Effects of reconstruction layer profiles on atmospheric tomography in E-ELT AO systems

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ABSTRACT

In this paper, we will present new compression algorithms to determine optimal layer heights and turbulence weights for the tomographic reconstruction in wide field AO systems. Among other approaches, a new compression method based on discrete optimization of collecting atmospheric layers to subgroups is discussed. Furthermore, studies of the influence of layer heights and $c_n^2$-profiles on the reconstruction quality for different reconstruction algorithms and atmospheric profiles will be shown. Our comparison suggests that reconstructions on fewer atmospheric layers yield comparable quality with lower computational effort, if an appropriate compression algorithm is used. The numerical results were obtained on the ESO end-to-end simulation tool OCTOPUS.

Keywords: compression algorithm, atmospheric tomography, reconstruction algorithm

1. INTRODUCTION

Wide field of view Adaptive Optics (AO) systems, such as Multi-Conjugate AO (MCAO), Laser-Tomography AO (LTAO) or Multi-Object AO (MOAO) systems, are designed to correct atmospheric distortions in a large part of the sky. Such systems depend heavily on a good tomographic reconstruction. Therefore, an accurate atmospheric profile model is needed. However, in particular in the new generation of Extremely Large Telescopes (ELT), the reconstruction on many atmospheric layers represents a challenge to existing computational power. Therefore, there is a growing need for compression algorithms which accurately estimate a reduced number of reconstruction layers that still guarantee a good reconstruction quality. In order to design such algorithms, the influence of reconstruction profile variation on the tomographic quality is of interest\textsuperscript{1,2}. In this paper, we test a new compression algorithm, the optimal grouping, that yields superior results. Additionally, we present first results of a novel strategy for profile compression during the tomographic reconstruction\textsuperscript{3}. We study and compare the influence on the reconstruction quality for different compression algorithms in an ELT Multi-Object AO system for differently sized fields of view.

The paper is structured as follows: In Section 2, we present a new and describe existing compression algorithms used for our simulations. Section 3 reviews the reconstruction algorithms used for atmospheric tomography and explains the strategy of joint optimization of tomography and profile. Section 4 states the numerical results, and Section 5 gives the conclusions and outlook.

2. COMPRESSION ALGORITHMS

In atmospheric tomography, one aims at a good reconstruction quality in a large field of view. The choice of the reconstruction profile, i.e. the layer heights $h_l$ and $c_n^2$ weights $\gamma_l$, are crucial parameters. Obviously, using all layers of the atmospheric model for reconstruction would be best. However, due to time limited computational resources, there is a general interest in using only a few reconstruction layers without loosing too much quality. A classical approach to choose a downsampled profile are compression algorithms. In the following, we compare a new compression algorithm – the method of “optimal grouping” – with several existing ones. Let us shortly describe all methods used in the results section 4. In the following, $N$ denotes the number of layers at heights $H_1, \ldots, H_N$ in the atmosphere model and $L$ the number of reconstruction layers at heights $h_1, \ldots, h_L$.
2.1 Equidistant Grouping

This is a very simple method of downsampling. The new altitudes are set at equidistant heights (between 0 and the altitude of the highest layer), and the $c_n^2$ values are set to one half at the ground layer and identically on all other layers, i.e.

$$h_l = H_1 + (l-1) \cdot \left( \frac{H_N - H_1}{L - 1} \right), \quad \text{and} \quad \gamma_1 = 0.5, \quad \gamma_l = \frac{0.5}{L - 1}, \quad l \geq 2.$$  

2.2 Exponential Grouping

This compression algorithm has been introduced in. The reconstruction layer heights are chosen as an exponential sequence, i.e.

$$h_1 = 0, \quad h_{l>1} = H_1 \cdot q^{l-2} \quad \text{with} \quad q = \left( \frac{H_N}{H_1} \right)^{1/(L-2)}$$

Around each height a Voronoi-interval is defined

$$I_1 = \left[ 0, \frac{h_1 + h_2}{2} \right], \quad I_{1<l<L} = \left[ \frac{h_{l-1} + h_l}{2}, \frac{h_l + h_{l+1}}{2} \right], \quad I_L = \left[ \frac{h_{L-1} + h_L}{2}, 2h_L \right]$$

and the $c_n^2$ weights $\gamma_l$ are calculated as

$$\gamma_l = \sum_{m: H_m \in I_l} C_n^2(H_m)$$

2.3 Mean-weighted compression

This method has been introduced by T. Fusco. Hereby, the original profile is cut into equidistant slabs $s_1, \ldots, s_L$. The new heights are calculated as mean-weighted altitudes in each slab, and the new $c_n^2$-values are simply integrated over each slab, i.e.

$$h_l = \frac{\int_{s_l} (C_n^2(H))^5/3 dH}{\int_{s_l} C_n^2(H) dH}, \quad \text{and} \quad \gamma_l = \int_{s_l} C_n^2(H) dH.$$  

2.4 Optimal Grouping

For a given atmosphere profile $A = (H_1, H_2, \ldots, H_N; C_n^2(H_1), C_n^2(H_2), \ldots, C_n^2(H_N))$ and desired number $L$ of reconstruction layers, an index grouping $G_1, G_2, \ldots, G_L$ and a reconstruction layer profile $R = (h_1, h_2, \ldots, h_L; \gamma_1, \gamma_2, \ldots, \gamma_L)$ are found such that the target function $F(A, R)$ is minimized.

$$F(A, R) = \sum_{j=1}^{L} \sum_{k \in G_j} C_n^2(H_k) \cdot |H_k - h_j|$$

$F$ estimates the unavoidable error of any tomographic reconstruction algorithm, caused by the shifts due to $H_k \neq h_j$. Post-processing: $\gamma_j := \sum_{k \in G_j} C_n^2(H_k)$. (Publication of the underlying theory and algorithmic details is in progress)
2.5 Comparison of compression methods

In Figure 1, we compare the compression from 40 to 9 layers for all 4 methods. We can observe that the optimal grouping and the mean-weighted method approximate the real profile. In the choice of layer heights, the latter is limited by the fact that the slabs are equidistantly chosen. The exponential grouping tends to put more layers near the ground by definition, such that very few reconstruction layers at high altitudes are chosen.

![Compressed profile for 9 layers](image)

Figure 1. Comparison of compression methods: 40 layer to 9 layer.

3. ATMOSPHERIC TOMOGRAPHY

Atmospheric tomography comprises the problem of reconstructing turbulent atmospheric layers from data. Typically, there are two approaches how to reconstruct a layered atmosphere $\Phi = (\Phi^{(1)}, \ldots, \Phi^{(L)})$ from (Shack-Hartmann) wavefront sensor data $s = (s_g^x, s_g^y)_{g=1}^G$, from $G$ guide stars. Commonly, one uses an algorithm that yields $\Phi$ directly from sensor data (one could call this a 2-step method, as the DM shape is calculated in 2 steps). A recent approach is to use an intermediate step and reconstruct incoming wavefronts from sensor data before calculating the turbulent atmosphere (called 3-step method). For a comparison of the two approaches, see Ramlau et al.

3.1 Reconstruction of the atmosphere

In the following, the majority of the results have been obtained with the 3-step method using a Gradient-based iteration to reconstruct turbulent layers from incoming wavefronts $\varphi = (\varphi_1, \ldots, \varphi_G)$. The first step – the wavefront reconstruction – has been performed with the CuReD method. The second step is also referred to as atmospheric tomography (on wavefronts), and comprises the solution of a system of equations

$$A\Phi = \varphi,$$

where $A = (A_1, \ldots, A_G)$ are the projection operators in $G$ guide star directions. The Gradient-based method solves (1) by means of minimization of the least squares functional

$$\Phi^{rec} = \arg\min_{\Phi} \|A\Phi - \varphi\|^2,$$

with a Gradient iteration. The third step in the 3-step approach, the DM fitting, is done by projection of the reconstructed atmosphere onto the DM(s).

Additional verification of the numerical results has been obtained with the FEWHA and FrIM. The Finite Element-Wavelet Hybrid Algorithm (FEWHA) relies on the (preconditioned) CG method to solve the maximum a posteriori estimation with dual domain composition. This approach includes the turbulence...
statistics of the atmosphere $C_\Phi$, which is efficiently approximated by a diagonal matrix $D$\cite{6,16,17}. Additionally, the noise statistics for spot elongation $C_\eta$\cite{19,20} are taken into account. The Fractal Iterative Method (FrIM)\cite{7,18} is a CG based method with sparse factorization of the inverse turbulence statistics $C_\Phi^{-1}$. Similar to FEWHA, it solves the maximum a posteriori estimate with an efficient diagonal preconditioner.

All three methods are fast iterative algorithms which perform linearly in $O(n)$ and are well parallelizable. In the three step approach, the wavefront reconstructor CuReD is a direct solver which is also pipelineable.

### 3.2 Joint optimization of tomography and profile

In addition to the compression algorithms in Section 2, we compare some preliminary results for a novel analytical method of joint optimization of tomographic reconstruction and reconstruction profile, described in Helin et al.\cite{3} In the tomography step (of the 3-step approach), the turbulent atmosphere and the optimal reconstruction layer profile are calculated simultaneously via solving

$$(\Phi_{rec}, C_{n_{rec}}) = \arg\!\min_{\Phi, C_n} \left( \| A\Phi - \varphi \|^2 + \alpha \| C_\Phi^{-1/2} \Phi \|^2 \right).$$

The minimizer is obtained via a gradient-based iteration with shrinkage and an analytic update step for the $c_n^2$ values. This yields a sparse reconstruction profile as many (depending on the choice of $\alpha$) $c_n^2$ values are set to zero. As it is a costly method, one would prefer to perform the analytical $c_n^2$ update only at times, and offline, to adapt the reconstruction profile for changing atmospheric conditions. One way to use this analytical model reduction, is to run one timestep of joint optimization such that $L$ layers have non-zero weights $\gamma > 0$. Then, one can utilize the downsampled atmosphere as fixed reconstruction profile for the reconstruction method.

### 4. NUMERICAL RESULTS

In the following, we show the simulation results obtained on OCTOPUS, the ESO end-to-end simulation tool. To test a variety of settings in a large field of view, we chose a Multi-Object AO system with one DM in zenith direction. The simulation and reconstruction parameters are listed in Table 1 and the schematic guide star asterism can be seen in Figure 2. As evaluation criteria we use long exposure (LE) Strehl in K band (for a wavelength of 2200nm) in the center direction.

![Figure 2. Guide star asterism](image)

![Table 1. Simulation and reconstruction parameters](table)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Telescope</td>
<td>42m E-ELT, 28% central obstruction</td>
</tr>
<tr>
<td>Atm.profiles</td>
<td>9-, 35-, 40-layer, $r_0 = 0.129$</td>
</tr>
<tr>
<td>MOAO</td>
<td>1 DM in open loop, $85 \times 85$, zenith direction</td>
</tr>
<tr>
<td>SH-WFS</td>
<td>$84 \times 84$, w/o spot elongation, highflux</td>
</tr>
<tr>
<td>FoV Variation</td>
<td>LGS radius: $0.75'$, $1.75'$, $2.75'$, $3.75'$</td>
</tr>
<tr>
<td></td>
<td>NGS radius: $1'$, $7/3'$, $11/3'$, $5'$</td>
</tr>
<tr>
<td>Rec.layers</td>
<td>$L = 6, 9, 14, 20, 25, 30, 35, 40$</td>
</tr>
<tr>
<td>Rec.method</td>
<td>Gradient, FEWHA, FrIM</td>
</tr>
</tbody>
</table>

All reconstructors are run with a fixed set of parameters for different numbers of reconstruction profiles. In particular, no tuning (gain, regularization parameters, number of iterations) was performed.
4.1 Variation of reconstruction profile

In Figure 3, we compare the four compression algorithms – optimal, exponential, equidistant grouping and mean-weighted compression – and the joint optimization. For the latter, we run one timestep of joint optimization such that $L$ layers have non-zero weights $\gamma_l$. Then, we utilize the downsampled atmosphere as input for the standard Gradient method with the fixed new profile for all time steps. For 40 reconstruction layers, the original (simulated) profile has been used for reconstruction, thus, the difference in quality between 35 and 40 reconstruction layers is most pronounced for the equidistant grouping.

Note that the reconstructors yield slightly different levels of quality only due to tuning issues, their behaviour for the different compression algorithms is similar.

![Figure 3. Variation of reconstruction profile: GS separation 0.75 arcmin (left), GS separation 3.75 arcmin (right).](image)

For the optimal and exponential grouping as well as for the mean-weighted compression, we additionally varied the $\gamma_l$-values, see Figure 4. A very simple scenario is to chose $\gamma_1 = 1/L$ equally for all layers. A better choice is the ground layer enforcing variant $\gamma_1 = 0$ and $\gamma_l = 0.5/(L-1)$ for $l \geq 2$.

![Figure 4. Additional variation of $c^2_n$-profile along with reconstruction profile: GS separation 0.75 arcmin (left), GS separation 3.75 arcmin (right). Reconstruction method: Gradient.](image)

4.2 Variation of field of view

So far, we observed two cases of guide star separation, a very large one with a circle of radius 3.75 (LGS) and 5 (NGS) arcmin and a very narrow one with a radius of 0.75 (LGS) and 1 (NGS) arcmin. Figure 5 shows that
in between the reconstruction quality drops more than linearly, in particular for the equidistant and exponential grouping which generally yield lower quality.

Figure 5. Variation of FoV: LGS radius vs. LE strehl, 6 reconstruction layers (left), 25 reconstruction layers (right).

5. CONCLUSION AND OUTLOOK

The above results allow several observations: First, the choice of the reconstruction profile has similar influence on all reconstructors, although they are of completely different nature. Thus, we can conclude that all our observations are independent of the choice of the particular reconstruction method. Second, the choice of the compression method has a strong influence on the reconstruction quality. With a good guess for the reconstruction profile, few layers suffice, even for large guide star separations. However, if one estimates the compressed profile badly, quality drops dramatically, in particular for large guide star separations. Third, although the $c_n^2$-profile has a significant influence on the reconstruction quality, the layer altitudes are much more important.

So far, all results have been obtained with the 40-layer model. We expect that the underlying simulated profile influences the level of reconstruction quality, thus, other atmospheric models have to be tested in the future. Additionally, further investigation of algorithms such as the optimal grouping (a publication of the underlying theory and algorithmic details of optimization is in preparation) and the joint optimization is planned. Moreover, compression algorithms are not only useful in the derivation of an optimal reconstruction profile in atmospheric tomography but can also be used to estimate the optimal DM conjugation heights in Multi-Conjugate AO.

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REFERENCES


