 0 \) is \( eh \) while the manager’s expected payoff from choosing \( k_1^{ND} = eh/2 \) is \( eh/2 + eE(\lambda_2) V_1^{ND} = eh/2 + e((l + h)/2)h/(l + h). \) This makes the manager indifferent between choosing \( k_1^{ND} = 0 \) and choosing \( k_1^{ND} = eh/2. \)

Recall if \( P_1^{ND} \leq 4/(l + h) \) then \( S_1^{ND} = S_2^{ND} + S_3^{ND} = P_1^{ND} (l + h - 4)/(4 - P_1^{ND}). \) When \( \lambda_1 = l, \) the manager chooses \( k_1^{ND} = el/2 \) with probability \( S_3^{ND} \) (and \( k_1^{ND} = 0 \) with complementary probability) that makes the investor indifferent between choosing \( m_2^{ND} = e \) and choosing \( m_2^{ND} = 0. \) When \( \lambda_1 = h, \) the manager chooses \( k_1^{ND} = el/2 \) with probability \( S_2^{ND} \) (\( k_1^{ND} = eh/2 \) with probability \( S_1^{ND} \) and \( k_1^{ND} = 0 \) with probability \( 1 - S_1^{ND} - S_2^{ND} \)) that makes the investor indifferent between choosing \( m_2^{ND} = e \) and choosing \( m_2^{ND} = 0. \)

To verify the investor’s indifference, note that the investor is indifferent between choosing \( m_2^{ND} = e \) and choosing \( m_2^{ND} = 0 \) if \( P_2^{ND} = 4/(l + h). \) When \( k_1^{ND} = el/2 \) Bayes Rule gives

\[
P_2^{ND} = P_1^{ND} (1 - P_1^{ND}) (S_3^{ND} + S_2^{ND}) / \left( P_1^{ND} + (1 - P_1^{ND}) (S_3^{ND} + S_2^{ND}) \right)
\]

Inserting \( S_2^{ND} + S_3^{ND} = P_1^{ND} (1 - 4/(l + h))(4/(l + h))(1 - P_1^{ND}) \) and simplifying, \( P_2^{ND} = 4/(l + h). \) When \( k_1^{ND} = eh/2 \) Bayes Rule gives

\[
P_2^{ND} = P_1^{ND} (1 - P_1^{ND}) S_1^{ND} / \left( P_1^{ND} + (1 - P_1^{ND}) S_1^{ND} \right)
\]

Inserting \( S_1^{ND} = P_1^{ND} (1 - 4/(l + h))(4/(l + h))(1 - P_1^{ND}) \) and simplifying, \( P_2^{ND} = 4/(l + h). \)

This completes the proof that the set of beliefs and strategies described earlier constitutes a perfect Bayesian equilibrium.