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Prior, Gideon

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Optimal Control Of Continuous Time Systems With Quantized Actuators

A dissertation submitted in partial satisfaction of the requirements for the degree Doctor of Philosophy

in

Electrical Engineering (Intelligent Systems, Robotics and Control)

by

Gideon Prior

Committee in charge:

Professor Miroslav Krstic, Chair
Professor Massimo Francescetti, Co-Chair
Professor Robert Bitmead
Professor Raymond de Callafon
Professor Vitaliy Lomakin
Professor George Papen

2013
The dissertation of Gideon Prior is approved, and it is acceptable in quality and form for publication on microfilm and electronically:

Co-Chair

Chair

University of California, San Diego

2013
DEDICATION

For my wife Shannon, whose patience and support appear to grow without bound, and for my children Sophia, Aidan and Jack for removing failure from the set of realizable choices.
EPIGRAPH

“When the only tool you own is a hammer, every problem begins to resemble a nail.”
– Abraham Maslow

"I have never let my schooling interfere with my education.”
– Mark Twain
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VITA

2003-2004  Research Assistant, Computer Vision Group  
            Jet Propulsion Laboratory, Pasadena California

2006   Engineering Internship, Unmanned Systems Group  
            Space and Naval Warfare Systems Center, San Diego California

2006-2013  Research and Development Engineer, Power Electronics and Controls  
            General Atomics, San Diego California

2007   B. S. in Electrical and Computer Engineering *cum laude*  
            University of California, San Diego

2009   M. S. in Electrical Engineering  
            (Intelligent Systems, Robotics and Control)  
            University of California, San Diego

2007-2013  Graduate Teaching Assistant, University of California, San Diego  
            Introduction to Power Electronics (ECE 188)  
            Introduction to Active Circuit Design (ECE 102)  
            Linear Systems Fundamentals (ECE 101)  
            Introduction to Analog Design (ECE 35)  
            Introduction to Digital Design (ECE 25)  
            Experimental Techniques (MAE 170)  
            Linear Control (MAE 143B)  
            Linear Circuits (MAE 140)

2013   Ph. D. in Electrical and Computer Engineering  
            (Intelligent Systems, Robotics and Control)  
            University of California, San Diego

PUBLICATIONS


ABSTRACT OF THE DISSERTATION

Optimal Control Of Continuous Time Systems With Quantized Actuators

by

Gideon Prior

Doctor of Philosophy in Electrical Engineering
(Intelligent Systems, Robotics and Control)

University of California, San Diego, 2013

Professor Miroslav Krstic, Chair
Professor Massimo Francescetti, Co-Chair

Continuous time plants acted upon by quantized actuators are an important class of systems and arise in numerous control applications in the fields of power electronics and motor drives. This thesis presents a new method for stabilizing these systems by selecting input sequences through the evaluation of an energy related control Lyapunov function. Through this approach multiple performance objectives can be targeted including switching frequency reduction and minimization of torque ripple while maintaining fast transient response and providing stability guarantees. Additionally, by abandoning the heuristics used in popular state of the art methods in favor of the proposed stability based approach, non-optimal inputs can be quickly removed from consideration with little computational investment which provides a significant benefit to real time model predictive control strategies. Due to the relatively simple implementa-
tion requirements and the flexibility in setting control objectives, the proposed control theory is potentially well suited for a broad range of applications outside of power electronics and motor drives.
Chapter 1

Introduction

While the majority of this thesis is written for an audience with a control theoretic back-
ground, the fundamental characteristics and limitations of the systems under study as well as
the methods used to steer the systems to where we want them to go are fairly straightforward.
It is therefore the purpose of this chapter to introduce a non-specialized reader to the notion of
continuous systems with quantized inputs through several, hopefully illuminating examples. Ad-
ditionally, energy conversion, electric machinery and the increasingly important role of power
electronics will be discussed including a high level description of various control policies that
have been ultimately designed with the far-reaching purpose of improving our quality of life.
Though later chapters present somewhat complicated analysis and design, the theme linking this
body of work together is simple and is representative of a process that human beings experience
every day.

1.1 Quantized Input Systems

1.1.1 Classes of systems encountered in control engineering

Typical dynamic systems that we seek to control are composed of continuous valued
state variables that are steered to some desired state through the application of force. While this
actuation force that is applied has dynamics of its own, we usually expect a 'good' actuator to
be capable of producing an arbitrary, precise force upon the system (within its mechanical lim-
itations) to enable the controlled system to achieve the desired output. Examples of continuous
systems acted upon by continuous actuators are abound. Some common examples include a
cruise control system regulating a vehicles desired speed by actuating the gas pedal, or a con-
trol system in a drug manufacturing plant that regulates the rate of a reaction by adjusting the electrical current flowing through heating coils in the reaction vessel.

Many systems of interest to control theorists are fundamentally discrete but can be well approximated as continuous systems. Consider as an example a financial control system with the state of the system represented as the net worth of some portfolio and the investment decisions made over time acting as an input sequence. While the value of the state and inputs can not change continuously, they can change in increments that are very small compared to their current values. Furthermore the range of values that are reachable is very large depending on the investor. Many other, perhaps more common discrete control systems with a wide range of potential inputs and outputs are continuous systems controlled by microprocessors whose finite bit-depth analog to digital converters discretize sensor readings, state estimates and actuator commands.

There is another class of systems however in which continuous valued state variables are acted on by a small, finite set of possible control actions. An iconic example of this type of system is the home thermostat based temperature control system. While the user is allowed to select from a continuum of desired settings, the actual furnace providing the control action is either on or off. The objective of the control system is then to determine when to turn the furnace on and when to turn it off so that the mean temperature over time converges to the set point. However, regulating the mean temperature is not enough if we ever expect to be comfortable in our home.

Imagine our living room is a frigid $0^\circ C$ when the heater is left off for a while and assume we have a furnace which can bring the temperature up to a sweltering $50^\circ C$ when left on. If the controller turns the heat on for a full day and then turns it off the following day, the mean temperature would be close to the $25^\circ C$ we set as a reference temperature despite the miserable instantaneous temperatures we would experience. In this sense, it can be seen that there is a utility to controlling not just the mean error of the system, but also its variance about the mean. In our thermostat example this can be achieved by switching the actuator between on and off more frequently. Due to the inertia of the system, which in our case stems from the air in the room not being able to instantaneously heat up or cool down, high frequency switching will help the controller produce a more consistent temperature with smaller deviations about the mean set point.

However, as nothing comes for free there are costs associated with every switching event. One type of cost is associated with the lifetime of the actuator. Switching the furnace on and off more frequently puts more thermal stress on the individual components which can
lead to premature failure. Other costs associated with switching losses are due to start up as
the furnace operates at reduced efficiency until the full operating temperature is reached. In
addition there are strict limitations due to actuator dynamics that prevent switching arbitrarily
fast. These dynamics are characterized by the time delay between sending the ‘on’ command
and the moment the furnace actually delivers heat.

What is perhaps the most compelling feature of the example discussed above is that the
control system is attempting to achieve a complicated objective with a binary decision. At every
instant the tasks of rapidly regulating the mean temperature of a room with minimal variance
about the set point, incurring minimal wear on the furnace and achieving maximal operating ef-
ficiency all while respecting the mechanical limitations of the system all comes down to making
a decision on whether to open or close a switch.

1.1.2 Quantized input control processes encountered in daily life

While the above example describes a specific type of quantized input control system
typically designed by engineers, there are quantized input processes that we all encounter on a
daily basis which are controlled by the decisions we make. Consider the task of driving your
car from one location to another. If you happen to be in a rural, undeveloped part of the country
and the only thing between you and your destination is an open field, the best course of action
in terms of minimizing driving time would likely be to simply drive in a straight line. However,
most of the time we are forced to remain on a fixed path determined by the network of streets and
freeways available to us while obeying the traffic laws specific to each path segment. While our
location evolves continuously as we drive, the path we take and consequently the time we arrive
at our destination is determined by the sequence of choices we make at each intersection. Though
some choices such as deciding to take the freeway may get us in the vicinity of where we want
to go in less time compared to taking city streets, the reduced number of choices on a freeway
where drivers can enter and exit makes it likely that you will over or undershoot your target.
None the less, we often decide to take faster paths even though they can lead us temporarily
out of the way if we believe the overall time savings will work out in our favor. When solving
this problem, we take into account much more than just the speed limits of the potential paths.
We predict traffic patterns, road conditions, proximity to gas stations and even the beauty of the
surrounding scenery when we are optimizing the path we wish to take. For a computer, solving
these types of optimization problems requires a sufficiently rich model of the decision space, a
cost function to assign a value to each candidate decision and an explicit routine for finding a
solution. Human beings however are very adept at finding near optimal solutions in a very small amount of time, particularly when the total number of options can be pruned down to just a few choices.

### 1.1.3 Decisions with limited choices

Being a thesis on advancements in control theory and engineering, relating an algorithmic decision process occurring in a microprocessor to patterns in human decision making may seem off topic if not gratuitous. However, this thesis has also been written to partially meet the requirements for a doctorate in philosophy so it seems reasonable that some attempt should be made to briefly generalize the results of this exposition to some higher level philosophical principles which we can all relate to.

An active area of research among sociologists is the study of how the human decision making process changes as more choices become available. Most of us agree that freedom is good, and therefore maximizing freedom by maximizing choice must also be good. However, in his book ”The Paradox of Choice: Why More Is Less” philosopher Barry Schwartz asserts that as people are offered more choices their satisfaction from the choices they make decreases, and in some cases can lead to ‘decision paralysis’ where no decision is made out of fear of making the wrong choice [67]. In an analogous fashion, many types of control algorithms also suffer when given an excessive amount of choices to make even if the number of options at any one point in time is small.

Consider the instructions required for a computer to find the best route to take for your morning commute. An exhaustive search of all the options would require determining the cost of every possible route, including those that take you through small residential streets and back alley ways. Though the causes are different, decision paralysis can still result when the size of the search space overwhelms the computational capacity of the computer. Human beings deal with this by throwing out options that we predetermine to be unlikely to achieve our goals without fully examining the associated cost, allowing us to only consider options that we preconceive to be contenders for a winning choice. There are similar techniques used in computer science and in predictive control that allows the search for a solution to be made tractable by limiting the available options. A novel method for pruning a decision tree for this purpose is the subject of Chapter 4.
Figure 1.1: Some common applications that are enabled by power electronics.

1.2 The Power Electronics Revolution

1.2.1 Power electronic devices and their applications

As opposed to common electrical circuits whose purpose is to manipulate and transmit information in the form of electrical signals, power electronic circuits are composed of devices capable of manipulating the flow of electrical power. Though somewhat less salient to the general public than their low power counterparts, power electronic systems are truly ubiquitous and are fundamentally necessary to support modern civilization. Common power electronic based circuits include DC to DC converters, DC to AC inverters, AC to DC rectifiers and AC to AC cycloconverters and matrix converters. Each of these circuit topologies are enabling technologies
for energy conversion and control with applications to renewable energy sources, variable speed
motor drives, uninterruptable power supplies, battery chargers, HVDC transmission systems,
pulsed power, electric vehicles, grid stabilization, directed energy weapons, large scale data
centers, mass public transportation and radar systems just to name a few.

Beginning in 1902 with the invention of the mercury arc rectifier by Peter Cooper Hewitt,
our ability to transform and control electrical energy has been steadily escalating and over the
past two decades has improved to such a degree many researchers suggest that we are in the
midst of a power electronics revolution whose effects will have a similar impact on society as
the information age. A commonality between power electronic devices including insulated gate
bipolar transistors (IGBTs), power MOSFETs, silicon controlled rectifiers (SCRs), gate turn off
thyristors (GTOs) and integrated gate commutated thyristors (IGCTs) is that they all have the
ability to act as an electrical switch. In the ideal sense, a switch does not consume any power
when it is open as no electrical current can flow through it. Similarly, when it is closed it also
does not consume any power as the resistance of an ideal closed switch is zero. Though no
ideal power electronic switches exist, modern devices are nearly ideal and allow the design of
energy conversion circuits that commonly exceed ninety five percent efficiency. Through high
speed switching between conducting and non-conducting states highly efficient amplifiers can
also be constructed which rely on the inductance of the load to filter out the generated pulses to
produce an averaged waveform of the desired amplitude. Compared to analog amplifiers who’s
instantaneous output values are in proportion to their input, power electronic amplifiers do not
operate as continuously variable resistors in the partially 'on' mode which consumes power in
the device.

In spite of the excellent efficiencies attainable by operating these devices only in fully
on or fully off modes, the control of systems incorporating these devices becomes less intuitive
compared to linear amplifier based systems. Linear amplifiers are capable of producing any output
within their operating range by scaling a continuously varying input signal. In the thermostat
example given previously, if your home furnace operated in this fashion any desired temperature
could be reached through continuous adjustments to the amount of fuel being consumed. Power
electronic systems on the other hand operate in a very similar way to the thermostats we are all
familiar with, requiring a sequence of switching instants to be determined in order to regulate
the room temperature.
1.2.2 Power electronics and the actuation of electric machinery

One of the primary reasons for the unprecedented growth and sustained academic interest in the field of power electronics is the high dynamic performance and efficient operation of variable speed motors made possible by DC to AC power inverters. Electric motors are omnipresent machines that make modern society possible. They are likely the most common device in every household and provide the mechanical power to operate refrigerators, dish washers, garage doors, washers and dryers, heating and air conditioning systems, blenders, fans, microwave ovens and hard drives. However their utility goes far beyond household applications. Electric motors are the enabling technology for almost all industry including textile, manufacturing, agriculture, mining, oil and gas, iron and steel production and just about any other industry imaginable. Along with continuing pressure from industry, modern high efficiency inverters, advanced control algorithms and the availability of low cost high speed digital signal processors have contributed to a recent boom in motor system improvements despite being a very mature field well over 150 years old. To put a ‘value to the world’ perspective on the continuing developments in this field, consider the staggering findings by the U.S. Department of Energy and the Electrical Power Research Institute (EPRI) who estimate that sixty to sixty five percent of all the power generated in the United State is consumed by an electric motor [30]. In some industries including mining, oil and gas, water supply, irrigation and sewage, motor systems consume over 90 percent of the total supplied electrical energy [82]. With this in mind, even incremental efficiency improvements have the potential for a significant impact on the world.

1.3 Energy Shaping and Control Lyapunov Functions

It is well known that any unforced dynamic mechanical system will evolve towards the minimum energy solution of the governing differential equations. Using the argument that energy is conserved when ignoring heat losses and strictly decreasing otherwise, it is possible to design system models and analyze their stability properties based on minimizing the system’s stored energy over time with the advantage that position and force vectors required by a Newtonian framework are replaced by a scalar valued energy function.

In 1892 Aleksandr Lyapunov showed that other scalar valued functions could be used instead of energy to analyze the stability of an equilibrium point [46]. Since the state of a system that corresponds to the unforced minimum energy solution is usually of no interest, Lyapunov’s theory on stability analysis has been augmented to a control design methodology using what
Figure 1.2: A visual analogy of energy shaping methods. Due to friction and gravity the ball settles to the position corresponding to minimum energy. By applying an input the minimum energy solution can be changed so that the ball naturally falls into the desired location.

Figure 1.3: One of the pioneers in the field of stability analysis of dynamic systems, Aleksandr Lyapunov circa 1900.

is known as control Lyapunov functions (CLFs). As shown in Figure 1.2 the unforced resting position of the ball due to friction and gravity corresponds to the minimum energy solution of the system. However, if we wish to manipulate the position of the ball, an energy shaping approach would be one that finds the control input $u$ that alters the closed loop systems dynamics so the minimum energy position is where we want the ball to go. Rather than being bound to shaping the true energy of the system, Lyapunov showed that any function that has energy like qualities, specifically functions that are positive everywhere and non-increasing, can be used in the stability analysis and consequently in the control design for a dynamic system.
1.4 Putting It All Together

With the motivation behind this work given and the main ingredients of the system and approach introduced, a summary of the central idea linking the chapters of this thesis together can now be stated.

Given a permanent magnet synchronous motor (PMSM) with the electrical phase currents and the rotational velocity of the shaft being the components of the continuously evolving state vector, we wish to control the state to achieve fast dynamic response, rejection of disturbances and smoothness of operation in steady state while simultaneously maximizing efficiency. However, due to the quantized nature of the motor’s actuator, a voltage source power inverter composed of IGBTs operating as nearly ideal switches, impressing an arbitrary voltage on the motor’s phase windings to produce the required currents is not possible. Given this continuous system with quantized inputs, we are left with constructing a decision making process that determines which switches to open or close at every instant so that the end control objectives are achieved. The basis for this decision making algorithm is built around a control Lyapunov function. While typical CLF based control methods begin with a Lyapunov function and seek to determine an input value that will stabilize the system, the approach taken in this thesis instead begins with the small number of available input choices and works backwards to determine which will correspond to a stabilizing Lyapunov function. In this fashion, input selections can be made based on the relative degree of stability each input is capable of providing which is the subject of Chapter 3. Furthermore, as discussed in Chapter 4, destabilizing inputs can be immediately removed from consideration when searching for optimal sequences over a predictive horizon in order to reduce computational burden and increase the frequency of control decisions. Finally, upon removal of destabilizing inputs from the set of input candidates the remaining inputs can be combined in a convex fashion to provide a smoother response in steady state. Discussed in Chapter 5 this process can be described as providing the system a ‘soft landing’ on the desired trajectory by nudging it in the desired direction from multiple locations with inputs that have been scaled by their effect on the stability of the system.
Chapter 2

Plant and Actuator Models

This chapter presents a model of a permanent magnet synchronous motor (PMSM) that will be used throughout the remainder of this thesis. While typical synchronous motors have three phases, they are connected in a wye configuration which allows any phase current to be determined from the sum of the other two. Using this fact along with a rotational transformation, the two independent phase currents are attached to the rotor’s coordinate system to transform a time varying three phase system into what appears to be a simple, two phase DC motor. The process of transforming the motor’s phase currents and voltages is achieved through the Park and Clark transformations. Upon completing the derivation of the motor model, the model of a power inverter required for actuating the motor will then be given in both the stationary and rotating reference frames.

2.1 Model of a PMSM in the $dq$ Reference Frame

Using Kirchoff’s voltage law and Newton’s second law in rotational form, a PMSM motor model can be derived and transformed into the standard direct and quadrature field-oriented rotating reference frame [40]. The $dq$ frame model of a PMSM is given as

$$\frac{di_d}{dt} = \frac{v_d}{L_d} - \frac{R}{L_d} i_d + p\omega_r \frac{L_q}{L_d} i_q$$

$$\frac{di_q}{dt} = \frac{v_q}{L_q} - \frac{R}{L_q} i_q - p\omega_r \frac{L_d}{L_q} i_d - p\omega_r \frac{\phi_m}{L_q}$$

$$\frac{d\omega_r}{dt} = \frac{3p\phi_m}{2J} i_q + \frac{3p}{2J} (L_q - L_d) i_d i_q - \frac{\beta}{J} \omega_r - \frac{\tau}{J}$$
with direct and quadrature axis currents \( i_d, i_q \), shaft rotational velocity \( \omega_r \), direct and quadrature axis inductances \( L_d, L_q \), stator resistance \( R \), number of magnetic pole pairs \( p \), magnetic flux \( \phi_m \), damping coefficient \( \beta \), load torque \( \tau \) and rotor moment of inertia \( J \). Note that the PMSM is a multi-input single output (MISO) system with inputs \( v_d \) and \( v_q \) and the output defined as the mechanical rotor velocity \( \omega_r \).

While the control methodology proposed in the following chapters of this thesis can be trivially extended to incorporate the generalized motor model given by (2.1)-(2.3), the motor used in our application has a salient, isotropic rotor which renders the direct and quadrature inductances equal and simplifies the dynamics to

\[
\frac{di_d}{dt} = \frac{v_d}{L} - \frac{R}{L} i_d + p \omega_r i_q - \frac{di_d^*}{dt} \tag{2.4}
\]

\[
\frac{di_q}{dt} = \frac{v_q}{L} - \frac{R}{L} i_q - p \omega_r i_d - p \omega_r \phi_m \frac{di_q^*}{dt} \tag{2.5}
\]

\[
\frac{d\omega_r}{dt} = \frac{3p \phi_m}{2J} i_q - \frac{\beta}{J} \omega_r - \frac{\tau}{J}, \tag{2.6}
\]

In order to drive the closed-loop system to the origin we define the state errors as

\[
\bar{\omega}_r = \omega_r - \omega_r^* \tag{2.7}
\]

\[
\bar{i}_d = i_d - i_d^* \tag{2.8}
\]

\[
\bar{i}_q = i_q - i_q^* \tag{2.9}
\]

where \( \omega_r^* \) is the desired mechanical rotor speed and \( i_d^*, i_q^* \) are the target currents.

In addition, we also introduce a speed error integral term \( \bar{\theta}_r \), related to the speed error by \( \frac{d\bar{\theta}_r}{dt} = \bar{\omega}_r \), to provide zero steady state error despite an unknown load torque and damping coefficient. The complete error dynamic system used for control design is now given as

\[
\frac{d\bar{i}_d}{dt} = \frac{v_d}{L} - \frac{R}{L} i_d + p \omega_r i_q - \frac{di_d^*}{dt} \tag{2.10}
\]

\[
\frac{d\bar{i}_q}{dt} = \frac{v_q}{L} - \frac{R}{L} i_q - p \omega_r i_d - p \omega_r \phi_m \frac{di_q^*}{dt} \tag{2.11}
\]

\[
\frac{d\bar{\omega}_r}{dt} = \frac{3p \phi_m}{2J} i_q - \frac{\beta}{J} \omega_r - \frac{\tau}{J} - \frac{d\omega_r^*}{dt} \tag{2.12}
\]

\[
\frac{d\bar{\theta}_r}{dt} = \bar{\omega}_r \tag{2.13}
\]
2.2 Power Inverters and Quantized Input Voltage Realization

2.2.1 The two-level power inverter

Input voltages to the motor drive are realized by selecting the switching configuration of a two-level power inverter whose generalized architecture is shown in Figure 2.1. The inverter consists of three half bridges connected in parallel, with each half bridge containing a high side and low side complementary pair of switches. Because the high side and low side switches can never be active at the same time, the inverter can be fully described by a binary ordered triple indicating the position of the high side switch for each inverter leg. Given three legs, each being in one of two positions, there are a total of $2^3$ possible configurations available as inputs to the motor with two of the configurations [111] and [000] both applying zero line to line voltage. The collective on/off status of the individual switches will be from here on referred to as the 'switching state' in order to remain consistent with current literature, though it should be noted that this is not truly a state of the system as it has no memory. Notating the set of switching states as $s$, given the $k^{th}$ switching state

$$s^k = [s_a^k s_b^k s_c^k]^T$$

$$s_i^k \in \{0, 1\} \quad \text{(2.15)}$$

$$k \in \{0, 7\} \quad \text{(2.16)}$$
the phase to ground voltages generated by that state can be represented as

\[ [v_{ag} \ v_{bg} \ v_{cg}]^T = V_{dc} s^k. \] (2.17)

### 2.2.2 The two-level inverter in the synchronous reference frame

In order to pose the entire motor-inverter system in the rotating reference frame, the realizable phase to neutral voltages are transformed as follows.

Assuming a wye connected motor, the phase to neutral voltages can be computed as

\[
\begin{bmatrix}
  v_{an} \\
  v_{bn} \\
  v_{cn}
\end{bmatrix} = \frac{1}{3} \begin{bmatrix}
  2 & -1 & -1 \\
  -1 & 2 & -1 \\
  -1 & -1 & 2
\end{bmatrix}
\begin{bmatrix}
  v_{ag} \\
  v_{bg} \\
  v_{cg}
\end{bmatrix}. \] (2.18)

With knowledge of the electrical angle of the rotor \( \theta_e \), the phase to neutral voltages are rotated into the synchronous reference frame with the Park transform as follows [40], [17]

\[
\begin{bmatrix}
  v_{qn} \\
  v_{dn} \\
  v_{0n}
\end{bmatrix} = \frac{2}{3}
\begin{bmatrix}
  \cos(\theta_e) & \cos(\theta_e - \frac{2\pi}{3}) & \cos(\theta_e + \frac{2\pi}{3}) \\
  \sin(\theta_e) & \sin(\theta_e - \frac{2\pi}{3}) & \sin(\theta_e + \frac{2\pi}{3}) \\
  \frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix}
\begin{bmatrix}
  v_{an} \\
  v_{bn} \\
  v_{cn}
\end{bmatrix}. \] (2.19)

Enumerating each possible switching state as \( s^i, i = 1, 2, \ldots, n \) the transformation from the inverter switching state to a rotating vector pair \( v_{dq}^i = [v_{d}^i, v_{q}^i]^T \) will be condensed to a single function through out this dissertation represented as

\[ v_{dq}^i = h(s^i, \theta_e, V_{dc}). \] (2.20)

This chapter, in part, is a reprint of the material as it appears in IEEE Transactions on Control Systems Technology, 2012 [63]. The dissertation author was the primary investigator and author of this chapter.
Chapter 3

Control Lyapunov Methods for Systems with Quantized Inputs

In this chapter we present a new method for the generation of input switching sequences in a synchronous motor control system based on the evaluation of a control Lyapunov function over a discrete set of realizable inputs. Typical reference input realization methods such as space vector modulation rely on high frequency state space averaging which can yield unnecessary switching events and increased switching losses. Alternative input selection strategies such as look up table based direct torque control rely on heuristically chosen hysteresis bands to determine switching instants and often results in a suboptimal choice between switching frequency and other performance measures. In this work we use an energy related Lyapunov function to guide the input selection process by making switching decisions based on the stabilizing effect each input has on the closed loop system. We provide a theoretical analysis of a motor-inverter system leading to a stability proof for a quantized input control law. The controller performance is verified through computer simulations and experimental results.

3.1 Introduction

High torque permanent magnet synchronous motors (PMSM) are becoming an increasingly popular choice in low to medium speed industrial motor applications where torque density and efficiency are critical. In comparison with induction motors, permanent magnet machines typically have lower inductance and a greater number of poles, yielding a small electrical time constant that leads control designers towards high frequency commutation solutions as a means
to generate sufficiently smooth stator currents. However, as the phase currents become large as is typical in low speed high power applications, the maximum attainable switching frequency shrinks due to the thermal limitations of the switching elements. Thus, the control engineer is faced with the problem of maximizing the closed loop performance of the system using a highly constrained actuator. This chapter presents a conceptually simple field oriented control algorithm with intrinsic commutation that is capable of minimizing switching events while providing fast dynamic response, robustness to parametric uncertainty and stability guarantees.

Being a fairly mature field, myriad techniques have been proposed over the years to improve the performance and efficiency of electric machines. Possibly the most notable of these is based on field oriented control (FOC) first proposed in [12] and further detailed in [40]. FOC allowed controllers to be developed similarly to well established techniques applied to separately excited DC motors by attaching a rotating reference frame to the rotor via the Park transform. Once posed in this reference frame, trigonometric non-linearities are removed, resulting in a bi-linear system that allows for independent regulation of torque producing and magnetizing currents. Now dealing with DC quantities proportional plus integral (PI) feedback loops are used for regulation which would otherwise have trouble tracking sinusoidal references.

Many researchers have proposed improvements to field oriented control. Extended state observers (ESOs) have been developed for motor control applications to compensate for unmodeled dynamics and disturbances [43], [69], including the use of ESOs to enable the use of active disturbance rejection control in passivity based designs [74]. In [60] the authors model the non-ideal characteristics of a PMSM responsible for producing torque ripple and build an adaptive current controller nested inside a linear speed loop to minimize the non-DC components of the generated torque. A similar approach is shown in [3] where converter dynamics are included in addition to the Fourier series based model of the torque spectra. Adaptive output feedback controllers have also been developed for classes of synchronous motors where the states to be controlled do not coincide with the measurable outputs [49]. Other nonlinear methods based on control Lyapunov functions have been reported in [55], as well as the related adaptive backstepping technique [56], [81], [19] and [18] which includes experimental comparisons between adaptive backstepping and robust control. These methods have been further refined in [14] where the authors design a continuous controller that achieves asymptotic tracking of a nonlinear system through the use of adaptive backstepping to address parametric uncertainties while employing a robust control technique to compensate for additive disturbances.

Field oriented controllers are used to generate idealized, continuous reference vectors
and rely on standard pulse width modulation techniques such as sine triangle PWM (ST-PWM) or space vector modulation (SVM) to approximate the desired input via high speed switching. The design and optimization of switching methods has been widely studied and continues to be an active area of research [27], [77]. In [8], the authors exploit the non-uniqueness of the standard SVM switching sequence to subdivide switching regions in a way to determine the best available sequence for torque ripple minimization. In [78] a variation on the ST-PWM technique is presented where the carrier frequency is modulated by the slope of the modulator, yielding a dynamic switching frequency with fewer commutations on average. The need for advanced control of power electronic converters is not limited to motor drives but rather applies to a wide class of systems requiring energy conversion. Backstepping techniques for PWM control are explored in [32] where the authors, through the use of state space averaging, formulate an expression for the duty cycle of a switch that drives the desired current and voltage from a photovoltaic array to target values determined by an extremum-seeking algorithm in order to maximize output power. Other improvements to standard FOC employ variable sampling techniques [75], randomized PWM [13], [37], [45] and online parameter estimation [76] which have been developed to address design specific objectives.

A popular alternative to FOC+PWM is direct torque control (DTC). As first described in [73], DTC does not compute reference vectors in the $dq$ space for tracking but instead uses hysteresis bands and a switching lookup table to keep estimates of the stator flux and the impressed torque in bounds. DTC has fast dynamic response and notably faster flux control capabilities when compared to FOC [21], [42], but can suffer from increased torque ripple and high switching losses depending on the size of the hysteresis bands [10], [8]. To mitigate these drawbacks the use of multiple time scale control laws has been proposed to slow down system trajectories near switching surfaces [80], while others have proposed augmenting DTC with model predictive control (MPC) to optimize competing switching signals over a finite horizon [26], [50], [54], [10].

In this chapter we incorporate a model of a PMSM in the $dq$ space and propose a control Lyapunov plus backstepping approach that provides stability guarantees. We then extend the proposed control law to govern the switching sequence of a class of quantized input systems. Rather than using continuous reference inputs and state space averaging assumptions as used in PWM generation, we use stability information from a control Lyapunov function evaluated at each input to select the switching configuration of the inverter, thus preserving the stability guarantees given by the continuous controller. Given more than one stabilizing input it is possible
to design input selection rules that minimize switching losses, randomize the switching spectra
or maximize dynamic response.

The rest of this chapter is organized as follows. In Section 3.2 we develop a continuous
speed controller using the integrator backstepping method. In Section 3.4 we derive the quanti-
tized input control law and discuss various methods of switching sequence optimization. Section
3.5 contains simulation results including speed and current regulation as well as a visualization
of the input selection process. The controller performance and implementation feasibility are
verified experimentally in Section 3.6. Section 3.7 provides a summary and some conclusory
remarks.

3.2 Continuous Space-Time Speed Controller via Integrator Backstepping

3.2.1 Stabilization of the mechanical subsystem

We begin our control design by introducing a control Lyapunov function (CLF) based
solely on the output of the mechanical subsystem (2.12), (2.13) defined as

$$V_1 = \frac{1}{2} \hat{\omega}_r^2 + \frac{1}{2} K_{\theta_r} \hat{\theta}_r^2$$

(3.1)

where $K_{\theta_r}$ is a positive design gain. Evaluating the time derivative of $V_1$ along the trajectories
of the state using the chain rule yields the following expression.

$$\dot{V}_1 = \dot{\omega}_r \frac{d\omega_r}{dt} + K_{\theta_r} \dot{\theta}_r \frac{d\theta_r}{dt}$$

$$= \dot{\omega}_r \left[ \frac{3p\phi_m}{2J} i_q - \frac{\beta}{J} \omega_r - \frac{\tau}{J} - \frac{d\omega^*_q}{dt} + K_{\theta_r} \hat{\theta}_r \right]$$

(3.2)

Focusing on the regulation of the state errors to zero, we assume constant or slowly
varying reference signals, rendering $\frac{d\omega_q^*}{dt} = 0$. Because the inputs to the system are not available
at the output we treat $i_q$ as a virtual input to the motor speed dynamics and back-step through an
integrator into the electrical dynamics governing the torque producing currents. Introducing an
additional design gain $K_{\omega_r} \geq 0$, choosing the stabilizing function $i_q^*$ as

$$i_q^* = -K_{\omega_r} \hat{\omega}_r + \frac{\beta}{J} (\hat{\omega}_r + \omega_r^*) + \frac{\tau}{J} - K_{\theta_r} \hat{\theta}_r$$

(3.3)
will render the mechanical subsystem stable about the origin. This is seen by substituting \( i_q^* \) for \( i_q \) in (3.2) and evaluating \( \dot{V}_1 \)

\[
\dot{V}_1 = -K_{\omega_r} \dot{\omega}_r^2 \leq 0
\]  

(3.4)

While the above expression is only negative semidefinite, upon completion of the backstepping procedure we will show that the closed loop system is globally exponentially stable.

### 3.2.2 Stabilization of the electrical subsystem

The backstepping procedure has been used to ensure the actual control inputs \( v_d \) and \( v_q \) drive the speed error to zero. We are now able to extend the control Lyapunov function (3.5), (3.6) by including states \( \tilde{i}_q \) and \( \tilde{i}_d \).

\[
V_2 = V_1 + \frac{1}{2} K_q \tilde{i}_q^2 + \frac{1}{2} K_d \tilde{i}_d^2
\]  

(3.5)

\[
\dot{V}_2 = \dot{V}_1 + K_q \dot{\tilde{i}}_q \frac{\tilde{i}_q}{dt} + K_d \dot{\tilde{i}}_d \frac{\tilde{i}_d}{dt}
\]  

(3.6)

In order to express the derivative of the quadrature current error \( \frac{\tilde{i}_q}{dt} \), (3.3) is differentiated as follows.

\[
\frac{d\tilde{i}_q}{dt} = \frac{di_q}{dt} = \frac{-K_{\omega_r} \dot{\omega}_r + \beta J \omega_r^2 - K_{\theta_r} \dot{\omega}_r}{3p \frac{\phi_m}{2J}}
\]  

(3.7)

The derivative of the direct current \( \frac{\tilde{i}_d}{dt} \) is considerably more simple to express after noting that

\[
i_d^* = \frac{di_d}{dt} = 0
\]  

(3.8)

which implies

\[
\tilde{i}_d = i_d
\]  

(3.9)

\[
\frac{d\tilde{i}_d}{dt} = \frac{di_d}{dt}
\]  

(3.10)

since the direct component of the current produces no torque and is not needed to magnetize the rotor in a PMSM. However, the direct component can be easily made non-zero without loss of generality to allow for field weakening if a boost in top speed at the expense of torque is desired.
Expanding (3.7) with the dynamics given in (2.12) and substituting into (2.10) we arrive at a transformed system in terms of state errors suitable for evaluating the total time derivative of the extended CLF (3.6).

\[
\begin{align*}
\frac{d\tilde{i}_d}{dt} &= \frac{v_d}{L} - \frac{R}{L} \tilde{i}_d + p(\tilde{\omega}_r + \omega_r^*) (\tilde{i}_q + i_q^*) \\
\frac{d\tilde{i}_q}{dt} &= \frac{v_q}{L} - \frac{R}{L} (\tilde{i}_q + i_q^*) \\
&\quad - p(\tilde{\omega}_r + \omega_r^*) \left( \frac{\tilde{i}_d + \phi_m}{L} \right) + \frac{2JK\tilde{\omega}_r}{3p\phi_m} \\
&\quad - \frac{2J}{3p\phi_m} \left( (K_{\omega_r} + K_{\theta_r} - \frac{3p\phi_m}{2J} \tilde{i}_q) (K_{\omega_r} - \frac{\beta}{J}) \right) \\
\frac{d\tilde{\omega}_r}{dt} &= \frac{3p\phi_m}{2J} \tilde{i}_q - K_{\omega_r} \tilde{\omega}_r - K_{\theta_r} \tilde{\theta}_r \\
\frac{d\tilde{\theta}_r}{dt} &= \tilde{\omega}_r
\end{align*}
\] (3.11)

Using these governing differential equations, we repeat the process performed in Section 3.2.1 by evaluating (3.6) and selecting the control inputs \(v_d\) and \(v_q\) that ensure \(\dot{V}_2\) is negative definite. By substituting (3.11) and (3.12) into (3.6), we can choose our control inputs so that they cancel out all positive and indefinite terms while leaving useful negative definite terms that provide damping. If \(v_q\) and \(v_d\) are chosen as

\[
\begin{align*}
v_q &= -K_q \tilde{i}_q + Ri_q^* + p\omega_r (L\tilde{i}_d + \phi_m) + L \left( \frac{di_q^*}{dt} - \frac{3p\phi_m\tilde{\omega}_r}{2K_qJ} \right) \\
v_d &= -K_q \tilde{i}_d - Lp\omega_r i_q^* - L\tilde{i}_q p\omega_r
\end{align*}
\] (3.15) (3.16)
the resulting closed loop error system is

\[
\begin{align*}
\frac{d\tilde{i}_d}{dt} &= - \left( \frac{K_d + R}{L} \right) \tilde{i}_d \\
\frac{d\tilde{i}_q}{dt} &= - \left( \frac{K_q + R}{L} \right) \tilde{i}_q - \frac{3p\phi_m\tilde{\omega}_r}{2K_qJ} \\
\frac{d\tilde{\omega}_r}{dt} &= \frac{3p\phi_m}{2J} \tilde{i}_q - K_{\omega_r} \tilde{\omega}_r - K_{\theta_r} \tilde{\theta}_r \\
\frac{d\tilde{\theta}_r}{dt} &= \tilde{\omega}_r
\end{align*}
\] (3.17) (3.18) (3.19) (3.20)
Using (3.17) - (3.20) to evaluate (3.6) shows

\[ \dot{V}_2 = -K_\omega \ddot{\omega}_r - \left( \frac{K_q + R}{L} \right) \dddot{i}_q - \left( \frac{K_d + R}{L} \right) \dddot{i}_d \]

(3.21)

is only negative semi-definite since the state \( \tilde{\theta}_r \) has been cancelled out by the control inputs. To prove asymptotic stability note that \( V_2 \) is radially unbounded and observe that

\[ \dot{V}_2 = 0 \rightarrow \dot{i}_d = \frac{d\dot{i}_d}{dt} = \dot{i}_q = \frac{d\dot{i}_q}{dt} = \ddot{\omega}_r = \dot{\omega}_r = 0. \]

(3.22)

Evaluating (3.19) at \( \dot{V}_2 = 0 \) results in the expression

\[ 0 = -K_\theta \tilde{\theta}_r. \]

(3.23)

Therefore, since no solution can stay identically in the set \( S = \{ [\tilde{i}_d, \tilde{i}_q, \tilde{\omega}_r, \tilde{\theta}_r]^T | \dot{V}_2 = 0 \} \) other than the trivial solution, and observing that the error dynamic system (3.11)-(3.14) is autonomous as it is only dependant on initial conditions, by the theorem of Barbashin and Krasovskii [35] the origin is globally asymptotically stable. Furthermore, because the closed loop error system (3.17)-(3.20) is linear, the origin is in fact globally exponentially stable.

### 3.3 Space Vector Modulation

A fundamental issue in motor control design concerns bridging the gap between the continuous-valued inputs optimally suited for a motor and the small, finite set of seven unique inputs that are capable of being generated by a two-level inverter. Because the method we present is in the class of vector controls it is insightful to describe how other algorithms in this class address this issue. Possibly the most popular algorithm used in modern motor control research to approximate continuous valued input commands is Space Vector Modulation (SVM) due to its relative simplicity, single switch event per commutation and higher line to line voltage capability when compared to traditional sine-triangle PWM [34]. SVM is a method of state space averaging, where the duty cycles of realizable switching states bounding the desired reference input are computed such that the on-times of the switching states average to the continuous valued input. Figure 3.1 shows the available states of a two-level inverter in the stationary reference frame with \( \theta = 0 \), along with a desired space vector with coordinates \( v^\alpha_{\alpha,\beta} = (v^\alpha_\alpha, v^\alpha_\beta) \). A fast average, time-based convex combination of the input sequence to approximate the continuous
Figure 3.1: Quantized input space of a two-level power inverter in the stationary reference frame

The quantized input depicted in Figure 3.1 is given by

\[
(v^*_\alpha, v^*_\beta) \approx T_s - \eta - \gamma \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T + \eta \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T + \gamma \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T + \frac{T_s - \eta - \gamma}{2} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T.
\]

where \( T_s \) is the sampling frequency and \( \eta, \gamma \) are the duty cycles assigned to each non-zero switching state and must satisfy \( \eta, \gamma \geq 0 \) and \( \eta + \gamma \leq T_s \). It should be noted that other nearly equivalent sequences exist, including those that only use a single zero vector for what is known as discontinuous PWM [34]. However, all of the different variants of SVM involve fast switching between the two non-zero states that bound the desired input to approximate the command angle and switching to either one or both zero states to approximate the command magnitude. Typical FOC methods compute input commands in the synchronously rotating \( dq \) frame and use nested PI controllers for regulation of phase currents and rotor speed. In this reference frame the synchronous motor behaves like a DC motor where non-sinusoidal reference inputs improve the quality of integral action on the steady state error [40]. Once the inputs \( v_q \) and \( v_d \) are calculated, they are rotated back into the stationary, two-phase reference frame by the inverse Park transform resulting in \( v_\alpha \) and \( v_\beta \). With the stationary reference frame mapped to the switching states of the inverter, the sector that bounds the input is computed to determine which three switching states to modulate.
3.4 Stabilization with Quantized Inputs

3.4.1 Proof of Stability

It is well known that if a stable convex combination of unstable systems exists, then there exists a stabilizing switching rule between the systems [44]. An often non-trivial task noted in [44] is finding such a convex combination. We will show that the control law derived in Section 3.2 can be expressed as a stable convex combination of realizable inputs. Moreover, explicitly computing the control inputs given by (3.15) and (3.16) is unnecessary as simply knowing the convex combination exists is enough formulate a switching rule that preserves the stability guarantees given by the continuous case.

**Lemma 3.4.1.** Denoting the state vector \( x = [\tilde{i}_d, \tilde{i}_q, \tilde{\omega}_r, \tilde{\theta}_r] \) \(^T\) and input vector \([v_d(x), v_q(x)]\) \(^T\) satisfying the feasibility requirement that \(|v_{dq}(x)| \leq \frac{2}{3} V_{dc}\), expressing the dynamic system defined by (3.11)-(3.14) as

\[
\dot{x} = F(x) + G(x) v_{dq}(x)
\]  

(3.25)

define a Lyapunov function as in (3.5) by

\[
V(x) = \frac{1}{2} K x^2
\]  

(3.26)

with weighting matrix \( K \) defined as

\[
K = \text{diag}[K_d, K_q, 1, K_{\theta_r}].
\]  

(3.27)

Suppose for all feasible \( x \) there exists \( v_{dq}^*(x) \) such that

\[
\frac{\partial V(x)}{\partial x} \left( F(x) + G(x)v_{dq}^*(x) \right) \leq 0.
\]  

(3.28)

Given discrete input pairs \( v_{dq}^k \) with \( k = 0, 1, \ldots, 6 \) corresponding to the seven unique, realizable two-level inverter voltages, the quantized-input control Lyapunov function derivative

\[
\frac{\partial V(x)}{\partial x} \left( F(x) + G(x)v_{dq}^k \right)
\]  

(3.29)
satisfies
\[
\min_k \frac{\partial V(x)}{\partial x} \left( F(x) + G(x)v^k_{dq} \right) \leq 0 \tag{3.30}
\]
\[
\forall x : |v^*_{dq}(x)| \leq \frac{2}{3} V_{dc}.
\]

Proof. As illustrated in Figure 3.1, observe that any reference vector \( v^*_{dq}(x) \) within the region of feasibility (having magnitude less than \( \frac{2}{3} V_{dc} \)) is contained within one of the six non-zero switching sectors of width \( \frac{\pi}{3} \) with vertices \( (v^0_d, v^0_q), (v^i_d, v^i_q), (v^j_d, v^j_q) \) where \( i, j \in \{1, 2, ..., 6\} \) are the non-zero switching states to the left and right of the reference vector and \( (v^0_d, v^0_q) \) is one of the two zero vectors. Containment within a switching sector ensures the existence of coefficients \( \gamma, \eta \) satisfying \( \gamma, \eta \geq 0, \gamma + \eta \leq 1 \) such that the reference vector is expressible as a convex combination of the realizable inputs, given by
\[
v^*_{dq}(x) = \gamma v^i_{dq} + \eta v^j_{dq} + (1 - \gamma - \eta)v^0_{dq}. \tag{3.31}
\]

Plugging (3.31) into (3.28) and noting that the system is control affine, we see that
\[
\frac{\partial V(x)}{\partial x} \left( F(x) + G(x)v^*_{dq}(x) \right) =
\]
\[
\frac{\partial V(x)}{\partial x} \left( F(x) + G(x) \left( \gamma v^i_{dq} + \eta v^j_{dq} + (1 - \gamma - \eta)v^0_{dq} \right) \right) =
\]
\[
\gamma \frac{\partial V(x)}{\partial x} \left( F(x) + G(x)v^i_{dq} \right) + \eta \frac{\partial V(x)}{\partial x} \left( F(x) + G(x)v^j_{dq} \right)
\]
\[
+ (1 - \gamma - \eta) \frac{\partial V(x)}{\partial x} \left( F(x) + G(x)v^0_{dq} \right) \leq 0. \tag{3.32}
\]

Because \( \gamma, \eta \) and \( (1 - \gamma - \eta) \) are all non-negative, (3.32) implies that at least one of the following inequalities holds:
\[
\frac{\partial V(x)}{\partial x} \left( F(x) + G(x)v^i_{dq} \right) \leq 0 \tag{3.33}
\]
\[
\frac{\partial V(x)}{\partial x} \left( F(x) + G(x)v^j_{dq} \right) \leq 0 \tag{3.34}
\]
\[
\frac{\partial V(x)}{\partial x} \left( F(x) + G(x)v^0_{dq} \right) \leq 0, \tag{3.35}
\]

which completes the proof of the lemma. \( \square \)
3.4.2 Summary of the proposed control law

A block diagram of the proposed control method is given in Figure 3.2. With Lemma 3.4.1 in hand we are now able to define numerous input selection methods to optimize some performance metric while maintaining stability guarantees provided by the continuous Lyapunov methods from Section 3.2 so long as a sufficiently fast evaluation of the quantized input stability function given by (3.29) is possible for each input candidate. While the proof given above asserts at least one of the three inputs bounding a stabilizing reference vector are themselves stabilizing, we will see in Section 3.5 that other inputs not bounding the reference can also be stabilizing and can be used in the optimization of the switching sequence. For example, it is possible to penalize switching events by selecting the input state that results in the most negative value of the stability function and monitor the degree of stability this input provides as the motor evolves in time without performing any additional switching or optimization. As the stability function approaches zero and the motor becomes marginally stable, re-minimize (3.29) over \( k \) by selecting the input that again yields the most negative result. In this fashion, switching only occurs when it is necessary; only when stability would otherwise be lost if a new input was not selected. The step by step description of a minimum switching control law is given as follows, where for notational brevity, the quantized input stability function is denoted as \( \dot{V}^k \).

1. Measure the state of the motor and express in the synchronous frame \( \omega_r, \theta_r, i_d, i_q \).

**Figure 3.2:** Block Diagram of the Proposed Control Method.
2. Following the procedure outlined in Section 3.2, compute \( \tilde{\omega}_r, \tilde{\theta}_r, i^*_q, \frac{d i^*_q}{dt} \).

3. Use the measured electrical angle \( \theta_e = p\dot{\theta}_r \) and (2.19) to express the seven unique input voltages in the synchronous frame.

4. Use the dynamics of the system given by (3.11) - (3.14) to evaluate all seven values of the stability function \( \dot{V}^k \).

5. Select input \( k \) that satisfies

\[
\min_k \dot{V}^k \leq 0.
\]

Because of Lemma 3.4.1 we know at any instant at least one of the input states will produce a negative valued stability function.

6. Without switching, continue to measure the state and update the value of \( \dot{V}^k \) until stability would be lost if a switching event does not occur.

7. Re-evaluate the stability function and again choose the minimizing input.

One point worth noting is that this algorithm is optimal in the sense that it minimizes the average switching frequency under stability constraints so long as the evaluation of the stability function is much faster than the electrical time constant of the motor. Other control laws can be formulated that leverage the stabilizing effect of each input in order to optimize other metrics, such as minimizing torque ripple, maximizing torque response or minimizing \( \frac{dV}{dt} \) stress on the inverter.

### 3.5 Simulation Results

The motor-inverter system was simulated in MATLAB with continuous dynamics given in (2.4) computed using a classical Runge-Kutta (RK4) integrator with a time step equal to ten times the control frequency. Measurements are simulated by adding Gaussian noise to the states evolved by the RK4 method and are subsequently marched forward by one time step corresponding to the control period. These state prediction estimates are used along with the closed form expressions of \( i^*_q, \frac{d i^*_q}{dt} \) and the the model dynamics to estimate the stability function \( \dot{V}^k, k = 0, 1, ..., 7 \) at the next control period to allow for a switching decision before stability is lost. The initial model parameters are given in Table 3.1.
Table 3.1: Motor parameters and control gains used for simulation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )</td>
<td>9</td>
<td>pole pairs</td>
</tr>
<tr>
<td>( L )</td>
<td>8.0 mH</td>
<td>stator inductance per phase</td>
</tr>
<tr>
<td>( R )</td>
<td>2.0 mΩ</td>
<td>stator resistance per phase</td>
</tr>
<tr>
<td>( J )</td>
<td>1 kgm(^2)</td>
<td>rotor moment of inertia</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.5 Nm/s/Rad</td>
<td>viscous damping coefficient</td>
</tr>
<tr>
<td>( \tau )</td>
<td>25 Nm</td>
<td>external load torque</td>
</tr>
<tr>
<td>( \phi )</td>
<td>0.44 Nm/Amp</td>
<td>magnetic flux</td>
</tr>
<tr>
<td>( K_{\omega_r} )</td>
<td>1</td>
<td>speed error gain</td>
</tr>
<tr>
<td>( K_{\theta_r} )</td>
<td>10</td>
<td>speed error integrator gain</td>
</tr>
<tr>
<td>( K_q )</td>
<td>1</td>
<td>quadrature current error gain</td>
</tr>
<tr>
<td>( K_d )</td>
<td>0.75</td>
<td>quadrature current error gain</td>
</tr>
<tr>
<td>( V_{dc} )</td>
<td>200 V</td>
<td>DC link voltage</td>
</tr>
<tr>
<td>( f_s )</td>
<td>10 kHz</td>
<td>maximum switching frequency</td>
</tr>
</tbody>
</table>

Before examining the traditional step response characteristics we first look at an alternative evaluation of the controller that is performed by plotting the values of the stability function \( \dot{V}^k \) over all inputs while denoting the actual switching sequence selected by the control law as a means of showing what the stability of the system would be at a given time for each input had that input been selected. Figure 3.3 shows the stabilizing effect of each input as the motor just begins to rotate from a standstill, where for readability purposes every 50\(^{th}\) decision instance has been designated with a marker. At time \( t = 0 \) the controller is initialized to state 000, which immediately loses stability as the errors begin to increase. The controller then selects input 100 as the best input and remains there for quite some time as the inertia of the motor is overcome and during this period approximately 500 switching events are skipped. This demonstrates a very intuitive behavior of the control law; do not switch away from the input that generates the most torque when acceleration is required.

Continuing with this method of evaluating the control law, Figure 3.4 shows the selection process of the input sequence after the initial input no longer provides stability. Given the current state of the motor over the time interval shown, the inverter input 110 is stabilizing with exception to four instances where switching occurs. Referring to Figure 3.4, consider time \( t = 0.4958 = t_0 \) seconds. The controller applies the input 010 for the next four decision instances after which it predicts that remaining at 010 will yield instability at the next step and thus switches to a new input. This input is predicted to make the value of the stability function positive in the next time step so the controller again selects 010. Over the next six time steps \( t_0 + 4\Delta T \) through \( t_0 + 9\Delta T \)
Figure 3.3: Stabilizing effect of each input as the motor accelerates. Initially at rest with $\theta = 0$ aligned with $I_{\text{phaseA}} = I_q$, the switching state producing the maximal torque is selected (100). Note that the two other non-zero states (101 and 110) where phase A is connected to $V_{dc}$ are also stabilizing but to a lesser extent and produce the same value of the stability metric. Additionally the two states (001 and 010) with phase A connected to ground and one other phase connected to $V_{dc}$ are not stabilizing and also produce identical stability curves. Finally, the top-most curve verifies the intuition that the most unstable switching choice 011 is the opposite (largest distance in the $L_1$ sense) switching state of the most stabilizing choice.

this input remains stabilizing and therefore no switching occurs, allowing the stability function to be evaluated using only the current input. However, at $t_0 + 10\Delta T$ the stability function at time $t_0 + 11\Delta T$ is predicted to become positive. At this point the stability function is again minimized and a new input is selected as it yields the minimizing value. This input is again determined to be stabilizing for one time step, where again input 010 is selected for six more time steps before further switching becomes necessary. This behavior is not always periodic as can be seen at $t = .497$ seconds. At this point the state 011 is stabilizing by a small margin yet is predicted to maintain stability for fourteen time steps. The advantage to this method is that switching is
Figure 3.4: Switching sequence selection based on predicted stability. At each time step the value of the control Lyapunov function is computed for each input, representing the stabilizing effect on the system had that input been selected. Only occurring when it is necessary from a stability viewpoint. After an input $k$ is selected, we simply follow $\dot{V}_k$ to zero before making another selection, allowing for the reduction in switching losses without prohibiting fast switching when it is required as is typical of static switching frequency methods. Additionally, the control law randomizes the switching pattern to an extent. While switching at a non-uniform, minimum average frequency can increase torque ripple and total acoustic noise, spreading out the spectral content by decreasing coherency can be useful in certain applications such as reducing the detectability of a submersible vehicle motor drive without employing noise injection techniques used in random PWM methods [13], [45].
3.6 Experimental Results

3.6.1 Description of the experimental testbed

The experimental test bed shown in Figure 3.6 has been designed to verify the effectiveness of the proposed control law. The algorithm depicted by Figure 3.2 has been developed in Simulink and implemented in real time on dSpace hardware through the Real Time Workshop software package available from the Mathworks. The modular DS1006 system from dSpace has been augmented with the DS5202 FPGA base board equipped with the ACMC motor control card capable of PWM synchronized phase current and DC link measurements, position measurements from QEP encoder signals, and the generation of gate drive signals from zero to one hundred percent duty cycle. Actuation is achieved with an IAPL600T120 900 Volt, 600 Amp two-level inverter from Applied Power Systems optically connected to the ACMC through a custom made laser gate driver board to minimize the coupling of electromagnetic noise and the drive signals. The inverter independently inserts a 5μs deadtime between the high side and low side devices of each inverter leg as the corresponding phase transitions from high to low. This
Table 3.2: Model parameters and control gains used for experimental results

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>5</td>
<td>pole pairs</td>
</tr>
<tr>
<td>$L$</td>
<td>2.8 $mH$</td>
<td>stator inductance per phase</td>
</tr>
<tr>
<td>$R$</td>
<td>20 $m\Omega$</td>
<td>stator resistance per phase</td>
</tr>
<tr>
<td>$J$</td>
<td>.69 $kgm^2$</td>
<td>rotor moment of inertia</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.1763 $Nms/Rad$</td>
<td>viscous damping coefficient</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0 $Nm$</td>
<td>estimated load torque</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.08 $Nm/Amp$</td>
<td>magnetic flux</td>
</tr>
<tr>
<td>$K_{\omega_r}$</td>
<td>1</td>
<td>speed error gain</td>
</tr>
<tr>
<td>$K_{\theta_r}$</td>
<td>10</td>
<td>speed error integrator gain</td>
</tr>
<tr>
<td>$K_q$</td>
<td>1</td>
<td>quadrature current error gain</td>
</tr>
<tr>
<td>$K_d$</td>
<td>.75</td>
<td>quadrature current error gain</td>
</tr>
<tr>
<td>$V_{dc}$</td>
<td>100 $V$</td>
<td>DC link voltage</td>
</tr>
<tr>
<td>$f_s$</td>
<td>10 $kHz$</td>
<td>maximum switching frequency</td>
</tr>
</tbody>
</table>

research is motivated by the desire to control high current, high torque motors where the switching frequency is severely limited which led us to a 540kW inverter solution. However, initial experiments are conducted on a scaled down bench top Kollmorgen AKM64P five pole-pair, three phase brushless servo motor. A viscous load is applied through a tension brake applied to an aluminum disk attached to the rotor as well as an eddy current magnetic brake used to apply load disturbances, which consists of a pair of NdFeB magnets attached to a plate that can be positioned at variable distances from the rotor disk. The load provided by both of these mechanisms, shown in Figure 3.7 is unknown and allows the demonstration of the controllers disturbance rejection capabilities and insensitivity to parametric uncertainty in the damping coefficient $\beta$ as well as the load torque $\tau$. Unless otherwise noted, the model parameters provided to the controller are given in Table 3.2.

3.6.2 Step response, switching frequency reduction and tracking a sinusoidal reference given and unknown asymmetric load

We begin experimental verification of the proposed controller by applying a series of step commands over a variety of operating points without any applied load. As seen in Figure 3.8, as the speed of the motor increases faster switching is required to maintain speed regulation. While the data exhibits a small overshoot at the command transitions, the controller achieves rapid speed regulation despite the low switching frequency.

In addition to speed regulation, the ability to accurately control acceleration and de-
Figure 3.6: Test bed used for controller evaluation.

celeration is an important feature to examine in the evaluation of a motor controller. While the stability proof presented in Section 3.4 assumes an autonomous system which somewhat restricts our focus to the speed regulation problem, if a time varying input whose variations are slow compared to the dynamics of the system the input commands will appear approximately constant and the theoretical stability analysis will hold. To investigate this further, a slowly varying sinusoidal input reference with $\omega_r = 300\sin(0.5t)$ is used as a reference speed command and a viscous load is applied to the rotor using the tension and eddy current braking mechanisms. The load is both unknown and asymmetric as it applied a different amount of force in the clockwise and counter-clockwise directions. Figure 3.9 demonstrates the ability of the proposed control method to track slowly varying commands in both directions with an uncertain load while maintaining a very low switching frequency.
3.6.3 Robustness to parametric uncertainty

Another desirable attribute of a motor control system is a robustness to parametric uncertainty. Sensitivity to parameters in model based designs can greatly inhibit the implementation of the controller, particularly when the parameter count is high as is the case in our proposed method. While a rigorous analysis of the robustness to parametric uncertainty is beyond the scope of this work, it is conceptually worthwhile to point out that the input selection algorithm discussed in Section 3.4.2 reduces the apparent parametric sensitivity by evaluating the stabilizing effect of each input using the same parameters for each evaluation. Because every parameter is intrinsically positive, parametric error in the electrical subsystem will only scale the values of $\hat{V}^k$ and will not change the ordering of the inputs in terms of which one is the most stabilizing. One caveat worth noting is that parametric error in the mechanical subsystem can influence controller performance as the mechanical parameters are used to determine the target reference
Figure 3.8: Experimentally measured step response and average switching frequency for various speed commands. A window of 500 samples is used to determine the localized average switching frequency of each leg of the inverter which is then averaged across all three legs. The variable switching frequency allowed by the input selection method increases as necessary with increasing speed, however the overall switching frequency remains relatively small.

current $i_q^*$. While this has been observed experimentally to add a small degree of overshoot to the response, the steady state behavior remains unchanged due to the integral action incorporated into the mechanical component of the CLF. To demonstrate this robustness four experiments are conducted where the values provided to the controller for the damping coefficient $\beta$, the inductance $L$, the rotor moment of inertia $J$ and the DC link voltage $V_{dc}$ are changed by a factor of two or more. The effects of this parametric variation are shown as a series of plots given in Figure 3.10.

The first plot shows the tracking response with the inductance more than tripled from the nominal value, from 2.8mH to 10mH. This parameter is used in the evaluation of each input and the net result on the performance of the controller is negligible. The next two plots show the effects of doubling the damping coefficient $\beta$ from 0.1763 Nms/Rad to 0.3526 Nms/Rad and more than doubling the rotor moment of inertia $J$ from 0.69 kgm$^2$ to 3 kgm$^2$. As these two parameters are part of the mechanical subsystem and influence the reference current $i_q^*$ derived from the
Figure 3.9: Experimentally measured tracking of a sinusoidal speed reference. The motor controller is capable of accurate control of acceleration and deceleration while given an unknown, asymmetric load. Similar to the step response data, the switching frequency increases with rotor speed as needed. However it also increases with increased load. The envelope of the phase currents shows positive (clockwise) rotation requires more torque to maintain speed tracking than negative rotation due to asymmetries in the braking mechanism. The peak switching frequency during positive rotation is 2930 Hz while the peak switching frequency during negative rotation is 2628 Hz, a difference of about 300 Hz.

Backstepping procedure, parametric uncertainty $\beta$ and $J$ can contribute to adding overshoot to the response. However these transient effects are short lived due to the incorporation of integral action in the mechanical component of the CLF which allows the input selection method to adapt to mechanical parametric error. The last plot shows the response of the system when the DC link measurement is doubled from its actual value of 100V to 200V. Similarly to parametric errors in the inductance, measurement errors in the DC link voltage have no effect on the system as it does not change the relative stabilizing effect of each input. It was found experimentally that as long as the actual DC link voltage and current sourcing capability were sufficient to achieve the desired speed and torque, any overestimate of the DC link voltage had no effect on the controller, while an underestimate occasionally resulted degraded performance, likely due to the controller falsely determining that no inputs will provide the necessary amount of torque. This attribute
can allow the elimination of a DC link sensor so long as a lower bound on the link voltage is known.

### 3.6.4 Performance and actuator longevity issues that can arise due to non-constant switching frequency

One of the features of fixed frequency commutation strategies that differentiate them from variable switching frequency techniques is their ability to generate approximately sinusoidal phase currents which in turn produce minimal torque ripple. Unlike variable frequency controllers such as DTC that use hysteresis bands to control flux and torque as opposed to directly controlling the phase currents, the proposed controller incorporates a model of the motor to direct the minimum energy solution of the closed loop system to the origin of the error dynamic system through the use of control Lyapunov functions and it is therefore plausible that the generated currents will more closely match the characteristic sinusoidal back EMF waveforms of the motor despite the variable switching frequency behavior. Figure 3.11 verifies the controllers emergent capability to generate sinusoidal phase currents by allowing energy related...
control Lyapunov functions to guide the input selection process.

Another advantage to space vector modulation and other fixed frequency commutation methods is that they yield the same degree of stress on each of the power electronic switching elements which can contribute to greater actuator longevity. While variable switching frequency methods can not guarantee this behavior it is intuitive that a smoothly running motor with sinusoidal phase currents should equally employ each power electronic switch in the inverter. To verify this the localized (100 point moving average) switching frequency of each leg of the inverter is used to generate a histogram detailing how often each switch operates at each frequency. This method of analysis is chosen over simply counting the total number of switching events per device because it is possible for a switching element to operate at a locally higher frequency than the remaining devices while being utilized comparatively less often than others at larger time scales, resulting in the false conclusion that the switching effort is balanced. Figure 3.12 shows the actuator stress is evenly distributed across each leg of the inverter at every frequency, implying that the low average switching frequency seen in Figure 3.8 is not due to some inverter states rarely being selected, but is rather due to an even reduction in total switching events.

Figure 3.11: Experimentally measured phase currents generated by the quantized input method controlling an unknown load at 500 RPM and a switching frequency of about 5 kHz.
Figure 3.12: A histogram derived from experimental data depicting how frequently the power electronic switching devices operate at different frequencies. The data shows an even distribution of actuator stress across each leg of the inverter.

3.7 Conclusions

We have presented a new method for closed-loop control of a motor-inverter system that utilizes the advantages of field orientation while accounting for the quantized input nature of the actuator with an energy related commutation strategy directly build into the control law. The proposed controller uses model-based stability measures to compare each realizable input in a decision process designed to preserve stability while minimizing switching events. The control law generates a low switching frequency that naturally varies as demands on the motor change yet it is capable of producing sinusoidal phase currents and an even distribution of actuator stress, it is parametrically insensitive and yields fast torque response while maintaining independent control of the torque and flux producing currents.

This chapter, in part, is a reprint of the material as it appears in IEEE Transactions on Control Systems Technology, 2012 [63]. The dissertation author was the primary investigator and author of this paper.
Chapter 4

Extensions to Finite Control Set MPC with Dynamic Pruning

In this chapter we present a novel model predictive control scheme that incorporates stability information derived from a control Lyapunov function (CLF) to dynamically prune suboptimal sequences from the search space and decrease the computational burden placed on the controller. The CLF used for pruning is then incorporated into a cost function that penalizes energy in the error system as well as energy loss due to switching. Despite the very small control periods allowed due to dynamic pruning, experimental results are given showing the resulting controller generates low switching frequencies and low total harmonic distortion.

4.1 Introduction

Recent advances in power electronics and high speed, low cost digital microcontrollers have dramatically improved the performance of power converters and electric drive systems. Partly due to their discrete nature, modern power electronic based energy conversion systems such as active front ends, boost/buck converters and power inverters typically operate at efficiencies exceeding 95% which has led to large scale adoption in industry and sustained academic research interest. However, the discrete operation of power electronic switching elements also provides numerous control challenges as they are most often used to control continuous phase currents. In this chapter we present a novel power inverter input vector realization method for a permanent magnet synchronous motor control application.

A popular technique for addressing the hybrid nature of controlling continuous currents
with quantized actuators is the adoption of a modulation scheme for input vector realization. In a widely analyzed approach for motor drive applications the stator currents and the shaft rotational speed are used in the feedback loops within a cascaded set of classical linear controllers for the determination of the desired phase voltages. Relying on the phase inductance for smoothing, a high frequency modulation algorithm such as space vector modulation (SVM) or its predecessor sine triangle pulse width modulation (ST-PWM) then computes the switching angles and duty cycles required so the time averaged phase voltage approximates the desired voltage over one control period. There has been a significant amount of research conducted within this framework, much of it focused on improving the modulation techniques to reduce switching losses [64], [78], [75] the reduction of acoustic noise [13], [45], [37] and the reduction of torque ripple [8], [68], [77]. [2].

Alternatively, hysteresis based control methods have also gained significant attention within the academic community. Rather than relying on high frequency averaging assumptions, hysteresis based approaches such as direct torque control (DTC) directly control the switching state of the inverter so that estimates of the stator flux and the rotor torque are held within predefined error bands. Controlling the switching state directly exploits some of the benefits of a quantized actuator by avoiding the time delay associated with nested control loops which results in a faster torque response. However, in tuning the width of the hysteresis bands the control designer is required to choose between high frequency switching associated with narrow bands and excessive torque ripple resulting from wide error bands. Compounding this issue, typical modulator-free input vector realization methods do not employ the zero states through a duty cycle as in SVM or ST-PWM. To prevent the torque ripple from exceeding acceptable limits, hysteretic control methods require much smaller control periods than duty cycle based modulators. This is particularly evident in the low speed region where the generated back EMF is small compared to the DC link voltage making the change in the phase current per switching event more pronounced. Despite these drawbacks, the simplicity of implementation and the fast dynamic response of DTC and other related methods has made this approach very popular in industry including ABB who has reportedly spent over one hundred man years developing DTC drives [1]. Areas of current research interest include hybrid DTC-SVM methods for reduced torque ripple [87], [85], [41] applications to multilevel inverters [22], [86], adaptive flux estimation [72] and control of high phase count motors [88], [72].

One of the most promising developments in modulation-free approaches is a computationally efficient form of model predictive control (MPC) [24], [15], [38]. Also known as
constrained optimal control with a receding horizon, traditional MPC selects at each time step the sequence of control actions that minimizes a cost function based on predictions of the state over a horizon $N$. The first element of this minimizing sequence is applied to the system and the optimization process is then repeated forming a receding horizon control policy. While MPC provides a relatively simple framework for including nonlinearities and constraints, applications of this method to systems with fast dynamics have been limited due to the large number of calculations required over small sampling periods. To overcome this limitation many researchers have adopted an explicit method where the optimal solution, given as a piecewise affine function defined over a partitioning of the feasible states, is computed offline to reduce computational burden. Based upon the pioneering work in [11], explicit MPC approaches have been increasingly adopted, many of which incorporate modulators for reduced harmonic distortion at the expense of requiring hybrid continuous time models or linear constrained parameter varying models in order to maintain tractability [48], [9], [4]. One disadvantage of using an explicitly solved control law is the potential for exponential growth in the number of regions where the control law is defined though there has been some recently reported results where a combination of explicit MPC and online optimization is employed to limit both storage space requirements and computation time [83].

An alternative approach is to leverage the inherently discrete nature of a power inverter to reduce the optimization space to a small number of candidate sequences and thus reduce the computational requirements to a feasible level. The resulting control method, sometimes referred to as finite control set model predictive control (FCS-MPC), has recently lead to numerous efforts from within the DTC community mostly aimed at reducing switching frequency [16], [26], [58], [61], [50] though other improvements have been achieved including the reduction of torque ripple [47], [23], [6], balancing capacitor voltages in multilevel inverters [15], [26] and the inclusion of a modulator within the FCS-MPC framework to achieve a fixed switching frequency [66].

While the adoption of FCS-MPC has greatly facilitated the use of predictive control techniques to power electronic systems, even the relatively small number of realizable actuator inputs can still present computational challenges when input sequences are evaluated over very small control periods. This has led some researchers to reduce the number of inputs considered in the optimization process. For example, in [5] and [6] the authors present a multilevel six phase predictive controller where the number of inputs are reduced from 64 to the largest 12 vectors and a single zero vector in order to make the control law computationally feasible. Other ap-
proaches for addressing this issue have been recently proposed including those that use explicitly solved offline controllers [47], [25] branch and bound search methods for discarding suboptimal sequences [24] and those that impose limits on the number of transistors that are allowed to switch during each state transition [61]. In this chapter we extend our previous developments of a control Lyapunov (CLF) based approach that incorporates stability information in the input selection process [63]. This method has been implemented within a finite control set model predictive control architecture where candidate inputs are first vetted according to their stabilizing influence on the system before they are used in integrating the closed loop system dynamics for predicting future states. The resulting controller provides stability guarantees while dynamically pruning suboptimal input sequences for the reduction of computational burden.

The rest of this chapter is organized as follows. Section 4.2 gives an overview of our previous results on using CLFs evaluated over a finite control set to guide the input selection process. Section 4.3 extends our results to include prediction and optimization over a receding horizon. Section 4.4 gives experimental results documenting the usefulness of this approach, and Section 4.5 gives concluding remarks and proposals for future work.

### 4.2 CLF Based Quantized Input Selection

Chapter 3 presented a novel input selection method for the control of a PMSM connected to a two level power inverter which is briefly recapitulated here for reference. While we skip the formulation of the target states for brevity, it should be noted that the target speed \( \omega_r^* \) and its derivative are assumed given while the target direct current \( i_d^* \) and its derivative are set to zero to achieve maximum torque per ampere within the non field weakening regime. The target quadrature current \( i_q^* \) and its derivative can be derived through the backstepping procedure as done in [63] or can be determined from the output of an outer speed loop. Since the value of \( \frac{di_q^*}{dt} \) is determined by the relatively slow mechanical dynamics particularly when compared to the extremely small time intervals used for MPC, many researchers assume a constant quadrature current (or torque) reference over the prediction horizon. The previously proposed input selection algorithm is as follows.

Given the system dynamics (2.10)-(2.12) we begin by creating a control Lyapunov function (CLF)

\[
V = \frac{1}{2} K_\omega \dot{\omega}_r^2 + \frac{1}{2} K_i i_q^2 + \frac{1}{2} K_d i_d^2
\]  

(4.1)
with derivative

\[
\dot{V} = K_\omega \ddot{\omega}_r \frac{d\ddot{\omega}_r}{dt} + K_q \ddot{i}_q \frac{d\ddot{i}_q}{dt} + K_d \dddot{i}_d \frac{d\dddot{i}_d}{dt}.
\] (4.2)

Using (2.17)-(2.19) to rotate the seven realizable unique inputs into the rotor reference frame, (4.2) is then evaluated using each input pair \(v_{dq}\). It is shown in Chapter 3 that at each sampling instant at least one realizable voltage vector will render (4.2) negative given the desired state of the system is within the region of feasibility determined by the \(dc\) link voltage. This result directly addresses an open question posed in [28] where the authors discuss the difficulty in providing a certificate of stability for closed loop systems with quantized inputs due to the optimal, stabilizing input \(v_{dq}^*\) not laying within the finite control set. Several novel switching rules can be constructed with knowledge of the stabilizing effect of each candidate input and a guarantee that at least one input will always work to stabilize the system. Examples include a time optimal switching rule where the input providing maximum stability is selected at each control period or a switching loss minimization rule where no switching occurs until the current input is no longer providing stability.

4.3 Finite Control Set Model Predictive Control

4.3.1 Generalized FCS-MPC

The finite control set model predictive control approach is well suited for motor drive applications due to the finite number of control actions that are allowed at each time step. The problem can be defined as the determination of the next inverter switching state based on the cost evaluation of candidate switching sequences over a receding time horizon. Consider the state of the system sampled at time \(t_k\) defined as

\[
x(t_k) = [\omega_r(t_k), i_d(t_k), i_q(t_k)]^T
\] (4.3)

\[
t_k = kT_s
\] (4.4)

\[
k \in \mathbb{N}
\] (4.5)

where \(T_s\) is the sampling and control period. Using a discrete time model of the system \(f(x, v_{dq})\) the value of the next state can be predicted under the action of each of the \(n\) available inputs
\[ v^i_{dq}(t_k), i = 1, \ldots, n \]

where the superscript \( i \) designates the predicted state due to the \( i^{th} \) input and the subscript \( p \) denotes that this is a predicted value. The resulting cost associated with each potential control action is then computed and stored in memory and the process is repeated using each predicted state as the starting point for a new set of predictions. This process continues until the desired prediction horizon \( N \) is reached at which point the total cost of each sequence is summed and the first control action of the sequence with the lowest cost is applied at time \( t_k \). However, due to the finite time required to perform these calculations, to implement this scheme the initial measured state is first marched forward in time by one time step using the currently active control input and the input sequence optimization is then carried out from \( t_k+1 \) to \( t_{k+1+N} \). An example state propagation graph including this initial prediction step for a system with \( n \) inputs and a horizon of \( N = 2 \) is given in Figure 4.1

4.3.2 Cost function design

A cost function designed to properly capture and weight the metrics to be optimized is critical to the successful implementation of a model predictive controller. Common cost functions used for power converter control are

\[ J = |x_p - x^*| \]

\[ J = (x_p - x^*)^2 \]

\[ J = \frac{1}{T_s} \int^{T_s} [x_p(t) - x(t)^*] dt \]

with the effects on system performance due to each of these functions being detailed in [38]. As we are already using a control Lyapunov function to determine stabilizing switching states, a natural choice for the cost of a given state is

\[ J(t_k) = V(x_p(t_k) - x^*(t_k)) = V(\bar{x}_p(t_k)). \]

It should be noted that in order to condense the remainder of this presentation and avoid notational obfuscation, the time step \( (t_k) \) will not be expressly written except for in instances
where it adds clarity.

To minimize numerical errors that can arise when comparing components of a cost function with different magnitudes, the values of $K_\omega$, $K_d$ and $K_q$ are selected so the Lyapunov function is expressed in units of average energy in the error dynamic system over one control period $\Delta t$ given by

$$\bar{E} = \frac{1}{2} J \ddot{\omega}_r^2 + \left(R \Delta t + \frac{1}{2} L \right) \ddot{\theta}^2.$$  \hspace{1cm} (4.12)

The control gains given in (4.1) are chosen as

$$K_\omega = J$$  \hspace{1cm} (4.13)
Figure 4.2: Combinatorial growth of the search space as the horizon is increased for standard FCS-MPC applied to a two level inverter.
Figure 4.3: The improved FCS-MPC algorithm with reduced search space growth through dynamic pruning of candidate input sequences.
\[
K_d = 2R\Delta t + L
\]
\[
K_q = 2R\Delta t + L
\]

which yields

\[
V = \frac{1}{2}K_\omega \tilde{\omega}_r^2 + \frac{1}{2}K_d \tilde{i}_d^2 + \frac{1}{2}K_q \tilde{i}_q^2
\]  
(4.16)

\[
= \frac{1}{2}J\tilde{\omega}_r^2 + \frac{1}{2}(2R\Delta t + L)\tilde{i}_d^2 + \frac{1}{2}(2R\Delta t + L)\tilde{i}_q^2
\]  
(4.17)

\[
= \frac{1}{2}\begin{bmatrix} \tilde{\omega}_r, \tilde{i}_d, \tilde{i}_q \end{bmatrix} \begin{bmatrix} J & 0 & 0 \\ 0 & 2R\Delta t + L & 0 \\ 0 & 0 & 2R\Delta t + L \end{bmatrix} \begin{bmatrix} \tilde{\omega}_r \\ \tilde{i}_d \\ \tilde{i}_q \end{bmatrix}
\]  
(4.18)

\[
= \frac{1}{2}x^T P\tilde{x}
\]  
(4.19)

\[ P > 0. \]
(4.20)

It is also desirable to minimize the number of switching events that occur over a path under evaluation due to the associated switching losses that result from each state transition. Given the relationship between the \(i^{th}\) rotating reference frame input at time \(t_k\) and its implementable representation as a binary ordered triple in the fixed reference frame

\[
v_{dq}^i(t_k) = h(s^i, \theta_e(t_k), V_{dc})
\]  
(4.21)

the associated switching loss incurred by transitioning from state \(i\) to state \(j\) is proportional to the difference between the states given as

\[
\Delta s_{ji} \triangleq s^j - s^i
\]  
(4.22)

\[
= \begin{bmatrix} s^j_a - s^i_a \\ s^j_b - s^i_b \\ s^j_c - s^i_c \end{bmatrix}
\]  
(4.23)

Noting that for each transition in a given phase leg one switching element turns on while its compliment turns off, the energy lost per switching event is given by

\[
E_{loss}(x) = \frac{V_{dc}}{V_T} \left[ E_{on}(x) + E_{off}(x) \right]
\]  
(4.24)

where \(V_T\) is the test voltage used by the device manufacturer to measure the switching losses.
$E_{on}$ and $E_{off}$ have a nearly linear dependence on the current flowing through the device when they transition and can be well approximated from curves provided on the device data sheet.

Combining 4.22 and 4.24 the total energy lost during a state transition can be expressed as

$$L(x) = \frac{1}{2}\Delta s^{ji}T Q(x)\Delta s^{ji}$$

with

$$Q(x) = 2 \begin{bmatrix} E_{\text{loss}}(x) & 0 & 0 \\ 0 & E_{\text{loss}}(x) & 0 \\ 0 & 0 & E_{\text{loss}}(x) \end{bmatrix} > 0.$$

The two energy related cost metrics can now be put together along with a convex weighting function $\alpha \in [0, 1]$ to form the cost function used in the remainder of this paper given as

$$J = \frac{1}{2} \tilde{x}^T P \tilde{x} + (1 - \alpha) \frac{1}{2} \Delta s^{ji}T Q(x)\Delta s^{ji}.$$  (4.27)

Given the discrete time equivalent of (2.10),(2.11) along with the input rotation function (4.21) given together as

$$\tilde{x}^i(t_{k+1}) = f(\tilde{x}(t_k), v_{dq}^i(t_k))$$
$$= f(\tilde{x}(t_k), h(s^i, \theta_e(t_k), V_{dc}))$$  (4.28)

the minimum value path over a horizon of length $N$ is

$$J^* = \min_{s^i} \sum_{\ell=k+1}^{k+1+N} \alpha \tilde{x}^T(t_\ell) P \tilde{x}(t_\ell) + (1 - \alpha) \Delta s^{ji}T (t_\ell) Q(x)\Delta s^{ji}(t_\ell).$$

### 4.3.3 Pruning the search space

While the discrete nature of the actuator in a motor drive system limits the size of the search space, as depicted in Figure 4.2 due to the combinatorial explosion of candidate sequences as the horizon increases even two level inverters can impose tremendous computational effort on the controller and limit the length of the implementable horizon for a given sampling time. To address this we propose a new method for pruning the search space based on the predicted stability of each future state. As shown in Figure 4.3, once a state is determined to be unstable,
the sequence that led to this state is removed from consideration and subsequent paths originating from it are not explored. The remaining stabilizing sequences are then compared and the one with the lowest accumulated cost is chosen. From the results in [63] at least one input will lead to a stable state at every prediction interval which guarantees at least one sequence will be found in which every element is stabilizing. Additionally, heuristically chosen weighting parameters typical in cost functions are not capable of destabilizing the system since before any cost is computed along a path the input being evaluated must first be determined to be stabilizing.

4.3.4 The improved FCS-MPC algorithm

The proposed control algorithm is described as follows. Labeling the currently active control input as \( v_{dq}^\dagger \), the controller first marches the system forward using the discrete time model of the system before input optimization begins. Using this predicted state each realizable input is used to evaluate the CLF and the dynamics are marched forward again for each input that provides stability. Using each of these predicted states as a starting point this process is then repeated over the next interval, again only considering inputs that pass a stability test. While the target currents are not assumed constant as is commonly done, to further reduce computational cost they are assumed to be linear over the prediction window. A block diagram description of the FCS-MPC method incorporating control Lyapunov functions for optimal input sequence selection is presented in Figure 4.4. For clarity and readability of the block diagram the notation has been somewhat condensed. Without loss of generality let the \( n^{th} \) prediction step \( t_{k+n} \) be notated as simply \( n \). The predicted state at the \( n^{th} \) point along the receding horizon under the influence of input \( i \) will be denoted as \( \tilde{x}_n^i \) and the corresponding derivative of the Lyapunov function under the same input will be denoted \( \dot{V}_n^i \). The cost as given in (4.27) at this point along the horizon is denoted as \( J_n^i \) and the lowest cost is given as \( J_n^i_* \). Finally, the available inputs expressed in the \( dq \) reference frame at point \( n \) given by \( v_n^i \) are computed using (2.17)-(2.19), the predicted electrical angle \( \theta_{e_n} \) and the \( dc \) link voltage \( V_{dc} \).

4.4 Experimental Results

4.4.1 Description of the experimental testbed for FCS-MPC evaluation

An experimental test bed has been designed to verify the effectiveness of the proposed control law. The algorithm depicted in Figure 4.4 has been developed in Simulink and implemented in real time on dSpace hardware through the Real Time Workshop software package.
Figure 4.4: A block diagram description of the proposed FCS-MPC algorithm.
Table 4.1: Model parameters and control gains used for experimental results

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )</td>
<td>5</td>
<td>pole pairs</td>
</tr>
<tr>
<td>( L )</td>
<td>2.8 ( mH )</td>
<td>stator inductance per phase</td>
</tr>
<tr>
<td>( R )</td>
<td>20 ( m\Omega )</td>
<td>stator resistance per phase</td>
</tr>
<tr>
<td>( J )</td>
<td>.69 ( kgm^2 )</td>
<td>rotor moment of inertia</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.1763 ( Nms/Rad )</td>
<td>viscous damping coefficient</td>
</tr>
<tr>
<td>( \phi )</td>
<td>0.08 ( Nm/Amp )</td>
<td>magnetic flux</td>
</tr>
<tr>
<td>( V_{dc} )</td>
<td>60 ( V )</td>
<td>DC link voltage</td>
</tr>
<tr>
<td>( f_s )</td>
<td>50 ( kHz )</td>
<td>control frequency</td>
</tr>
</tbody>
</table>

available from the Mathworks. The modular DS1006 system from dSpace has been augmented with the DS5202 base board equipped with the ACMC motor control card capable of PWM synchronized phase current and DC link measurements, position measurements from QEP encoder signals, and the generation of gate drive signals from zero to one hundred percent duty cycle. All experiments were conducted on a Kollmorgen AKM64P five pole-pair, three phase brushless servo motor actuated with an IAPL600T120 two-level inverter from Applied Power Systems which has been optically connected to the ACMC through a custom made laser gate driver board. The inverter independently inserts a \( 5\mu s \) deadtime between the high side and low side devices of each inverter leg. Total harmonic distortion is measured with a Fluke Norma 4000 three phase power analyzer connected in series between the inverter and motor with the internal low pass filter set to limit the THD calculation to the first twenty three harmonics. The switching frequency of the inverter is measured with a frequency measurement block provided by the DS1106 library which is then filtered by a 100 point moving average to provide an average switching frequency. It should be noted that the data presented in Section 4.4.2 has been downsampled by a factor of five due to limitations in the capabilities of dSpace to simultaneously capture each sample for a large number of signals at \( 20\mu S \). However, the THD is computed from raw data by placing the Fluke Norma power analyzer in series with the physical phase currents making these results independent of the sampling frequency of the data acquisition system.

4.4.2 The effects of increasing the prediction horizon and switching penalties

Experimental analysis of the proposed control method has been broken into nine experiments. For each experiment a speed command of 500 \( rpm \) has been supplied as a reference command while the prediction horizon is changed from \( N = 0 \) to \( N = 1 \) to \( N = 2 \). For each selected horizon three weighting factor values \( \alpha = 0.01, 0.5, 0.99 \) are applied to examine the
controllers effectiveness in penalizing switching events versus phase current distortion. For each horizon distance chosen the state is first marched forward in time by one time step using the previously selected input to account for the inherent computation delay associated with a discrete time system. Given this initial prediction step the case \( N = 0 \) predicts which inputs stabilize the system and then only computes and compares the costs associated with applying these inputs. If \( N > 0 \) each of these candidate inputs are used as a starting point for another set of predictions and the process continues until the horizon level is reached. All predictions are made by integrating the system dynamics with a three step Adams-Bashforth-Moulton predictor-corrector method seeded by a fourth order Runge-Kutta integrator.

The results of these experiments are given in Figure 4.5 with the corresponding waveforms and measured switching frequencies given in Figures 4.6 - 4.8. It is observed from the data that for each value of \( \alpha \) progressively increasing the prediction horizon from 0 to 2 significantly decreases the measured THD, suggesting longer horizons favor smoother phase currents given the proposed cost function.

Due to the well known inverse relationship between THD and switching losses, the decrease in THD as the horizon grows is typically provided at the expense of increased switching frequency. Consider the equal weighting case of \( \alpha = 0.5 \) given in Figure 4.7. Increasing the horizon from \( N = 0 \) to \( N = 1 \) decreases the THD by 37.45% while increasing the switching frequency by 28.18%. While increasing the horizon from \( N = 1 \) to \( N = 2 \) decreases the THD by an additional 10.1%, the switching frequency is only increased by 3.91%. To provide a means of comparison between the change in THD and switching frequency as the horizon is incremented, a horizon length penalty metric \( H(\Delta N) \) is defined as the ratio between the percent increase in switching frequency versus the percent decrease in total harmonic distortion, given as

\[
H(\Delta N) = \frac{\% \uparrow F_{sw}}{\% \downarrow THD}.
\]

Increasing the horizon from 0 to 1 provides much more improvement to the THD than is achieved by further increasing the horizon from 1 to 2, suggesting a decrease in return on computational investment for longer horizons. This is in agreement with the findings in [52] where the authors report that MPC methods using a horizon of 1 often achieve near optimal performance. However, as shown in Table 4.2 the improvements gained by using longer horizons
Table 4.2: $H(\Delta N)$ for $\Delta N_{0\rightarrow 1}$ and $\Delta N_{1\rightarrow 2}$.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$H(\Delta N_{0\rightarrow 1})$</th>
<th>$H(\Delta N_{1\rightarrow 2})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>1.575</td>
<td>0.4650</td>
</tr>
<tr>
<td>0.5</td>
<td>0.7524</td>
<td>0.3875</td>
</tr>
<tr>
<td>0.99</td>
<td>0.5069</td>
<td>-0.3448</td>
</tr>
</tbody>
</table>

Figure 4.5: Comparison of THD and switching frequency as a function of horizon and weighting factor $\alpha$.

4.5 Conclusions

We have presented a new method for inverter input selection that is well suited for application within a finite control set model predictive control framework. The proposed method dynamically eliminates suboptimal candidate input sequences before they are fully evaluated by predicting the stabilizing effect of each node in the input sequence and removing it and all that come at a proportionally lower switching cost. This is particularly evident in the $\alpha = 0.99$ case where a horizon of $N = 2$ simultaneously reduces the switching frequency and the THD, resulting in a negative value of $H(\Delta N)$. 
Figure 4.6: Phase currents in the stationary reference frame, rotating reference frame and average switching frequency for $N = 0, 1, 2$ with weighting factor $\alpha = 0.01$.

Figure 4.7: Phase currents in the stationary reference frame, rotating reference frame and average switching frequency for $N = 0, 1, 2$ with weighting factor $\alpha = 0.5$.

subsequent child nodes from consideration if it is predicted to be destabilizing. This method of pruning has enabled the real time implementation of a horizon 2 (3 prediction step) MPC controller using a multistep predictor-corrector integration method while operating at $50kHz$. While
it is shown that most of the performance benefits occur at the first level of prediction ($N = 1$), some arguments are given as to why an additional prediction step adds further benefit due to its relatively small impact on the switching cost. Although horizon lengths greater than 1 have shown moderate improvements in THD levels while only slightly increasing switching costs, this method would likely show even greater utility when applied to systems with large numbers of inputs such as a five or seven level inverter where the size of the decision space can be more drastically reduced to allow very small control periods without preemptively eliminating inputs from consideration.

This chapter, in full, has been submitted for publication of the material as it may appear in ASME Journal of Dynamic Systems and Control, 2013. The dissertation author was the primary investigator and author of this paper.
Chapter 5

Control Lyapunov Modulation Methods for Quantized Input Systems

This chapter presents a new modulation technique for motor drives. It has been shown in previous work that control Lyapunov functions can be used to generate optimal input sequences, providing fast torque response and stability guarantees. However, similarly to direct torque control, this technique applies non-zero inputs over the entire control period which increases torque and current ripple. To remedy this, a novel modulation strategy that averages inputs in proportion to their stabilizing effect is proposed. Additionally an active-null modulator is incorporated into the CLF-based controller. Experimental results show improvements in harmonic ripple and non-harmonic chatter without degradation of torque response.

5.1 Introduction

Improving waveform quality of the phase currents generated by a voltage source inverter has been and continues to be an active area of interest power electronics literature. Waveform distortion reduces energy conversion efficiency and contributes to torque ripple in electric drives. A typical approach for generating low distortion waveforms is the application of high switching frequencies and a reliance on the inductance of the applied load to provide adequate filtering. While this is an acceptable solution in some applications, the resulting switching losses begin to dominate the total losses in the system when the switching frequency is excessively high. Moreover, in high power applications the thermal limitations of the individual power electronic elements impose a hard limit on the maximum attainable frequency.
Over the past decade there has been a sustained interest in developing techniques that minimize ripple currents in grid connected inverters and motor drive systems. In [2] the author extends a Fourier coefficient compensation technique for BLDC motors to accommodate finite amplifier bandwidth limitations that occur at high rotor velocities. An alternative to this approach is reported in [51] where repetitive control is used to decrease the bandwidth required to actively compensate for ripple harmonics. In [59] the authors minimize torque ripples in a BLDC by optimizing the reference torque producing current while allowing for three phase unbalanced conditions. Incorporating DC link measurements to reduce commutation ripple by balancing the slopes of the incoming and outgoing phase currents has been reported in [71] and further developments to this idea have been reported in [53] where the DC link voltage is modulated during commutation instants. While aiming to minimize current distortion, many other contributions aim to minimize complexity and parameter dependence such as reported in [84], [87] and [70] where the authors incorporate a proportional-resonant controller to achieve zero steady state error without requiring coordinate transformations.

While many of these methods are focused on improving feedback control loops, a considerable amount of attention has also been given to optimizing the modulation strategies that these controllers employ to realize the desired reference inputs. Control and modulation strategies can be optimized independently by assuming the control process is quasi-continuous. Recently reported in [39] the authors use parameter free evolutionary methods to determine optimal switching angles, switching frequency and filter parameters while using online “perturb and observe” methods to optimize the reference voltage vector generated by the control law. Continuous and discontinuous PWM techniques have also been compared, and hybrid methods have been proposed that seek to minimize torque ripples by switching between continuous and discontinuous modulation indexes based upon the location of the reference voltage vector [7], [29], [36].

In contrast to control methods that first generate continuous valued reference signals which are then approximated by discrete-space modulation techniques, other approaches including direct torque control (DTC) [73], [20], [31], [57], and direct power control (DPC) [79] employ a lookup table to directly select switching states from a small quantized set of available inverter inputs without relying on averaging assumptions necessary for the development of continuous controllers. Directly controlling an inverter’s switching sequence gives these methods fast dynamic response at the expense of increased ripple, high sampling frequency requirements and a non-constant switching frequency. To remedy this several methods have been proposed to incorporate a duty cycle into the switching sequence with the introduction of a null inverter
vector. In [87] the authors consider the influence of the flux and torque error amplitudes in determining the angular increment of the reference vector while using space vector modulation (SVM) to achieve a constant switching frequency from within the DTC framework. An alternative method of introducing a duty cycle known as active-null modulation [65], [33], [84], [85] focuses on minimizing the deviation of the torque from a constant reference over each control period by computing the optimal instant to switch from an active vector to a null vector. The switching instant is typically chosen to achieve one of three criteria; the instantaneous torque equals the reference torque at the end of the period, the mean torque is equal to the reference over the entire period, or the torque ripple RMS value over the period is minimized.

In [63] a novel method for directly controlling the switching sequence of an inverter is presented by employing a control Lyapunov function (CLF) that evaluates the stabilizing effect of each realizable input over each control period. Rather than relying on switching tables and heuristically chosen hysteresis bands as is typical in DTC methods, this method uses stability information to guide the input selection process. With this information novel objective functions can be formulated including those that minimize switching frequency, maximize transient response or minimize switching stresses across the power electronic devices. Additionally, knowledge of the stabilizing effect of each realizable input can be used within an online model predictive controller to prune the search space and reduce the computational effort typically associated with MPC methods [62]. However, similarly to DTC/DPC, the methods proposed in [63] and [62] apply an active vector over the entire control period which can lead to high torque ripples, particularly at low speeds. This work improves upon previous results by introducing two distinct methods for incorporating a duty cycle within a CLF based hybrid feedback control law.

The rest of the chapter is organized as follows. Section 5.2.2 presents an active-null modulation strategy based on minimizing the squared error of the torque producing current over a control period and Section 5.2.3 develops a new modulation strategy based on forming a time based convex combination of stabilizing inputs. Section 5.3 presents experimental results and Section 5.4 gives some concluding remarks.

5.2 CLF Based Modulation Strategies for Reducing Distortion

5.2.1 Quantized input control without modulation: a baseline for comparison

The control law developed in Section 3 guarantees a GES closed loop system provided the continuous valued inputs $v_q, v_d$ are realizable. However, power inverters are only capable
of producing a finite number of line to line phase voltages. To address this controllers are often split into two functional blocks, a continuous valued block that generates desired phase voltages and a discrete modulator that seeks to average the realizable inputs to the desired value over each control period through high speed switching. Alternatively modulation free techniques such as DTC typically use a continuous speed controller to produce a reference torque. This reference is then used within a look up table based sliding mode controller to keep the flux and torque within the desired hysteresis bands. In [63] it is proven that if a stabilizing continuous valued reference voltage vector exists and is within the region of feasibility of the inverter, meaning that the vector would be realizable if infinite switching frequencies were permissible, than at each instant at least one realizable voltage vector is also stabilizing. With this result it is possible to eliminate unnecessary switching events associated with averaging modulation schemes while using stability information rather than heuristically derived look up tables and hysteresis bands to determine the inverter switching sequence. Moreover, knowing the stabilizing influence each inverter state has on the system as it evolves provides a natural choice for a cost function in predictive control as the optimal input search space can be quickly pruned when a given input is deemed destabilizing.

5.2.2 Active-null modulation

Constantly selecting the most stabilizing vector as discussed in Chapter 3 ensures the non zero inputs are applied over the entire control period which under utilizes the null states \([0 \ 0 \ 0]\) and \([1 \ 1 \ 1]\). This can result in excessive torque ripple, particularly in the low speed regime where the slopes of the currents under active inputs become large. To remedy this it is possible to first select the most stabilizing input at the onset of a control period and then using a predicted trajectory of the current compute a fraction of the control period for this input to be applied before switching to a null vector. This can be achieved as follows. The discrete form of the quadrature current dynamics given by

\[
i_q(k+1) = i_q(k) + t_s \left[ \frac{v_q(k)}{L} - \frac{R}{L} i_q(k) - p \omega_r(k) \left( i_d(k) + \frac{\phi_m}{L} \right) \right]
\]  

(5.1)

can be broken into two pieces, one corresponding to the application of an active input and another corresponding to a null input. Defining the slopes of the current trajectories as

\[
f_1 = \frac{v_q(k)}{L} - \frac{R}{L} i_q(k) - p \omega_r(k) \left( i_d(k) + \frac{\phi_m}{L} \right), \quad v_q \neq 0
\]  

(5.2)
As depicted in Figure 5.1, the evolution of the torque producing current over a control period subject to an active for a length of time $t_a$ and null vector for a length of time $t_s - t_a$ can be expressed as

$$i_q(k + 1) = i_q(k) + t_a f_1 + (t_s - t_a) f_2$$

$$0 \leq t_a \leq t_s.$$  \hspace{1cm} (5.4)

If the control period is sufficiently small compared to the time constant of the motor the slopes $f_1$ and $f_2$ can be considered approximately constant allowing the square of the RMS ripple in the quadrature current to be expressed as

$$\tilde{i}_{qRMS}^2 = \frac{1}{t_s} \int_0^{t_a} \left( f_1 t + \tilde{i}_q(k) \right)^2 dt$$

$$+ \frac{1}{t_s} \int_{t_a}^{t_s} \left( f_1 t_a + f_2 (t - t_a) + \tilde{i}_q(k) \right)^2 dt.$$  \hspace{1cm} (5.6)

To determine the optimal switching instant, (5.6) is differentiated with respect to $t_a$ and set to zero which results in

$$t_a = \frac{2 \tilde{i}_q + f_2 t_s}{f_2 - 2 f_1}.$$  \hspace{1cm} (5.7)

In the event that the motor is not in steady state it is possible for the duty cycle to go out of bounds, i.e. $t_a < 0$ or $t_a > t_s$. In this case the null vector (for $t_a < 0$) or the active vector (for $t_a > t_s$) is applied over the entire interval. A block diagram depiction of the proposed controller including the active-null modulation approach (method 1) is given in Figure 5.2.

### 5.2.3 Modulation via convex combination of stabilizing inputs

The active-null modulation strategy just described optimizes the duty cycle based on the RMS error in the torque producing current. One potential limitation to this approach is that only a single duty cycle value is computed at each time step meaning that each non-zero element of the switching state $s$ will always be active for the same amount of time. An alternative method of introducing a duty cycle can be achieved by applying every input that renders $\dot{V} < 0$ for a duration of time proportional to the degree of stability that they provide. Creating an auxiliary
Figure 5.1: A graphical depiction of the active-null modulation principle.

function $\dot{V}^\dagger$ defined as

$$\dot{V}^\dagger_k = \begin{cases} \dot{V}^k & \dot{V}^k < 0 \\ 0 & \text{otherwise} \end{cases} \quad (5.8)$$

and considering the collection of all realizable inputs $s$, the duty cycles $d^* = [d_a, d_b, d_c]^T$ can be computed as

$$d^* = \frac{\sum_k \dot{V}^\dagger_k s^k}{\sum_k \dot{V}^\dagger_k}. \quad (5.9)$$

Referring to Figure 5.3 it is seen that at the onset of each sampling interval several inputs will render $\dot{V} < 0$. While choosing the input corresponding to the most negative value of the CLF derivative pushes the error system to the origin the hardest, this approach does not allow for a ‘soft landing’ as the error gets small, and due to the computation constraints limiting the minimum size of the control period, overshoot occurs during the inter-sample regions. By incorporating the less stabilizing inputs the system is not pushed as hard as the error gets small and as will be shown in Section 5.3 results in less current ripple in steady state when compared to the full duty cycle case.

As an example refer again to Figure 5.3 and observe at time $t = 3.408s$ that evaluating the CLF derivative with the inverter state $[1\ 0\ 0]^T$ yields a stability value of $-50.6$. Three other
Table 5.1: Motor parameters used for experimental results

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
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<td>$R$</td>
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</tr>
<tr>
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<td>.69 kgm$^2$</td>
<td>rotor moment of inertia</td>
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<td>0.1763 Nms/Rad</td>
<td>viscous damping coefficient</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.08 Nm/Amp</td>
<td>magnetic flux</td>
</tr>
<tr>
<td>$V_{dc}$</td>
<td>100 V</td>
<td>DC link voltage</td>
</tr>
<tr>
<td>$f_s$</td>
<td>12.5 kHz</td>
<td>maximum switching frequency</td>
</tr>
</tbody>
</table>

inputs (inverter states) are also stabilizing, $[1 \ 1 \ 0]^T$, $[1 \ 0 \ 1]^T$ and the zero line to line voltage inputs $[0 \ 0 \ 0]^T$ and $[1 \ 1 \ 1]^T$ with stability values $-27.5$, $-25.0$ and $-1.851$ respectively. The proposed method of converting this information into a duty cycle consists of multiplying each inverter state by its corresponding stability value, normalizing each product by the sum of all stability values and summing them together, which in this example is shown as

$$d^* = \frac{-50.6}{104.951}[1 \ 0 \ 0]^T + \frac{-27.5}{104.951}[1 \ 1 \ 0]^T$$
$$+ \frac{-25.0}{104.951}[1 \ 0 \ 1]^T + \frac{-1.851}{104.951}[0 \ 0 \ 0]^T \text{ or } [1 \ 1 \ 1]^T$$

(5.10)

$$= .48213[1 \ 0 \ 0]^T + .26203[1 \ 1 \ 0]^T$$
$$+ .23821[1 \ 0 \ 1]^T + \frac{0.1764}{2}[0 \ 0 \ 0]^T + \frac{0.1764}{2}[1 \ 1 \ 1]^T$$

(5.11)

$$= \left[ .99119 \ .27085 \ .24703 \right]^T$$

(5.12)

where the duty cycle corresponding to the redundant zero states $[0 \ 0 \ 0]^T$ and $[1 \ 1 \ 1]^T$ has been evenly divided between the two. Notice that unlike the active-null modulation strategy, each element of $D^*$ can have independent duration’s. While the optimality of this approach is not yet proven and the controller evaluation is left to experimental comparisons, the idea of using additional inputs with smaller CLF slopes to ease the error system into the origin is fairly intuitive and despite the somewhat complicated control formulation is very simple to implement. A block diagram depiction of the proposed controller with modulation through a convex combination of stabilizing inputs (method 2) is given in Figure 5.4.
5.3 Experimental Results

5.3.1 Description of the experimental testbed for CLF based modulation

The experimental test bed shown in Figure 5.5 has been designed to verify the effectiveness of the proposed control law. The algorithms depicted in Figures 5.2 and 5.4 have been developed in Simulink and implemented in real time on dSpace hardware through the Real Time Workshop software package available from the Mathworks. The modular DS1006 system from dSpace has been augmented with the DS5202 FPGA base board equipped with the ACMC motor control card capable of PWM synchronized phase current and DC link measurements, position measurements from QEP encoder signals, and the generation of gate drive signals from zero to one hundred percent duty cycle. Actuation is achieved with an IAPL600T120 two-level inverter from Applied Power Systems optically connected to the ACMC through a custom made laser gate driver board to minimize the coupling of electromagnetic noise and the drive signals. Experiments are conducted on a bench top Kollmorgen AKM64P five pole-pair, three phase brushless servo motor connected to a custom built nine pole-pair axial flux motor through a torque coupling shown in Figure 5.6. The three phases of the axial flux motor can be connected together by closing a switch in order to impart a load step on the system. To increase the resolution of the current measurement an external three phase current measurement board incorporating 100 Amp LEM sensors was used. Because the expected phase currents are in the two to five Amp range each phase was wrapped through the transducer ten times and the resulting measurement was then divided by ten to account for the ten fold increase in the apparent current. While in previous experiments the manufacturer’s data sheet values for inductance was used, with a total of twenty turns added between phases the line to line inductance of the system was instead measured with an LCR meter. The measured value was larger than what was previously used partly due to the differences in the current measurement method and also due to using a line to line value rather than a line to neutral. While a line to neutral value could be inferred from this measurement it was found that using the line to line value yielded slightly lower THD ($\approx 1\%$) so this parameter estimate was used instead. Unless otherwise noted, the model parameters provided to the controller are given in Table 5.1.

5.3.2 Steady state controller comparison

The first set of experiments consist of comparing the steady state performance of the two proposed methods for incorporating a duty cycle from within the CLF framework outlined
Table 5.2: Steady state controller performance comparisons.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>THD</th>
<th>MSE $i_q$</th>
<th>MSE $i_d$</th>
<th>VAR $i_q$</th>
<th>VAR $i_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Duty</td>
<td>4.54%</td>
<td>.0779</td>
<td>.0605</td>
<td>.00320</td>
<td>.00340</td>
</tr>
<tr>
<td>Method 1</td>
<td>2.69%</td>
<td>.0381</td>
<td>.0356</td>
<td>3.83e-5</td>
<td>7.95e-4</td>
</tr>
<tr>
<td>Method 2</td>
<td>2.16%</td>
<td>.0304</td>
<td>.0341</td>
<td>8.55e-4</td>
<td>9.83e-4</td>
</tr>
</tbody>
</table>

in Sections 5.2.2 and 5.2.3 (hereafter referred to as method 1 and method 2 respectively) with the full duty controller discussed in Section 5.2.1. For each method the motor is brought up to 500 RPM while under load. After the motor has reached steady state the THD for each phase is measured and averaged together to serve as a metric for controller performance. In order to quantify the non-harmonic errors the mean square error and the error variance of the direct and quadrature currents are also computed after the data has been collected. A visual comparison is provided by the recorded phase currents in the $ABC$ and $dq$ reference frames. In addition to the phase currents the computed raw and filtered duty cycles are also given as it was found that these data give insight to the switching behavior that is not readily apparent from the phase currents alone. To minimize the phase shift that arises from filtering the raw duty cycles, a non-causal filter of length $\ell = 60$ is employed. Notating the raw duty cycle for phase $A$ as $d^a$, the filtered duty cycle as $\hat{d}^a$ and a window length $\ell = 60$, the filtered data point for the sample at $k + \frac{\ell}{2}$ is given as

$$\hat{d}^a(k + \frac{\ell}{2}) = \frac{1}{\ell} \sum_{i=k}^{k+\ell} d^a(i). \quad (5.13)$$

The baseline full duty cycle method shown in Figure 5.7 demonstrates the overall effectiveness of a CLF based input selection strategy with relatively low phase current distortion. However there is a chattering effect that is more noticeable when viewed in the $dq$ frame which arises due to the controller selecting only nonzero inputs. The introduction of a duty cycle achieved by applying the maximally stabilizing input for the length of time $t_a$ given by (5.7) reduces chattering as shown in Figure 5.8. This control and modulation strategy (method 1) achieves a very low THD and the lowest error current ripple among the three cases as seen by the $dq$ frame variance measurements given in Table 5.2. However, this method is based on the prediction of the torque slope over the control period and is thus more susceptible to errors arising from parametric mismatches between the model and the plant which can be seen by observing the small steady state error in quadrature current tracking. By computing a duty cycle
based on a combination of each stabilizing input as shown in Figure 5.9, modulation method 2 yields slightly lower THD and MSE than the other two approaches and does not exhibit a steady state quadrature tracking error, however the switching sequence generated is somewhat more aggressive as can be seen by the increased current error variance.

An alternative comparison of controller switching behavior is given by examining the raw and filtered duty cycles as shown in Figures 5.10 - 5.12. It is interesting to observe that while the baseline full duty method in Figure 5.10 generates only 0% or 100% duty switching signals corresponding to the maximally stabilizing realizable inverter input, the filtered duty cycles appear somewhat sinusoidal. This behavior is even more evident when a duty cycle is introduced using method 1 as shown in Figure 5.11. Note that this modulation strategy produces slightly lower average duty cycles with switching only occurring on at most two of the three legs at any time in a similar fashion to trapezoidal control. This is due to a single duty cycle value being computed for a given input, meaning any nonzero elements of the input will have identical durations. Comparing the duty cycles produced by method 2 in Figure 5.12 with the other two approaches shows a highly sinusoidal variation in duty cycles which is evident without filtering. Unlike sine-triangle modulation methods which explicitly generate sinusoidal variations in duty cycles, this is an emergent behavior of this control strategy as it is designed to seek the minimum energy solution of an error system with sinusoidal dynamics.

The next set of experiments have been designed to examine the response to load and current step commands for the three modulation methods under consideration. The load step responses, shown in Figures 5.13, 5.14 and 5.15 are generated by issuing a constant speed command with the load motor’s phases open circuited, followed by closing a switch that shorts the phases together producing an opposing current through the load motor’s windings. Consistent with earlier experiments, all three methods track the changing reference current quite well although the full duty method produces the most chattering followed by method 2 and coming to a minimum with method 1 which produces the smoothest current waveforms.

To remove the influence of the speed error on reference current generation, the speed to current backstepping method described in [63] has been bypassed for the remaining experiments and replaced with an explicit \(dq\) reference current. Figure 5.16 shows the quadrature current step response for each method. Each approach yields sub-millisecond rise time in response to a doubling of the reference current while modulation method 2 maintains it’s trend as an improvement to the full duty approach in regards to current ripple while producing a smaller steady state error than method 1.
To further explore the sensitivity to parametric uncertainty the estimated inductance used in the model, denoted \( \hat{L} \), has been varied over a wide range and the corresponding current regulation performance of each controller has been recorded. While the baseline, full duty method is relatively insensitive to variations in inductance, particularly for a controller using a high parameter count coupled nonlinear plant model, it can be seen in Figure 5.17 that for large overestimates of the inductance current chattering becomes much more pronounced. Comparing this to the results shown in Figures 5.18 and 5.19, the introduction of a duty cycle significantly reduces this effect. However, an noticeable difference between modulation methods 1 and 2 is observed for large underestimates of the inductance with method 1 resulting in a relatively large steady state error for \( \hat{L} = 5\text{mH} \). This is due to the additional role of \( \hat{L} \) used in method 1 to compute the quadrature current slope. As the estimated inductance becomes excessively small the predicted quadrature current slope increases, resulting in duty cycle values that are too small to reach the reference current over the control period. While method 1 does generate the smoothest currents when the parametric mismatch is minimal, method 2 has the advantage of performing nearly identically across a wide range of parameter estimates.

5.4 Conclusions

This chapter presents two novel methods for incorporating a duty cycle into an inverter input selection algorithm based on control Lyapunov functions. Both modulation strategies produce smoother phase currents than those shown in previous results while maintaining fast response to varying torque current demands. The first modulation method presented is based on similar approaches reported from within the DTC community and it has been shown that this technique can be easily modified to fit within a CLF based motor controller. The second modulation method is an entirely new approach which uses a time-weighted combination of all inputs within a stabilizing set making it unique to CLF based control designs. This method produces low harmonic distortion and has a low sensitivity to parametric mismatch although there is an increased amount of non-harmonic chatter compared to the first method. While the length of time a stabilizing input is impressed on the motor has been chosen to be proportional to its relative stability, this is not a necessarily optimal choice. The design of alternative methods for weighting viable inputs to meet various optimality metrics including the reduction of voltage stress and switching losses remains an open opportunity for future work.
This chapter, in full, has been submitted for publication of the material as it may appear in IFAC Control Engineering Practice, 2013. The dissertation author was the primary investigator and author of this paper.
Figure 5.2: A block diagram of the CLF method with active-null modulation (method 1).
Figure 5.3: Experimentally measured evolution of $\frac{dV}{dt}$ evaluated for each input.
Figure 5.4: A Block diagram of the CLF method with modulation through a convex combination of stabilizing inputs (method 2).
Figure 5.5: Testbed used for verification of the proposed modulation strategies.

Figure 5.6: Kollmorgen AKM64P servo motor and a custom axial flux motor acting as a load.
Figure 5.7: Full duty cycle CLF controller: $ABC$ and $dq$ currents at steady state.

Figure 5.8: CLF modulation method 1: $ABC$ and $dq$ currents at steady state.
Figure 5.9: CLF modulation method 2: $ABC$ and $dq$ currents at steady state.

Figure 5.10: Full duty cycle CLF controller: Duty cycles produced at steady state.
Figure 5.11: CLF modulation method 1: Duty cycles produced at steady state.

Figure 5.12: CLF modulation method 2: Duty cycles produced at steady state.
Figure 5.13: Full duty cycle CLF controller: $ABC$ and $dq$ currents subject to a load step

Figure 5.14: CLF modulation method 1: $ABC$ and $dq$ currents subject to a load step
Figure 5.15: CLF modulation method 2: $ABC$ and $dq$ currents subject to a load step

Figure 5.16: Comparison of controller responses to a quadrature current step command
Figure 5.17: Full duty cycle CLF controller: Current regulation performance using estimated inductance values of $\hat{L} = 200 \, mH$, $20 \, mH$ and $5 \, mH$

Figure 5.18: CLF modulation method 1: Current regulation performance using estimated inductance values of $\hat{L} = 200 \, mH$, $20 \, mH$ and $5 \, mH$
Figure 5.19: CLF modulation method 2: Current regulation performance using an estimated inductance values of $\hat{L} = 200 mH$, $20 mH$ and $5 mH$
Appendix A

Custom Control Hardware and Sensor Conditioning Design

During the course of this work several pieces of hardware have been developed for experimental controller validation. The first test bed design was used for implementing hall effect based trapezoidal modulation as well as a field oriented controller which allowed me to become familiar with standard control techniques and to go through the process of debugging sensor signal polarities, filter values and real time operation constraints. Additionally, it was found to be beneficial to the overall understanding of the system to design all of the major components from a circuit board level. The bench top test bed consisted of a 2kW power inverter, an analog conditioning unit, a hall effect and QEP encoder interface and a power distribution board. While these could have all easily fit on to a single PCB it was decided to separate these units to ease debugging and to allow reuse of selected boards as the test stand became more refined.

In addition to the initial test stand development an optical interface circuit was also designed and fabricated. The higher power inverters from Applied Power Systems used for generating the experimental data in this thesis are actuated through AVAGO HFBR optical links. In order to interface these units with the dSpace system an electrical to optical converter board was required. The resulting design with variable laser diode gain has found use in multiple laboratory experiments outside of motor control including bidirectional buck boost converters and 5kV, 7kA high speed DC breaker controllers.

All schematic design and PCB layout has been conducted in Altium Designer.
A.1 Optical Interface Development

Each of the six channels of the optical interface is powered by a V7805 5 Volt regulator as well as a LM317 adjustable regulator for varying the power delivered to a laser diode. 3.3 to 5 Volt control signals are isolated from the laser driver circuitry through a FOD8001 optoisolator. The laser diode used for transmitting optical pulses to the inverter are controlled by an IXDD509SIA laser driver capable of sourcing up to 9 amps. The schematic of a single channel of the optical interface board is shown in Figure A.1 and the fully assembled PCB is shown in Figure A.2.
A.2 Power Inverter Design

At the heart of the testbed used for initial experiments is a small, bench-top power inverter capable of sourcing 10 Amps from a 200 Volt DC link. All switching signals are isolated from the gate drive circuitry through ADUM1200 digital isolators with transient suppression diodes at the inputs to protect the connected digital signal processor (DSP). The power electronic switching elements selected for this design are the International Rectifier IRG4PC50FD Insulated Gate Bipolar Transistor (IGBT) driven by the IR2183 gate driver chip. Control of the high side (upper switch) IGBTs in a power inverter is more difficult than controlling the low side switches as their emitters are not directly connected to ground. As the gate voltage of the device needs to be above the emitter potential by about 15 Volts depending on the specific device being used, a floating high side emitter requires large gate voltages compared to the circuit ground. Options for remedying this include using p type devices for the high side switches, using separate isolated gate drive power supplies for each high side switch, coupling the high side switches through transformers or using bootstrap capacitor circuits which was the solution chosen for this design. The bootstrap capacitor works as a charge pump, charging a capacitor to the required gate voltage as referenced to ground while the low side switch is active. When the low side device releases the negative side of the capacitor is pushed up to the high side switches emitter voltage and applies the required turn on potential for the switch. A diode is placed in between the capacitor and the charging voltage to prevent current from flowing back through the supply when the low side switch is off. A schematic of the power inverter is shown in Figure A.3.

A.3 Analog Signal Conditioner

The phase currents of the motor are measured by a LEM current transducer which produces an output current proportional to the current flowing through the sensor. To facilitate sampling of this signal by a microcontroller an analog signal conditioner circuit was designed. The output current from the sensor is first converted to a +/- 3V voltage by dropping the signal to ground through a 30 Ohm resistor. The voltage difference across this resistor is fed to the INA148UA differential amplifier with a 1.5V bias for buffering and level shifting the signal to a range from 0 to 3V. The signal is then passed to a unity gain Sallen Key second order low pass anti-aliasing filter before being passed on the the analog to digital converter on the DSP. The DSP inputs are further protected from over and under voltage conditions through a BAS40-04 fast acting dual Shottky Barrier diodes. The schematic of the analog conditioning circuit is given
in Figure A.4.

A.4 Hall Sensor and QEP Interface

The Hall effect sensors and quadrature encoder attached to the motor are measured by the DSP through a Hall Sensor and QEP interface circuit shown in Figure A.5. The motor QEP outputs are given by differential signals to increase noise immunity. To allow the use of the DSPs dedicated single-ended QEP peripheral, the encoder signals are sent to a AM26LS32ACJ differential line transducer. Both Hall signals and the single-ended encoder signals are then routed to an ADUM1300 digital isolator to separate the motor system from the DSPs inputs.

A.5 Power Distribution Circuit

To provide power to each of the control and sensor conditioning circuits a power distribution board was designed and is shown in Figure A.6. This circuit is powered by a single 24V input and outputs the voltages required by each board through isolated DC/DC converters from XP Power. The output voltages are passed through CLC filters to ensure smooth, low ripple power delivery.

A.6 Testbed Design

After fabrication and individual testing the sensor conditioning boards and the power inverter were stacked together as shown in Figures A.7, A.8. The IGBTs in the inverter have been mounted on the bottom of the PCB to allow contact with a heat sink and fan assembly.
Figure A.3: Schematic of 2kW Prototype Inverter
Figure A.4: Schematic of Signal Conditioning Circuit
Figure A.5: Schematic of the Encoder Interface Circuit
Figure A.6: Schematic of the Power Distribution Circuit
Figure A.7: Stack of Assembled Motor Control Boards

Figure A.8: Stack of Assembled Motor Control Boards
Bibliography


